Today I would talk about how viscosity and speed of sound affect spatial asymmetries in heavy ion collisions.

Our motivation was to analyze how simple effects, which can't be discussed analytically, affect the time evolution of spatial asymmetries. With existing analytical solutions of hydrodynamics can't be discussed all effects that, so we must use numerical methods. With numerical hydrodynamics we can discuss very realistic the flow of quark-gluon plasma, but if we do that all effects will mixing, and we can't answer for our simple questions. So for that we use only hydrodynamics with initial conditions that are very close to exact solutions, but are more realistic.

In my presentation firstly I will talk about the equations that describe the collective motion of quark-gluon plasma. After that I present shortly a new multipole solution of relativistic hydrodynamics made by Mate Csanad and Andras Szabo. In my work I made code which solve the equations of hydrodynamics, so after showing an exact solution I will present the numerical scheme that we had used. In the following I justify why we think that our code work well. After that I will present our results about time evolution of spatial asymmetries in quark-gluon plasma. In final I show a simple model of hadronization.

In hydrodynamics we assume that the conservation of mass, momentum and energy is true locally. In nonrelativistic case that can be described with these equations, where the first express the conservation of mass, the conservation of momentum, the last one the conservation of energy. We also need an equation of state, that makes link between pressure and energy-density. When we want to describe the quark-gluon plasma, we can use this equation, the kappa is equal with the inverse of square of speed of sound. If we want a realistic model then the kappa depends on temperature. In relativistic case the equations come from that the divergence of stress-energy tensor is 0. The stress-energy tensor of perfect fluid is that. The description of non perfect fluid in special relativity isn't clear yet.

Firstly, I will show a multipole solution of relativistic hydrodynamics. The solution was made by Máté Csanád and András Szabó, this year was published in Physical Review Letters C. The big advantage of solution is that this work's any spacial asymmetries. But it's not an accelerating solution. In the solution we write basic quantities this form, where tau is the coordinate-proper time, and tauf is the freez-out proper time. The scale variable with optional asymmetries can be written in this form, the meaning of this is in the top picture, where is some initial condition with different asymmetries. but we can combine several asymmetries. That will be a solution if the R satisfy this equation. In the bottom pictures we can see a data fit for v2,v3,v4 parameters, calculated from this solution. The datas are from PHENIX. The different pictures shows   fit in different centrality.

In the following I will show an option to solve the equations of hydrodynamics numerically. According to experiments, all distributions have maximum at midrapidity, and in environment are constant these distributions. So it's enough for us to solve the equations in 2+1 dimension. The equations of hydrodynamics can be transformed to advection form, which is this. The relativistic equations are in this form automatically. The purpose is that to solve this partial-differential-system with numeric methods. To solve the equations we have to discretize them. For that we can use the finite volume method, in this method we make control volumes around grid pints and we integrate the equations for that. When we do that the difficulties comes from that we need the fluxes between grid points and for that we have to use approximation. And the most important moment of solving equation is that how we approximate these fluxes. If we solve partial differential equations we will meet with instability problems. That comes from that if we add a function to solution, which is null at grid points, that will be solution too. The approximation of fluxes can make the amplitude of this fake solution bigger and giber, and after some time this will dominate the solution and not the real

one. So we must check what will do our approximation with a fake solution and we have to make conditions which provides that won't happen. The conditions is named CFL conditions.
The 2+1 dimension still too difficult, we can simplify the problem if we use operator splitting, which is a good approximation. This means we can solve the 2+1 dimensional equitation system in more step, solving 1+1 dimensional equations. We used a method called Lie splitting, which means that we solved the equations in 2 step, solving 1+1 dimensional equations, first x direction, after the y direction. When we solve the non relativistic equations then we have the spatial derivates of speed in fluxes so we need to determine that. We can mange that if firstly we do an ideal substep, that means with solve equations with no viscosity, in this step we calculate the derivates of speed. After that we do a 2 step only with viscous fluxes.

For approximating the fluxes we used a method called MUSTA, the name comes from Multi Stage Approach, this was published by E.F. Toro at Jorunal of Computational Physics and related articles in several papers. In this method we use some stages to approximate the fluxes, that means we do some steps and in each one step we manage corrections in the flux approximation with predicted Q values. Lets say that the lth predicted values are this. Initially we start from Q which is in grid point, we define an intermediate value and flux. Which that we can approximate the fluxes between grid points. After that by using the equation what we want to solve we predict the next values of Q which can be only used in flux approximation.

So we had written a code that solve the hydro equations using MUSTA method. After all we had tested the code, and it passed the test so we believe that the results are correct. For testing we used a 3+1 dimensional solution of nonrelativistic hydro which is made by Tamas Csorgo, and it was Published in Physical Review Letters C in 2003. The solution in 2+1 dimension can be written in this form. In the plots we can see the time evolution of relative difference between numerical and exact solution, the left one shows the symmetric case and the right one the asymmetric case. We also had some other checks.

We can characterize the asymmetry with epsilon defined by this equation. The rho means matter density or number density, the p pressure or energy density and the w is defined to calculate the speed distribution asymmetry. The reason for this definition is that in this way we can reduce the numerical error. The new epsilon and the epsilon that is in the scale variable is not the same. Initially we can approximate the link between the 2 epsilon with Taylor-series. The links are these. We can see that if we have eps2, eps3 or eps2, eps4 in the scale variable that will generate eps1 in our asymmetry parameter.

In the following I will present our results. In first section I will talk about nonrelativistic results. In the left plot we can see that the viscosity makes slower the disappearance of asymmetries in energy and mass density. So if we have viscosity the asymmetries remain for longer time in energy and mass. The reason can be that, the viscosity makes the flow slower, and if the quark-gluon plasma flows slower the change of asymmetries will flow slow. As we can see in the right plot in the speed distribution the effect is opposite. There the viscosity makes faster the disappearance of asymmetries. The reason of this can be that the viscosus force contains the second space derivate of speed, that means the parts which have bigger asymmetires feels stronger force, the parts whith less asymmetires feels smaller force, so the difference between parts will dissapear faster.

In this slide we can see some frame of time evolution mass density.
In this one some frame about time evolution of velocity.

In the left plot we can see the time evolution of asymmetries calculated in mass density, the right one shows the same in speed distribution. We can say that generally the reduction of speed of sound makes the asymmetries dissapearance slower. The reason could be that in fluids the speed of pressure waves is the speed of sound, so if we reduce it the equalization of perssure will happaen slower.

In the left plot we see the mass density, right velocity distribution. The space dependent term of pressure is exponential with -scale variable. So if we multiply in presure the scale variable with ec then we change the pressure gradient. The plots shows that bigger pressure gradient makes the dissapearance of asymmetries will be faster. That could be beacause if the pressure gradient grows then the velocity grows too, and that means faster flow which means that we see.

In relativistic case the initial value of distributions is different from the nonrel ones. In nonrel case we don't have speed distribution initially, in rel case we do. In these picture we can see that in relativistic case the effect of speed of sounds is the same, the reason possibly is the same. In the realatvisitic case the calculation wasn't fixed, when the whole quark-gluon plasma hadronized the calulation stoped. In these plots we can see when we increase of kappa, which means decrease of speed of sounds, then hadronization will be latter. The reason could be that because the flow is slower, the QGP cools slower.

In the relativistic case we made hadronization. We had use for source function the M-J type source function. With source function we can simply calculate the measurable quantities. The plot show the v2 v3 parameters for pions, different kappas. We can see that parameters is very sensitive to the value of kappa. The reason may be that the time of end of QGP state depends on kappa. So if the kappa is bigger the freez-out will be later so the the system will freez-out with different asymmetries. That we can see in plot.