Composition and Mass of observable universe

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The size, complexity and composition of the universe have bewildered mankind for millenniums. Only in the late 20th century have we been able to unravel the mysteries, understand, explain and confidently speak about it. Cosmology moved from the realm of meta-physics to precision physics. We understand and speak with a high degree of certainty about the origin, age, dimension and composition of the universe.

In this paper I will compute the mass and composition of the observable universe explain the theory behind the numbers and provide supporting evidence (observational facts) .

Edwin Hubble meticulously measured the velocities and distances of galaxies and discovered the galaxies moving away from one another with the farther galaxies moving faster. He captured this observation in the Hubble's Law $V = H_0 d$ where V is velocity of a Galaxy, d the distance and H_0 the Hubble's constant. The Hubble constant also evolves with time, H_0 is its current value.

General Relativity tells us that matter (or energy) curves space around it. So the total amount of matter in the universe determines its curvature. Friedmann equations connect the Mass of the universe to its curvature.Consider a test particle on the surface of an expanding sphere of radius R. Newton's equation of motion gives us

$$\ddot{R} = -\frac{GM}{R^2} \qquad (1)$$

$\dot{R}\ddot{R} = -\frac{GM}{R^2}\dot{R}$ Multiply both-sides by $\dot{R}$ (2)

$(R^2)' = 2R\dot{R} \Rightarrow (\dot{R}^2)' = 2\dot{R}\ddot{R}$ (3)

$(\frac{1}{R})' = -\frac{\dot{R}}{R^2}$ (4)

$\frac{(\dot{R}^2)'}{2} = GM(\frac{1}{R})'$ (5)

$[\frac{(\dot{R}^2)}{2} - GM\left(\frac{1}{R}\right)]' = 0$ (6)

$\frac{(\dot{R}^2)}{2} - \frac{GM}{R} = K$ Integrating both sides (7)

$\frac{(\dot{R}^2)}{2} - \frac{G\ 4\pi R^3.\rho}{3R} = K$ where ρ is the density (8)

$\frac{(\dot{R}^2)}{R^2} - \frac{8\pi G.\rho}{3} = \frac{K}{R^2}$ Multiply by $\frac{2}{R^2}$ (9)

$H^2 - \frac{8\pi G.\rho}{3} = \frac{K}{R^2}$ Since $H = \frac{\dot{R}}{R}$ (10)

Let $\rho_c = \frac{3H^2}{8\pi G} \Rightarrow \frac{H^2}{\rho_c} = \frac{8\pi G}{3}$ where ρ_c is the critical density (11)

$H^2 - \frac{H^2\rho}{\rho_c} = \frac{K}{R^2}$ Substituting (11) in (10) (12)

$H^2 + \frac{K}{R^2} = \frac{H^2\rho}{\rho_c}$ re-arranging (13)

$\rho_c \quad + \frac{K\rho_c}{R^2H^2} = \rho$ Multiply (13) by $\frac{\rho_c}{H^2}$ (14)

$\rho = \rho_c \quad + \frac{3K}{8\pi G R^2}$ Substituting (11) (15)

The friedmann equation (15) connects there quantities, the density ρ, the space curvature $\frac{K}{R^2}$ and the expansion rate H of the universe. The nature of the curvature depends upon the density ρ as follows

$\rho < \rho_c => K < 0$ Then the universe has a negative curvature like a saddle.

$\rho > \rho_c => K > 0$ Then the universe has a positive curvature like a balloon.

$\rho = \rho_c => K = 0$ Then the universe has a 0 curvature and is flat as a sheet.

Experiments (BOOMERANG, COBE, WMAP etc) on CMB have proven the universe to be flat with $\Omega = \rho / \rho_c = 1$. So the density of the universe equal to its critical density. is $\rho \quad = \rho_c = \frac{3H^2}{8\pi G}$.

The current radius R of the observable universe is given by

$$R = \int_0^{t_0} c \left(\frac{t}{t_0}\right)^{-2/3} dt = 3ct_0 = 3c/H \qquad (16)$$

Given the radius and density we can derive Mass of universe as

$$M = \frac{4\pi R^3 \rho}{3} \qquad (17)$$

$$M = \frac{4\pi(\frac{3c}{H})^3}{3} \frac{3H^2}{8\pi G} \qquad (18)$$

$$M = \frac{27c^3}{2HG} \qquad (19)$$

Using the current values of H from the WMAP satellite we compute M =~ $24x10^{53}$ kg.

Once the amount of matter (or density) of the universe is known, one can begin inventory of what constitutes the universe. It is customary to express the current density as a fraction of the critical density $\Omega = \rho / \rho_c$. At a high level we can consider this density is comprised densities of matter (both luminous and non-luminous), radiation (photons) and anything else (we shall elaborate more later on this) .

The sum of matter (electrons, baryons, neutrinos etc), radiation (photons) and everything else is equal to the

$$\rho = \rho_m + \rho_r + \rho_\Lambda \tag{20}$$

$$\frac{\rho}{\rho_c} = \frac{\rho_m}{\rho_c} + \frac{\rho_r}{\rho_c} + \frac{\rho_\Lambda}{\rho_c} \tag{21}$$

$$\Omega = \Omega_m + \Omega_r + \Omega_\Lambda \tag{22}$$

Where Ω_m, Ω_r is the density of matter and radiation respectively.

The density of the radiation Ω_r has been measured by calculating the number of photons in a cubic centimeter, then converting the energy of these photons to mass using $E = mc^2$. This energy is mostly due to CMP and cosmic neutrinos. WMAP assessed this to be =~ $8x10^{-5}$ which is very low compared to the Ω_m. Using the current CMB temperature of 2.7^0K in Bose-Einstein energy and Fermi-Dirac distribution equations a similar low value for $\Omega_r$can be derived. Most of the observed radiation is from CMB. Although stars are intense sources of radiation, they contribute a tiny fraction because they are miniscule compared to the size of the universe. The CMB is very weak and cold, but the total CMB contains more energy than has been emitted by all the stars and galaxies that have ever existed.

Variety of techniques is used to determine the density of matter. Some are highlighted below.

1. Direct Observation: Using the mass-to-light ratio.
2. Measuring the amount of light absorbed quantifies the amount of hydrogen light encounters along its way pass a galaxy.
3. Computing the baryon contents of universe from the anisotropies of the CMB.
4. Gravitational Lensing.
5. Inferring the baryonic content from the abundance of the light elements in the universe. Models for big bang Nucleosynthesis predict certain amount of light elements in the universe and these predictions match very well with the observation when $\Omega_m = 0.045$ as shown in Figure 1 BBN and Abundance of lighter elements

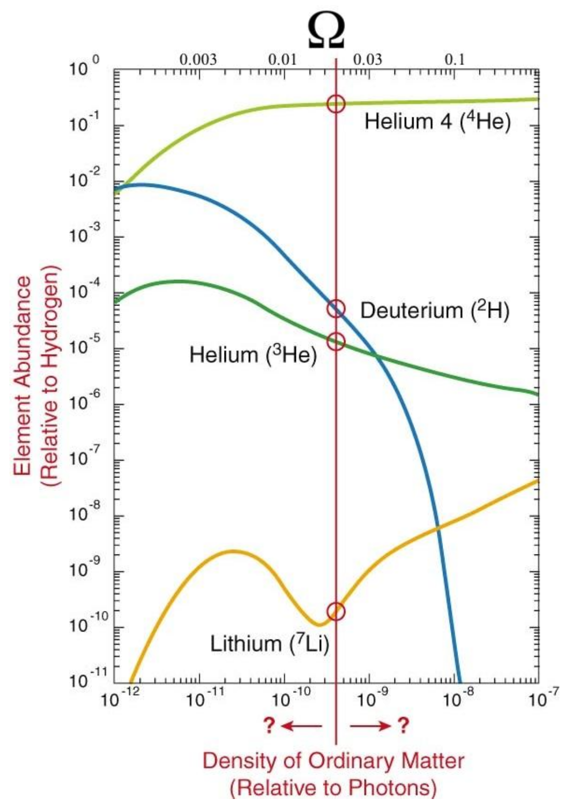
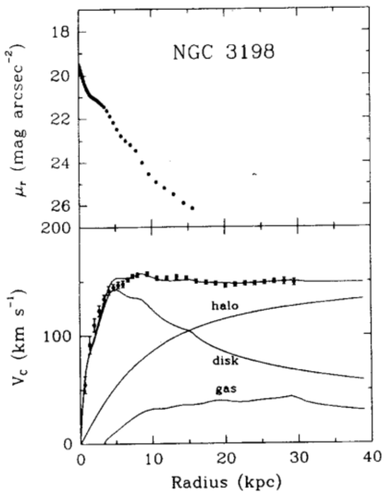


Figure 1 BBN and Abundance of lighter elements

A consensus emerges on $\Omega_m = 0.045$. So normal matter atoms constitute only 4% of the observable universe.

Galaxies rotate and the speed of rotation depends upon the distribution of mass inside the galaxies. Gravity keeps stars bound to galaxies as they rotate. The orbital velocity of any object (star or planet) is given by $V^2 = \frac{GM}{R}$. One would expect the orbital velocities to decrease for stars on the fringes of galaxies. But orbital velocities measured and plotted by Vera Rubin and others show orbital velocities flattening out near the peripheries of the galaxies. Further given the observable mass of a galaxy, the velocity of stars at the peripheries is too large for them to be in an orbit within the galaxy. The star should have flown away from the galaxy. So there is some additional un-seen matter that gravitationally binds these fast moving stars to the galaxies. This un-seen matter is called dark matter.



The upper panel shows the R-band radial surface brightness distribution of the spiral galaxy NGC 3198. The lower panel shows its H I rotation curve (points). The curve labelled *disk shows the expected rotation curve if the surface* density distribution followed the surface brightness distribution in the upper panel. The curve labelled *gas is the contribution to* the rotation curve from the observed gas. Together, the gas and the disk cannot reproduce the observed flat rotation curve at large radius. An extra gravitating component, the dark halo, is needed. The curve labelled *halo is the rotation curve of the* adopted dark halo model: the three labelled rotation curves, when added in quadrature, produce the total rotation curve that passes through the observed points. (From Begeman K 1987 *PhD Thesis University of Groningen.)*

Figure 2 Rotation Curve of Galaxy NGC 3198

The curve indicates that the mass of galaxies is not concentrated in the center but has a mass distributed in a halo surrounding the galaxy. One of the many models for the mass distribution is $\rho = \frac{\rho(0)}{1+({}^{r}/_{a})^{\alpha}}$ where *r* - radius, *a* - halo length, ρ the density. There are other models too. All the luminous content including the inter-galactic gas does not account for the unseen matter. The velocity rise appears to flatten out around 50Kpc and one can infer that the dark matter is 5-7 times more than the luminous matter thereby making $\Omega_m = 0.28$. Dark matter thus interacts gravitationally, but does not interact electromagnetically (no light emission/absorption), holds matter in galaxies together. Evidence for Dark Matter include

1. Rotational curves described above
2. Inter galactic gases are too hot for gravity to hold them in galaxy clusters. Without dark matter these gases would have evaporated.

Hot X-ray emitting gas (shown in purple) was discovered by the ROSAT satellite to be present in this group of galaxies. The presence of the gas provides evidence for the existence of dark matter. (NASA image).

3. Distorted images of background galaxies behind a cluster due to weak lensing

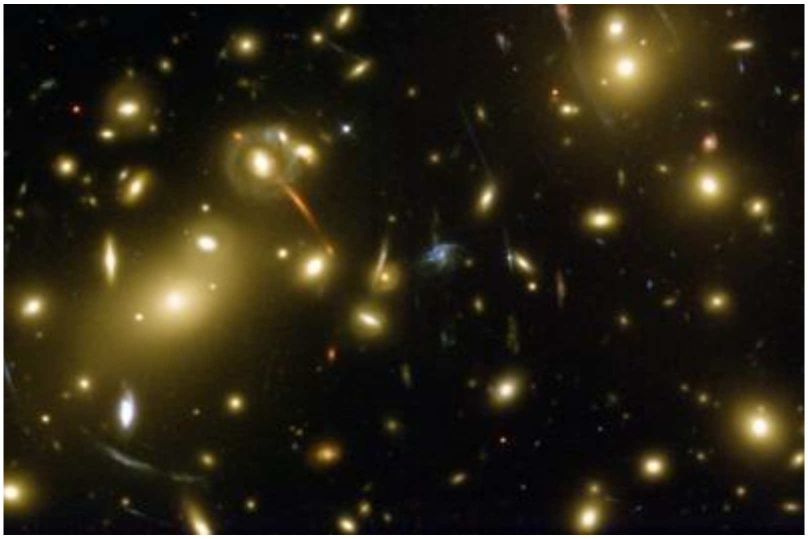


Figure 3 Abell 2218 image

4. Collision of enormous cluster of galaxies captured by the Chandra observatory shows normal matter(hot gas shown in pink) wrenched away from the dark matter (blue)

Figure 4 Bullet cluster showing dark matter

While existence of dark matter has been demonstrated, its constituent components are unknown. It may be a new form of matter. Some potential candidates for this new form of matter are

- WIMPs (Weak interactive massive particle)
- MACHOs (Massive compact halo objects)
- Neutrinos

The standard candle technique has been used for ages to measure distances, but this method suffers from calibration and standardization problems. One is hard pressed to determine the absolute luminosity of the candle. The Type Ia supernovae offer an effective alternative. These are bright explosions that occur when a white dwarf upon acquiring sufficient mass from a companion star crosses the Chandrasekhar limit and explodes. Its intrinsic luminosity is well understood both theoretically and empirically, hence serving as excellent standard candle. The shift in the peak luminosity of the SNe over weeks is a great indication of the rate of expansion of the universe. Further Goobar & Perlmutter (1995) showed the possibility of separating the relative contributions of the Ω_m to the changes in expansion rate by studying supernovae red shifts.

Two independent teams initially studied the SNe Ia and discovered the Hubble curve having a slight bent upwards indicating an accelerating universe pointing to existence of energy that is driving this acceleration against the pull of gravity. Their results captured in the famous charts reproduced in Figure 5 Hubble charts showing an accelerating universe.

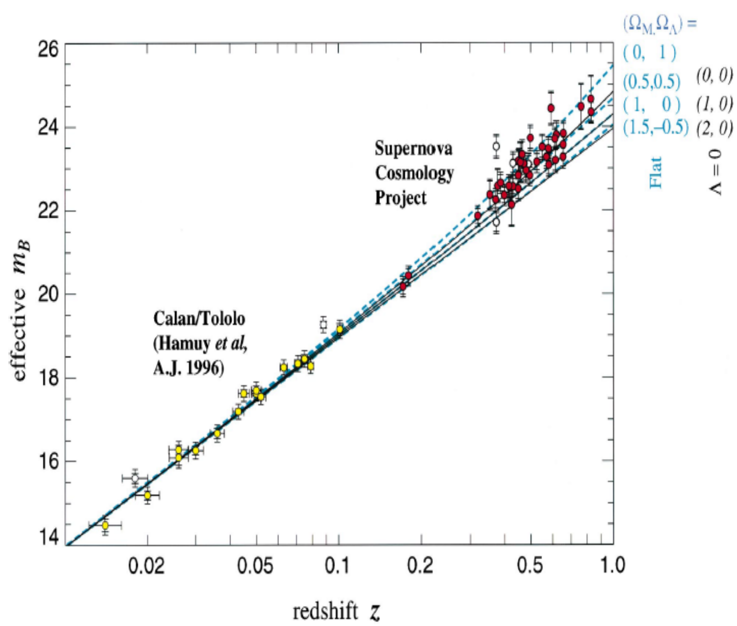
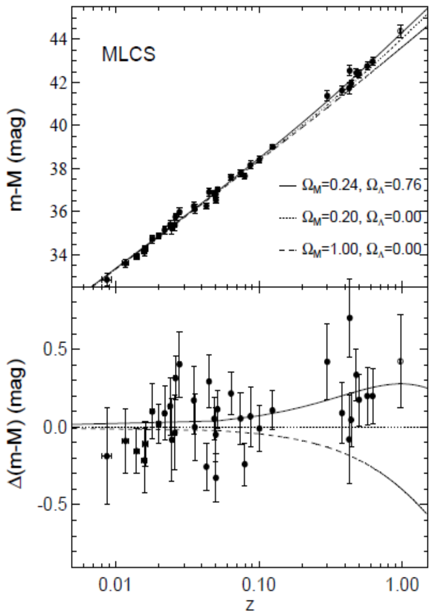


FIG. 1.—Hubble diagram for 42 high-redshift type Ia supernovae from the Supernova Cosmology Project and 18 low-redshift type Ia supernovae from the Calán/Tololo Supernova Survey after correcting both sets for the SN Ia light-curve width-luminosity relation. The inner error bars show the uncertainty due to measurement errors, while the outer error bars show the total uncertainty when the intrinsic luminosity dispersion, 0.17 mag, of light-curve–width-corrected type Ia supernovae is added in quadrature. The unfilled circles indicate supernovae not included in fit C. The horizontal error bars represent the assigned peculiar velocity uncertainty of 300 km s^{-1}. The solid curves are the theoretical $m_B^{eff}(z)$ for a range of cosmological models with zero cosmological constant: $(\Omega_M, \Omega_\Lambda) = (0, 0)$ on top, (1, 0) in middle, and (2, 0) on bottom. The dashed curves are for a range of flat cosmological models: $(\Omega_M, \Omega_\Lambda) = (0, 1)$ on top, (0.5, 0.5) second from top, (1, 0) third from top, and (1.5, −0.5) on bottom.

Figure 5 Hubble charts showing an accelerating universe



This hitherto unknown energy is called dark energy and it acts like a repulsive gravity causing the universe to accelerate. Examining (22) we see

$$\frac{\rho}{\rho_c} = \frac{\rho_m}{\rho_c} + \frac{\rho_r}{\rho_c} + \frac{\rho_\Lambda}{\rho_c} \tag{21}$$

$$\Omega = \Omega_m + \Omega_r + \Omega_\Lambda \tag{22}$$

$$\Omega = 1; \;\; \Omega_m = 0.28 + \quad \Omega_r = \sim 10^{-5} => \quad \Omega_\Lambda = 0.71 \tag{23}$$

Dark energy contributes to the $\Omega_\Lambda = 0.71$ and thus accounts for 71% of the observable universe.

Friedmann equation and GR can be analyzed to understand the theoretical origins of the acceleration.

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{\kappa}{a^2} \tag{24}$$

$$\dot{a}^2 = \frac{8\pi G a^2 \rho}{3} - \kappa \tag{25}$$

κ is the factor contributing to the dark energy. As the universe expands ρ ∝ a^{-4} in a radiation dominated universe and ρ ∝ a^{-3} in a matter dominated universe. (Figure 6 Mass density over timeFigure 6 Mass density over time). In either case the RHS in (25) will decrease assuming is κ constant. So we would expect the universe to be decelerating. But since the SNe Ia data shows an accelerating universe, Feiedmann's equation has to be modified to include a dark-energy term

$$\dot{a}^2 \quad \alpha \qquad \frac{\rho_{matter}}{a} + a^2\rho_{dark\_energy} + constant \qquad (26)$$

The mysterious phantom dark-energy is causing the expansion to accelerate, is smoothly distributed and varies very slowly (if at all) with time.

One candidate for the dark energy component is the energy of Vacuum. It is possible that the dark energy is property of space itself. Another theory called Super symmetric proposes that for every normal matter particle there is a corresponding s-particle (electrons vs selectron). Or maybe we need a new theory of gravity that goes beyond general relativity. Currently there is no comprehensive and compelling theory that explains dark energy. Turner et.al succinctly summarized and assessed these theories as

- Cosmological constant: simple, but no underlying physics.
- Vacuum energy: Mathematically equivalent to a cosmological constant w = −1 is consistent with all data, but all attempts to estimate its size are at best orders of magnitude too large.
- Scalar Fields: Temporary period of cosmic acceleration,w varies between−1 and 1 (and could also be < −1), possibly related to

inflation, but does not address the cosmological constant problem and may lead to new long-range forces.

- New gravitational physics: Cosmic acceleration could be a clue to going beyond GR, but no self-consistent model has been put forth.
- Old gravitational physics: It may be possible to find an inhomogeneous solution that is observationally viable, but such solutions do not yet seem compelling

How did the composition of the universe evolve over time? As the universe expanded it cooled. Going backwards in time, reverses the expansion, the red shift of the CMP becomes blue shift and the CMB gets hotter (E =hc/λ). Closer to the birth (big bang), the radiation is so got that it breaks matter and the universe becomes plasma of fundamental particles and photons. The universe thus evolved to its current form (matter dominated) from hot, dense plasma (radiation dominated). As it expanded and cooled the densities of matter Ω_m and radiation Ω_r changed too. In a radiation dominated universe $\lambda \propto R$ i.e. wavelength stretches, the density is derived as

$$E = h\nu = \frac{hc}{\lambda} \propto R^{-1} \tag{26}$$

$$\rho_r = \frac{E}{R^3} \propto R^{-4} \tag{27}$$

While in a matter dominated universe the density is $E = mc^2$ and the density is

$$\rho_m = \frac{mc^2}{R^3} \propto R^{-3} \tag{28}$$

Since the radius of the universe is function of time a plot of the density over time looks like

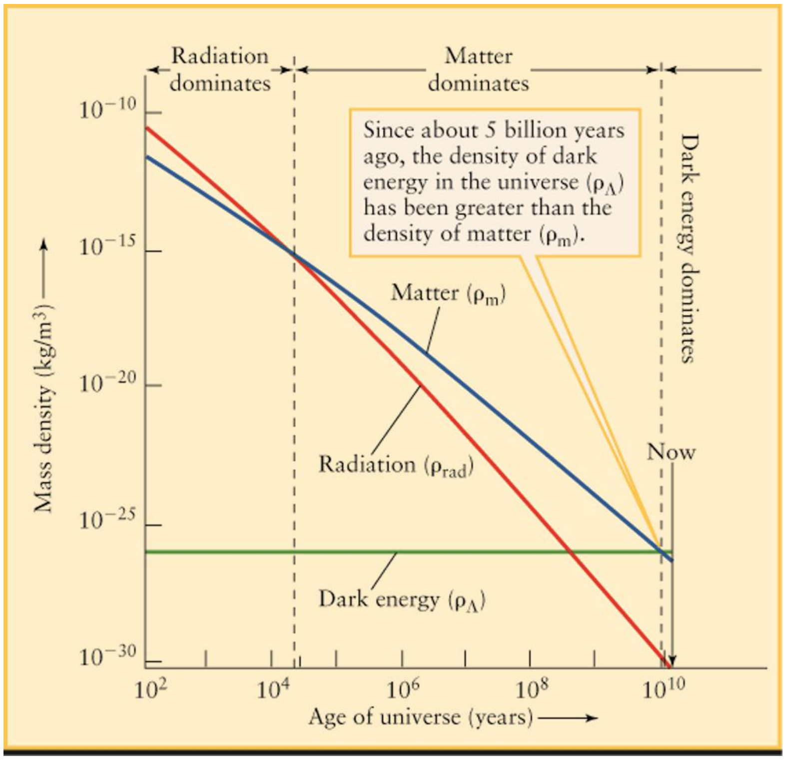


Figure 6 **Mass density over time**

In conclusion we can say that the universe started its life in a radiation dominated era as a hot dense radiation, as it expanded and cooled matter formed, as it continued expansion the density of radiation fell faster than the density of matter and universe entered a matter dominated era. We are now at an interesting juncture where the matter density is same as the dark energy density (Cosmic Coincidence problem) and the future era will be one in which the dark energy will dominate. The matter of the universe is spread among luminous matter, non-luminous (dark matter), radiation energy and dark energy with

Luminous matter = $4 \pm 1\%$

Dark matter = 24%

Dark energy = 71%

The pie in Figure 7 Composition of the Universe shows just that.

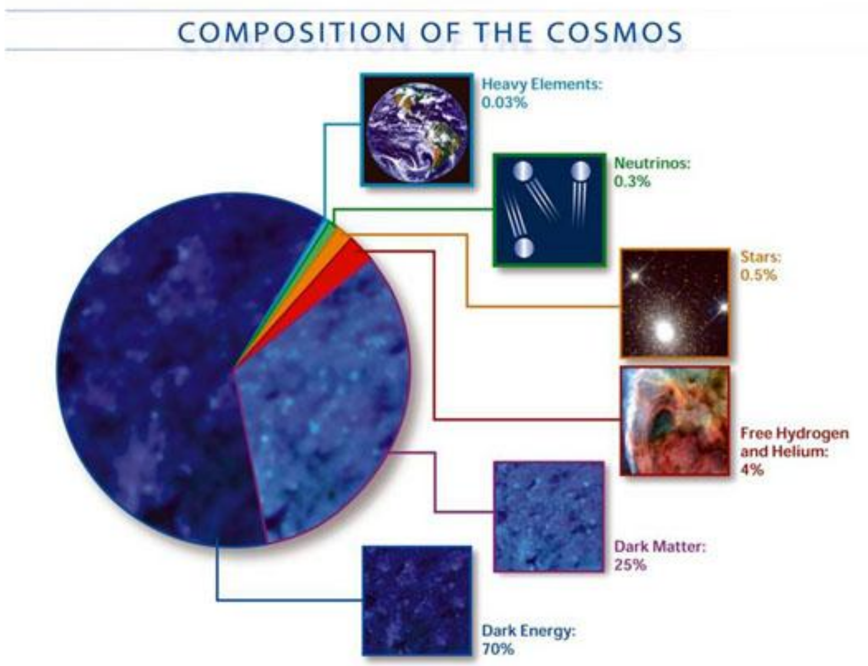


Figure 7 Composition of the Universe

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