$$=2\arctan\sqrt{\frac{1-x}{1+x}}+\ln\left|\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right|+C.$$

方法二

$$\int\sqrt{\frac{1-x}{1+x}}\,\frac{dx}{x}=\int\frac{1-x}{x\sqrt{1-x^2}}dx\xrightarrow{x=\sin u}\int\frac{1-\sin u}{\sin u}du=\int\csc u\,du-\int du$$

$$=\ln|\csc u-\cot u|-u+C=\ln\frac{1-\sqrt{1-x^2}}{|x|}-\arcsin x+C.$$

24. $\int\frac{dx}{\sqrt[3]{(x+1)^2(x-1)^4}}=\int\frac{1}{x^2-1}\sqrt[3]{\frac{x+1}{x-1}}dx,$

令 $u=\sqrt[3]{\frac{x+1}{x-1}}$，即 $x=\frac{u^3+1}{u^3-1}$，得到

$$\int\frac{dx}{\sqrt[3]{(x+1)^2(x-1)^4}}=\int\frac{u}{\left(\frac{u^3+1}{u^3-1}\right)^2-1}\cdot\frac{-6u^2}{(u^3-1)^2}du=-\frac{3}{2}\int du$$

$$=-\frac{3}{2}u+C=-\frac{3}{2}\sqrt[3]{\frac{x+1}{x-1}}+C.$$

第五节　积分表的使用

（略）

习题 4—5 解答

利用积分表计算下列不定积分：

1. $\int\frac{dx}{\sqrt{4x^2-9}}$.
2. $\int\frac{1}{x^2+2x+5}dx$.
3. $\int\frac{dx}{\sqrt{5-4x+x^2}}$.
4. $\int\sqrt{2x^2+9}\,dx$.
5. $\int\sqrt{3x^2-2}\,dx$.
6. $\int e^{2x}\cos x\,dx$.
7. $\int x\arcsin\frac{x}{2}dx$.
8. $\int\frac{dx}{(x^2+9)^2}$.
9. $\int\frac{dx}{\sin^3 x}$.
10. $\int e^{-2x}\sin 3x\,dx$.
11. $\int\sin 3x\sin 5x\,dx$.
12. $\int\ln^3 x\,dx$.
13. $\int\frac{1}{x^2(1-x)}dx$.
14. $\int\frac{\sqrt{x-1}}{x}dx$.
15. $\int\frac{1}{(1+x^2)^2}dx$.
16. $\int\frac{1}{x\sqrt{x^2-1}}dx$.
17. $\int\frac{x}{(2+3x)^2}dx$.
18. $\int\cos^6 x\,dx$.
19. $\int x^2\sqrt{x^2-2}\,dx$.
20. $\int\frac{1}{2+5\cos x}dx$.
21. $\int\frac{dx}{x^2\sqrt{2x-1}}$.
22. $\int\sqrt{\frac{1-x}{1+x}}dx$.
23. $\int\frac{x+5}{x^2-2x-1}dx$.
24. $\int\frac{x\,dx}{\sqrt{1+x-x^2}}$.

25. $\int \frac{x^4}{25+4x^2}\mathrm{d}x$.

解 注意：下列各题中最后括号内所标的是所用积分公式在教材上册附录Ⅳ积分表中的编号.

1. $\int \frac{\mathrm{d}x}{\sqrt{4x^2-9}}=\frac{1}{2}\int \frac{\mathrm{d}(2x)}{\sqrt{(2x)^2-3^2}}=\frac{1}{2}\ln|2x+\sqrt{(2x)^2-3^2}|+C$

$=\frac{1}{2}\ln|2x+\sqrt{4x^2-9}|+C.$ (45)

2. $\int \frac{1}{x^2+2x+5}\mathrm{d}x=\int \frac{1}{(x+1)^2+2^2}\mathrm{d}(x+1)=\frac{1}{2}\arctan\frac{x+1}{2}+C.$ (19)

3. $\int \frac{\mathrm{d}x}{\sqrt{5-4x+x^2}}=\int \frac{\mathrm{d}(x-2)}{\sqrt{(x-2)^2+1}}=\ln[x-2+\sqrt{(x-2)^2+1}]+C$

$=\ln(x-2+\sqrt{5-4x+x^2})+C.$ (31)

4. $\int \sqrt{2x^2+9}\,\mathrm{d}x=\frac{1}{\sqrt{2}}\int \sqrt{(\sqrt{2}x)^2+3^2}\,\mathrm{d}(\sqrt{2}x)=\frac{1}{\sqrt{2}}\left\{\frac{\sqrt{2}x}{2}\sqrt{(\sqrt{2}x)^2+3^2}+\frac{3^2}{2}\ln[\sqrt{2}x+\sqrt{(\sqrt{2}x)^2+3^2}]\right\}+C$

$=\frac{x}{2}\sqrt{2x^2+9}+\frac{9\sqrt{2}}{4}\ln(\sqrt{2}x+\sqrt{2x^2+9})+C.$ (39)

5. $\int \sqrt{3x^2-2}\,\mathrm{d}x=\frac{1}{\sqrt{3}}\int \sqrt{(\sqrt{3}x)^2-(\sqrt{2})^2}\,\mathrm{d}(\sqrt{3}x)$

$=\frac{1}{\sqrt{3}}\left[\frac{\sqrt{3}x}{2}\sqrt{(\sqrt{3}x)^2-(\sqrt{2})^2}-\frac{(\sqrt{2})^2}{2}\ln|\sqrt{3}x+\sqrt{(\sqrt{3}x)^2-(\sqrt{2})^2}|\right]+C$

$=\frac{x}{2}\sqrt{3x^2-2}-\frac{\sqrt{3}}{3}\ln|\sqrt{3}x+\sqrt{3x^2-2}|+C.$ (53)

6. $\int \mathrm{e}^{2x}\cos x\mathrm{d}x=\frac{1}{2^2+1^2}\mathrm{e}^{2x}(\sin x+2\cos x)+C=\frac{1}{5}\mathrm{e}^{2x}(\sin x+2\cos x)+C.$ (129)

7. $\int x\arcsin\frac{x}{2}\mathrm{d}x=\left(\frac{x^2}{2}-\frac{2^2}{4}\right)\arcsin\frac{x}{2}+\frac{x}{4}\sqrt{2^2-x^2}+C$

$=\left(\frac{x^2}{2}-1\right)\arcsin\frac{x}{2}+\frac{x}{4}\sqrt{4-x^2}+C.$ (114)

8. $\int \frac{\mathrm{d}x}{(x^2+9)^2}=\int \frac{\mathrm{d}x}{(x^2+3^2)^2}=\frac{x}{2(2-1)3^2(x^2+3^2)}+\frac{2\times2-3}{2(2-1)3^2}\int \frac{\mathrm{d}x}{x^2+3^2}$

$=\frac{x}{18(x^2+9)}+\frac{1}{18}\cdot\frac{1}{3}\arctan\frac{x}{3}+C=\frac{x}{18(x^2+9)}+\frac{1}{54}\arctan\frac{x}{3}+C.$ (20,19)

9. $\int \frac{\mathrm{d}x}{\sin^3 x}=-\frac{1}{2}\cdot\frac{\cos x}{\sin^2 x}+\frac{1}{2}\int \frac{\mathrm{d}x}{\sin x}=-\frac{\cos x}{2\sin^2 x}+\frac{1}{2}\ln\left|\tan\frac{x}{2}\right|+C.$ (97,88)

10. $\int \mathrm{e}^{-2x}\sin 3x\mathrm{d}x=\frac{1}{(-2)^2+3^2}\mathrm{e}^{-2x}(-2\sin 3x-3\cos 3x)+C=-\frac{\mathrm{e}^{-2x}}{13}(2\sin 3x+3\cos 3x)+C.$ (128)

11. $\int \sin 3x\sin 5x\mathrm{d}x=-\frac{1}{2(3+5)}\sin(3+5)x+\frac{1}{2(3-5)}\sin(3-5)x+C=-\frac{1}{16}\sin 8x+\frac{1}{4}\sin 2x+C.$ (101)

12. $\int \ln^3 x\mathrm{d}x=x(\ln x)^3-3\int \ln^2 x\mathrm{d}x=x(\ln x)^3-3\left[x(\ln x)^2-2\int \ln x\mathrm{d}x\right]$

$=x(\ln x)^3-3x(\ln x)^2+6\int \ln x\mathrm{d}x=x(\ln x)^3-3x(\ln x)^2+6(x\ln x-x)+C$

$=x\ln^3 x-3x\ln^2 x+6x\ln x-6x+C.$ (135,132)

13. $\int \frac{1}{x^2(1-x)}\mathrm{d}x=-\frac{1}{x}-\ln\left|\frac{1-x}{x}\right|+C.$ (6)

14. $\int \frac{\sqrt{x-1}}{x}\mathrm{d}x=2\sqrt{x-1}-\int \frac{1}{x\sqrt{x-1}}\mathrm{d}x=2\sqrt{x-1}-2\arctan\sqrt{x-1}+C.$ (17,15)

15. $\int \frac{1}{(1+x^2)^2}dx=\frac{x}{2(1+x^2)}+\frac{1}{2}\int \frac{1}{1+x^2}dx=\frac{x}{2(1+x^2)}+\frac{1}{2}\arctan x+C.$（20,19）

16. $\int \frac{1}{x\sqrt{x^2-1}}dx=\arccos\frac{1}{|x|}+C.$（51）

17. $\int \frac{x}{(2+3x)^2}dx=\frac{1}{9}\left(\ln|2+3x|+\frac{2}{2+3x}\right)+C.$（7）

18. $\int\cos^6 x dx=\frac{1}{6}\cos^5 x\sin x+\frac{5}{6}\int\cos^4 x dx=\frac{1}{6}\cos^5 x\sin x+\frac{5}{6}\left(\frac{1}{4}\cos^3 x\sin x+\frac{3}{4}\int\cos^2 x dx\right)$

$=\frac{1}{6}\cos^5 x\sin x+\frac{5}{24}\cos^3 x\sin x+\frac{5}{8}\int\cos^2 x dx$

$=\frac{1}{6}\cos^5 x\sin x+\frac{5}{24}\cos^3 x\sin x+\frac{5}{8}\left(\frac{1}{2}\cos x\sin x+\frac{1}{2}\int dx\right)$

$=\frac{1}{6}\cos^5 x\sin x+\frac{5}{24}\cos^3 x\sin x+\frac{5}{16}\cos x\sin x+\frac{5}{16}x+C.$（96）

19. $\int x^2\sqrt{x^2-2}dx=\frac{x}{8}(2x^2-2)\sqrt{x^2-2}-\frac{4}{8}\ln\left|x+\sqrt{x^2-2}\right|+C$

$=\frac{x}{4}(x^2-1)\sqrt{x^2-2}-\frac{1}{2}\ln|x+\sqrt{x^2-2}|+C.$（56）

20. $\int \frac{1}{2+5\cos x}dx=\frac{1}{7}\sqrt{\frac{7}{3}}\ln\left|\frac{\tan\frac{x}{2}+\sqrt{\frac{7}{3}}}{\tan\frac{x}{2}-\sqrt{\frac{7}{3}}}\right|+C=\frac{1}{\sqrt{21}}\ln\left|\frac{\sqrt{3}\tan\frac{x}{2}+\sqrt{7}}{\sqrt{3}\tan\frac{x}{2}-\sqrt{7}}\right|+C.$（106）

21. $\int \frac{dx}{x^2\sqrt{2x-1}}=-\frac{\sqrt{2x-1}}{-x}-\frac{2}{-2}\int \frac{dx}{x\sqrt{2x-1}}=\frac{\sqrt{2x-1}}{x}+2\arctan\sqrt{2x-1}+C.$（16,15）

22. **方法一**

$\int\sqrt{\frac{1-x}{1+x}}dx=\int\frac{1-x}{\sqrt{1-x^2}}dx=\int\frac{1}{\sqrt{1-x^2}}dx-\int\frac{x}{\sqrt{1-x^2}}dx=\arcsin x+\sqrt{1-x^2}+C.$（59,61）

方法二

$\int\sqrt{\frac{1-x}{1+x}}dx=(x+1)\sqrt{\frac{1-x}{1+x}}-2\arcsin\sqrt{\frac{1-x}{2}}+C=\sqrt{1-x^2}-2\arcsin\sqrt{\frac{1-x}{2}}+C.$（80）

23. $\int \frac{x+5}{x^2-2x-1}dx=\int\frac{x}{x^2-2x-1}dx+5\int\frac{1}{x^2-2x-1}dx$

$=\frac{1}{2}\ln|x^2-2x-1|-\frac{-2}{2}\int\frac{1}{x^2-2x-1}dx+5\int\frac{1}{x^2-2x-1}dx$

$=\frac{1}{2}\ln|x^2-2x-1|+6\cdot\frac{1}{\sqrt{(-2)^2-4\cdot1\cdot(-1)}}\cdot\ln\left|\frac{2x-2-\sqrt{(-2)^2-4\cdot1\cdot(-1)}}{2x-2+\sqrt{(-2)^2-4\cdot1\cdot(-1)}}\right|+C$

$=\frac{1}{2}\ln|x^2-2x-1|+\frac{3}{\sqrt{2}}\ln\left|\frac{x-(\sqrt{2}+1)}{x+(\sqrt{2}-1)}\right|+C.$（30,29）

24. $\int \frac{xdx}{\sqrt{1+x-x^2}}=-\sqrt{1+x-x^2}+\frac{1}{2}\arcsin\frac{2x-1}{\sqrt{5}}+C.$（78）

25. $\int \frac{x^4}{25+4x^2}dx=\int\left(\frac{1}{4}x^2-\frac{25}{16}+\frac{625}{16}\cdot\frac{1}{25+4x^2}\right)dx=\frac{x^3}{12}-\frac{25}{16}x+\frac{625}{32}\int\frac{1}{5^2+(2x)^2}d(2x)$

$=\frac{x^3}{12}-\frac{25}{16}x+\frac{625}{32}\cdot\frac{1}{5}\arctan\frac{2x}{5}+C=\frac{x^3}{12}-\frac{25}{16}x+\frac{125}{32}\arctan\frac{2x}{5}+C.$（19）