Statistical & Mathematical Methods for Data Science

Assignment 1

Instructions: You are required to solve all the following problems and upload their scanned/camera captured solutions in a **single pdf file**. The name of the pdf file should be your roll number e.g MSDS2000x.pdf .

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1) Two jurors are selected from 4 alternates to serve at a murder trial. Using the notation A_1A_3, for example, to denote the simple event that alternates 1 and 3 are selected, list the 6 elements of the sample space S.

2) If S = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} and A = {0, 2, 4, 6, 8}, B = {1, 3, 5, 7, 9}, C = {2, 3, 4, 5}, and D = {1, 6, 7}, list the elements of the sets corresponding to the following events:

(a) A U C
(b) A ∩ B
(c) C'
(d) (C' ∩ D) U B
(e) (S ∩ C)'
(f) A ∩ C ∩ D'

3) A developer of a new subdivision offers a prospective home buyer a choice of 4 designs, 3 different heating systems, a garage or carport, and a patio or screened porch. How many different plans are available to this buyer?

4) The probability that an American industry will locate in Shanghai, China, is 0.7, the probability that it will locate in Beijing, China, is 0.4, and the probability that it will locate in either Shanghai or Beijing or both is 0.8. What is the probability that the industry will locate

(a) in both cities?
(b) in neither city?

5) Which of the following events are equal?

(a) A = {1, 3};
(b) B = {x | x is a number on a die};
(c) C = {x | x2 − 4x +3=0};
(d) D = {x | x is the number of heads when six coins are tossed}.

6) List the elements of each of the following sample spaces:

(a) the set of integers between 1 and 50 divisible by 8;
(b) the set S = {x | x2 + 4x − 5=0};
(c) the set of outcomes when a coin is tossed until a tail or three heads appear;
(d) the set S = {x | x is a continent};
(e) the set S = {x | 2x − 4 ≥ 0 and x < 1}.

7) The probabilities that a hotel will serve food to 0, 1, 2, 3, 4, or 5 or more customers during a certain – time of the day are 0.03, 0.18, 0.24, 0.28, 0.10, and 0.17, respectively. Find the probability that in this time of the day (a) more than 2 customers receive food; (b) at most 4 customers receive food; (c) 4 or more customers receive food.

8) In a certain federal prison, it is known that 2/3 of the inmates are under 25 years of age. It is also known that 3/5 of the inmates are male and that 5/8 of the inmates are female or 25 years of age or older. What is the probability that a prisoner selected at random from this prison is female and at least 25 years old?

9) The probability that an automobile being filled with gasoline also needs an oil change is 0.25; the probability that it needs a new oil filter is 0.40; and the probability that both the oil and the filter need changing is 0.14. (a) If the oil has to be changed, what is the probability that a new oil filter is needed? (b) If a new oil filter is needed, what is the probability that the oil has to be changed?

10) The probability that a doctor correctly diagnoses a particular illness is 0.7. Given that the doctor makes an incorrect diagnosis, the probability that the patient files a lawsuit is 0.9. What is the probability that the doctor makes an incorrect diagnosis and the patient sues?

11) A manufacturer of a flu vaccine is concerned about the quality of its flu serum. Batches of serum are processed by three different departments having rejection rates of 0.10, 0.08, and 0.12, respectively. The inspections by the three departments are sequential and independent. (a) What is the probability that a batch of serum survives the first departmental inspection but is rejected by the second department? (b) What is the probability that a batch of serum is rejected by the third department?

12) In an experiment to study the relationship of hypertension and smoking habits, the following data are collected for 180 individuals:

	Non smoker	Moderate smoker	Heavy Smoker
H	21	36	30
NH	48	26	19

H and NH in the table stand for Hypertension and Nonhypertension, respectively. If one of these individuals is selected at random, find the probability that the person is (a) experiencing

hypertension, given that the person is a heavy smoker; (b) a nonsmoker, given that the person is experiencing no hypertension.

13) If each coded item in a catalog begins with 3 distinct letters followed by 4 distinct nonzero digits, find the probability of randomly selecting one of these coded items with the first letter a vowel and the last digit even

14) Suppose that in a senior college class of 500 students it is found that 210 smoke, 258 drink alcoholic beverages, 216 eat between meals, 122 smoke and drink alcoholic beverages, 83 eat between meals and drink alcoholic beverages, 97 smoke and eat between meals, and 52 engage in all three of these bad health practices. If a member of this senior class is selected at random, find the probability that the student (a) smokes but does not drink alcoholic beverages; (b) eats between meals and drinks alcoholic beverages but does not smoke; (c) neither smokes nor eats between meals.

15) A box contains 500 envelopes, of which 75 contain $100 in cash, 150 contain $25, and 275 contain $10. An envelope may be purchased for $25. What is the sample space for the different amounts of money? Assign probabilities to the sample points and then find the probability that the first envelope purchased contains less than $100.

16) Find the errors in each of the following statements:

(a) The probabilities that an automobile salesperson will sell 0, 1, 2, or 3 cars on any given day in February are, respectively, 0.19, 0.38, 0.29, and 0.15.

(b) The probability that it will rain tomorrow is 0.40, and the probability that it will not rain tomorrow is 0.52.

(c) The probabilities that a printer will make 0, 1, 2, 3, or 4 or more mistakes in setting a document are, respectively, 0.19, 0.34, −0.25, 0.43, and 0.29.

(d) On a single draw from a deck of playing cards, the probability of selecting a heart is 1/4, the probability of selecting a black card is 1/2, and the probability of selecting both a heart and a black card is 1/8.

17) Three cards are drawn in succession, without replacement, from an ordinary deck of playing cards. Find the probability that the event $A1 \cap A2 \cap A3$ occurs, where A1 is the event that the first card is a red ace, A2 is the event that the second card is a 10 or a jack, and A3 is the event that the third card is greater than 3 but less than 7