一、C C B A B

二、(6) $\frac{1}{4}+\frac{1}{2}\ln 2$　(7)

X	1	2	3	…
P_k	0.76	$0.76\cdot0.24$	$0.76\cdot0.24^2$	

$P\{X=k\}=0.76\times(0.24)^{k-1}$

$k=1,2,\cdots$

(8) a5　(9) 集齐108将分别为 $X_1, X_2, \cdots, X_{108}$

设需购X包方便面

$X=X_1+X_2+X_3+\cdots+X_{108}$

$P\{X_n=k\}=\left(\frac{n-1}{108}\right)^{k-1}\frac{108-(n-1)}{108}$

X_n 服从参数为 $p_n=\frac{109-n}{108}$ 的几何分布

$E(X_n)=\sum_{k=1}^{n} k\,P\{X_n=k\}=\frac{1}{p_n}=\frac{108}{109-n}$

$E(X)=108\left(1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{108}\right)$

$=108\times5.264$

$=568.5$

(10) $Cov(X_1,Y)=\frac{1}{n}\delta^2$

1)

解：设 A_1：电压不超过200伏； A_2：200–240之间； A_3：超过240

B：元件损坏 $X\sim N(220,25^2)$

$$P(A_1)=P\{X\le 200\}=P\left\{\frac{X-220}{25}\le\frac{200-220}{25}\right\}=\Phi(-0.8)=0.212$$

$$P(A_2)=P\{200<X<240\}=\Phi(0.8)-\Phi(-0.8)=0.576$$

$$P(A_3)=P\{X>240\}=1-0.212-0.576=0.212$$

(1) $$P(B)=\sum_{i=1}^{3}P(A_i)P(B|A_i)=0.064$$

(2) $$P(A_2|B)=\frac{P(A_2)P(B|A_2)}{P(B)}\approx 0.009$$

2) 解：①由 $\int_{-\infty}^{+\infty}f(x)dx=1$ 得 $\left[\int_0^3 kx+\int_3^4\left(2-\frac{x}{2}\right)\right]dx=1$

得 $k=\frac{1}{6}$

$$f(x)=\begin{cases}\frac{x}{6} & 0\le x<3\\ 2-\frac{x}{2} & 3\le x<4\\ 0 & \text{其它}\end{cases}$$

②

$0\le x<3$ 时 $F(x)=\int_0^x\frac{t}{6}dt=\frac{x^2}{12}$

$3\le x<4$ 时 $F(x)=\int_0^3 f(t)dt+\int_3^x f(t)dt=-\frac{x^2}{4}+2x-3$

$4\le x$ 时 $F(x)=\int_0^3 f(t)dt+\int_3^4 f(t)dt+\int_4^x f(t)dt=1$

$$F(x)=\begin{cases}0 & x<0\\ \frac{x^2}{12} & 0\le x<3\\ -\frac{x^2}{4}+2x-3 & 3\le x<4\\ 1 & 4\le x\end{cases}$$

3)

(13) ① $A=\frac{1}{\pi^2}$ $B=\frac{\pi}{2}$ $C=\frac{\pi}{2}$

② $f(x,y)=\frac{\partial^2 F(x,y)}{\partial x\partial y}=\frac{6}{\pi^2(4+x^2)(9+y^2)}$ $x,y\in R$

③ $f_X(x)=\frac{2}{\pi(4+x^2)}$ $f_Y(y)=\frac{3}{\pi(9+y^2)}$

④ 为相独立

(14) 解：① $E(Z)=E(\frac{X}{3}+\frac{Y}{2})=\frac{1}{3}E(X)+\frac{1}{2}E(Y)=\frac{1}{3}$

$D(Z)=D(\frac{X}{3}+\frac{Y}{2})=\frac{1}{9}D(X)+\frac{1}{4}D(Y)=5$

② $\rho_{XY}=\frac{Cov(X,Y)}{\sqrt{D(X)}\sqrt{D(X)}}$

$Cov(X,Z)=Cov(X,\frac{X}{3}+\frac{Y}{2})=\frac{1}{3}D(X)+\frac{1}{2}Cov(X,Y)=0$

$\rho_{XY}=0-\frac{1}{2}$ $Cov(X,Y)=\rho_{XY}\sqrt{D(X)}\sqrt{D(Y)}=-6$

$\rho_{XZ}=\frac{Cov(X,Z)}{\sqrt{D(X)}\sqrt{D(Y)}}=0$

③ X与Y不相关，(X,Y)为维正态，得 X与Y相互独立

5)解: ① $Y=X_1+X_2$

$$\begin{cases}0<x_1<1\\0<x_2<1\end{cases}\Rightarrow\begin{cases}0<x_1<1\\0<y-x_1<1\end{cases}\Rightarrow\begin{cases}0<x_1<1\\y-1<x_2<y\end{cases}$$

$$f_Y(y)=\begin{cases}\int_0^y 1\,dx\\ \int_{y-1}^1 1\,dx\end{cases}\Rightarrow f_Y(y)=\begin{cases}y & 0<y<1\\ 2-y & 1\le y<2\\ 0 & \text{其它}\end{cases}$$

② $Z=Y+X_3$

$$\begin{cases}0<x_3<1\\0<y<1\end{cases}\Rightarrow\begin{cases}0<y<1\\z-1<y<z\end{cases}$$

$$f_z(z)=\begin{cases}\int_0^z y\,dy & 0<z<1\\ \int_{z-1}^1 y\,dy & 1\le z<2\end{cases}\Rightarrow f_Z(z)=\begin{cases}\frac{z^2}{2} & 0<z<1\\ -\frac{z^2}{2}+z & 1\le z<2\end{cases}$$

$$\begin{cases}0<x_3<1\\1\le y<2\end{cases}\Rightarrow\begin{cases}1\le y<2\\z-1<y<z\end{cases}$$

$$f_Z(z)=\begin{cases}\int_1^z 2-y\,dy & 1\le z<2\\ \int_{z-1}^z (2-y)\,dy & 2\le z<3\end{cases}\Rightarrow f_Z(z)=\begin{cases}2z-\frac{3}{2}-\frac{z^2}{2} & 1\le z<2\\ \frac{(z-3)^2}{2} & 2\le z<3\end{cases}$$

合并 $1\le z<2$ 得

$$f_Z(z)=\begin{cases}\frac{z^2}{2} & 0<z<1\\ -(z-\frac{3}{2})^2+\frac{3}{4} & 1\le z<2\\ \frac{(z-3)^2}{2} & 2\le y<3\\ 0 & \text{其它}\end{cases}$$

16. (1) $P\{X^2=Y^2\}=1 \Rightarrow P\{X^2\neq Y^2\}=0$

$P\{X=0,Y=1\}=P\{X=0,Y=-1\}=P\{X=1,Y=0\}=0$

$P\{X=1,Y=1\}=\frac{1}{3}$

X \ Y	-1	0	1
0	0	$\frac{1}{3}$	0
1	$\frac{1}{3}$	0	$\frac{1}{3}$

$P\{XY=-1\}=P\{X=1,Y=-1\}=\frac{1}{3}$

(2)

Z	-1	0	1
P_k	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

$P\{XY=0\}=P\{X=0,Y=0\}+P\{X=0,Y=-1\}$
$+P\{X=0,Y=1\}=\frac{1}{3}$

(3) $E(X)=\frac{2}{3}$ $E(Y)=0$ $D(X)=\frac{2}{9}$ $D(Y)=0$

$\rho_{XY}=0$