

Table VII: GNY Expression

Notation	Description
$P, Q$	Parties of the session
$(X, Y)$	Conjunction of $X$ and $Y$
$H(X)$	$H$ is a one-way function of $X$
$F(X_1, \dots, X_n)$	$F$ is a many-to-one computationally feasible function for any $X_i$
$*X$	$X$ is a not-originated-here
$P \triangleleft X$	$P$ is told $X$
$P \ni X$	$P$ possesses, or is capable of possessing $X$
$P \sim X$	$P$ once conveyed formula $X$
$P \equiv \#(X)$	$X$ has not been used for the same purpose at any time before the current run of the protocol
$P \equiv \emptyset(X)$	$P$ would recognize $X$ if $P$ has certain expectations about the contents of $X$ before actually receiving $X$
$P \xleftrightarrow{K} Q$	$K$ is a shared secret between $P$ and $Q$
$LR$	$LR$ achieves leakage-resilient according to Dziembowski <i>et al.</i> [22], [31](additional expression)

## APPENDIX B

### CLR-EGNY LOGIC ANALYSIS

The notations and statements are summarized in Table VII.

#### 1) Description

The parser algorithm would produce the following description of LLRA based on above paraphrases:

$Msg_1 : S_B \triangleleft *X_A, *Y_A, *T_A, *C_A, *Auth;$

$Msg_2 : U_A \triangleleft *X_B, *Y_B, *T_B, *E_B;$

$Msg_3 : S_B \triangleleft *E_A.$

#### 2) Goal

The shared session keys between  $U_A$  and  $U_B$  shall achieve the following goals:

**Goal 1:**  $U_A \equiv \#SK;$

**Goal 2:**  $U_A \equiv \phi SK;$

**Goal 3:**  $U_A \equiv U_B \ni SK;$

**Goal 4:**  $U_B \equiv \#SK;$

**Goal 5:**  $U_B \equiv \phi SK;$

**Goal 6:**  $U_B \equiv U_A \ni SK.$

#### 3) Initial Assumptions

Referring to LLRA's registration phrase, we have several initialization assumptions:

A1:  $U_A \equiv \#y_A;$

A2:  $U_A \equiv \phi y_A;$

A3:  $U_A \ni y_A, ID_A, hw_A, D_A, \overrightarrow{r_{AL}}, \overrightarrow{r_{AR}}, G, Z_A, Z_B;$

A4:  $S_B \equiv \#y_B;$

A5:  $S_B \equiv \phi y_B;$

A6:  $S_B \ni y_B, ID_B, hw_B, D_B, \overrightarrow{r_{BL}}, \overrightarrow{r_{BR}}, G, Z_A, Z_B;$

A7:  $U_A \equiv S_B \ni Z_A, Z_B;$

A8:  $S_B \equiv U_A \ni Z_A, Z_B;$

A9:  $U_A \equiv U_A \xleftrightarrow{pw} S_B;$

A10:  $S_B \equiv S_B \xleftrightarrow{pw} U_A;$

A11:  $U_A \equiv S_B \implies S_B \equiv *;$

A12:  $S_B \equiv U_A \implies U_A \equiv *;$

A13:  $U_A \equiv S_B \implies S_B \ni (u_B, y_B);$

A14:  $S_B \equiv U_A \implies U_A \ni (u_A, y_A);$

A15:  $U_A \equiv \Lambda(r_A);$

A16:  $S_B \equiv \Lambda(r_B).$

#### 4) Proof

Then, we start the formal proof of **Goal 1** to **Goal 6** in GNY logic.

According to rules T1 and P1, we can get that  $U_A$  possesses  $X_B, Y_B, T_B, E_B, X_A, Y_A, T_A, E_A.$

$$\frac{U_A \triangleleft *X_B, *Y_B, *T_B, *E_B}{U_A \triangleleft X_B, Y_B, T_B, E_B} (T1)$$

$$\frac{U_A \triangleleft X_B, Y_B, T_B, E_B}{U_A \ni X_B, Y_B, T_B, E_B} (P1)$$

According to rules T1 and P1, we can get that  $U_B$  possesses  $X_A, Y_A, T_A, C_A, Auth, E_A.$

$$\frac{U_B \triangleleft *X_A, *Y_A, *T_A, *C_A, *Auth, *E_A}{U_B \triangleleft X_A, Y_A, T_A, C_A, Auth, E_A} (T1)$$

$$\frac{U_B \triangleleft X_A, Y_A, T_A, C_A, Auth, E_A}{U_B \ni X_A, Y_A, T_A, C_A, Auth, E_A} (P1)$$

**Goal 1:** According to A1 and the rule F1, we can get that  $U_A$  believes that  $((u_A + y_A) \cdot G)$  is fresh. and  $Y_A = (u_A + y_A) \cdot G.$

$$\frac{U_A \equiv \#y_A}{U_A \equiv \#((u_A + y_A) \cdot G)} (F1)$$

According to the rule F1, we can get that  $U_A$  believes that  $(X_A || Y_A || X_B || Y_B)$  is fresh. and  $sid_A = Z_A || Y_A || Z_B || Y_B.$

$$\frac{U_A \equiv \#Y_A}{U_A \equiv \#(Z_A || Y_A || Z_B || Y_B)} (F1)$$

According to the rule F1, we can get that  $U_A$  believes that  $(sk_A || sid_A)$  is fresh.

$$\frac{U_A \equiv \#sid_A}{U_A \equiv \#(sk_A || sid_A)} (F1)$$

According to A3 and the rule P2, we can get that  $U_A \ni (\overrightarrow{v_A}, \overrightarrow{w_A}).$

$$\frac{U_A \ni y_A, \overrightarrow{r_{AL}}, \overrightarrow{r_{AR}}}{U_A \ni (\overrightarrow{v_A}, \overrightarrow{w_A})} (P2)$$

$$\frac{U_A \ni (\overrightarrow{v_A}, \overrightarrow{w_A})}{U_A \ni u_A} (P2)$$

According to A3 and the rule P4, we can get that  $U_A \ni H_2(s_A).$

$$\frac{U_A \ni s_A}{U_A \ni H_2(s_A)} (P4)$$

According to A3 and the rule P2, we can get that  $U_A \ni H_2(s_A) \oplus ID_A$ , so we can get that  $U_A \ni D_A.$

$$\frac{U_A \ni H_2(s_A), ID_A}{U_A \ni H_2(s_A) \oplus ID_A} (P2)$$

According to the A3 and rule P2, we can get that  $U_A \ni hw_A \oplus D_A \oplus u_A \oplus y_A.$

$$\frac{U_A \ni hw_A, D_A, u_A, y_A}{U_A \ni hw_A \oplus D_A \oplus u_A \oplus y_A} (P2)$$

According to the rule P4, we can get that  $U_A \ni H_2(hw_A \oplus D_A \oplus u_A \oplus y_A).$  so we can get that  $U_A \ni X_A.$

$$\frac{U_A \ni hw_A \oplus D_A \oplus u_A \oplus y_A}{U_A \ni H_2(hw_A \oplus D_A \oplus u_A \oplus y_A)} (P2)$$

According to the A3 and the rule P2, we can get that  $U_A \ni ((u_A + y_A) \cdot Y_B)$ , so we can get that  $U_A \ni V_A$ .

$$\frac{U_A \ni u_A, y_A, Y_B}{U_A \ni ((u_A + y_A) \cdot Y_B)} (P2)$$

According to the rule P4, we can get that  $U_A \ni H_3(V_A)$ .

$$\frac{U_A \ni V_A}{U_A \ni H_3(V_A)} (P4)$$

According to the A3 and the rule P2, we can get that  $U_A \ni ((u_A + y_A) \cdot G)$ , so we can get that  $U_A \ni Y_A$ .

$$\frac{U_A \ni u_A, y_A, G}{U_A \ni ((u_A + y_A) \cdot G)} (P2)$$

According to the rule P2, we can get that  $U_A \ni (X_A || Y_A || X_B || Y_B)$ , so we can get that  $U_A \ni sid_A$ .

$$\frac{U_A \ni Z_A, Y_A, Z_B, Y_B}{U_A \ni Z_A || Y_A || Z_B || Y_B} (P2)$$

According to the A3 and the rule P2, we can get that  $U_A \ni (X_A \oplus X_B \oplus H_3(V_A))$ , so we can get that  $U_A \ni sk_A$ .

$$\frac{U_A \ni X_A, X_B, H_3(V_A)}{U_A \ni (X_A \oplus X_B \oplus H_3(V_A))} (P2)$$

According to the rule P2, we can get that  $U_A \ni (sk_A || sid_A)$ .

$$\frac{U_A \ni sk_A, sid_A}{U_A \ni sk_A || sid_A} (P2)$$

According to the rule F10, we can get that  $U_A$  believes that  $SK$  is fresh.

$$\frac{U_A \models \#(sk_A || sid_A), U_A \ni (sk_A || sid_A)}{U_A \models \#H(sk_A || sid_A)} (F10)$$

$$\frac{U_A \models \#H(sk_A || sid_A)}{U_A \models \#SK}$$

**Goal 2:** According to A1 and the rule R1, we can get that  $U_A$  believes that  $((u_A + y_A) \cdot G)$  is recognizable.

$$\frac{U_A \models \phi y_A}{U_A \models \phi((u_A + y_A) \cdot G)} (R1)$$

According to the rule R1, we can get that  $U_A$  believes that  $(Z_A || Y_A || Z_B || Y_B)$  is recognizable.

$$\frac{U_A \models \phi Y_A}{U_A \models \phi(Z_A || Y_A || Z_B || Y_B)} (R1)$$

According to the rule R1, we can get that  $U_A$  believes that  $(sk_A || sid_A)$  is recognizable.

$$\frac{U_A \models \phi sid_A}{U_A \models \phi(sk_A || sid_A)} (R1)$$

According to the rule R10, we can get that  $U_A$  believes that  $SK$  is recognizable.

$$\frac{U_A \models \phi(sk_A || sid_A), U_A \ni (sk_A || sid_A)}{U_A \models \phi H(sk_A || sid_A)} (R5)$$

$$\frac{U_A \models \phi H(sk_A || sid_A)}{U_A \models \phi SK}$$

**Goal 3:** According to A1 and the rule F10, we can get that  $S_A$  believes that  $X_A$  is fresh, and  $X_A = H_2(hw_A \oplus D_A \oplus u_A \oplus y_A)$ .

$$\frac{U_A \models \#y_A, U_A \ni (hw_A \oplus D_A \oplus u_A \oplus y_A)}{U_A \models \#H_2(hw_A \oplus D_A \oplus u_A \oplus y_A)} (F10)$$

According to the rule F1, we can get that  $U_A$  believes that  $sk_A$  is fresh, and  $sk_A = X_B \oplus X_A \oplus H_3(V_A)$ .

$$\frac{U_A \models \#X_A}{U_A \models \#(X_B \oplus X_A \oplus H_3(V_A))} (F1)$$

According to A2 and the rule R5, we can get that  $U_A$  believes that  $X_A$  is recognizable, and  $X_A = H_2(hw_A \oplus D_A \oplus u_A \oplus y_A)$ .

$$\frac{U_A \models \phi y_A, U_A \ni (hw_A \oplus D_A \oplus u_A \oplus y_A)}{U_A \models \phi H_2(hw_A \oplus D_A \oplus u_A \oplus y_A)} (R5)$$

According to the rule R1, we can get that  $U_A$  believes that  $sk_A$  is recognizable.

$$\frac{U_A \models \phi X_A}{U_A \models \phi(T_B, hw_B, y_B, X_A, V_B, T_A)} (R1)$$

According to A9 and the rule I1, we can get that  $U_A$  believes that  $S_B$  once conveyed  $(T_B, hw_B, y_B, X_A, V_B, T_A)$ , and  $U_A$  owns  $sk_B$ .

$$\frac{U_A \triangleleft^* E_B, U_A \ni sk_B, U_A \models U_A \xrightarrow{pw} S_B, U_A \models \phi(T_B, hw_B, y_B, X_A, V_B, T_A), U_A \models \#sk_B}{U_A \models S_B \sim (T_B, hw_B, y_B, X_A, V_B, T_A), U_A \models S_B \ni sk_B} (I1)$$

According to the rule I7, we can get that  $U_A$  believes that  $S_B$  once conveyed  $V_B$ .

$$\frac{U_A \models S_B \sim (T_B, hw_B, y_B, X_A, V_B, T_A)}{U_A \models S_B \sim V_B} (I7)$$

According to the rule F1, we can get that  $U_A$  believes that  $V_B$  is fresh, and  $V_B = (u_B + y_B) \cdot Y_A$ .

$$\frac{U_A \models \#Y_A}{U_A \models \#(u_B + y_B) \cdot Y_A} (F1)$$

According to our new GNY expression, we can get that  $U_A \models \Lambda(r_A)$ .

$$\frac{U_A \ni Encode_{Z_q^*}^{n,m}(r_A), U_A \sim (f_1(r_{AL}), f_2(r_{AR})), U_A \ni SK_A, U_A \ni Refresh_{Z_q^*}^{n,m}(r_{AL}, r_{AR})}{U_A \models \Lambda(r_A)}$$

According to A11 and the rule LRJ2, we can get that  $U_A$  believes that  $S_B$  believes that  $S_B$  possesses  $u_B$ .

$$\frac{U_A \models S_B \implies S_B \models *, U_A \models S_B \sim (V_B \rightsquigarrow (S_B \ni u_B)), U_A \models \#V_B, U_A \models \Lambda(r_A), S_B \models \Lambda(r_B)}{U_A \models S_B \models (S_B \ni u_B)} (LRJ2)$$

According to A13 and the rule LRJ1, we can get that  $U_A$  believes that  $S_B$  possesses  $u_B$ .

$$\frac{U_A \models S_B \implies S_B \ni u_B, U_A \models S_B \models (S_B \ni u_B), U_A \models \Lambda(r_A), S_B \models \Lambda(r_B)}{U_A \models S_B \ni u_B} (LRJ1)$$

According to the rationality rule from postulate P3, we can get that  $U_A \models S_B \ni y_B$ .

$$\frac{U_A \models S_B \ni u_B}{U_A \models S_B \ni y_B} (RP3)$$

According to the rationality rule from postulate P2, we can get that  $U_A \models S_B \ni (u_B + y_B) \cdot G$ , so we can obtain that  $U_A \models S_B \ni Y_B$ .

$$\frac{U_A \models S_B \ni u_B, y_B, G}{U_A \models S_B \ni (u_B + y_B) \cdot G} (RP2)$$

According to the rule I6, we can get that  $U_A \models S_B \ni V_B$ .

$$\frac{U_A \models S_B \ni V_B, U_A \models \#V_B}{U_A \models S_B \ni V_B} (I6)$$

According to the rationality rule from postulate P5, we can get that  $U_A \models S_B \ni Y_A$ .

$$\frac{U_A \models S_B \ni ((u_B + y_B) \cdot Y_A), U_A \models S_B \ni u_B, y_B}{U_A \models S_B \ni Y_A} (RP5)$$

According to the rationality rule from postulate P2, we can get that  $U_A \models S_B \ni Z_A \parallel Z_B \parallel Y_A \parallel Y_B$ , so we can obtain that  $U_A \models S_B \ni sid_B$ .

$$\frac{U_A \models S_B \ni Z_A, Z_B, Y_A, Y_B}{U_A \models S_B \ni Z_A \parallel Z_B \parallel Y_A \parallel Y_B} (RP2)$$

According to the rationality rule from postulate P2 and P4, we can get that  $U_A \models S_B \ni H(sk_B \parallel sid_B)$ , so we can obtain that  $U_A \models S_B \ni SK_B$ .

$$\frac{U_A \models S_B \ni sk_B, sid_B}{U_A \models S_B \ni sk_B \parallel sid_B} (RP2)(RP4)$$

#### Goal 4:

According to the rule F1 and A4, we can get that  $S_B$  believes that  $Y_B$  is fresh, and  $Y_B = (u_B + y_B) \cdot G$ .

$$\frac{S_B \models \#y_B}{S_B \models \#((u_B + y_B) \cdot G)} (F1)$$

According to the rule F1, we can get that  $S_B$  believes that  $sid_B$  is fresh, and  $sid_B = Z_A, Y_A, Z_B, Y_B$ .

$$\frac{S_B \models \#Y_B}{S_B \models \#(Z_A, Y_A, Z_B, Y_B)} (F1)$$

According to the rule F1, we can get that  $S_B$  believes that  $(sk_B, sid_B)$  is fresh.

$$\frac{S_B \models \#sid_B}{S_B \models \#(sk_B, sid_B)} (F1)$$

According to A6 and the rule P2, we can get that  $S_B$  possesses  $\vec{v}_B$  and  $\vec{w}_B$ , and  $\vec{v}_B = (y_B, r_{BL_1}, \dots, r_{BL_n}), \vec{w}_B = (1, r_{BR_1}, \dots, r_{BR_n})$ .

$$\frac{S_B \ni \vec{r}_{BL}, S_B \ni y_B}{S_B \ni \vec{v}_B} (P2)$$

$$\frac{S_B \ni \vec{r}_{BR}}{S_B \ni \vec{w}_B} (P2)$$

According to the rule P2, we can get that  $S_B$  possesses  $u_B$ , and  $u_B = \vec{v}_B \cdot \vec{w}_B$ .

$$\frac{S_B \ni \vec{v}_B, S_B \ni \vec{w}_B}{S_B \ni \vec{v}_B \cdot \vec{w}_B} (P2)$$

According to the rule P4, we can get that  $S_B$  possesses  $X_B$ , and  $X_B = H_2(hw_B \oplus D_B \oplus u_B \oplus y_B)$ .

$$\frac{S_B \ni hw_B, S_B \ni D_B, S_B \ni u_B, S_B \ni y_B}{S_B \ni H_2(hw_B \oplus D_B \oplus u_B \oplus y_B)} (P4)$$

According to A6 and the rule P2, we can get that  $S_B$  possesses  $Y_B$ , and  $Y_B = (u_B + y_B) \cdot G$ .

$$\frac{S_B \ni u_B, S_B \ni y_B, S_B \ni G}{S_B \ni (u_B + y_B) \cdot G} (P2)$$

According to the rule P2, we can get that  $S_B$  possesses  $V_B$ , and  $V_B = (u_B + y_B) \cdot Y_A$ .

$$\frac{S_B \ni Y_A, S_B \ni u_B, S_B \ni y_B}{S_B \ni (u_B + y_B) \cdot Y_A} (P2)$$

According to the rule P4, we can get that  $S_B$  possesses  $H_3(V)$ .

$$\frac{S_B \ni v}{S_B \ni H_3(V)} (P4)$$

According to the rule P2, we can get that  $S_B$  possesses  $sk_B$ , and  $sk_B = X_A \oplus X_B \oplus H_3(V)$ .

$$\frac{S_B \ni X_A, S_B \ni X_B, S_B \ni H_3(V)}{S_B \ni X_A \oplus X_B \oplus H_3(V)} (P2)$$

According to the rule P2, we can get that  $S_B$  possesses  $sk_B, sid_B$ , and  $sid_B = Z_A, Y_A, Z_B, Y_B$ .

$$\frac{S_B \ni Z_A, S_B \ni Y_A, S_B \ni Z_B, S_B \ni Y_B}{\frac{S_B \ni (Z_A \parallel Y_A \parallel Z_B \parallel Y_B)}{S_B \ni (sk_B \parallel sid_B)}} (P2)$$

According to the rule F10, we can get that  $S_B$  believes that  $SK$  is fresh, and  $SK = H(sk_B \parallel sid_B)$ . Goal 4 is proved.

$$\frac{S_B \models \#(sk_B \parallel sid_B), S_B \ni (sk_B \parallel sid_B)}{S_B \models \#H(sk_B \parallel sid_B)} (F10)$$

**Goal 5:** According to the rule R1 and A5, we can get that  $S_B$  believes that  $Y_B$  is recognizable, and  $Y_B = (u_B + y_B) \cdot G$ .

$$\frac{S_B \models \phi y_B}{S_B \models \phi((u_B + y_B) \cdot G)} (R1)$$

According to the rule R1, we can get that  $S_B$  believes that  $sid_B$  is recognizable, and  $sid_B = Z_A, Y_A, Z_B, Y_B$ .

$$\frac{S_B \models \phi Y_B}{S_B \models \phi(Z_A, Y_A, Z_B, Y_B)} (R1)$$

According to the rule R1, we can get that  $S_B$  believes that  $(sk_B, sid_B)$  is recognizable.

$$\frac{S_B \models \phi sid_B}{S_B \models \phi(sk_B, sid_B)} (R1)$$

According to the rule R5, we can get that  $S_B$  believes that  $SK$  is recognizable, and  $SK = H(sk_B || sid_B)$ . Goal 5 is proved.

$$\frac{S_B \models \phi(sk_B || sid_B), S_B \ni (sk_B || sid_B)}{S_B \models \phi H(sk_B || sid_B)} (R5)$$

**Goal 6:** According to A4 and the rule F10, we can get that  $S_B$  believes that  $X_B$  is fresh, and  $X_B = H_2(hw_B \oplus D_B \oplus u_B \oplus y_B)$ .

$$\frac{S_B \models \#y_B, S_B \ni (hw_B \oplus D_B \oplus u_B \oplus y_B)}{S_B \models \#H_2(hw_B \oplus D_B \oplus u_B \oplus y_B)} (F10)$$

According to the rule F1, we can get that  $S_B$  believes that  $sk_B$  is fresh, and  $sk_B = X_A \oplus X_B \oplus H_3(V_B)$ .

$$\frac{S_B \models \#X_B}{S_B \models \#(X_A \oplus X_B \oplus H_3(V_B))} (F1)$$

According to A5 and the rule R5, we can get that  $S_B$  believes that  $X_B$  is recognizable, and  $X_B = H_2(hw_B \oplus D_B \oplus u_B \oplus y_B)$ .

$$\frac{S_B \models \phi y_B, S_B \ni (hw_B \oplus D_B \oplus u_B \oplus y_B)}{S_B \models \phi H_2(hw_B \oplus D_B \oplus u_B \oplus y_B)} (R5)$$

According to the rule R1, we can get that  $S_B$  believes that  $sk_B$  is recognizable.

$$\frac{S_B \models \phi X_B}{S_B \models \phi(T_B, hw_A, y_A, u_A, X_B, V_A)} (R1)$$

According to A10 and the rule I1, we can get that  $S_B$  believes that  $U_A$  once conveyed  $(T_B, hw_A, y_A, u_A, X_B, V_A)$ , and  $U_A$  owns  $sk_A$ .

$$\frac{S_B \triangleleft *E_A, S_B \ni sk_A, S_B \models S_B \xrightarrow{pw} U_A, S_B \models \phi(T_B, hw_A, y_A, u_A, X_B, V_A), S_B \models \#sk_A}{S_B \models U_A \sim (T_B, hw_A, y_A, u_A, X_B, V_A), S_B \models U_A \ni sk_A} (I1)$$

According to the rule I7, we can get that  $S_B$  believes that  $U_A$  once conveyed  $V_A$ .

$$\frac{S_B \models U_A \sim (T_B, hw_A, y_A, u_A, X_B, V_A)}{S_B \models U_A \sim V_A} (I7)$$

According to the rule F1, we can get that  $S_B$  believes that  $V_A$  is fresh, and  $V_A = (u_A + y_A) \cdot Y_B$ .

$$\frac{S_B \models \#Y_B}{S_B \models \#(u_A + y_A) \cdot Y_B} (F1)$$

According to our new GNY expression we can get that  $S_B$  achieves leakage-resilient.

$$\frac{S_B \ni Encode_{\mathbb{Z}_q^*}^{n,m}(r_B), S_B \sim (f_1(r_{BL}), f_2(r_{BR})), S_B \ni SK_B, S_B \ni Refresh_{\mathbb{Z}_q^*}^{n,m}(r_{BL}, r_{BR})}{S_B \models \Lambda(r_B)}$$

According to A12 and the rule LRJ2, we can get that  $S_B$  believes that  $U_A$  believes that  $U_A$  possesses  $u_A$ .

$$\frac{S_B \models U_A \implies U_A \models *, S_B \models U_A \sim (V_A \rightsquigarrow (U_A \ni u_A)), S_B \models \#V_A, U_A \models \Lambda(r_A), S_B \models \Lambda(r_B)}{S_B \models U_A \models (U_A \ni u_A)} (LRJ2)$$

According to A14 and the rule LRJ1, we can get that  $S_B$  believes that  $U_A$  possesses  $u_A$ .

$$\frac{S_B \models U_A \implies U_A \ni u_A, S_B \models U_A \models (U_A \ni u_A), U_A \models \Lambda(r_A), S_B \models \Lambda(r_B)}{S_B \models U_A \ni u_A} (LRJ1)$$

According to the rationality rule from postulate P3, we can get that  $S_B \models U_A \ni y_A$ .

$$\frac{S_B \models U_A \ni u_A}{S_B \models U_A \ni y_A} (RP3)$$

According to the rationality rule from postulate P2, we can get that  $S_B \models U_A \ni (u_A + y_A) \cdot G$ , so we can obtain that  $S_B \models U_A \ni Y_A$ .

$$\frac{S_B \models U_A \ni u_A, y_A, G}{S_B \models U_A \ni (u_A + y_A) \cdot G} (RP2)$$

According to the rule I6, we can get that  $S_B \models U_A \ni V_A$ .

$$\frac{S_B \models U_A \sim V_A, S_B \models \#V_A}{S_B \models U_A \ni V_A} (I6)$$

According to the rationality rule from postulate P5, we can get that  $S_B \models U_A \ni Y_B$ .

$$\frac{S_B \models U_A \ni ((u_A + y_A) \cdot Y_B), S_B \models U_A \ni u_A, y_A}{S_B \models U_A \ni Y_B} (RP5)$$

According to the rationality rule from postulate P2, we can get that  $S_B \models U_A \ni Z_A || Z_B || Y_A || Y_B$ , so we can obtain that  $S_B \models U_A \ni sid_A$ .

$$\frac{S_B \models U_A \ni Z_A, Z_B, Y_A, Y_B}{S_B \models U_A \ni Z_A || Z_B || Y_A || Y_B} (RP2)$$

According to the rationality rule from postulate P2 and P4, we can get that  $S_B \models U_A \ni H(sk_A || sid_A)$ , so we can obtain that  $S_B \models U_A \ni SK_A$ .

$$\frac{S_B \models U_A \ni sk_A, sid_A}{S_B \models U_A \ni sk_A || sid_A} (RP2)(RP4)$$