

# Statistics Formulas

Statistics is  
the grammar of science.  
*Karl Pearson*

# Descriptive Statistics

## Population

Size  $N$

Mean  $\mu = \frac{\sum_{i=1}^N x_i}{N}$

Variance  $\sigma^2 = \frac{\sum_{i=1}^N (x_i - \mu)^2}{N}$

Standard deviation  $\sigma = \sqrt{\frac{\sum_{i=1}^N (x_i - \mu)^2}{N}}$

## Sample

Sample size  $n$

Mean  $\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$

Variance  $S^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}$

Standard deviation  $S = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}$

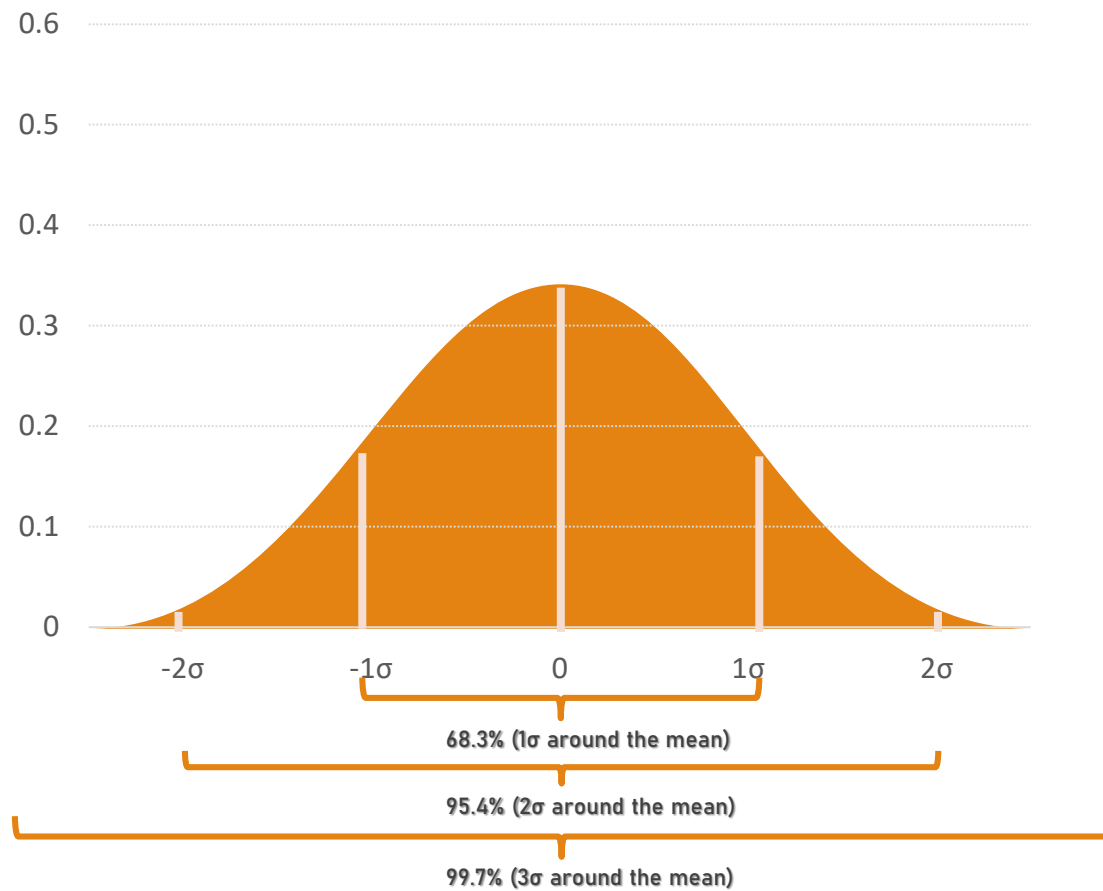
## Mode

The mode is defined by the most common value of the sample or the population. If there are two most frequent values we may say the dataset is bimodal.

## Median

The median is the value exactly in the middle of a sample or a population.

# Normal distribution



## Empirical rule

For normal distributions, about

- 68% of data points fall within 1 standard deviation of the mean.
- 95% of data points fall within 2 standard deviations of the mean.
- 99.7% of data points fall within 3 standard deviations of the mean.

## Z-score

$$Z = \frac{x - \mu}{\sigma}$$

## Z-Score Table (positive)

Z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.00034	0.00032	0.00031	0.00030	0.00029	0.00028	0.00027	0.00026	0.00025	0.00024
-3.3	0.00048	0.00047	0.00045	0.00043	0.00042	0.00040	0.00039	0.00038	0.00036	0.00035
-3.2	0.00069	0.00066	0.00064	0.00062	0.00060	0.00058	0.00056	0.00054	0.00052	0.00050
-3.1	0.00097	0.00094	0.00090	0.00087	0.00084	0.00082	0.00079	0.00076	0.00074	0.00071
-3	0.00135	0.00131	0.00126	0.00122	0.00118	0.00114	0.00111	0.00107	0.00104	0.00100
-2.9	0.00187	0.00181	0.00175	0.00169	0.00164	0.00159	0.00154	0.00149	0.00144	0.00139
-2.8	0.00256	0.00248	0.00240	0.00233	0.00226	0.00219	0.00212	0.00205	0.00199	0.00193
-2.7	0.00347	0.00336	0.00326	0.00317	0.00307	0.00298	0.00289	0.00280	0.00272	0.00264
-2.6	0.00466	0.00453	0.00440	0.00427	0.00415	0.00402	0.00391	0.00379	0.00368	0.00357
-2.5	0.00621	0.00604	0.00587	0.00570	0.00554	0.00539	0.00523	0.00508	0.00494	0.00480
-2.4	0.00820	0.00798	0.00776	0.00755	0.00734	0.00714	0.00695	0.00676	0.00657	0.00639
-2.3	0.01072	0.01044	0.01017	0.00990	0.00964	0.00939	0.00914	0.00889	0.00866	0.00842
-2.2	0.01390	0.01355	0.01321	0.01287	0.01255	0.01222	0.01191	0.01160	0.01130	0.01101
-2.1	0.01786	0.01743	0.01700	0.01659	0.01618	0.01578	0.01539	0.01500	0.01463	0.01426
-2	0.02275	0.02222	0.02169	0.02118	0.02068	0.02018	0.01970	0.01923	0.01876	0.01831
-1.9	0.02872	0.02807	0.02743	0.02680	0.02619	0.02559	0.02500	0.02442	0.02385	0.02330
-1.8	0.03593	0.03515	0.03438	0.03362	0.03288	0.03216	0.03144	0.03074	0.03005	0.02938
-1.7	0.04457	0.04363	0.04272	0.04182	0.04093	0.04006	0.03920	0.03836	0.03754	0.03673
-1.6	0.05480	0.05370	0.05262	0.05155	0.05050	0.04947	0.04846	0.04746	0.04648	0.04551
-1.5	0.06681	0.06552	0.06426	0.06301	0.06178	0.06057	0.05938	0.05821	0.05705	0.05592
-1.4	0.08076	0.07927	0.07780	0.07636	0.07493	0.07353	0.07215	0.07078	0.06944	0.06811
-1.3	0.09680	0.09510	0.09342	0.09176	0.09012	0.08851	0.08691	0.08534	0.08379	0.08226
-1.2	0.11507	0.11314	0.11123	0.10935	0.10749	0.10565	0.10383	0.10204	0.10027	0.09853
-1.1	0.13567	0.13350	0.13136	0.12924	0.12714	0.12507	0.12302	0.12100	0.11900	0.11702
-1	0.15866	0.15625	0.15386	0.15151	0.14917	0.14686	0.14457	0.14231	0.14007	0.13786
-0.9	0.18406	0.18141	0.17879	0.17619	0.17361	0.17106	0.16853	0.16602	0.16354	0.16109
-0.8	0.21186	0.20897	0.20611	0.20327	0.20045	0.19766	0.19489	0.19215	0.18943	0.18673
-0.7	0.24196	0.23885	0.23576	0.23270	0.22965	0.22663	0.22363	0.22065	0.21770	0.21476
-0.6	0.27425	0.27093	0.26763	0.26435	0.26109	0.25785	0.25463	0.25143	0.24825	0.24510
-0.5	0.30854	0.30503	0.30153	0.29806	0.29460	0.29116	0.28774	0.28434	0.28096	0.27760
-0.4	0.34458	0.34090	0.33724	0.33360	0.32997	0.32636	0.32276	0.31918	0.31561	0.31207
-0.3	0.38209	0.37828	0.37448	0.37070	0.36693	0.36317	0.35942	0.35569	0.35197	0.34827
-0.2	0.42074	0.41683	0.41294	0.40905	0.40517	0.40129	0.39743	0.39358	0.38974	0.38591
-0.1	0.46017	0.45620	0.45224	0.44828	0.44433	0.44038	0.43644	0.43251	0.42858	0.42465
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

## Z-Score Table (negative)

Z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976

## Bayes' theorem

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)}$$

## At least one success or failure

$$\begin{aligned} P(\text{at least 1 success}) &= 1 - P(\text{all failure}) \\ P(\text{at least 1 failure}) &= 1 - P(\text{all successes}) \end{aligned}$$

## Probability of an event

$$P(\text{event}) = \frac{\text{number of outcome that meets our criteria (success)}}{\text{number of all possible outcomes}}$$

For events where every outcome has the same probability.

## Addition rule

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

## Multiplication rule

For independent events:  $P(A \text{ and } B) = P(A) \cdot P(B)$

For dependent events:  $P(A \text{ and } B) = P(A) \cdot P(B|A)$

## Binomial coefficient

$$\binom{n}{k} = \frac{n!}{k! (n - k)!}$$

## Binomial density function

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{(n-k)}$$

"Probability of k successes in n attempts"

$$E(X) = \mu = np$$

$$\sigma^2 = np(1 - p)$$

## Poisson density function

$$P(X = k) = \frac{e^{-\lambda} \lambda^k}{k!}$$

$$E(X) = \mu = \lambda$$

$$\sigma^2 = \lambda$$

## Poisson cumulative density function (c.d.f.)

$$P(X \leq n) = e^{-\lambda} \sum_{k=0}^n \frac{\lambda^k}{k!}$$

## Sampling distribution of the sample mean

Mean:  $\mu_{\bar{x}} = \mu$

Standard deviation:  $SE_{\bar{x}} = \sigma_{\bar{x}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

## Sampling distribution of the sample proportion

Mean:  $\mu_{\hat{p}} = p$

Standard deviation:  $SE_{\hat{p}} = \sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

## Confidence interval

$$\hat{p} \pm z^* SE_{\hat{p}}$$
$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

## Margin of error

$$z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$



# t-table

## Significance level

	One-sided	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
	Two-sided	0.5	0.40	0.3	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df	1	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
	2	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
	3	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
	4	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
	5	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
	6	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
	7	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
	8	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
	9	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
	10	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
	11	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
	12	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
	13	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
	14	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
	15	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
	16	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
	17	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
	18	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
	19	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
	20	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
	21	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
	22	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
	23	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
	24	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
	25	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
	26	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
	27	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
	28	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
	29	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
	30	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

*t-score* where *s* is the sample standard deviation

## Regression line

$$m = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$b = \frac{(\sum y) - m(\sum x)}{n}$$

## Correlation coefficient

$$r_{xy} = \frac{n \sum x_i y_i - \sum x_i \sum y_i}{\sqrt{n \sum x_i^2 - (\sum x_i)^2} \sqrt{n \sum y_i^2 - (\sum y_i)^2}}$$

## Residual

residual = actual value – predicted value

$$e_i = a_i - p_i$$

## MSE (Mean-Square-Error)

$$MSE = \frac{\sum e_i^2}{n}$$

## MAE (Mean-Absolute-Error)

$$MAE = \frac{\sum |e_i|}{n}$$

## RMSE (Root-Mean-Square-Error)

$$RMSE = \frac{\sqrt{\sum e_i^2}}{n}$$

# F Table for $\alpha=0.10$

(see next table below for 12 to  $\infty$ )

	DF1									
DF2	1	2	3	4	5	6	7	8	9	10
1	39.86346	49.5	53.59324	55.83296	57.24008	58.20442	58.90595	59.43898	59.85759	60.19498
2	8.52632	9	9.16179	9.24342	9.29263	9.32553	9.34908	9.36677	9.38054	9.39157
3	5.53832	5.46238	5.39077	5.34264	5.30916	5.28473	5.26619	5.25167	5.24	5.23041
4	4.54477	4.32456	4.19086	4.10725	4.05058	4.00975	3.97897	3.95494	3.93567	3.91988
5	4.06042	3.77972	3.61948	3.5202	3.45298	3.40451	3.3679	3.33928	3.31628	3.2974
6	3.77595	3.4633	3.28876	3.18076	3.10751	3.05455	3.01446	2.98304	2.95774	2.93693
7	3.58943	3.25744	3.07407	2.96053	2.88334	2.82739	2.78493	2.75158	2.72468	2.70251
8	3.45792	3.11312	2.9238	2.80643	2.72645	2.66833	2.62413	2.58935	2.56124	2.53804
9	3.3603	3.00645	2.81286	2.69268	2.61061	2.55086	2.50531	2.46941	2.44034	2.41632
10	3.28502	2.92447	2.72767	2.60534	2.52164	2.46058	2.41397	2.37715	2.34731	2.3226
11	3.2252	2.85951	2.66023	2.53619	2.45118	2.38907	2.34157	2.304	2.2735	2.24823
12	3.17655	2.8068	2.60552	2.4801	2.39402	2.33102	2.28278	2.24457	2.21352	2.18776
13	3.13621	2.76317	2.56027	2.43371	2.34672	2.28298	2.2341	2.19535	2.16382	2.13763
14	3.10221	2.72647	2.52222	2.39469	2.30694	2.24256	2.19313	2.1539	2.12195	2.0954
15	3.07319	2.69517	2.48979	2.36143	2.27302	2.20808	2.15818	2.11853	2.08621	2.05932
16	3.04811	2.66817	2.46181	2.33274	2.24376	2.17833	2.128	2.08798	2.05533	2.02815
17	3.02623	2.64464	2.43743	2.30775	2.21825	2.15239	2.10169	2.06134	2.02839	2.00094
18	3.00698	2.62395	2.41601	2.28577	2.19583	2.12958	2.07854	2.03789	2.00467	1.97698
19	2.9899	2.60561	2.39702	2.2663	2.17596	2.10936	2.05802	2.0171	1.98364	1.95573
20	2.97465	2.58925	2.38009	2.24893	2.15823	2.09132	2.0397	1.99853	1.96485	1.93674
21	2.96096	2.57457	2.36489	2.23334	2.14231	2.07512	2.02325	1.98186	1.94797	1.91967
22	2.94858	2.56131	2.35117	2.21927	2.12794	2.0605	2.0084	1.9668	1.93273	1.90425
23	2.93736	2.54929	2.33873	2.20651	2.11491	2.04723	1.99492	1.95312	1.91888	1.89025
24	2.92712	2.53833	2.32739	2.19488	2.10303	2.03513	1.98263	1.94066	1.90625	1.87748
25	2.91774	2.52831	2.31702	2.18424	2.09216	2.02406	1.97138	1.92925	1.89469	1.86578
26	2.90913	2.5191	2.30749	2.17447	2.08218	2.01389	1.96104	1.91876	1.88407	1.85503
27	2.90119	2.51061	2.29871	2.16546	2.07298	2.00452	1.95151	1.90909	1.87427	1.84511
28	2.89385	2.50276	2.2906	2.15714	2.06447	1.99585	1.9427	1.90014	1.8652	1.83593
29	2.88703	2.49548	2.28307	2.14941	2.05658	1.98781	1.93452	1.89184	1.85679	1.82741
30	2.88069	2.48872	2.27607	2.14223	2.04925	1.98033	1.92692	1.88412	1.84896	1.81949
40	2.83535	2.44037	2.22609	2.09095	1.99682	1.92688	1.87252	1.82886	1.7929	1.76269
60	2.79107	2.39325	2.17741	2.04099	1.94571	1.87472	1.81939	1.77483	1.73802	1.70701
120	2.74781	2.34734	2.12999	1.9923	1.89587	1.82381	1.76748	1.72196	1.68425	1.65238
$\infty$	2.70554	2.30259	2.0838	1.94486	1.84727	1.77411	1.71672	1.6702	1.63152	1.59872

# F Table for $\alpha=0.10$

(table for 12 to  $\infty$ )

DF2	DF1								
	12	15	20	24	30	40	60	120	$\infty$
1	60.70521	61.22034	61.74029	62.00205	62.26497	62.52905	62.79428	63.06064	63.32812
2	9.40813	9.42471	9.44131	9.44962	9.45793	9.46624	9.47456	9.48289	9.49122
3	5.21562	5.20031	5.18448	5.17636	5.16811	5.15972	5.15119	5.14251	5.1337
4	3.89553	3.87036	3.84434	3.83099	3.81742	3.80361	3.78957	3.77527	3.76073
5	3.26824	3.23801	3.20665	3.19052	3.17408	3.15732	3.14023	3.12279	3.105
6	2.90472	2.87122	2.83634	2.81834	2.79996	2.78117	2.76195	2.74229	2.72216
7	2.66811	2.63223	2.59473	2.57533	2.55546	2.5351	2.51422	2.49279	2.47079
8	2.50196	2.46422	2.42464	2.4041	2.38302	2.36136	2.3391	2.31618	2.29257
9	2.37888	2.33962	2.29832	2.27683	2.25472	2.23196	2.20849	2.18427	2.15923
10	2.28405	2.24351	2.20074	2.17843	2.15543	2.13169	2.10716	2.08176	2.05542
11	2.20873	2.16709	2.12305	2.10001	2.07621	2.05161	2.02612	1.99965	1.97211
12	2.14744	2.10485	2.05968	2.03599	2.01149	1.9861	1.95973	1.93228	1.90361
13	2.09659	2.05316	2.00698	1.98272	1.95757	1.93147	1.90429	1.87591	1.8462
14	2.05371	2.00953	1.96245	1.93766	1.91193	1.88516	1.85723	1.828	1.79728
15	2.01707	1.97222	1.92431	1.89904	1.87277	1.84539	1.81676	1.78672	1.75505
16	1.98539	1.93992	1.89127	1.86556	1.83879	1.81084	1.78156	1.75075	1.71817
17	1.95772	1.91169	1.86236	1.83624	1.80901	1.78053	1.75063	1.71909	1.68564
18	1.93334	1.88681	1.83685	1.81035	1.78269	1.75371	1.72322	1.69099	1.65671
19	1.9117	1.86471	1.81416	1.78731	1.75924	1.72979	1.69876	1.66587	1.63077
20	1.89236	1.84494	1.79384	1.76667	1.73822	1.70833	1.67678	1.64326	1.60738
21	1.87497	1.82715	1.77555	1.74807	1.71927	1.68896	1.65691	1.62278	1.58615
22	1.85925	1.81106	1.75899	1.73122	1.70208	1.67138	1.63885	1.60415	1.56678
23	1.84497	1.79643	1.74392	1.71588	1.68643	1.65535	1.62237	1.58711	1.54903
24	1.83194	1.78308	1.73015	1.70185	1.6721	1.64067	1.60726	1.57146	1.5327
25	1.82	1.77083	1.71752	1.68898	1.65895	1.62718	1.59335	1.55703	1.5176
26	1.80902	1.75957	1.70589	1.67712	1.64682	1.61472	1.5805	1.54368	1.5036
27	1.79889	1.74917	1.69514	1.66616	1.6356	1.6032	1.56859	1.53129	1.49057
28	1.78951	1.73954	1.68519	1.656	1.62519	1.5925	1.55753	1.51976	1.47841
29	1.78081	1.7306	1.67593	1.64655	1.61551	1.58253	1.54721	1.50899	1.46704
30	1.7727	1.72227	1.66731	1.63774	1.60648	1.57323	1.53757	1.49891	1.45636
40	1.71456	1.66241	1.60515	1.57411	1.54108	1.50562	1.46716	1.42476	1.37691
60	1.65743	1.60337	1.54349	1.51072	1.47554	1.43734	1.3952	1.34757	1.29146
120	1.6012	1.545	1.48207	1.44723	1.40938	1.3676	1.32034	1.26457	1.19256
$\infty$	1.54578	1.48714	1.4206	1.38318	1.34187	1.29513	1.23995	1.1686	1

# F Table for $\alpha=0.05$

(see next table below for 12 to  $\infty$ )

DF2	DF1									
	1	2	3	4	5	6	8	9	10	
1	161.4476	199.5	215.7073	224.5832	230.1619	233.986	238.8827	240.5433	241.8817	
2	18.5128	19	19.1643	19.2468	19.2964	19.3295	19.371	19.3848	19.3959	
3	10.128	9.5521	9.2766	9.1172	9.0135	8.9406	8.8452	8.8123	8.7855	
4	7.7086	6.9443	6.5914	6.3882	6.2561	6.1631	6.041	5.9988	5.9644	
5	6.6079	5.7861	5.4095	5.1922	5.0503	4.9503	4.8183	4.7725	4.7351	
6	5.9874	5.1433	4.7571	4.5337	4.3874	4.2839	4.1468	4.099	4.06	
7	5.5914	4.7374	4.3468	4.1203	3.9715	3.866	3.7257	3.6767	3.6365	
8	5.3177	4.459	4.0662	3.8379	3.6875	3.5806	3.4381	3.3881	3.3472	
9	5.1174	4.2565	3.8625	3.6331	3.4817	3.3738	3.2296	3.1789	3.1373	
10	4.9646	4.1028	3.7083	3.478	3.3258	3.2172	3.0717	3.0204	2.9782	
11	4.8443	3.9823	3.5874	3.3567	3.2039	3.0946	2.948	2.8962	2.8536	
12	4.7472	3.8853	3.4903	3.2592	3.1059	2.9961	2.8486	2.7964	2.7534	
13	4.6672	3.8056	3.4105	3.1791	3.0254	2.9153	2.7669	2.7144	2.671	
14	4.6001	3.7389	3.3439	3.1122	2.9582	2.8477	2.6987	2.6458	2.6022	
15	4.5431	3.6823	3.2874	3.0556	2.9013	2.7905	2.6408	2.5876	2.5437	
16	4.494	3.6337	3.2389	3.0069	2.8524	2.7413	2.5911	2.5377	2.4935	
17	4.4513	3.5915	3.1968	2.9647	2.81	2.6987	2.548	2.4943	2.4499	
18	4.4139	3.5546	3.1599	2.9277	2.7729	2.6613	2.5102	2.4563	2.4117	
19	4.3807	3.5219	3.1274	2.8951	2.7401	2.6283	2.4768	2.4227	2.3779	
20	4.3512	3.4928	3.0984	2.8661	2.7109	2.599	2.4471	2.3928	2.3479	
21	4.3248	3.4668	3.0725	2.8401	2.6848	2.5727	2.4205	2.366	2.321	
22	4.3009	3.4434	3.0491	2.8167	2.6613	2.5491	2.3965	2.3419	2.2967	
23	4.2793	3.4221	3.028	2.7955	2.64	2.5277	2.3748	2.3201	2.2747	
24	4.2597	3.4028	3.0088	2.7763	2.6207	2.5082	2.3551	2.3002	2.2547	
25	4.2417	3.3852	2.9912	2.7587	2.603	2.4904	2.3371	2.2821	2.2365	
26	4.2252	3.369	2.9752	2.7426	2.5868	2.4741	2.3205	2.2655	2.2197	
27	4.21	3.3541	2.9604	2.7278	2.5719	2.4591	2.3053	1.7306	1.6717	
28	4.196	3.3404	2.9467	2.7141	2.5581	2.4453	2.2913	2.236	2.19	
29	4.183	3.3277	2.934	2.7014	2.5454	2.4324	2.2783	2.2229	2.1768	
30	4.1709	3.3158	2.9223	2.6896	2.5336	2.4205	2.2662	2.2107	2.1646	
40	4.0847	3.2317	2.8387	2.606	2.4495	2.3359	2.1802	2.124	2.0772	
60	4.0012	3.1504	2.7581	2.5252	2.3683	2.2541	2.097	2.0401	1.9926	
120	3.9201	3.0718	2.6802	2.4472	2.2899	2.175	2.0164	1.9588	1.9105	
$\infty$	3.8415	2.9957	2.6049	2.3719	2.2141	2.0986	1.9384	1.8799	1.8307	

# F Table for $\alpha=0.05$

(table for 12 to  $\infty$ )

DF2	DF1								
	12	15	20	24	30	40	120	$\infty$	
1	243.906	245.9499	248.0131	249.0518	250.0951	251.1432	253.2529	254.3144	
2	19.4125	19.4291	19.4458	19.4541	19.4624	19.4707	19.4874	19.4957	
3	8.7446	8.7029	8.6602	8.6385	8.6166	8.5944	8.5494	8.5264	
4	5.9117	5.8578	5.8025	5.7744	5.7459	5.717	5.6581	5.6281	
5	4.6777	4.6188	4.5581	4.5272	4.4957	4.4638	4.3985	4.365	
6	3.9999	3.9381	3.8742	3.8415	3.8082	3.7743	3.7047	3.6689	
7	3.5747	3.5107	3.4445	3.4105	3.3758	3.3404	3.2674	3.2298	
8	3.2839	3.2184	3.1503	3.1152	3.0794	3.0428	2.9669	2.9276	
9	3.0729	3.0061	2.9365	2.9005	2.8637	2.8259	2.7475	2.7067	
10	2.913	2.845	2.774	2.7372	2.6996	2.6609	2.5801	2.5379	
11	2.7876	2.7186	2.6464	2.609	2.5705	2.5309	2.448	2.4045	
12	2.6866	2.6169	2.5436	2.5055	2.4663	2.4259	2.341	2.2962	
13	2.6037	2.5331	2.4589	2.4202	2.3803	2.3392	2.2524	2.2064	
14	2.5342	2.463	2.3879	2.3487	2.3082	2.2664	2.1778	2.1307	
15	2.4753	2.4034	2.3275	2.2878	2.2468	2.2043	2.1141	2.0658	
16	2.4247	2.3522	2.2756	2.2354	2.1938	2.1507	2.0589	2.0096	
17	2.3807	2.3077	2.2304	2.1898	2.1477	2.104	2.0107	1.9604	
18	2.3421	2.2686	2.1906	2.1497	2.1071	2.0629	1.9681	1.9168	
19	2.308	2.2341	2.1555	2.1141	2.0712	2.0264	1.9302	1.878	
20	2.2776	2.2033	2.1242	2.0825	2.0391	1.9938	1.8963	1.8432	
21	2.2504	2.1757	2.096	2.054	2.0102	1.9645	1.8657	1.8117	
22	2.2258	2.1508	2.0707	2.0283	1.9842	1.938	1.838	1.7831	
23	2.2036	2.1282	2.0476	2.005	1.9605	1.9139	1.8128	1.757	
24	2.1834	2.1077	2.0267	1.9838	1.939	1.892	1.7896	1.733	
25	2.1649	2.0889	2.0075	1.9643	1.9192	1.8718	1.7684	1.711	
26	2.1479	2.0716	1.9898	1.9464	1.901	1.8533	1.7488	1.6906	
27	2.1323	2.0558	1.9736	1.9299	1.8842	1.8361	1.7306	1.6717	
28	2.1179	2.0411	1.9586	1.9147	1.8687	1.8203	1.7138	1.6541	
29	2.1045	2.0275	1.9446	1.9005	1.8543	1.8055	1.6981	1.6376	
30	2.0921	2.0148	1.9317	1.8874	1.8409	1.7918	1.6835	1.6223	
40	2.0035	1.9245	1.8389	1.7929	1.7444	1.6928	1.5766	1.5089	
60	1.9174	1.8364	1.748	1.7001	1.6491	1.5943	1.4673	1.3893	
120	1.8337	1.7505	1.6587	1.6084	1.5543	1.4952	1.3519	1.2539	
$\infty$	1.7522	1.6664	1.5705	1.5173	1.4591	1.394	1.2214	1	

# F Table for $\alpha=0.025$

(see next table below for 12 to  $\infty$ )

DF2	DF1								
	1	2	3	4	5	6	8	9	10
1	647.789	799.5	864.163	899.5833	921.8479	937.1111	956.6562	963.2846	968.6274
2	38.5063	39	39.1655	39.2484	39.2982	39.3315	39.373	39.3869	39.398
3	17.4434	16.0441	15.4392	15.101	14.8848	14.7347	14.5399	14.4731	14.4189
4	12.2179	10.6491	9.9792	9.6045	9.3645	9.1973	8.9796	8.9047	8.8439
5	10.007	8.4336	7.7636	7.3879	7.1464	6.9777	6.7572	6.6811	6.6192
6	8.8131	7.2599	6.5988	6.2272	5.9876	5.8198	5.5996	5.5234	5.4613
7	8.0727	6.5415	5.8898	5.5226	5.2852	5.1186	4.8993	4.8232	4.7611
8	7.5709	6.0595	5.416	5.0526	4.8173	4.6517	4.4333	4.3572	4.2951
9	7.2093	5.7147	5.0781	4.7181	4.4844	4.3197	4.102	4.026	3.9639
10	6.9367	5.4564	4.8256	4.4683	4.2361	4.0721	3.8549	3.779	3.7168
11	6.7241	5.2559	4.63	4.2751	4.044	3.8807	3.6638	3.5879	3.5257
12	6.5538	5.0959	4.4742	4.1212	3.8911	3.7283	3.5118	3.4358	3.3736
13	6.4143	4.9653	4.3472	3.9959	3.7667	3.6043	3.388	3.312	3.2497
14	6.2979	4.8567	4.2417	3.8919	3.6634	3.5014	3.2853	3.2093	3.1469
15	6.1995	4.765	4.1528	3.8043	3.5764	3.4147	3.1987	3.1227	3.0602
16	6.1151	4.6867	4.0768	3.7294	3.5021	3.3406	3.1248	3.0488	2.9862
17	6.042	4.6189	4.0112	3.6648	3.4379	3.2767	3.061	2.9849	2.9222
18	5.9781	4.5597	3.9539	3.6083	3.382	3.2209	3.0053	2.9291	2.8664
19	5.9216	4.5075	3.9034	3.5587	3.3327	3.1718	2.9563	2.8801	2.8172
20	5.8715	4.4613	3.8587	3.5147	3.2891	3.1283	2.9128	2.8365	2.7737
21	5.8266	4.4199	3.8188	3.4754	3.2501	3.0895	2.874	2.7977	2.7348
22	5.7863	4.3828	3.7829	3.4401	3.2151	3.0546	2.8392	2.7628	2.6998
23	5.7498	4.3492	3.7505	3.4083	3.1835	3.0232	2.8077	2.7313	2.6682
24	5.7166	4.3187	3.7211	3.3794	3.1548	2.9946	2.7791	2.7027	2.6396
25	5.6864	4.2909	3.6943	3.353	3.1287	2.9685	2.7531	2.6766	2.6135
26	5.6586	4.2655	3.6697	3.3289	3.1048	2.9447	2.7293	2.6528	1.878
27	5.6331	4.2421	3.6472	3.3067	3.0828	2.9228	2.7074	2.6309	2.5676
28	5.6096	4.2205	3.6264	3.2863	3.0626	2.9027	2.6872	2.6106	2.5473
29	5.5878	4.2006	3.6072	3.2674	3.0438	2.884	2.6686	2.5919	2.5286
30	5.5675	4.1821	3.5894	3.2499	3.0265	2.8667	2.6513	2.5746	2.5112
40	5.4239	4.051	3.4633	3.1261	2.9037	2.7444	2.5289	2.4519	2.3882
60	5.2856	3.9253	3.3425	3.0077	2.7863	2.6274	2.4117	2.3344	2.2702
120	5.1523	3.8046	3.2269	2.8943	2.674	2.5154	2.2994	2.2217	2.157
INF	5.0239	3.6889	3.1161	2.7858	2.5665	2.4082	2.1918	2.1136	2.0483

# F Table for $\alpha=0.025$

(table for 12 to  $\infty$ )

DF2	DF1							
	12	15	20	24	30	40	120	$\infty$
1	976.7079	984.8668	993.1028	997.2492	1001.414	1005.598	1014.02	1018.258
2	39.4146	39.4313	39.4479	39.4562	39.465	39.473	39.49	39.498
3	14.3366	14.2527	14.1674	14.1241	14.081	14.037	13.947	13.902
4	8.7512	8.6565	8.5599	8.5109	8.461	8.411	8.309	8.257
5	6.5245	6.4277	6.3286	6.278	6.227	6.175	6.069	6.015
6	5.3662	5.2687	5.1684	5.1172	5.065	5.012	4.904	4.849
7	4.6658	4.5678	4.4667	4.415	4.362	4.309	4.199	4.142
8	4.1997	4.1012	3.9995	3.9472	3.894	3.84	3.728	3.67
9	3.8682	3.7694	3.6669	3.6142	3.56	3.505	3.392	3.333
10	3.6209	3.5217	3.4185	3.3654	3.311	3.255	3.14	3.08
11	3.4296	3.3299	3.2261	3.1725	3.118	3.061	2.944	2.883
12	3.2773	3.1772	3.0728	3.0187	2.963	2.906	2.787	2.725
13	3.1532	3.0527	2.9477	2.8932	2.837	2.78	2.659	2.595
14	3.0502	2.9493	2.8437	2.7888	2.732	2.674	2.552	2.487
15	2.9633	2.8621	2.7559	2.7006	2.644	2.585	2.461	2.395
16	2.889	2.7875	2.6808	2.6252	2.568	2.509	2.383	2.316
17	2.8249	2.723	2.6158	2.5598	2.502	2.442	2.315	2.247
18	2.7689	2.6667	2.559	2.5027	2.445	2.384	2.256	2.187
19	2.7196	2.6171	2.5089	2.4523	2.394	2.333	2.203	2.133
20	2.6758	2.5731	2.4645	2.4076	2.349	2.287	2.156	2.085
21	2.6368	2.5338	2.4247	2.3675	2.308	2.246	2.114	2.042
22	2.6017	2.4984	2.389	2.3315	2.272	2.21	2.076	2.003
23	2.5699	2.4665	2.3567	2.2989	2.239	2.176	2.041	1.968
24	2.5411	2.4374	2.3273	2.2693	2.209	2.146	2.01	1.935
25	2.5149	2.411	2.3005	2.2422	2.182	2.118	1.981	1.906
26	2.4908	2.3867	2.2759	2.2174	2.157	2.093	1.954	1.878
27	2.4688	2.3644	2.2533	2.1946	2.133	2.069	1.93	1.853
28	2.4484	2.3438	2.2324	2.1735	2.112	2.048	1.907	1.829
29	2.4295	2.3248	2.2131	2.154	2.092	2.028	1.886	1.807
30	2.412	2.3072	2.1952	2.1359	2.074	2.009	1.866	1.787
40	2.2882	2.1819	2.0677	2.0069	1.943	1.875	1.724	1.637
60	2.1692	2.0613	1.9445	1.8817	1.815	1.744	1.581	1.482
120	2.0548	1.945	1.8249	1.7597	1.69	1.614	1.433	1.31
INF	1.9447	1.8326	1.7085	1.6402	1.566	1.484	1.268	1



# F Table for $\alpha=0.01$

(see next table below for 12 to  $\infty$ )

DF2	DF1								
	1	2	3	4	5	6	8	9	10
1	4052.181	4999.5	5403.352	5624.583	5763.65	5858.986	5981.07	6022.473	6055.847
2	98.503	99	99.166	99.249	99.299	99.333	99.374	99.388	99.399
3	34.116	30.817	29.457	28.71	28.237	27.911	27.489	27.345	27.229
4	21.198	18	16.694	15.977	15.522	15.207	14.799	14.659	14.546
5	16.258	13.274	12.06	11.392	10.967	10.672	10.289	10.158	10.051
6	13.745	10.925	9.78	9.148	8.746	8.466	8.102	7.976	7.874
7	12.246	9.547	8.451	7.847	7.46	7.191	6.84	6.719	6.62
8	11.259	8.649	7.591	7.006	6.632	6.371	6.029	5.911	5.814
9	10.561	8.022	6.992	6.422	6.057	5.802	5.467	5.351	5.257
10	10.044	7.559	6.552	5.994	5.636	5.386	5.057	4.942	4.849
11	9.646	7.206	6.217	5.668	5.316	5.069	4.744	4.632	4.539
12	9.33	6.927	5.953	5.412	5.064	4.821	4.499	4.388	4.296
13	9.074	6.701	5.739	5.205	4.862	4.62	4.302	4.191	4.1
14	8.862	6.515	5.564	5.035	4.695	4.456	4.14	4.03	3.939
15	8.683	6.359	5.417	4.893	4.556	4.318	4.004	3.895	3.805
16	8.531	6.226	5.292	4.773	4.437	4.202	3.89	3.78	3.691
17	8.4	6.112	5.185	4.669	4.336	4.102	3.791	3.682	3.593
18	8.285	6.013	5.092	4.579	4.248	4.015	3.705	3.597	3.508
19	8.185	5.926	5.01	4.5	4.171	3.939	3.631	3.523	3.434
20	8.096	5.849	4.938	4.431	4.103	3.871	3.564	3.457	3.368
21	8.017	5.78	4.874	4.369	4.042	3.812	3.506	3.398	3.31
22	7.945	5.719	4.817	4.313	3.988	3.758	3.453	3.346	3.258
23	7.881	5.664	4.765	4.264	3.939	3.71	3.406	3.299	3.211
24	7.823	5.614	4.718	4.218	3.895	3.667	3.363	3.256	3.168
25	7.77	5.568	4.675	4.177	3.855	3.627	3.324	3.217	3.129
26	7.721	5.526	4.637	4.14	3.818	3.591	3.288	3.182	3.094
27	7.677	5.488	4.601	4.106	3.785	3.558	3.256	3.149	3.062
28	7.636	5.453	4.568	4.074	3.754	3.528	3.226	3.12	3.032
29	7.598	5.42	4.538	4.045	3.725	3.499	3.198	3.092	3.005
30	7.562	5.39	4.51	4.018	3.699	3.473	3.173	3.067	2.979
40	7.314	5.179	4.313	3.828	3.514	3.291	2.993	2.888	2.801
60	7.077	4.977	4.126	3.649	3.339	3.119	2.823	2.718	2.632
120	6.851	4.787	3.949	3.48	3.174	2.956	2.663	2.559	2.472
$\infty$	6.635	4.605	3.782	3.319	3.017	2.802	2.511	2.407	2.321

# F Table for $\alpha=0.01$

(table for 12 to  $\infty$ )

DF2	DF1							
	12	15	20	24	30	40	120	$\infty$
1	6106.321	6157.285	6208.73	6234.631	6260.649	6286.782	6339.391	6365.864
2	99.416	99.433	99.449	99.458	99.466	99.474	99.491	99.499
3	27.052	26.872	26.69	26.598	26.505	26.411	26.221	26.125
4	14.374	14.198	14.02	13.929	13.838	13.745	13.558	13.463
5	9.888	9.722	9.553	9.466	9.379	9.291	9.112	9.02
6	7.718	7.559	7.396	7.313	7.229	7.143	6.969	6.88
7	6.469	6.314	6.155	6.074	5.992	5.908	5.737	5.65
8	5.667	5.515	5.359	5.279	5.198	5.116	4.946	4.859
9	5.111	4.962	4.808	4.729	4.649	4.567	4.398	4.311
10	4.706	4.558	4.405	4.327	4.247	4.165	3.996	3.909
11	4.397	4.251	4.099	4.021	3.941	3.86	3.69	3.602
12	4.155	4.01	3.858	3.78	3.701	3.619	3.449	3.361
13	3.815	3.665	3.587	3.507	3.425	3.341	3.165	3.1748
14	3.8	3.656	3.505	3.427	3.348	3.266	3.094	3.004
15	3.666	3.522	3.372	3.294	3.214	3.132	2.959	2.868
16	3.553	3.409	3.259	3.181	3.101	3.018	2.845	2.753
17	3.455	3.312	3.162	3.084	3.003	2.92	2.746	2.653
18	3.371	3.227	3.077	2.999	2.919	2.835	2.66	2.566
19	3.297	3.153	3.003	2.925	2.844	2.761	2.584	2.489
20	3.231	3.088	2.938	2.859	2.778	2.695	2.517	2.421
21	3.173	3.03	2.88	2.801	2.72	2.636	2.457	2.36
22	3.121	2.978	2.827	2.749	2.667	2.583	2.403	2.305
23	3.074	2.931	2.781	2.702	2.62	2.535	2.354	2.256
24	3.032	2.889	2.738	2.659	2.577	2.492	2.31	2.211
25	2.993	2.85	2.699	2.62	2.538	2.453	2.27	2.169
26	2.958	2.815	2.664	2.585	2.503	2.417	2.233	2.131
27	2.926	2.783	2.632	2.552	2.47	2.384	2.198	2.097
28	2.896	2.753	2.602	2.522	2.44	2.354	2.167	2.064
29	2.868	2.726	2.574	2.495	2.412	2.325	2.138	2.034
30	2.843	2.7	2.549	2.469	2.386	2.299	2.111	2.006
40	2.665	2.522	2.369	2.288	2.203	2.114	1.917	1.805
60	2.496	2.352	2.198	2.115	2.028	1.936	1.726	1.601
120	2.336	2.192	2.035	1.95	1.86	1.763	1.533	1.381
$\infty$	2.185	2.039	1.878	1.791	1.696	1.592	1.325	1

Statistical thinking  
will one day be  
as necessary a qualification  
for efficient citizenship  
as the ability  
to read and write.

*Herbert George “H. G.” Wells  
(1866 – 1946, English writer)*