

# Operating Systems

## 21. Cryptographic Systems: An Introduction

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# Cryptography $\neq$ Security

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Cryptography may be a component of a secure system

Adding cryptography may not make a system secure

# Cryptography: what is it good for?

- **Authentication**
  - determine origin of message
- **Integrity**
  - verify that message has not been modified
- **Nonrepudiation**
  - sender should not be able to falsely deny that a message was sent
- **Confidentiality**
  - others cannot read contents of the message

# Terms

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**Plaintext** (cleartext) message  $P$

**Encryption**  $E(P)$

Produces **Ciphertext**,  $C = E(P)$

**Decryption**,  $P = D(C)$

**Cipher** = cryptographic algorithm

# Terms: types of ciphers

- Types
  - restricted cipher
  - symmetric algorithm
  - public key algorithm
- Stream vs. Block
  - Stream cipher
    - Encrypt a message a character at a time
  - Block cipher
    - Encrypt a message a chunk at a time

# Restricted cipher

## Secret algorithm

- Vulnerable to:
  - Leaking
  - Reverse engineering
    - HD DVD (Dec 2006) and Blu-Ray (Jan 2007)
    - RC4
    - All digital cellular encryption algorithms
    - DVD and DIVX video compression
    - Firewire
    - Enigma cipher machine
    - Every NATO and Warsaw Pact algorithm during Cold War
- Hard to validate its effectiveness (who will test it?)
- Not a viable approach!

# Symmetric-key algorithm

- Same secret key,  $K$ , for encryption & decryption

$$C = E_K(P) \quad P = D_K(C)$$

- Examples: AES, 3DES, IDEA, RC5
- Key length
  - Determines number of possible keys
    - DES: 56-bit key:  $2^{56} = 7.2 \times 10^{16}$  keys
    - AES-256: 256-bit key:  $2^{256} = 1.1 \times 10^{77}$  keys
  - *Brute force attack*: try all keys

# The power of 2

- Adding one extra bit to a key doubles the search space.
- Suppose it takes 1 second to search through all keys with a 20-bit key

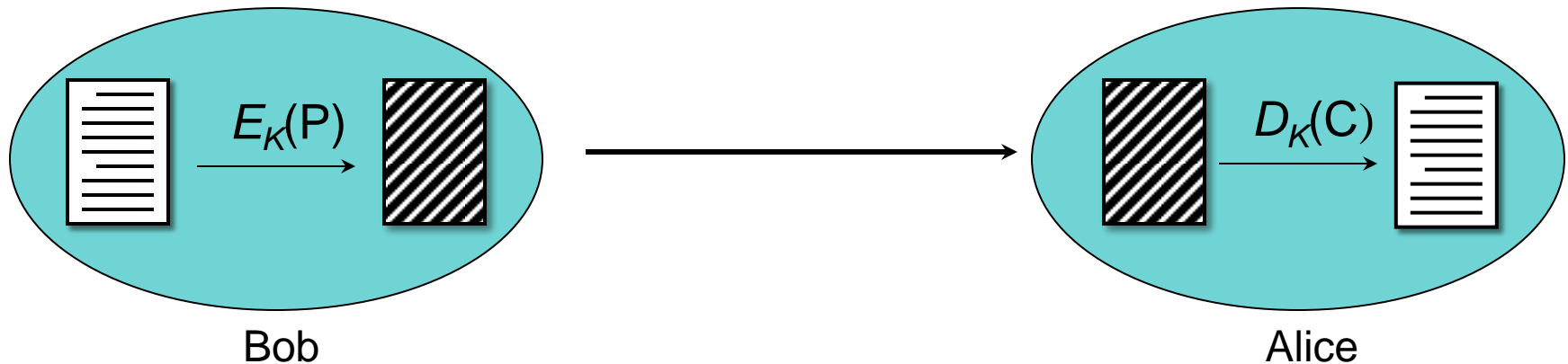
| key length | number of keys       | search time                |
|------------|----------------------|----------------------------|
| 20 bits    | 1,048,576            | 1 second                   |
| 21 bits    | 2,097,152            | 2 seconds                  |
| 32 bits    | $4.3 \times 10^9$    | ~ 1 hour                   |
| 56 bits    | $7.2 \times 10^{16}$ | 2,178 years                |
| 64 bits    | $1.8 \times 10^{19}$ | > 557,000 years            |
| 256 bits   | $1.2 \times 10^{77}$ | $3.5 \times 10^{63}$ years |

Distributed & custom hardware efforts typically allow us to search between 1 and >100 billion 64-bit (e.g., RC5) keys per second



# Communicating with symmetric cryptography

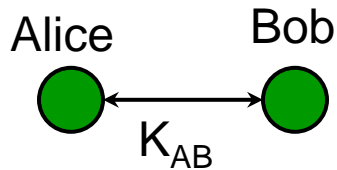
- Both parties must agree on a secret key,  $K$
- Message is encrypted, sent, decrypted at other side



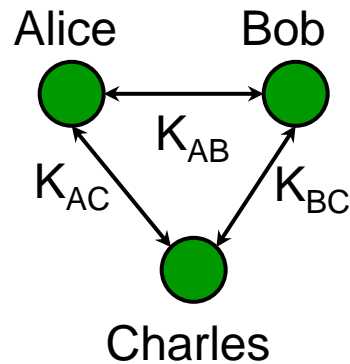
- Key distribution must be secret
  - otherwise messages can be decrypted
  - users can be impersonated

# Key explosion

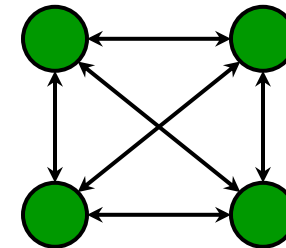
Each pair of users needs a separate key for secure communication



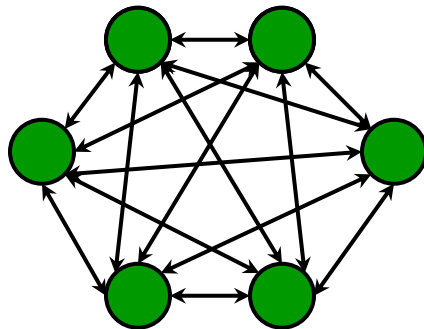
**2 users: 1 key**



**3 users: 3 keys**



**4 users: 6 keys**



**6 users: 15 keys**

100 users: 4,950 keys

1000 users: 399,500 keys

$$n \text{ users: } \frac{n(n-1)}{2} \text{ keys}$$

# Key distribution

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Secure key distribution is the biggest problem with symmetric cryptography

# Public-key algorithm

- Two related keys.

$$C = E_{K_1}(P) \quad P = D_{K_2}(C)$$

$$C' = E_{K_2}(P) \quad P = D_{K_1}(C')$$

$K_1$  is a **public** key

$K_2$  is a **private** key

- Examples:

- RSA, Elliptic curve algorithms  
DSS (digital signature standard),  
Diffie-Hellman (key exchange only!)

- Key length

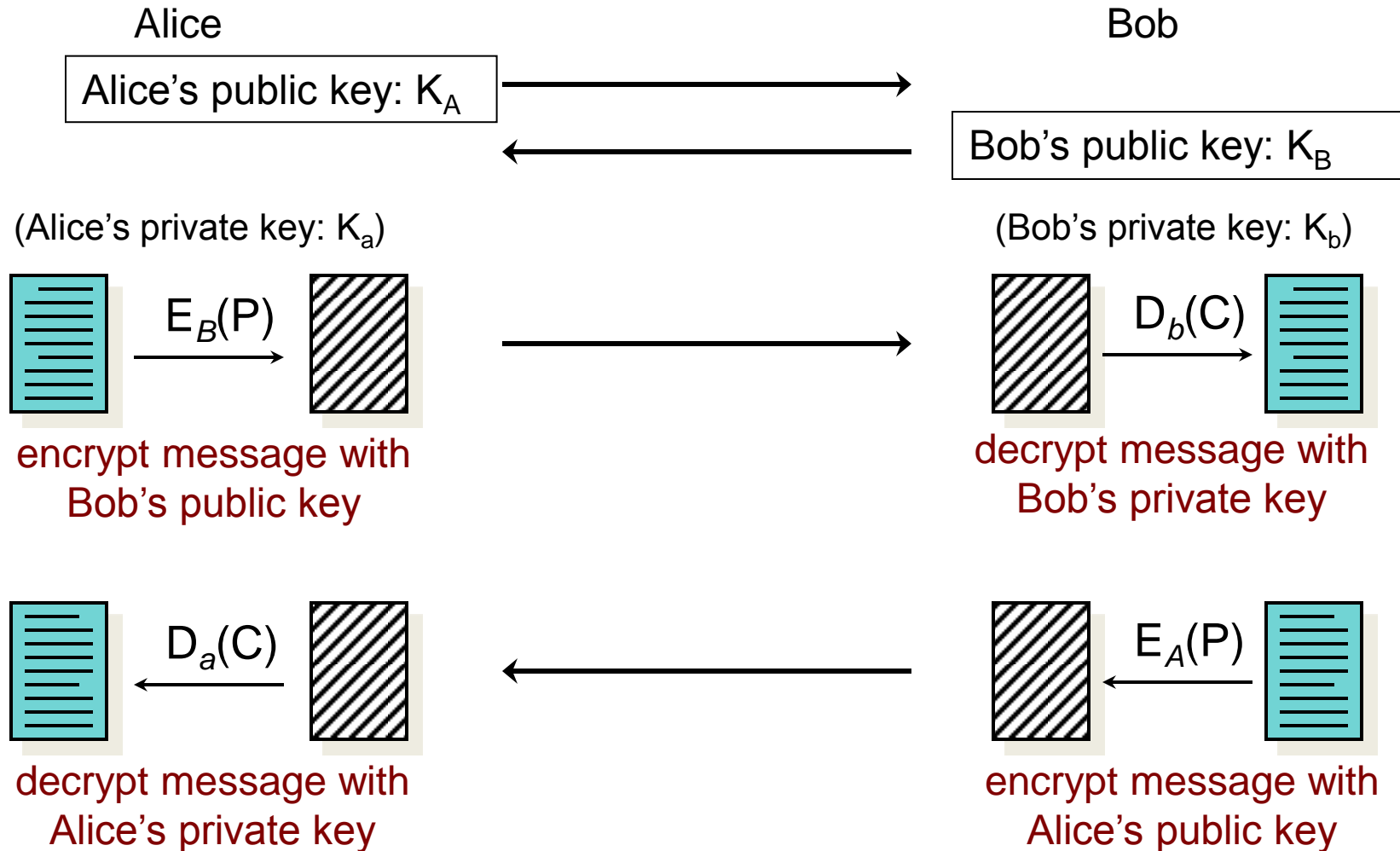
- Unlike symmetric cryptography, not every number is a valid key
- 3072-bit RSA = 256-bit elliptic curve = 128-bit symmetric cipher
- 15360-bit RSA = 521-bit elliptic curve = 256-bit symmetric cipher

# Communication with public key algorithms

Different keys for encrypting and decrypting

- No need to worry about key distribution

# Communication with public key algorithms



# Hybrid Cryptosystems

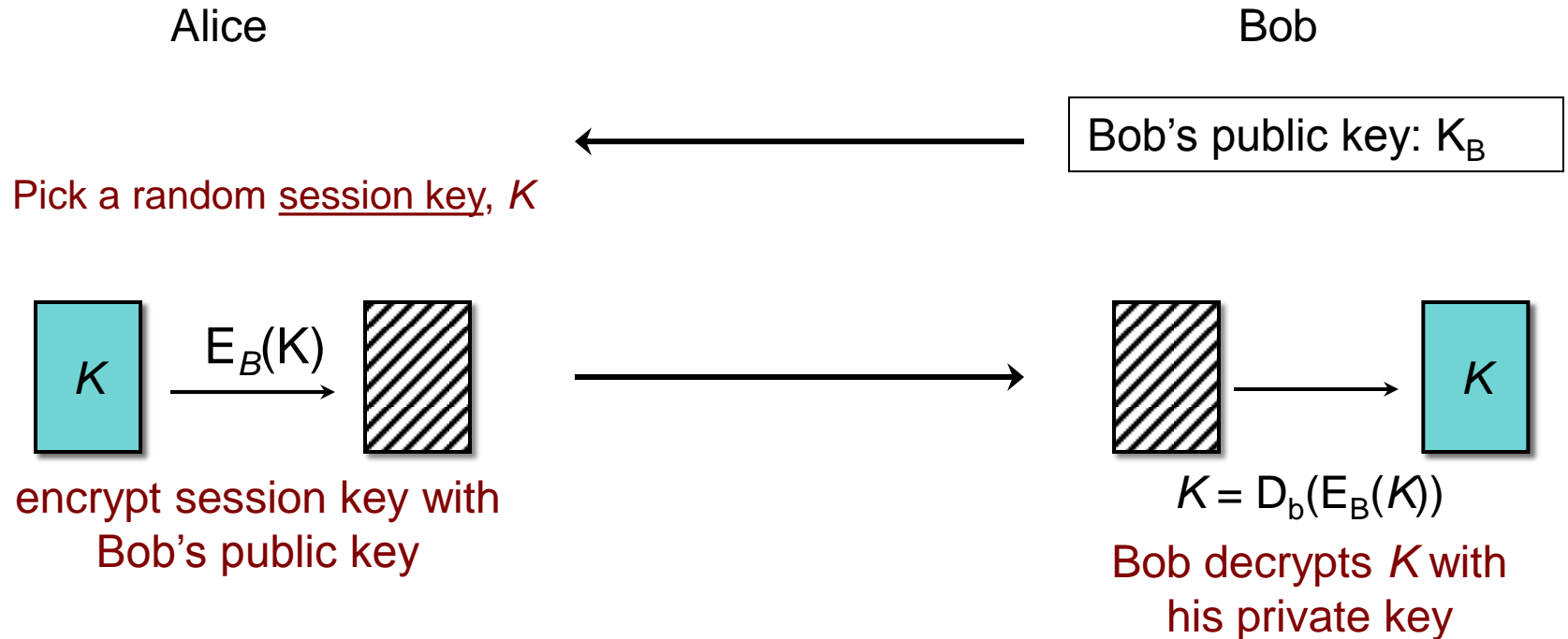
**Session key**: randomly-generated key for one communication session

- Use a **public key algorithm** to send the session key
- Use a **symmetric algorithm** to encrypt data with the session key

Public key algorithms are almost never used to encrypt messages

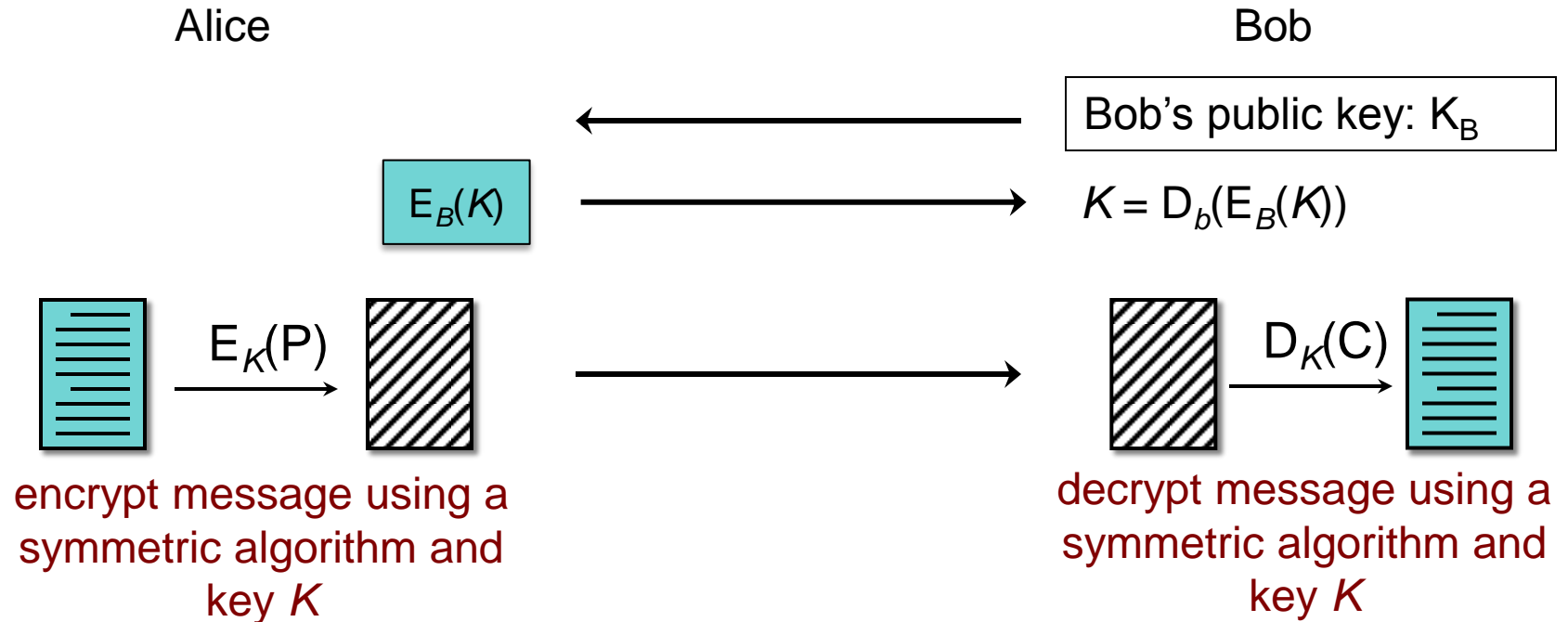
- MUCH slower; vulnerable to *chosen-plaintext attacks*
- RSA-2048 approximately 55x slower to encrypt and 2000x slower to decrypt than AES-256.

# Communication with a hybrid cryptosystem





# Communication with a hybrid cryptosystem



# Message Integrity & Authentication

# One-way functions

- Easy to compute in one direction
- Difficult to compute in the other

Examples:

## Factoring:

$$pq = N$$

EASY

find  $p, q$  given  $N$

DIFFICULT

## Discrete Log:

$$a^b \bmod c = N$$

EASY

find  $b$  given  $a, c, N$

DIFFICULT

# Example of a one-way function

Example with an 18 digit number

$A = 289407349786637777$

$A^2 = 83756614110525308948445338203501729$

Middle square,  $B = 110525308948445338$

Given  $A$ , it is easy to compute  $B$

Given  $B$ , it is difficult to compute  $A$

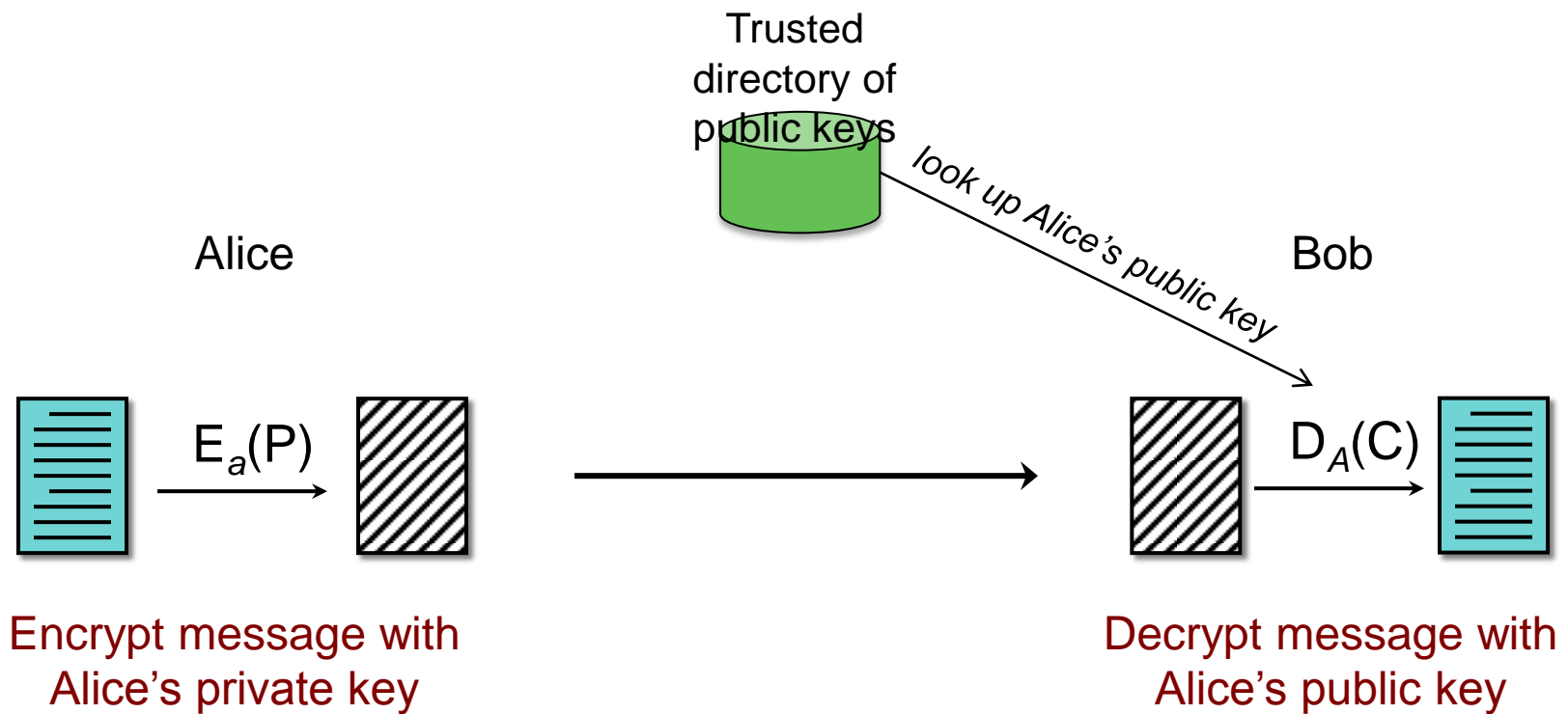
“Difficult” = no known short-cuts; requires an exhaustive search

# Message Integrity: Digital Signatures

- Validate the creator (signer) of the content
- Validate the the content has not been modified since it was signed
- The content itself does not have to be encrypted

# Digital Signatures: Public Key Cryptography

Encrypting a message with a private key is the same as signing it!



# But...

- Not quite what we want
  - We don't want to permute or hide the content
  - We just want Bob to verify that the content came from Alice
- Moreover...
  - Public key cryptography is much slower than symmetric encryption
  - What if Alice sent Bob a multi-GB file – she didn't want to encrypt it but wants Bob to be able to validate that it hasn't been modified

# Hashes to the rescue!

- **Cryptographic hash function** (also known as a **digest**)
  - Input: arbitrary data
  - Output: fixed-length bit string
- **Properties**
  - **One-way function**
    - Given  $H=\text{hash}(M)$ , it should be difficult to compute  $M$ , given  $H$
  - **Collision resistant**
    - Given  $H=\text{hash}(M)$ , it should be difficult to find  $M'$ , such that  $H=\text{hash}(M')$
    - For a hash of length  $L$ , a perfect hash would take  $2^{(L/2)}$  attempts
  - **Efficient**
    - Computing a hash function should be computationally efficient



# Popular hash functions

- **SHA-2**
  - Designed by the NSA; published by NIST
  - SHA-224, SHA-256, SHA-384, SHA-512
    - e.g., Linux passwords used MD5 and now SHA-512
- **SHA-3**
  - NIST standardization still in progress
- **MD5**
  - 128 bits (not often used now since weaknesses were found)
- Derivations from ciphers:
  - **Blowfish** (used for password hashing in OpenBSD)
  - **3DES** – used for old Linux password hashes

# Digital signatures using hash functions

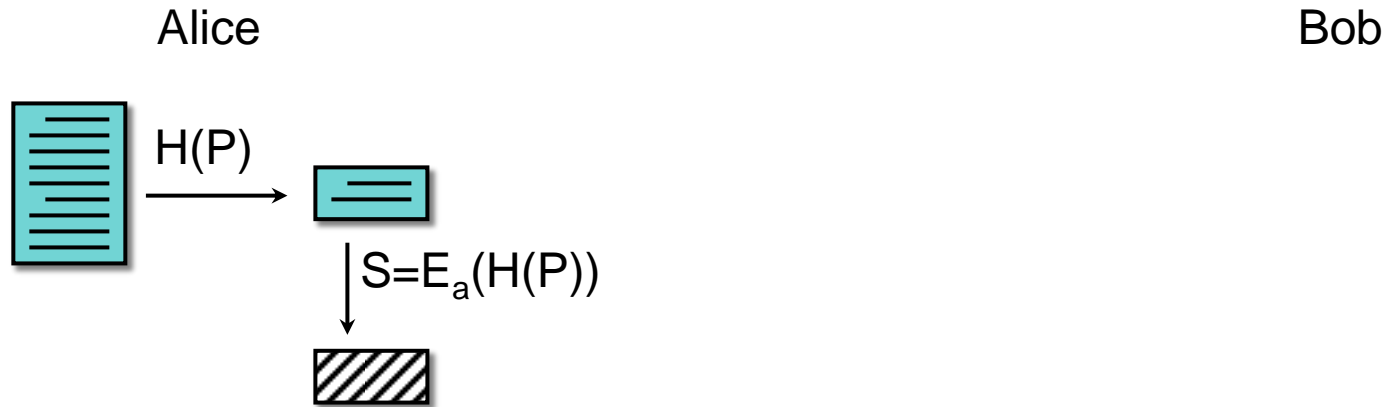
- You do this to create a signature:
  - Create a hash of the message
  - Encrypt the hash with your private key & send it with the message
- Recipient does this to validate the message:
  - Decrypts the encrypted hash using your public key
  - Computes the hash of the received message
  - Compares the decrypted hash with the message hash
  - If they're the same then the message has not been modified

# Digital signatures: public key cryptography



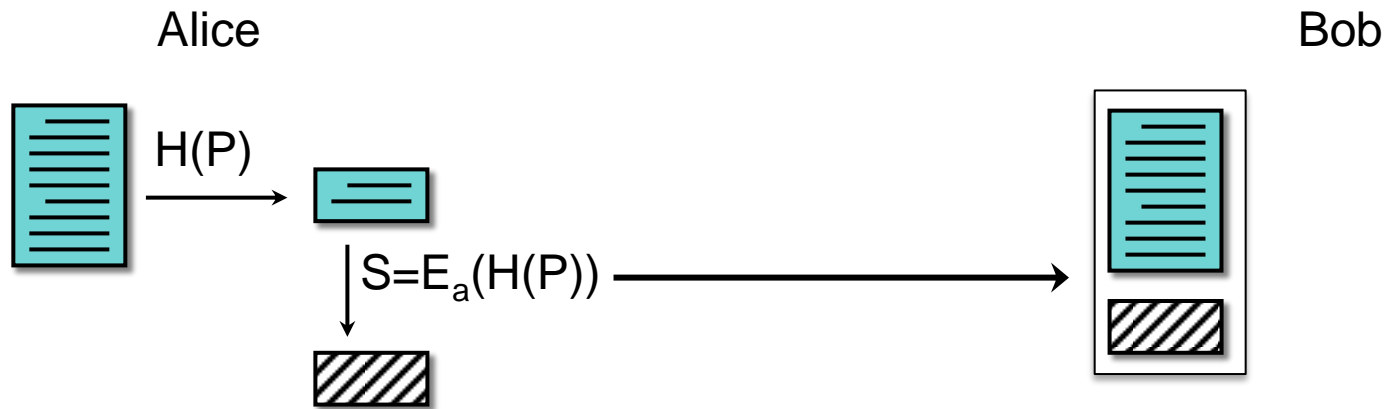
Alice generates a hash of the message

# Digital signatures: public key cryptography



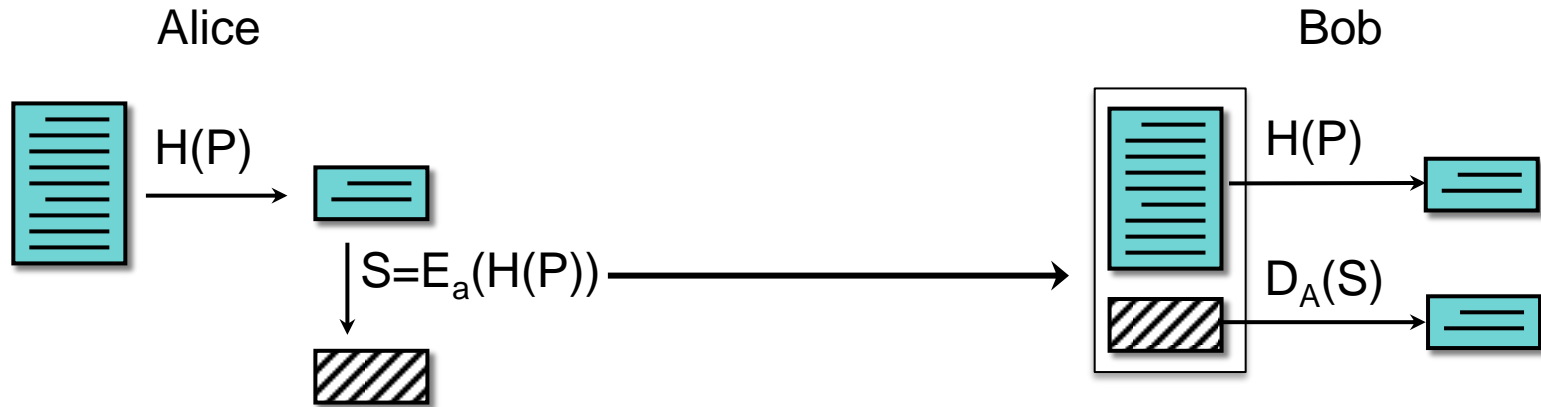
Alice encrypts the hash with her private key  
This is her **signature**.

# Digital signatures: public key cryptography



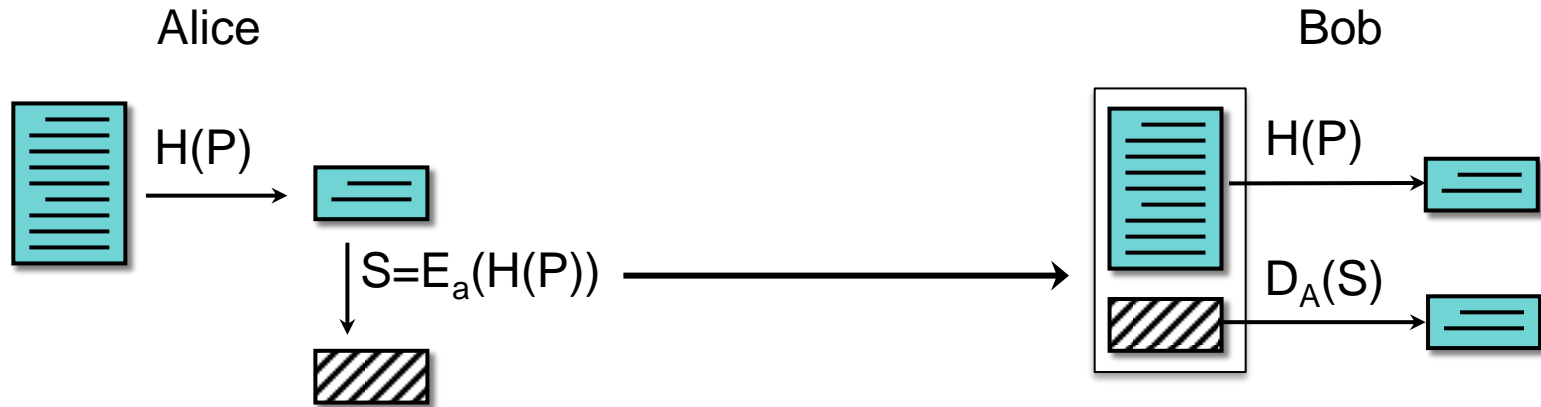
Alice sends Bob the message & the encrypted hash

# Digital signatures: public key cryptography



1. Bob decrypts the hash using Alice's public key
2. Bob computes the hash of the message sent by Alice

# Digital signatures: public key cryptography



If the hashes match, the signature is valid  
– the encrypted hash *must* have been generated by Alice

# Cryptographic toolbox

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- Symmetric encryption
- Public key encryption
- One-way hash functions
- Random number generators



# The End