Chapter 2 solution

Automata Theory (University of the Punjab)

Step 1 of 2

Given that, Language S = {a b}

So, the language S* with different words with the language S and different lengths will be

$$S^* = \{\Lambda, \text{a, b, aa, ab, ba, bb, aaa, aab, aba, } ....\}$$

Comment

Step 2 of 2

The language S contains n (here n = 2 i.e., a, b) words

If we consider L to be the length of the words from the language S*, we can derive n^L combinations with the words of language S.

Thus to get words of length 2, L becomes 2.

Total words of Length 2 = n^L

= 2^2

= 4 which were nothing but {aa, ab, ba, bb}

Similarly, to get words of length 3, L becomes 3.

Total words of Length 3 = n^L

= 2^3

= 8 which were nothing but {aaa, aab, aba … …}

In general, total words of Length L = n^L

Comments (4)

Chapter 2, Problem 2P

6 Bookmarks

Show all steps: OFF

Problem

Consider the language S^*, where $S = \{aa\ b\}$.

How many words does this language have of length 4? of length 5? of length 6? What can be said in general?

Step-by-step solution

Step 1 of 4

Consider the language S^* over $S = \{aa\ b\}$.

Words of length 4:

To form the words of length 4, add words of length 2 to words '*aa*' and add words of length 3 to '*b*'.

Thus, the words of length 4 are *baab*, *bbaa*, *bbbb*, *aaaa*, and *aabb*.

Hence, there are 5 words of length 4 in the language S^*.

Comments (1)

Chapter 2, Problem 2P

6 Bookmarks

Show all steps: OFF

Step 2 of 4

Words of length 5:

To form the words of length 5, add words of length 3 to words '*aa*' and add words of length 4 to '*b*'.

Thus, the words of length 5 are *aaaab*, *aabaa*, *aabbb*, *bbaab*, *bbbaa*, *bbbbb*, *baaaa*, and *baabb*.

Hence, there are 8 words of length 5 in the language S^*.

Comment

Step 3 of 4

Words of length 6:

To form the words of length 6, add words of length 4 to words '*aa*' and add words of length 5 to '*b*'.

Thus, the words of length 6 are *aabaab*, *aabbaa*, *aabbbb*, *aaaaaa*, *aaaabb*, *baaaab*, *baabaa*, *baabbb*, *bbbaab*, *bbbbaa*, *bbbbbb*, *bbaaaa*, and *bbaabb*.

Hence, there are 13 words of length 6 in the language S^*.

Comment

Chapter 2, Problem 2P

6 Bookmarks

Show all steps: OFF

Comment

Step 4 of 4

In general, words of length *n* can be formed by adding words of length $(n-2)$ to *aa* and words of length $(n-1)$ to *b*. It can be written as a Fibonacci series as, $f(n)=f(n-2)+f(n-1)$ where $f(0)=1$ and $f(1)=1$.

Comment

Was this solution helpful? 4 0

Chapter 2, Problem 3P

3 Bookmarks

Show all steps: OFF

Problem

Consider the language *S**, where *S* = {*ab ba*}. Write out all the words in *S** that have seven or fewer letters. Can any word in this language contain the substrings *aaa* or *bbb* ? What is the smallest word that is *not* in this language?

Step-by-step solution

Step 1 of 3

Consider the language $S=\{ab\ ba\}$.

The words in *S** that have seven or fewer letters

$S^*=\{\Lambda,\ ab,\ ba,\ abab,\ abba,\ baab,\ baba,\ ababab,\ ababba,\ abbaab,\ abbaba,\ baabab,\ baabba,\ babaab,\ bababa\}$

Comment

Step 2 of 3

Chapter 2, Problem 3P

3 Bookmarks

Show all steps: OFF

Step 2 of 3

The language does not contain the substrings *"aaa"* and *"bbb"*, because the language set S is defined over {*ab ba*} does not contain *aa* or *bb*. In the language *S*, any substring can have at most two consecutive *a*'s or *b*'s.

Comment

Step 3 of 3

The smallest word that is not in this language is either '*a*' or '*b*'.

Comment

Was this solution helpful? 28 0

Chapter 2, Problem 4P

1 Bookmark

Show all steps: OFF

Problem

Consider the language *S**, where *S* = {*a ab ba*}. Is the string (*abbba*) a word in this language? Write out all the words in this language with six or fewer letters. What is another way in which to describe the words in this language? Be careful, this is not simply the language of all words without *bbb*.

Step-by-step solution

Step 1 of 4

Language $\mathrm{S} = \{\mathrm{a\ ab\ ba}\}$ then the language

$S^* = \{\Lambda, \mathrm{a, aa, ab, ba, aab, aba, baa, abab, abba, baab, baba} \ldots\}$

Comment

Step 2 of 4

No, the "abbba" is not a word in this language

Chapter 2, Problem 4P

1 Bookmark

Show all steps: OFF

Step 3 of 4

S* = { Λ, a, aa ,ab, ba ,aaa, aab, aba, baa, aaaa, aaab, aaba, abaa ,abab, abba, baaa, baab, baba, aaaaa, aaaab, aaaba, aabaa, aabab, aabba, abaaa, abaab, ababa, abaaa, abaab ,ababa, abbaa, baaaa, baaab, baaba, babaa, aaaaaa, aaaaab, aaaaba, aaabaa, aaabab, aaabba, aabaaa, aabaab, aababa, aabaaa, aabaab, aababa, aabbaa, abaaaa, abaaab, abaaba, ababaa abaaaa, abaaab, abaaba, ababaa, ababab, ababba, abbaaa ,abbaab, abbaba, baaaaa, baaaab, baaaba, baabaa, baabab, baabba, babaaa, babaab, bababa,……..}

Comment

Step 4 of 4

Another way:

Another way to represent the words of this language is

$(a+ab+ba)^{*}$

That means the words that contain either ‘a’ or ‘ab’ or ‘ba’ and the strings of their combination.

Comment

Chapter 2, Problem 5P

2 Bookmarks

Show all steps: OFF

Problem

Consider the language *S**, where *S*= {*aa aba baa*}. Show that the words *aabaa*, *baaabaaa*, and *baaaaababaaaa* are all in this language. Can any word in this language be interpreted as a string of elements from *S* in two different ways? Can any word in this language have an odd total number of *a*'s?

Step-by-step solution

Step 1 of 3

Language $S=\{\text{aa aba baa}\}$ then, the language

S^{*}= { Λ, aa, aba, baa, aaaa, aaaba, aabaa, abaaa, abaaba, ababaa, baaaa, baaaba, baabaa, aaaaaa, aaaaaba, aaaabaa, aaabaaa, aaabaaba, aaababaa, aabaaaa, aabaaaba, aabaabaa, abaaaaa, abaaaaba, abaaabaa, abaabaaa, abaabaaba, abaababaa, ababaaaa, ababaaaba, ababaabaa, baaaaaa, baaaaaba, baaaabaa, baaabaaa, baaabaaba, baaababaa, baabaaaa, baabaaaba, baabaabaa ………….}

Comment

Chapter 2, Problem 5P

2 Bookmarks

Show all steps: OFF

Step 2 of 3

The word aabaa is divided into (aa)(baa) which belongs to Language S, so the word (aabaa) is in this language .

The word baaabaaa is divided into (baa)(aba)(aa) which belongs to Language S,so the word(baaabaaa) is in this language.

The word baaaaababaaaa is divided into (baa)(aa)(aba)(baa)(aa) which belongs to Language S ,so the word (baaaaababaaaa) is in this language.

Comment

Step 3 of 3

No interpretation for string of elements from S in two different ways.

No odd of total number of a's in this language because each word in the given language contains even number of a's. So, by concatenating these words we will not get odd number of a's

Comment

Chapter 2, Problem 6P

5 Bookmarks

Show all steps: OFF

Product of words and factorizations

Factorization is the different possible ways of representing a number with the given set of words.

For example, x^{19} can be factorized into $\{x^1, x^{18}\}, \{x^2, x^{17}\}, \ldots, \{x^9, x^{10}\}$.

Consider the language $S^* = \{\Lambda\ xx\ xxx\ xxxx \ldots\}$

where $S = \{xx\ xxx\}$ is the set of words in the language.

Calculate number of ways representing the product of words for x^{19}.

The lengths of the words in the language are $xx = x^2 = 2$ and $xxx = x^3 = 3$.

This can also be understood as, factors of 19 with multiples of 2 and 3, or

$$(x^2)^a + (x^3)^b = x^{19}$$

Then, the possible sets are {16, 3}, {10, 9} and {4, 15}

1. 8 number of xx and 1 xxx

$$(x^2)^8 + (x^3)^1 = x^{19}$$

$$x^{16} + x^3 = x^{19}$$

The total number of words that can be formed using

8 words of length 2 + 1 word of length 3= (8+1) = 9 ....(1)

2. 5 number of xx and 3 number of xxx

$$(x^2)^5 + (x^3)^3 = x^{19}$$

$$x^{10} + x^9 = x^{19}$$

The total number of words that can be formed using

5 words of length 2 + 3 word of length 3= (5+3) = 8 ....(2)

3. 2 number of xx and 5 number of xxx

$$(x^2)^2 + (x^3)^5 = x^{19}$$

$$x^4 + x^{15} = x^{19}$$

The total number of words that can be formed using

2 words of length 2 + 5 word of length 3= (2+5) = 7 ....(3)

Chapter 2, Problem 6P

5 Bookmarks Show all steps: OFF

Step 2 of 2

Calculate the total number of ways or factorizations, representing x^{19}.

Use the formula of combinations to find the total number of possible different combinations of the given word:

$$C(n,r)=\frac{n!}{r!(n-r)!}$$

This can be interpreted as the factorial of total number of factors possible divided by the number of repetitions in each type.

From the equation (1) calculate for 9,

$$\begin{aligned}C(9,1)&=\frac{9!}{1!(9-1)!}\\&=\frac{9!}{8!}\\&=9\,\text{combinations}\end{aligned}$$

From the equation (2) calculate for 8,

$$\begin{aligned}C(8,3)&=\frac{8!}{3!(8-3)!}\\&=\frac{8\times7\times6\times5\times4\times3\times2\times1}{(3\times2\times1)(5\times4\times3\times2\times1)}\\&=56\,\text{combinations}\end{aligned}$$

From the equation (3) calculate for 7,

$$\begin{aligned}C(7,5)&=\frac{7!}{5!(7-5)!}\\&=\frac{7\times6\times5\times4\times3\times2\times1}{(5\times4\times3\times2\times1)(2\times1)}\\&=21\text{ combinations}\end{aligned}$$

Calculate total combinations from equation solvation,

$$\begin{aligned}&=9+56+21\\&=86\end{aligned}$$

Hence, total number of combinations possible for x^{19} is $\boxed{86}$.

Chapter 2, Problem 7P

12 Bookmarks Show all steps: OFF

Step-by-step solution

Step 1 of 5

Consider the language PALINDROME over the alphabet $\{a\ \ b\}$.

(i)

The string *x* is said to be palindrome if the reverse of the string *x* is equal to *x* i.e., $(x)^R=x$.

Consider the string *x* is PALINDROME.

It is true for *n* = 1 by assumption x^n is in palindrome. Assume that it is true for *n*-1, namely, x^{n-1} is in PALINDROME and $\left(x^{n-1}\right)^R=x^{n-1}$. From the definition of the palindrome, it is very clear that $(xy)^R=y^Rx^R$ Here 'R' means reverse of the string.

$$\begin{aligned}\left(x^n\right)^R&=\left(x^{n-1}x\right)^R\\&=x^R\left(x^{n-1}\right)^R\\&=xx^{n-1}\\&=x^n\end{aligned}$$

Therefore, x^n is in PALINDROME.

Comment

Therefore, x^n is in PALINDROME.

Comment

Step 2 of 5

(ii)

Consider y^3 is in PALINDROME. From the definition of the palindrome, $y^3 = \text{reverse}\left(y^3\right)$.

$$y^3 = \text{reverse}\left(y^3\right)$$
$$y^3 = \left(\text{reverse}(y)\right)^3$$
$$y.y.y = \text{reverse}(y).\text{reverse}(y).\text{reverse}(y)$$

Here, *y* and reverse(*y*) have the same length. From the above equation, $y = reverse\left(y\right)$. Thus, *y* is in PALINDROME.

Therefore, *y* is in PALINDROME.

Comment

Step 3 of 5

Chapter 2, Problem 7P
12 Bookmarks
Show all steps: OFF

$y.y.y = \text{reverse}(y).\text{reverse}(y).\text{reverse}(y)$

Here, *y* and reverse(*y*) have the same length. From the above equation, $y = reverse\left(y\right)$. Thus, *y* is in PALINDROME.

Therefore, *y* is in PALINDROME.

Comment

Step 3 of 5

(iii)

Consider z^n is in PALINDROME. From the definition of the palindrome, $z^n = \text{reverse}\left(z^n\right)$.

$$z^n = \text{reverse}\left(z^n\right)$$
$$z^n = \left(\text{reverse}(z)\right)^n$$
$$z.z.z...z = \text{reverse}(z).\text{reverse}(z).\text{reverse}(z)...\text{reverse}(z)$$

Here, *z* and reverse(*z*) have the same length. From the above equation, $z = reverse\left(z\right)$. Thus, *z* is in PALINDROME.

Therefore, *z* is in PALINDROME.

Comment

Comment

Step 4 of 5

(iv)

The three-letter PALINDROME words that can be formed over the alphabet $\{a \quad b\}$ are of the form *ABA*. Here, the *A* can be replaced by *a*, *b* and *B* can be replaced by *a*, *b*. Thus, the total number of possible words of length 3 is 4. The words are *aaa*, *aba*, *bab*, *bbb*.

The four-letter PALINDROME words that can be formed over the alphabet $\{a \quad b\}$ are of the form *ABBA*. Here, the *A* can be replaced by *a*, *b* and *B* can be replaced by *a*, *b*. Thus, the total number of possible words of length 4 is 4. The words are *aaaa*, *abba*, *baab*, *bbbb*.

Therefore, it is proven that PALINDROME has as many words of length 4 as it does of length 3.

Comment

Step 5 of 5

Chapter 2, Problem 7P

12 Bookmarks Show all steps: OFF

Therefore, it is proven that PALINDROME has as many words of length 4 as it does of length 3.

Comment

Step 5 of 5

(v)

The words in PALINDROME of length 2*n* when *n*=1 are $\{aa,\ bb\}$.

The words in PALINDROME of length 2*n* when *n*=2 are $\{aaaa,\ abba,\ baab,\ bbbb\}$.

The words in PALINDROME of length 2*n*-1 when *n*=1 are $\{a,\ b\}$.

The words in PALINDROME of length 2*n*-1 when *n*=2 are $\{aaa,\ aba,\ bab,\ bbb\}$.

Therefore, it is proven that PALINDROME has as many words of length 2*n* as it has of length 2*n*-1.

Comment

Chapter 2, Problem 8P

4 Bookmarks

Show all steps: OFF

Step 1 of 2

1. If the lengths of x and y are equal, then there exists a word z that equals either x or y. If x, y $\in$ PALINDROME, and xy $\in$ PALINDROME, then if length(x) = length(y), x=y=z.

That is $x_1 = x_k = y_k,\ x_2 = x_{(k-1)} = y_{(k-1)},$ etc....

2. If the length of y is equal to n * length(x), then there exists a word z that equals y, and each string of length(x) / n would equal z.

If length(x) ≠ length(y).

3. If the length of x is less than y, the length of x or y is not 0, and the leftmost portion of length x from string y meets the criteria of 1, then make y equal to the unused rightmost portion of string y and repeat steps 1 and 2 (m and n would equal 1).

Let x be the shorter word:

If xy is in PALINDROME, then it follows that:

$x_1 = y^m$ (the first and the last letters of xy

$$x_2 = y^{(m-1)}$$
$$x^k = y^{(m-k)}\left(m > k \textit{ because } y \textit{ is longer than } x\right).$$

Because y is also in PALINDROME, it follows that:

$$y_1 = y^m$$
$$y_2 = y^{(m-1)}$$
$$y^k = y^{(m-k+1)} etc.$$

Because xy is also in PALINDROME, it follows that:

$$y_1 = y^{(m-k)}$$
$$y_2 = y^{(m-k-1)}$$
$$y^k = y^{(m-2k+1)}$$

Chapter 2, Problem 8P

4 Bookmarks

Show all steps: OFF

$$y_1 = y$$
$$y_2 = y^{(m-k-1)}$$
$$y^k = y^{(m-2k+1)}$$

Comment

Step 2 of 2

• Here, it can be observed that each time k elements of y_1 are equated until the middle element is reached.

• Then using palindromes of length k (the shorter word at a time), it follows that the longer word must be some power of x. (x might in turn also be a power of a shorter word).

• Only words produced by 1 and 2 are allowed in the language, such that if x()=reverse of some variable, then x= z^n and y = z^m.

Comment

Was this solution helpful? 1 0

(i)

Consider the following data:

$S = \{ab, bb\}$
$T = \{ab, bb, bbbb\}$

The Kleene closure of S is as follows:

$S^* = \{\wedge, ab, bb, abbb, bbab, bbbb, abab, ...\}$

The Kleene closure of T is as follows:

$T^* = \{\wedge, ab, bb, bbbb, abbb, bbbbbb, bbbbab, abab, bbbbbbbb......\}$

The language generated by S* and T* are the same, so S* = T* as $S^* \subseteq T^*$ **and** $T^* \subseteq S^*$**.**

(ii)

Consider the following data:

$S = \{ab, bb\}$
$T = \{ab, bb, bbb\}$

The Kleene closure of S is as follows:

$S^* = \{\wedge, ab, bb, abbb, bbab, bbbb, abab, ...\}$

The Kleene closure of T is as follows:

$T^* = \{\wedge, ab, bb, bbbb, abbb, bbbbbb, bbbbab, abab, bbbbbbbb......\}$

The set S is the subset of T, $S \subset T$ **, so every string that belongs to S* also belongs to T* but the string bbb belongs to T* and it does not belong to S*.**

So, $S^* \neq T^*$ **and** $S^* \subset T^*$ **but** $T^* \not\subset S^*$ **.**

Chapter 2, Problem 9P 7 Bookmarks Show all steps: OFF

Comment

Step 2 of 2

(iii)

The principle illustrated as strings in one language are same as another language and strings made-up of concatenation of strings in language are same as another language than both languages are same closure.

Example 1:

The part (i) is example of this principle, because the both languages of S^* and T^* are same.

Example 2:

The part (ii) is 'unfortunate situation', because word or words are present in one language (T) but not in the other (S).

Comment

Was this solution helpful? 2 0

Chapter 2, Problem 10P

Bookmark

Show all steps: OFF

If we replace the operator * with the operator + in problem 9 in this chapter, then

I) Both are Symmetry($S^+ = T^+$) by the following proof:

S={ab bb} then S^+ ={ab, bb, abab, abbb, bbab, bbbb, bbbbab.........}

T={ab bb bbbb} then T^+={ab, bb, abab, abbb, bbab, bbbb, bbbbab........}

Since, $S^+ \subseteq T^+ \,\&\, T^+ \subseteq S^+$

$\therefore S^+ = T^+$

Here S^+ without $\wedge$,so this is does not allow the concatenation of no words of S.

Comment

Step 2 of 3

II) S = {ab bb} then S^+ = { ab, bb, abab, bbbb,........}

T = {ab bb bbb} then T^+ = {ab, bb, bbb, abbb, bbbab,..........}

Here $S^+ \subseteq T^+$ but $T^+ \not\subset S^+$ because S^+ does not contain the word with 'bbb'

Thus, $S^+ \neq T^+$

Comment

Chapter 2, Problem 10P

Bookmark

Show all steps: OFF

Step 2 of 3

II) S = {ab bb} then S^+ = { ab, bb, abab, bbbb,........}

T = {ab bb bbb} then T^+ = {ab, bb, bbb, abbb, bbbab,..........}

Here $S^+ \subseteq T^+$ but $T^+ \not\subset S^+$ because S^+ does not contain the word with 'bbb'

Thus, $S^+ \neq T^+$

Comment

Step 3 of 3

III) This is called unfortunate situation from part II.

Comment

Was this solution helpful? 0 0

Chapter 2, Problem 11P

4 Bookmarks Show all steps: OFF

Problem

Prove that for all sets *S*,

(i) (*S*+)* = (*S**)*

(ii) (*S*+)+ = *S*+

(iii) Is (*S**)+ = (*S*+)* for all sets *S*?

Step-by-step solution

Step 1 of 3

Proof:

i) (S^+)* = (S*)*

Every factor from S+ is made up of factors from S, excluding if it's not in the set. Every factor from (S^+)* is made up of factors from S^+, including .

Every factor from S* is made up of factors from S, including . Every factor from (S*)* is made up of factors from S*, including .

Therefore, every word in (S^+)* and (S*)* is made up of factors from S. Therefore, every word in (S^+)* is also a word in (S*)*.

Comment

Chapter 2, Problem 11P

4 Bookmarks Show all steps: OFF

ii) (S+)+ = S^+

Every factor from S^+ is made up of factors from S, excluding if it's not in the set. Every factor from (S+)+ is made up of factors from S^+, excluding again.

Therefore, every word in (S+)+ and S^+ is made up of factors from S excluding . Therefore, every word in (S+)+ is also a word in S^+.

Comment

Step 3 of 3

iii) Is $(S^*)^+$ = (S^+)* for all sets S?

Every factor from S* is made up of factors from S, including . Every factor from $(S^*)^+$ is made up of factors from S^+, excluding if it doesn't exist. However, since S* will always contain , $(S^*)^+$ will also always contain .

Every factor from S^+ is made up of factors from S, excluding if it's in the set. Every factor from (S^+)* is made up of factors from S^+, including even if it doesn't exist in S^+.

Therefore, every word in $(S^*)^+$ is made up of factors of S including , and every word in (S^+)* is made up of factors from S including . Therefore, every word in (S^+)* is also a word in $(S^*)^+$

Chapter 2, Problem 12P

2 Bookmarks Show all steps: OFF

Problem

Let *S* = {*a bb bab abaab*}. Is abbabaabab in *S**?. Is abaabbabbaabb? Does any word in *S** have an odd total number of *b*'s?

Step-by-step solution

Step 1 of 4

Consider $S=\{a\ \ bb\ \ bab\ \ abaab\}$. The Kleene star of *S* i.e., $S*$ contain the infinite set of all possible strings that are made of the set of strings of *S*. The $S*$ contains the empty string (ε) as well.

Comment

Step 2 of 4

Consider the string $abbabaabab$. The string can be divided into groups as $(a)(bb)(abaab)ab$. Except the substring *ab*, remaining all substrings are the part of *S*.

Therefore, $\boldsymbol{abbabaabab}$ does not belongs to $\boldsymbol{S*}$.

Comment

Chapter 2, Problem 12P

2 Bookmarks Show all steps: OFF

Comment

Step 3 of 4

Consider the string $abaabbabbaabb$. The string can be divided into groups as $(abaab)(bab)baa(bb)$. Except the substring *baa*, remaining all substrings are the part of *S*.

Therefore, $\boldsymbol{abaabbabbaabb}$ does not belongs to $\boldsymbol{S*}$.

Comment

Step 4 of 4

Each string in *S* contains the even number of *b*'s. The strings that can be formed with the strings of *S* contain the even number of *b*'s.

Therefore, there is no possibility to have odd number of *b* 's in the strings (words) that belongs to $\boldsymbol{S*}$.

Comment

Was this solution helpful? 3 0

Chapter 2, Problem 13P

1 Bookmark

Show all steps: OFF

Problem

Suppose that for some language *L* we can always concatenate two words in *L* and get another word in *L* if and only if the words are *not* the same. That is, for any words *w*1 and *w*2 in *L* where *w*1 ≠ *w*2, the word *w*1,*w*2 is in *L* but the word *w*1,*w*2, is not in *L* Prove that this cannot happen.

Step-by-step solution

Step 1 of 1

Given theorem is contradiction, because

For example consider, Language S = {w1 w2}

Then S* = { ^, w1, w2, w1w1, w1w2, w2w2, w1w1w1,……..}

Thus, the language contains both the strings w1w2 (for w1 $\neq$ w2) and w1w1. That is the language set is closed under concatenation.

And hence the given statement is false.

Comment

Was this solution helpful? 20 2

Chapter 2, Problem 14P

Bookmark

Show all steps: OFF

Problem

Let us define

$(S^{**})^* = S^{***}$

Is this set bigger than S^*? Is it bigger than S?

Step-by-step solution

Step 1 of 1

$$(S^{**})^* = S^{***}$$

$$(S^*)^* = S^{***} \quad (\because S^{**} = S^*)$$

$$S^* = S^{***}$$

∴ This set is equal to S*, and is bigger than S.

Comment

Was this solution helpful? 2 0

Let w be a string of letters and let the language T be defined as adding w to the language S. Suppose further that $T^* = S^*$.

(i) Is it necessarily true that $w \in S$?

(ii) Is it necessarily true that $w \in S^*$?

Step-by-step solution

Step 1 of 2

(I) We know that S and T are languages such that

T=S + w and T*=S*

e.g.

Let S = {a aa bb}

Let w = aaa

Then, T = {aa bb aaa}

Still, S* = T*, as S belongs to T*, so S* also belongs to T* and are equal.

But it does not mean, that w belongs to S, as stated in this part.

Chapter 2, Problem 15P

3 Bookmarks

Show all steps: OFF

Still, S* = T*, as S belongs to T*, so S* also belongs to T* and are equal.

But it does not mean, that w belongs to S, as stated in this part.

Comment

Step 2 of 2

(II) **True**, because S* = T*, as w belongs to T* hence it also should belong to S*

Comment

Was this solution helpful? 4 0

Chapter 2, Problem 16P

1 Bookmark

Show all steps: OFF

Problem

Give an example of a set *S* such that the language has more six-letter words than seven-letter words. Give an example of an *S** that has more six-letter words than eight-etter words. Does there exist an *S** such that it has more six-letter words than twelve-letter words?

Step-by-step solution

Step 1 of 1

For a set e.g. S = {aabbaa} we get a closure of:

S*={aabbaa aabbaaaabbaa aabbaaaabbaaaabbaa ...}

Chapter 2, Problem 16P

1 Bookmark

Show all steps: OFF

We see we hava one 6-letter long word and none 7 or 8 long letters.

Other example for 7 letters might be S = {aa bb}, as we can create many 6-letter words and none 7-letter.

Other example of 8 ltters might be S = {abb bbb} - many six letter words, none 8-letter words

It is not possible to have more six-letter words than 12-letter words, as 6 is a divisor of 12. In other words, we can concatenate 3-letter words to get 12-letter one, we can concatenate two 6-letter words, 4x3-letter words etc.

Chapter 2, Problem 17P

3 Bookmarks

Show all steps: OFF

Problem

Consider language S^*, where $S = \{aa\ ab\ ba\ bb\}$. Give another description of this language.

(i) Give an example of a set S such that S^* only contains all possible strings of a's and b s that have length divisible by 3.

(ii) Let S be all strings of a's and b's with odd length. What is S^*?

Step-by-step solution

Step 1 of 3

I) S={aa ab ba bb}

Language representation for S* is $S^*=\{\Lambda\ aa\ ab\ ba\ bb\ aaaa\ aaab\ aaba\ bbaa\ abbb..\}$

S*={λ Plus all the words composed of aa ab ba bb where length of all words is even}

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Step 2 of 3

II) S*={aba abb baa bba aab aba aabbbb aababa............}

The S * only contains all possible strings of a's and b's this have length divisible by 3.

$S=\{aaa\ aba\ baa\ bbb\ abb\ \ bba\ aab\ bab\}$

Comment

Step 3 of 3

III) S={a b}

S = {a b aaa aab aba abb baa bab bba bbb aaaaa aaaab ...}

S* = {a b aa ab ba bb aaa ...} = all strings of a's and b's

Note: $S \subset S^*$, S contains only odd length strings. S* contains all possible even length and odd length strings.

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(i) If $S = \{a\ b\}$ and $T^* = S^*$, prove that T must contain S.

(ii) Find another pair of sets S and T such that if $T^* = S^*$, then $S \subset T$.

Step-by-step solution

Step 1 of 1

i) For a given language S = {a b} we might create its closure:

S*={^ a b aa ab ba bb aaa aab aba ...}

We clearly see that S* contains all the possible strings cocatenated of a's and b's. If T*=S*, then T must also contain a and b (with perhaps sth additional). This is the only way, when we create all possible strings using a and b - to have them in language.

e.g. T={a b aa} or T={a b aabb} --> T contains S

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ii) Another example of these kind of sets:

S={a} T = {a aa aaaa}

S={aa bb} T = {aa bb aabbaa}

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Problem

One student suggested the following algorithm to test a string of *a*'s and *b*'s to see if it is a word in S^*, where S = {*aa ba aba abaab*}. Step 1, cross off the longest set of characters from the front of the string that is a word in *S*. Step 2, repeat step 1 until it is no longer possible. If what remains is the string Λ, the original string was a word in S^*.If what remains is not Λ (this means some letters are left, but we cannot find a word in S at the beginning), the original string was not a word in S^*. Find a string that disproves this algorithm.

Step-by-step solution

Step 1 of 1

For set S = { aa ba aba abaab} we have to find a word, which does belong to S* but does not follow the crossing algorithm (cross the longest factor in S from left repeatedly, until left with null string or non-empty string, which means the word either is in S* or not).

As an example let's take word abaaba, which is a concatenation of aba and aba. If we cross out the longest factor of S from left side, meaning abaab, we are left with a, which is not a factor of S and cannot be cross out longer. It would mean, that abaaba does not belong to S*, which is fault statement.

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Problem

A language *L*1 is smaller than another language *L*2 if $L1 \subset L2$ and $L1 \neq L2$. Let *T* be any language closed under concatenation; that is, if $t1 \in T$ and $t2 \in T$, then *t*1*t*2 is also an ele-ment of *T*. Show that if *T* contains *S* but $T \neq S^*$, then S^* is smaller than *T*. We can summarize this by saying that S^* is the smallest closed language containing *S*.

Step-by-step solution

Step 1 of 1

We have to show that for a given language T, of property:

if t1 belongs to T, and t2 belongs to T, then t1t2 belongs to T,

we get:

if $S \subset T$, and $T \neq S^*$, Then S^* is smaller than T.

Firstly, we see that for any given words belonging to T, the concatenation of those two also belongs to T. Next, the concatenation of the created word with another in the set also belongs to T.

As we see, **S belongs to T,** therefore, all the word in S are part of set T. Using the given property, we can concatenate any of the words belonging to S with each other and the word stil belongs to T. If we state, that the concatenation goes forever, then we would eventually get S^*.

We now see, that $S^* \subset T$. We also know that $T \neq S^*$. It means then, that S^* is a proper subset of T and therefore is smaller than T.

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