



Previous finals

Question 1 [Final 2017]

Suppose in an I agent, 2 good world, prices are normalized such that $p_1 + p_2 = 1$, where both p_1 and p_2 are non-negative, and excess demand for good 1 $x_1(p_1) : (0, 1) \rightarrow \mathbb{R}$ is continuous. Let $P_1(p_1) \subset [0, 1]$ specify Debreu's correspondence for proving existence.

- What is the set of $P_1(0)$? What is the set of $P_1(1)$?
- What is the set $P_1(p_1)$ if $x_1(p_1) > 0$? What is the set $P_1(p_1)$ if $x_1(p_1) < 0$?
- What is the set $P_1(p_1)$ if $x_1(p_1) = 0$?
- Graph the correspondence assuming $x_1(p_1) > 0$ for some $p_1 \in (0, 1)$, $x_1(p_1) < 0$ for some $p_1 \in (0, 1)$ and $x_1(\cdot)$ is decreasing. Is this enough for the existence of a fixed point?

Question 2 [Final 2017]

Consider a 2 good, 2 agent world. Good one, x denotes oranges. Good two, y , denotes orange juice. Agent $i = 1$ has utility function $u_1(x, y)$ and agent $i = 2$ has a utility function $u_2(x, y)$. Suppose each agent is endowed with 2 orange and no orange juice. Further assume there exist two identical firms which can turn oranges into orange juice according to the production function $f(x) = x$.

- Define a competitive equilibrium.
- Assuming u_1 and u_2 satisfy the usual properties (strictly increasing in both arguments, concave, differentiable), derive a set of necessary equations for equilibrium (do not try to solve the system).
- Discuss which objects will be determined in equilibrium and which will not.

Question 3 [Final 2017]

Consider a complete markets economy with I agents, $T + 1$ dates (from $t = 0$ to $t = T$), where at each date, a publicly observable random variable $s \in S$ is realized. Each agent i 's endowment of the single consumption good at date t depends only on the realization of s at date t . If $c_{it}(s^t)$ denotes agent's i consumption at date t after history $s^t = (s_0, \dots, s_t)$, his preferences are represented by $\sum_{t=1}^T \beta^t \sum_{s^t} \pi(s^t) u_i(c_{it}(s^t))$

- Define a feasible allocation.
- Sketch out what is necessary for the FWT to hold.
- Assuming the FWT holds and that the utility possibilities set is strictly convex, show that in any equilibrium if two agents have the same preferences, if agent i consumes more than agent j for any date t and history s^t then agent i consumes more than agent j at every date t and history s^t .

Question 4 [Final 2018]

Consider an I agent, M good world, with prices are normalized such that $\sum_m p_m = 1$ (with all p_m non-negative). Debreu's existence proof showed, under certain conditions, that a particular correspondence, $P(p)$, mapping $p \in \Delta^{M-1}$ to subsets of Δ^{M-1} was guaranteed to have a fixed point p^* such that $p^* \in P(p^*)$ and that $x_m(p^*) = 0$ for all goods m , where $x_m(p^*)$ is the excess demand for good m .

- Suppose $p_m > 0$ for all m except good 1, with $p_1 = 0$. What is $P(p)$?
- Suppose $p_m > 0$ for all m and that $x_1(p) \neq x_m(p)$ for all $m \neq 1$. What is $P(p)$?
- Debreu assumes preferences are such that if one considers a sequence of price vectors such that the price of one good goes to zero (say good 1), while the prices of all other goods stay positive, $x_1(p)$ goes to infinity (while $x_m(p)$ does not for all $m > 1$). What role does this assumption play in the proof?

Question 5 [Final 2018]

Consider a 3 good, I agent, 4 firm world. Firm 1 can turn good 1 into good 2 such that if it destroys x units of good 1, it produces x units of good 2. Firm 2 can turn good 1 into good 2 such that if it destroys x units of good 1, it produces $2x$ units of good 2. Firms 3 and 4 can turn good 1 into good 3 such that if they destroy x units of good 1, they produce \sqrt{x} units of good 3. Each agent is endowed with 1 unit of good 1, no units of good 2 or 3, and owns an equal share of each firm.

- a) Carefully define a Competitive Equilibrium.
- b) Discuss which objects will be determined in equilibrium and which won't.

Question 6 [Final 2018]

Consider a complete markets economy with 2 agents $i \in \{1, 2\}$, a single consumption good, and $T + 1$ dates ($t = 0$ to $t = T$). Agent 1's preferences are represented by $\sum_{t=0}^T \beta^t \sum_{s^t} \pi_t(s^t) c_{1,t}(s^t)$. Agent 2's preferences are represented by $\sum_{t=0}^T \delta^t \sum_{s^t} \pi_t(s^t) \log(c_{2,t}(s^t))$, where $\delta < \beta < 1$ and $c_{i,t}(s^t)$ represents agent i 's consumption at date t after history s^t .

- a) Suppose agent 1's endowment $e_{1,t} = 2$ for all t , while agent 2's endowment $e_{2,t} = 0$ with probability .5 and $e_{2,t} = 4$ with probability .5 at each date, i.i.d. Assume endowments are observable and there is no ability to store the good over time.
 - a) Define a feasible allocation in terms of transfers.
 - b) Characterize the set of efficient allocations.
- b) Now suppose agent 2's endowment $e_{2,t} = 2$ for all t , while agent 1's endowment $e_{1,t} = 0$ with probability .5 and $e_{1,t} = 4$ with probability .5 at each date, i.i.d. (and still assume endowments are observable and there is no ability to store the good over time.).
 - a) Define a feasible allocation in terms of transfers.
 - b) Characterize the set of efficient allocations.
- c) Now again suppose agent 2's endowment $e_{2,t} = 2$ for all t , while agent 1's endowment $e_{1,t} = 0$ with probability .5 and $e_{1,t} = 4$ with probability .5 at each date, i.i.d., and still assume there is no ability to store the good over time, but that agent 1's endowment realization is private.
 - a) Define an incentive compatible allocation in terms of transfers.
 - b) Characterize the set of efficient allocations. (hint: what does incentive compatibility at period T imply? What then does incentive compatibility in period $T - 1$ imply?)

Question 7 [Final 2019]

Suppose (c, y, p) is a Competitive Equilibrium with production given initial endowments $\left\{ \{e_{i,m}\}_{m=1}^M \right\}_{i=1}^I$ and ownership of firms $\left\{ \{\theta_{i,h}\}_{h=1}^H \right\}_{i=1}^I$. Further suppose for all \hat{c} s.t. $\hat{c}_i \succeq_i c_i$ for all i and $\hat{c}_j \succ_j c_j$ for at least one j . $\sum_m p_m \hat{c}_{i,m} \geq \sum_m p_m e_{i,m} + \sum_h \theta_{i,h} \sum_m p_m y_{h,m}$ for all i . Define a Competitive Equilibrium and a Pareto Efficient Allocation with production. Show that (c, y) must be a Pareto efficient allocation.

Question 8 [Final 2019]

(25 points) Consider a 2 good, 2 agent world, Good one, x , denotes oranges and good two, y denotes orange juice. Each agent has utility function $u(x, y) = \ln(x) + \ln(y)$. Suppose each agent is endowed with 1 orange and no orange juice. Further assume there exist two identical firms which can turn oranges into orange juice according to the production function $f(x) = x$

- a) Draw the production sets for each firm.
- b) Define and find ALL competitive equilibrium over all possible specifications of firm ownership.

Question 9 [Final 2019]

Consider an economy with a finite I number of agents, each of whom receives an endowment of 1 with probability $\frac{1}{2}$ and an endowment of 2 with probability $\frac{1}{2}$, independent of the realizations of the other agents. (That is, each flips his own coin.) Let the state of the world m specify the endowment realization of each agent and let $c_{i,m}$ and $e_{i,m}$ denote the consumption and endowment of agent i in state of the world m , respectively, with $c_i = (c_{i,1}, \dots, c_{i,M})$ and $e_i = (e_{i,1}, \dots, e_{i,M})$. Suppose each person i ranks consumption vectors c_i such that $c_i \succeq_i \hat{c}_i$ if and only if $\sum_m \pi_m \frac{c_{i,m}^{1-\sigma}}{1-\sigma} \geq \sum_m \pi_m \frac{\hat{c}_{i,m}^{1-\sigma}}{1-\sigma}$, where π_m denotes the probability of the state of the world m .

- a) Define an allocation, being careful about the number of states of the world. When is such an allocation feasible?
- b) Characterize the set of Pareto efficient allocations. In particular, prove that each agent's consumption is a constant fraction of the aggregate endowment in any Pareto efficient allocation.
- c) Finally assume every agent's endowment of potatoes is private to him. Under what conditions (if any) is a Pareto efficient allocation assuming full information (or full observability) incentive compatible if endowments are, in fact, not answer). observable? (Justify your