MSDS 20075 Medical Image Computing

Q No 1

Acording to definition of affine

Scaling rotation and translation form affine so

rotation = $\begin{matrix} \cos(a) & -\sin(a) & 0 \\ \sin(a) & \cos(a) & 0 \\ 0 & 0 & 1 \end{matrix}$

scaling = $\begin{matrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{matrix}$

translation = $\begin{matrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{matrix}$

Resultant = $\begin{matrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{matrix}$

Hence 6 variable are formed by 3 points

Q No 2

Solution

$$\text{Error} = (mx + c - y)^2$$

diff w.r.t m & c

diff w.r.t m

$$2\sum (mx_i + c - y_i)x_i = 0$$

$$\sum mx_i^2 + \sum cx_i - \sum x_i y_i = 0$$

$$m\sum x_i^2 + c\sum x_i = \sum x_i y_i \quad — (1)$$

differentiate w.r.t c

$$2\sum (mx_i + c - y_i) = 0$$

$$\sum mx_i + \sum c - \sum y_i = 0$$

$$m\sum x_i + c\sum 1 = \sum y_i \quad — (2)$$

According to eq (1) & (2)

$$\begin{bmatrix} \sum x_i^2 & \sum x_i \\ \sum x_i & \sum 1 \end{bmatrix} \begin{bmatrix} m \\ c \end{bmatrix} = \begin{bmatrix} \sum x_i y_i \\ \sum y_i \end{bmatrix}$$

Q No 3

$$E = \sum_{j=1}^{n} \left[(a_1 x_j + a_2 y_j + a_3 - x_j')^2 + (a_4 x_j + a_5 y_j + a_6 - y_j')^2 \right]$$

diff w.r.t a_1, a_2, a_3, a_4, a_5 & a_6

=) w.r.t a_1

$$2\sum_{j=1}^{n} (a_1 x_j + a_2 y_j + a_3 - x_j')x_j + 0 = 0$$

$$a_1 \sum_{j=1} x_j^2 + a_2 \sum_{j=1} y_j x_j + a_3 \sum_{j=1} x_j - \sum_{j} x_j x_j' = 0 \quad — (1)$$

=) w.r.t a_2

$$2\sum_{j=1}^{n} (a_1 x_j + a_2 y_j + a_3 - x_j')1 = 0$$

$$a_1 \sum_{j=1} x_j y_j + a_2 \sum_{j=1} y_j^2 + a_3 \sum y_j - \sum x_j' y_j = 0 \quad — (2)$$

w.r.t a_3

$$2\sum_{j=1}(a_1x_j + a_2y_j + a_3 - x_j')\,1 = 0$$

$$a_1\sum x_j + a_2\sum y_j + a_3\sum 1 - \sum x_j' = 0 \quad \text{—— (3)}$$

w.r.t a_4

$$0 + 2\sum_{j=1}(a_4x_j + a_5y_j + a_6 - y_j')y_j = 0$$

$$a_4\sum_{j=1} x_j^2 + a_5\sum_{j=1} x_j y_j + a_6\sum_{j=1} x_j - \sum x_j y_j' = 0 \quad \text{—— (4)}$$

w.r.t a_5

$$0 + 2\sum(a_4x_j + a_5y_j + a_6 - y_j')y_j = 0$$

$$a_4\sum_{j=1} x_jy_j + a_5\sum y_j^2 + a_6\sum y_j - \sum y_j y_j' = 0 \quad \text{—— (5)}$$

w.r.t a_6

$$0 + 2\sum_{j=1}(a_4x_j + a_5y_j + a_6 - y_j')\,1 = 0$$

$$a_4\sum_j x_j + a_5\sum_{j=1} y_j + a_6\sum_{j=1} 1 - \sum y_j' = 0 \quad \text{—— (6)}$$

These 6 equations can be written together in matrix format as

$$\begin{bmatrix} \sum_j x_j^2 & \sum_j x_jy_j & \sum_j x_j & 0 & 0 & 0 \\ \sum_j x_jy_j & \sum_j y_j^2 & \sum_j y_j & 0 & 0 & 0 \\ \sum_j x_j & \sum_j y_j & \sum_j 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sum_j x_j^2 & \sum_j x_jy_j & \sum_j x_j \\ 0 & 0 & 0 & \sum_j x_jy_j & \sum_j y_j^2 & \sum_j y_j \\ 0 & 0 & 0 & \sum_j x_j & \sum_j y_j & \sum_j 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} = \begin{bmatrix} \sum_j x_jx_j' \\ \sum_j y_j x_j' \\ \sum_j x_j' \\ \sum_j x_j y_j' \\ \sum_j y_j y_j' \\ \sum_j y_j' \end{bmatrix}$$