Qasim Ali ↳ Rought work is attached at the end.

P20-0070

Section BS-CS-3B

Assignment #2

Due: 15th of October, 2021

NOTE: Read this please

1 - Take a print of this assignment (If there is no nearby printer then write it again in the same format along with the spaces provided for answer).
2 - Some questions may need you to do some rough work. Do it on a blank sheet and write on the top of the sheet 'Rough Work'
3 - Solve it using pen
4 – In case you are not on campus and not come then take snaps and make a pdf of the solved assignment and submit via slate.

Q1: Use the principle of resolution to show that the hypothesis "Chohan works hard", "If Chohan works hard then he is a dull boy", "if Chohan is a dull boy, he will not get a job" imply the conclusion "Chohan will not get the job". **(Marks 3)**

Solution:

★ Solve in rough work done.

P1: p	C1: p			
P2: $p \to q$	C2: $\neg p \vee q$			
P3: $q \to \neg r$	C3: $\neg q \vee \neg r$			
C: $\neg r$	C4: r			
	C5: q	From 1 and 2		
	C6: $\neg r$	From 3 and 5		
	C7:	From 4 and 6		

Q2: Write the negation of the following statements in English using the logical equivalence of $\sim\forall x\, P(x) \equiv \exists x \sim P(x)$ and $\sim\exists x\, P(x) \equiv \forall x \sim P(x)$. No credit will be given if you didn't use these logical equivalences. **(6 marks)**

a. $\forall x \forall y\, (P(x,y) \to \sim Q(x,y))$

Solution: $\exists x \exists y\, P(x,y) \wedge Q(x,y)$

There exist x for some y for which P of (x,y) and Q of (x,y)

b. $\exists x \forall y\, (P(x,y) \vee \sim Q(x,y))$

Solution: $\forall x \exists y \neg P(x,y) \wedge Q(x,y)$

For all x there exist y where P of (x,y) is not and Q of (x,y).

Note: When you are done with simplification of the quantifiers then also use the equivalences of $P \to Q \equiv \sim P \vee Q$ and Demorgan law to simplify further your answer. I will deduct marks if you ignore this.

Q3: Write a logical expression corresponding to the following using only the predicates, logical conjunctions, disjunction and nothing else. Assume the domain of x,y = {1,2,3} **(Marks 9)**

$\sim\forall x\ \forall y\ P(x,y) =$

No R.S.

$\exists x\ \neg\forall y\ P(x,y) =$

No Rough sheets.

$\forall x\ \exists y\ \neg P(x,y) =$

Done No Rough sheets.

Q4: The following question relates to the inhabitants of the island of knights and knaves created by Smullyan. **The knights only speak the truth when they are happy. The knaves always speak lie regardless they are happy or sad**. You encounter two people A and B. Determine if possible what A and B are if they address you in the way. If you cannot determine what these two people are, can you draw any conclusions? **(Marks 20)**

A says "The two of us are both knights" and

B says "A is a knave"

Solution:

P = A is a knight	~P = A is a knave
Q = B is a knight	~Q = B is a knave

Scenario 1: Assume all knights to be happy

CASE 1: knight, knight

1) $P \wedge Q = T$ $\quad T \wedge T$

2) $\neg P = F$ $\quad F$

CASE 2: knight, knave

1) $P \wedge Q$ $\quad T \wedge F \neq T$

2) $\neg P = F$

CASE 3: knave knight

1) $P \wedge Q = F$ $\quad F \wedge T = F$

2) $\neg P = T$ $\quad T = T$] case hold

CASE 4: knave, knave

1) $P \wedge Q = F$ $\quad F \wedge F$

2) $\neg P = F$ $\quad T \neq F$

Conclusion: A is knave, B is knight

Scenario 2: Assume all knights to be sad (means the knights will speak lies now)

CASE 1: Knight, Knight

1) $P \wedge Q = F$ $T \wedge T \neq F$ $P = T$ $\neg P = F$

2) $\neg P = F$ $q = T$ $\neg Q = F$

CASE 2: Knight, Knave

1) $P \wedge Q = F$ $F \wedge T = F$] Case hold. $P = T$ $\neg P = F$

2) $\neg P = F$ $F = F$ $q = F$ $\neg Q = T$

CASE 3: Knave, knight

1) $P \wedge Q = F$ $F \wedge T = F$ $P = F$ $\neg P = T$

2) $\neg P = F$ $F \neq F$ $q = T$ $\neg q = F$

CASE 4: Knave, Knave

1) $P \wedge Q = F$ $F \wedge F = F$ $P = F$ $\neg P = T$

2) $\neg P = F$ $T \neq F$ $q = F$ $\neg q = T$

Conclusion: A is Knight, B is Knave

Q5: Assume that the statement "if it is sunny day then I will not go to beach" is in contrapositive form. Make the following forms of this statement using English sentences **(3 marks)**

Converse: If it is not sunny then I will go to beach $(\neg P - q)$

Contrapositive:

If it is sunny day them I will not go to beach $(P \rightarrow \neg q)$

Inverse:

I will not go to beach if it is sunny

OR

If I not go to beach than it ill be a sunny day $(\neg q - q)$

Scanned with CamScanner

→ Solove Rough Work

Question 01:-

Sol:→

Let p = Chohan work hard

Let q = Chohan is a dull boy.

Let r = Chohan will get a job.

$P1: p$	$C1: p \longrightarrow (1)$
$P_2: p \to q$	$C2: \neg p \vee q \to (2)$
$P_3: q \to \neg r$	$C3: \neg q \vee \neg r \to (3)$
	$C4: r \longrightarrow (4)$

$$\frac{\begin{array}{c} P1: p \\ P_2: p \to q \\ P_3: q \to \neg r \end{array}}{C: \neg r}$$

from 1 and 2

$$\frac{\begin{array}{c} p \\ \neg p \vee q \end{array}}{q \to (5)}$$

from 5 and 3

$$\frac{\begin{array}{c} q \\ \neg q \vee \neg r \end{array}}{\neg r \to (6)}$$

From 6 & 4

$$\frac{\begin{array}{c} \neg r \\ r \end{array}}{\square \checkmark}$$

X ———————— X

02
Question:

P = 02

Sol:-

$\cancel{\exists_x \exists_y\, p(x,y) \wedge (x,y)}$

a: $\forall_x \forall_y\, (p(x,y) \longrightarrow \neg Q(x,y))$

know Apply a negation

$\neg(\forall_x \forall_y (p(x,y) \longrightarrow \neg Q(x,y))$

$\exists_x \neg(\forall_y (p(x,y) \longrightarrow \neg Q(x,y))$

$\exists_x \exists_y\, \neg p(x,y) \rightarrow \neg Q(x,y)$

$\exists_x \exists_y\, \neg(\neg p(x,y) \rightarrow \neg Q(x,y))$

$\exists_x \exists_y\, p(x,y) \wedge Q(x,y)$

b:- $\exists_x \forall_y\, (p(x,y) \vee \sim Q(x,y))$

Apply a negation

$\neg(\exists_x \forall_y (p(x,y) \vee \sim Q(x,y))$

$\forall_x \exists_y\, \neg p(x,y) \wedge \neg(\neg Q(x,y))$

$\forall_x \exists_y \neg p(x,y) \wedge Q(x,y)$

03:

Question $\sim \forall_x \forall_y \; p(x,y)$:

P = 03

Sol:

a:- $\sim \forall_x \forall_y \; p(x,y)$

Domain : $x, y = \{1,2,3\}$

$= \exists_x \exists_y \; \neg P(x,1) \vee \neg P(x,2) \vee \neg P(x,3)$

Let ~~xx~~ putting x value

$= (\neg P(1,1) \vee \neg P(1,2) \vee \neg P(1,3)) \vee$

$(\neg P(2,1) \vee \neg P(2,2) \vee \neg P(2,3)) \vee$

$(\neg P(3,1) \vee \neg P(3,2) \vee \neg P(3,3))$

b:- $\exists_x \neg \forall_y \; p(x,y)$ $\quad x, y = \{1,2,3\}$

Sol: $\exists_x \neg \forall_y \; p(x,y)$

$\Rightarrow \exists_x (\neg P(x,1) \vee \neg P(x,2) \vee \neg P(x,3))$

$= (\neg P(1,1) \vee \neg P(1,2) \vee \neg P(1,3)) \vee$

$(\neg P(2,1) \vee \neg P(2,2) \vee \neg P(2,3)) \vee$

$(\neg P(3,1) \vee \neg P(3,2) \vee \neg P(3,3))$

C :- $\forall x \; \exists y \; \neg P(x,y)$ $\quad x, y \; \{1,2,3)$

Sol :- $\forall x \; \exists y \; \neg P(x,y)$

$\cancel{\forall x \; \exists}$

$\Rightarrow \forall x \left(\neg P(x,1) \vee \neg P(x,2) \vee \neg P(x,3) \right)$

$= [\neg P(1,1) \vee \neg P(1,2) \vee \neg P(1,3)] \wedge$

$[\neg P(2,1) \vee \neg P(2,2) \vee \neg P(2,3)] \wedge$

$[\neg P(3,1) \vee \neg P(3,2) \vee \neg P(3,3)]$