Chapter 3 solution

Automata Theory (University of the Punjab)

Chapter 3, Problem 1P

1 Bookmark

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Problem

Write another recursive definition for the language *L*1 of Chapter 2.

Step-by-step solution

Step 1 of 1

Recursive definition:

$L_1 = \{x, xx, xxx, xxxx, xxxxx, \ldots\ldots\ldots\}$

The language Rule 1 : x is in L_1

Rule 2 : If x is in L_1, then so x^{n+1} for n = 0,1,2,34.....

So, the alternative form of L_1 is,

$L_1 = \{ x^{n+1}$ for $n = 0,1,2,3,\ldots\ldots\}$

Comments (1)

Chapter 3, Problem 2P

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Problem

Using the second recursive definition of the set EVEN, how many different ways can we prove that 14 is in EVEN?

Step-by-step solution

Step 1 of 2

According to the second recursive definition for the set EVEN, if a and b are even then $a+b$ is also even.

Rule 1: 2 is an even number.

Rule 2: If a is an EVEN number and b is an EVEN number, then $a+b$ is also an EVEN number.

Comment

Step 2 of 2

Chapter 3, Problem 2P

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Step 2 of 2

Based on the above definition, there are different ways to prove that 14 is EVEN. Some of them are as follows:

I)

Rule 1: 2 is EVEN

Rule 2: x=2, y=2, the sum 2+2 = 4 is EVEN

Rule 2: x=2, y=4, the sum 2+4 = 6 is EVEN

Rule 2: x=4, y=4, the sum 4+4 = 8 is EVEN

Rule 2: x=6, y=8, the sum 6+8 = 14 is EVEN

II)

Rule 1: 2 is EVEN

Rule 2: x=2, y=2, the sum 2+2 = 4 is EVEN

Rule 2: x=4, y=4, the sum 4+4 = 8 is EVEN

Rule 2: x=8, y=4, the sum 8+4 = 12 is EVEN

Rule 2: x=12, y=2, the sum 12+2 = 14 is EVEN

Chapter 3, Problem 2P

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III)

Rule 1: 2 is EVEN

Rule 2: x=2, y=2, the sum 2+2 = 4 is EVEN

Rule 2: x=4, y=4, the sum 4+4 = 8 is EVEN

Rule 2: x=2, y=8, the sum 2+8 = 10 is EVEN

Rule 2: x=10, y=4, the sum 10+4 = 14 is EVEN

IV)

Rule 1: 2 is EVEN

Rule 2: x=2, y=2, the sum 2+2 = 4 is EVEN

Rule 2: x=2, y=4, the sum 2+4 = 6 is EVEN

Rule 2: x=6, y=6, the sum 6+6 = 12 is EVEN

Rule 2: x=12, y=2, the sum 12+2 = 14 is EVEN

Comments (1)

Chapter 3, Problem 3P

1 Bookmark

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Using the second recursive definition of EVEN, what is the smallest number of steps required to prove that 100 is EVEN? Describe a good method for showing that 2*n* is in EVEN.

Step-by-step solution

Step 1 of 3

The second recursive definition of EVEN is:

Rule 1 2 is in EVEN.

Rule 2 If x and y are both in EVEN then so is

x+y (even if x and y have the same number)

Hence, for the number 100,

By Rule 1 2 is in the set EVEN

By Rule 2 x=2, y=2 $\rightarrow$4 is in the set EVEN

By Rule 2 x=4, y=2 $\rightarrow$6 is in the set EVEN

By Rule 2 x=6, y=4 $\rightarrow$10 is in the set EVEN

By Rule 2 x=10, y=10 $\rightarrow$20 is in the set EVEN

By Rule 2 x=20, y=20 $\rightarrow$ 40 is in the set EVEN

By Rule 2 x=40, y=40 $\rightarrow$80 is in the set EVEN

By Rule 2 x=80, y=20 $\rightarrow$100 is in the set EVEN

Chapter 3, Problem 3P

1 Bookmark

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Step 2 of 3

$\therefore$ 100 is in set EVEN The total number of steps required to prove that 100 is in the set EVEN is $\boxed{8}$

Comment

Step 3 of 3

To show 2n is even,

For n=1, 2n = 2 which is EVEN

For n=2, 2n = 4, By Rule 2: x=2, y=2, 2+2 = 4 is in EVEN

For n=3, 2n = 6, By Rule 2: x=2, y=4, 2+4 = 6 is in EVEN

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If 'n' is even

x=2, y=n $\rightarrow$2n is EVEN

If 'n' is odd

x=2, y=n+1 $\rightarrow$ 2n is EVEN

Comment

Show that the following is another recursive definition of the set EVEN:

Rule 1 2 and 4 are in EVEN.

Rule 2 If *x* is in EVEN, then so is *x* + 4.

Step-by-step solution

Step 1 of 2

Recursive definition of set EVEN

Recursive definition of the set EVEN is,

Rule 1: 2 and 4 are in EVEN

Rule 2: If *x* is in EVEN, then so $x+4$

Define EVEN set for all positive integers divisible by 2.

EVEN defined by three rules

Rule 1: 2 and 4 are in EVEN,

Rule 2: If *x* is in EVEN, then so $x+4$

Rule 3: The elements in the set EVEN can produced from above two rules

Suppose proving of 18 is in the set EVEN

By Rule 1: **2** and **4** are in EVEN

By Rule 2: 2 is in EVEN, then **2+4 = 6** is in EVEN

By Rule 2: 6 is in EVEN, then **6+4 = 10** is in EVEN

By Rule 2: 10 is in EVEN, then **10+4 = 14** is in EVEN

By Rule 2: 14 is in EVEN, then **14+4 = 18** is in EVEN

By Rule 2: 10 is in EVEN, then **10+4 = 14** is in EVEN

By Rule 2: 14 is in EVEN, then **14+4 = 18** is in EVEN

Comment

Step 2 of 2

We can make another definition of the set EVEN

Rule 1: 2 and 4 are in EVEN,

Rule 2: *x* and *y* are in EVEN, then so $x+y$ or $x+y+4$ are in EVEN

Rule 3: The elements in the set EVEN can produced from above two rules

Suppose proving the same 18 is in EVEN by causing new implemented rules

By Rule 1: **2** and **4** are in EVEN

By Rule 2: 2 and 4 are in EVEN, then **2+4 = 6** or **2+4+4 = 10** is in EVEN

By Rule 2: 6 and 4 are in EVEN, then **6+4 = 10** or 6**+4+4 = 14** is in EVEN

By Rule 2: 10 and 4 are in EVEN, then **10+4 = 14 or 10+4+4 =18** is in EVEN

By Rule 2: 14 and 4 are in EVEN, then **14+4 = 18 or 14+4+4=22** is in EVEN

Hence it is another recursive definition of the set EVEN

Comment

Was this solution helpful? 0 0

Chapter 3, Problem 5P

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Problem

Show that there are infinitely many different recursive definitions for the set EVEN.

Step-by-step solution

Step 1 of 7

Recursive definitions for any set can be defined by the following three steps,

1. Specify the base case as basic words in the set.

2. Construct new cases or words by applying recursive rules.

3. No other words/objects constructed by rule 1 and Rule 2 will be in this language.

Comment

Step 2 of 7

Recursive definitions for EVEN set are:

Case 1 : Base case is in EVEN set is that 2 is the even number.

Case 2 : EVEN = { All positive whole numbers divisible by 2}

Chapter 3, Problem 5P

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Step 3 of 7

The set EVEN is defined by these rules,

Rule 1 : Base case : 2 is EVEN.

Rule 2 : Construct new words : If x is in EVEN, then so a + 2

Rule 3 : The set EVEN will contain only the words constructed by the rule 1 and rule 2.

Comment

Step 4 of 7

There would be many different recursive approaches for the set EVEN.

Example 1:

To prove this, take an example 10 is in EVEN

Rule 1: 2 is EVEN

Rule 2: a = 2, 2 + 2 = 4 EVEN

Rule 2: a = 4, 4 + 2 = 6 EVEN

Rule 2: a = 6, 6 + 2 = 8 EVEN

Rule 2: a = 8, 8 + 2 = 10 EVEN

Recursively Rule 2 is followed.

Comment

Recursively Rule 2 is followed.

Comment

Step 5 of 7

Example 2:

Another recursive approach, to show that 10 is in EVEN.

Rule 1 : 2 is EVEN

Rule 2 : If a is any whole number multiply it with 2. Then a * 2 = EVEN.

Eg : 10 is in EVEN

Rule 1: 2 is EVEN

Consider, x = 5, 5 * 2 = 10 EVEN (Direct Method)

Rule 2: a = 2, 2 + 2 = 2 EVEN

Rule 2: a = 2, 2 + 2 = 4 EVEN

Rule 2: a = 4, 4 + 2 = 6 EVEN

Rule 2: a = 6, 6 + 2 = 8 EVEN

Rule 2: a = 8, 8 + 2 = 10 EVEN

Use Recursive definition approach, 5 times recursively call the Rule 2.

Comment

Chapter 3, Problem 5P

Step 6 of 7

Example 3:

Another recursive approach, to show that 10 is in EVEN.

Rule 1: 2 is EVEN

Rule 2: If a is any whole number and b is also a whole number and both are EVEN,

then a + b = EVEN

Rule 2: a = 2, b = 2, 2 + 2 = 4 EVEN

Rule 2: a = 4, b = 2, 4 + 2 = 6 EVEN

Rule 2: a = 6, b = 4,6 + 4 = 10 EVEN

Comment

Step 7 of 7

From the example1, example2 and example 3, it is proved that there are many approaches to prove EVEN using recursive approach.

Thus, there are many different recursive definitions for the set EVEN

Comment

Problem

Using any recursive definition of the set EVEN, show that all the numbers in it end in the digits 0, 2, 4, 6, or 8.

Step-by-step solution

Step 1 of 3

1) Basic step:

$0 \in$EVEN by definition, therefore the property is true of the zero' the step since $0 \in \{0,2,4,6,8\}$

Comment

Step 2 of 3

2) Induction hypothesis:

Assume that the last digit of $(m+2) \in \{0,2,4,6,8\}$ for $0 < m < n$

Comment

Chapter 3, Problem 6P

Step 3 of 3

3) Induction step:

n	∈ EVEN
0	2
1	4
2	6
3	8
4	10

n	∈ EVEN
n	2n+2
n+1	(2n+2)+2
n+1	2(n+1)+2

∴ 0+2=2, 2+2=4, 4+2=6, 6+2=8, 8+2=0 ∈ {0, 2, 4, 6, 8}

Chapter 3, Problem 7P

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Problem

The set POLYNOMIAL defined in this chapter contains only the polynomials in the one variable *x*. Write a recursive definition for the set of all polynomials in the two variables *x* and *y*.

Step-by-step solution

Step 1 of 1

The recursive definition for set of polynomials for two variables x and y is defined as,

Rule 1 : Any number is in POLINOMIAL

Rule 2 : Variable ‘x’ is in POLINOMIAL

Rule 3 : Variable ‘y’ is in POLINOMIAL

Rule 4 : If p and q are in POLINOMIALS, then so are p+q, p-q, p and pq

Now consider an example to illustrate this definition,

$5x^2+9y^2+7y+3x-9$

Rule 1 : 5 is in POLINOMIAL

Rule 2 : x is in POLINOMIAL

Chapter 3, Problem 7P

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$5x^2+9y^2+7y+3x-9$

Rule 1 : 5 is in POLINOMIAL

Rule 2 : x is in POLINOMIAL

Rule 4 : (5x)(x) is in POLINOMIAL ; call it $5x^2$

Rule 1 : 9 is in POLINOMIAL

Rule 3 : y is in POLINOMIAL

Rule 4 : (9y)(y) is in POLINOMIAL ; call it $9y^2$

Rule 1 : 3 is in POLINOMIAL

Rule 4 : (3)(x) is in POLINOMIAL

Rule 1 : 7 is in POLINOMIAL

Rule 4 : (7)(y) is in POLINOMIAL

Rule 1 : -9 is in POLINOMIAL

Rule 4 : $5x^2+9y^2+7y+3x-9$ is in POLINOMIAL

Comment

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Chapter 3, Problem 8P

3 Bookmarks

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Define the set of valid algebraic expressions ALEX as follows:

Rule 1 All polynomials are in ALEX.

Rule 2 If $f(x)$and $g(x)$ are in ALEX, then so are:

(i) $(f(x))$

(ii) $-(f(x))$

(iii) $f(x) + g(x)$

(iv) $f(x) - g(x)$

(v) $f(x)g(x)$

(vi) $f(x)/g(x)$

(vii) $f(g(x))$

(viii) $f(g(x))$

a) Show that $(x + 2)\text{-}3x$ is in ALEX.

b) Show that elementary calculus contains enough rules to prove the theorem that all algebraic expressions can be differentiated.

c) Is Rule 2 (viii) really necessary?

Chapter 3, Problem 8P

3 Bookmarks

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Step-by-step solution

Step 1 of 4

Consider the following rules:

Rule 1: All polynomials are in ALEX.

Rule 2: If $f(x)$ and $g(x)$ are in ALEX, then so are:

(i) (x)

(ii) $-(x)$

(iii) $f(x) + g(x)$

(iv) $f(x) - g(x)$

(v) $f(x)g(x)$

(vi) $f(x)/g(x)$

(vii) $f(x)^{g(x)}$

(viii) $f(g(x))$

Comment

Comment

Step 2 of 4

(a)

Consider the following polynomial: $(x+2)^{3x}$

Assume, $x+2=f(x)$ and $3x=g(x)$.

• Both expressions $f(x)$ and $g(x)$ are polynomials. By Rule 1, $(x+2)$ and $3x$ are in ALEX.

• From Rule 2, $(x+2)^{3x}$ is in the form of $f(x)^{g(x)}$, where $f(x)=x+2$ and $g(x)=3x$.

Comment

Step 3 of 4

(b)

The statement of the theorem is that "All algebraic expressions can be differentiated".

Algebraic expressions are the expressions containing constants, variable, elementary arithmetic operations, factorial, integer and rational exponent and nth roots.

Chapter 3, Problem 8P

3 Bookmarks

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Step 3 of 4

(b)

The statement of the theorem is that "All algebraic expressions can be differentiated".

Algebraic expressions are the expressions containing constants, variable, elementary arithmetic operations, factorial, integer and rational exponent and nth roots.

Constants:

• The expression containing constants are the expressions containing only numbers.

• These are the expressions containing the terms without variables.

• The value of constant expression never changes.

Ex: 3, 4, 5, -2 etc.

Variables:

• The expression containing variables are the expressions containing symbol for representing a number.

• The value of variable can be changed when required.

Ex: x, y, z, a etc.

Elementary arithmetic expressions:

• The mathematical expressions containing variables, numbers and operations are elementary arithmetic expressions.

• The arithmetic expressions are addition, subtraction, multiplication and division.

Ex: $x+y$, $3+5$, $2x+3y$, $x-5$, $3/2$, $5x\cdot 3y$ etc.

Factorial:

• The factorial of an integer is the product of integers less than or equal to it.

• The multiplication between two constant values is also a constant which can be represented in an expression.

Ex: $5!=5\times4\times3\times2\times1=120$ etc.

• The mathematical expressions containing variables, numbers and operations are elementary arithmetic expressions.

• The arithmetic expressions are addition, subtraction, multiplication and division.

Ex: $x + y,\ 3 + 5,\ 2x + 3y,\ x - 5,\ 3\ /\ 2,\ 5x \cdot 3y$ etc.

Factorial:

• The factorial of an integer is the product of integers less than or equal to it.

• The multiplication between two constant values is also a constant which can be represented in an expression.

Ex: $5! = 5\times4\times3\times2\times1 = 120$ etc.

Integer and rational exponents:

• The integer exponent of an expression is the product of the expression specified by the integer exponential value.

• The rational exponent of an expression is the product of the expression specified by the fractional exponential value.

Ex: $3^2 = 3\times3,\ x^3 = x\times x\times x,\ 5^{-3} = \frac{1}{5^3} = \frac{1}{5\times5\times5}$ etc.

n th roots:

• The nth roots of an expression is the number that should be multiplied *n* times itself to equal a given value.

Ex:

$$\begin{aligned}\sqrt[5]{32} &= (32)^{\frac{1}{5}} \\ &= (2\times2\times2\times2\times2)^{\frac{1}{5}} \\ &= (2^5)^{\frac{1}{5}} \\ &= 2\end{aligned}$$

As the algebraic expression for elementary calculus satisfies the enough rules required for differentiating the theorem is satisfied.

• The nth roots of an expression is the number that should be multiplied *n* times itself to equal a given value.

Ex:

$$\begin{aligned}\sqrt[5]{32} &= (32)^{\frac{1}{5}} \\ &= (2\times2\times2\times2\times2)^{\frac{1}{5}} \\ &= (2^5)^{\frac{1}{5}} \\ &= 2\end{aligned}$$

As the algebraic expression for elementary calculus satisfies the enough rules required for differentiating the theorem is satisfied.

Comment

Step 4 of 4

(c)

The rule $f(g(x))$ is not really necessary because, the expression is in the form of $f(x)$, with the value of input x is $g(x)$. Here, the output of $g(x)$ is the input of $f(x)$.

Comment

Was this solution helpful? 0 0

Using the fact that $3x^2 + 7x - 9 = ((((3)x) + 7)x) - 9)$,show how to produce this polynomial from the rules for POLYNOMIAL using multiplication only twice. What is the smallest number of steps needed for producing *x*8 + *x*4? What is the smallest number of steps needed for $7x^7 + 5x^5 + 3x^3 + x$?

Step-by-step solution

Step 1 of 3

$3x^2+7x-9=\left(\left(\left(\left((3)x\right)+7\right)x\right)-9\right)$

Rules for POLYNOMIAL

Rule 1: Any number is in POLYNOMIAL

Rule 2: The variable x is in POLYNOMIAL

Rule 3: If p and q are in POLYNOMIAL, then so are p + q, p – q, (p), and p q

By rule 1: 3 is in POLYNOMIAL

By rule 3: (3) is in POLYNOMIAL

By rule 2: x is in POLYNOMIAL

By rule 3: ((3)x) is in POLYNOMIAL; call it 3x

By rule 1: 7 is in POLYNOMIAL

By rule 3: ((3)x)+7 is in POLYNOMIAL; call it 3x+7

By rule 3: (((3)x)+7) is in POLYNOMIAL; call it (3x+7)

By rule 3: ((((3)x)+7)x) is in POLYNOMIAL; call it (3x+7)x i.e., $3x^2+7x$

By rule 1: -9 is in POLYNOMIAL

By rule 3: $3x^2+7x-9$ is in POLYNOMIAL

Chapter 3, Problem 9P

2 Bookmarks Show all steps: OFF

By rule 3: (((3)x)+7) is in POLYNOMIAL; call it (3x+7)

By rule 3: ((((3)x)+7)x) is in POLYNOMIAL; call it (3x+7)x i.e., $3x^2+7x$

By rule 1: -9 is in POLYNOMIAL

By rule 3: $3x^2+7x-9$ is in POLYNOMIAL

Comment

Step 2 of 3

x^8+x^4

By rule 2: x is in POLYNOMIAL

By rule 3: $(x)(x)$ is in POLYNOMIAL; x^2

By rule 3: $(x^2)(x^2)$ is in POLYNOMIAL; x^4

By rule 3: x^4+1 is in POLYNOMIAL

By rule 3: $(x^4)(x^4+1)$ is in POLYNOMIAL; x^8+x^4

By rule 3: x^8+x^4 is in POLYNOMIAL

Comment

Step 3 of 3

$7x^7+5x^5+3x^3+x$

Chapter 3, Problem 9P

2 Bookmarks Show all steps: OFF

Step 3 of 3

$7x^7+5x^5+3x^3+x$

By rule 1: 7 is in POLYNOMIAL

By rule 2: x is in POLYNOMIAL

By rule 3: $(7)(x)$ is in POLYNOMIAL

By rule 3: $(7x)(x)$ is in POLYNOMIAL

By rule 3: $(7x^2)(x^2)$ is in POLYNOMIAL

By rule 1: (x^3) is in POLYNOMIAL

By rule 3: $(7x^4)(x^3)$ is in POLYNOMIAL

By rule 3: $(7x^7)$ is in POLYNOMIAL

By rule 1: +5 is in POLYNOMIAL

By rule 3: + $(5)(x)$ is in POLYNOMIAL

By rule 3: + $(5)(x^4)(x)$ is in POLYNOMIAL

By rule 3: + $(5x^5)$ is in POLYNOMIAL

By rule 1: +3 is in POLYNOMIAL

By rule 3: x is in POLYNOMIAL

By rule 1: (x^2) is in POLYNOMIAL

By rule 3: + $(3)(x)(x^2)$ is in POLYNOMIAL

By rule 3: + $3x^3$ is in POLYNOMIAL

By rule 3: $7x^7+5x^5+3x^3$ is in POLYNOMIAL

By rule 3: $7x^7+5x^5+3x^3+x$ is in POLYNOMIAL

Chapter 3, Problem 10P

4 Bookmarks Show all steps: OFF

Problem

Show that if *n* is less than 31, then *x* n can be shown to be in POLYNOMIAL in fewer than eight steps.

Step-by-step solution

Step 1 of 1

By rule 2: x is in POLYNOMIAL

By rule 3: $(x)(x)$ is in POLYNOMIAL

By rule 3: $(xx)(xx)$ is in POLYNOMIAL

By rule 3: $(xxxx)(xxxx)$ is in POLYNOMIAL

By rule 3: $(xxxxxxxx)(xxxxxxxx)$ is in POLYNOMIAL

By rule 3 : $(xxxxxxxxxxxxxxxx)(xxxxxxxxxx)$ is in POLYNOMIAL

By rule 3: $(xxxxxxxxxxxxxxxxxxxxxxxxxx)(xxxx)$ is in POLYNOMIAL

Only it takes less then 8 steps.

Comment

Was this solution helpful? 3 0

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Problem

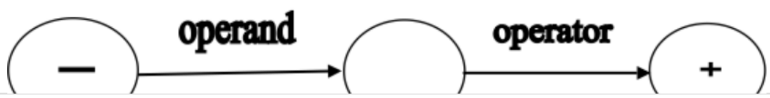
In this chapter, we mentioned several substrings of length 2 that cannot occur in arithmetic expressions, such as (/, +), //, and */. What is the complete list of substrings of length 2 that cannot occur?

Step-by-step solution

Step 1 of 1

The main idea to exclude the substrings of length 2 that cannot occur is that two operators, or two operands cannot occur together.

It means that if the substring is like: (operand) operator (operand) then it is accepted.



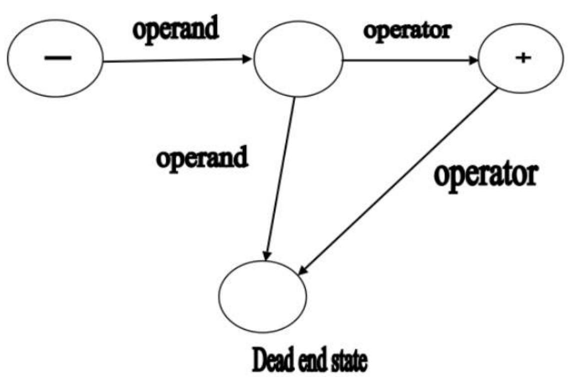
☰ Chapter 3, Problem 11P | 1 Bookmark | Show all steps: OFF

Step-by-step solution

Step 1 of 1

The main idea to exclude the substrings of length 2 that cannot occur is that two operators, or two operands cannot occur together.

It means that if the substring is like: (operand) operator (operand) then it is accepted.



Thus, (operator)(operator) and (operand)(operand) is rejected. The substrings such as: (/,+), //, */ will be rejected in the same way.

Chapter 3, Problem 12P

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Problem

Are there any substrings of length 3 that cannot occur that do not contain forbidden substrings of length 2? (This means that /// is already known to be illegal because it contains the forbidden substring //.) What is the longest forbidden substring that does not contain a shorter forbidden substring?

Step-by-step solution

Step 1 of 1

There are no substrings of length 3 that cannot occur that do not contain forbidden strings of length 2.

The longest forbidden substring that does not contain a shorter forbidden substring is:

$\{xx, xxx\}^*$

Comment

Chapter 3, Problem 13P

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Problem

The rules given earlier for the set AE allow for the peculiar expressions

(((((9))))) and -(-(-(-(9))))

It is not really harmful to allow these in AE, but is there some modified definition of AE that el iminates this problem?

Step-by-step solution

Step 1 of 2

Consider the two arithmetic expressions $(((((9)))))$ and $-(-(-(-(9))))$. These two arithmetic expressions are allowed as per the rules specified for the Arithmetic Expressions (AE).

Comment

Step 2 of 2

Consider the two arithmetic expressions $(((((9)))))$ and $-(-(-(-(9))))$. These two arithmetic expressions are allowed as per the rules specified for the Arithmetic Expressions (AE).

Comment

Step 2 of 2

The strings $(((((9)))))$ and $-(-(-(-(9))))$ are not harmful to allow in AE because these two strings are not ambiguous.

The Rule 2 in the definition of AE should be modified to eliminate the peculiar expressions. The following is the modified Rule 2 for the AE:

Rule 2 If x is in AE, then so are

(i) (x) (Provided x does not already start with a '(' and does not end with a ')')

(Provided x does not already start with '-(' and does not end with a ')')

(ii) $-x$ (Provided x does not already start with a minus sign)

The remaining two rules (Rule 1 and Rule 3) remains same.

Comment

Was this solution helpful? 6 0

Problem

i) Write out the full recursive definition for the propositional calculus that contains the symbols ∨ and ∧ as well as ¬ and →

ii) What are all the forbidden substrings of length 2 in this language?

Step-by-step solution

Step 1 of 3

Recursive definitions for any set can be defined by the following three steps:

1. Specify the base case as basic words in the set.

2. Construct new cases or words by applying recursive rules.

3. No other words/objects constructed by rule 1 and Rule 2 will be in this language.

Comment

Step 2 of 3

Step 2 of 3

The full recursive definition for the propositional calculus that contains the symbols $\vee$ and $\wedge$ as well as $\neg$ and $\rightarrow$ is as follows:

(i)

$$((\neg x \vee x) \rightarrow (\neg y \wedge y))$$

Now follow the rules of recursive definitions for the prepositional calculus:

Rule 1: Any letter is a WFF.

Thus, from the rule 1, x and y are the WFF.

Rule 2: If x is a WFF, then (x) and $\neg x$ are also WFF.

Rule 2: If y is a WFF, then (x) and $\neg y$ are also WFF.

Rule 2: If x and $\neg x$ are WFF, then $\neg x \vee x$ are also WFF.

Rule 2: If y and $\neg y$ are WFF, then $\neg y \wedge y$ are also WFF.

Rule 2: If $\neg x \vee x$ and $\neg y \wedge y$ are WFF, then $(\neg x \vee x) \rightarrow (\neg y \wedge y)$ are also WFF.

Here, x, y, $\wedge, \wedge, \neg$ are in calculus.

Comment

Chapter 3, Problem 14P
1 Bookmark
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Comment

Step 3 of 3

(ii)

Forbidden sub-string is in simple words, counting the parentheses. In the above string, there is no forbidden sub-string in this language.

- In the above string, $(\neg x \vee x)$ is a sub-string that is complete.
- Another sub-string is $(\neg y \wedge y)$ that is also complete.
- At the end, take the last sub-string, $((\neg x \vee x) \rightarrow (\neg y \wedge y))$ that is also complete.

Thus, it is proved that the above string $((\neg x \vee x) \rightarrow (\neg y \wedge y))$ does not contain any forbidden sub-string.

Comment

Was this solution helpful? 1 0

Chapter 3, Problem 15P

3 Bookmarks

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Problem

(i) When asked to give a recursive definition for the language PALINDROME over the alphabet Σ = {*a b*}, a student wrote:

Rule 1 *a* and *b* are in PALINDROME.

Rule 2 If *x* is in PALINDROME, then so are *axa* and *bxb*.

Unfortunately, all the words in the language defined above have an odd length and so it is not all of PALINDROME. Fix this problem.

(ii) Give a recursive definition for the language EVENPALINDROME of all palindromes of even length.

Next

Step-by-step solution

Step 1 of 2

i) Rule 1 : , a, b are in PALINDROME

Chapter 3, Problem 15P

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Step-by-step solution

Step 1 of 2

i) Rule 1 : **,** a, b are in PALINDROME

Rule 2 : If x is in PALINDROME, so axa, bxb

The set of PALINDROME = { ^, a, b, aa, bb, aaa, aba, bab, bbb, aaaa, abba,…..}

We have added the Null string to rule 1 it allows us to have strings of the form aa since Null string can be substituted for x in axa (per Rule 2 x is in PALINDROME and Null string meets that requirement). Once we prove that aa is in PALINDROME we can build all the other EVEN PALINDROMES as well.

Consider example for even length Palindrome

axa

aaxaa [from rule 2]

aaaa [from rule 1]

Hence it supports both even and odd length PALINDROME.

Chapter 3, Problem 15P | 3 Bookmarks | Show all steps: OFF

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Step 2 of 2

ii) The set of EVEN PALINDROME is as shown,

{ ^, aa, bb, aaaa, bbbb, abba, baab,………}

Rule 1 : aa and bb are in EVENPALINDROME.

Rule 2 : If x is in EVENPALINDROME, then so are axa, bxb.

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Problem

(i) Give a recursive definition for the set ODD = {1 3 5 7 . . .},

(ii) Give a recursive definition for the set of strings of digits 0, 1, 2, 3, . . . 9 that cannot start with the digit 0.

Step-by-step solution

Step 1 of 2

Recursive function to generate the ODD positive integers:

$$Base\,step: \quad f(1) = 1$$

$Recursive\,Step:$

Chapter 3, Problem 16P

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Step 1 of 2

Recursive function to generate the ODD positive integers:

$$Base\,step: \quad f(1)=1$$

$$\mathrm{Re}\,cursive\,Step:$$

$$\begin{aligned} f(n+1)+1 &= f(n)+f(1)+1 \\ &= f(n)+1+1 \\ &= f(n)+2 \end{aligned}$$

Consider two rules:

Rule 1: 1 is odd

Rule 2: If x is odd, then x+2 is odd

Proof:

$$\begin{aligned} &1\,is\,odd -> Rule1 \\ &3\,is\,odd -> Rule2(1+2=3) \\ &5\,is\,odd -> Rule2(3+2=5) \\ &7\,is\,odd -> Rule2(5+2=7)\ ...... \end{aligned}$$

$$\therefore \{1,3,5,7......\}\,are\,ODD\ Integers.$$

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Chapter 3, Problem 16P

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Step 2 of 2

Recursive function to generate the positive integers cannot start with 0:

Assume that two strings 0 and 1 are POSITIVE

Consider the two rules

Rule 1: 0 and 1 are in positive

Rule 2: if x is not zero and in positive, then x + 1 are positive.

Proof:

$$\begin{aligned} &0\ and\ 1\,are\ Positive -> Rule1 \\ &1\,is\ \ positive -> Rule1,\ But\,not\,0 \\ &2\ is\ in\ positive -> Rule\ 2\ (1+1\ =2) \\ &3\ is\ in\ positive -> Rule\ 2\ (2+1\ =3) \\ &4\ is\ in\ positive -> Rule\ 2\ (3+1\ =4)...... \end{aligned}$$

$$\therefore \{1,2,3,4,5.......n\}\,are\ Positive\ Integers.$$

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javascript:;

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