EE227 – Digital Logic Design

- **Lecture Slides**
- **Week 4**

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Outline

- KARANUGH Map (K – Map) Method
  - With 2 variables
  - With 3 variables
  - With 4 variables
- Product of Sum Simplification
- Don’t Care Conditions

KARANUGH Map (K – Map)

- K – Map is a pictorial method used to minimize Boolean expressions without having to use Boolean algebra theorems and equation manipulations
- It is thought as a special version of a truth table
- Using a K-map, expressions with two to four variables are easily minimized

2-Variable K-Map:

A	B	F
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3

A \ B	0	1
0	m_0	m_1
1	m_2	m_3

Recall:

Decimal	Binary	Gray Code
0	0 0	0 0
1	0 1	0 1
2	1 0	1 1
3	1 1	1 0

3-Variable K-Map:

A	B	C	F
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

A \ BC	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

4- Variable K-Map:

A	B	C	D	F
0	0	0	0	m_0
0	0	0	1	m_1
0	0	1	0	m_2
0	0	1	1	m_3
0	1	0	0	m_4
0	1	0	1	m_5
0	1	1	0	m_6
0	1	1	1	m_7
1	0	0	0	m_8
1	0	0	1	m_9
1	0	1	0	m_{10}
1	0	1	1	m_{11}
1	1	0	0	m_{12}
1	1	0	1	m_{13}
1	1	1	0	m_{14}
1	1	1	1	m_{15}

AB \ CD	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

Rule for K-Map Simplification : SOP Form

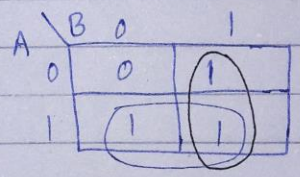
① Cover the maximum number of 1's using minimum number of boxes. (overlapping is allowed)

② Boxes of 1's can have size of 16, 8, 4, 2, 1.

③ While writing simplified expression ignore variable which changes in a box/group.

Ex1: Find simplest expression using K-Map:

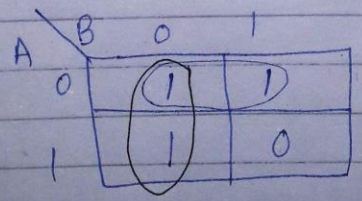
A	B	F
0	0	0
0	1	1
1	0	1
1	1	1



$F = A + B$

EX2:

$F(A,B) = \Sigma(0,1,2)$



$F = \overline{A} + \overline{B}$

(21)

How to draw Boxes / or Make Groups of 1's?

(1) You are allowed to make groups of 1's along Horizontal and Vertical direction of adjacent cells.

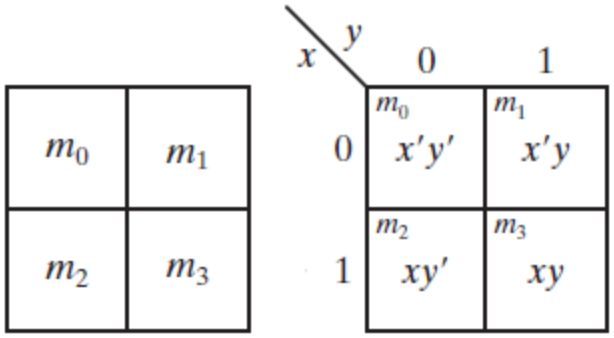
(2) Folding is also allowed.

(3) You can not make group of 1's which are not adjacent to each other.

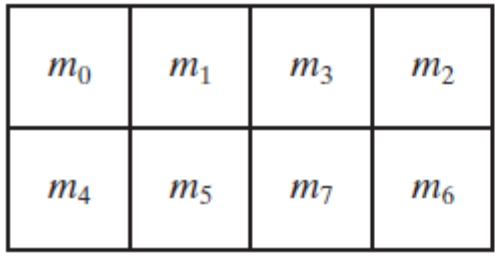
(4) You can not make group of 1's diagonalwise.

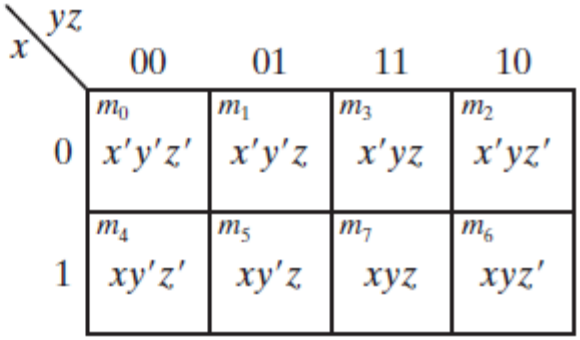
Generating K – Map (1/2)

- A k map will contain 2^n squares
  - Where n is the number of variables in the expression
- While generating a k-map one should keep in mind that **any two adjacent squares in the map differ by only one bit**



With two variables





With three variables

Generating K – Map (2/2)

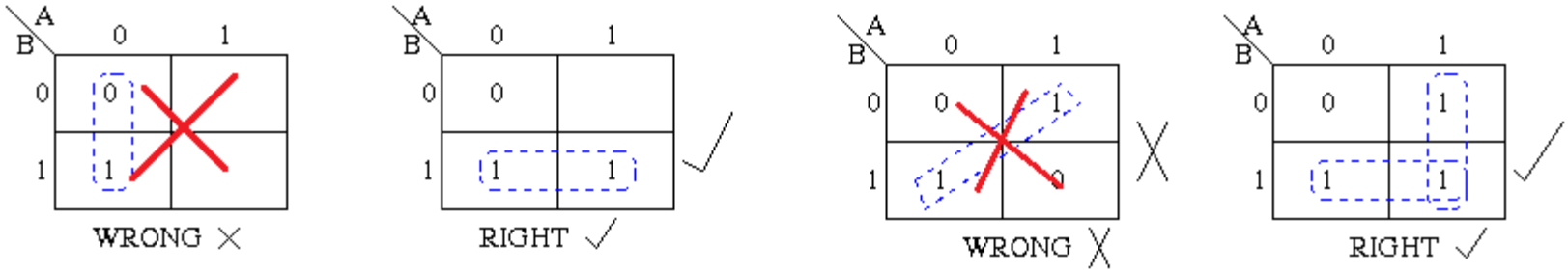
- With four variables

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

wx \ yz	00	01	11	10
00	m_0 $w'x'y'z'$	m_1 $w'x'y'z$	m_3 $w'x'yz$	m_2 $w'x'yz'$
01	m_4 $w'xy'z'$	m_5 $w'xy'z$	m_7 $w'xyz$	m_6 $w'xyz'$
11	m_{12} $wxy'z'$	m_{13} $wxy'z$	m_{15} $wxyz$	m_{14} $wxyz'$
10	m_8 $wx'y'z'$	m_9 $wx'y'z$	m_{11} $wx'yz$	m_{10} $wx'yz'$

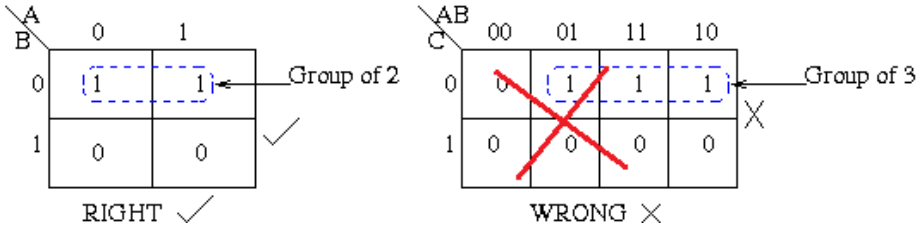
K-Map - Rules of Simplification (1/4)

- While using k-map we group the 1's present in the k-map table and note the variables which are not changing
- Here are some rules for simplification and grouping of 1's
  - Groups may not include any cell containing a zero
  - Groups may be horizontal or vertical, but not diagonal

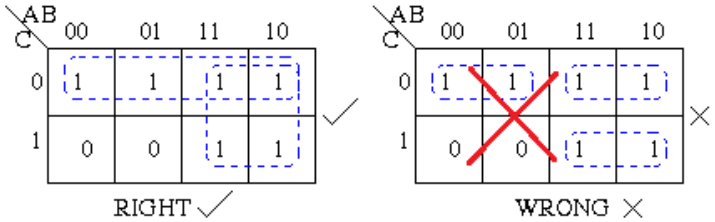


K-Map - Rules of Simplification (2/4)

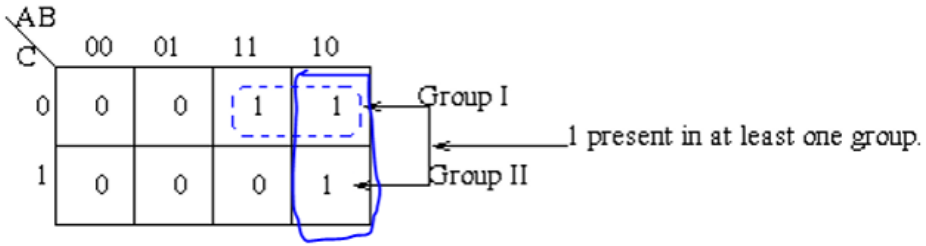
- Groups must contain 1, 2, 4, 8, or in general 2^n cells



- Each group should be as large as possible

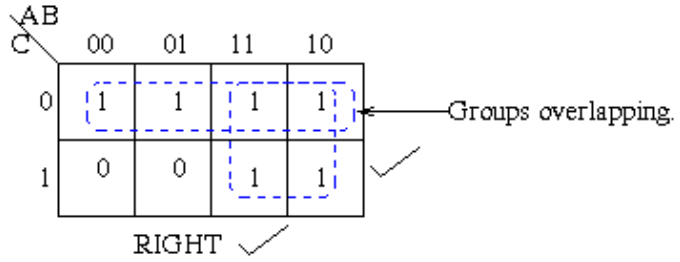


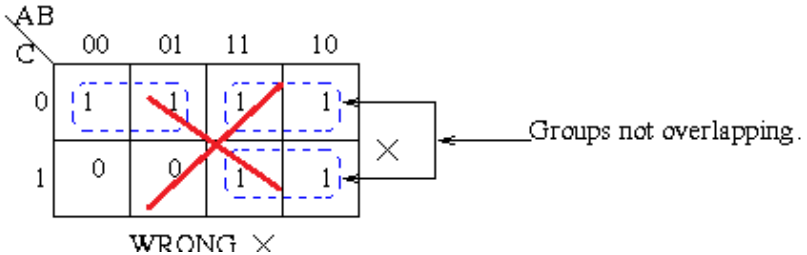
- Each cell containing a one must be in at least one group



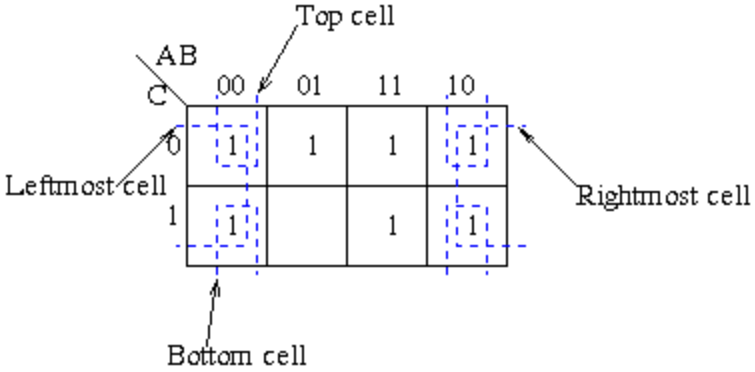
K-Map - Rules of Simplification (3/4)

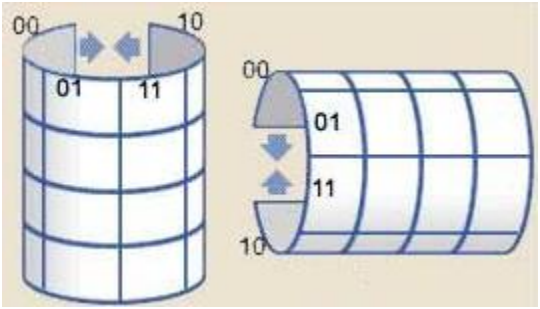
- Groups may overlap





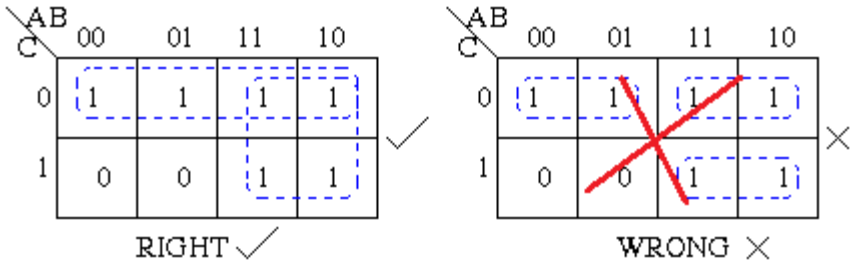
- Groups may wrap around the table. The leftmost cell in a row may be grouped with the rightmost cell and the top cell in a column may be grouped with the bottom cell





K-Map - Rules of Simplification (4/4)

- There should be as few groups as possible, as long as this does not contradict any of the previous rules



K – Map Simplification Steps

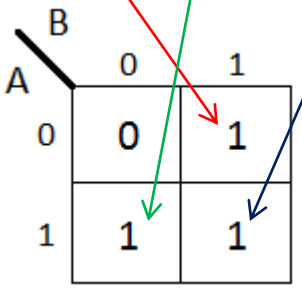
1. Populate the k-map
   - By using truth table
   - Or by using minterms
     - Introduce all the variables in each term
2. Form the groups by considering the rules
3. Note the constant variables for each group
4. Multiply the constant variables of each group to form a term
5. Add all the terms to obtain the simplified Boolean function

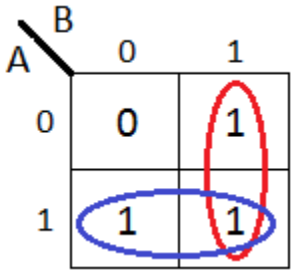
K – Map with Two Variables

Simplify the following expression by using k-map

$$F = A'B + AB' + AB$$

A	B	A'	B'	A'B	AB'	AB	F
0	0	1	1	0	0	0	**0**
0	1	1	0	1	0	0	**1**
1	0	0	1	0	1	0	**1**
1	1	0	0	0	0	1	**1**





The simplified expression is

$$F = A + B$$

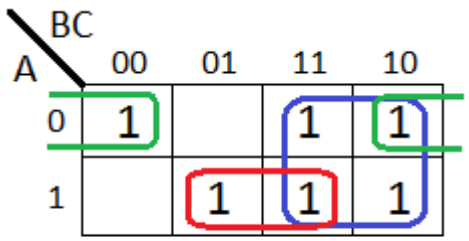
K – Map with Three Variables

Simplify the following Boolean function by using k-map

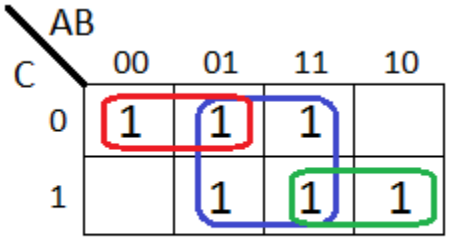
F = A’B’C’ + A’B + ABC’ + AC

F = A’B’C’ + A’B(C + C’) + ABC’ + AC(B + B’)

F = A’B’C’ + A’BC + A’BC’ + ABC’ + ABC + AB’C



F = B + AC + A'C'

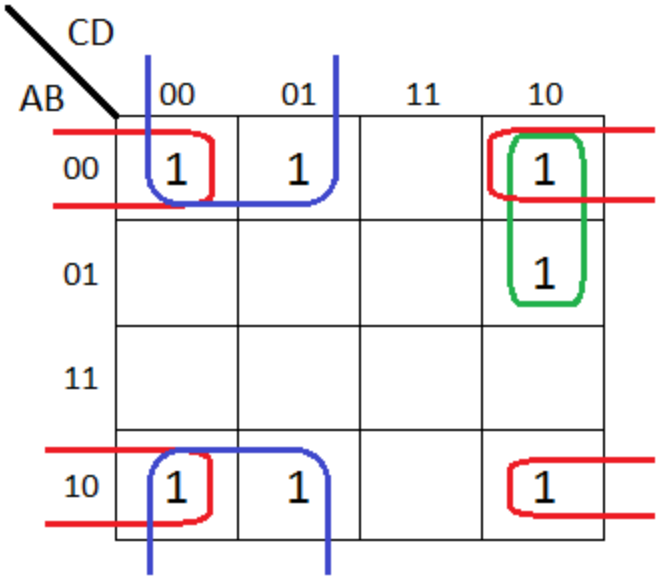


F = B + AC + A'C'

K – Map with Four Variables (1/2)

Simplify the following Boolean function by using k-map

F = A’B’C’D’ + A’B’C’D + A’B’CD’ + A’BCD’ + AB’C’D’ + AB’C’D + AB’CD’

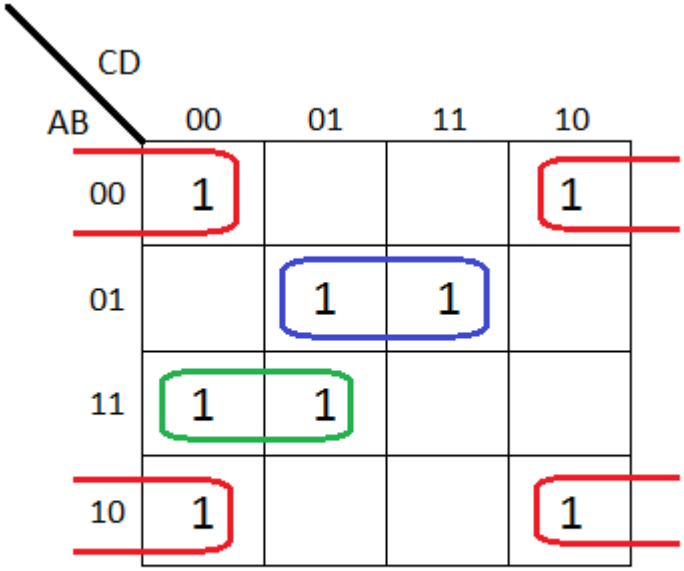


F = B'D' + B'C' + A'CD'

K – Map with Four Variables (2/2)

Simplify the following Boolean function by using k-map

F = A’B’C’D’ + A’B’CD’ + A’BC’D + A’BCD + ABC’D’ + ABC’D + AB’C’D’ + AB’CD’



F = B'D' + ABC' + A'BD

K-Map - Rules of Simplification

- Groups may not include any cell containing a zero
- Groups may be horizontal or vertical, but not diagonal
- Groups must contain 1, 2, 4, 8, or in general 2^n cells
- Each group should be as large as possible
- Each cell containing a one must be in at least one group
- Groups may overlap
- Groups may wrap around the table
- There should be as few groups as possible

4 Variables K-Map Examples (SOP-Form)

Simple simplification rules are;

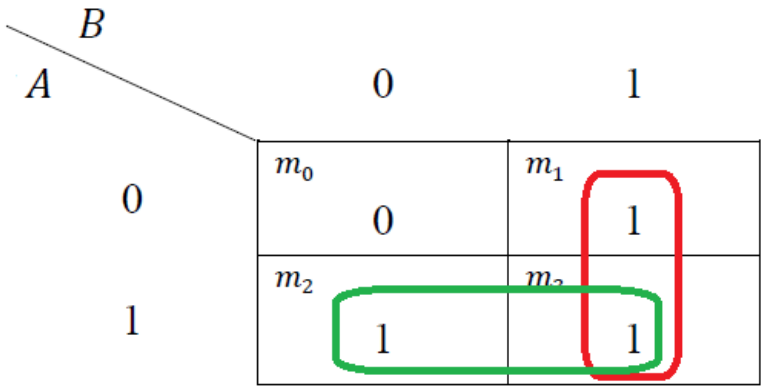
1. Allowed Groups of size are: 16, 8, 4, 2, 1 (Priority Order)
2. Grouping of adjacent 1's is allowed along horizontal and vertical direction. Folding and overlapping is allowed.
3. Cover maximum number of 1's with minimum number of groups.

4-Variable K-map is;

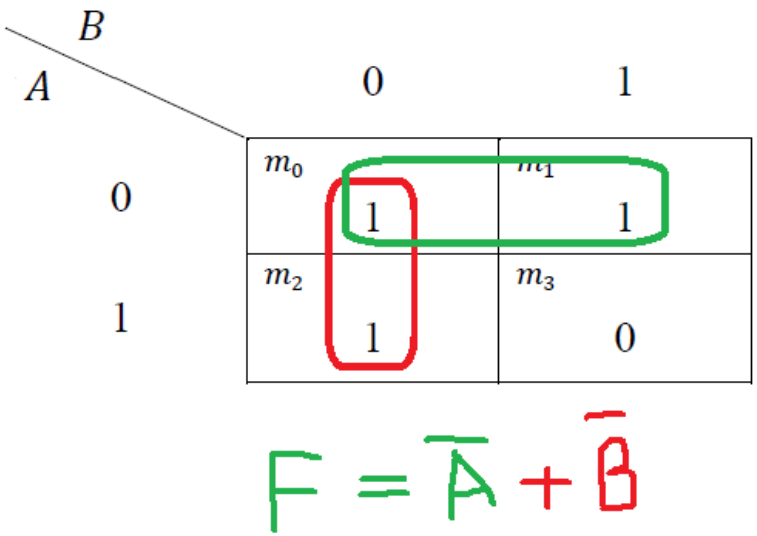
	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

K-Map Examples

Ex # 1: $F(A,B) = \sum(1,2,3)$



Ex # 2: $F(A,B) = \sum(0,1,2)$ **23**



Ex # 3: $F(A,B,C) = \sum(2,3,4,5)$

A \ BC	00	01	11	10
0	m_0 0	m_1 0	m_3 1	m_2 1
1	m_4 1	m_5 1	m_7 0	m_6 0

$$F = A\bar{B} + \bar{A}B$$

Ex # 4: $F(A,B,C) = \sum(2,3,4,5,6,7)$

25

A \ BC	00	01	11	10
0	m_0 0	m_1 0	m_3 1	m_2 1
1	m_4 1	m_5 1	m_7 1	m_6 1

$F = B + A$

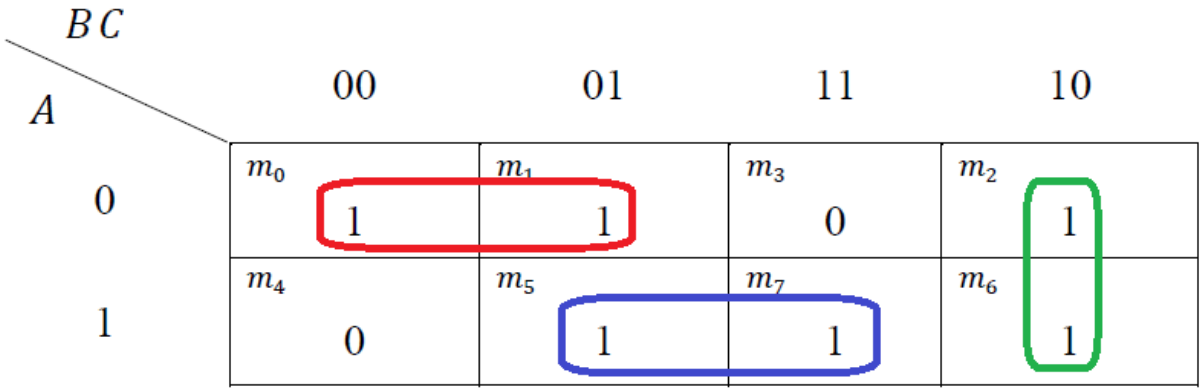
Ex # 5: $F(A,B,C) = \sum(1,3,4,5,7)$ **26**

A \ BC	00	01	11	10
0	m_0 0	m_1 1	m_3 1	m_2 0
1	m_4 1	m_5 1	m_7 1	m_6 0

$F = A\bar{B} + C$

Ex # 6: $F(A,B,C) = \sum(0,1,2,5,6,7)$

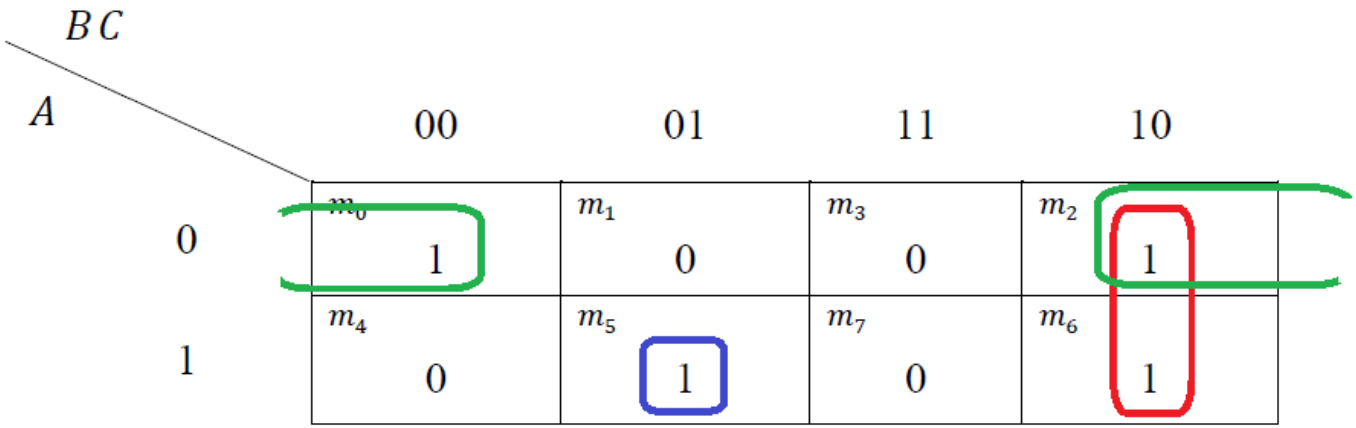
27



$$F = AC + \bar{A}\bar{B} + B\bar{C}$$

Ex # 7: $F(A,B,C) = \sum(0,2,5,6)$

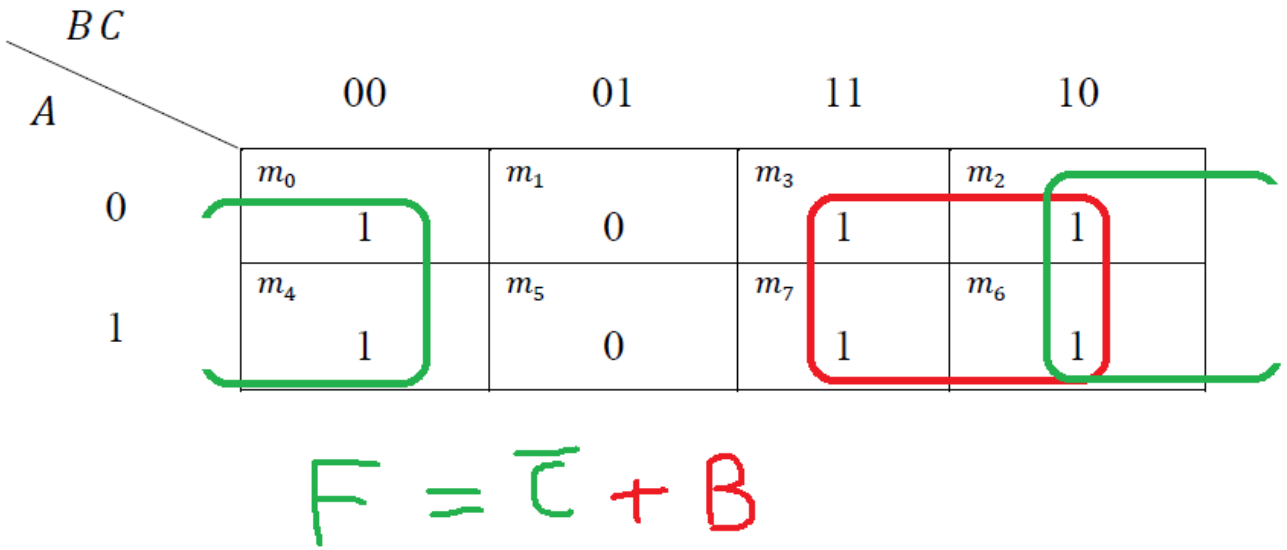
28



$F = A\bar{B}C + \bar{A}\bar{C} + B\bar{C}$

Ex # 8: $F(A,B,C) = \sum(0,2,3,4,6,7)$

29



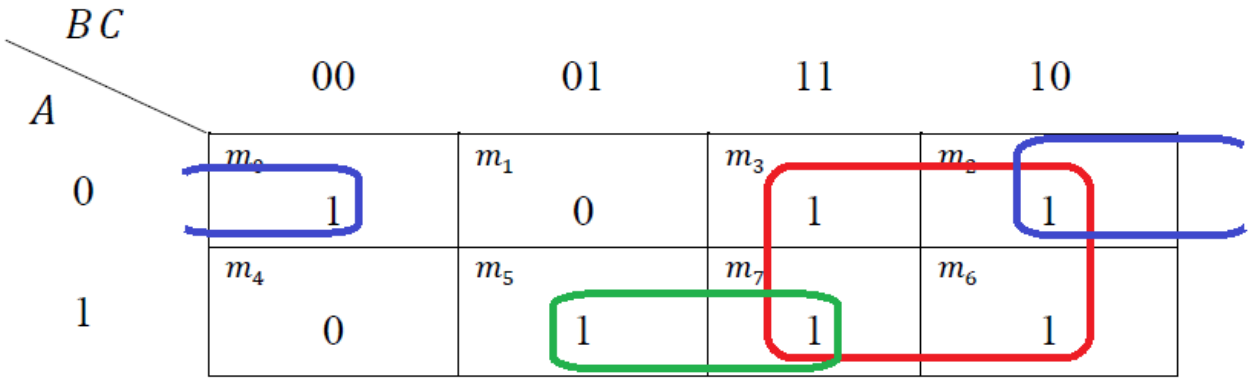
Ex # 9: $F(A,B,C) = \sum(1,2,3,5,7)$ **30**

A \ BC	00	01	11	10
0	m_0 0	m_1 1	m_3 1	m_2 1
1	m_4 0	m_5 1	m_7 1	m_6 0

$F = \bar{A}B + C$

Ex # 10: $F(A,B,C) = \sum(0,2,3,5,6,7)$

31



$F = \bar{A}\,\bar{C} + AC + B$

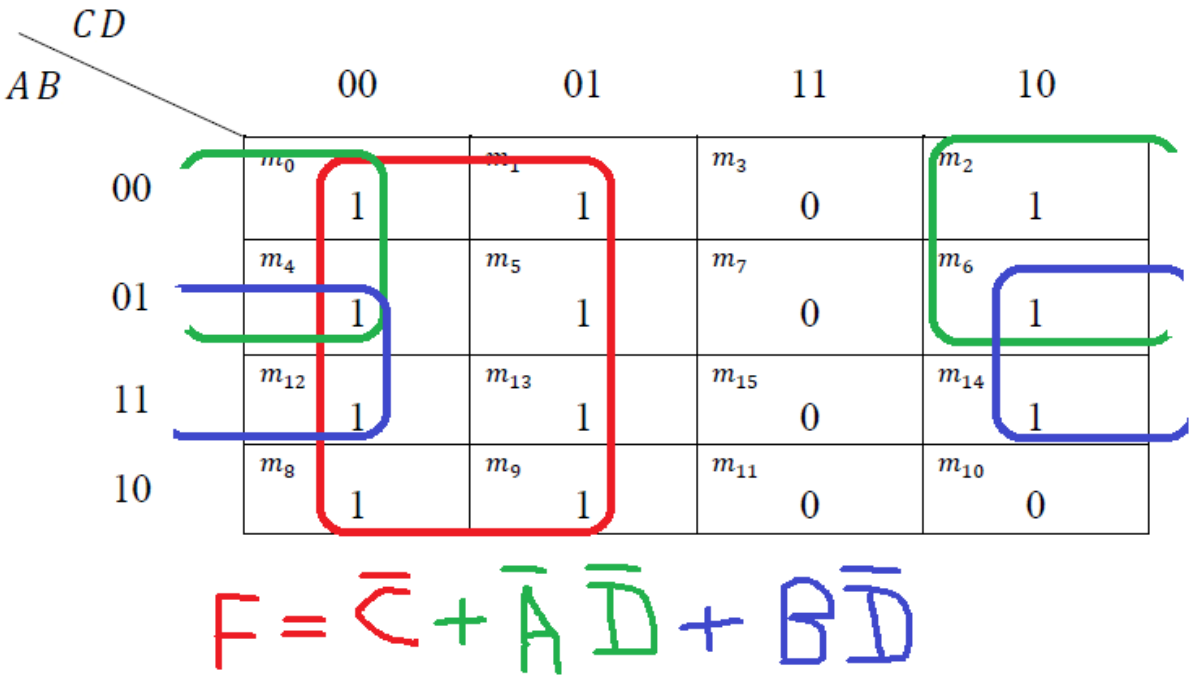
Ex # 11: $F(A,B,C) = \sum(1,2,4,7)$

32

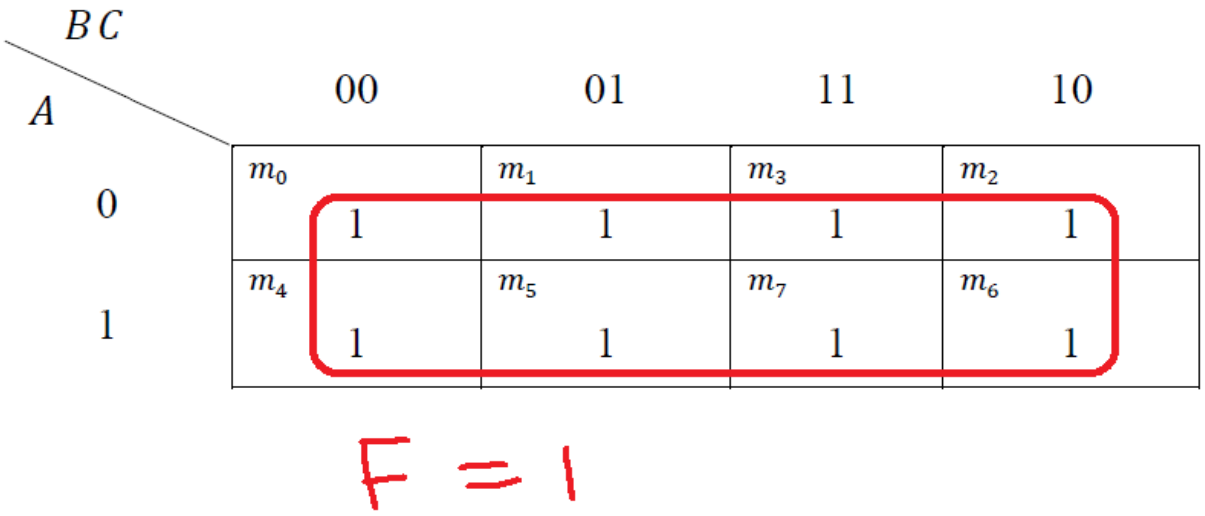
A \ BC	00	01	11	10
0	m_0 0	m_1 1	m_3 0	m_2 1
1	m_4 1	m_5 0	m_7 1	m_6 0

$F = \bar{A}B\bar{C} + A\bar{B}\bar{C} + \bar{A}\bar{B}C + ABC$

Ex # 1: $F(A,B,C,D) = \sum(0,1,2,4,5,6,8,9,12,13,14)$ **35**

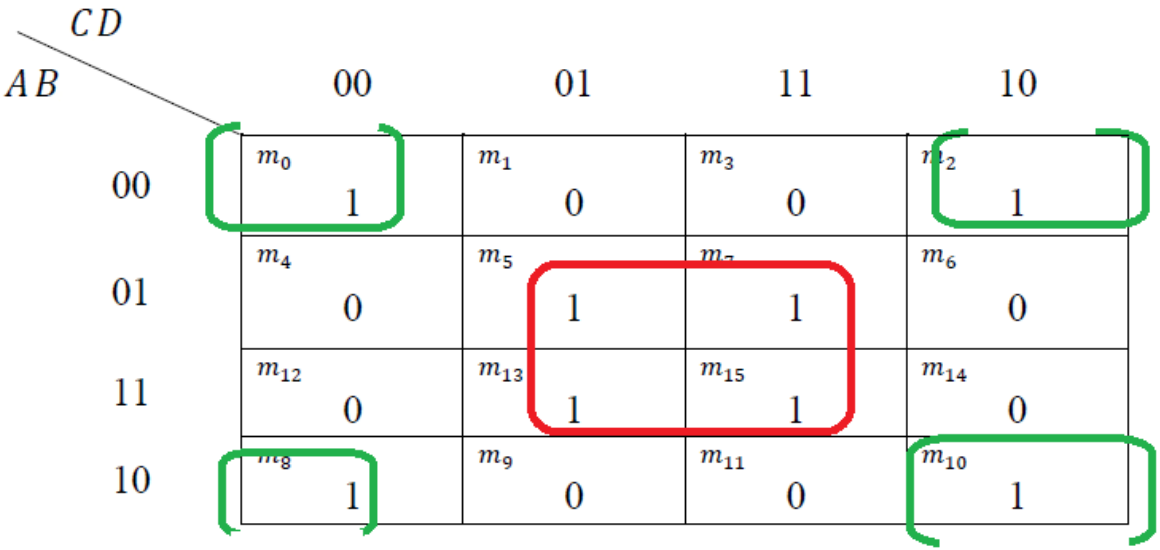


Ex # 12: $F(A,B,C) = \sum(0,1,2,3,4,5,6,7)$ **33**



Ex # 2: $F(A,B,C,D) = \sum(0,2,5,7,8,10,13,15)$

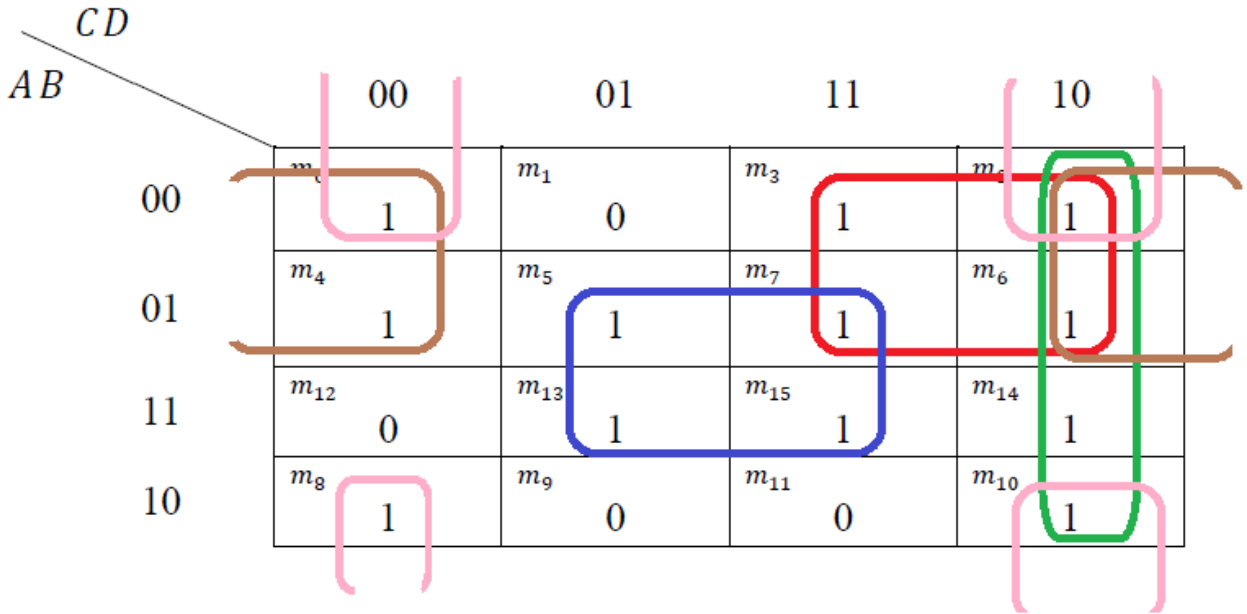
36



$F = \bar{B}\bar{D} + BD$

Ex # 3: $F(A,B,C,D) = \sum(0,2,3,45,6,7,8,10,13,14,15)$

37



$$F = \bar{B}\bar{D} + BD + \bar{A}C + C\bar{D} + \bar{A}\bar{D}$$

Ex # 4: $F(A,B,C,D) = \sum(1,2,3,4,6,7,9,10,11,12,13,14)$

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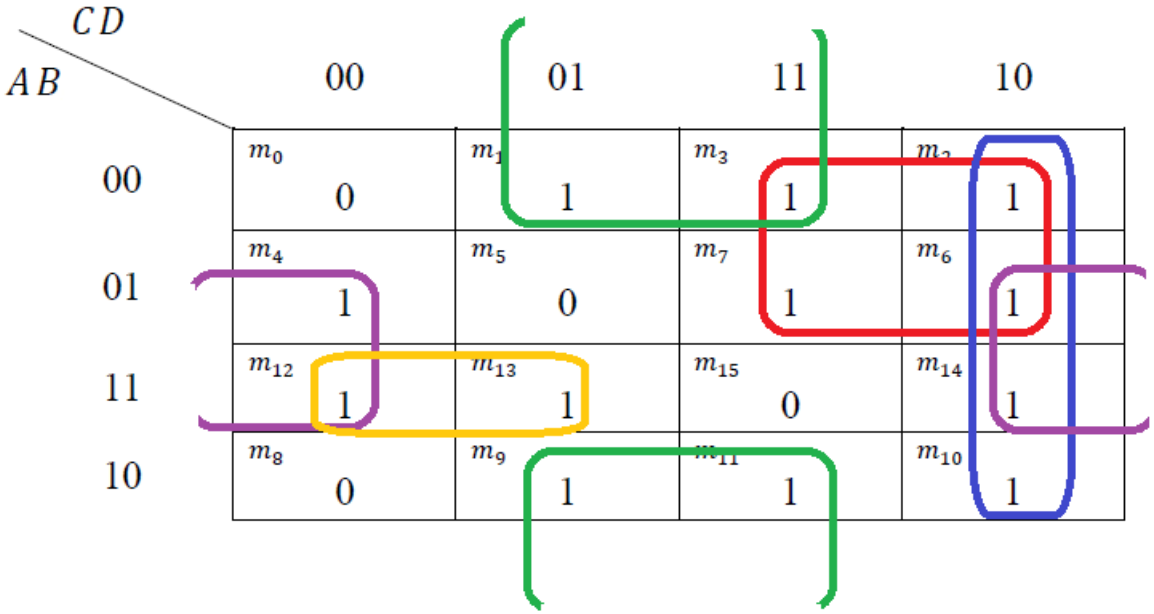

$$F = AB\bar{C} + B\bar{D} + \bar{B}D + \bar{A}C + C\bar{D}$$

Ex # 5: $F(A,B,C,D) = \sum(2,3,7,8,9,10,11,13,15)$

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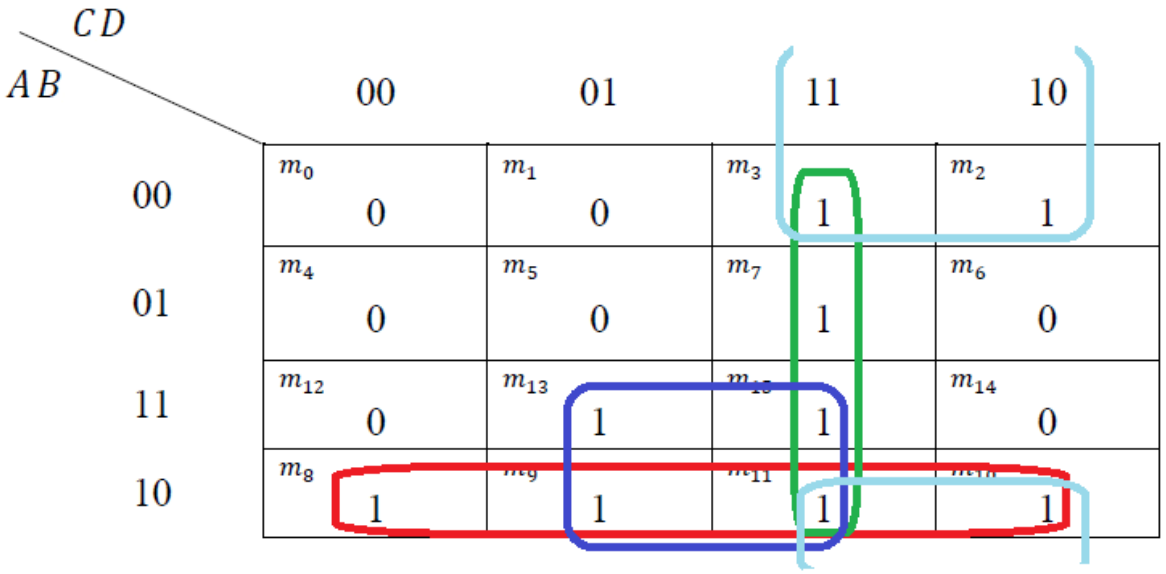
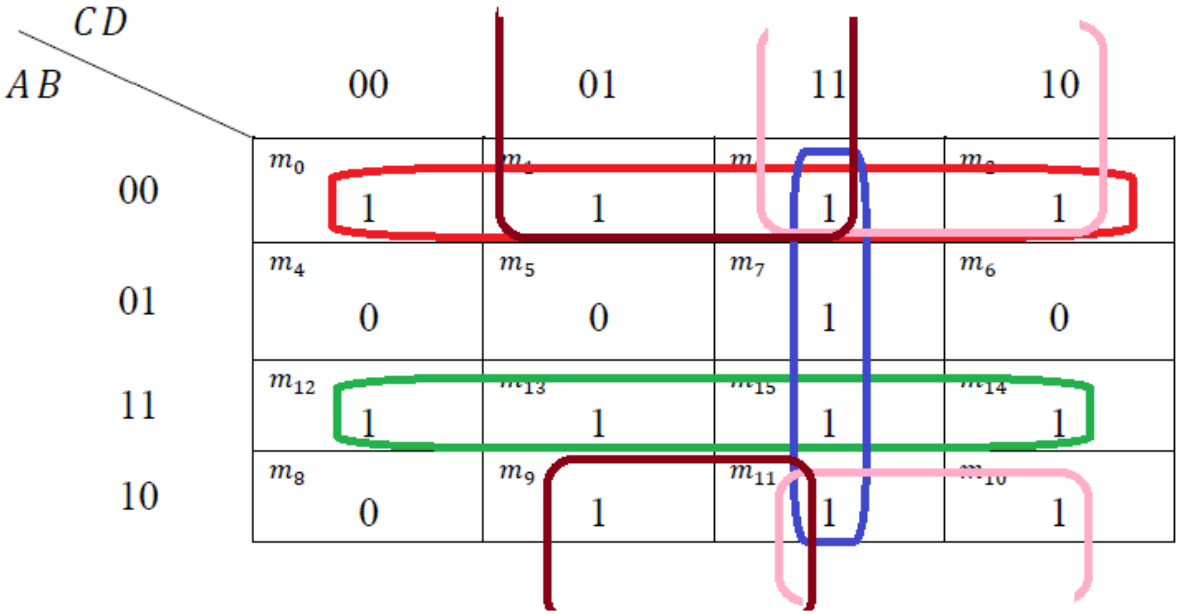


$F = \bar{B}C + A\bar{B} + AD + CD$

Ex # 6: $F(A,B,C,D) = \sum(0,1,2,3,7,9,10,11,12,13,14,15)$

40



$$F = \bar{B}D + \bar{A}\bar{B} + AB + CD + \bar{B}C$$

Product of Sum Simplification

- The 1's placed in the k-map represent the minterms present in the function
- The 0's represent the minterms not included in the function and hence is the complement of the same function
- In order to present the function as product of sum
  1. Combine squares containing 0's
  2. Then compute the complement of the obtained example
  - The simplified function in now in product of sum

Don’t Care Conditions

- The logical sum of the minterms associated with a Boolean function specifies the conditions under which the function is equal to 1
- The function is equal to 0 for the rest of the minterms
- Functions that have unspecified outputs for some input combinations are called incompletely specified functions
- A don’t-care minterm is a combination of variables whose logical value is not specified
- Don’t care conditions are indicated with X in k-map and truth tables and can be taken as 0 or 1

K-Map Examples (POS-Form)

Simple simplification rules are;

1. Allowed Groups of size are: 16, 8, 4, 2, 1 (Priority Order for 4 variables)
2. Grouping of adjacent 0's is allowed along horizontal and vertical direction. Folding and overlapping is allowed.
3. Cover maximum number of 0's with minimum number of groups.

Ex # 1: $F(A,B,C) = \prod(0,1,6,7)$

	00	01	11	10
0	m_0 0	m_1 0	m_3 1	m_2 1
1	m_4 1	m_5 1	m_7 0	m_6 0

Ex # 2: $F(A,B,C) = \prod(2,3,4,5,6,7)$

	00	01	11	10
0	m_0 1	m_1 1	m_3 0	m_2 0
1	m_4 0	m_5 0	m_7 0	m_6 0

Ex # 1: $F(A,B,C) = \prod(0,1,6,7)$

A \ $B\,C$	00	01	11	10
0	m_0 0	m_1 0	m_3 1	m_2 1
1	m_4 1	m_5 1	m_7 0	m_6 0

$$F = (\bar{A} + \bar{B}) \cdot (A + B)$$

Ex # 2: $F(A,B,C) = \prod(2,3,4,5,6,7)$

A \ $B\,C$	00	01	11	10
0	m_0 1	m_1 1	m_3 0	m_2 0
1	m_4 0	m_5 0	m_7 0	m_6 0

$F = (\overline{B}) \cdot (\overline{A})$

Ex # 3: $F(A,B,C,D) = \prod(0,1,2,3,4,5,6,8,9,10,11,14,15)$

$A\,B$ \ $C\,D$	00	01	11	10
00	m_0 0	m_1 0	m_3 0	m_2 0
01	m_4 0	m_5 0	m_7 1	m_6 0
11	m_{12} 1	m_{13} 1	m_{15} **0**	m_{14} 0
10	m_8 0	m_9 0	m_{11} 0	m_{10} 0

$F = (A + C) \cdot (\bar{C} + D) \cdot (B) \cdot (\bar{A} + \bar{C})$

Ex # 4: $F(A,B,C) = \sum(0,1,6,7) + d(2,3)$

A \ BC	00	01	11	10
0	m_0 1	m_1 1	m_3 X	m_2 X
1	m_4 0	m_5 0	m_7 1	m_6 1

$F = B + \bar{A}$ (SoP)

A \ BC	00	01	11	10
0	m_0 1	m_1 1	m_3 X	m_2 X
1	m_4 0	m_5 0	m_7 1	m_6 1

$F = \bar{A} + B$ (PoS)

Ex # 5: $F(A,B,C,D) = \sum(1,3,7,11,15) + d(0,2,5)$

$A\,B$ \ $C\,D$	00	01	11	10
00	m_0 X	m_1 1	m_3 1	m_2 X
01	m_4 0	m_5 X	m_7 1	m_6 0
11	m_{12} 0	m_{13} 0	m_{15} 1	m_{14} 0
10	m_8 0	m_9 0	m_{11} 1	m_{10} 0

$F = CD + \bar{A}\bar{B}$ (SOP)

Ex # 5: $F(A,B,C,D) = \sum(1,3,7,11,15) + d(0,2,5)$

$A\,B$ \ $C\,D$	00	01	11	10
00	m_0 X	m_1 1	m_3 1	m_2 X
01	m_4 0	m_5 X	m_7 1	m_6 0
11	m_{12} 0	m_{13} 0	m_{15} 1	m_{14} 0
10	m_8 0	m_9 0	m_{11} 1	m_{10} 0

$F = (\bar{A} + C) \cdot D$ (POS)

Practice Problem 1

- Simplify the following Boolean functions by using k-map method
  a) $F_1 = xy + x'y'z' + x'yz'$
  b) $F_2 = A'B + BC' + BC + AB'C'$
  c) $F_3 = M + MC + AC + AM + AMC$
  d) $F(W,X,Y,Z) = \sum(1, 4, 5, 6, 12, 14, 15)$

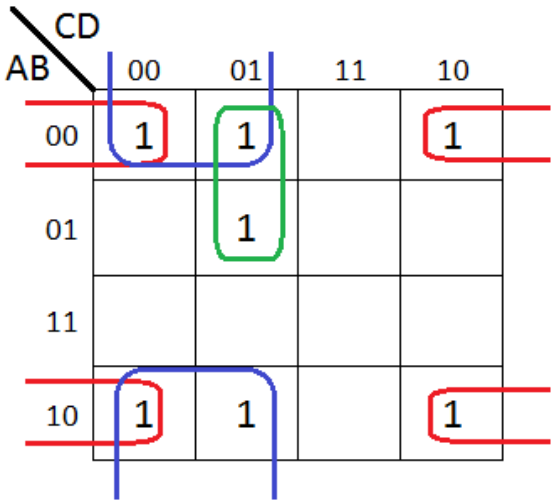
Answers

a) F1 = x'z' + xy
b) F2 = B + AC'
c) F3 = M + AC
d) F(w,x,y,z) = w'y'z + wxy + xz'

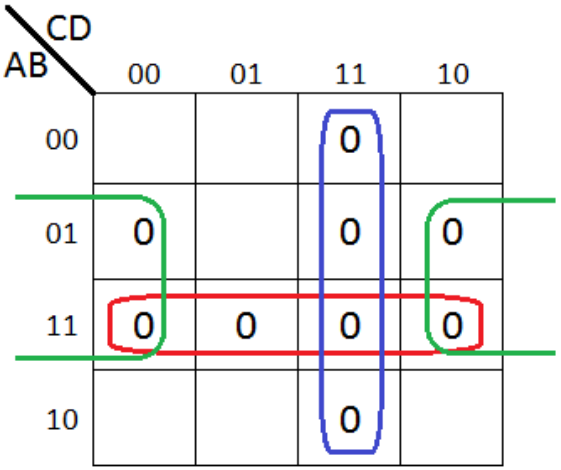
Practice Problem 2

- Simplify the following Boolean function into
  a) Sum-of-products form
  b) Product-of-sums form

$$F(A,B,C,D) = \sum(0, 1, 2, 5, 8, 9, 10)$$



F = B'D' + B'C' + A'C'D



F' = AB + CD + BD'

F = (A' + B)(C' + D')(B' + D)

Practice Problem 3

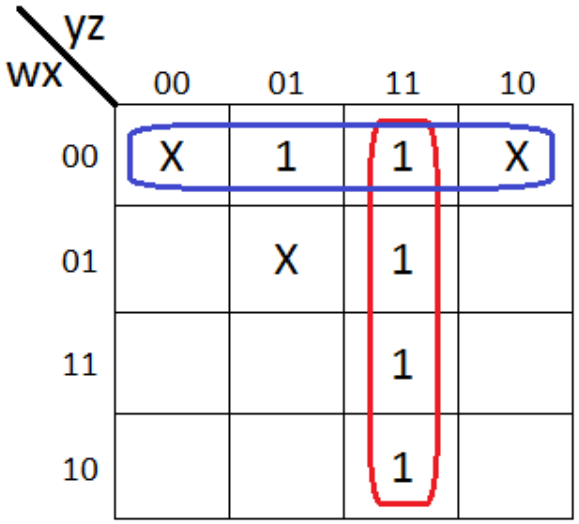
- Simplify the Boolean Function

$$F(w,x,y,z) = \sum(1, 3, 7, 11, 15)$$

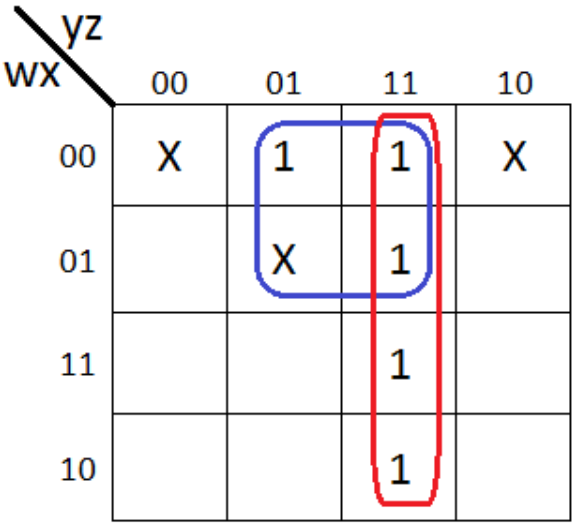
- Which has the don't care conditions

$$d(w,x,y,z) = \sum(0,2,5)$$

Both Results are Valid



F = wx + yz



F = w'z + yz

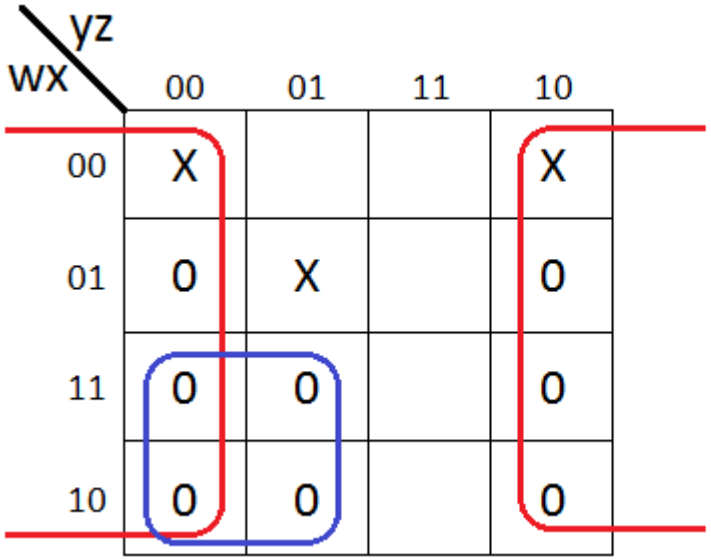
Practice Problem 4

- Simplify the Boolean Function in POS form

$$F(w,x,y,z) = \sum(1, 3, 7, 11, 15)$$

- Which has the don't care conditions

$$d(w,x,y,z) = \sum(0,2,5)$$



F' = z' + wy'

F = (z)(w' + y)

Practice Problem 5

- Find all the prime implicants for the following Boolean functions, and determine which are essential
  a) $F(w, x, y, z) = \Sigma(0, 2, 4, 5, 6, 7, 8, 10, 13, 15)$
  b) $F(A, B, C, D) = \Sigma(0, 2, 3, 5, 7, 8, 10, 11, 14, 15)$

Practice Problem 6

- Simplify the following Boolean functions F, together with the don't-care conditions d

a) $F(x, y, z) = \Sigma(0, 1, 4, 5, 6)$
$d(x, y, z) = \Sigma(2, 3, 7)$

b) $F(A, B, C, D) = \Sigma(0, 6, 8, 13, 14)$
$d(A, B, C, D) = \Sigma(2, 4, 10)$

c) $F(A, B, C, D) = \Sigma(5, 6, 7, 12, 14, 15)$
$d(A, B, C, D) = \Sigma(3, 9, 11)$