9.056/12.702 ≈ 0.72, and so on. The points on the horizontal axis correspond to the letters in order of decreasing frequency.

Figure 2.6 also shows the frequency distribution that results when the text is encrypted using the Playfair cipher. To normalize the plot, the number of occurrences of each letter in the ciphertext was again divided by the number of occurrences of e in the plaintext. The resulting plot therefore shows the extent to which the frequency distribution of letters, which makes it trivial to solve substitution ciphers, is masked by encryption. If the frequency distribution information were totally concealed in the encryption process, the ciphertext plot of frequencies would be flat, and cryptanalysis using ciphertext only would be effectively impossible. As the figure shows, the Playfair cipher has a flatter distribution than does plaintext, but nevertheless, it reveals plenty of structure for a cryptanalyst to work with. The plot also shows the Vigenère cipher, discussed subsequently. The Hill and Vigenère curves on the plot are based on results reported in [SIMM93].

Hill Cipher[5]

Another interesting multiletter cipher is the Hill cipher, developed by the mathematician Lester Hill in 1929.

Concepts from Linear Algebra Before describing the Hill cipher, let us briefly review some terminology from linear algebra. In this discussion, we are concerned with matrix arithmetic modulo 26. For the reader who needs a refresher on matrix multiplication and inversion, see Appendix E.

We define the inverse $\mathbf{M}^{-1}$ of a square matrix $\mathbf{M}$ by the equation $\mathbf{M}(\mathbf{M}^{-1}) = \mathbf{M}^{-1}\mathbf{M} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix. $\mathbf{I}$ is a square matrix that is all zeros except for ones along the main diagonal from upper left to lower right. The inverse of a matrix does not always exist, but when it does, it satisfies the preceding equation. For example,

$$\mathbf{A} = \begin{pmatrix} 5 & 8 \\ 17 & 3 \end{pmatrix} \qquad \mathbf{A}^{-1} \bmod 26 = \begin{pmatrix} 9 & 2 \\ 1 & 15 \end{pmatrix}$$

$$\mathbf{A}\mathbf{A}^{-1} = \begin{pmatrix} (5 \times 9) + (8 \times 1) & (5 \times 2) + (8 \times 15) \\ (17 \times 9) + (3 \times 1) & (17 \times 2) + (3 \times 15) \end{pmatrix}$$

$$= \begin{pmatrix} 53 & 130 \\ 156 & 79 \end{pmatrix} \bmod 26 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

To explain how the inverse of a matrix is computed, we begin with the concept of determinant. For any square matrix ($m \times m$), the **determinant** equals the sum of all the products that can be formed by taking exactly one element from each row

[5] This cipher is somewhat more difficult to understand than the others in this chapter, but it illustrates an important point about cryptanalysis that will be useful later on. This subsection can be skipped on a first reading.

and exactly one element from each column, with certain of the product terms preceded by a minus sign. For a 2×2 matrix,

$$\begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix}$$

the determinant is $k_{11}k_{22} - k_{12}k_{21}$. For a 3×3 matrix, the value of the determinant is $k_{11}k_{22}k_{33} + k_{21}k_{32}k_{13} + k_{31}k_{12}k_{23} - k_{31}k_{22}k_{13} - k_{21}k_{12}k_{33} - k_{11}k_{32}k_{23}$. If a square matrix $\mathbf{A}$ has a nonzero determinant, then the inverse of the matrix is computed as $[\mathbf{A}^{-1}]_{ij} = (\det \mathbf{A})^{-1}(-1)^{i+j}(\mathrm{D}_{ji})$, where (D_{ji}) is the subdeterminant formed by deleting the jth row and the ith column of $\mathbf{A}$, $\det(\mathbf{A})$ is the determinant of $\mathbf{A}$, and $(\det \mathbf{A})^{-1}$ is the multiplicative inverse of $(\det \mathbf{A}) \bmod 26$.

Continuing our example,

$$\det\begin{pmatrix} 5 & 8 \\ 17 & 3 \end{pmatrix} = (5 \times 3) - (8 \times 17) = -121 \bmod 26 = 9$$

We can show that $9^{-1} \bmod 26 = 3$, because $9 \times 3 = 27 \bmod 26 = 1$ (see Chapter 4 or Appendix E). Therefore, we compute the inverse of $\mathbf{A}$ as

$$\mathbf{A} = \begin{pmatrix} 5 & 8 \\ 17 & 3 \end{pmatrix}$$

$$\mathbf{A}^{-1} \bmod 26 = 3\begin{pmatrix} 3 & -8 \\ -17 & 5 \end{pmatrix} = 3\begin{pmatrix} 3 & 18 \\ 9 & 5 \end{pmatrix} = \begin{pmatrix} 9 & 54 \\ 27 & 15 \end{pmatrix} = \begin{pmatrix} 9 & 2 \\ 1 & 15 \end{pmatrix}$$

The Hill Algorithm This encryption algorithm takes m successive plaintext letters and substitutes for them m ciphertext letters. The substitution is determined by m linear equations in which each character is assigned a numerical value ($a = 0, b = 1, \ldots, z = 25$). For $m = 3$, the system can be described as

$$\begin{aligned} c_1 &= (k_{11}p_1 + k_{21}p_2 + k_{31}p_3) \bmod 26 \\ c_2 &= (k_{12}p_1 + k_{22}p_2 + k_{32}p_3) \bmod 26 \\ c_3 &= (k_{13}p_1 + k_{23}p_2 + k_{33}p_3) \bmod 26 \end{aligned}$$

This can be expressed in terms of row vectors and matrices:[6]

$$(c_1\ c_2\ c_3) = (p_1\ \ p_2\ \ p_3)\begin{pmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{pmatrix} \bmod 26$$

or

$$\mathbf{C} = \mathbf{PK} \bmod 26$$

[6] Some cryptography books express the plaintext and ciphertext as column vectors, so that the column vector is placed after the matrix rather than the row vector placed before the matrix. Sage uses row vectors, so we adopt that convention.

where **C** and **P** are row vectors of length 3 representing the plaintext and ciphertext, and **K** is a 3 × 3 matrix representing the encryption key. Operations are performed mod 26.

For example, consider the plaintext "paymoremoney" and use the encryption key

$$\mathbf{K} = \begin{pmatrix} 17 & 17 & 5 \\ 21 & 18 & 21 \\ 2 & 2 & 19 \end{pmatrix}$$

The first three letters of the plaintext are represented by the vector (15 0 24). Then(15 0 24)**K** = (303 303 531) mod 26 = (17 17 11) = RRL. Continuing in this fashion, the ciphertext for the entire plaintext is RRLMWBKASPDH.

Decryption requires using the inverse of the matrix **K**. We can compute det **K** = 23, and therefore, $(\det \mathbf{K})^{-1} \bmod 26 = 17$. We can then compute the inverse as[7]

$$\mathbf{K}^{-1} = \begin{pmatrix} 4 & 9 & 15 \\ 15 & 17 & 6 \\ 24 & 0 & 17 \end{pmatrix}$$

This is demonstrated as

$$\begin{pmatrix} 17 & 17 & 5 \\ 21 & 18 & 21 \\ 2 & 2 & 19 \end{pmatrix}\begin{pmatrix} 4 & 9 & 15 \\ 15 & 17 & 6 \\ 24 & 0 & 17 \end{pmatrix} = \begin{pmatrix} 443 & 442 & 442 \\ 858 & 495 & 780 \\ 494 & 52 & 365 \end{pmatrix} \bmod 26 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

It is easily seen that if the matrix $\mathbf{K}^{-1}$ is applied to the ciphertext, then the plaintext is recovered.

In general terms, the Hill system can be expressed as

$$\mathbf{C} = \mathrm{E}(\mathbf{K}, \mathbf{P}) = \mathbf{PK} \bmod 26$$
$$\mathbf{P} = \mathrm{D}(\mathbf{K}, \mathbf{C}) = \mathbf{CK}^{-1} \bmod 26 = \mathbf{PKK}^{-1} = \mathbf{P}$$

As with Playfair, the strength of the Hill cipher is that it completely hides single-letter frequencies. Indeed, with Hill, the use of a larger matrix hides more frequency information. Thus, a 3 × 3 Hill cipher hides not only single-letter but also two-letter frequency information.

Although the Hill cipher is strong against a ciphertext-only attack, it is easily broken with a known plaintext attack. For an $m \times m$ Hill cipher, suppose we have m plaintext–ciphertext pairs, each of length m. We label the pairs $\mathbf{P}_{\mathrm{j}} = (p_{1j}p_{1j} \ldots p_{mj})$ and $\mathbf{C}_{\mathrm{j}} = (c_{1j}\, c_{1j} \ldots c_{mj})$ such that $\mathbf{C}_j = \mathbf{P}_j\mathbf{K}$ for $1 \leq j \leq m$ and for some unknown key matrix **K**. Now define two $m \times m$ matrices $\mathbf{X} = (p_{ij})$ and $\mathbf{Y} = (c_{ij})$. Then we can form the matrix equation **Y** = **XK**. If **X** has an inverse, then we can determine $\mathbf{K} = \mathbf{X}^{-1}\mathbf{Y}$. If **X** is not invertible, then a new version of **X** can be formed with additional plaintext–ciphertext pairs until an invertible **X** is obtained.

[7] The calculations for this example are provided in detail in Appendix E.

Consider this example. Suppose that the plaintext “hillcipher” is encrypted using a 2 × 2 Hill cipher to yield the ciphertext HCRZSSXNSP. Thus, we know that $(7 \quad 8)\mathbf{K} \bmod 26 = (7 \quad 2)$; $(11 \quad 11)\mathbf{K} \bmod 26 = (17 \quad 25)$; and so on. Using the first two plaintext–ciphertext pairs, we have

$$\begin{pmatrix} 7 & 2 \\ 17 & 25 \end{pmatrix} = \begin{pmatrix} 7 & 8 \\ 11 & 11 \end{pmatrix} \mathbf{K} \bmod 26$$

The inverse of **X** can be computed:

$$\begin{pmatrix} 7 & 8 \\ 11 & 11 \end{pmatrix}^{-1} = \begin{pmatrix} 25 & 22 \\ 1 & 23 \end{pmatrix}$$

so

$$\mathbf{K} = \begin{pmatrix} 25 & 22 \\ 1 & 23 \end{pmatrix} \begin{pmatrix} 7 & 2 \\ 17 & 25 \end{pmatrix} = \begin{pmatrix} 549 & 600 \\ 398 & 577 \end{pmatrix} \bmod 26 = \begin{pmatrix} 3 & 2 \\ 8 & 5 \end{pmatrix}$$

This result is verified by testing the remaining plaintext–ciphertext pairs.

Polyalphabetic Ciphers

Another way to improve on the simple monoalphabetic technique is to use different monoalphabetic substitutions as one proceeds through the plaintext message. The general name for this approach is **polyalphabetic substitution cipher**. All these techniques have the following features in common:

1. A set of related monoalphabetic substitution rules is used.
2. A key determines which particular rule is chosen for a given transformation.

VIGENÈRE CIPHER The best known, and one of the simplest, polyalphabetic ciphers is the Vigenère cipher. In this scheme, the set of related monoalphabetic substitution rules consists of the 26 Caesar ciphers with shifts of 0 through 25. Each cipher is denoted by a key letter, which is the ciphertext letter that substitutes for the plaintext letter a. Thus, a Caesar cipher with a shift of 3 is denoted by the key value 3.[8]

We can express the Vigenère cipher in the following manner. Assume a sequence of plaintext letters $P = p_0, p_1, p_2, \ldots, p_{n-1}$ and a key consisting of the sequence of letters $K = k_0, k_1, k_2, \ldots, k_{m-1}$, where typically $m < n$. The sequence of ciphertext letters $C = C_0, C_1, C_2, \ldots, C_{n-1}$ is calculated as follows:

$$\begin{aligned} C &= C_0, C_1, C_2, \ldots, C_{n-1} = \mathrm{E}(K, P) = \mathrm{E}[(k_0, k_1, k_2, \ldots, k_{m-1}), (p_0, p_1, p_2, \ldots, p_{n-1})] \\ &= (p_0 + k_0) \bmod 26, (p_1 + k_1) \bmod 26, \ldots, (p_{m-1} + k_{m-1}) \bmod 26, \\ &\quad (p_m + k_0) \bmod 26, (p_{m+1} + k_1) \bmod 26, \ldots, (p_{2m-1} + k_{m-1}) \bmod 26, \ldots \end{aligned}$$

Thus, the first letter of the key is added to the first letter of the plaintext, mod 26, the second letters are added, and so on through the first m letters of the plaintext. For the next m letters of the plaintext, the key letters are repeated. This process

[8] To aid in understanding this scheme and also to aid in it use, a matrix known as the Vigenère tableau is often used. This tableau is discussed in a document in the Premium Content Web site for this book.

continues until all of the plaintext sequence is encrypted. A general equation of the encryption process is

$$C_i = (p_i + k_{i \bmod m}) \bmod 26 \tag{2.3}$$

Compare this with Equation (2.1) for the Caesar cipher. In essence, each plaintext character is encrypted with a different Caesar cipher, depending on the corresponding key character. Similarly, decryption is a generalization of Equation (2.2):

$$p_i = (C_i - k_{i \bmod m}) \bmod 26 \tag{2.4}$$

To encrypt a message, a key is needed that is as long as the message. Usually, the key is a repeating keyword. For example, if the keyword is *deceptive*, the message "we are discovered save yourself" is encrypted as

key: *deceptivedeceptivedeceptive*
plaintext: wearediscoveredsaveyourself
ciphertext: ZICVTWQNGRZGVTWAVZHCQYGLMGJ

Expressed numerically, we have the following result.

key	3	4	2	4	15	19	8	21	4	3	4	2	4	15
plaintext	22	4	0	17	4	3	8	18	2	14	21	4	17	4
ciphertext	25	8	2	21	19	22	16	13	6	17	25	6	21	19

key	19	8	21	4	3	4	2	4	15	19	8	21	4
plaintext	3	18	0	21	4	24	14	20	17	18	4	11	5
ciphertext	22	0	21	25	7	2	16	24	6	11	12	6	9

The strength of this cipher is that there are multiple ciphertext letters for each plaintext letter, one for each unique letter of the keyword. Thus, the letter frequency information is obscured. However, not all knowledge of the plaintext structure is lost. For example, Figure 2.6 shows the frequency distribution for a Vigenère cipher with a keyword of length 9. An improvement is achieved over the Playfair cipher, but considerable frequency information remains.

It is instructive to sketch a method of breaking this cipher, because the method reveals some of the mathematical principles that apply in cryptanalysis.

First, suppose that the opponent believes that the ciphertext was encrypted using either monoalphabetic substitution or a Vigenère cipher. A simple test can be made to make a determination. If a monoalphabetic substitution is used, then the statistical properties of the ciphertext should be the same as that of the language of the plaintext. Thus, referring to Figure 2.5, there should be one cipher letter with a relative frequency of occurrence of about 12.7%, one with about 9.06%, and so on. If only a single message is available for analysis, we would not expect an exact match of this small sample with the statistical profile of the plaintext language. Nevertheless, if the correspondence is close, we can assume a monoalphabetic substitution.

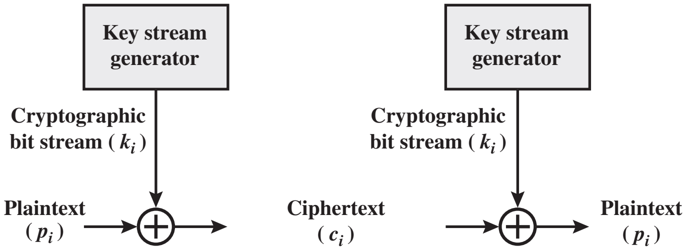


Figure 7. Vernam Cipher

His system works on binary data (bits) rather than letters. The system can be expressed succinctly as follows (Figure 7):

$$c_i = p_i \oplus k_i$$

where

p_i = ith binary digit of plaintext
k_i = ith binary digit of key
c_i = ith binary digit of ciphertext
$\oplus$ = exclusive-or (XOR) operation

Compare this with Equation (2.3) for the Vigenère cipher.

Thus, the ciphertext is generated by performing the bitwise XOR of the plaintext and the key. Because of the properties of the XOR, decryption simply involves the same bitwise operation:

$$p_i = c_i \oplus k_i$$

which compares with Equation (2.4).

The essence of this technique is the means of construction of the key. Vernam proposed the use of a running loop of tape that eventually repeated the key, so that in fact the system worked with a very long but repeating keyword. Although such a scheme, with a long key, presents formidable cryptanalytic difficulties, it can be broken with sufficient ciphertext, the use of known or probable plaintext sequences, or both.

One-Time Pad

An Army Signal Corp officer, Joseph Mauborgne, proposed an improvement to the Vernam cipher that yields the ultimate in security. Mauborgne suggested using a random key that is as long as the message, so that the key need not be repeated. In addition, the key is to be used to encrypt and decrypt a single message, and then is discarded. Each new message requires a new key of the same length as the new message. Such a scheme, known as a **one-time pad**, is unbreakable. It produces random output that bears no statistical relationship to the plaintext. Because the ciphertext contains no information whatsoever about the plaintext, there is simply no way to break the code.

An example should illustrate our point. Suppose that we are using a Vigenère scheme with 27 characters in which the twenty-seventh character is the space character, but with a one-time key that is as long as the message. Consider the ciphertext

```
ANKYODKYUREPFJBYOJDSPLREYIUNOFDOIUERFPLUYTS
```

We now show two different decryptions using two different keys:

```
ciphertext: ANKYODKYUREPFJBYOJDSPLREYIUNOFDOIUERFPLUYTS
key:        pxlmvmsydofuyrvzwc tnlebnecvgdupahfzzlmnyih
plaintext:  mr mustard with the candlestick in the hall

ciphertext: ANKYODKYUREPFJBYOJDSPLREYIUNOFDOIUERFPLUYTS
key:        pftgpmiydgaxgoufhklllmhsqdqogtewbqfgyovuhwt
plaintext:  miss scarlet with the knife in the library
```

Suppose that a cryptanalyst had managed to find these two keys. Two plausible plaintexts are produced. How is the cryptanalyst to decide which is the correct decryption (i.e., which is the correct key)? If the actual key were produced in a truly random fashion, then the cryptanalyst cannot say that one of these two keys is more likely than the other. Thus, there is no way to decide which key is correct and therefore which plaintext is correct.

In fact, given any plaintext of equal length to the ciphertext, there is a key that produces that plaintext. Therefore, if you did an exhaustive search of all possible keys, you would end up with many legible plaintexts, with no way of knowing which was the intended plaintext. Therefore, the code is unbreakable.

The security of the one-time pad is entirely due to the randomness of the key. If the stream of characters that constitute the key is truly random, then the stream of characters that constitute the ciphertext will be truly random. Thus, there are no patterns or regularities that a cryptanalyst can use to attack the ciphertext.

In theory, we need look no further for a cipher. The one-time pad offers complete security but, in practice, has two fundamental difficulties:

1. There is the practical problem of making large quantities of random keys. Any heavily used system might require millions of random characters on a regular basis. Supplying truly random characters in this volume is a significant task.
2. Even more daunting is the problem of key distribution and protection. For every message to be sent, a key of equal length is needed by both sender and receiver. Thus, a mammoth key distribution problem exists.

Because of these difficulties, the one-time pad is of limited utility and is useful primarily for low-bandwidth channels requiring very high security.

The one-time pad is the only cryptosystem that exhibits what is referred to as *perfect secrecy*. This concept is explored in Appendix F.

TRANSPOSITION TECHNIQUES

All the techniques examined so far involve the substitution of a ciphertext symbol for a plaintext symbol. A very different kind of mapping is achieved by performing some sort of permutation on the plaintext letters. This technique is referred to as a transposition cipher.

The simplest such cipher is the **rail fence** technique, in which the plaintext is written down as a sequence of diagonals and then read off as a sequence of rows. For example, to encipher the message "meet me after the toga party" with a rail fence of depth 2, we write the following:

```
m e m a t r h t g p r y
 e t e f e t e o a a t
```

The encrypted message is

MEMATRHTGPRYETEFETEOAAT

This sort of thing would be trivial to cryptanalyze. A more complex scheme is to write the message in a rectangle, row by row, and read the message off, column by column, but permute the order of the columns. The order of the columns then becomes the key to the algorithm. For example,

```
Key:           4 3 1 2 5 6 7
Plaintext:     a t t a c k p
               o s t p o n e
               d u n t i l t
               w o a m x y z
Ciphertext:    TTNAAPTMTSUOAODWCOIXKNLYPETZ
```

Thus, in this example, the key is 4312567. To encrypt, start with the column that is labeled 1, in this case column 3. Write down all the letters in that column. Proceed to column 4, which is labeled 2, then column 2, then column 1, then columns 5, 6, and 7.

A pure transposition cipher is easily recognized because it has the same letter frequencies as the original plaintext. For the type of columnar transposition just shown, cryptanalysis is fairly straightforward and involves laying out the ciphertext in a matrix and playing around with column positions. Digram and trigram frequency tables can be useful.

The transposition cipher can be made significantly more secure by performing more than one stage of transposition. The result is a more complex permutation that is not easily reconstructed. Thus, if the foregoing message is reencrypted using the same algorithm,

```
Key:      4 3 1 2 5 6 7
Input:    t t n a a p t
          m t s u o a o
          d w c o i x k
          n l y p e t z
Output:   NSCYAUOPTTWLTMDNAOIEPAXTTOKZ
```

To visualize the result of this double transposition, designate the letters in the original plaintext message by the numbers designating their position. Thus, with 28 letters in the message, the original sequence of letters is

```
01 02 03 04 05 06 07 08 09 10 11 12 13 14
15 16 17 18 19 20 21 22 23 24 25 26 27 28
```

After the first transposition, we have

```
03 10 17 24 04 11 18 25 02 09 16 23 01 08
15 22 05 12 19 26 06 13 20 27 07 14 21 28
```

which has a somewhat regular structure. But after the second transposition, we have

```
17 09 05 27 24 16 12 07 10 02 22 20 03 25
15 13 04 23 19 14 11 01 26 21 18 08 06 28
```

This is a much less structured permutation and is much more difficult to cryptanalyze.

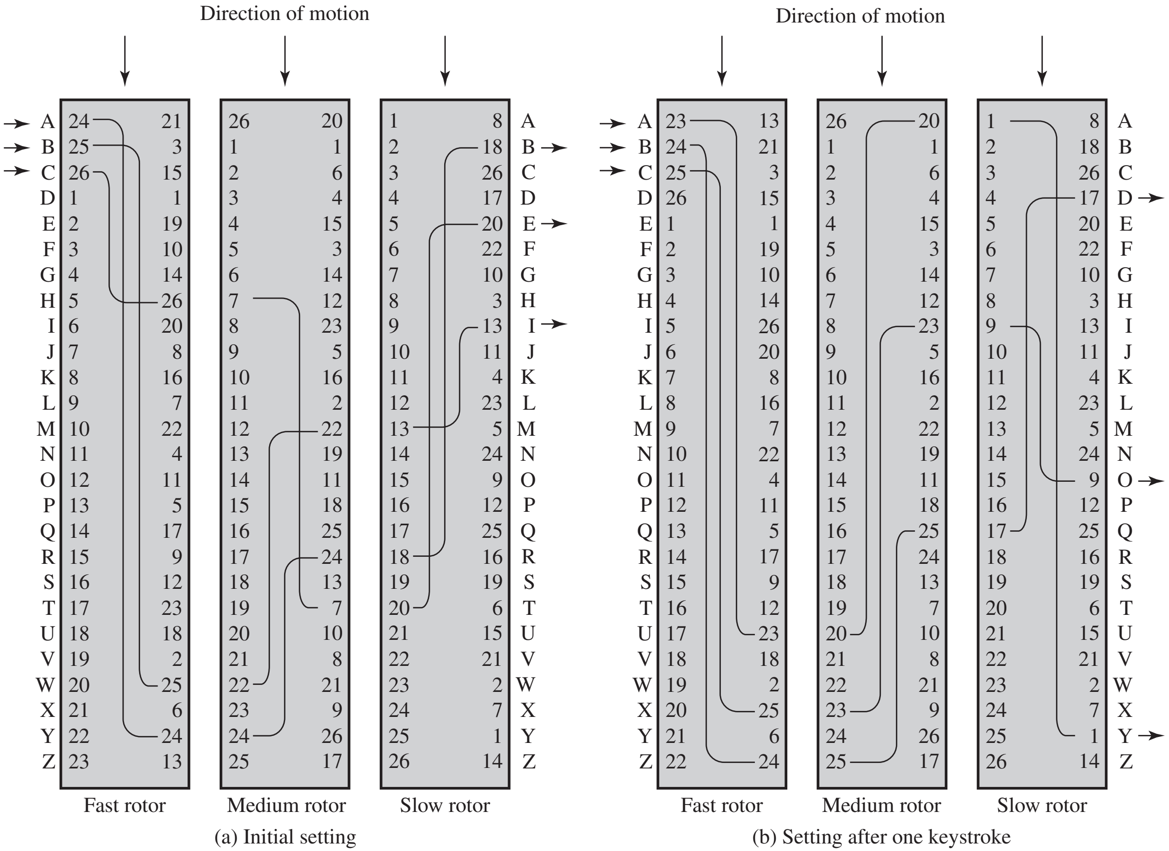
ROTOR MACHINES

The example just given suggests that multiple stages of encryption can produce an algorithm that is significantly more difficult to cryptanalyze. This is as true of substitution ciphers as it is of transposition ciphers. Before the introduction of DES, the most important application of the principle of multiple stages of encryption was a class of systems known as rotor machines.[10]

The basic principle of the rotor machine is illustrated in Figure 8. The machine consists of a set of independently rotating cylinders through which electrical pulses can flow. Each cylinder has 26 input pins and 26 output pins, with internal wiring that connects each input pin to a unique output pin. For simplicity, only three of the internal connections in each cylinder are shown.

If we associate each input and output pin with a letter of the alphabet, then a single cylinder defines a monoalphabetic substitution. For example, in Figure 8, if an operator depresses the key for the letter A, an electric signal is applied to

[10] Machines based on the rotor principle were used by both Germany (Enigma) and Japan (Purple) in World War II. The breaking of both codes by the Allies was a significant factor in the war's outcome.



(a) Initial setting

	Fast rotor		Medium rotor		Slow rotor		
→ A	24	21	26	20	1	8	A
→ B	25	3	1	1	2	18	B →
→ C	26	15	2	6	3	26	C
D	1	1	3	4	4	17	D
E	2	19	4	15	5	20	E →
F	3	10	5	3	6	22	F
G	4	14	6	14	7	10	G
H	5	26	7	12	8	3	H
I	6	20	8	23	9	13	I →
J	7	8	9	5	10	11	J
K	8	16	10	16	11	4	K
L	9	7	11	2	12	23	L
M	10	22	12	22	13	5	M
N	11	4	13	19	14	24	N
O	12	11	14	11	15	9	O
P	13	5	15	18	16	12	P
Q	14	17	16	25	17	25	Q
R	15	9	17	24	18	16	R
S	16	12	18	13	19	19	S
T	17	23	19	7	20	6	T
U	18	18	20	10	21	15	U
V	19	2	21	8	22	21	V
W	20	25	22	21	23	2	W
X	21	6	23	9	24	7	X
Y	22	24	24	26	25	1	Y
Z	23	13	25	17	26	14	Z

(b) Setting after one keystroke

	Fast rotor		Medium rotor		Slow rotor		
→ A	23	13	26	20	1	8	A
→ B	24	21	1	1	2	18	B
→ C	25	3	2	6	3	26	C
D	26	15	3	4	4	17	D →
E	1	1	4	15	5	20	E
F	2	19	5	3	6	22	F
G	3	10	6	14	7	10	G
H	4	14	7	12	8	3	H
I	5	26	8	23	9	13	I
J	6	20	9	5	10	11	J
K	7	8	10	16	11	4	K
L	8	16	11	2	12	23	L
M	9	7	12	22	13	5	M
N	10	22	13	19	14	24	N
O	11	4	14	11	15	9	O →
P	12	11	15	18	16	12	P
Q	13	5	16	25	17	25	Q
R	14	17	17	24	18	16	R
S	15	9	18	13	19	19	S
T	16	12	19	7	20	6	T
U	17	23	20	10	21	15	U
V	18	18	21	8	22	21	V
W	19	2	22	21	23	2	W
X	20	25	23	9	24	7	X
Y	21	6	24	26	25	1	Y →
Z	22	24	25	17	26	14	Z

Figure 8. Three-Rotor Machine with Wiring Represented by Numbered Contacts

the first pin of the first cylinder and flows through the internal connection to the twenty-fifth output pin.

Consider a machine with a single cylinder. After each input key is depressed, the cylinder rotates one position, so that the internal connections are shifted accordingly. Thus, a different monoalphabetic substitution cipher is defined. After 26 letters of plaintext, the cylinder would be back to the initial position. Thus, we have a polyalphabetic substitution algorithm with a period of 26.

A single-cylinder system is trivial and does not present a formidable cryptanalytic task. The power of the rotor machine is in the use of multiple cylinders, in which the output pins of one cylinder are connected to the input pins of the next. Figure 2.8 shows a three-cylinder system. The left half of the figure shows a position in which the input from the operator to the first pin (plaintext letter a) is routed through the three cylinders to appear at the output of the second pin (ciphertext letter B).

With multiple cylinders, the one closest to the operator input rotates one pin position with each keystroke. The right half of Figure 2.8 shows the system's configuration after a single keystroke. For every complete rotation of the inner cylinder, the middle cylinder rotates one pin position. Finally, for every complete rotation of the middle cylinder, the outer cylinder rotates one pin position. This is the same type of operation seen with an odometer. The result is that there are $26 \times 26 \times 26 = 17{,}576$ different substitution alphabets used before the system repeats. The addition of fourth and fifth rotors results in periods of 456,976 and 11,881,376 letters, respectively. Thus, a given setting of a 5-rotor machine is equivalent to a Vigenère cipher with a key length of 11,881,376.

Such a scheme presents a formidable cryptanalytic challenge. If, for example, the cryptanalyst attempts to use a letter frequency analysis approach, the analyst is faced with the equivalent of over 11 million monoalphabetic ciphers. We might need on the order of 50 letters in each monalphabetic cipher for a solution, which means that the analyst would need to be in possession of a ciphertext with a length of over half a billion letters.

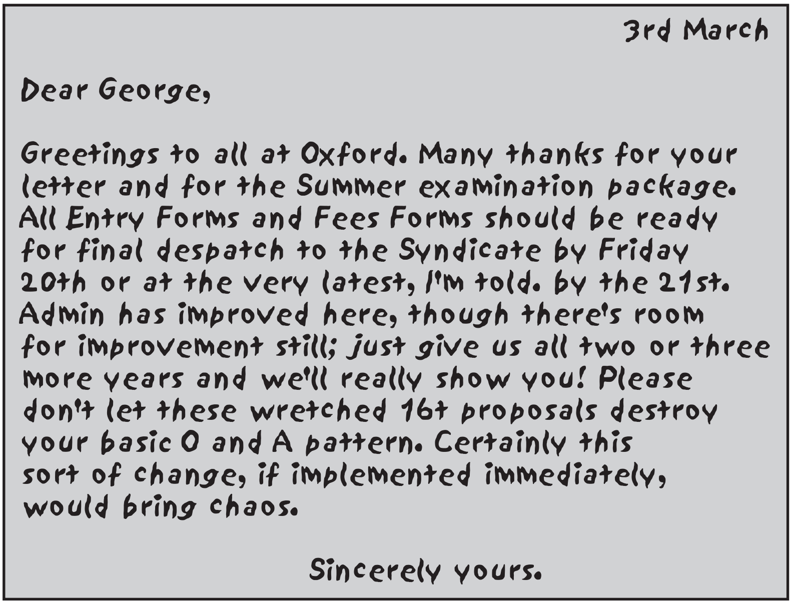
The significance of the rotor machine today is that it points the way to the most widely used cipher ever: the Data Encryption Standard (DES).

STEGANOGRAPHY

We conclude with a discussion of a technique that (strictly speaking), is not encryption, namely, **steganography**.

A plaintext message may be hidden in one of two ways. The methods of **steganography** conceal the existence of the message, whereas the methods of cryptography render the message unintelligible to outsiders by various transformations of the text.[11]

[11] *Steganography* was an obsolete word that was revived by David Kahn and given the meaning it has today [KAHN96].



3rd March

Dear George,

Greetings to all at Oxford. Many thanks for your
letter and for the Summer examination package.
All Entry Forms and Fees Forms should be ready
for final despatch to the Syndicate by Friday
20th or at the very latest, I'm told. by the 21st.
Admin has improved here, though there's room
for improvement still; just give us all two or three
more years and we'll really show you! Please
don't let these wretched 16+ proposals destroy
your basic O and A pattern. Certainly this
sort of change, if implemented immediately,
would bring chaos.

Sincerely yours.

Figure 9. A Puzzle for Inspector Morse
(From The Silent World of Nicholas Quinn, *by Colin Dexter)*

A simple form of steganography, but one that is time-consuming to construct, is one in which an arrangement of words or letters within an apparently innocuous text spells out the real message. For example, the sequence of first letters of each word of the overall message spells out the hidden message. Figure 9 shows an example in which a subset of the words of the overall message is used to convey the hidden message. See if you can decipher this; it's not too hard.

Various other techniques have been used historically; some examples are the following [MYER91]:

- **Character marking:** Selected letters of printed or typewritten text are overwritten in pencil. The marks are ordinarily not visible unless the paper is held at an angle to bright light.
- **Invisible ink:** A number of substances can be used for writing but leave no visible trace until heat or some chemical is applied to the paper.
- **Pin punctures:** Small pin punctures on selected letters are ordinarily not visible unless the paper is held up in front of a light.
- **Typewriter correction ribbon:** Used between lines typed with a black ribbon, the results of typing with the correction tape are visible only under a strong light.

KORN96 Korner, T. *The Pleasures of Counting.* Cambridge, England: Cambridge University Press, 1996.

KUMA97 Kumar, I. *Cryptology.* Laguna Hills, CA: Aegean Park Press, 1997.

NICH96 Nichols, R. *Classical Cryptography Course.* Laguna Hills, CA: Aegean Park Press, 1996.

NICH99 Nichols, R., ed. *ICSA Guide to Cryptography.* New York: McGraw-Hill, 1999.

SING99 Singh, S. *The Code Book: The Science of Secrecy from Ancient Egypt to Quantum Cryptography.* New York: Anchor Books, 1999.

SINK09 Sinkov, A., and Feil, T. *Elementary Cryptanalysis: A Mathematical Approach.* Washington, D.C.: The Mathematical Association of America, 2009.

WAYN09 Wayner, P. *Disappearing Cryptography.* Boston: AP Professional Books, 2009.

KEY TERMS, REVIEW QUESTIONS, AND PROBLEMS

Terms

block cipher
brute-force attack
Caesar cipher
cipher
ciphertext
computationally secure
conventional encryption
cryptanalysis
cryptographic system
cryptography
cryptology
deciphering
decryption
digram
enciphering
encryption
Hill cipher
monoalphabetic cipher
one-time pad
plaintext
Playfair cipher
polyalphabetic cipher
rail fence cipher
single-key encryption
steganography
stream cipher
symmetric encryption
transposition cipher
unconditionally secure
Vigenère cipher

Review Questions

What are the essential ingredients of a symmetric cipher?

What are the two basic functions used in encryption algorithms?

How many keys are required for two people to communicate via a cipher?

What is the difference between a block cipher and a stream cipher?

What are the two general approaches to attacking a cipher?

List and briefly define types of cryptanalytic attacks based on what is known to the attacker.

What is the difference between an unconditionally secure cipher and a computationally secure cipher

Briefly define the Caesar cipher.

Briefly define the monoalphabetic cipher.

Briefly define the Playfair cipher.

What is the difference between a monoalphabetic cipher and a polyalphabetic cipher?2.12
What are two problems with the one-time pad?
What is a transposition cipher?
What is steganography?

Problems

A generalization of the Caesar cipher, known as the affine Caesar cipher, has the following form: For each plaintext letter p, substitute the ciphertext letter C:

$$C = E([a, b], p) = (ap + b) \bmod 26$$

A basic requirement of any encryption algorithm is that it be one-to-one. That is, if $p \neq q$, then $\mathrm{E}(k, p) \neq \mathrm{E}(k, q)$. Otherwise, decryption is impossible, because more than one plaintext character maps into the same ciphertext character. The affine Caesar cipher is not one-to-one for all values of a. For example, for $a = 2$ and $b = 3$, then $\mathrm{E}([a, b], 0) = \mathrm{E}([a, b], 13) = 3$.

a. Are there any limitations on the value of b? Explain why or why not.
b. Determine which values of a are not allowed.
c. Provide a general statement of which values of a are and are not allowed. Justify your statement.

How many one-to-one affine Caesar ciphers are there?

A ciphertext has been generated with an affine cipher. The most frequent letter of the ciphertext is "B," and the second most frequent letter of the ciphertext is "U." Break this code.

The following ciphertext was generated using a simple substitution algorithm.

```
53‡‡†305))6*;4826)4‡.)4‡);806*;48†8¶60))85;;]8*;:‡*8†83
(88)5*†;46(;88*96*?;8)*‡(;485);5*†2:*‡(;4956*2(5*—4)8¶8*
;4069285);)6†8)4‡‡;1(‡9;48081;8:8‡1;48†85;4)485†528806*81
(‡9;48;(88;4(‡?34;48)4‡;161;:188;‡?;
```

Decrypt this message.

Hints:

1. As you know, the most frequently occurring letter in English is e. Therefore, the first or second (or perhaps third?) most common character in the message is likely to stand for e. Also, e is often seen in pairs (e.g., meet, fleet, speed, seen, been, agree, etc.). Try to find a character in the ciphertext that decodes to e.
2. The most common word in English is "the." Use this fact to guess the characters that stand for t and h.
3. Decipher the rest of the message by deducing additional words.

Warning: The resulting message is in English but may not make much sense on a first reading.

One way to solve the key distribution problem is to use a line from a book that both the sender and the receiver possess. Typically, at least in spy novels, the first sentence of a book serves as the key. The particular scheme discussed in this problem is from one of the best suspense novels involving secret codes, *Talking to Strange Men*, by Ruth Rendell. Work this problem without consulting that book!

Consider the following message:

SIDKHKDM AF HCRKIABIE SHIMC KD LFEAILA

This ciphertext was produced using the first sentence of *The Other Side of Silence* (a book about the spy Kim Philby):

> The snow lay thick on the steps and the snowflakes driven by the wind looked black in the headlights of the cars.

A simple substitution cipher was used.

a. What is the encryption algorithm?
b. How secure is it?
c. To make the key distribution problem simple, both parties can agree to use the first or last sentence of a book as the key. To change the key, they simply need to agree on a new book. The use of the first sentence would be preferable to the use of the last. Why?

In one of his cases, Sherlock Holmes was confronted with the following message.

534 C2 13 127 36 31 4 17 21 41
DOUGLAS 109 293 5 37 BIRLSTONE
26 BIRLSTONE 9 127 171

Although Watson was puzzled, Holmes was able immediately to deduce the type of cipher. Can you?

This problem uses a real-world example, from an old U.S. Special Forces manual (public domain). The document, filename *SpecialForces.pdf,* is available at the Premium Content site for this book.

a. Using the two keys (memory words) *cryptographic* and *network security*, encrypt the following message:

   Be at the third pillar from the left outside the lyceum theatre tonight at seven.
   If you are distrustful bring two friends.

   Make reasonable assumptions about how to treat redundant letters and excess letters in the memory words and how to treat spaces and punctuation. Indicate what your assumptions are. *Note:* The message is from the Sherlock Holmes novel, *The Sign of Four*.
b. Decrypt the ciphertext. Show your work.
c. Comment on when it would be appropriate to use this technique and what its advantages are.

A disadvantage of the general monoalphabetic cipher is that both sender and receiver must commit the permuted cipher sequence to memory. A common technique for avoiding this is to use a keyword from which the cipher sequence can be generated. For example, using the keyword *CIPHER*, write out the keyword followed by unused letters in normal order and match this against the plaintext letters:

```
plain:   a b c d e f g h i j k l m n o p q r s t u v w x y z
cipher:  C I P H E R A B D F G J K L M N O Q S T U V W X Y Z
```

If it is felt that this process does not produce sufficient mixing, write the remaining letters on successive lines and then generate the sequence by reading down the columns:

```
C I P H E R
A B D F G J
K L M N O Q
S T U V W X
Y Z
```