Recitations 12

JAKUB PAWELCZAK

MIN II

FALL 2020

12/03/20 RECITATION 12

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Today:

- MIDTERM
- HW 3
- Complete Markets

Recitations 12

[Definitions used today]

- State dependent allocations, Time 0 trade, Arrow Debreu securities

Question 1 [Ex2 Midterm 2020]

Consider the following 2 agent, 2 good, endowment economy. Both agents, $i \in \{1,2\}$ have utility function $u_i(c_{i,1}, c_{i,2}) = 2\min(c_{i,1}, c_{i,2})$., where $c_{i,m}$ is the amount of good $m \in \{1,2\}$ agent i consumes. There is 1 divisible unit of each good in the world, and each agent is able to consume any non-negative amount of either good.

1. What is the set of Pareto efficient allocation for this economy?
2. Derive the utility possibilities set for this economy.
3. Specify the Arrow problem here, carefully specifying what is taken as a parameter. Is every solution Pareto efficient? Is every Pareto efficient allocation the solution to some Arrow problem?
4. Set up the Negishi problem here, again carefully specifying what is taken as a parameter. Is every solution Pareto efficient? Is every Pareto efficient allocation the solution to some Negishi problem?
5. Derive the set of Competitive Equilibria. Are they all Pareto Efficient?
6. Are all Pareto efficient allocations achievable for some set of initial endowments?
7. Suppose an allocation c is Pareto efficient. Describe the set $B(c)$.

Question 2 [Ex 1 Homework 3]

Consider a 2 good, 2 agent world. Good one x, denotes oranges and good two, y denotes orange juice. Agent $i = 1$ has utility function $u_1(x, y) = 2\log(x) + \log(y)$ and agent $i = 2$ has utility function $u_2(x, y) = \log(x) + 2\log(y)$. Suppose each agent is endowed with 1 orange and no orange juice. Further assume there exist two identical firms which can turn oranges into orange juice according to the production function $f(x) = \sqrt{x}$

1. Define and find the Competitive Equilibrium. For what planner weights (if any) does this solve the Negishi problem (with production)?
2. Now suppose agent 1 is endowed with 1 orange (and no orange juice) and agent2 is endowed with 0 oranges. Each agent owns half of each firm. Find the competitive equilibrium and the weights (if any) for which this is a solution to the Negishi Problem.
3. Do again but assume agent 1 is endowed with 0 oranges (and no orange juice)and agent 2 is endowed with 1 orange (and zero juice) (with again each owning half of each firm).

Question 3 [Ex 2 Homework 3]

Consider a two period economy where all agents are endowed with 1 unit of the single consumption good at date $t = 0$ and no units of the single consumption good at date $t = 1$. There exist two firms which can store the consumption good from the first to the second period where each unit stored today becomes $0 < \alpha < 1$ units tomorrow

1. Draw the production set for each firm
2. If $u(c_1, c_2) = \log(c_1) + \beta\log(c_2)$ for each agent, find ALL competitive equilibrium.

Question 4

Suppose $t \in \{0, \ldots, T\}$. At each date t, nature flips a coin. With 50% probability, agent 1 has an endowment of 2 bananas and agent 2 has an endowment of zero bananas, and with 50% probability, agent 1 has an endowment of 0 bananas and agent 2 has an endowment of 2 bananas. There is no production and all endowments are observable. Let s_t be the joint endowment realization at date t, and $s^t = \{s_0, \ldots, s_t\}$ Assume preferences are characterized by $\sum_{t=0}^{T} \beta^t \sum_{s^t} \pi_t(s^t) u(c_t(s^t))$ where $\pi(s^t)$ is the (obvious) probability of sequence s^t and u is some strictly concave function. 0

1. Characterize the set of feasible allocations.
2. Characterize the set of Pareto efficient allocations.
3. Characterize the competitive equilibrium from these endowments. instead there are N agents each of whom at each date flips a fair coin and if heads, has an endowment of 2 bananas, and if tails has an endowment of zero bananas. Redo the previous parts to this question. What happens as $N \to \infty$?

Question 5 [Final 2017]

Consider a complete markets economy with I agents, $T+1$ dates ($t=0$ to $t=T$) where at each date, a publicly observable random variable $s \in S$ is realized. Each agent i 's endowment of the single consumption good at date t depends only on the realization of s at date t. If $c_{i,t}(s^t)$ denotes agent i 's consumption at date t after history $s^t = (s_0, \ldots, s_t)$, his preferences are represented by $\sum_{t=0}^{T} \beta^t \sum_{s^t} \pi(s^t) u_i(c_{i,t}(s^t))$

1. Define a feasible allocation.
2. Sketch out what is necessary for the for the first welfare theorem to hold.
3. Assuming the first welfare theorem holds and that the utility possibilities set is strictly convex, show that in any equilibrium, if two agents have the same preferences, if agent i consumes more than agent j for any date t and history s^t, then agent i consumes more than agent j at every date t and history s^t

- Adding time
- state contingent structure of markets
- I can be $|I| < +\infty$
  $|I| = +\infty$
  $i \in [0,1]$

MIDTERM

2 (a): $u_i = 2 \cdot \min(c_{i1}, c_{i2})$

$UPS = \{ (u_1, u_2)$

$u_1 = 2 \cdot \min(c_{11}, c_{12})$

$u_2 = 2 \cdot \min(c_{21}, c_{22})$

$c_{11} + c_{21} = 1$

$c_{12} + c_{22} = 1$

$\}$

① $c_{11} \geq c_{12}$

$c_{21} \geq c_{22}$

$u_1 = 2 c_{12}$

$u_2 = 2 \cdot c_{22}$

$u_1 + u_2 = 2 (c_{12} + c_{22}) \leq 2$

② $c_{11} \leq c_{12}$

$c_{21} \leq c_{22}$

...

$u_1 + u_2 \leq 2$

③ $c_{11} \geq c_{12} \rightarrow u_1 = 2 \cdot c_{12}$

$c_{21} \leq c_{22} \rightarrow u_2 = 2 \cdot c_{21}$

$u_1 + u_2 = 2(c_{12} + c_{21}) \leq 2(c_{12} + c_{22})$

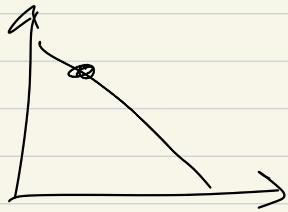
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \| $

④ $c_{11} \leq c_{12}$

$c_{21} \geq c_{22} \rightarrow u_1 + u_2 \leq 2$ $\qquad 2$

1(a) PE

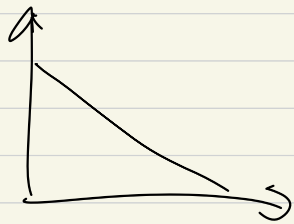
$(x, y, 1-x, 1-y) \quad y, x \in [0,1]$



2(a)

$(x, x, 1-x, 1-x)$

$x \in [0,1]$



1, 2 common mistake

If $p_1 < p_2$ $\quad c_1 = (\cdot, 0)$

$c_2 = (\cdot, 0)$ $\quad$ ↯

If $p_1 > p_2$ $\quad c_1 = (0, \cdot)$

$c_2 = (0, \cdot)$ $\quad$ ↯

If $p_1 = p_2$ $\quad c_1 = (\cdot, 0)$

$c_2 = (0, \cdot)$

$p_1 = p_2$

$p_1 c_{11} + p_2 c_{12} \le p_1 e_{11} + p_2 e_{12}$

$p_1 (c_{11} + c_{12}) \le p_1 e_{11} + p_2 e_{12}$

$(\cdot, 0)$

$(0, \cdot)$

π_i is not str. monotone
& weakly convex.
Both 1st 2nd goes through

MIDTERM

- $\bar{x} = 85$
  st. dev = 10

- ≥ 90 ≈ 14

Ex. 2 [Ex 1 HW3]

Def. CE :

- allocations z^M, z^F
- profits π^1, π^2
- prices $P = (p_1, p_2)$

given $\{e_{ij}, \theta_{ij}\}_{i,j=1,2}$

- $\forall j = 1,2$ firm j picks (y_{j1}, y_{j2}) given P s.t.

$$\pi^j = \max \; p_1 \cdot y_{j1} + p_2 y_{j2}$$

$$y_{j2} \le \sqrt{-y_{j1}}$$

$$y_{j2} \ge 0 \ge y_{j1}$$

$$p_1 - \frac{1}{2}(-y_{j1})^{-1/2} \cdot p_2 = 0$$

$$y_{j1} = -\left(\frac{p_2}{2p_1}\right)^2$$

$$\pi^j = p_1 y_{j1} + p_2 y_{j2} = \frac{p_2^2}{4p_1}$$

Let $p_1 = 1$.

Denote $\Theta = \theta_{11} + \theta_{12}$

$$2 - \Theta = 1 - \theta_{11} + 1 - \theta_{12}$$

- agent 1 picks (c_{11}, c_{12}) given $P, \pi^j, \theta_{1,1}, \theta_{1,2}, e_{11}, e_{12}$

$$\max \underbrace{2\ln c_{11} + \ln c_{12}}_{u_1}$$

(λ_1)
$$p_1 c_{11} + p_2 c_{12} \leq p_1 e_{11} + p_2 e_{12} + \theta_{11}\pi^1 + \theta_{12}\pi^2$$

~~$\nabla u = [\nabla u_1, \nabla u_2]$~~

$u_2 = \ln c_{21} + 2\ln c_{22}$

- $\nabla u_1 = \left[\frac{2}{c_{11}}, \frac{1}{c_{12}}\right] > 0$
- $\nabla u_2 = \left[\frac{1}{c_{21}}, \frac{2}{c_{22}}\right] > 0$
- $D^2 u_1 = \begin{bmatrix} -\frac{2}{c_{11}^2} & 0 \\ 0 & -\frac{1}{c_{12}^2} \end{bmatrix} < 0$
- $D^2 u_2 = \begin{bmatrix} -\frac{1}{c_{21}^2} & 0 \\ 0 & -\frac{2}{c_{22}^2} \end{bmatrix} < 0$

We can FOCs KKT

$$\frac{2}{c_{11}} = \lambda_1 \cdot \qquad \frac{1}{c_{12}} = \lambda_1 p_2 \cdot$$

$$\frac{1}{c_{21}} = \lambda_2 \cdot \qquad \frac{2}{c_{22}} = \lambda_2 p_2 \cdot$$

$\lambda_1, \lambda_2 > 0$ so $BC = \pi^i$

Now BC

$$c_{11} + p_2 c_{12} = \underbrace{e_{11} + p_2 e_{12} + \theta \frac{p_2^2}{4}}_{\omega_1}$$

$$c_{21} + p_2 c_{22} = \underbrace{e_{21} + p_2 e_{22} + (2-\theta)\frac{p_2^2}{4}}_{\omega_2}$$

$$\frac{c_{11}}{2c_{12}} = p_2 = \frac{2c_{21}}{c_{22}}$$

$$c_{11} = 2 c_{12} \cdot p_2 \qquad (*)$$

$$c_{21} = p_2 \cdot c_{22} \cdot \frac{1}{2} \qquad (**)$$

$$c_{12} = \frac{1}{3} \frac{\omega_1}{p_1} = \frac{1}{3}\omega_1 \Rightarrow c_{11} = \frac{2}{3}\omega_1$$

$$c_{22} = \frac{2}{3} \frac{\omega_2}{p_2} \Rightarrow c_{21} = \frac{1}{3}\frac{\omega_2}{p_2}$$

From MCC

$$\begin{cases} \frac{2}{3} w_1 + \frac{1}{3}\omega_2 = -\frac{p_2^2}{2} + e_{11} + e_{21} \\ \frac{1}{3}\frac{\omega_1}{p_2} + \frac{2}{3}\frac{\omega_2}{p_2} = -\frac{p_2^2}{2} + e_{22} + e_{12} \end{cases}$$

$$e_{11} = 1 = e_{21}$$

$$(e_{12} = 0 = e_{22})$$

$$\frac{2}{3}\left(1 + 0 + \theta \frac{p_2^2}{4}\right) +$$

$$\frac{1}{3}\left(1 + 0 + (2-\theta)\frac{p_2^2}{4}\right)$$

$$= -\frac{p_2^2}{4} + \underbrace{1 + 1}$$

$$p_2 = \sqrt{\frac{12}{8+\theta}}$$

$$c_{11} = \frac{2}{3}\left[1 + \theta \, \cancel{\cdot} \, \frac{12}{8+\theta} \cdot \frac{1}{4}\right]$$

$$c_{12} = \frac{1}{3}\left[1 + \theta \frac{12}{8+\theta} \cdot \frac{1}{4}\right] \cdot \sqrt{\frac{8+\theta}{12}}$$

$$c_{21} = \frac{1}{3}\left[1 + \frac{12}{8+\theta} \cdot \frac{1}{4}(2-\theta)\right]$$

$$c_{22} = \frac{2}{3}\left[1 + \frac{12}{8+\theta} \cdot \frac{1}{4}(2-\theta)\right] \cdot \sqrt{\frac{8+\theta}{12}}$$

$$y_{11} = y_{21} = -\frac{1}{4} \cdot \frac{12}{8+\theta}$$

$$y_{12} = y_{22} = \frac{1}{2} \cdot \sqrt{\frac{12}{8+\theta}}$$

$$\pi^1 = \pi^2 = \frac{1}{4} \cdot \frac{12}{8+\theta}$$

$$p = \left(1, \sqrt{\frac{12}{8+\theta}}\right)$$

SPP

$$\lambda = \frac{4+2\theta}{\theta+11}$$

Ex. 5

$t=0 \qquad \pi_0(s_0)=1$

$t>0 \qquad s^t = [s_0, s_1, \dots, s_t]$

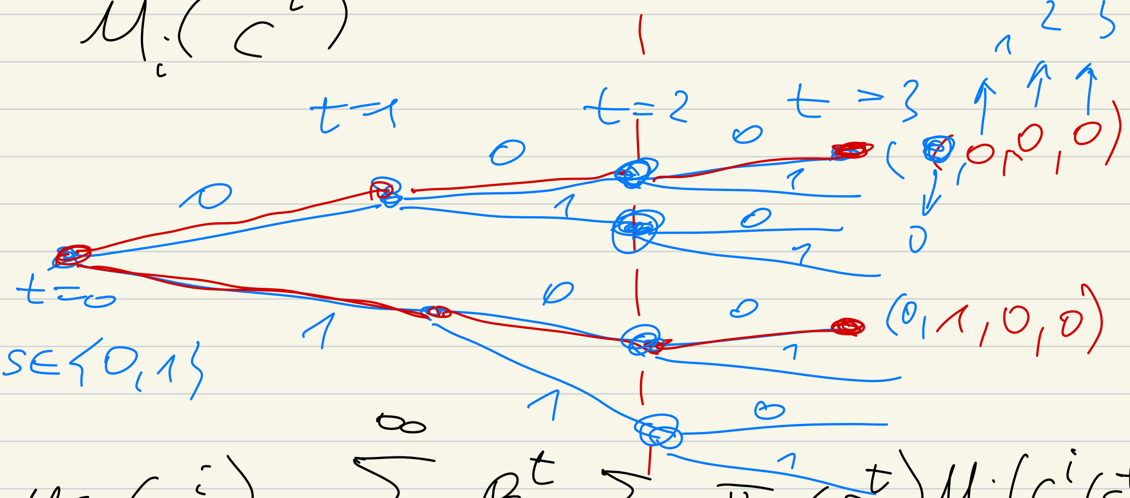
$\pi_t(s^t)$

$\pi_t(s^t \mid s^\tau) \qquad \tau > 0$

I agents $\qquad y_t^i(s^t)$

- s^t is observed by all agents
- $c^i = \{c_t^i(s^t)\}_{t=0}^{+\infty} \ \forall s^t$

$u_i(c^i)$



$$U_i(c^i) = \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \pi_t(s^t)\, u_i(c_t^i(s^t))$$

$t=2 \qquad s^t \quad 4$

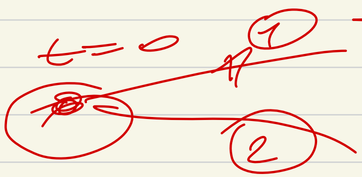
$$u^i(c^i) = \mathbb{E}_0 \sum_{t=0}^{+\infty} \beta^t u(c_t^i)$$

$$\mathbb{E}_0[\,\cdot\,] = \mathbb{E}[\quad | \sigma(s_0)]$$

FEASIBILITY

$$\sum_{i=1}^{I} c_t^i(s^t) \le \sum_{i=1}^{I} y_t^i(s^t)$$

$t=0 \rightarrow s_0$ 1 FEAS



$t=1$ 2 FEAS

$s \in \{0,1\}$

$t=T$ 2^T FEAS...

$s \in \{i_1 \dots i_K\}$

$t=T$ K^T FEAS

Lagrange multiplier

$\theta_t(s^t)$

SPP $\quad \max \sum \lambda_i \, u_i(c^i)$

$\theta_t(s^t) \quad \sum_{i=1}^{I} y_t^i(s^t) \geq \sum_{i=1}^{I} c_t^i(s^t)$

LNS $\quad$ ~~u_i~~ $\forall_i \quad u_i$

$u_i(c_t^i(s^t))$

$\underline{\underline{u_i(x)}} \Rightarrow$ ~~KE~~ 1st WThm

CE allocation $\Rightarrow$ PO allocation

UPS are str. convex:

$\cancel{\Leftarrow}$ SPP $\Longleftrightarrow$ PO

If u_i st. QC, monotone

FOCs:

$$\mathcal{L} = \sum_{t=0}^{+\infty} \sum_{s^t} \left(\sum \lambda_i u_i(c^i) \right) \beta^t + \theta_t(s^t) \left(\sum_{i=1}^{I} y_t^i(s^t) - c_t^i(s^t) \right)$$

$$\frac{\partial L}{\partial c_t^i(s^t)} : \lambda_i \beta^t u'(c_t^i(s^t)) \pi(s^t) = \theta_t(s^t)$$

$i = i, \quad i = 1$

$$\boxed{\frac{u'(c_t^i(s^t))}{u'(c_t^1(s^t))} = \frac{\lambda_1}{\lambda_i} \quad (*)}$$

(1) $I = 2 \quad \lambda_1 = \lambda_2$

(*) $c_t^1(s^t) = c_t^2(s^t) \quad \forall t, s^t$

(2) From (*)

$$u'(c_t^i(s^t)) = u'(c_t^1(s^t)) \cdot \frac{\lambda_1}{\lambda_i}$$

u' is invertible: $\exists (u')^{-1}$

$$c_t^i(s^t) = (u')^{-1}\left(\frac{\lambda_1}{\lambda_i} u'(c_t^1(s^t))\right)$$

$$\sum_{i=1}^{I} c_t^i(s^t) = \underbrace{\sum_{i=1}^{I} (u')^{-1}\left(\frac{\lambda_1}{\lambda_i} u'(c_t^1(s^t))\right)}_{LHS}$$

$$= \underbrace{\sum_{i=1}^{I} y_t^i(s^t)}_{RHS} \quad \forall t, s^t$$

. LHS $c_t^1(s^t)$ is unknown

$$c_t^i(s^t) = f\left(\sum_{i=1}^{I} y_t^i(s^t)\right)$$

$$\neq f(t) \neq g(s^t)$$

$$\sum y_t^i(s^t) = \sum y_t^i(\tilde{s}^t)$$

$$\Rightarrow c_t^i(s^t) = c_t^i(\tilde{s}^t)$$

$$\frac{u_i'(c_t^i(s^t))}{u_j'(c_t^j(s^t))} = \frac{\lambda_j}{\lambda_i} < 1 \quad (*)$$

$u = u_i = \cancel{g}\, u_j$

u' decreasing

$\exists\, \tau, s^\tau \qquad c_\tau^i(s^\tau) > c_\tau^j(s^\tau)$

Plug into $(*)$

$$u'(c_\tau^i(s^\tau)) < u'(c_\tau^j(s^\tau))$$

From $(*)$

$$1 > \frac{u'(c_\tau^i(s^\tau))}{u'(c_\tau^j(s^\tau))} = \frac{\lambda_j}{\lambda_i}$$

$$\Rightarrow \lambda_i > \lambda_j$$

$$u'(c_t^i(s^t)) < u'(c_t^j(s^t))$$

$$c_t^i(s^t) > c_t^j(s^t)$$

$$\forall t,\ s^t$$