Descriptive Statistics cont.

Measure of Relative Standing and Box Plots

Two Excel Files

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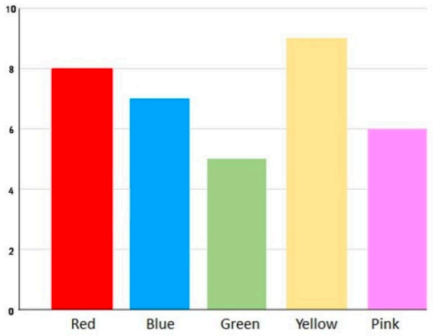
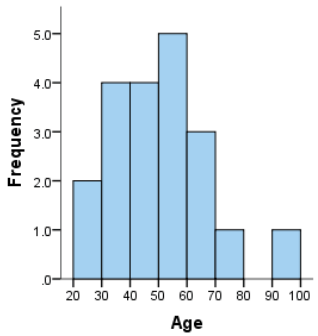
Graphical Methods in Descriptive Statistics

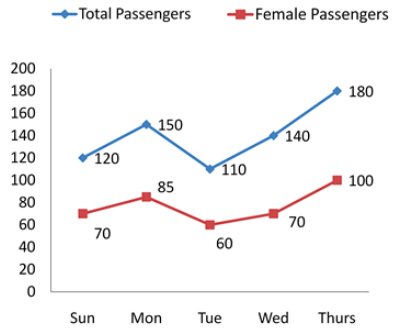
Bar Chart



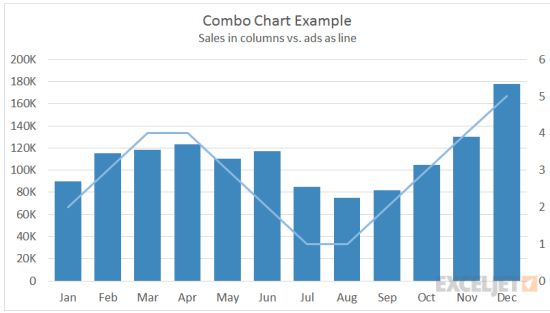
Histograms



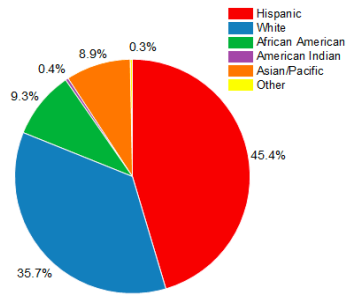
Line plot



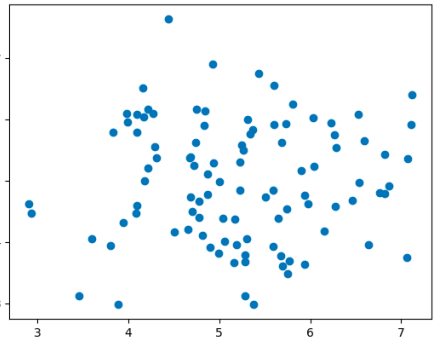
Combo Charts



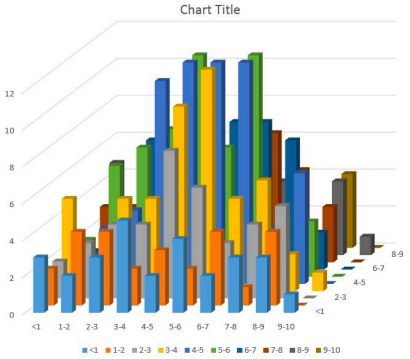
Pie Chart



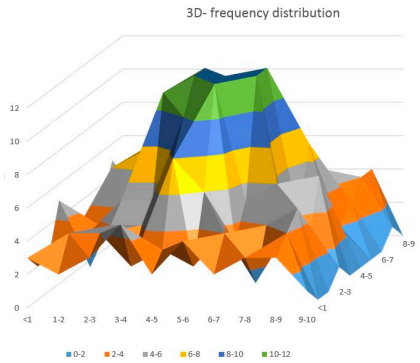
Scatter and bubble plot



3D Plot



Surface plot



Measures in Descriptive Statistics

Measure of Central Location

- We found the central location in the data
- ***Mean:*** Simple average
- ***Median:*** Middle value of the data
- ***Mode:*** Most frequently occurring value

μ	Population Mean
$\bar{x}$	Sample Mean
x_i	Values in the data
N	Total number of values
σ	Standard deviation

Measure of Variability

- We studied how to measure the spread of a dataset
- ***Range:*** Difference between the highest and lowest values
- **Standard Deviation:** measures the dispersion of a dataset relative to its mean

Roughly: "Average distance to the mean"

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2},$$

$$s = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}.$$

Measures of Relative Standing

Percentile P_k

- **P_k** percentile is a value which describes that ***k%*** of the data lies below this value.
- Remaining data that is above **P_k** is **100% – *k%***.

Quartiles

- Quartiles are percentiles that partition the data set into quarters.

Lower Quartile $Q_L = Q_1 = P_{25}$

Upper Quartile $Q_U = Q_3 = P_{75}$

Interquartile Range

- Difference between the Upper and Lower Quartile

$$IQR = Q_U - Q_L = Q_3 - Q_1 = P_{75} - P_{25}$$

- Let's take an example in excel worksheet.

Measures of Relative Standing

Locator of P_k

- Locator of $\boldsymbol{P_k}$ percentile in the data is:

$$\boldsymbol{L_k = \frac{(n+1)k}{100} = W + F}$$

where 'n' is the sample size, W is the integer part and F is the decimal part of L_k

Percentile Estimation

$$\boldsymbol{P_k = x_W + F(x_{W+1} - x_W)}$$

- If $F = 0$ then $L_k = W$ *and*
  $\boldsymbol{P_k} = \boldsymbol{x_W}$ = a value with the rank of $\boldsymbol{L_k}$

- If $F \neq 0$ then $L_k = W + F$ *and*
  $$\boldsymbol{P_k = x_W + F(x_{W+1} - x_W)}$$

Measures of Relative Standing

Example

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
6.8	7.4	7.9	8.2	**8.3**	**8.3**	8.4	8.8	9.1	9.8

We are required to find P_{25}?

To find P_{25}, we use the formula:

$$\boldsymbol{P_k = x_W + F(x_{W+1} - x_W)}$$

We require values of *F* and *W*.

$$L_{25} = \frac{\mathbf{(10+1)*25}}{\mathbf{100}} = \mathbf{2.75}$$

This implies *W* = 2 and *F* = 0.75. Finally,

$$\boldsymbol{P_{25} = Q_1} = x_2 + 0.75(x_3 - x_2) = 7.8$$

Find P_{50}

- $L_{50} = \frac{(10+1)50}{100} = \frac{550}{100} = 5.5$
- $$\begin{aligned}\boldsymbol{P_{50} = Q_2} &= x_5 + 0.5(x_6 - x_5)\\ &= 8.3 + 0.5(8.3 - 8.3)\\ &= 8.3 + 0.5\times 0\\ &= 8.3\end{aligned}$$

Find P_{75}

- $L_{75} = \frac{(10+1)75}{100} = \frac{825}{100} = 8.25$
- $$\begin{aligned}\boldsymbol{P_{75} = Q_3} &= x_8 + 0.25(x_9 - x_8)\\ &= 8.8 + 0.25(9.1 - 8.8)\\ &= 8.8 + 0.25\times 0.3\\ &= 8.875 \approx 8.9\end{aligned}$$

Interquartile range $= IQR = Q_U - Q_L = Q_3 - Q_1 = P_{75} - P_{25} = 8.9 - 7.8 = 1.1$

Measures of Relative Standing

Answer the following questions:

- Which *Measurement of Central Location (Mean, Median or Mode)* is equals to 50th Percentile or Q2?

  Answer: Median

- A percentile represents a value that is least amongst the *k*% of the data

  Answer: FALSE

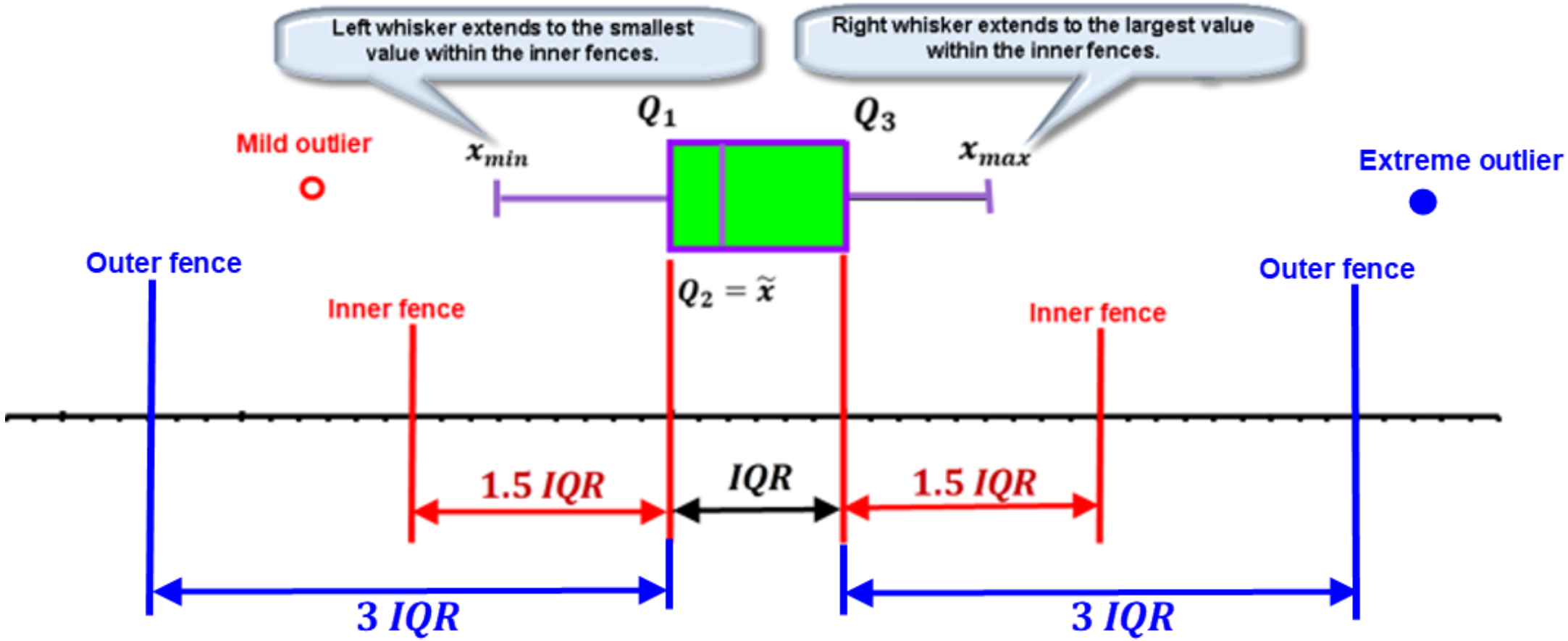
- In locator formula $L_k = W + F$, there are W values in a dataset that are less than the value of Percentile.

  Answer: TRUE

- Interquartile range is the difference between upper and lower quartile.

  Answer: TRUE

Box Plot


Left whisker extends to the smallest value within the inner fences.
Right whisker extends to the largest value within the inner fences.
Q_1
Q_3
Mild outlier
x_{min}
x_{max}
Extreme outlier
Outer fence
Inner fence
$Q_2 = \tilde{x}$
Inner fence
Outer fence
1.5 IQR
IQR
1.5 IQR
3 IQR
3 IQR

Box Plot

Example

Example

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}
6.8	7.4	7.9	8.2	8.3	8.3	8.4	8.8	9.1	9.8

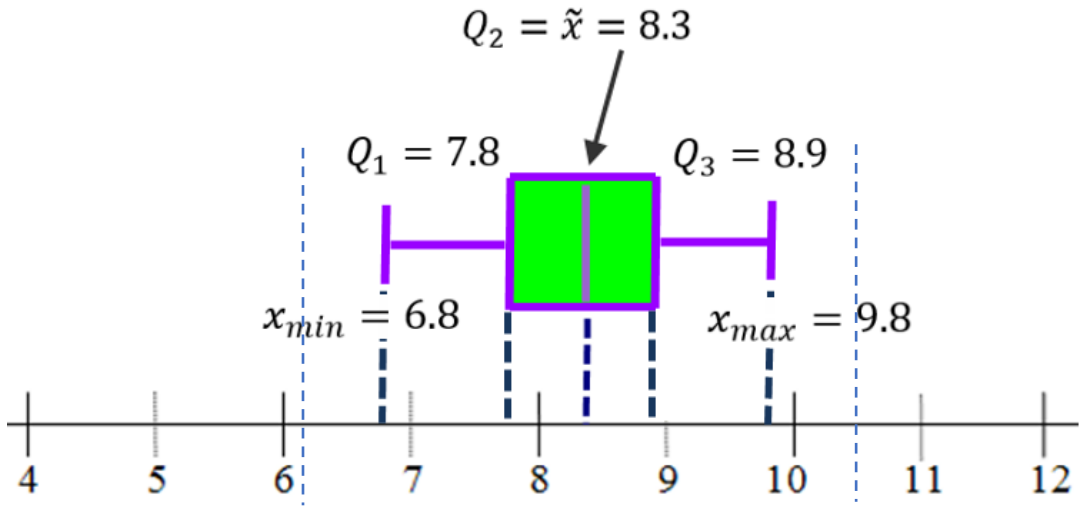
$x_{min} = 6.8, x_{max} = 9.8$

$Q_1 = 7.8, Q_2 = \tilde{x} = 8.3, Q_3 = 8.9$

1.5 Interquartile range $= 1.5\ IQR$

$= 1.5 \times (Q_3 - Q_1)$

$= 1.65$



Inner Fence:

Lower Limit $=$ LL $= Q_1 - 1.5IQR = 7.8 - 1.65 = 6.15$

Upper Limit $=$ UL $= Q_3 + 1.5IQR = 8.9 + 1.65 = 10.55$

Create a Box Plot in Excel

METHOD 1: EXCEL BOX PLOT OPTION (LINKED TO EXCEL FILE)

METHOD 2: STEP BY STEP APPROACH

To create your own box plot chart, the first step is to set up your data.

- Labels are not used in the chart. Let say the data is in column B and C with 13 values each.
- Enter the Box Plot Chart Formulas

Step 1: Calculate the quartile values

F4=MIN(B1:B13)	G4=MIN(C1:C13)
F5=QUARTILE(B1:B13,1)	G5=QUARTILE(C1:C13,1)
F6=MEDIAN(B1:B13)	G6=MEDIAN(C1:C13)
F7=QUARTILE(B1:B13,3)	G7=QUARTILE(C1:C13,3)
F8=MAX(B1:B13)	G8=MAX(C1:C13)

Step 2: Calculate quartile differences

=F5	=G5
=F6-F5	=G6-G5
=F7-F6	=G7-G6
=F8-F7	=G8-G7
=F5-F4	=G5-G4

Step 3: Create the Box Plot Chart

- Create a stacked column chart
- Convert the stacked column chart to the box plot style
- Hide the bottom data series
- Create whiskers for the box plot
- Color the middle areas

https://www.contextures.com/excelboxplotchart.html

https://support.microsoft.com/en-us/office/create-a-box-plot-10204530-8cdf-40fe-a711-2eb9785e510f

Box Plot

Answer the following questions:

- What is the basic use of box plot?

Answer: Skewness of Data