Medical Images- Pre-Processing

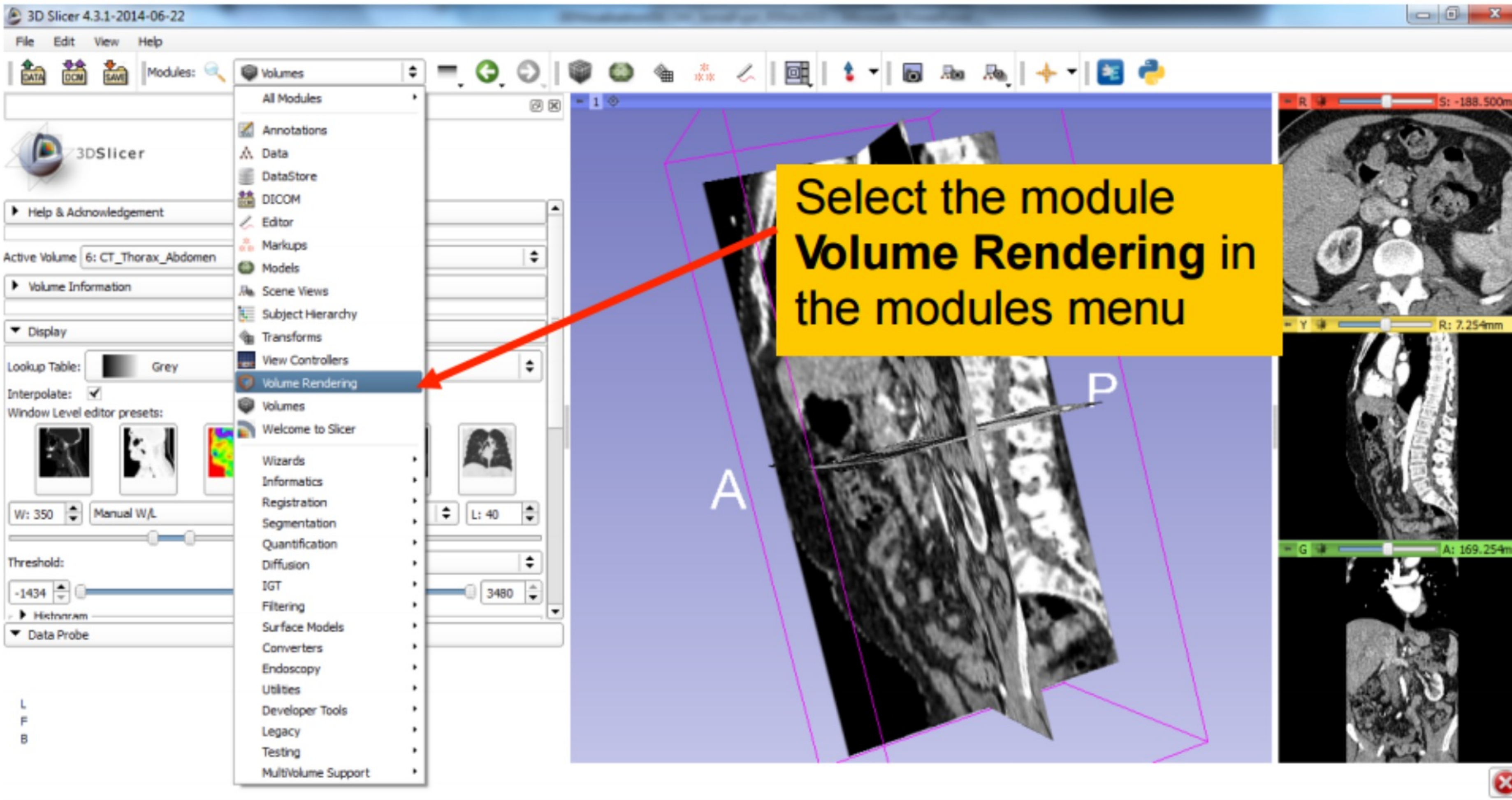
Lecture 3

Take home quiz

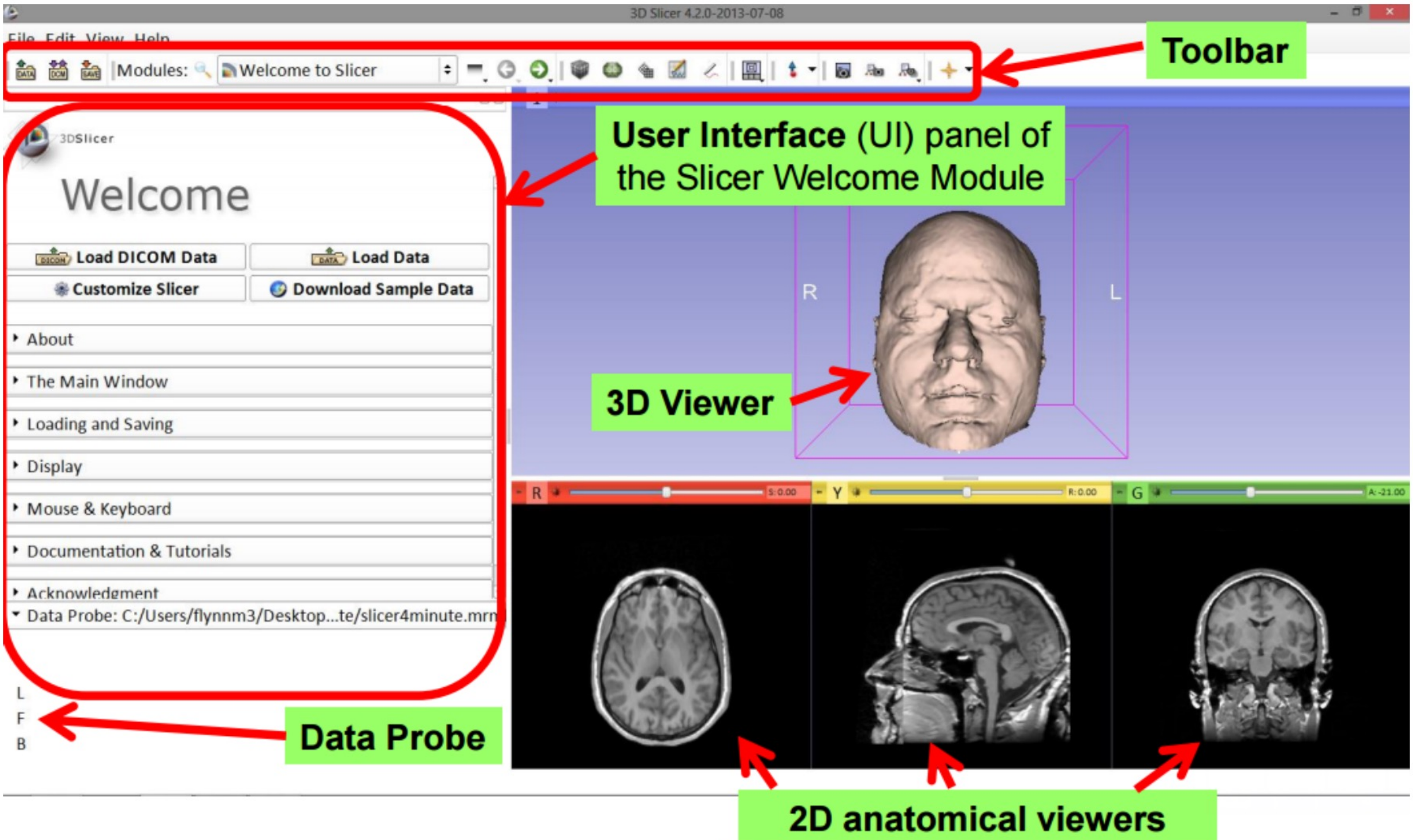
Explore the following!!

- ImageJ (and/or FIJI)
- ITK-Snap
- SimpleITK
- MITK
- FreeSurfer
- SLICER
- OsiriX
- An extensive list of software: www.idoimaging.com

Take home quiz



Slicer Welcome Module



CAVA: Computer Aided Visualization and Analysis

The science of underlying computerized methods of image processing, analysis, and visualizations to facilitate new therapeutic strategies, basic clinical research, education, and training.

CAD: Computer Aided Diagnosis

- The science of underlying computerized methods of image processing, analysis for the diagnosis of diseases via images.

Terminologies

- Object:

  An entity that is imaged and studied. May be rigid, deformable, static, or dynamic, physical or conceptual.

- Object system:

  A collection of related objects.

- Scanner:

  Any imaging device

- Body region:

  The support region of the imaged object system.

- Voxels (3D):

  Cuboidal elements into which body region is digitized by the imaging

Terminologies

- Scene:

  Multidimensional (2D, 3D, 4D, …) image of the body regions.
- Scene Intensity:

  Values assigned to voxels
- Binary Scene:

  A scene with intensities 0 and 1 only
- Structure:

  Geometric representation of an object in the object system derived from scenes.

Terminologies

- Body Coordinate System:

  A coordinate system associated with the imaged body region

- Scanner Coordinate System

  A coordinate system affixed to the scanner

- Scene Coordinate System

  A coordinate system affixed to the scene

- Display Coordinate System:

  A coordinate system associated with the display device

Image Pre-Processing

- Volume of Interest (VOI):

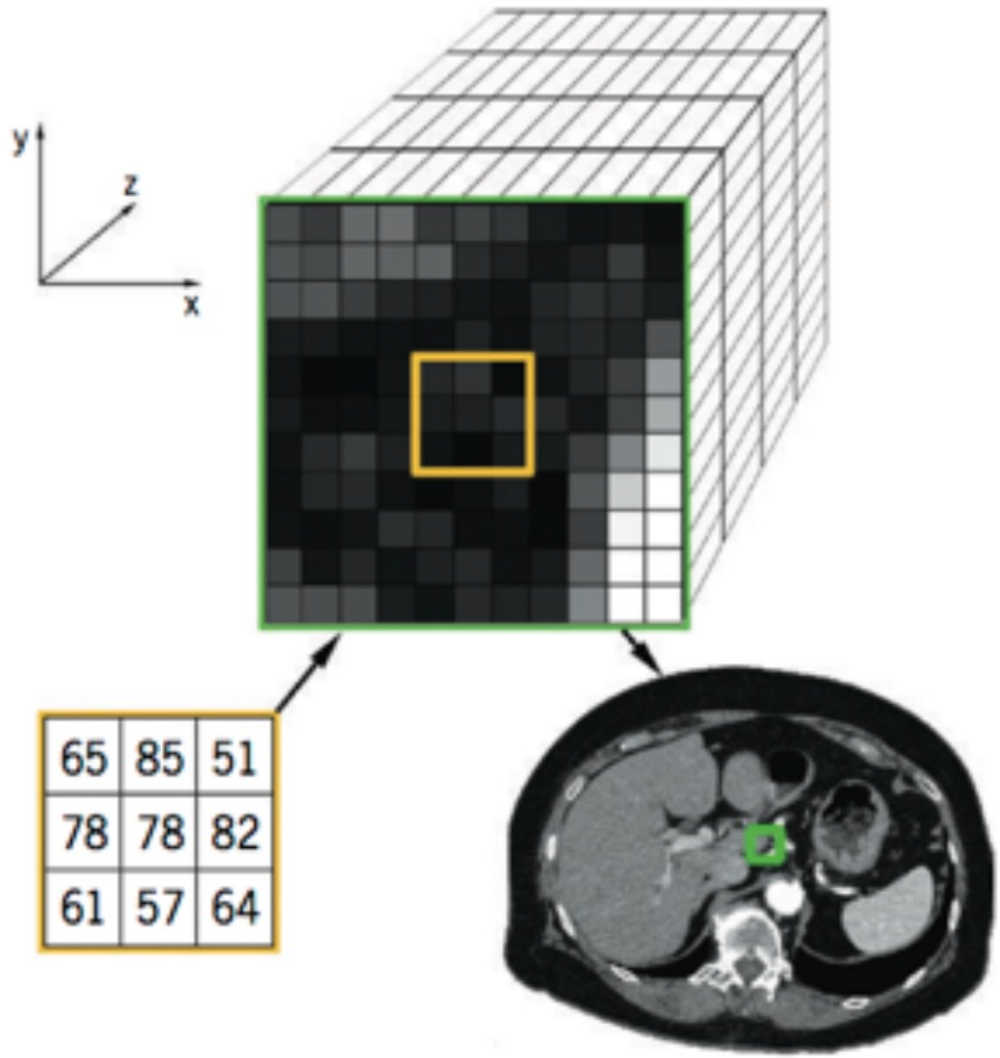
  Purpose: to reduce data for speeding up CAVA operations.
- Region of Interest (ROI):

  To specify a subset of the scene domain.
- Intensity of Interest (IOI):

  To specify a subset of the scene intensities.

- Storage requirement can be reduced by a factor of 2-10

Digital Images

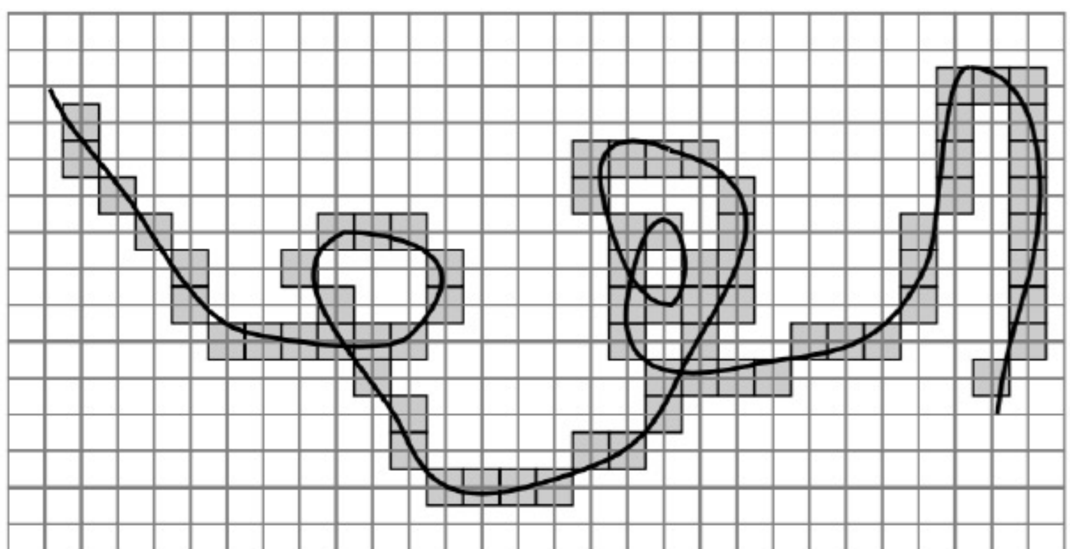


0	3	2	5	4	7	6	9	8
3	0	1	2	3	4	5	6	7
2	1	0	3	2	5	4	7	6
5	2	3	0	1	2	3	4	5
4	3	2	1	0	3	2	5	4
7	4	5	2	3	0	1	2	3
6	5	4	3	2	1	0	3	2
9	6	7	4	5	2	3	0	1
8	7	6	5	4	3	2	1	0

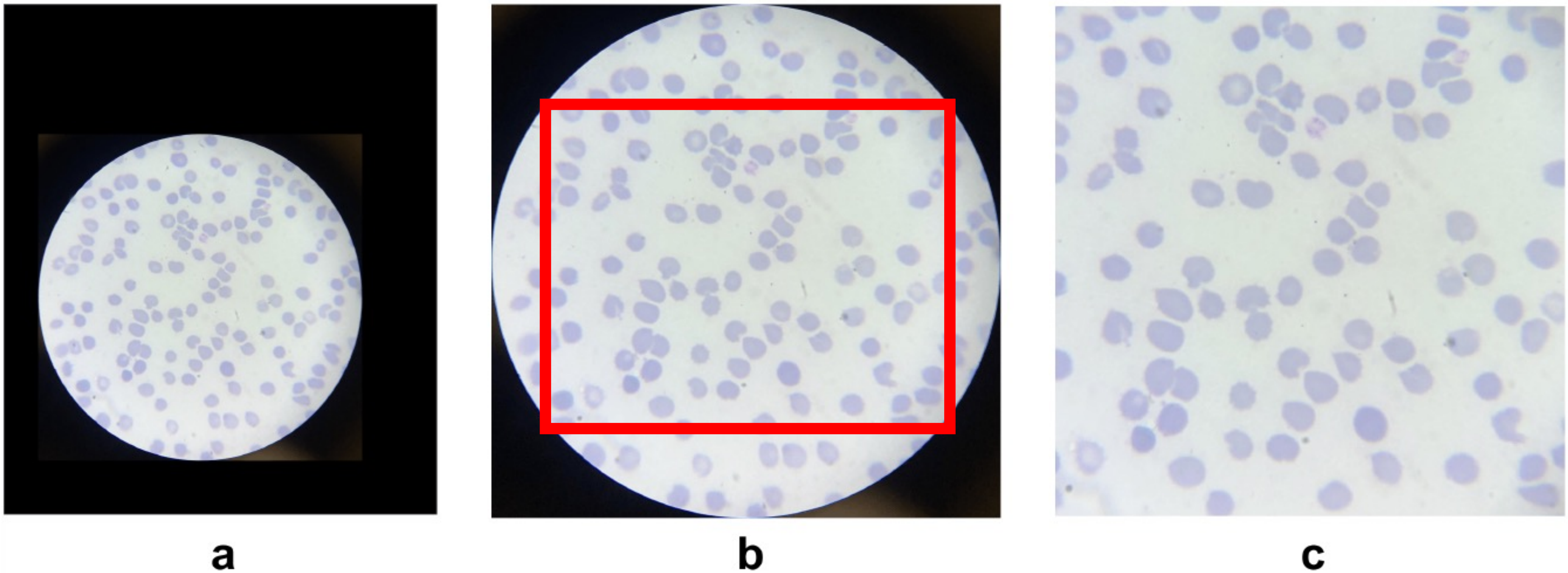
What computer sees!

Images → Sampling+ Quantization → Digital Images

- Computers use discrete form of the images
- The process of transforming continuous space into discrete space is called digitization

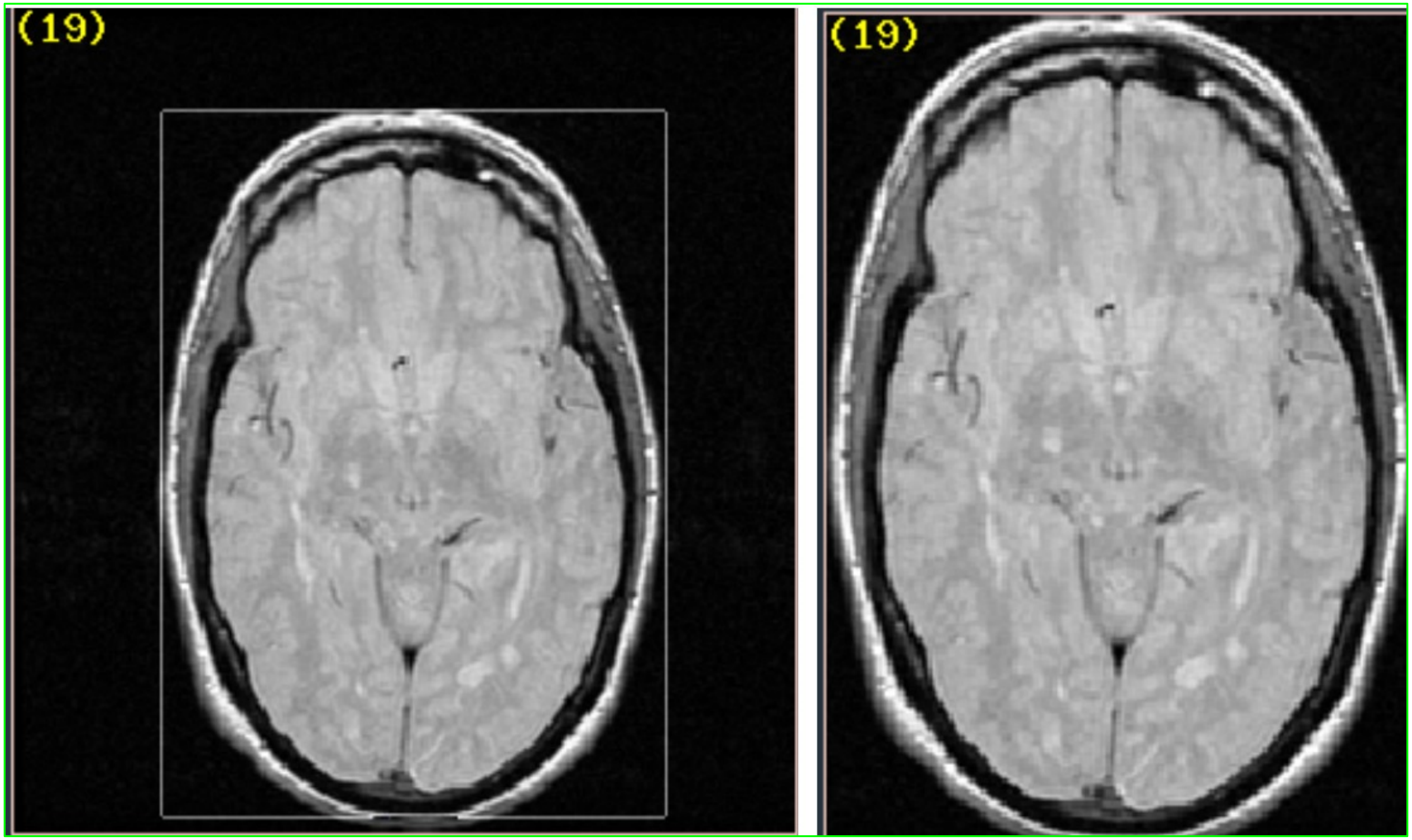


VOI / ROI (volume/region of interest)

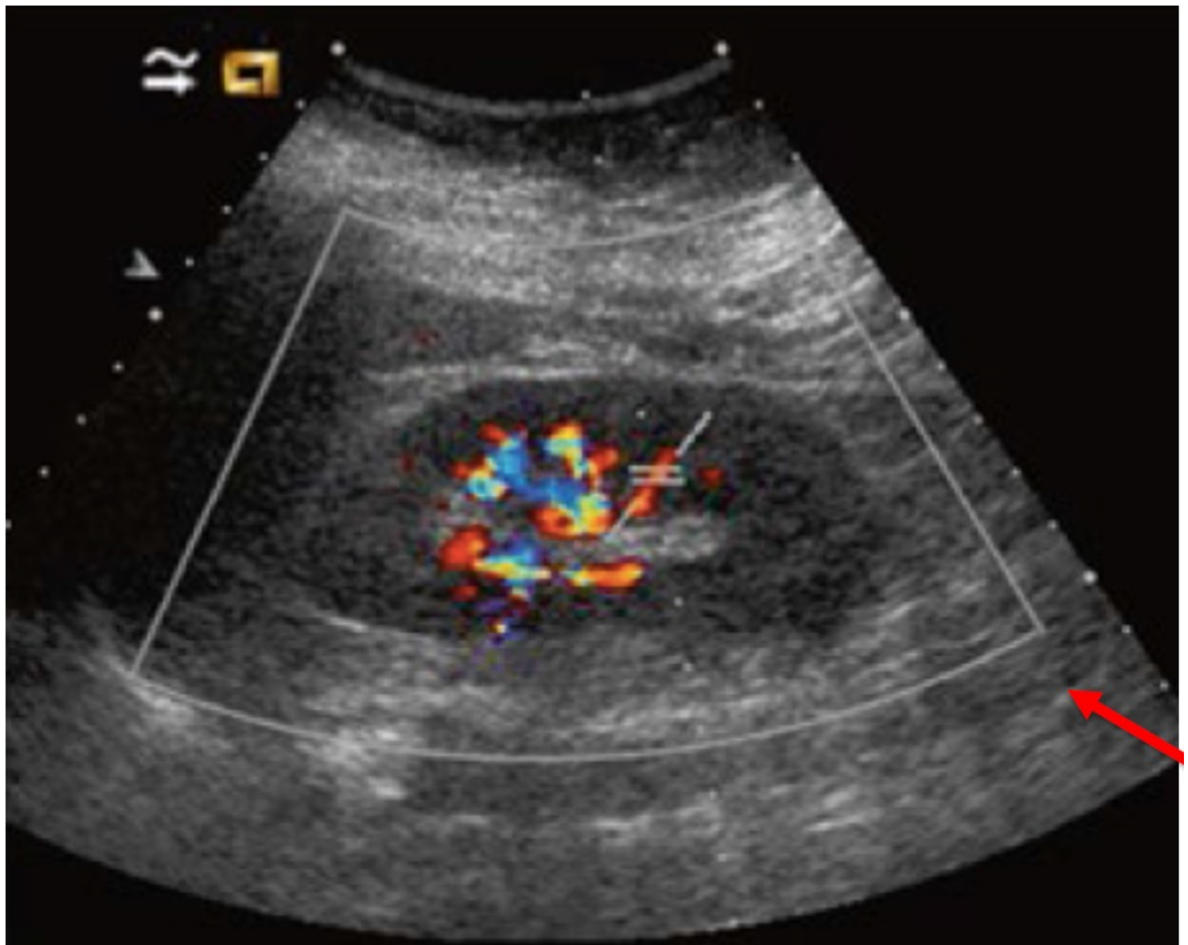


Process of removing unwanted darker regions images captured from microscope through mobile phone: a) shows the unwanted darker region around the original image captured with a mobile phone mounted over an optical microscope, b) shows the image after the first pre-processing step in which image is cropped by finding the largest contour on the original image, c) represents the image after applying automatic thresholding method of pre-processing to completely remove the darker region from the image.

VOI / ROI (volume/region of interest)



VOI / ROI (volume/region of interest)



VOI / ROI (volume/region of interest)

a b c

FIGURE 2.30 (a) Digital dental X-ray image. (b) ROI mask for isolating teeth with fillings (white corresponds to 1 and black corresponds to 0). (c) Product of (a) and (b).

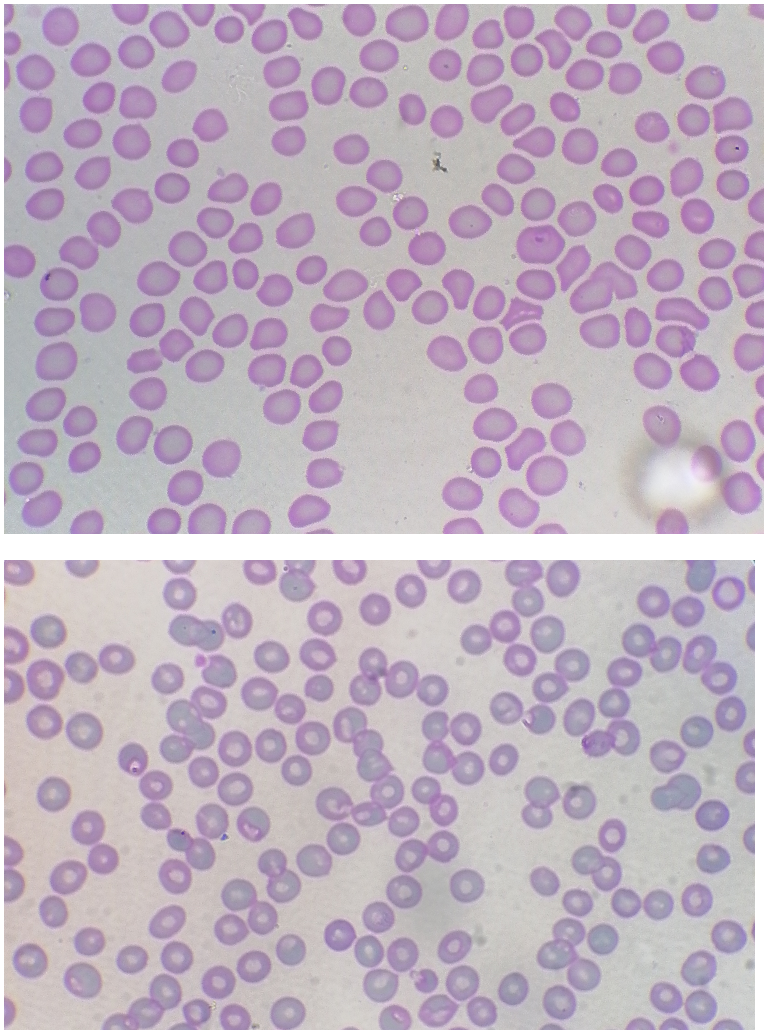
Image Noise

- Light Variations
- Camera Electronics
- Surface Reflectance
- Lens
- Noise is random, it occurs with some probability

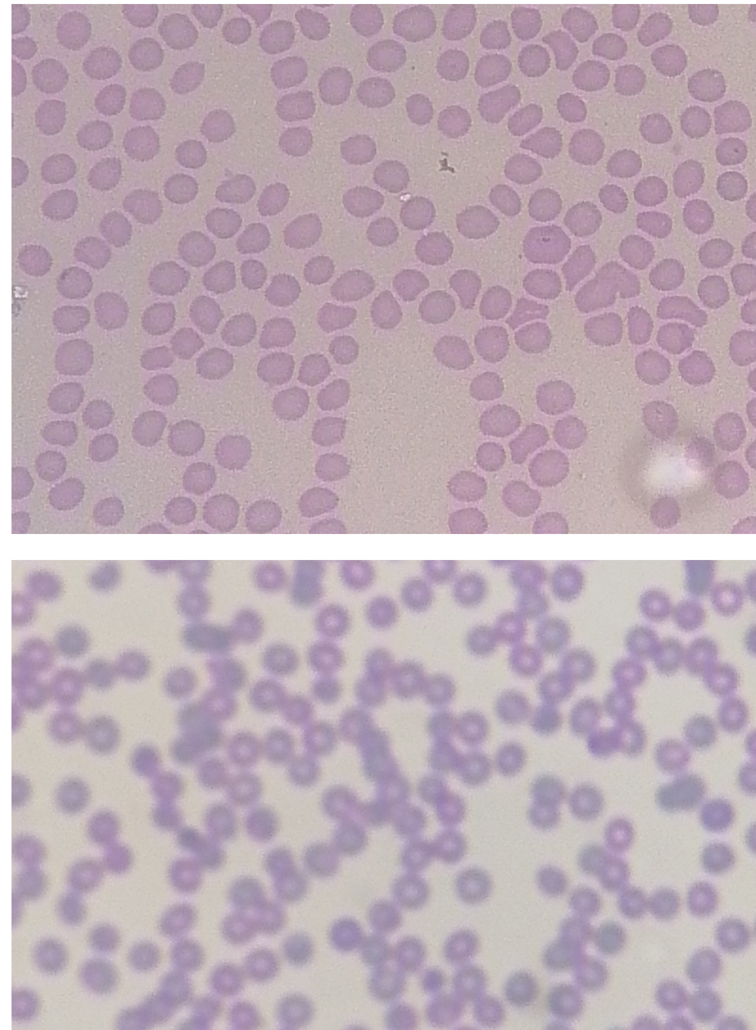
Noise

- $I(x,y)$: the true pixel values
- $n(x,y)$: the noise at pixel (x,y)
- $I'(x, y)=I(x, y) +n(x, y)$ Additive noise
- $I'(x, y)=I(x, y) \times n(x, y)$ Multiplicative noise

Example of Noisy Images

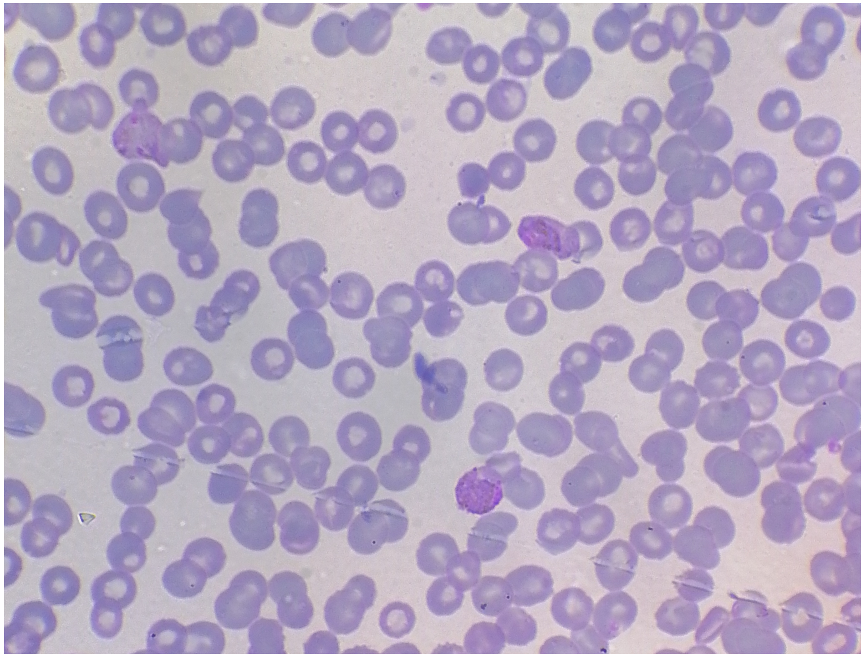


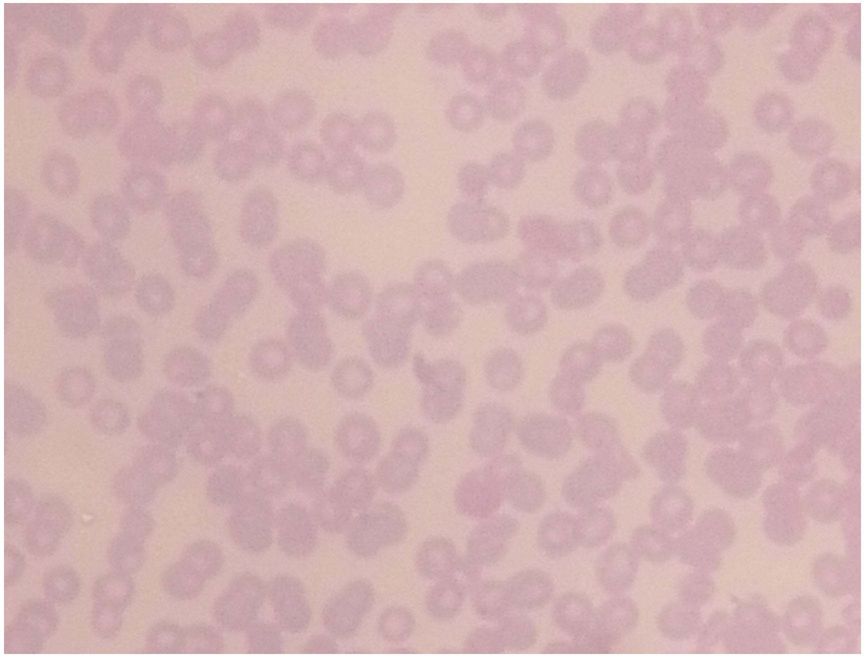
Good Quality Images



Noisy Images

Example of Noisy Images

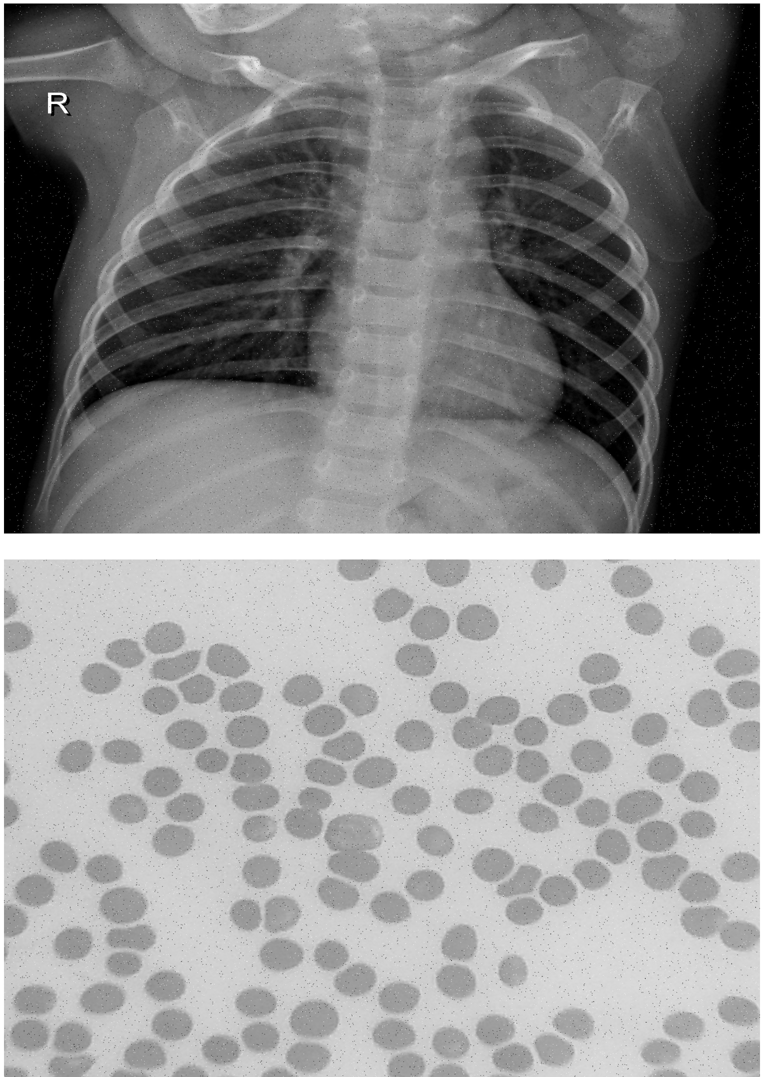


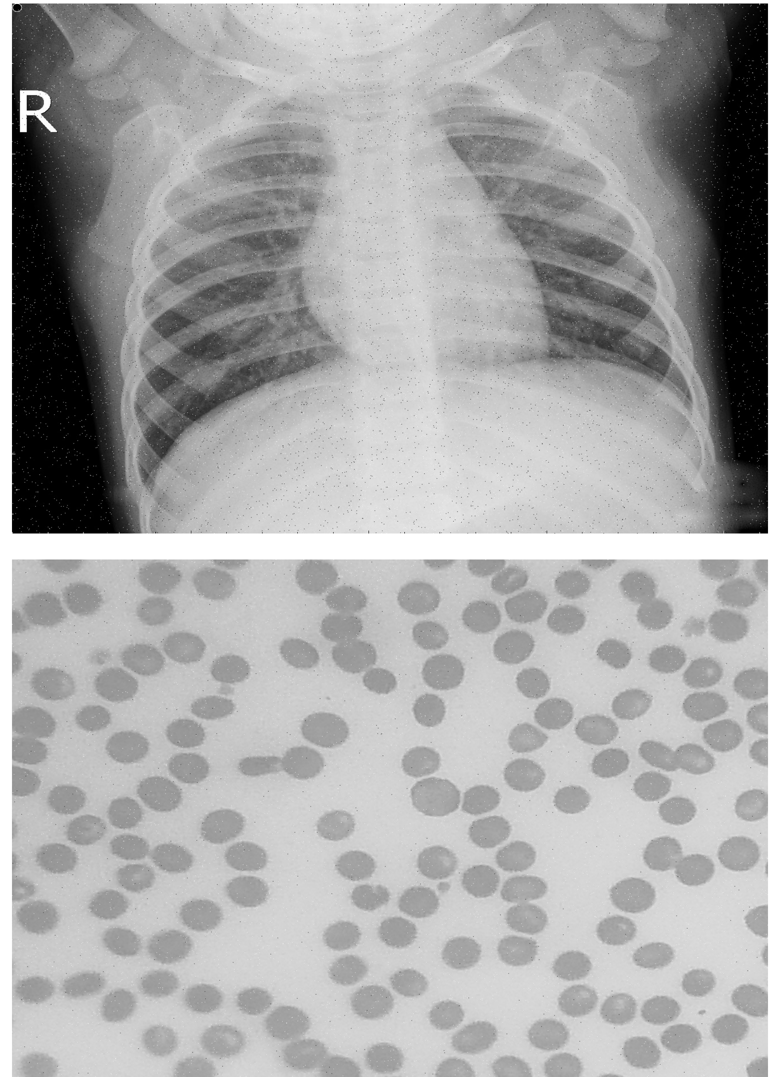


Good Quality Images

Noisy Images

Example of Noisy Images





Good Quality Images

Noisy Images

Image Enhancement

- Intensity Transformation

- Spatial Filtering

Histogram

- Histogram of an image provides the frequency of the brightness (intensity) value in the image.

0	3	2	5	4	7	6	9	8
3	0	1	2	3	4	5	6	7
2	1	0	3	2	5	4	7	6
5	2	3	0	1	2	3	4	5
4	3	2	1	0	3	2	5	4
7	4	5	2	3	0	1	2	3
6	5	4	3	2	1	0	3	2
9	6	7	4	5	2	3	0	1
8	7	6	5	4	3	2	1	0

Histogram

- Histogram of an image provides the frequency of the brightness (intensity) value in the image.

- Provides a natural bridge between images and a probabilistic description.

Ex. Probability of pixel (x,y) that has a brightness (intensity) value z

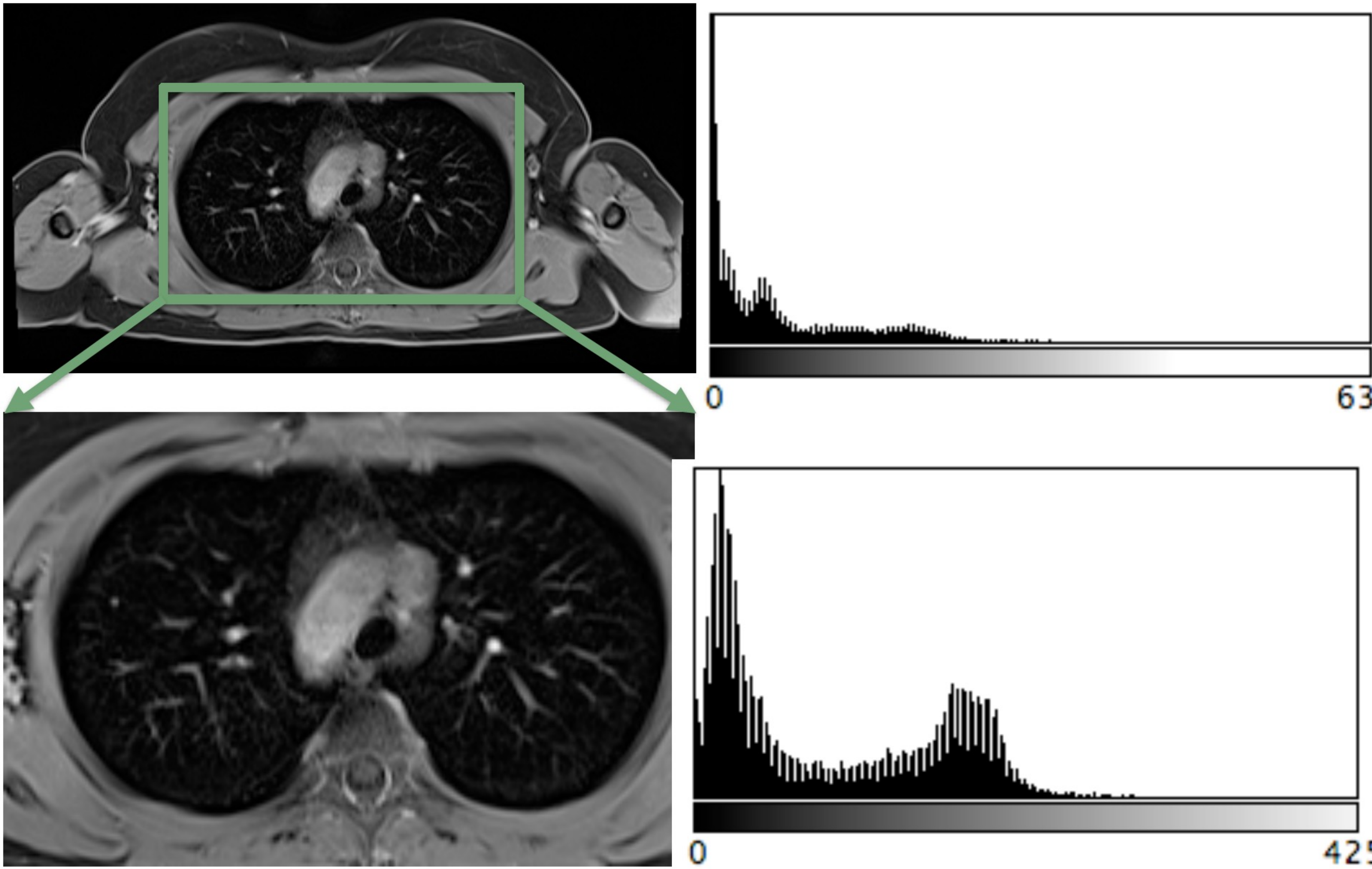
C Code:

```
for (i=0;i<m,i++)
   for (j=0;j<n,j++)
      hist[I[i,j]]++;
```

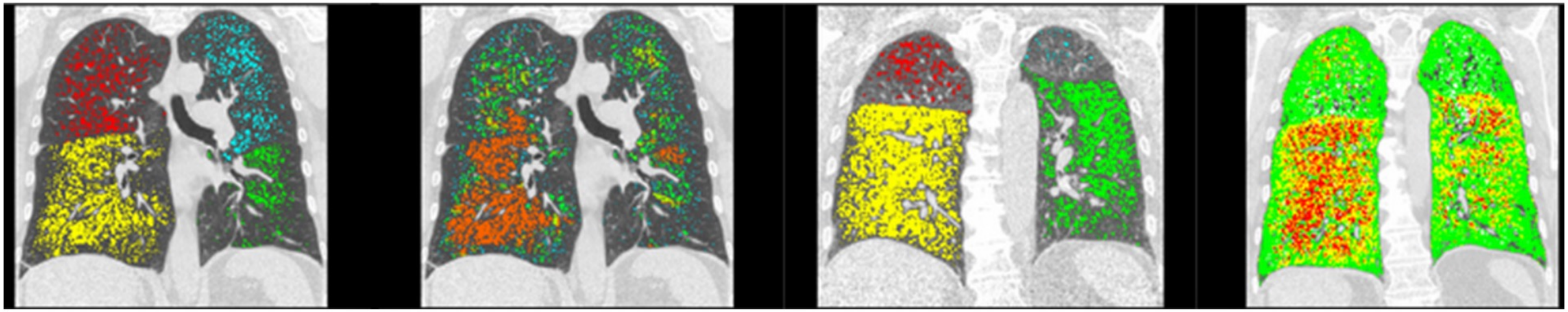
Pseudo-Code for Histogram

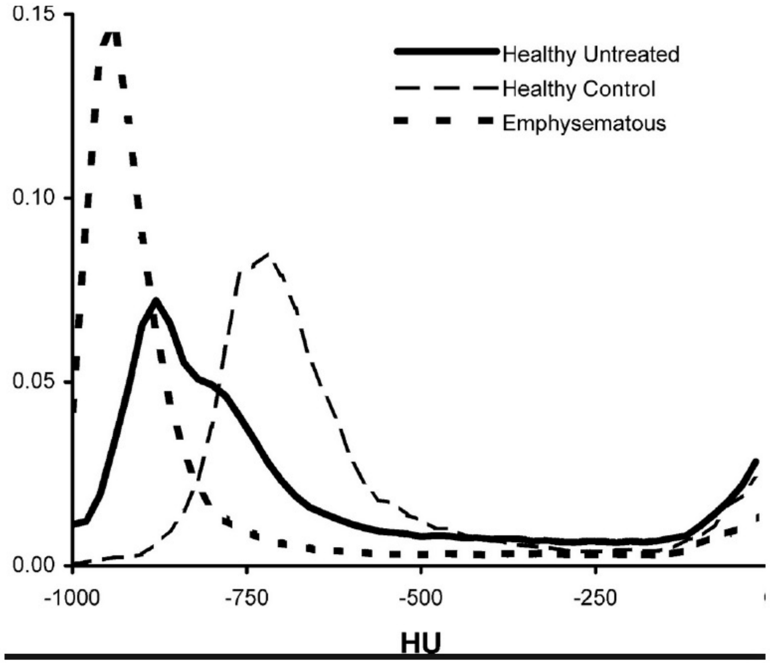
1.Create an array h with zero in its elements

2. For all pixel locations (x,y) of the image A, increment h(A(x,y)) by 1


0
639
0
425

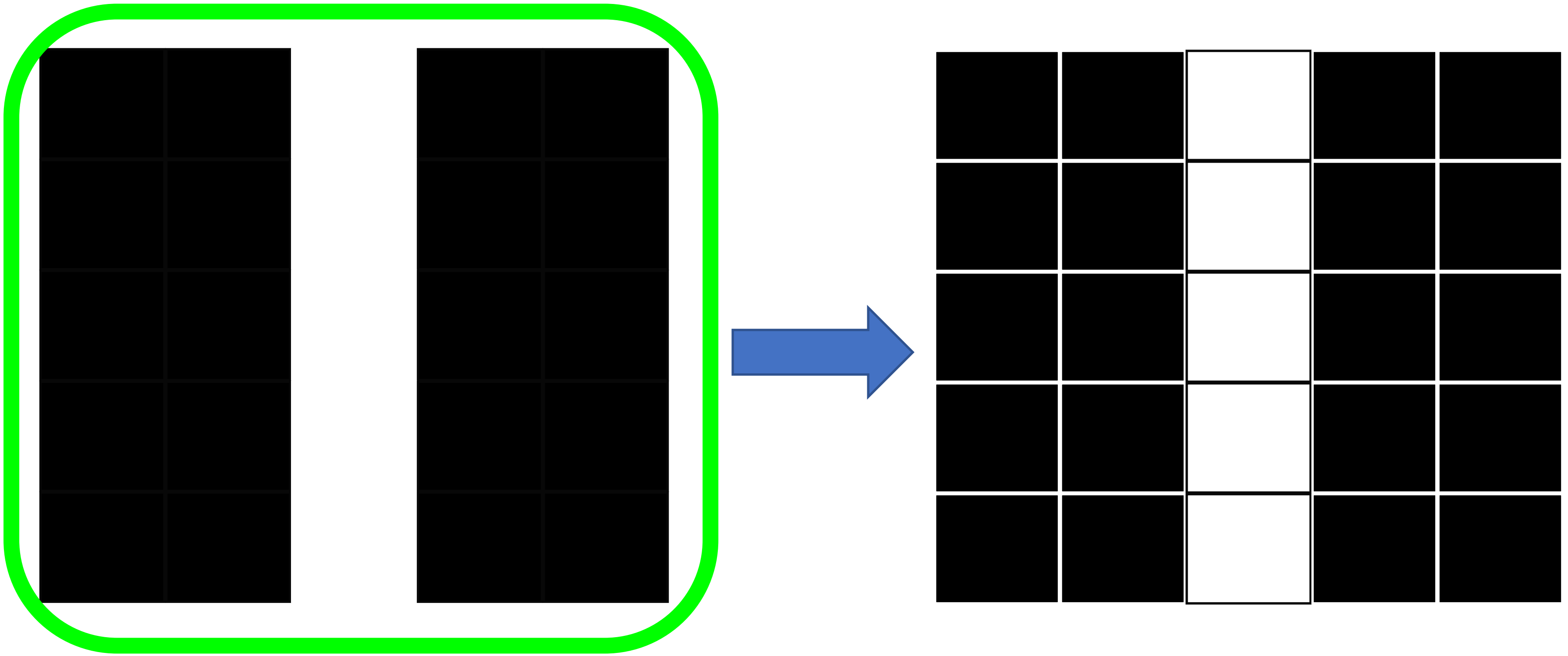
Ex: Histogram based analysis of Lung CT (credit: ***imbio***)

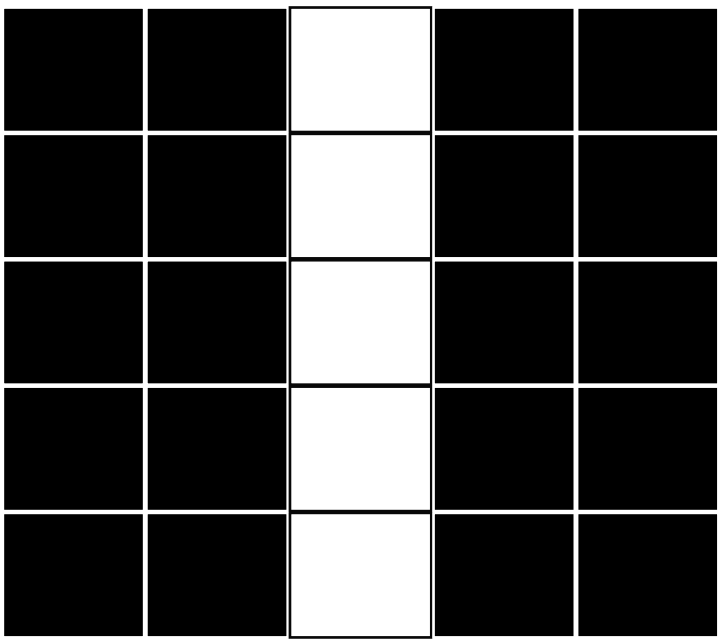




The **Hounsfield scale** named after Sir [Godfrey Hounsfield](), is a quantitative scale for describing [radiodensity]()

- Intensity Transformation
  - Negative Image





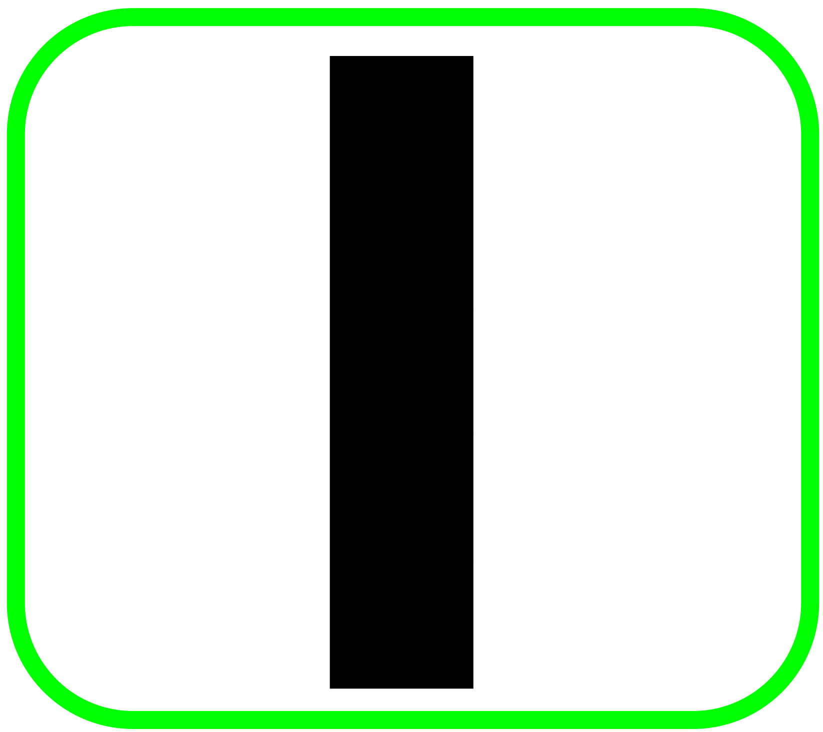
0	0	255	0	0
0	0	255	0	0
0	0	255	0	0
0	0	255	0	0
0	0	255	0	0

255-

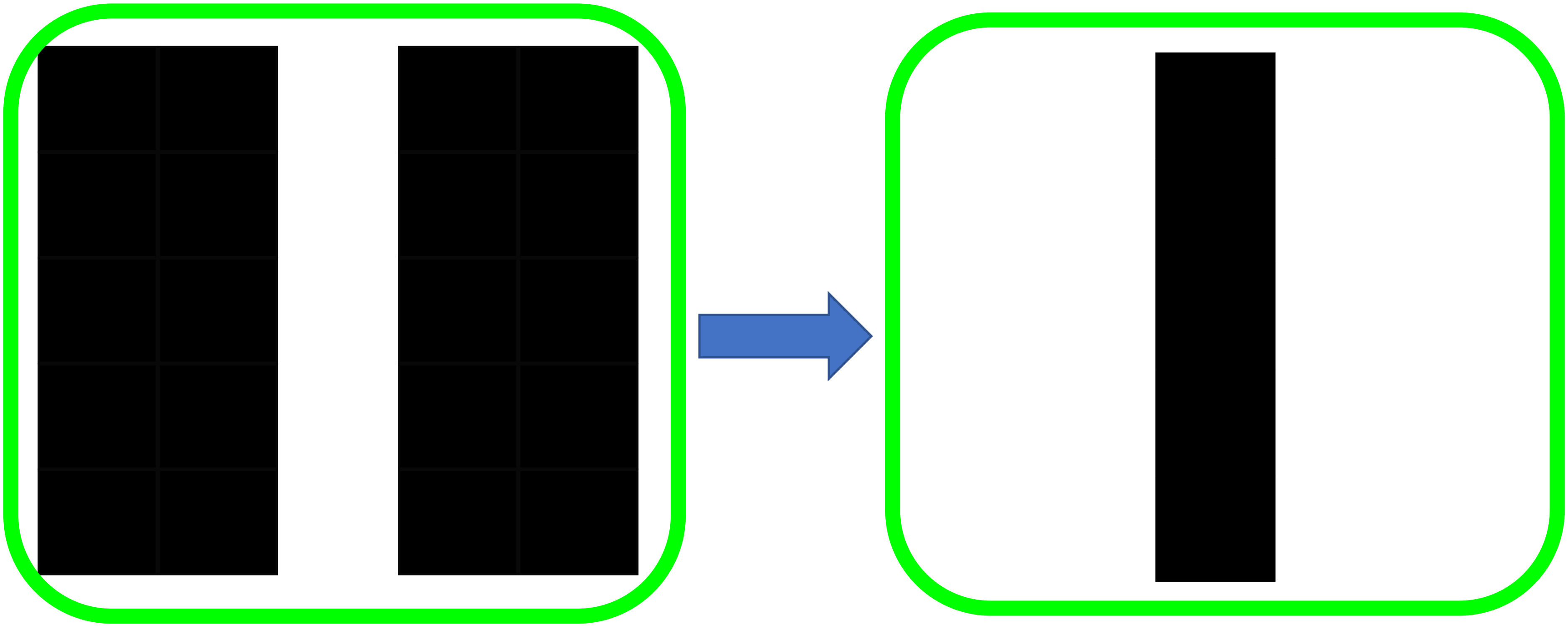
0	0	255	0	0
0	0	255	0	0
0	0	255	0	0
0	0	255	0	0
0	0	255	0	0

255	255	0	255	255
255	255	0	255	255
255	255	0	255	255
255	255	0	255	255
255	255	0	255	255

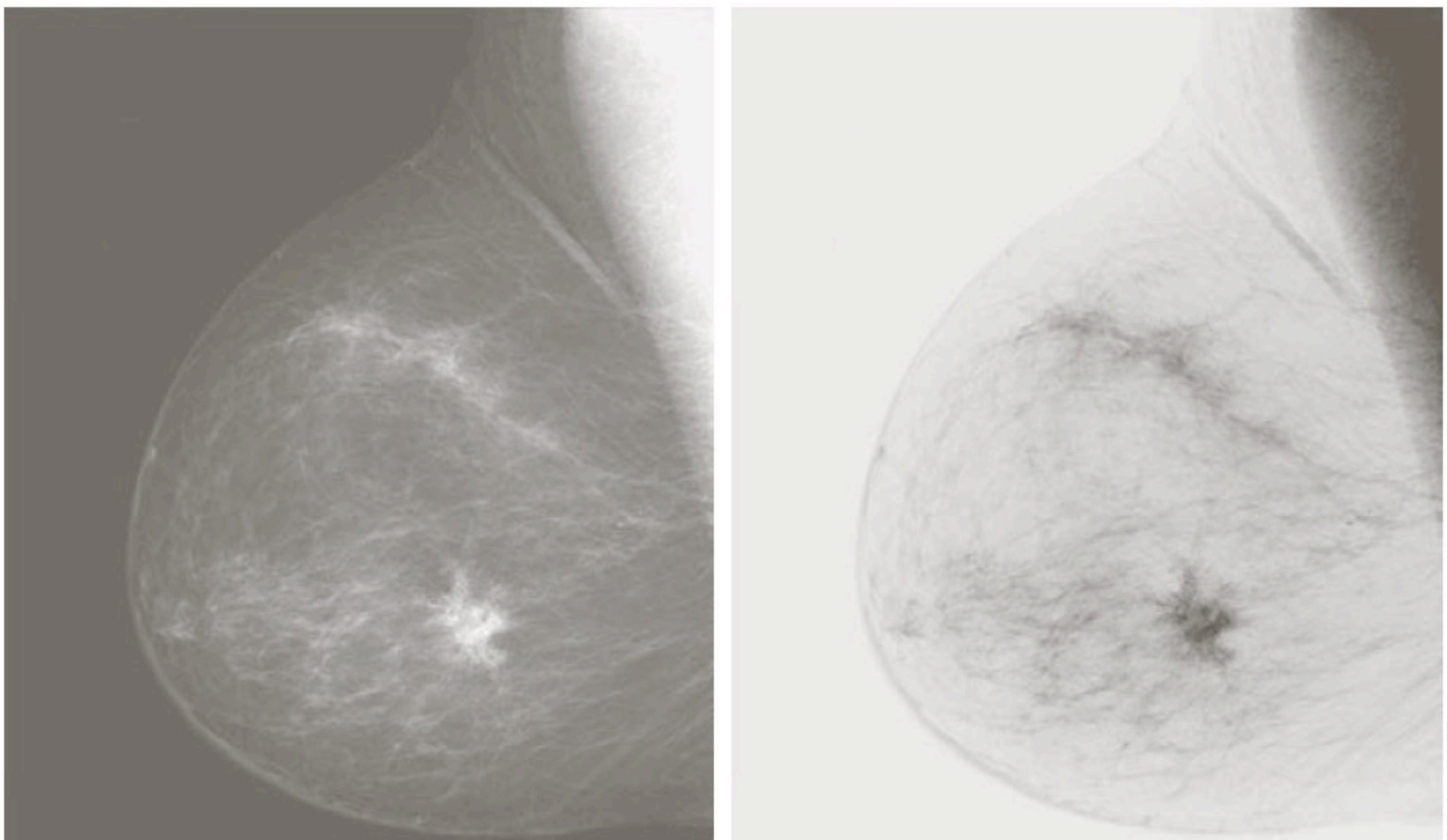
255	255	0	255	255
255	255	0	255	255
255	255	0	255	255
255	255	0	255	255
255	255	0	255	255



- Intensity Transformation
  - Negative Image



- Intensity Transformation



a b

FIGURE 3.4 (a) Original digital mammogram. (b) Negative image obtained using the negative transformation in Eq. (3.2-1). (Courtesy of G.E. Medical Systems.)

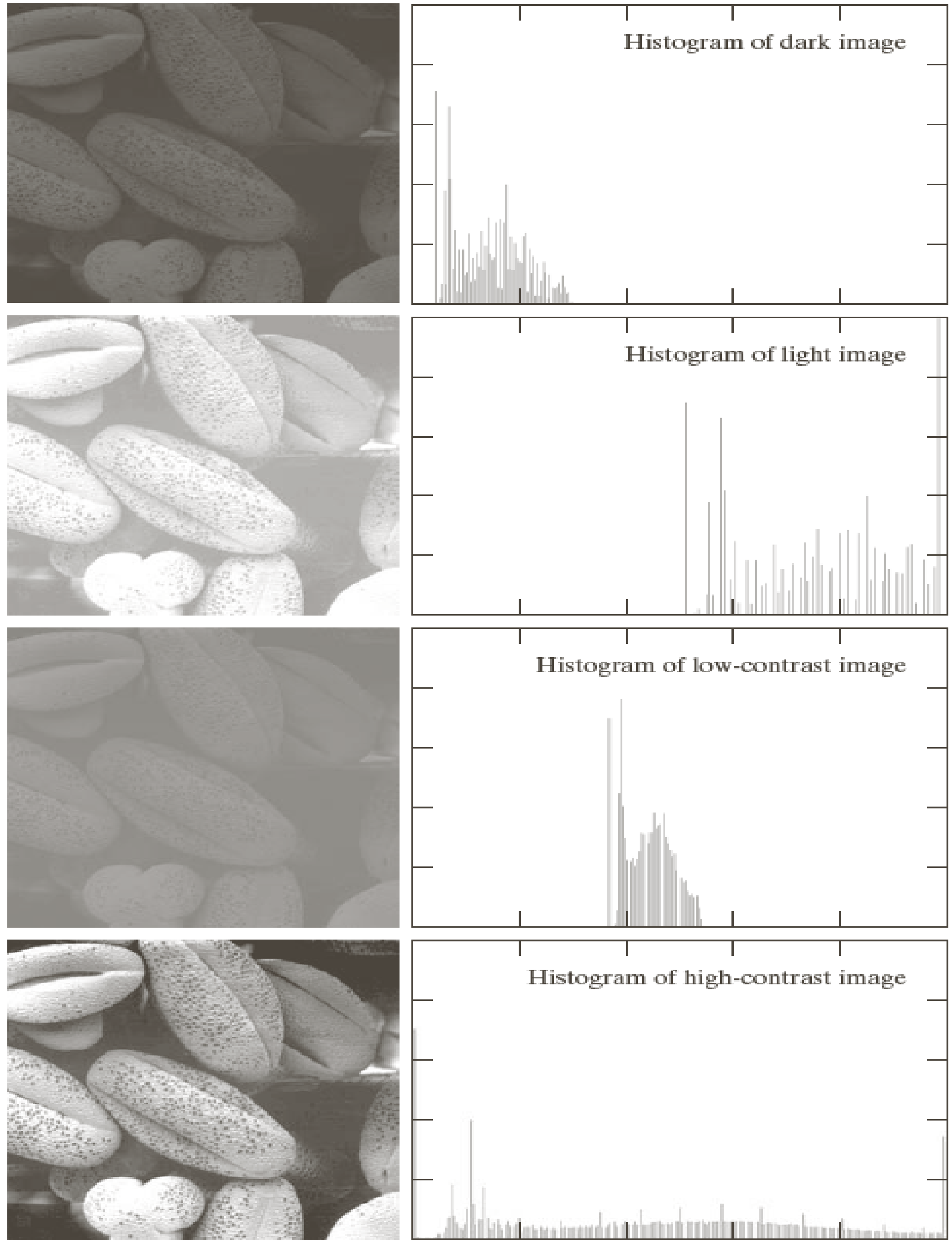


FIGURE 3.16 Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms.

Histogram Equalization

Example:

$$f(x,y) = \begin{bmatrix} 4 & 4 & 4 & 4 & 4 \\ 3 & 4 & 5 & 4 & 3 \\ 3 & 5 & 5 & 5 & 3 \\ 3 & 4 & 5 & 4 & 3 \\ 4 & 4 & 4 & 4 & 4 \end{bmatrix} \xrightarrow{H.E.} \begin{bmatrix} 6 & 6 & 6 & 6 & 6 \\ 2 & 6 & 7 & 6 & 2 \\ 2 & 7 & 7 & 7 & 2 \\ 2 & 6 & 7 & 6 & 2 \\ 6 & 6 & 6 & 6 & 6 \end{bmatrix}$$

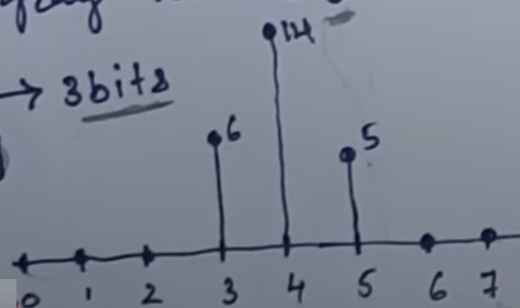
i/p Image

Gray levels	0	1	2	3	4	5	6	7
no of pixels n_K	0	0	0	6	14	5	0	0

highest gray value = 5

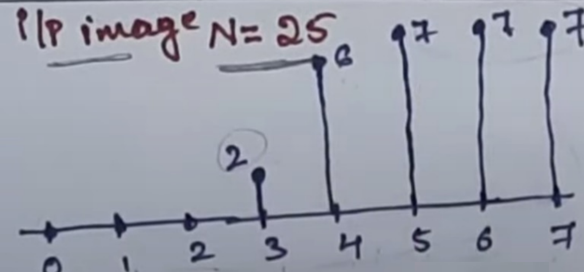
$2^3 = 8 \rightarrow$ 3 bits

[0 to 7]

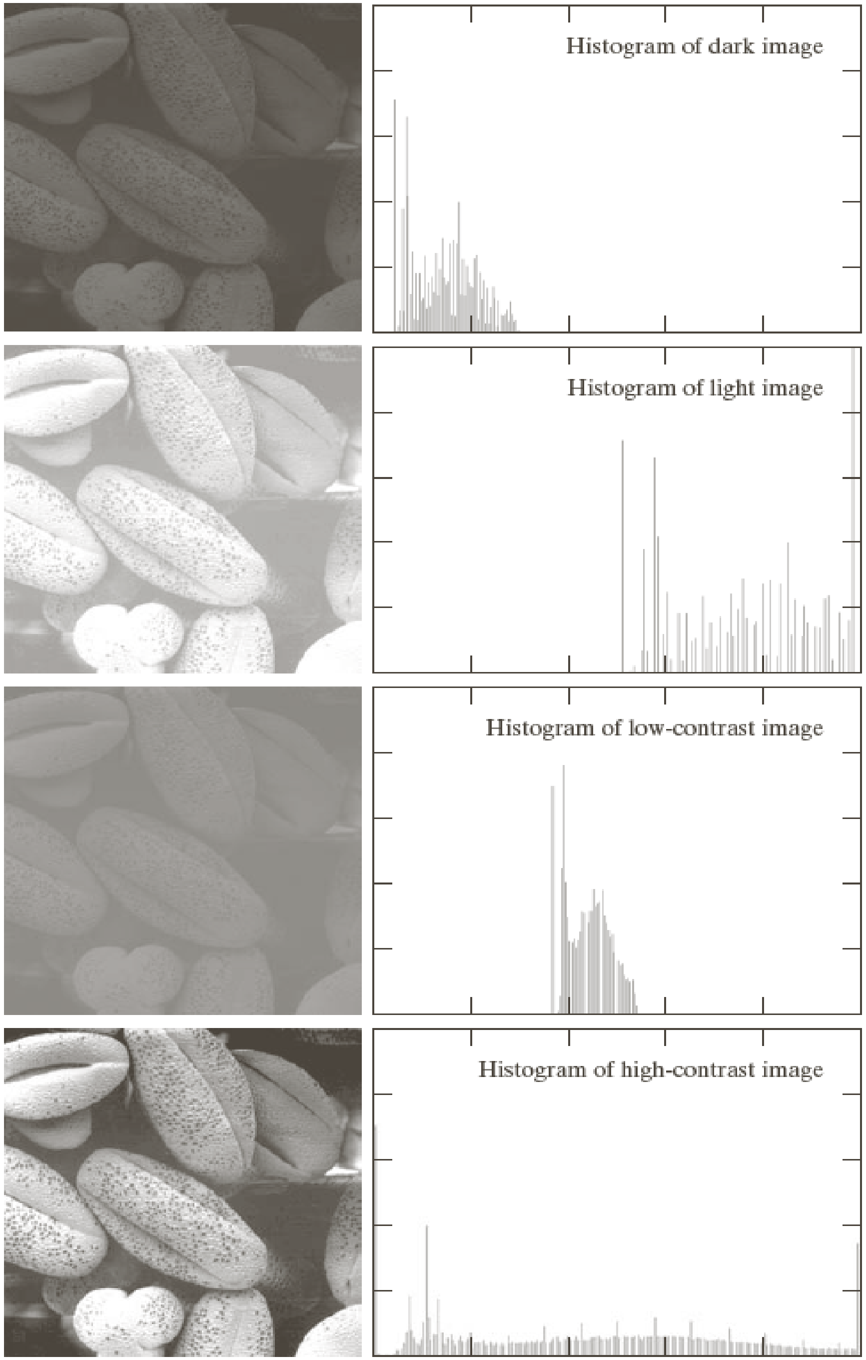


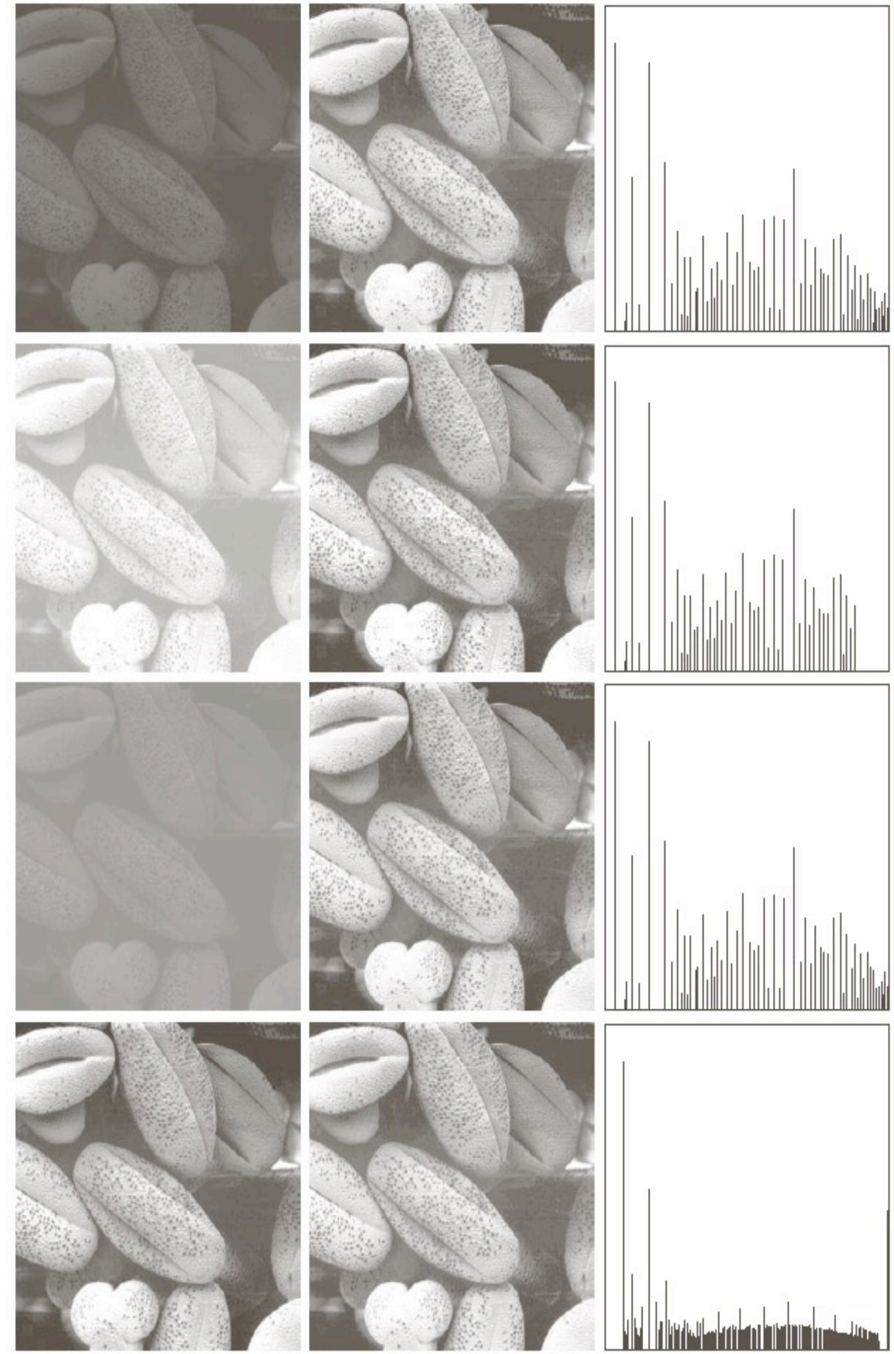
Gray level	no. of pixels n_K	PDF = n_K/sum	CDF = S_K	$S_K \times 7$	Histogram equal. level
0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	0	0	0
3	6	6/25 = 0.24	0.24	1.68	2
4	14	14/25 = 0.56	0.8	5.6	6
5	5	5/25 = 0.2	1.0	7	7
6	0	0	1.0	7	7
7	0	0	1.0	7	7

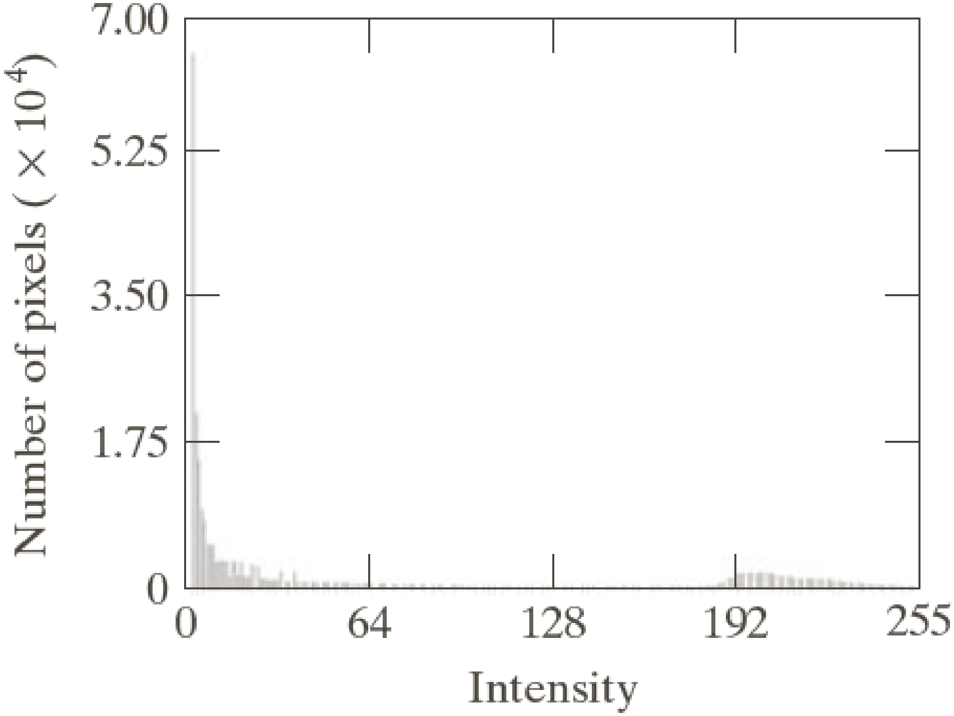
i/p image N = 25 — O/P



https://www.youtube.com/watch?v=uqeOrtAzSyU


Histogram of dark image
Histogram of light image
Histogram of low-contrast image
Histogram of high-contrast image



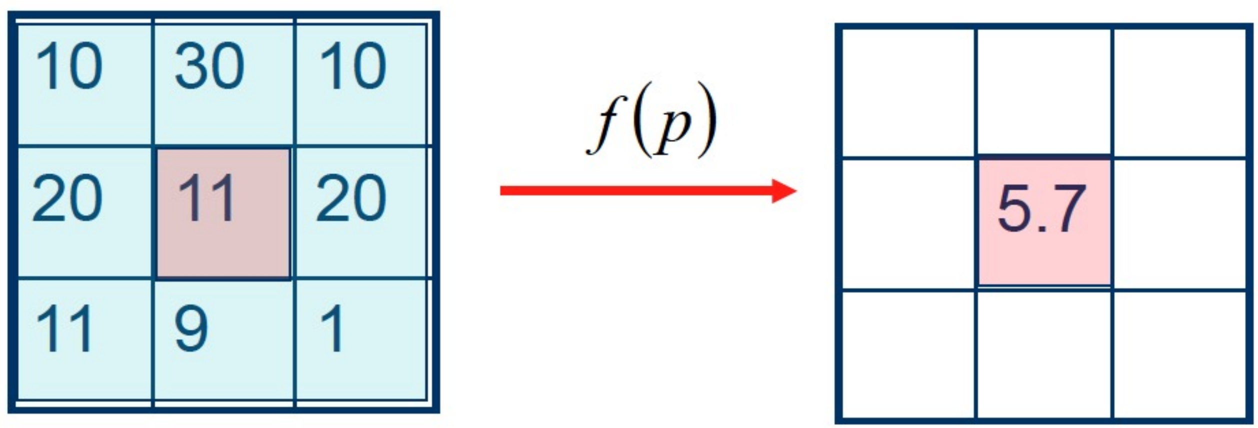


7.00
5.25
3.50
1.75
0
Number of pixels (× 10⁴)
0
64
128
192
255
Intensity

Image Filtering

Filtering

- Modify pixels based on some function of the neighborhood



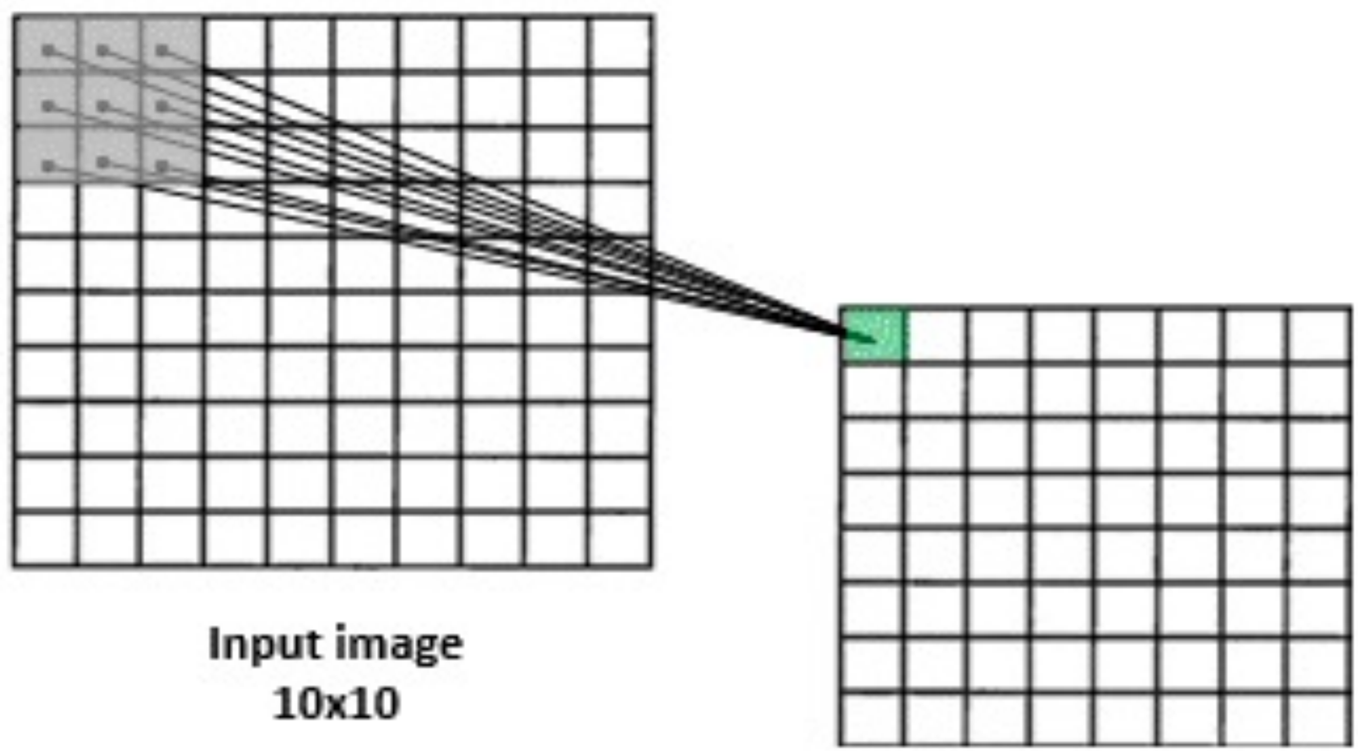
Linear Filtering

- The output is the linear combination of the neighborhood pixels

$$\begin{array}{|c|c|c|}\hline 1 & 3 & 0 \\ \hline 2 & 10 & 2 \\ \hline 4 & 1 & 1 \\ \hline \end{array} \otimes \begin{array}{|c|c|c|}\hline 1 & 0 & -1 \\ \hline 1 & 0.1 & -1 \\ \hline 1 & 0 & -1 \\ \hline \end{array} = \begin{array}{|c|c|c|}\hline & & \\ \hline & 5 & \\ \hline & & \\ \hline \end{array}$$

Image — Kernel — Filter Output

Alper Yilmaz, Mubarak Shah, UCF


Input image
10x10

Medical Image Filtering

Purpose:

- To suppress unwanted (non-object) info.
- To enhance wanted (object) information.

Enhancive:

- For enhancing edges, regions.
- For intensity scale standardization.
- For correcting background variation.

Suppressive:

- Mainly for suppressing random noise.

Medical Image Enhancement

- Medical images are often deteriorated by noise due to various sources of interference
  - ➢ affect measurement process in imaging and data acquisition systems
- Improvement in appearance and visual quality of the images may assist in interpretation of medical images
  - ➢ may affect diagnostic decision too!
- Some enhancement algorithms are developed for deriving images that are meant for use by a subsequent algorithm for computer processing!
  - ➢ Edge detection, object segmentation, etc.

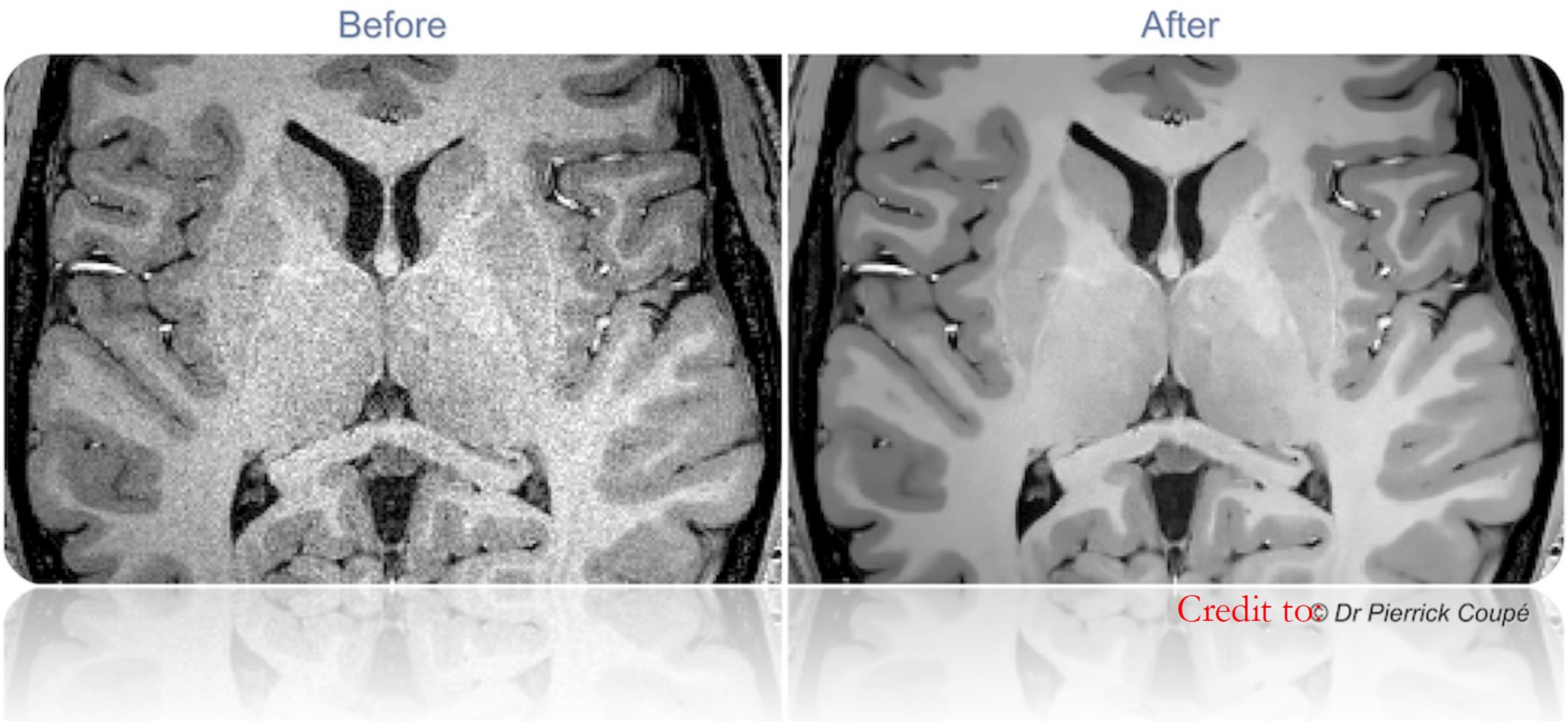
Inappropriate use of Enhancement Methods

- Enhancement methods themselves may increase noise while improving contrast!

- They may eliminate small details and edge sharpness while removing noise

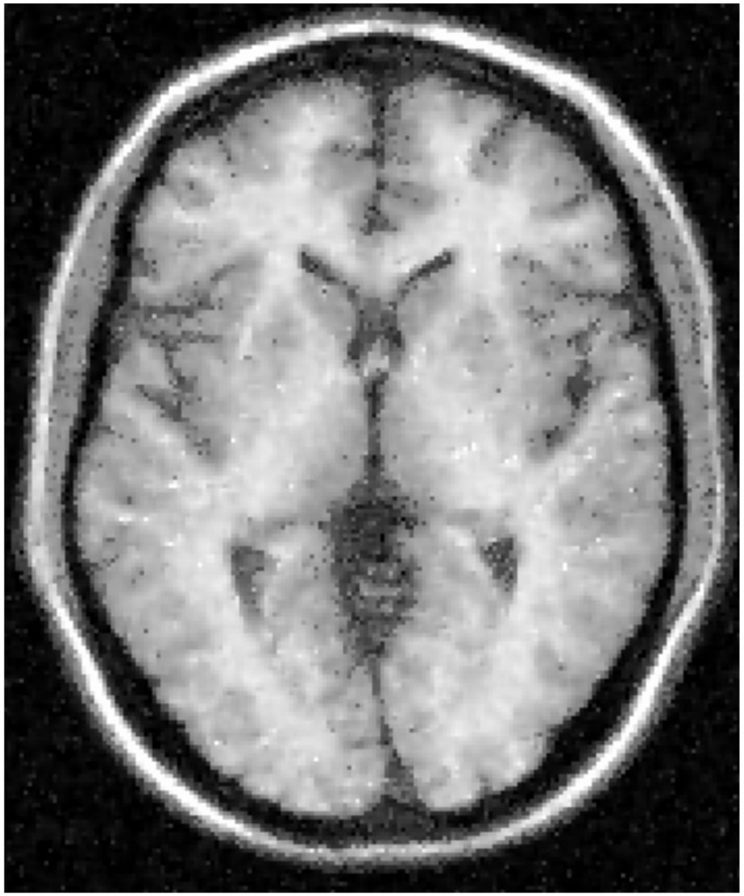
- They may produce artifacts in general.

Smoothing MRI

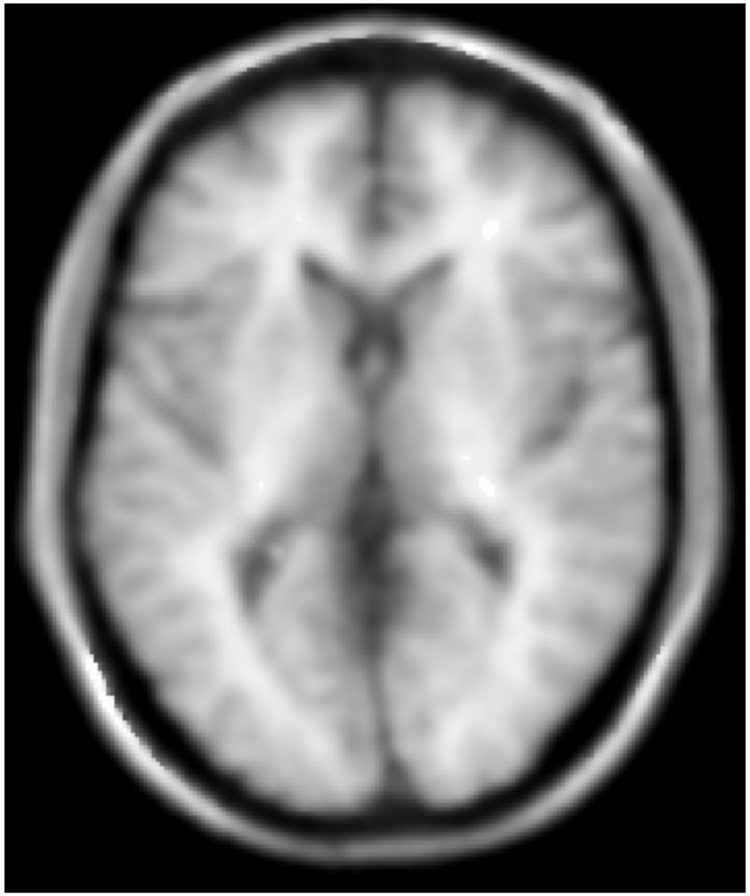


P. Coupé et al. An Optimized Blockwise NonLocal Means Denoising Filter for 3-D Magnetic Resonance Images. IEEE TMI 2008

Noise Supression



Higher noise, higher contrast



Lower noise, lower contrast

For best results, we need lower noise and higher contrast.

Filtering Operation (Spatial Domain)

- Example: Box Filtering (smoothing)

What does it do?

- Replaces each pixel with average of its neighbourhood
- Achieve smoothing effect

  (remove sharp features)

$$g[\cdot\,,\cdot\,]$$

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

Example: box filter

- Image filtering: compute function of local neighborhood at each position

$$g[\cdot\,,\cdot\,]$$

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Slide credit: David Lowe (UBC)

Filtering Operation (Spatial Domain)

$$g[\cdot,\cdot]\ \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

$$f[.,.]$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$h[.,.]$$

$$h[m,n]=\sum_{k,l} g[k,l]\, f[m+k,n+l]$$

Credit: S. Seitz

Filtering Operation (Spatial Domain)

$$g[\cdot\,,\cdot\,]\ \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10							

$$h[m,n]=\sum_{k,l} g[k,l]\, f[m+k,n+l]$$

Credit: S. Seitz

Filtering Operation (Spatial Domain)

$$g[\cdot,\cdot]\ \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

$$f[.,.]$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$h[.,.]$$

	0	10	20						

$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k, n+l]$$

Credit: S. Seitz

Filtering Operation (Spatial Domain)

$g[\cdot,\cdot]\ \frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20	30					

$$h[m,n]=\sum_{k,l} g[k,l]\, f[m+k,n+l]$$

Credit: S. Seitz

Filtering Operation (Spatial Domain)

$$g[\cdot,\cdot]\ \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

$$f[.,.]$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$h[.,.]$$

	0	10	20	30	30				

$$h[m,n]=\sum_{k,l} g[k,l]\, f[m+k,n+l]$$

Credit: S. Seitz

Filtering Operation (Spatial Domain)

$$g[\cdot\,,\cdot\,]\ \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20	30	30				
				?					

$$h[m,n]=\sum_{k,l} g[k,l]\, f[m+k,n+l]$$

Credit: S. Seitz

Filtering Operation (Spatial Domain)

$$g[\cdot,\cdot]\ \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20	30	30				
						?			
			50						

$$h[m,n]=\sum_{k,l} g[k,l]\, f[m+k,n+l]$$

Credit: S. Seitz

$g[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

Filtering Operation (Spatial Domain)

$f[.,.]$

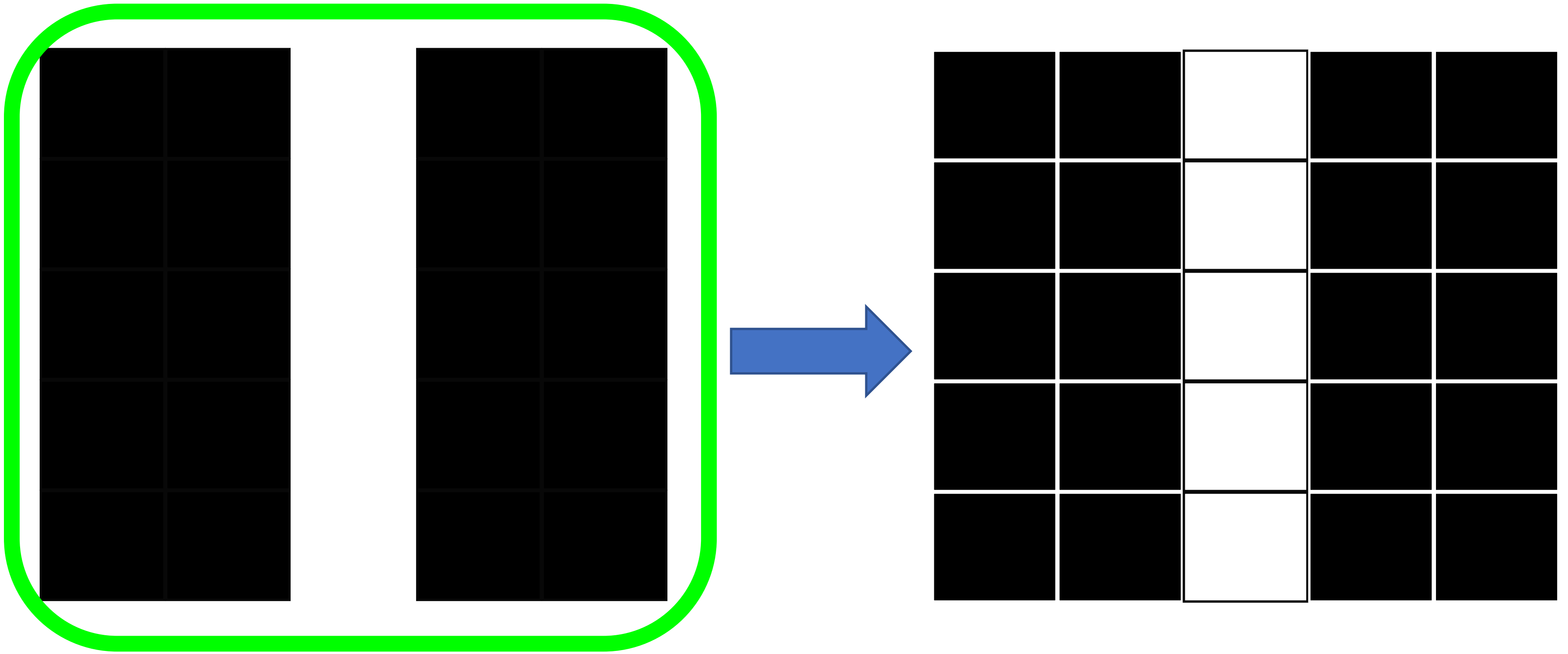
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

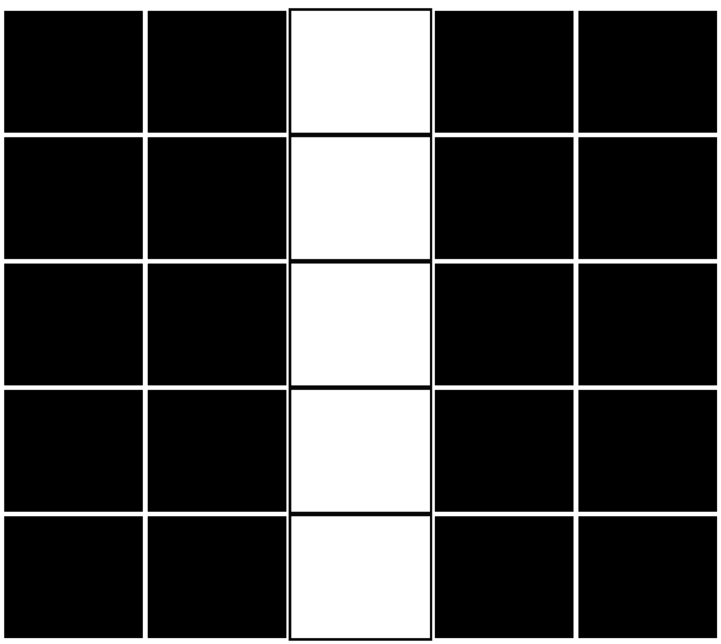
$h[.,.]$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k, n+l]$$

Credit: S. Seitz





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0	0	255	0	0
0	0	255	0	0

0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0
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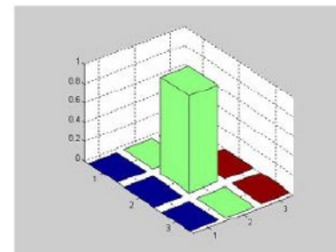
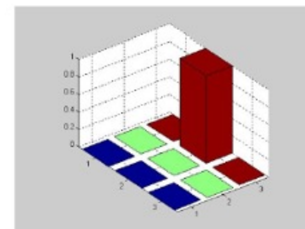
0	0	0	255	0	0	0



*

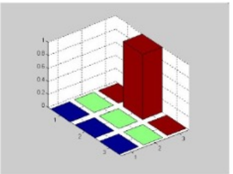
0	0	0
0	1	0
0	0	0

0	**0**	**0**	**255**	**0**	**0**	**0**
0	0	0	255	0	0	**0**
0	0	0	255	0	0	**0**
0	0	0	255	0	0	**0**
0	0	0	255	0	0	**0**
0	0	0	255	0	0	**0**
0	**0**	**0**	**255**	**0**	**0**	**0**



*

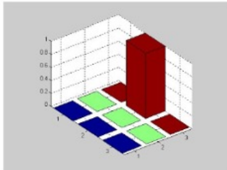
0	0	0
0	0	1
0	0	0



*

0	0	0
0	0	1
0	0	0

0	**0**	**0**	**255**	**0**	**0**	**0**
0	0	0	255	0	0	**0**
0	0	0	255	0	0	**0**
0	0	0	255	0	0	**0**
0	0	0	255	0	0	**0**
0	0	0	255	0	0	**0**
0	**0**	**0**	**255**	**0**	**0**	**0**

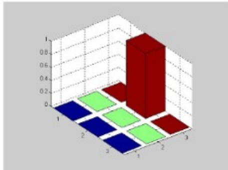


*

0	0	0
0	0	1
0	0	0

0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0

0				

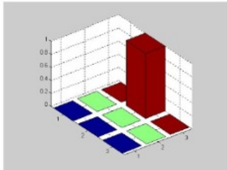


*

0	0	0
0	0	1
0	0	0

0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0

0	255			

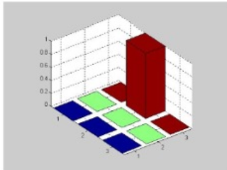


*

0	0	0
0	0	1
0	0	0

0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0

0	255	0		

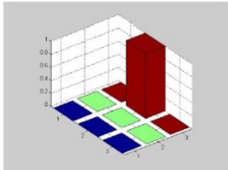


*

0	0	0
0	0	1
0	0	0

0	**0**	**0**	**255**	**0**	**0**	**0**
0	0	0	255	0	0	**0**
0	0	0	255	0	0	**0**
0	0	0	255	0	0	**0**
0	0	0	255	0	0	**0**
0	0	0	255	0	0	**0**
0	**0**	**0**	**255**	**0**	**0**	**0**

0	255	0	0	

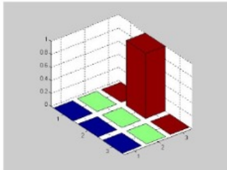


*

0	0	0
0	0	1
0	0	0

0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0

0	255	0	0	0

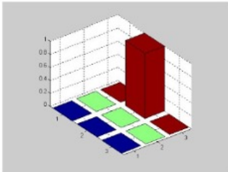


*

0	0	0
0	0	1
0	0	0

0	**0**	**0**	**255**	**0**	**0**	**0**
0	0	0	255	0	0	**0**
0	0	0	255	0	0	**0**
0	0	0	255	0	0	**0**
0	0	0	255	0	0	**0**
0	0	0	255	0	0	**0**
0	**0**	**0**	**255**	**0**	**0**	**0**

0	255	0	0	0
0				

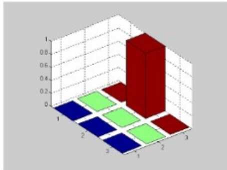


*

0	0	0
0	0	1
0	0	0

0	**0**	**0**	**255**	**0**	**0**	**0**
0	0	0	255	0	0	**0**
0	0	0	255	0	0	**0**
0	0	0	255	0	0	**0**
0	0	0	255	0	0	**0**
0	0	0	255	0	0	**0**
0	**0**	**0**	**255**	**0**	**0**	**0**

0	255	0	0	0
0	255			

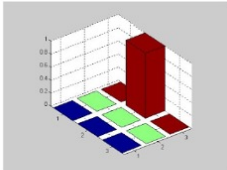


*

0	0	0
0	0	1
0	0	0

0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0

0	255	0	0	0
0	255	0		

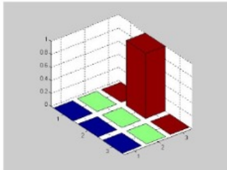


*

0	0	0
0	0	1
0	0	0

0	**0**	**0**	**255**	**0**	**0**	**0**
0	0	0	255	0	0	**0**
0	0	0	255	0	0	**0**
0	0	0	255	0	0	**0**
0	0	0	255	0	0	**0**
0	0	0	255	0	0	**0**
0	**0**	**0**	**255**	**0**	**0**	**0**

0	255	0	0	0
0	255	0	0	0



*

0	0	0
0	0	1
0	0	0

0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0
0	0	0	255	0	0	0

0	255	0	0	0
0	255	0	0	0
0	255	0	0	0
0	255	0	0	0
0	255	0	0	0

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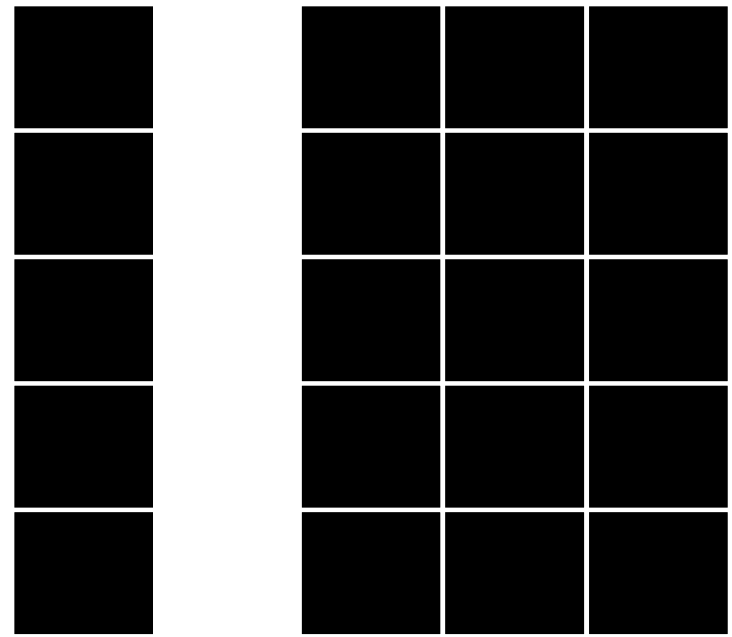
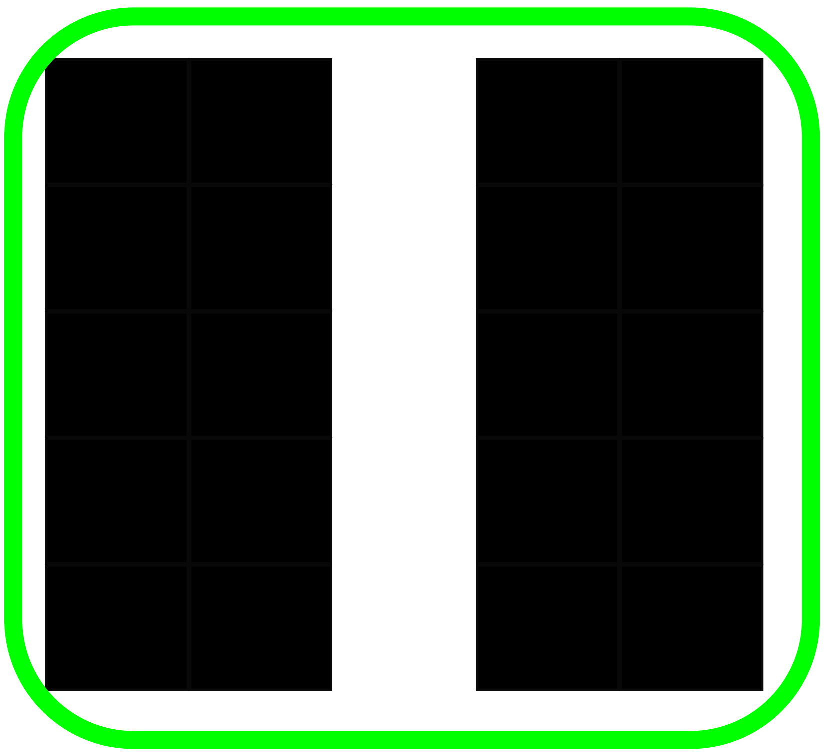
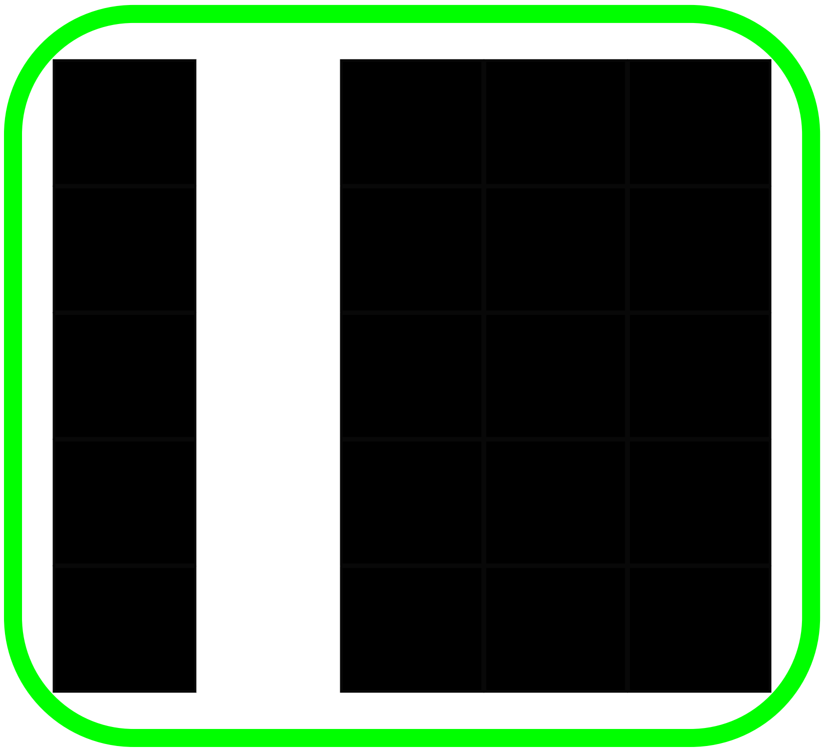
0	255	0	0	0
0	255	0	0	0
0	255	0	0	0
0	255	0	0	0





Original Image



Shifted Image

Summation
(capital sigma)

$$\sum_{n=0}^{4} 3n$$

```
sum = 0;
for( n=0; n<=4; n++ )
    sum += 3*n;
```

Product
(capital pi)

$$\prod_{n=1}^{4} 2n$$

```
prod = 1;
for( n=1; n<=4; n++ )
    prod *= 2*n;
```

Convolution

f = Image

h = Kernel

h

h_1	h_2	h_3
h_4	h_5	h_6
h_7	h_8	h_9

$X-flip$

h_7	h_8	h_9
h_4	h_5	h_6
h_1	h_2	h_3

$Y-flip$

f

f_1	f_2	f_3
f_4	f_5	f_6
f_7	f_8	f_9

*

h_9	h_8	h_7
h_6	h_5	h_4
h_3	h_2	h_1

$$f * h = f_1 h_9 + f_2 h_8 + f_3 h_7 + f_4 h_6 + f_5 h_5 + f_6 h_4 + f_7 h_3 + f_8 h_2 + f_9 h_1$$

Take Home quiz

- Write convolution equation and run it for a couple of iterations for a layman.

Correlation and Convolution

- Convolution is associative

$$F * (G * I) = (F * G) * I$$

Averages

- Mean

$$I = \frac{I_1 + I_2 + \ldots I_n}{n} = \frac{\sum_{i=1}^{n} I_i}{n}$$

- Weighted mean

$$I = \frac{w_1 I_1 + w_2 I_2 + \ldots + w_n I_n}{n} = \frac{\sum_{i=1}^{n} w_i I_i}{n}$$

- Weighted mean

$$I = \frac{w_1 I_1 + w_2 I_2 + \ldots + w_n I_n}{n} = \frac{\sum_{i=1}^{n} w_i I_i}{n}$$

$$g(x) = \begin{bmatrix} .011 & .13 & .6 & 1 & .6 & .13 & .011 \end{bmatrix}$$

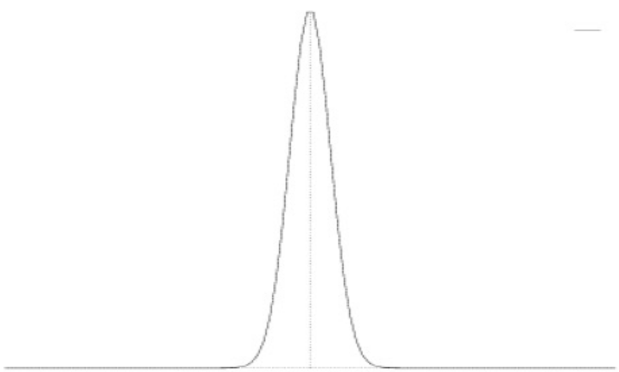
$$\frac{1}{9}$$

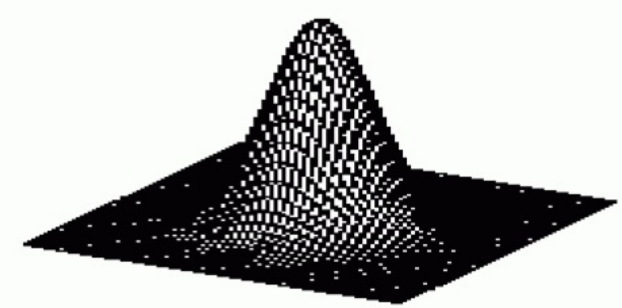
1	1	1
1	1	1
1	1	1

Gaussian

0.01	0.08	0.01
0.08	0.64	0.08
0.01	0.08	0.01

Gaussian Filter





$$g(x) = e^{\frac{-x^2}{2\sigma^2}}$$

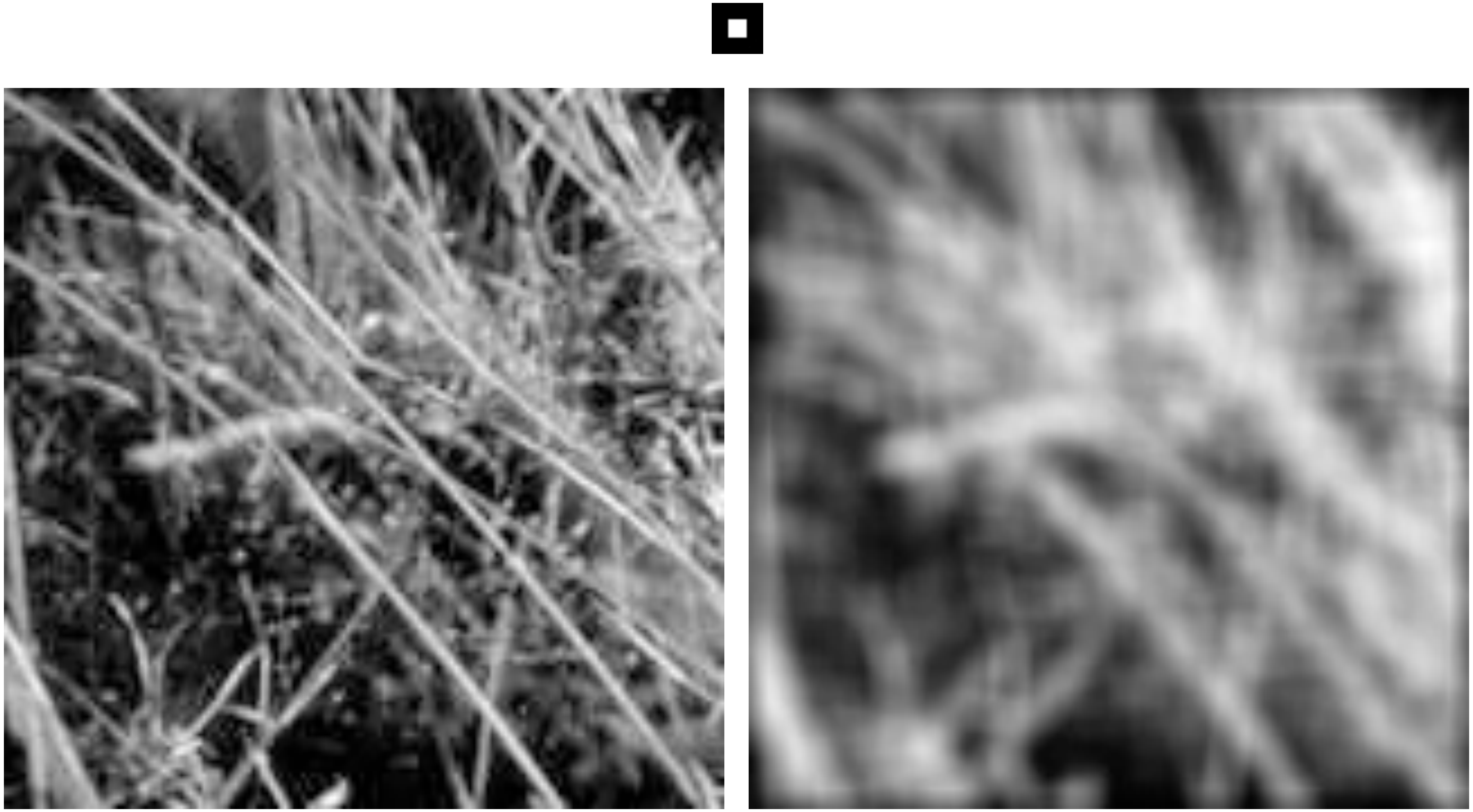
$$g(x,y) = e^{\frac{-(x^2+y^2)}{2\sigma^2}}$$

$$g(x) = [.011 \quad .13 \quad .6 \quad 1 \quad .6 \quad .13 \quad .011]$$

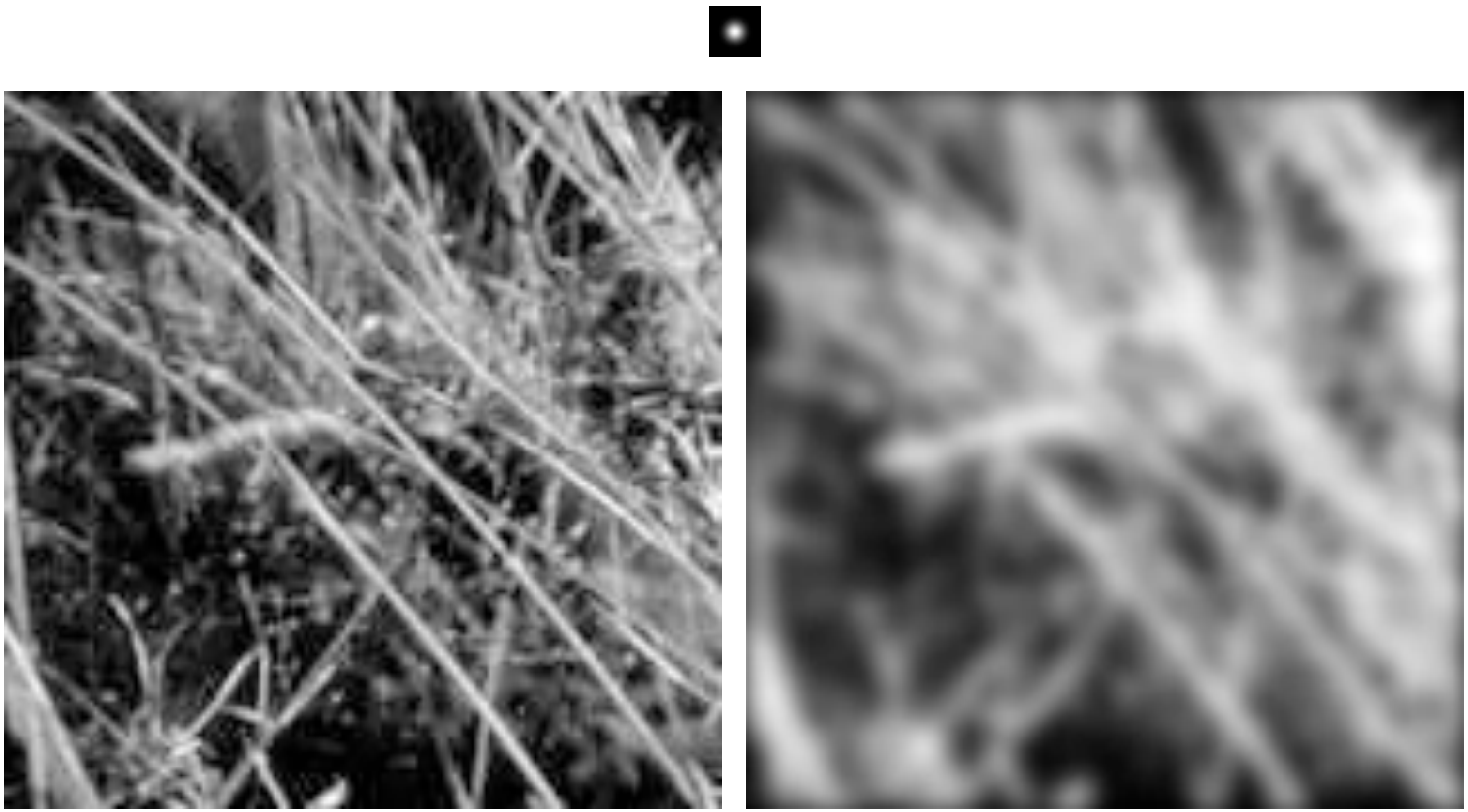
σ =1

Carl Friedrich Gauss
1777 to 1855

Smoothing with box filter



Smoothing with Gaussian filter



Gaussian filters

- Remove “high-frequency” components from the image (low-pass filter)
  - Images become more smooth

- *Separable* kernel
  – Factors into product of two 1D Gaussians

Source: K. Grauman

Separability of the Gaussian filter

$$G_\sigma(x, y) = \frac{1}{2\pi\sigma^2} \exp^{-\frac{x^2 + y^2}{2\sigma^2}}$$

$$= \left(\frac{1}{\sqrt{2\pi}\sigma} \exp^{-\frac{x^2}{2\sigma^2}} \right) \left(\frac{1}{\sqrt{2\pi}\sigma} \exp^{-\frac{y^2}{2\sigma^2}} \right)$$

The 2D Gaussian can be expressed as the product of two functions, one a function of x and the other a function of y

In this case, the two functions are the (identical) 1D Gaussian

Source: D. Lowe

Separability example

2D convolution
(center location only)

$$\begin{bmatrix}1&2&1\\2&4&2\\1&2&1\end{bmatrix} * \begin{bmatrix}2&3&3\\3&5&5\\4&4&6\end{bmatrix}$$

The filter factors
into a product of 1D
filters:

$$\begin{bmatrix}1&2&1\\2&4&2\\1&2&1\end{bmatrix} = \begin{bmatrix}1\\2\\1\end{bmatrix} \times \begin{bmatrix}1&2&1\end{bmatrix}$$

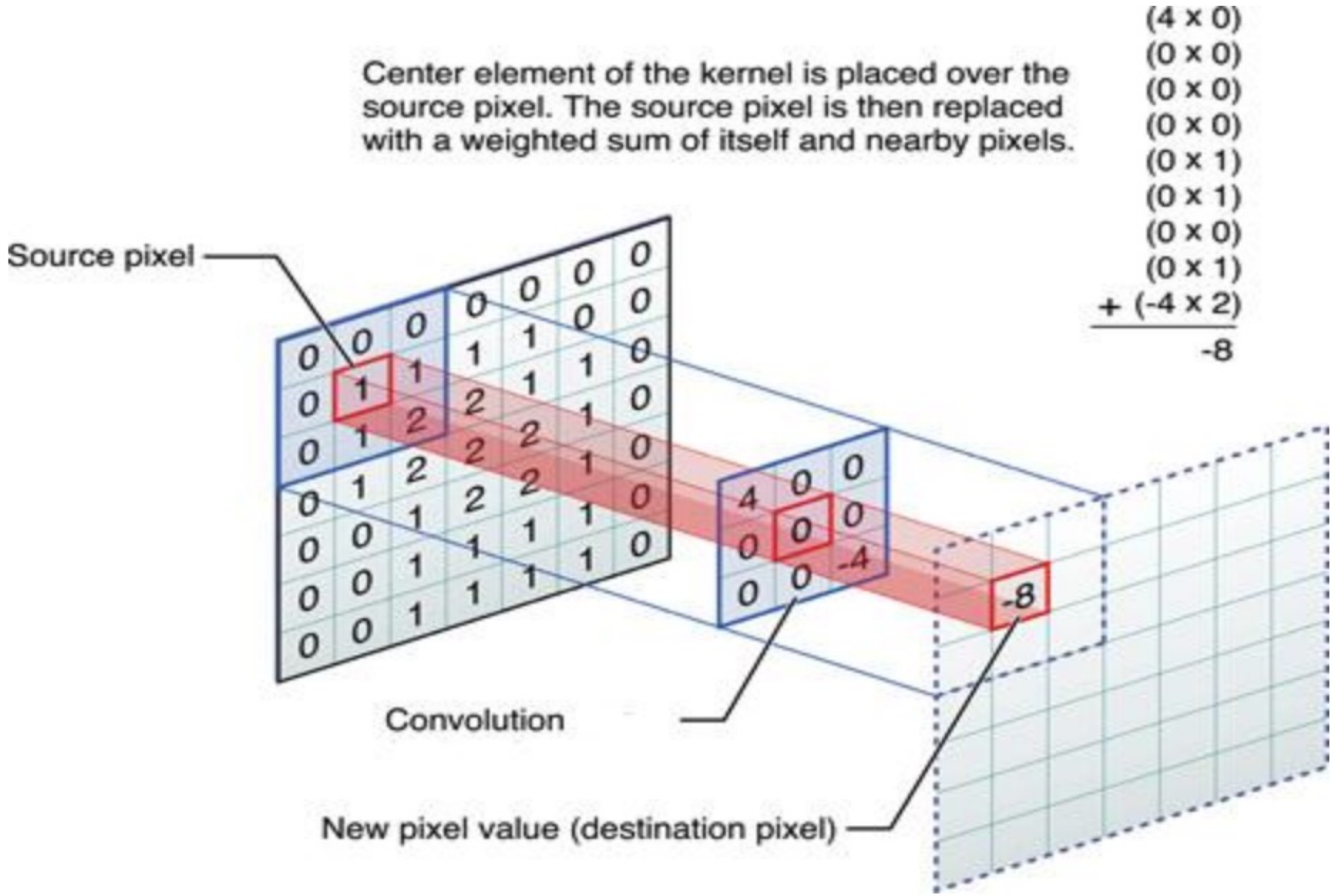
Perform convolution
along rows:

$$\begin{bmatrix}1&2&1\end{bmatrix} * \begin{bmatrix}2&3&3\\3&5&5\\4&4&6\end{bmatrix} = \begin{bmatrix} &11& \\ &18& \\ &18& \end{bmatrix}$$

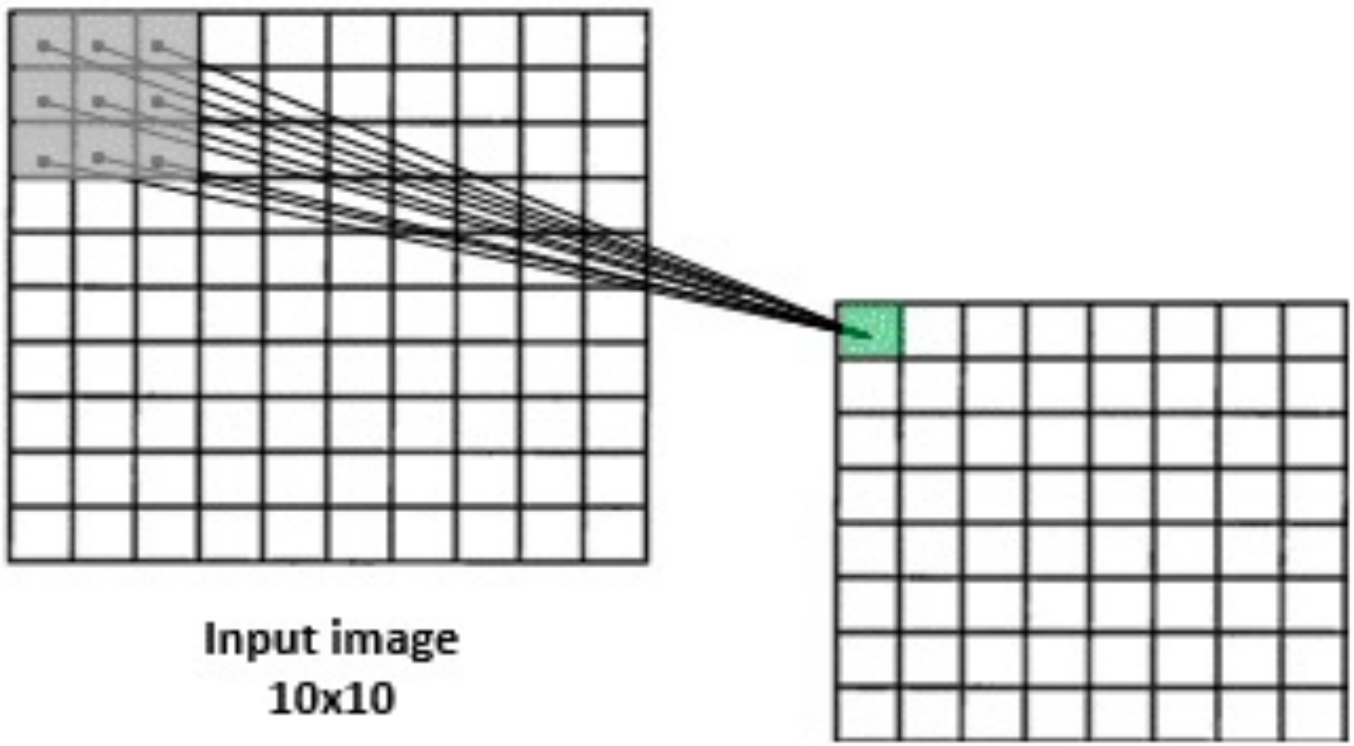
Followed by convolution
along the remaining column:

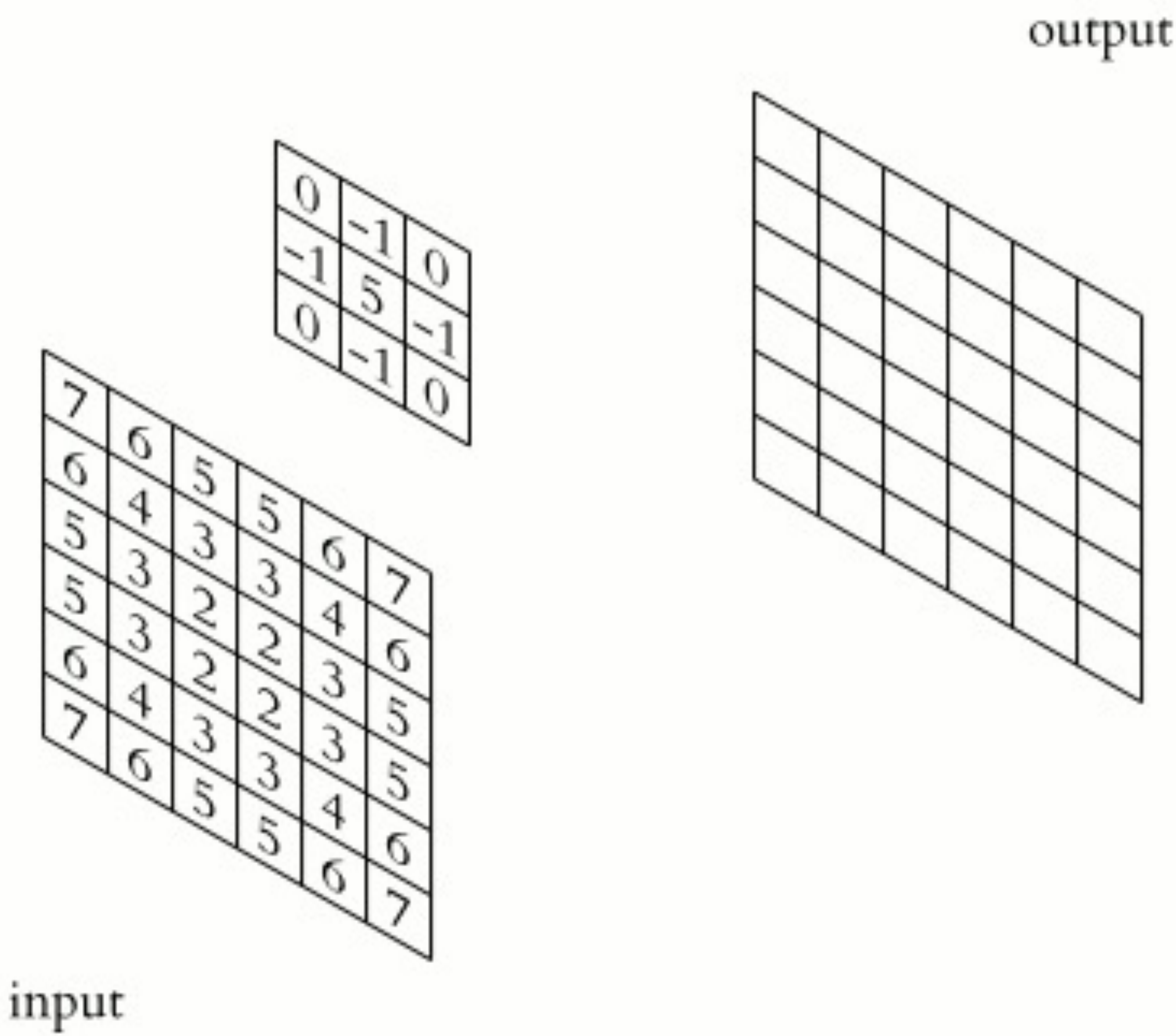
Source: K. Grauman

Convolution



Convolution Operation on a 7×7 matrix with a 3×3 kernel


Input image
10x10


output
0 -1 0
-1 5 -1
0 -1 0
7 6 5 5 6 7
6 4 3 3 4 6
5 3 2 2 3 5
5 3 2 2 3 5
6 4 3 3 4 6
7 6 5 5 6 7
input

Why Gaussian Assumption?

- Most common natural model
- Smooth function, it has infinite number of derivatives
- It is Symmetric
- Fourier Transform of Gaussian is Gaussian.
- Convolution of a Gaussian with itself is a Gaussian.
- Gaussian is separable; 2D convolution can be performed by two 1-D convolutions
- There are cells in eye that perform Gaussian filtering.

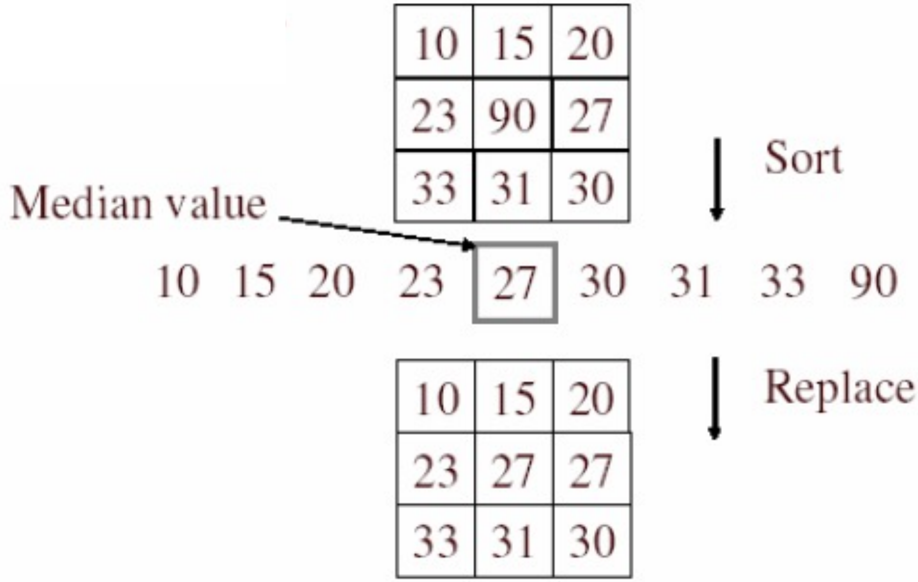
Median Filtering (Details)

[8 8 8 8 8 8 8 8 255]

median

Alternative idea: Median filtering

- A **median filter** operates over a window by selecting the median intensity in the window



- Is median filtering linear?

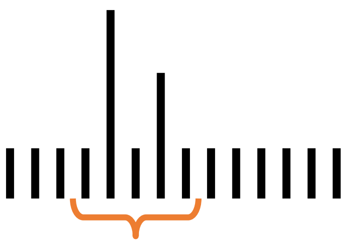
Source: K. Grauman

Median filter

- Replace each pixel by the median over N pixels (5 pixels, for these examples). Generalizes to “rank order” filters.

Median([1 7 1 5 1]) = 1
Mean([1 7 1 5 1]) = 2.8

In:

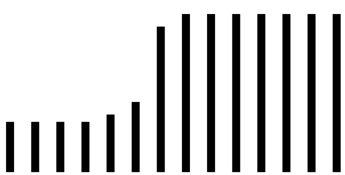


5-pixel neighborhood

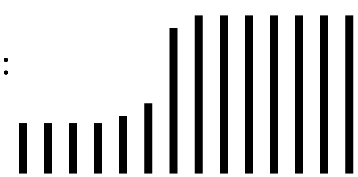
Out:

Spike noise is removed

In:



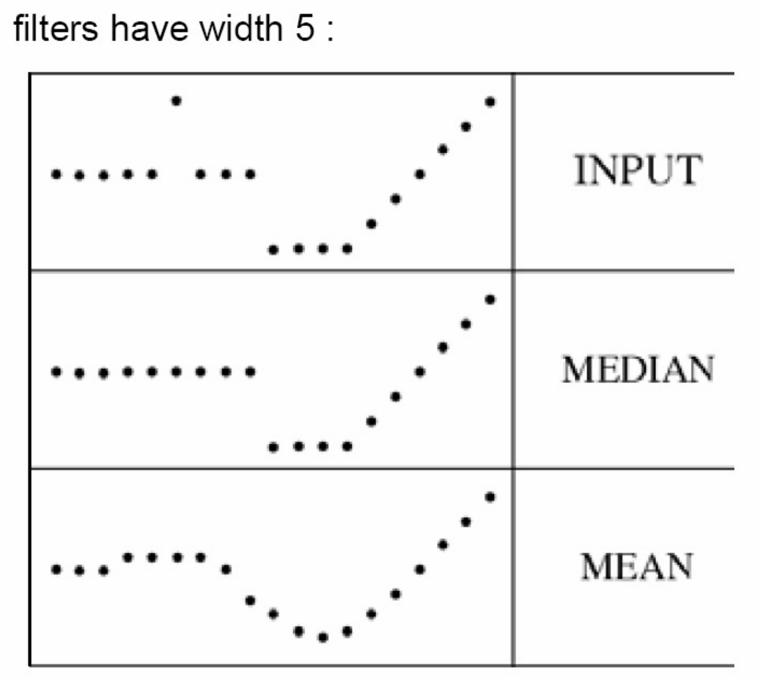
Out:



Monotonic edges remain unchanged

Median filter

- What advantage does median filtering have over Gaussian filtering?
  - Robustness to outliers



Source: K. Grauman

References and Slide Credits

- Jayaram K. Udupa, MIPG of University of Pennsylvania, PA.
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- CAP 5415 Computer Vision (Fall 2016) Lecture Presentations

- CAP 5937 (Fall 2016) Lecture Presentations

- Computer Vision (Lecture Presentations) by Dr. Mohsen Ali