AI 2002
Artificial Intelligence

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Perceptron Training Rule

- The ***perceptron training rule,*** which revises the weight w_i associated with input x_i according to the rule:

$$w_i \leftarrow w_i + \Delta w_i$$

where

$$\Delta w_i = \eta(t - o)x_i$$

Where:

- t is target value
- o is perceptron output
- η is small constant (e.g., 0.1) called *learning rate*

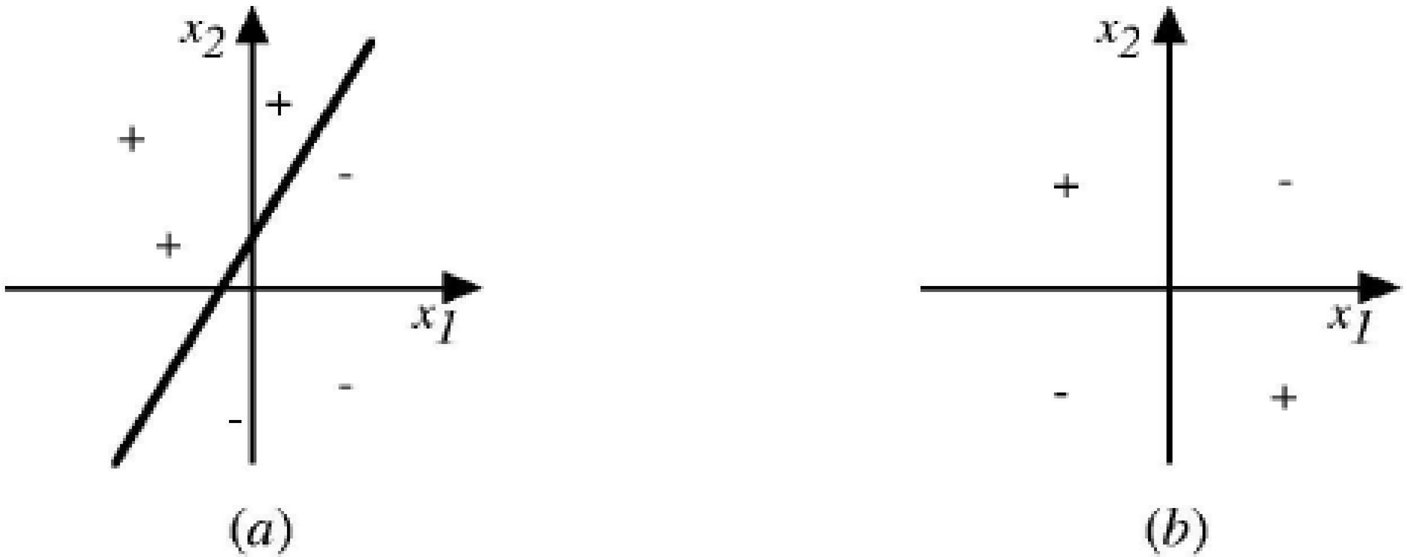
Perceptron Training Rule

- The **perceptron rule** finds a successful weight vector when the training examples are **linearly separable**,
- It fails to converge if the examples are **not linearly separable**.
- The solution is … **Delta Rule** also known as **(Widrow-Hoff Rule)**

Delta Rule

- use ***gradient descent*** to search the hypothesis space of possible weight vectors to find the weights that best fit the training examples.

Perceptron



The **decision surface** represented by a **two-input perceptron x_1** and x_2.
(a) A set of training examples and the decision surface of a perceptron that classifies them correctly. ***(b)*** A set of training examples that is not linearly separable.

Delta Rule

Delta Rule

- In **perceptron training rule** we employ ***thresholded perceptron***

$$o(\mathbf{x}) = \begin{cases} 1 & \text{if} \quad \mathbf{w}.\mathbf{x} > 0 \\ -1 & \quad \text{otherwise} \end{cases}$$

- The **delta training rule** is the task of training an ***unthresholded perceptron***;
  - a ***linear unit*** for which the output o is given by

$$o(\mathbf{x}) = \mathbf{w}.\mathbf{x}$$

  - A linear unit corresponds to the *first stage* of a perceptron, ***without*** the threshold.

Delta Rule

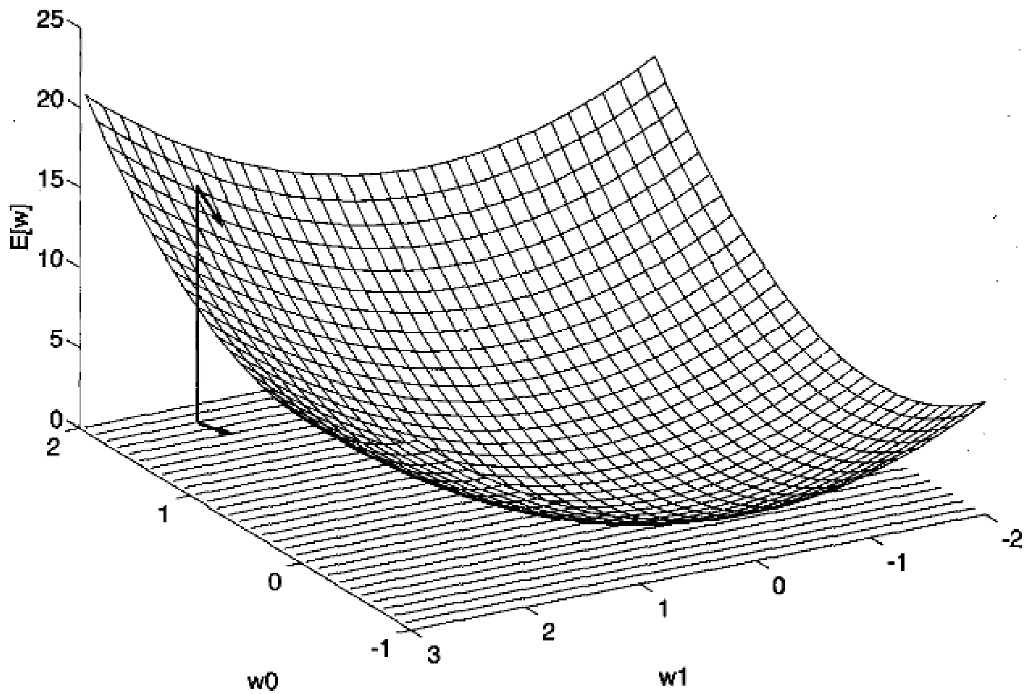
- In order to derive ***a weight learning rule for linear units***,
  - Specify a measure for the ***training error*** of a hypothesis (weight vector), relative to the training examples.

$$E(\vec{w}) \equiv \frac{1}{2} \sum_{d \in D} (t_d - o_d)^2$$

  - D is the set of training examples,
  - t_d is the target output for training example *d*,
  - o_d is the output of the linear unit for training example *d*
  - E is characterized as a function of ***w***, because the linear unit output *o* depends on this weight vector.

Hypothesis Space

- For a linear unit with two weights, the hypothesis space H is the $\boldsymbol{w_0}$, $\boldsymbol{w_1}$ plane.



Error of different hypotheses.

Gradient Descent

- Gradient descent search determines *a weight vector that minimizes **E*** by
  - Starting with an arbitrary initial weight vector,
  - Repeatedly modifying it in small steps.
  - At each step, the ***weight vector is altered in the direction*** that produces the **steepest descent** along the error surface,
  - This process continues until the **global minimum error** is reached.

Gradient Descent

How can we calculate the direction of steepest descent along the error surface?

- This direction can be found by computing the derivative of ***E*** with respect to each component of the vector $\boldsymbol{w}$.

$$\nabla E(\vec{w}) \equiv \left[\frac{\partial E}{\partial w_0}, \frac{\partial E}{\partial w_1}, \cdots, \frac{\partial E}{\partial w_n}\right]$$

- Notice $\nabla E(\vec{w})$ is itself a vector, whose components are the partial derivatives of ***E*** with respect to each of the $\boldsymbol{w_i}$.

Gradient Descent

- The **gradient specifies the direction** that produces the steepest increase in E**.** The training rule for gradient descent is,

$$\vec{w} \leftarrow \vec{w} + \Delta\vec{w}$$

where,

$$\Delta\vec{w} = -\eta \nabla E(\vec{w})$$

- The negative sign is present because we want to move the weight vector in the direction that ***decreases*** E.

Gradient Descent

- This training rule can also be written in its **component form**,

$$\vec{w} \leftarrow \vec{w} + \Delta\vec{w}$$

$$w_i \leftarrow w_i + \Delta w_i$$

where,

$$\Delta\vec{w} = -\eta \nabla E(\vec{w})$$

$$\Delta w_i = -\eta \frac{\partial E}{\partial w_i}$$

- The steepest descent is achieved by altering each component w_i in proportion to $\frac{\partial E}{\partial w_i}$

Gradient Descent

- The vector of derivatives $\frac{\partial E}{\partial w_i}$ that form the gradient can be obtained by differentiating *E* from delta rule

$$\boxed{E(\vec{w}) \equiv \frac{1}{2} \sum_{d \in D} (t_d - o_d)^2}$$

$$\boxed{o(\mathbf{x}) = \mathbf{w}.\mathbf{x}}$$

$$\begin{aligned}
\frac{\partial E}{\partial w_i} &= \frac{\partial}{\partial w_i} \frac{1}{2} \sum_{d \in D} (t_d - o_d)^2 \\
&= \frac{1}{2} \sum_{d \in D} \frac{\partial}{\partial w_i} (t_d - o_d)^2 \\
&= \frac{1}{2} \sum_{d \in D} 2(t_d - o_d) \frac{\partial}{\partial w_i} (t_d - o_d) \\
&= \sum_{d \in D} (t_d - o_d) \frac{\partial}{\partial w_i} (t_d - \vec{w} \cdot \vec{x}_d)
\end{aligned}$$

$$\boxed{\frac{\partial E}{\partial w_i} = \sum_{d \in D} (t_d - o_d)(-x_{id})}$$

Gradient Descent

- We now have an equation that gives $\frac{\partial E}{\partial w_i}$ in terms of
  - the linear unit inputs x_{id},
  - outputs o_d,
  - target values t_d associated with the training examples
- The weight update rule for gradient descent becomes,

$$\Delta w_i = -\eta \frac{\partial E}{\partial w_i}$$

$$\boxed{\Delta w_i = \eta \sum_{d \in D} (t_d - o_d)\ x_{id}}$$

Gradient Descent

GRADIENT-DESCENT($training\_examples, \eta$)

Each training example is a pair of the form $\langle \vec{x}, t \rangle$, where $\vec{x}$ is the vector of input values, and t is the target output value. η is the learning rate (e.g., .05).

- Initialize each w_i to some small random value
- Until the termination condition is met, Do
  - Initialize each Δw_i to zero.
  - For each $\langle \vec{x}, t \rangle$ in $training\_examples$, Do
    - Input the instance $\vec{x}$ to the unit and compute the output o
    - For each linear unit weight w_i, Do

$$\Delta w_i \leftarrow \Delta w_i + \eta (t - o) x_i$$

  - For each linear unit weight w_i, Do

$$\boxed{\Delta w_i = \eta \sum_{d \in D} (t_d - o_d)\, x_{id}}$$

$$w_i \leftarrow w_i + \Delta w_i$$

Gradient Descent

- Gradient descent is an important general paradigm for learning.
- It is a *strategy for searching through a **large or infinite** hypothesis space* that can be applied whenever
    1) the hypothesis space contains **continuously parameterized hypotheses** (e.g., the weights in a linear unit),
    2) the **error can be differentiated** with respect to these hypothesis parameters

Gradient Descent

- The **key practical difficulties** in applying gradient descent are:
  a) ***converging to a local minimum*** can sometimes be **quite slow** (i.e., it can require many thousands of gradient descent steps),

  b) ***if there are multiple local minima*** in the error surface, then there is no guarantee that the procedure will find the global minimum.

Stochastic Gradient Descent

- The idea behind stochastic gradient descent is to approximate the gradient descent search by **updating weights incrementally**, following the calculation of the error for ***each* individual example**.

$$\boxed{\Delta w_i = \eta \sum_{d \in D} (t_d - o_d)\ x_{id}}$$

$$\boxed{\Delta w_i = \eta (t - o)\ x_i}$$

Stochastic Gradient Descent

- One way to view this stochastic gradient descent is to consider a **distinct error function** defined for ***each*** *individual training example* ***d*** *as follows*.

$$E_d(\vec{w}) = \frac{1}{2}(t_d - o_d)^2$$

  - where t, and o_d are the target value and the unit output value for training example d.
  - Stochastic gradient descent iterates over the training examples d in D,
  - at each iteration altering weights according to the gradient

Stochastic Gradient Descent

GRADIENT-DESCENT(*training_examples*, η)

Each training example is a pair of the form $\langle \vec{x}, t \rangle$, where $\vec{x}$ is the vector of input values, and t is the target output value. η is the learning rate (e.g., .05).

- Initialize each w_i to some small random value
- Until the termination condition is met, Do
  - For each $\langle \vec{x}, t \rangle$ in *training_examples*, Do
    - Input the instance $\vec{x}$ to the unit and compute the output o
    - For each linear unit weight w_i, Do

$$w_i \leftarrow w_i + \eta(t - o)x_i$$

The Key Difference

- In **stochastic gradient descent,** weights are updated upon examining ***each*** training example.
- Whereas in **standard gradient descent**, the error is summed over ***all*** examples before updating weights,
  - **Standard gradient descent** requires **more computation** per weight update step.
  - **Standard gradient descent** is often used with a **larger step size** per weight update than stochastic gradient descent.

The Key Difference

- When there are multiple local minima with respect to $\boldsymbol{E}(\boldsymbol{w})$,
  - The **stochastic gradient descent** can sometimes *avoid falling into these local minima*,
  - It is due to the reason that it uses various $\nabla E_d(\vec{w})$ rather than $\nabla E(\vec{w})$ to guide its search.

Training Rules

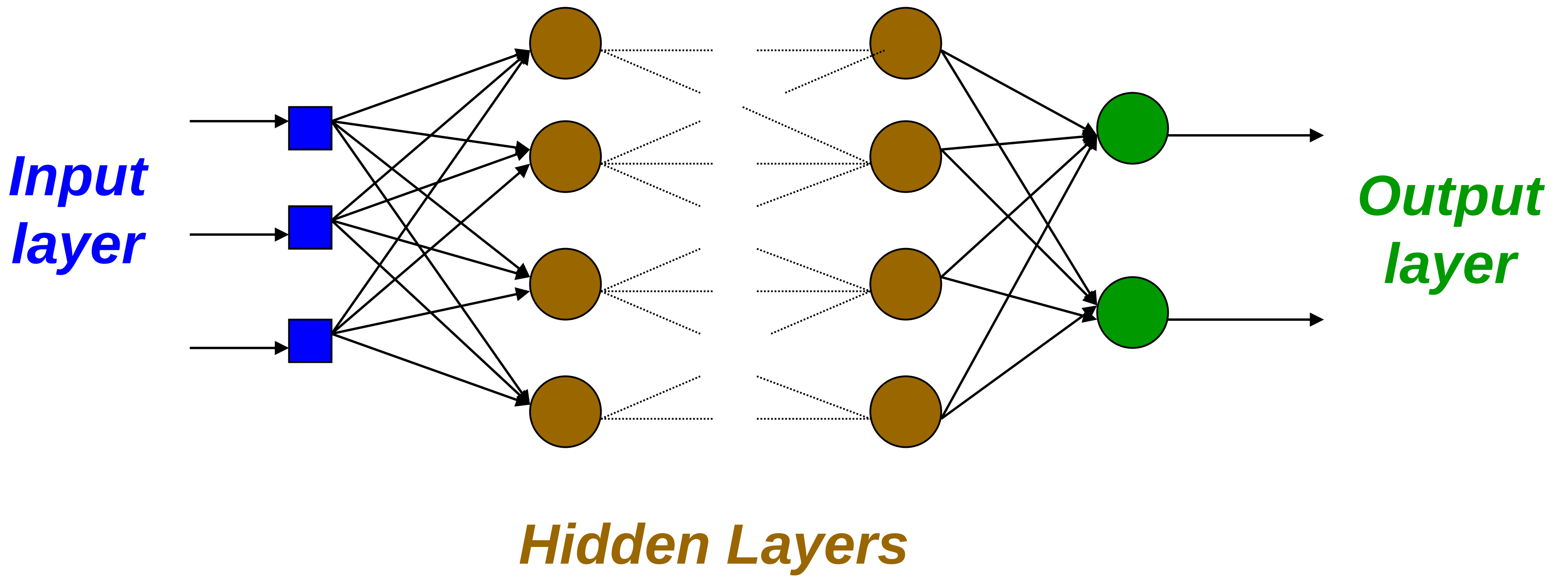
- **Perceptron Training Rule:** guarantee to succeed if
  - training examples are linearly separable
  - Sufficiently small learning rate

- **Delta Rule:**
  - use gradient descent
  - converges only asymptotically toward the minimum error hypothesis,
  - converges regardless of whether the training data are linearly separable.

Multilayer Networks

Multilayer Perceptron Architecture

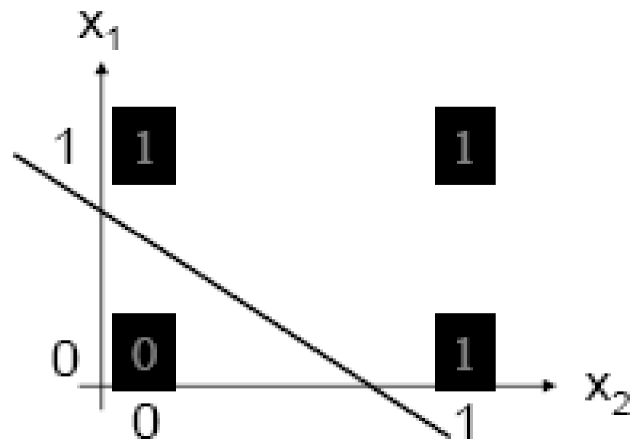
MLP used to describe any general feedforward (no recurrent connections) network



Multilayer Networks

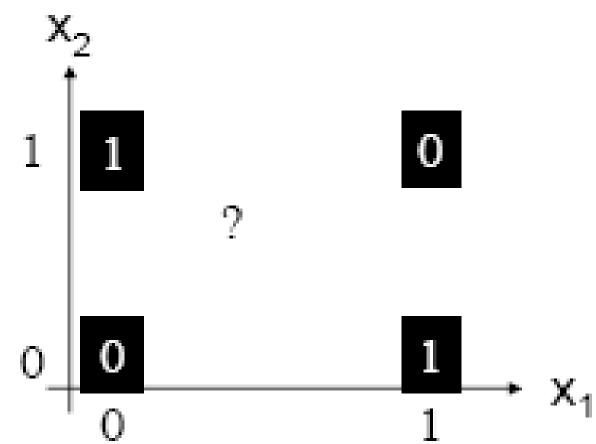
<u>OR function</u>

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	1

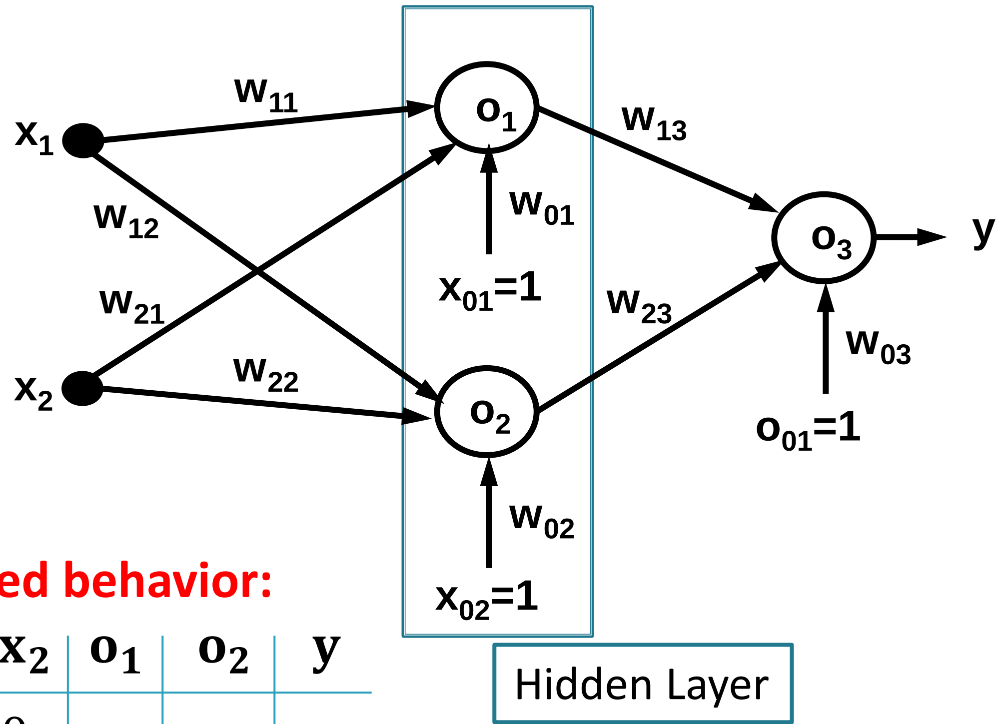


<u>XOR function</u>

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0



Multilayer Networks



Network Topology:

2 hidden nodes

1 output

Weights:

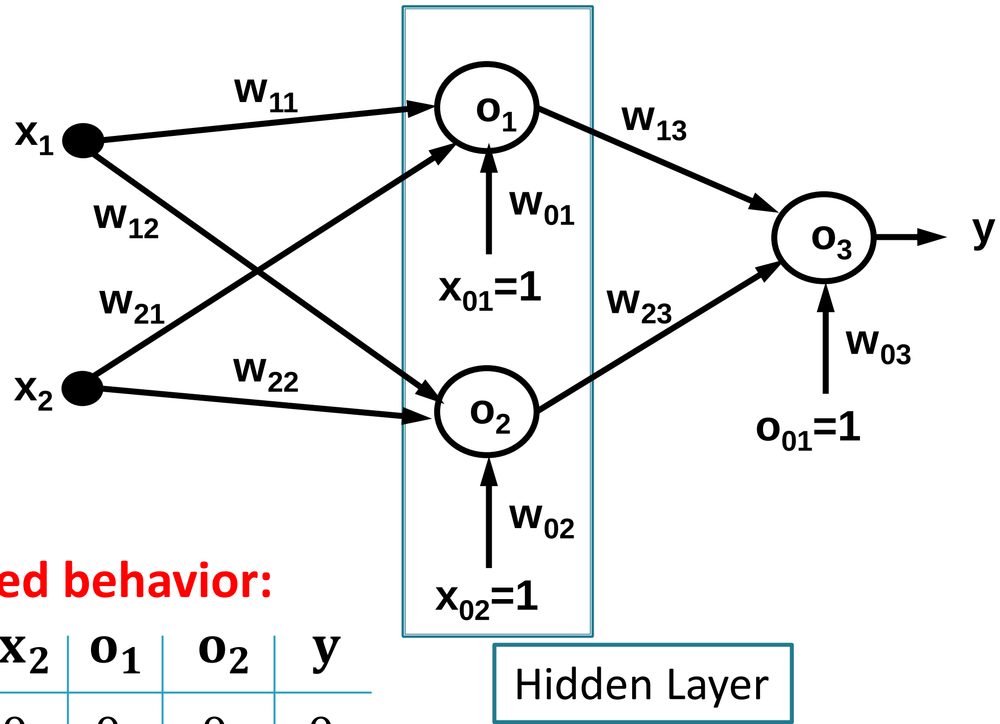
$$w_{11} = w_{12} = 1$$
$$w_{21} = w_{22} = 1$$
$$w_{01} = -1.5$$
$$w_{02} = -0.5$$
$$w_{13} = -1$$
$$w_{23} = 1$$
$$w_{03} = -0.5$$

Desired behavior:

x_1	x_2	o_1	o_2	y
0	0			
1	0			
0	1			
1	1			

Piecewise linear classification using an MLP with threshold (perceptron) units

Multilayer Networks



Network Topology:
2 hidden nodes
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$$w_{13} = -1$$
$$w_{23} = 1$$
$$w_{03} = -0.5$$

Desired behavior:

x_1	x_2	o_1	o_2	y
0	0	0	0	0
1	0	0	1	1
0	1	0	1	1
1	1	1	1	0

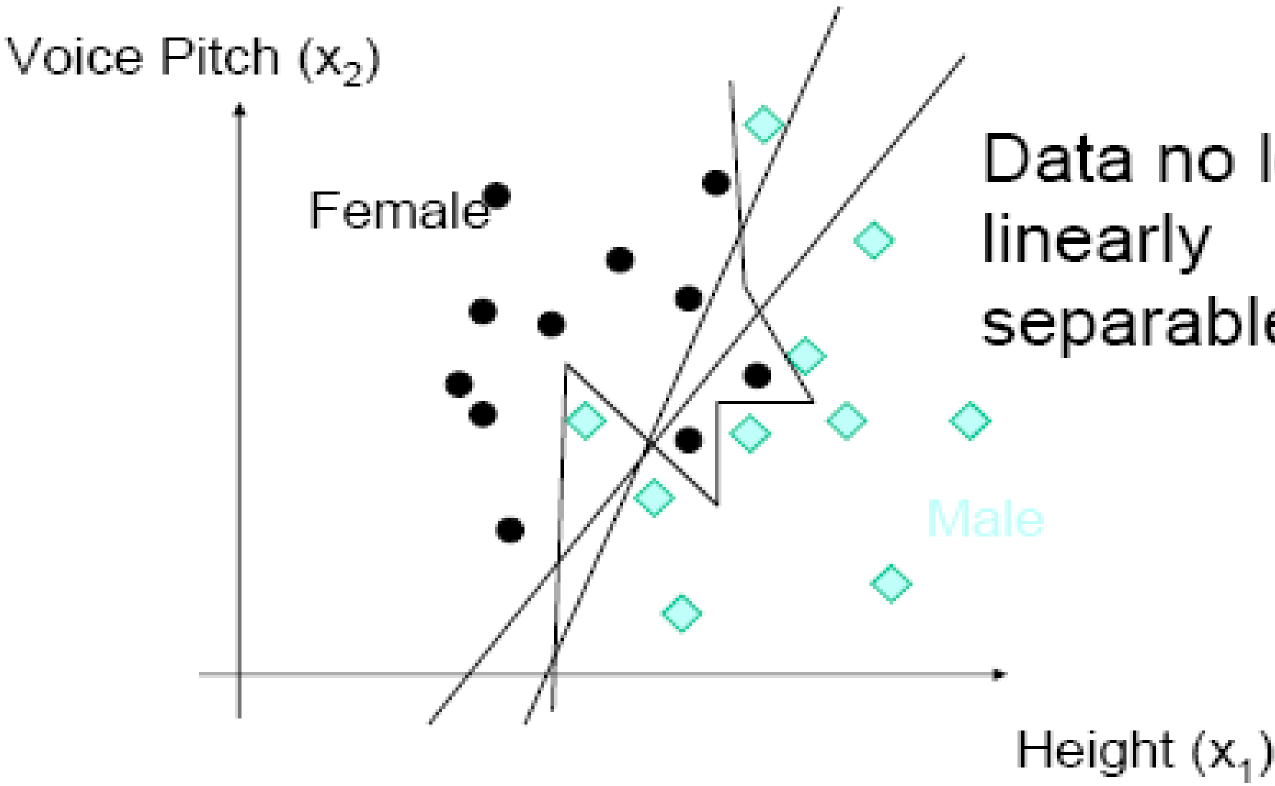
Piecewise linear classification using an MLP with threshold (perceptron) units

Multilayer Networks

- The single perceptron can only express **linear decision surfaces**.

- The kind of **multilayer networks** learned by the **back propagation** algorithm are capable of expressing a rich variety of **nonlinear decision surfaces**.

Multilayer Networks… Example



Data no longer linearly separable

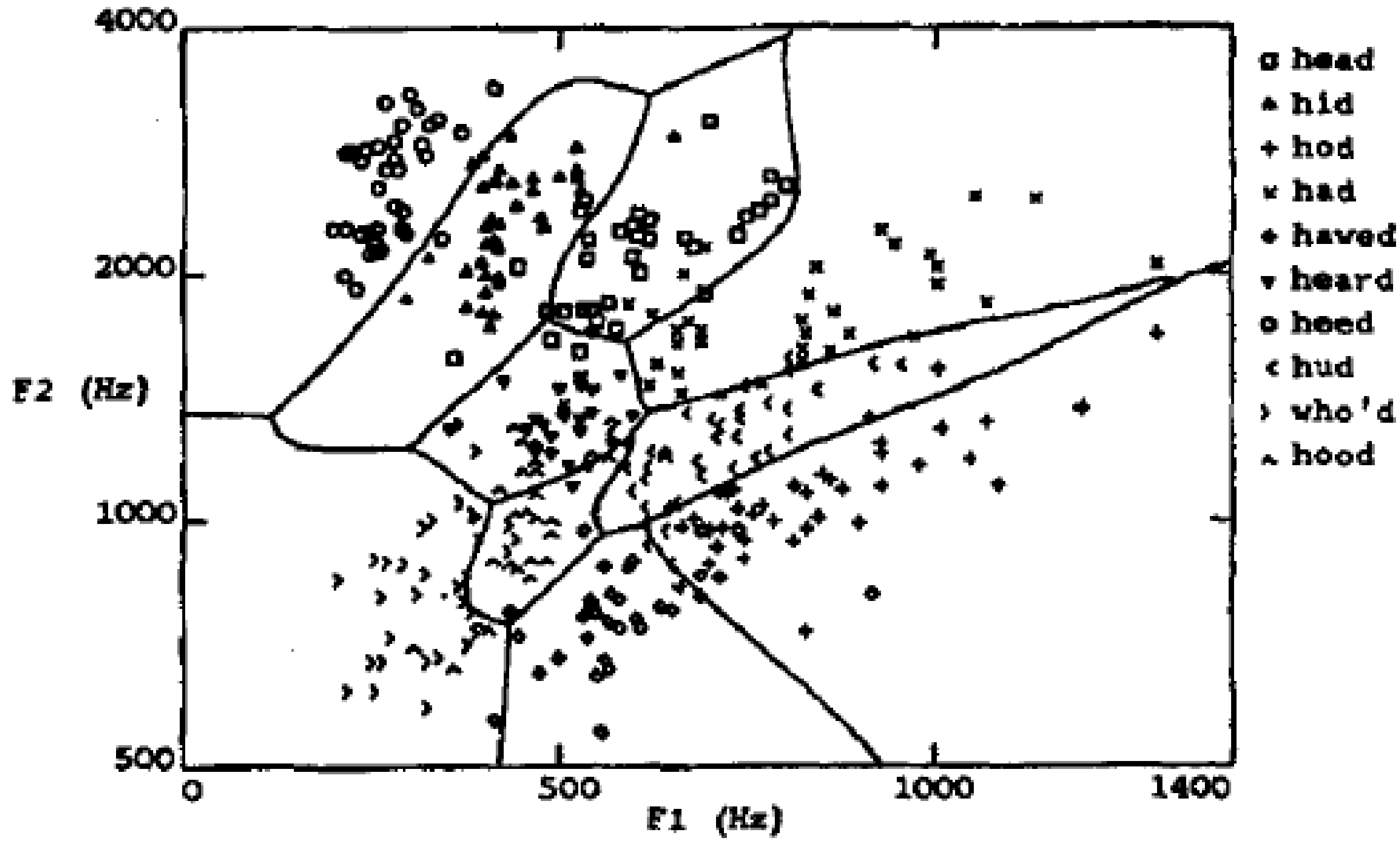
What is a good decision boundary ?

Multilayer Networks... Example

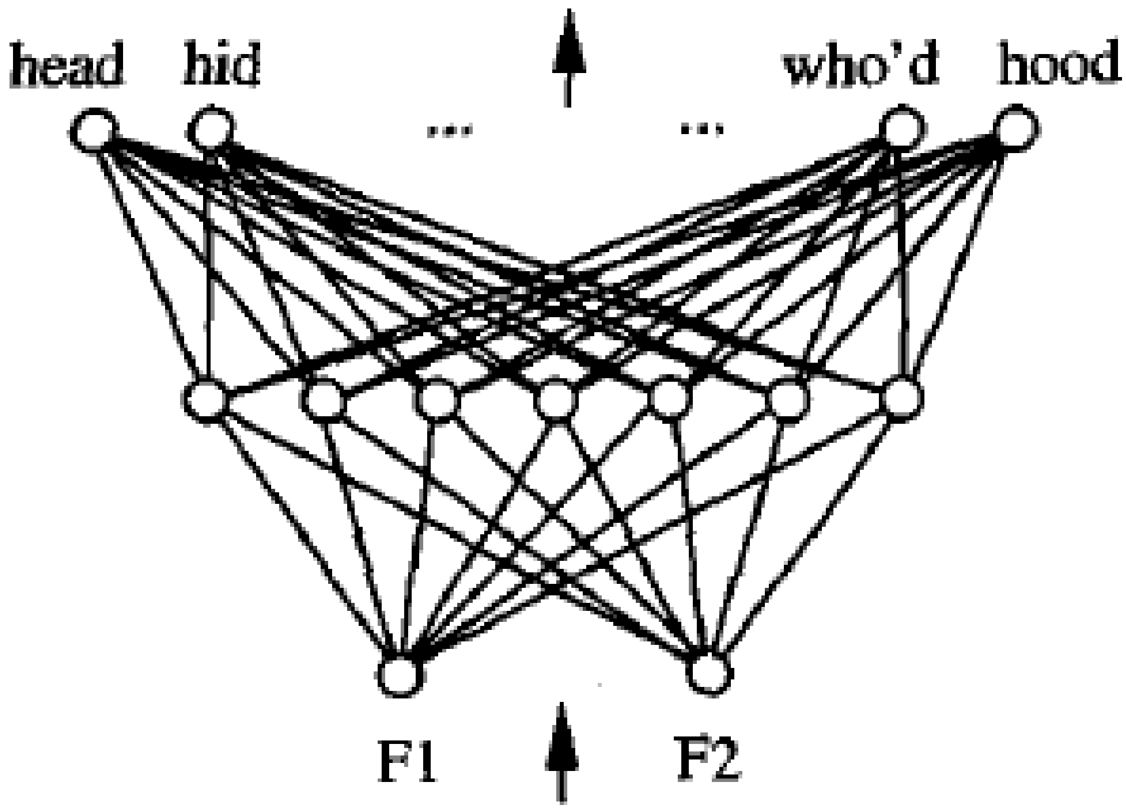
Example:

- The speech recognition task involves distinguishing among 10 possible vowels, all spoken in the context of "h-d" (i.e., "hid," "had," "head," "hood," etc.).

Multilayer Networks… Example



Multilayer Networks… Example



Reading Material

- **Artificial Intelligence,** A Modern Approach

  Stuart J. Russell and Peter Norvig
  - **Chapter 18.**
- **Machine Learning**

  Tom M. Mitchell
  - **Chapter 4.**