APPLIEDCALCULUS

1. The differential equation for electric current 'I' in an electric circuit containing an inductance 'L' and a resistor 'R' in series and acted on by an electromotive force '$E\sin wt$' is

   $$L\frac{dI}{dt} + RI = E\cos wt$$

   Verify that the current in circuit at any time 't' is $\frac{E}{(R^2+w^2L^2)}\left[R\cos wt + wL\sin wt\right]$. Whilst L, R, E and w are constants.

2. The length l meters of a certain metal rod at temperature $\theta^0 C$ is given by $l = \sqrt{3t + \sqrt{2+\sqrt{1-t}}}$. Determine the rate of change of length, in mm/^{0}C, when the temperature is (a) 100^{0}C and (b) 400^{0}C.
3. The voltage across the plates of a capacitor at any time t seconds is given by $v = Ve^{-t/CR}$, where V, C and R are constants. Given V=300 volts, $C = 0.12\times 10^{-6}$ F and $R = 4\times 10^6\,\Omega$, find (a) the initial rate of change of voltage and (b) the rate of change of voltage at 0.5s.
4. The average temperature in a laboratory can be approximated by the function $T = 42.5 - 17.4t + 3.57t^2 - 0.987t^3 + 0.0448t^4$, where T represents the temperature, in degrees Fahrenheit, and t is time (in months). Find the extreme time where the temperature of laboratory is maximum.
5. The temperature in a space is given by the function: $T(x,y,z) = 200xyz^2$. Find the hottest points on unit sphere.
6. A sinusoidal voltage $E\sin\omega t$, where t is time, is passed through a half-wave rectifier that clips the negative portion of the wave. The Fourier series of the resulting periodic function is $u(t) = a_0 + a_n\cos\omega t + b_n\sin\omega t$.

   Where, $a_0 = \frac{\omega}{2\pi}\int_0^{\pi/\omega} E\sin\omega t\,dt$, $\quad a_n = \frac{\omega}{\pi}\int_0^{\pi/\omega} E\sin\omega t\cos n\omega t\,dt$, $\quad b_1 = \frac{E}{2}$

   For simplicity take n=1, find a_0 and a_1 where the period $P = 4 = \frac{2\pi}{\omega}$
7. Find the RMS value for voltage defined by the following function over an interval [0,2T]:

   $$V(c) = \begin{cases} \frac{V_{pk}}{T}\sec c & 0 < c < T \\ -V_{pk} + \frac{V_{pk}}{T}(c-T) & T < c < 2T \end{cases}$$

   Where $V_{pk} = 200$V is the peak voltage and T = π