



# EE1005 – Digital Logic Design

- **Lecture Slides**
- **Week 3**

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# Signed Binary to Decimal Conversion (1/2)

- If MSB is 0
  - Convert the binary number to decimal like unsigned number

## Example

Convert signed  $(0100\ 1100)_2$  to decimal equivalent

## Solution

As MSB is 0 so number is positive

$$\text{Dec} = (1 \times 2^6) + (1 \times 2^3) + (1 \times 2^2)$$

$$\text{Dec} = 64 + 8 + 4$$

$$\text{Dec} = 76$$

$$\Rightarrow (0100\ 1100)_2 = (+76)_{10}$$

# Signed Binary to Decimal Conversion (2/2)

- If MSB is 1, then follow these three steps
  - Compute the 2's complement of number
  - Convert to decimal
  - Place a negative sign with answer

## Example

Convert signed  $(1100\ 1100)_2$  to decimal equivalent

## Solution

As MSB is 1 so number is negative

### **Step 1:**

Compute 2's complement of  $(1100\ 1100)_2$  which is  $(00110100)_2$

### **Step 2:**

Convert the complemented number to decimal

$$(00110100)_2 = (1 \times 2^5) + (1 \times 2^4) + (1 \times 2^2) = 32 + 16 + 4 = (52)_{10}$$

### **Step 3:**

Place a negative sign with answer

$$\Rightarrow (1100\ 1100)_2 = (-52)_{10}$$

# Signed Decimal to Binary Conversion (1/2)

- If the number is positive
  - Convert to binary by using repeated division method like unsigned numbers

## Example


Convert  $(+25)_{10}$  to binary equivalent

## Solution

As the number is positive, so we will use the repeated division method

$$\Rightarrow (+25)_{10} = (11001)_2$$

2	25		
2	12	-	1
2	6	-	0
2	3	-	0
2	1	-	1
	0	-	1



# Signed Decimal to Binary Conversion (2/2)

- If the number is negative
  - Convert the magnitude of the number to binary by using **correct number of bits**
  - Compute the 2's complement to get the final answer

## Example

Convert  $(-25)_{10}$  to binary equivalent

## Solution

As the number is negative so we need to follow two steps

### **Step 1:**

Convert the magnitude of  $(-25)$  to binary

And we need at least 6 bits to represent  $(-25)$

$$(25)_{10} = (011001)_2$$

### **Step 2:**

Zero Appended to Complete 6 bits

Compute the 2's complement to get the final answer

2's complement of  $(011001)_2$  is  $(100111)_2$

$$\Rightarrow (-25)_{10} = (100111)_2$$

In case we have used 5 bits in step 1, the final answer would be  $(00111)_2$ , which is obviously incorrect

# Arithmetic Subtraction in Signed Binary Numbers

- For subtraction we prefer 2's complement method
  - M-N case in which M>N
  - A carry out of the sign-bit position is discarded.

## Example

Subtract 14 from 26 by using 2's complement method.

## Solution

$$\begin{aligned}26 &= (11010)_2 \\14 &= (01110)_2 \\ \Rightarrow -14 &= (10010)_2\end{aligned}$$

$$\begin{array}{rcll}26 & = & 1 & 1 & 0 & 1 & 0 \\-14 & = & + & 1 & 0 & 0 & 1 & 0 \\+12 & = & & 1 & 0 & 1 & 1 & 0 & 0\end{array}$$

End Carry Discarded

Final Answer =  $(01100)_2 = (+12)_{10}$

# Arithmetic Addition in Signed Binary Numbers

- M-N case in which  $M < N$
- A carry out of the sign-bit position is discarded
- Take 2's complement and add a negative sign

## Example

Compute  $(+6) + (-13)$  in binary.

## Solution

$$+6 = (00110)_2$$

$$+13 = (01101)_2$$

$$\Rightarrow -13 = (10011)_2$$

$$\begin{array}{rcl} +6 & = & 0\ 0\ 1\ 1\ 0 \end{array}$$

$$\begin{array}{rcl} -13 & = & +\underline{1\ 0\ 0\ 1\ 1} \end{array}$$

$$\begin{array}{rcl} -7 & = & 1\ 1\ 0\ 0\ 1 \end{array}$$

•  $\Rightarrow$

$$\boxed{-(00111)_2 = (-7)_{10}}$$

# Outline

- Boolean Algebra
- Boolean Functions
- Canonical Forms
  - Minterms and Maxterms
- Standard forms
  - Sum of Products (SOP)
  - Product of Sums (POS)
- Digital Logic Gates
- Integrated Circuits



# Basic Definitions (1/2)

- Boolean algebra, like any other deductive mathematical system, may be defined with a set of elements, a set of operators, and a number of unproved axioms
- Set
  - A set of elements is any collection of objects, usually having a common property
  - If  $S$  is a set, and  $x$  and  $y$  are certain objects, then the notation  $x \in S$  means that  $x$  is a member of the set  $S$  and  $y \notin S$  means that  $y$  is not an element of  $S$
  - A set of elements is specified by braces:  $A = \{1, 2, 3, 4\}$

# Basic Definitions (2/2)

- Binary Operator
  - A binary operator defined on a set  $S$  of elements is a rule that assigns, to each pair of elements from  $S$ , a unique element from  $S$
  - As an example, consider the relation  $a * b = c$
  - We say that  $*$  is a binary operator if it specifies a rule for finding  $c$  from the pair  $(a, b)$  and also if  $a, b, c \in S$
  - However,  $*$  is not a binary operator if  $a, b \in S$ , and if  $c \notin S$

# The Postulates of a Mathematical System

## 1) Closure

- A set is said to be closed if the result obtained after applying a binary operator belongs to the same set

## 2) Associative Law

- $(x * y) * z = x * (y * z)$  for all  $x, y, z, \in S$

## 3) Commutative Law

- $(x * y) = (y * x)$  for all  $x, y, \in S$

## 4) Identity Element

- $e * x = x * e = x$  for every  $x \in S$

## 5) Inverse

- $x * y = e$  ( $y$  is inverse and  $e$  is identity element)

## 6) Distributive Law

- $x * (y . z) = (x * y) . (x * z)$

# Two Valued Boolean Algebra

- A two-valued Boolean algebra is defined on a set of two elements,  $B = \{0, 1\}$ , with rules for the two binary operators  $+$  and  $\cdot$  and complement as shown in the following operator tables

$x$	$y$	$x \cdot y$
0	0	0
0	1	0
1	0	0
1	1	1

$x$	$y$	$x + y$
0	0	0
0	1	1
1	0	1
1	1	1

$x$	$x'$
0	1
1	0

- These rules are exactly the same as the AND, OR, and NOT operations, respectively

# Basic Theorems and Properties of Boolean Algebra (1/2)

- Duality
  - It states that every algebraic expression deducible from the postulates of Boolean algebra remains valid if the operators and identity elements are interchanged
  - If the dual of an algebraic expression is desired, we simply interchange OR and AND operators and replace 1's by 0's and 0's by 1's

# Basic Theorems and Properties of Boolean Algebra (2/2)

## *Postulates and Theorems of Boolean Algebra*

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Postulate 2	(a)	$x + 0 = x$	(b)	$x \cdot 1 = x$
Postulate 3, commutative	(a)	$x + y = y + x$	(b)	$xy = yx$
Postulate 4, distributive	(a)	$x(y + z) = xy + xz$	(b)	$x + yz = (x + y)(x + z)$
Postulate 5	(a)	$x + x' = 1$	(b)	$x \cdot x' = 0$
Theorem 1	(a)	$x + x = x$	(b)	$x \cdot x = x$
Theorem 2	(a)	$x + 1 = 1$	(b)	$x \cdot 0 = 0$
Theorem 3, involution		$(x')' = x$		
Theorem 4, associative	(a)	$x + (y + z) = (x + y) + z$	(b)	$x(yz) = (xy)z$
Theorem 5, DeMorgan	(a)	$(x + y)' = x'y'$	(b)	$(xy)' = x' + y'$
Theorem 6, absorption	(a)	$x + xy = x$	(b)	$x(x + y) = x$

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# Operator Precedence

- The operator precedence for evaluating Boolean expressions
  1. Parentheses
  2. NOT
  3. AND
  4. OR

# Boolean Functions (1/2)

- A Boolean function described by an algebraic expression consists of binary variables, the constants 0 and 1, and the logic operation symbols

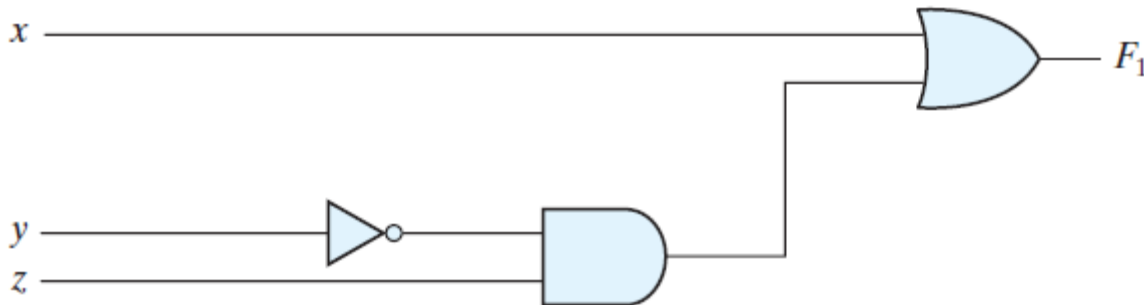
$$F_1 = x + y'z$$

- A Boolean function can be represented in a truth table
- The number of rows in the truth table is  $2^n$ , where  $n$  is the number of variables in the function
- A Boolean function can be transformed from an algebraic expression into a circuit diagram (also called schematic) composed of logic gates connected in a particular structure



# Boolean Functions (2/2)

$$F_1 = x + y'z$$

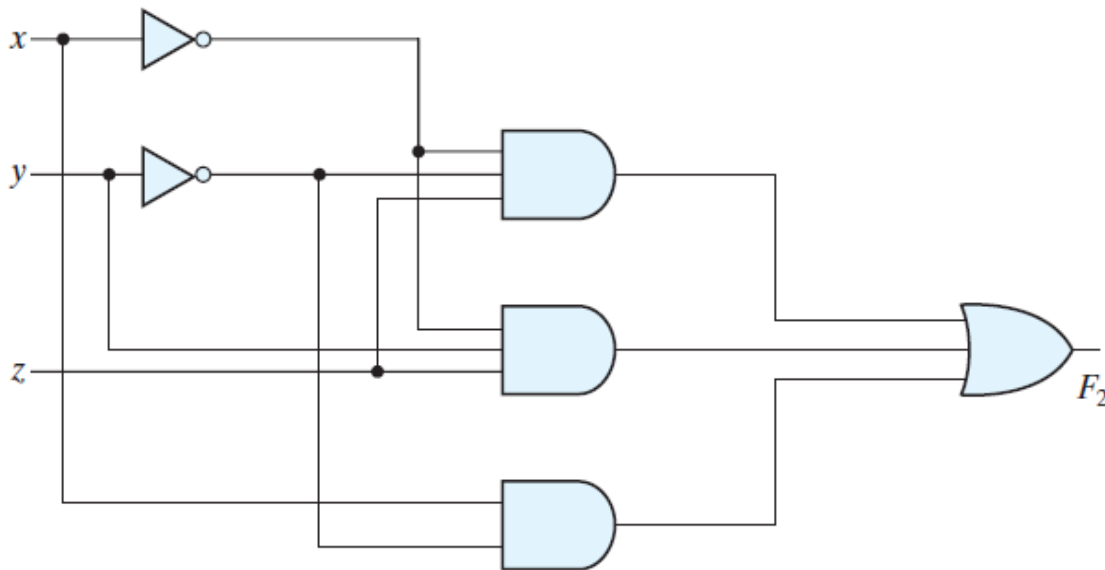


<b><i>x</i></b>	<b><i>y</i></b>	<b><i>z</i></b>	<b><i>F<sub>1</sub></i></b>
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

# Practice Problem

- Construct the truth table and circuit diagram for the following Boolean function

$$F_2 = x'y'z + x'yz + xy'$$



x	y	z	x'	y'	x'y'z	x'yz	xy'	F <sub>2</sub>
0	0	0	1	1	0	0	0	0
0	0	1	1	1	1	0	0	1
0	1	0	1	0	0	0	0	0
0	1	1	1	0	0	1	0	1
1	0	0	0	1	0	0	1	1
1	0	1	0	1	0	0	1	1
1	1	0	0	0	0	0	0	0
1	1	1	0	0	0	0	0	0

# Algebraic Manipulation (1/2)

- When a Boolean expression is implemented with logic gates, each term requires a gate and each variable within the term designates an input to the gate
- We define a literal to be a single variable within a term
- The function  $F_2 = x'y'z + x'yz + xy'$  has 3 terms and 8 literals
- By reducing the number of terms, the number of literals, or both in a Boolean expression, it is often possible to obtain a simpler circuit
- The manipulation of Boolean algebra consists mostly of reducing an expression for the purpose of obtaining a simpler circuit

# Algebraic Manipulation (2/2)

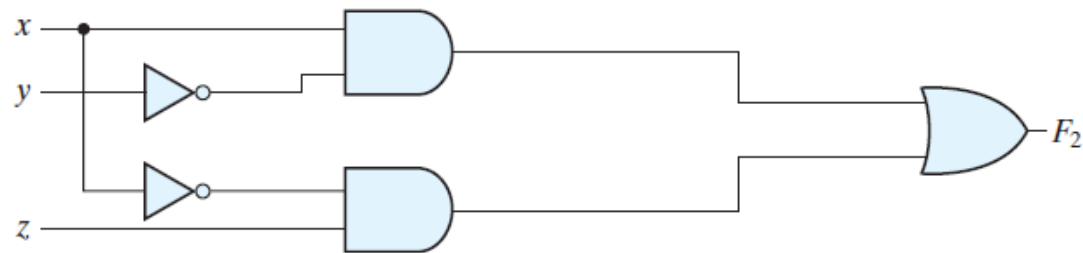
- The simple manipulation consists of applying basic theorems and relations
- The function  $F_2 = x'y'z + x'yz + xy'$  can be simplified in the following way

$$F_2 = x'y'z + x'yz + xy'$$

$$F_2 = x'z(y' + y) + xy'$$

$$F_2 = x'z(1) + xy'$$

$$F_2 = x'z + xy'$$



- The same function is reduced to two terms. The truth table will be the same and the schematic is now simplified

# Complement of a Function

- The complement of a function  $F$  is  $F'$  and can be obtained by an interchange of 1's for 0's and 0's for 1's in the value of the function
- The complement of a function may be derived algebraically through DeMorgan's theorems
- DeMorgan's theorems for any number of variables resemble the two-variable case

$$(A + B + C + \dots + F)' = A'B'C' \dots F'$$

$$(ABC \dots F)' = A' + B' + C' + \dots + F'$$

- The complement of a function is obtained by interchanging AND and OR operators and complementing each literal

# Practice Problem

- Find the complements of the following functions by applying Demorgan's laws

i.  $F_1 = x'yz' + x'y'z$

ii.  $F_2 = x(y'z' + yz)$

$$F_1' = (x'yz' + x'y'z)' = (x'yz')'(x'y'z)' = (x + y' + z)(x + y + z')$$

$$\begin{aligned} F_2' &= [x(y'z' + yz)]' = x' + (y'z' + yz)' = x' + (y'z')'(yz)' \\ &= x' + (y + z)(y' + z') \\ &= x' + yz' + y'z \end{aligned}$$

# Another Way to Compute the Complement of a Boolean Function

- A simpler procedure for deriving the complement of a function is to take the dual of the function and complement each literal

# Practice Problem

- Find the complements of the following functions by using duality

i.  $F_1 = x'yz' + x'y'z$

ii.  $F_2 = x(y'z' + yz)$

1.  $F_1 = x'yz' + x'y'z.$

The dual of  $F_1$  is  $(x' + y + z')(x' + y' + z).$

Complement each literal:  $(x + y' + z)(x + y + z') = F_1'$

2.  $F_2 = x(y'z' + yz).$

The dual of  $F_2$  is  $x + (y' + z')(y + z).$

Complement each literal:  $x' + (y + z)(y' + z') = F_2'$



Property # 9 :  $\overline{(x+y)} = \bar{x} \cdot \bar{y}$  De-Morgan Law

$x$	$y$	$\bar{x}$	$\bar{y}$	$x+y$	$\overline{x+y}$	$\bar{x} \cdot \bar{y}$
0	0	1	1	0	1	1
0	1	1	0	1	0	0
1	0	0	1	1	0	0
1	1	0	0	1	0	0
					L.H.S	R.H.S

Again: Property 9

$$\overline{xy} = \bar{x} + \bar{y} \quad \text{De-Morgan's Law}$$

x	y	$\bar{x}$	$\bar{y}$	xy	$\overline{xy}$	$\bar{x} + \bar{y}$
0	0	1	1	0	1	1
0	1	1	0	0	1	1
1	0	0	1	0	1	1
1	1	0	0	1	0	0
					L.H.S	R.H.S

It is verified that:

$$L.H.S = R.H.S$$

$$\overline{xy} = \bar{x} + \bar{y}$$

Example:  $F = A + \bar{B}C$  , Find  $\bar{F} = ?$

$$\begin{aligned}\bar{F} &= \overline{A + \bar{B}C} \\ &= \bar{A} \cdot \overline{\bar{B}C} \\ &= \bar{A} \cdot (\bar{\bar{B}} + \bar{C}) \\ &= \bar{A} \cdot (B + \bar{C})\end{aligned}$$

Another Example:  $F = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}$ , Find  $\bar{F}$

$$\bar{F} = \overline{\bar{A}\bar{B}C + \bar{A}BC + A\bar{B}}$$

$$\bar{F} = \overline{\bar{A}\bar{B}C} + \overline{\bar{A}BC} + \overline{A\bar{B}}$$

$$\bar{F} = \overline{\bar{A}\bar{B}C} \cdot \overline{\bar{A}BC} \cdot \overline{A\bar{B}}$$

$$= (\overline{\bar{A}} + \overline{\bar{B}} + \overline{\bar{C}}) \cdot (\overline{\bar{A}} + \overline{B} + \overline{\bar{C}}) \cdot (\overline{A} + \overline{\bar{B}})$$

$$= (A + B + \bar{C}) \cdot (A + \bar{B} + \bar{C}) \cdot (\bar{A} + B)$$



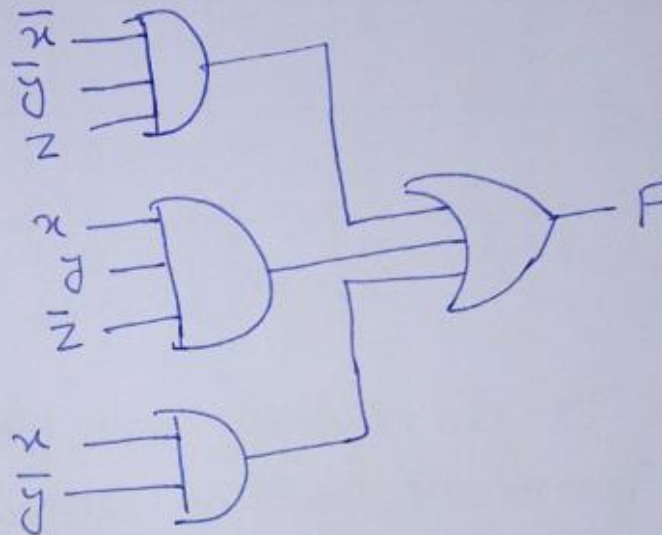
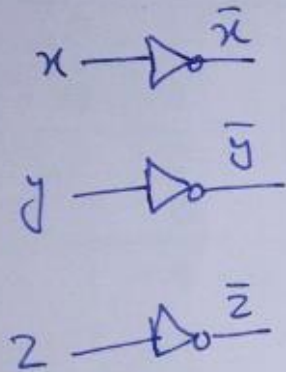
Example: Simplify  $F$  to minimum no. of literals.

$$\begin{aligned}\text{Given } F &= A(\bar{A} + B) \\ &= A\bar{A} + AB \\ &= 0 + AB = AB\end{aligned}$$

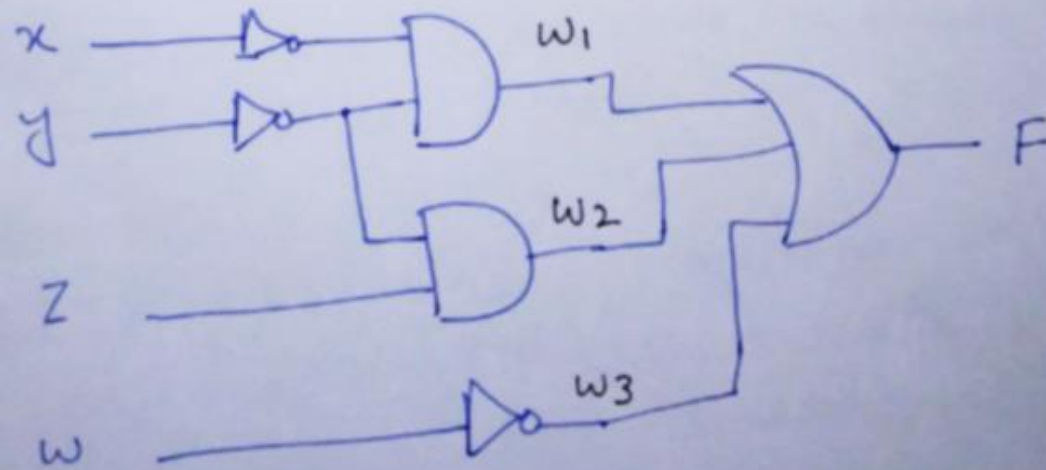
$$\begin{aligned}\text{Example: } F &= (A+B)(A+\bar{B}) \\ &= AA + A\bar{B} + BA + B\bar{B} \\ &= A + A\bar{B} + BA + 0 \\ &= A(1 + \bar{B} + B) = A \cdot 1 = A\end{aligned}$$

① Drawing Circuit Diagram From Boolean Function:

$$F = \bar{x} \bar{y} z + x y \bar{z} + x \bar{y}$$



② Writing Boolean Function from Given Circuit



$$w_1 = \bar{x} \bar{y} \quad ; \quad w_2 = \bar{y} z \quad ; \quad w_3 = \bar{w}$$

$$F = w_1 + w_2 + w_3 = \bar{x} \bar{y} + \bar{y} z + \bar{w}$$

③ Drawing Truth Table of Boolean Function

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$$F = \bar{x} \bar{y} z + x y \bar{z} + x y z$$

x	y	z	$\bar{x}$	$\bar{y}$	$\bar{z}$	$\bar{x} \bar{y} z$	$x y \bar{z}$	$x y z$	F
0	0	0	1	1	1	0	0	0	0
0	0	1	1	1	0	1	0	0	1
0	1	0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0	0	0
1	0	0	0	1	1	0	0	0	0
1	0	1	0	1	0	0	0	0	0
1	1	0	0	0	1	0	1	0	1
1	1	1	0	0	0	0	0	1	1



standard Form: When all variables are present in each literal.

Examples:

①  $F = \bar{x}\bar{y}z + xy\bar{z} + \bar{x}yz$  standard Form ✓

②  $G = \bar{x}\bar{y}z + x\bar{y}z + \bar{x}y$  Non standard Form ✗

③  $H = \bar{x}\bar{y}\bar{z} + \bar{x}y\bar{z} + \bar{x}\bar{y}z$  Standard Form ✓

④  $I = xy\bar{z} + x\bar{y}z + \bar{x}yz + x\bar{y}\bar{z}$  stand Form ✓

Drawing Directly Truth Table from standard function form

$$\textcircled{1} \quad F = \bar{x} \bar{y} z + x y \bar{z} + \bar{x} y z$$

$0 \ 0 \ 1$ 
 $1 \ 1 \ 0$ 
 $0 \ 1 \ 1$

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x	y	z	F	H	I
0	0	0	0	1	0
0	0	1	1	1	0
0	1	0	0	1	0
0	1	1	1	0	1
1	0	0	0	0	1
1	0	1	0	0	1
1	1	0	1	0	1
1	1	1	0	0	0

④ How to write Boolean Function from Given Truth Table

A	B	C	F <sub>1</sub>	F <sub>2</sub>
0	0	0	0	1
0	0	1	1	1
0	1	0	0	0
0	1	1	1	0
1	0	0	1	1
1	0	1	0	1
1	1	0	0	0
1	1	1	1	0

Note:

if A=0  $\Rightarrow \bar{A}$

if A=1  $\Rightarrow A$

$$F_1 = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + ABC$$

$$F_2 = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + A\bar{B}C + \bar{A}BC$$

# Canonical Form

- Boolean functions expressed as a sum of minterms or product of maxterms are said to be in canonical form
- Minterms
  - The AND of N variables such that they equals to 1 is called minterm or standard product
  - There are  $2^N$  possible minterms with N variables
  - Minterms are denoted by lower case m
- Maxterms
  - The OR of N variables such that the result is equal to 0 is called maxterm or standard sum
  - There are  $2^N$  possible maxterms with N variables
  - Maxterms are denoted by upper case M

# Calculating Minterms and Maxterms

- Each minterm is obtained from an AND term of the  $n$  variables, with each variable being primed if the corresponding bit of the binary number is a 0 and unprimed if a 1
- Each maxterm is obtained from an OR term of the  $n$  variables, with each variable being unprimed if the corresponding bit is a 0 and primed if a 1
- The complement of a minterm is equal to its corresponding maxterm

# Minterms and Maxterms for 3 Variables

x	y	z	Minterms		Maxterms	
			Term	Designation	Term	Designation
0	0	0	$x'y'z'$	$m_0$	$x + y + z$	$M_0$
0	0	1	$x'y'z$	$m_1$	$x + y + z'$	$M_1$
0	1	0	$x'yz'$	$m_2$	$x + y' + z$	$M_2$
0	1	1	$x'yz$	$m_3$	$x + y' + z'$	$M_3$
1	0	0	$xy'z'$	$m_4$	$x' + y + z$	$M_4$
1	0	1	$xy'z$	$m_5$	$x' + y + z'$	$M_5$
1	1	0	$xyz'$	$m_6$	$x' + y' + z$	$M_6$
1	1	1	$xyz$	$m_7$	$x' + y' + z'$	$M_7$

# Representing a Boolean Function in Canonical Form

- Boolean function can be represented in canonical form in two ways
  - As a sum of minterms
  - As a product of maxterms
- To represent a Boolean function in canonical form all the terms of that function must contain all the variables

Assume following truth table is given

	A	B	C	F <sub>1</sub>	F <sub>2</sub>
0	0	0	0	1	0
1	0	0	1	0	1
2	0	1	0	1	1
3	0	1	1	1	0
4	1	0	0	0	0
5	1	0	1	0	1
6	1	1	0	1	1
7	1	1	1	1	0



Simplest way of writing expression for  $F_1$ , see location of 1's in truth table.

$$F_1 = \sum (0, 2, 3, 6, 7)$$

$$= m_0 + m_2 + m_3 + m_6 + m_7$$

$$= \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C} + ABC$$

These individual terms are called minterms.  
i.e. terms corresponding to "1".

Note: if  $A=1 \Rightarrow A$   
 $A=0 \Rightarrow \bar{A}$

Consider again  $F_1$ , now write expression for  $F_1$  by looking at 0's location.

$$F_1 = \prod (1, 4, 5)$$

$$= M_1 \cdot M_4 \cdot M_5 = M_1 \cdot M_4 \cdot M_5$$

$$= (A+B+\bar{C}) \cdot (\bar{A}+B+C) \cdot (\bar{A}+B+\bar{C})$$

These individual terms are called maxterms  
i.e. terms corresponding to "0". Note: if  $A=1 \Rightarrow \bar{A}$   
 $A=0 \Rightarrow A$

Now, let us write minterm/max term express for  $F_2$ .

Minterm Expression for  $F_2$ :

$$F_2 = \sum (1, 2, 5, 6) \quad \leftarrow \text{1st way}$$

$$= m_1 + m_2 + m_5 + m_6 \quad \leftarrow \text{2nd way}$$

$$= \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}C + AB\bar{C} \quad \leftarrow \text{3rd way}$$

Max term Expression for  $F_2$ .

$$F_2 = \prod (0, 3, 4, 7) \quad \leftarrow \text{1st way}$$

$$= M_0 \cdot M_3 \cdot M_4 \cdot M_7 \quad \leftarrow \text{2nd way}$$

$$= (A+B+C) \cdot (A+\bar{B}+\bar{C}) \cdot (\bar{A}+B+C) \cdot (\bar{A}+\bar{B}+\bar{C})$$

SOP Expression: (Sum of Product)

Expression of function written using min terms

POS Expression: (Product of Sum)

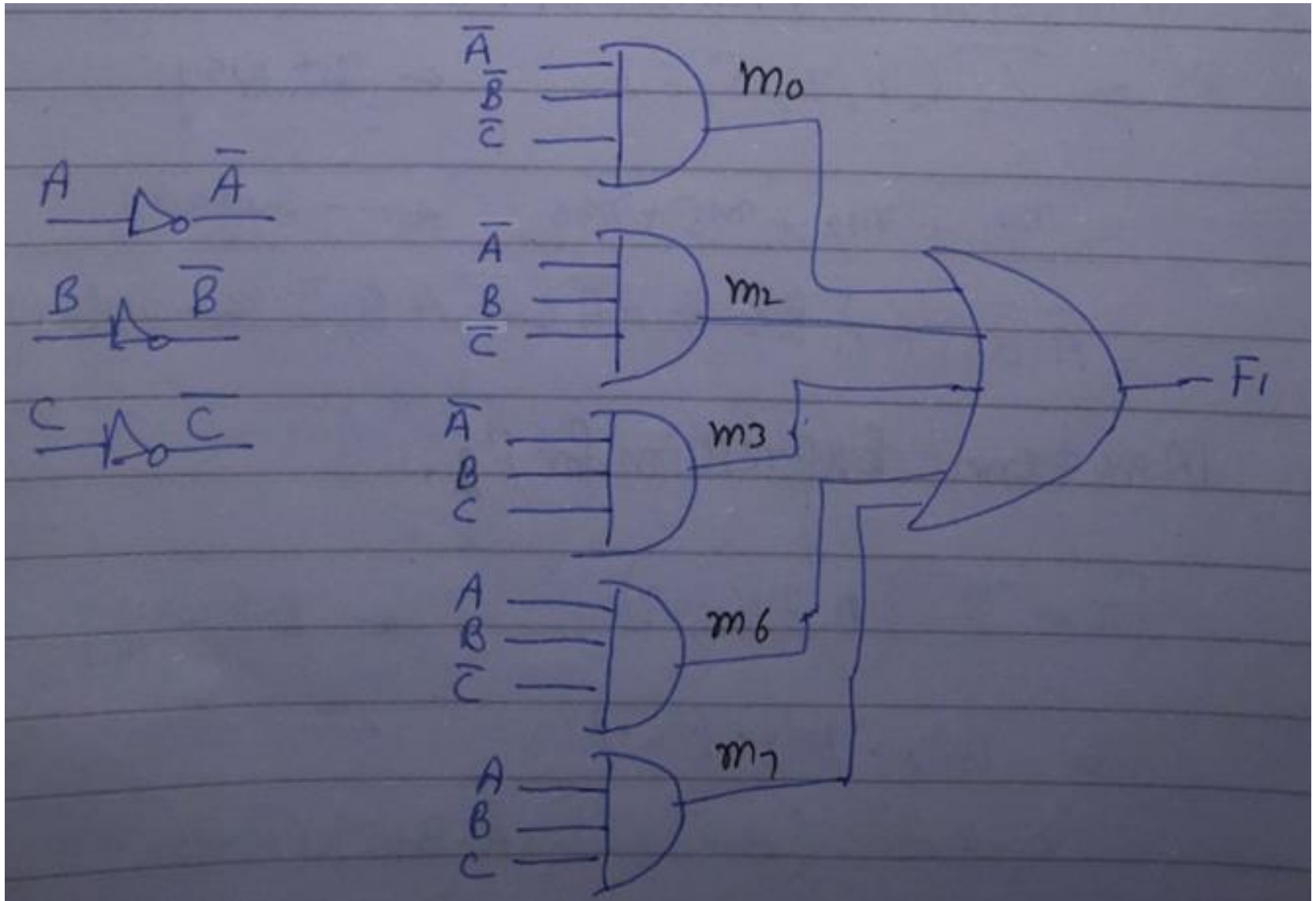
Expression of function using max terms.

⑥ Drawing Circuit Diagram using SOP/POS Expr.

SOP Expression Circuit:

$$\begin{aligned} F_1 &= \sum (0, 2, 3, 6, 7) = m_0 + m_2 + m_3 + m_6 + m_7 \\ &= \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + ABC \end{aligned}$$



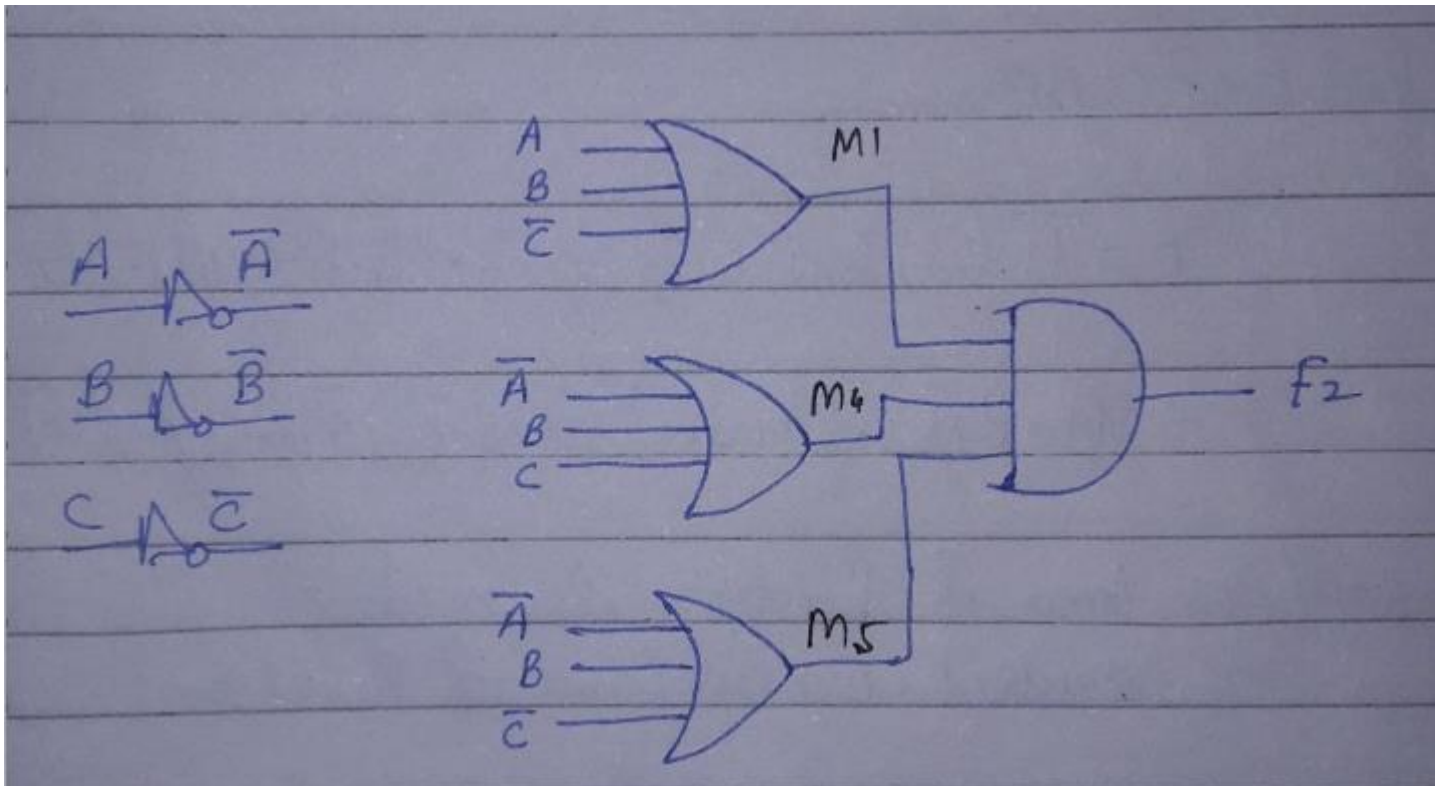


POS Expression Circuit:

$$F_1 = \prod (1, 4, 5)$$

$$= M_1 \cdot M_4 \cdot M_5$$

$$= (A+B+\bar{C}) \cdot (\bar{A}+B+C) \cdot (\bar{A}+B+\bar{C})$$





⑦ Conversion of given function to standard  
SOP / POS Form

$$F = A + \overline{B}C$$

A	B	C	$\overline{B}$	$\overline{B}C$	F
0	0	0	1	0	0
0	0	1	1	1	1
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	1	0	1
1	0	1	1	1	1
1	1	0	0	0	1
1	1	1	0	0	1

From above truth table, we can write;

SOP Expression:

$$F = \sum (1, 4, 5, 6, 7) = m_0 + m_4 + m_5 + m_6 + m_7$$
$$= \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC$$

POS Expression:



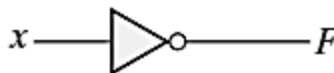

$$F = \prod (0, 2, 3) = M_0 \cdot M_2 \cdot M_3$$
$$= (A+B+C) \cdot (A+\bar{B}+C) \cdot (A+\bar{B}+\bar{C})$$

Relationship b/w SOP/POS Forms:





$$\text{SOP: } F_{\text{sop}} = \sum (1, 4, 5, 6, 7) = m_0 + m_4 + m_5 + m_6 + m_7$$

$$\text{POS: } F_{\text{pos}} = \prod (0, 2, 3) = M_0 \cdot M_2 \cdot M_3$$

# Digital Logic Gates (1/2)

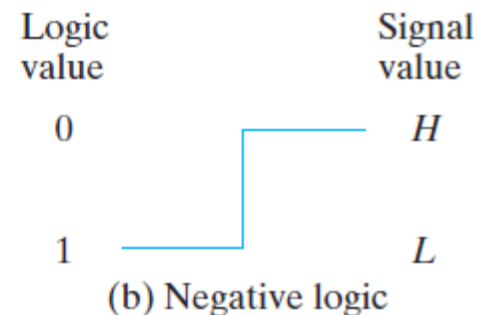
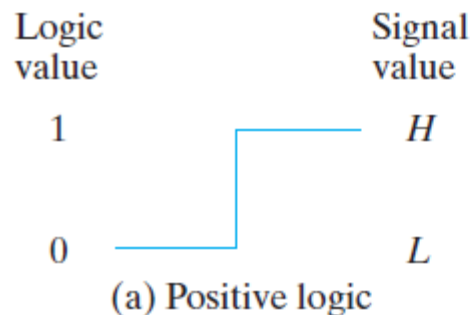
Name	Graphic symbol	Algebraic function	Truth table															
AND		$F = x \cdot y$	<table><tr><th><math>x</math></th><th><math>y</math></th><th><math>F</math></th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table>	$x$	$y$	$F$	0	0	0	0	1	0	1	0	0	1	1	1
$x$	$y$	$F$																
0	0	0																
0	1	0																
1	0	0																
1	1	1																
OR		$F = x + y$	<table><tr><th><math>x</math></th><th><math>y</math></th><th><math>F</math></th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table>	$x$	$y$	$F$	0	0	0	0	1	1	1	0	1	1	1	1
$x$	$y$	$F$																
0	0	0																
0	1	1																
1	0	1																
1	1	1																
Inverter		$F = x'$	<table><tr><th><math>x</math></th><th><math>F</math></th></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td></tr></table>	$x$	$F$	0	1	1	0									
$x$	$F$																	
0	1																	
1	0																	
Buffer		$F = x$	<table><tr><th><math>x</math></th><th><math>F</math></th></tr><tr><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td></tr></table>	$x$	$F$	0	0	1	1									
$x$	$F$																	
0	0																	
1	1																	

# Digital Logic Gates (2/2)

Name	Graphic symbol	Algebraic function	Truth table															
NAND		$F = (xy)'$	<table><tr><th><math>x</math></th><th><math>y</math></th><th><math>F</math></th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table>	$x$	$y$	$F$	0	0	1	0	1	1	1	0	1	1	1	0
$x$	$y$	$F$																
0	0	1																
0	1	1																
1	0	1																
1	1	0																
NOR		$F = (x + y)'$	<table><tr><th><math>x</math></th><th><math>y</math></th><th><math>F</math></th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table>	$x$	$y$	$F$	0	0	1	0	1	0	1	0	0	1	1	0
$x$	$y$	$F$																
0	0	1																
0	1	0																
1	0	0																
1	1	0																
Exclusive-OR (XOR)		$F = xy' + x'y$ $= x \oplus y$	<table><tr><th><math>x</math></th><th><math>y</math></th><th><math>F</math></th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table>	$x$	$y$	$F$	0	0	0	0	1	1	1	0	1	1	1	0
$x$	$y$	$F$																
0	0	0																
0	1	1																
1	0	1																
1	1	0																
Exclusive-NOR or equivalence		$F = xy + x'y'$ $= (x \oplus y)'$	<table><tr><th><math>x</math></th><th><math>y</math></th><th><math>F</math></th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table>	$x$	$y$	$F$	0	0	1	0	1	0	1	0	0	1	1	1
$x$	$y$	$F$																
0	0	1																
0	1	0																
1	0	0																
1	1	1																

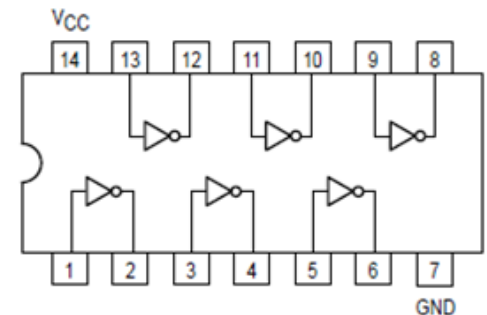
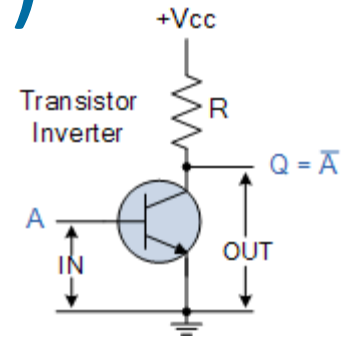
# Positive and Negative Logic

- The binary signal at the inputs and outputs of any gate has one of two values
- One signal value represents logic 1 and the other logic 0
- Since two signal values are assigned to two logic values, there exist two different assignments of signal level to logic value
- Choosing the high-level  $H$  to represent logic 1 defines a positive logic system
- Choosing the low-level  $L$  to represent logic 1 defines a negative logic system



# Integrated Circuits (1/4)

- An integrated circuit (IC) is fabricated on a die of a silicon semiconductor crystal, called a chip, containing the electronic components for constructing digital gates
- The various gates are interconnected inside the chip to form the required circuit
- The number of pins may range from 3 on a small IC package to several thousand on a larger package
- Each IC has a numeric designation printed on the surface of the package for identification



# Integrated Circuits (2/4)

- Level of Integration
  - Small Scale Integration (SSI)
    - The number of gates is usually fewer than 10
    - AND, OR, XOR etc ICs
  - Medium Scale Integration (MSI)
    - Have a complexity of approximately 10 to 1,000 gates in a single package
    - Decoders, adders, and multiplexers
  - Large Scale Integration (LSI)
    - These devices contain thousands of gates in a single package.
    - They include digital systems such as processors, memory chips, and programmable logic devices
  - Very Large Scale Integration (VLSI)
    - Contain millions of gates within a single package
    - Examples are large memory arrays and complex microcomputer chips



# Integrated Circuits (3/4)

- Digital Logic Families
  - The circuit technology of an IC is referred to as a digital logic family
  - TTL (transistor-transistor logic)
    - 50 years old and standard method
  - ECL (emitter coupled logic)
    - For high speed operations
  - MOS (metal oxide semiconductor)
    - Suitable for circuits that need high component density
  - CMOS (complementary metal-oxide semiconductor)
    - CMOS is preferable in systems requiring low power consumption

# Integrated Circuits (4/4)

- Fan-out
  - It specifies the number of standard loads that the output of a typical gate can drive without impairing its normal operation
- Fan-in
  - It is the number of inputs available in a gate
- Power dissipation
  - It is the power consumed by the gate that must be available from the power supply
- Propagation delay
  - It is the average transition delay time for a signal to propagate from input to output
- Noise margin
  - It is the maximum external noise voltage added to an input signal that does not cause an undesirable change in the circuit output