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# Recitations 10

**[Definitions used today]**

- Junge Economy, Welfare Theorems, Existence of Equilibrium

## Question 1 [Piccione & Rubinstein, 2006]

Some results on jungle equilibrium.

a) Define setting.

b) Define feasible allocation and jungle equilibrium.

c) Show that a jungle equilibrium exists.

d) Let $a = (a_1, \ldots, a_n)$ and $b = (b_1, \ldots, b_n)$ be strictly positive vectors, and suppose that $a \cdot x > 0$ and $b \cdot x < 0$, for some vector $x = (x_1, \ldots, x_n)$. Show that there exists a vector $y = (y_1, \ldots, y_n)$ such that:

- $y_k > 0$ for some $k$ for which $x_k > 0$
- $y_l < 0$ for some $l$ for which $x_l < 0$
- $y_h = 0$ for $h \neq kh \neq l$
- $a \cdot y > 0$ and $b \cdot y < 0$

e) Show that if a jungle is smooth then $\hat{z}$ is the unique jungle equilibrium.

f) Show that the allocation $\hat{z}$ is efficient.

g) Jungle equilibrium vs. competitive equilibrium: trading houses (with and without gold).

h) Can a jungle allocation be supported by a vector of prices in competitive equilibrium?

i) Suppose that the jungle is smooth. Show that in the exchange economy in which $w_i = \hat{z}_i, i \in \{1, \ldots, N\}$, there exists a sequence of price vectors $p_n$ such that, for every agent $i$, the sequence of demands of agent $i$ given $p_n$ converges to $z_i$.

j) Think about jungle equilibrium with production.

## Question 2 [Very Easy Existence Theorem]

Let $Z : \bar{\Delta} \to \mathbb{R}^l$ is a continuous function that satisfies Walras' law ($\forall p \in \bar{\Delta} : p \cdot Z(p) = 0$), then $\exists p^* \in \bar{\Delta}$ such that $Z(p^*) \leq 0$. Further, $Z(p^*) = 0$ only if $p^* \in \Delta$.

## Question 3 [Easy Existence Theorem]

Let $Z : \Delta \to \mathbb{R}^l$ be a continuous function that is bouded from below, satisfying Walras' Law and the boundary condition: $p_n \to p \in \partial\Delta \Rightarrow \|Z(p_n)\| \to \infty$. Then $\exists p^* \in \Delta$ such that $Z(p^*) = 0$.