Recitations 8

[Definitions used today]

- Arrow problem, Negishi problem, Social Planner Problem, Pareto efficient allocation, Competetive Equilibrium, Junge Economy

Question 1 [Midterm 2018]

Consider a pure exchange economy with two agents, $i \in \{1, 2\}$ and two goods $m \in \{1, 2\}$. There exists a total of 1 unit of each good. Agent 1's preference is represented by utility function $u_1(c_{1,1}, c_{1,2}) = \sqrt{c_{1,1}} + \sqrt{c_{1,2}}$. Agent 2 's preference is represented by the utility function $u(c_{2,1}, c_{2,2}) = 0$. Both agents have an unbounded ability to eat any non-negative amount of either good.

a) Sketch the utility possibilities frontier for this economy.

b) Set up the two "Arrow Problems" for this economy,

c) Are all Pareto efficient allocations a solution to each Arrow problem?

d) Are all solutions to each Arrow problem Pareto efficient? If so, prove why so. If not, argue why not.

e) Set up the class of "Negishi Problems" for this economy.

f) Are all Pareto efficient allocations a solution to a Negishi problem? (If so, which ones.)

g) Are all solutions to a Negishi problem Pareto efficient?

Question 2 [Piccione & Rubinstein, 2006]

Some results on jungle equilibrium.

a) Define setting.

b) Define feasible allocation and jungle equilibrium.

c) Show that a jungle equilibrium exists.

d) Let $a = (a_1, \ldots, a_n)$ and $b = (b_1, \ldots, b_n)$ be strictly positive vectors, and suppose that $a \cdot x > 0$ and $b \cdot x < 0$, for some vector $x = (x_1, \ldots, x_n)$. Show that there exists a vector $y = (y_1, \ldots, y_n)$ such that:

   - $y_k > 0$ for some k for which $x_k > 0$
   - $y_l < 0$ for some l for which $x_l < 0$
   - $y_h = 0$ for $h \neq kh \neq l$
   - $a \cdot y > 0$ and $b \cdot y < 0$

e) Show that if a jungle is smooth then $\hat{z}$ is the unique jungle equilibrium.

f) Show that the allocation $\hat{z}$ is efficient.

g) Jungle equilibrium vs. competitive equilibrium: trading houses (with and without gold).

h) Can a jungle allocation be supported by a vector of prices in competitive equilibrium?

i) Suppose that the jungle is smooth. Show that in the exchange economy in which $w_i = \hat{z}_i, i \in \{1, \ldots, N\}$, there exists a sequence of price vectors p_n such that, for every agent i, the sequence of demands of agent i given p_n converges to z_i.

j) Think about jungle equilibrium with production.

Question 3 [Homework 1]

Consider the following pure exchange economies with two agents (both of the agent consume nonnegative amount of goods):

1. 1 good world with total endowment $e = 4$. Person 1's utility function is $u_1(c_1) = c_1$, Person 2's utility function is $u_2(c_2) = [c_2]$.
2. 2 goods world with total endowment $e_1 = e_2 = 4$. Each person has the utility function $u_i(c_{i1}, c_{i2}) = c_{i1}^2 + c_{i2}^2$
3. 2 goods world with total endowment $e_1 = e_2 = 4$. Each person has the utility function $u_i(c_{i1}, c_{i2}) = \sqrt{c_{i1}} + \sqrt{c_{i2}}$. Agent 1 can eat any non-negative amount of both goods. Agent 1 however, cannot eat more than 1 unit of each good.

Answer following questions:

a) What is the utility possible set / frontier?

b) Show whether every solution to the Pareto Problem is Pareto efficient.

c) Show whether every Pareto efficient allocation is a solution to the Pareto problem.

d) Show whether every solution to the Negishi Problem is Pareto efficient.

e) Show whether every Pareto efficient allocation is a solution to the Negishi problem for some specification of Pareto weights λ_i

Question 4 [Prelim QII Fall 2020]

Consider the following pure exchange economies with two agents $i \in \{1, 2\}$ and two goods $m \in \{1, 2\}$. There exists a total of 25 units of each good in the economy. Agent 1 has preferences represented by utility $u_1(c_{11}, c_{12}) = \sqrt{c_{11}} + \sqrt{c_{12}}$. Agent 2 has preferences represented by utility $u_2(c_{21}, c_{22}) = \sqrt{\min\{c_{21}, 16\}} + \sqrt{\min\{c_{22}, 16\}}$. Both agents have an unbounded ability to eat any non-negative amount of wither good.

a) Carefully characterize the set of Pareto Efficient allocations for this economy and sketch utility possibility frontier for this economy

b) Set up Social Planner's (or Negishi Problem) for this economy

c) Are all Pareto Efficient allocations solutions to the Social Planner's problem? Are akk solutions to the Social Planner's problem Pareto Efficient? Explain

d) Define a Competetive Equilibrium and give the set of Competetive Equilibtria for this economy for all possible endowment specifications subject to the aggregate endowment beign 25 for each goood/ are they all Pareto Efficient? Are any Pareto Efficient. If not why not?

e) Can all Pareto Efficient Allocations in this environment be supported as a Competetive Equilibrium for some set of endowments (again where the aggregate endownment is 25 for each good)? If not, what assumption of the 2nd Welfare Theorem is violated?