ECON 8104

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Contents

1 Herding **2**

1.1 Bikchandani, Hirschleifer and Welch(1992) 3

1.2 Smith, Sorensen, Tian (2021) 4

*These notes are intended to summarize the main concepts, definitions and results covered in the first year of micro sequence for the Economics PhD of the University of Minnesota. The material is not my own. Please let me know of any errors that persist in the document. E-mail: pawel042@umn.edu .

1 Herding

We will look into learning by observing the past decisions of others can help explain some otherwise puzzling phenomena about human behavior. For example, why do people tend to converge on similar behavior, in what is known as 'herding'? Why is mass behavior prone to error and fads? Many of us identify restaurant quality by the fraction of seats occupied; perhaps not coincidentally, restaurants often close off back-room peak-load seating capacity until the main and most visible section becomes quite full. Advertisements report the fractions of doctors or dentists that use certain medications and health products.

In the standard herding model, privately informed individuals sequentially see prior actions and then act. An identical action herd eventually starts and public beliefs tend to "cascade sets" where social learning stops. What behaviour is socially efficient when actions ignore informational externalities? To see possible answers we go through following papers:

- Bikchandani, Hirschleifer and Welch(1992) JPE -examples, simple model observable actions and observable signals
- Banerjee (1992) - proposed a remedy for the social learning externality: conceal early actions
- Smith, Sorensen, Tian (2021) forthcoming REStud - main results shrinking cascade sets, contrarianism (the planner skews action choices at the margin)
- Smith, Sorensen (2000) Econometrica

The simplest and most basic cause of convergent behavior is that individuals face similar decision problems, by which we mean that people have similar information, face similar action alternatives, and face similar payoffs. As a result, they make similar choices.
Herding may arise when payoffs are similar even if initial information is not. In this case people communicate with each other or observe the actions of others or the consequences of these actions. The key issue is how individuals determine which alternative is better. Each individual could decide by direct analysis of the alternatives. However, this can be costly and time-consuming, so a plausible alternative is to rely on the information of others. Such influence may take the form of direct communication and discussion with, or observation of others. We will call influence resulting from

rational processing of information gained by observing others **observational learning or social learning**. We focuses mainly on the case where individuals learn by observing the actions of others.

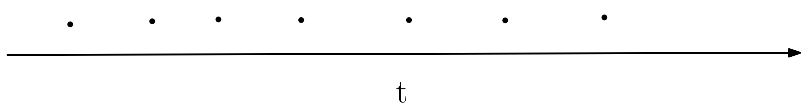
There are several other possible causes of conformity which do not require great similarity in individuals' decision problems. These include positive payoff externalities, which lead to conventions such as driving on the right-hand side of the road; preference interactions, as with everyone desiring to wear the more fashion- able' clothing as determined by what others are wearing; and sanctions upon deviants, as with a dictator punishing opposition behavior. Let's formalize our reasonings.

1.1 Bikchandani, Hirschleifer and Welch(1992)

Observable Actions versus Observable Signals

- N individuals starts with some private information,
- obtains some information from predecessors (ordered by time t),
- and then decides on particular action

In the observable actions scenario, individuals can observe the actions but not the signals of their predecessors. We compare this to a benchmark observable signals scenario in which individuals can observe both the actions and signals of predecessors.



$$\Omega = \{L, H\} \quad \text{prior} = (\frac{1}{2}, \frac{1}{2})$$

Conditionally IID signals$\in \{\underline{\sigma}, \bar{\sigma}\}$

$$q = P(\underline{\sigma}|L) > \frac{1}{2} > P(\bar{\sigma}|L)$$

$$q = P(\bar{\sigma}|H) > \frac{1}{2} > P(\underline{\sigma}|H)$$

$$\lambda = \frac{q}{1-q} > 1$$

what is probability of Low given low signal?

$$P(L|\underline{\sigma}) = P(\underline{\sigma}|L) \cdot \frac{P(L)}{P(\underline{\sigma})} = P(\underline{\sigma}|L) \cdot \frac{P(L)}{P(\underline{\sigma}|L)P(L) + P(\underline{\sigma}|H)P(H)}$$

$$= q \cdot \frac{\frac{1}{2}}{q\frac{1}{2} + (1-q)\frac{1}{2}} = \frac{1}{1 + \frac{1-q}{q}} =$$

$$= \frac{1}{1+\lambda} \geq \frac{1}{2}$$

So choose $l \iff$ observe $\underline{\sigma}$

1st player plays informatively.

Let's introduce second agent observing second player

$$P(L|\sigma_1, \sigma_2) \equiv P_1(L|\sigma_2)$$

$$P(L|\sigma_1, \sigma_2) = \frac{P(\sigma_2|L, \sigma_1)P(L|\sigma_1)}{P(\sigma_2|\sigma_1)}$$

$$P_1(L|\sigma_2) = \frac{P_1(\sigma_2|L)P_1(L)}{P_1(\sigma_2}$$

Suppose we observe opposite signals: $\sigma_1 = \underline{\sigma}$ and $\sigma_2 = \bar{\sigma}$

$$P_1(L|\bar{\sigma}) = \frac{P_1(\bar{\sigma}|L)P_1(L)}{P_1(\bar{\sigma})} = \frac{P(\bar{\sigma}|L)P(L|\underline{\sigma})}{P_1(\bar{\sigma}|L)P_1(L) + P_1(\bar{\sigma}|H)P_1(H)} =$$

$$= \frac{q\frac{1-q}{1+\frac{1-q}{q}}}{\frac{1-q}{1+\frac{1-q}{q}} + q(1 - \frac{1-q}{1+\frac{1-q}{q}})} = \frac{q(1-q)}{q(1-q) + q - q^2} = \frac{1}{2}$$

Let's add third player

$$a_1, a_2, \sigma_3 = l, l, \bar{\sigma}$$

1.2 Smith, Sorensen, Tian (2021)

Main results:

- shrinking cascade sets
- contrarianism

Assume that economic theory research fashion is captured by one of two unobserved states, either low-brow theory L or high-brow (intellectual) theory H. A Professor and a Student share a prior belief π on state H.

$$\Omega = \{L, H\} \qquad \pi = P(H)$$

Respectively, they observe conditionally independent draws σ_P, σ_S of a private signal, with cdf's $F^H(\sigma)$ and $F^L(\sigma)$ and densities $f^H(\sigma) = 2\sigma$ and $f^L(\sigma) = 1$, as in Figure 1 .

Two players : S P

Conditionally IID signals

CDFs: $\sigma \in [0, 1]$

$$F_H(\sigma) = \sigma^2. \qquad f_H(\sigma) = 2\sigma$$

$$F_L(\sigma) = \sigma. \qquad f_L(\sigma) = 1$$

$$\frac{f_H}{F_L}(\sigma) = 2\sigma$$

Since the signal likelihood ratio $f^H(\sigma)/f^L(\sigma) = 2\sigma$ in favor of state H increases, higher signals σ lead to higher posterior beliefs p in state H. Also, low $\sigma > 0$ are arbitrarily powerful indicators of state L, but all $\sigma < 1$ have bounded power for H.

After seeing his signal, the Professor either starts a low-brow paper l or high-brow paper h. His Student then learns from his paper choice, and makes his own paper selection. Research pays 1 if the paper and state match, and -1 otherwise (Figure 2). Actions $\{l, h\}$

$$u(\omega) = \begin{cases} 1, & \text{if } \omega = a \\ -1, & \text{otherwise} \end{cases}$$

If the Professor updates to the posterior belief q in state H, his expected payoff is $U(q) \equiv \max(2\pi - 1, 1 - 2\pi)$. We compare two extreme motivations for the Professor: he selfishly only cares about his own expected payoffs, or he is entirely motivated by his Student's expected payoffs.

$$q = P(H|\sigma)$$

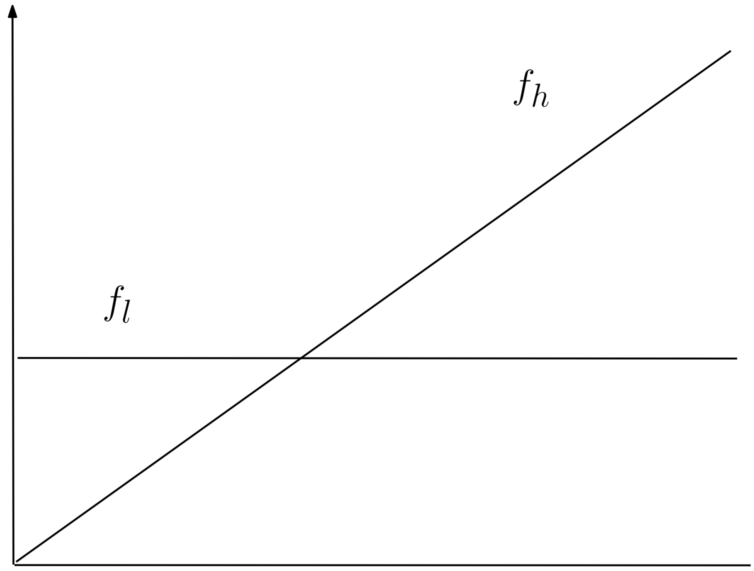
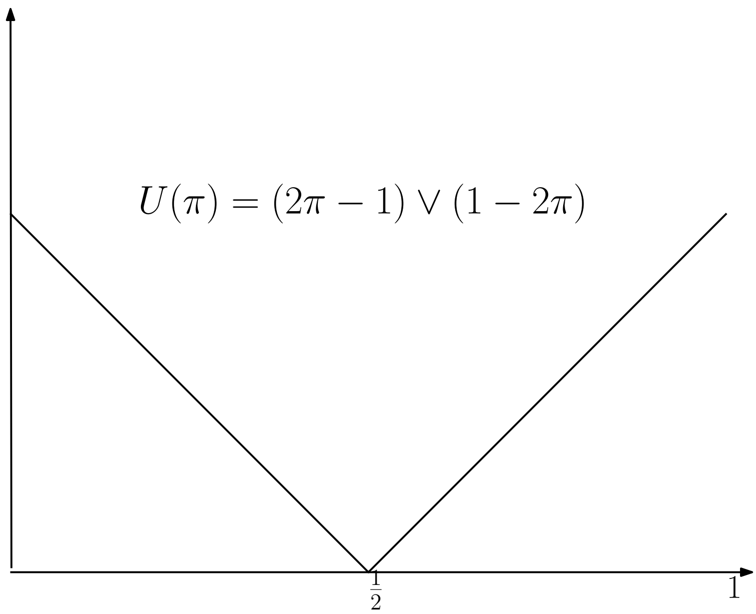


Figure 1: pdfs

Figure 2: payoffs



$$\mathbb{E}U_p = U(q)$$

Case 1: The Selfish Professor

Assume the Professor writes paper h when state H is most likely - i.e. for posterior beliefs $q \geq 1/2$. By Bayes rule, this happens when his posterior likelihood ratio of states H to L exceeds one, or $\left[f^H(\sigma)/f^L(\sigma)\right][\pi/(1-\pi)] \geq 1$

Lemma 1. *P writes h* $\iff$ $q \geq \frac{1}{2}$

$$q = P(H|\sigma) \geq \frac{1}{2} \iff \frac{f(\sigma|H)\pi}{F(\sigma|H)\pi + F(\sigma|L)(1-\pi)} \geq \frac{1}{2}$$

$$\frac{1}{1 + \frac{f_l(\sigma)(1-\pi)}{f_h(\sigma)\pi}} \geq \frac{1}{2}$$

$$\frac{f_h(\sigma)\pi}{f_l(\sigma)(1-\pi)} \geq 1$$

This happens for high private signals σ above a selfish threshold signal $\bar{\sigma}(\pi) \equiv (1-\pi)/(2\pi)$. For any prior belief $\pi < 1/3$ in state H, the threshold signal impossibly exceeds one $-$ in this case, the Professor always writes paper l. This event when the prior belief overwhelms all private signals is called a cascade.

This occurs when

$$\sigma \geq \bar{\sigma}(\pi): \quad \bar{\sigma}(\pi) = \frac{1-\pi}{2\pi}$$

$$\frac{f_h(\sigma)\pi}{f_l(\sigma)(1-\pi)} = 2\sigma\frac{\pi}{1-\pi} \geq 1 \quad \iff \quad \sigma \geq \frac{1-\pi}{2\pi}$$

$$\bar{\sigma}(\frac{1}{3}) = 1$$

$$\forall \quad \pi < \frac{1}{3} \quad \bar{\sigma}(\pi) > 1 \text{ which is unattainable since } \quad \sigma \in [0,1]$$

P writes l in this case $\forall\sigma$.

Here, the Professor's (prior expected) value - or highest expected payoff - is $V(\pi) = U(\pi)$ when $\pi < 1/3$. P's value:

$$V(\pi) = U(\pi) \quad \pi \leq \frac{1}{3}$$

$$\text{if} \quad \pi > \frac{1}{3}$$

$$V(\pi) = \underbrace{\pi}_{H}\underbrace{[(1-F_H(\bar{\sigma}(\pi)))}_{\sigma>\bar{\sigma}}\underbrace{\cdot(+1)}_{h} + \underbrace{F_H(\bar{\sigma}(\pi))}_{\sigma<\bar{\sigma}}\underbrace{\cdot(-1)}_{l}] + (1-\pi)[(1-F_L(\bar{\sigma}(\pi)))\cdot(-1) + F_L(\bar{\sigma}(\pi))\cdot(+1)]$$

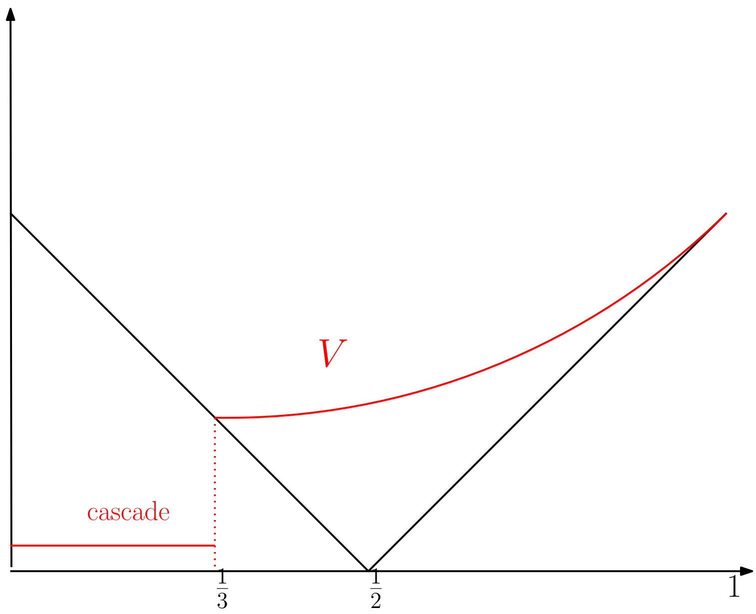
$$= \pi[1-2F_H] + (1-\pi)[2F_L - 1] = \pi[1 - 2(\frac{1-\pi}{2\pi})^2] + (1-\pi)[2\frac{1-\pi}{2\pi} - 1] = \cdots =$$

$$= \frac{1}{2}[5\pi - 4 + \frac{1}{\pi}]$$

which is strictly convex on $(\frac{1}{3}, 1)$

S updates beliefs to

$$p = p_l(\pi) \quad \text{or} \quad p_h(\pi)$$



If P sometimes writes l and. h

$$p_l(\pi) < \pi < p_h(\pi)$$

S writes l iff for S's signal τ

$$\tau < \bar{\sigma}(p) = \frac{1-p}{2p}$$

$$\text{S's payoff} \quad = V(p)$$

IF $\pi \leq \frac{1}{3}$ then

$$p = p_l(\pi) \leq \pi$$

P writes l $\Rightarrow$ S writes l too. S copies P!

Case 2: The Altruistic Professor

Consider next that the Professor chooses his paper genre to maximize his Student's expected value. Since $\bar{\sigma}(1/3) = 1$, with a prior belief at or just below 1/3, the selfish Professor always chooses the low brow paper, which sends the student a useless signal.

Suppose P chooses l or h to maximize S's utility

$$\bar{\sigma}(\frac{1}{3}) = 1 \quad \Rightarrow \quad \pi \leq \frac{1}{3}$$

it would lead P to choose l, useless for S.

To help the Student, by informing him of high signals, the altruistic Professor therefore leans against the prevailing prior belief, by choosing a lower altruistic threshold signal $\hat{\sigma} < \bar{\sigma}(\pi)$. In other words, he writes the high brow paper more often, yielding respectively lower continuation beliefs:

To help S, P chooses

$$\hat{\sigma} < \bar{\sigma}(\pi)$$

P writes h more often

$$\hat{p}_l(\pi, \hat{\sigma}) < p_l(\pi|\bar{\sigma}(\pi))$$

$$\hat{p}_h(\pi, \hat{\sigma}) < p_h(\pi|\bar{\sigma}(\pi))$$

$$\hat{p}_l(\pi, \hat{\sigma}) = \frac{P(l|H)P(H)}{P(l)} = \frac{\pi\hat{\sigma}^2}{\pi\hat{\sigma}^2 + (1-\pi)\hat{\sigma}} < \pi < \hat{p}_h(\pi, \hat{\sigma}) = \frac{\pi(1-\hat{\sigma}^2)}{\pi(1-\hat{\sigma}^2) + (1-\pi)(1-\hat{\sigma})}$$

P chooses $\hat{\sigma}$ to max S's value.

If $\hat{p}_l \geq \frac{1}{3}$

$$\mathbb{E}V(p) = \mathbb{E}[\frac{1}{2}(5p - 4 + \frac{1}{p})] = 5\pi - \frac{4}{2} + \frac{1}{2}\mathbb{E}[\frac{1}{p}]$$

$$\mathbb{E}[\frac{1}{p}] = \frac{(\pi\hat{\sigma}^2 + (1-\pi)\hat{\sigma})^2}{\pi\hat{\sigma}^2} + \frac{(\pi(1-\hat{\sigma}^2) + (1-\pi)(1-\hat{\sigma}))^2}{\pi(1-\hat{\sigma}^2)}$$

$$= \cdots = \frac{1}{\pi} + \frac{(1-\pi)^2}{\pi}\frac{1-\hat{\sigma}}{1+\hat{\sigma}}$$

and this function is decreasing in $\hat{\sigma}$ and it is maximized at $\hat{\sigma} = 0$

$$\hat{p}_l(\pi) < \frac{1}{3} \quad \forall \pi > \frac{1}{3}$$

$$\hat{p}_l(\pi) > \frac{1}{3} \quad \hat{\sigma} > 0 \quad \text{low enough}$$

then

$$\hat{p}_l(\pi) = \frac{1}{1 + \frac{1-\pi}{\pi\hat{\sigma}}} < \frac{1}{3}$$

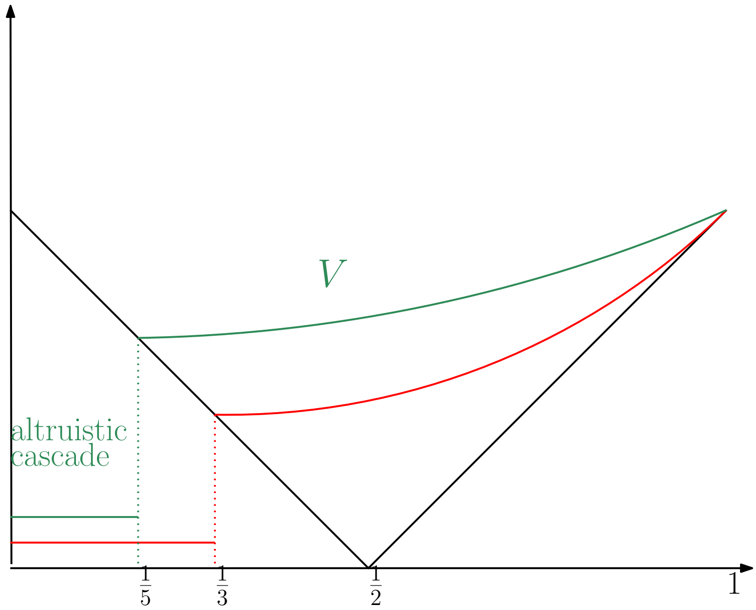
Unless $p_h(\pi) \geq \frac{1}{3}$, S is in a cascade so gets U

$$V(p) = \begin{cases} 1 - 2p, & \text{on } [0, \frac{1}{3}] \\ \\ 5p - 4 + \frac{1}{p}, & \text{on } [\frac{1}{3}, 1] \end{cases}$$

Optimal wrt to $\hat{\sigma}$

3)

Players $n = 1, 2 \ldots$



Prior $= (\frac{1}{2}, \frac{1}{2})$, $\omega \in \{L, H\}$,

$a \in \{1, 2, \ldots, A\}$

$$u_\omega(a) \quad \text{s.t.} u_L(1) > u_L(a) \forall a \neq 1, \quad u_H(A) > u_H(1) \quad \forall a \neq A$$

$$u_H(1) - u_L(1) < u_H(2) - u_L(2) < \cdots < u_H(A) - u_L(A)$$

$$\bar{u}(a, r) = (1 - r)u_L(a) + r u_H(a)$$

σ_n n-th private signal IID with

$$\sigma_n = P(H|\sigma_n) \sim F^w$$

F_L and F_H are mutually absolute ....

$$\sigma = P(H|\sigma) \Rightarrow \quad \frac{dF_H}{dF_L} = \frac{\sigma}{1 - \sigma} \quad \Rightarrow \quad F_H(\sigma) \leq F_L(\sigma)$$

$$\pi_n = P(H|a_1, \ldots a_n)$$

$$r = R(\pi, \sigma) = \frac{\pi\sigma}{\pi\sigma + (1 - \pi)(1 - \sigma)}$$

$$a = \psi(\sigma)$$

$$s_n : (a_1, \ldots a_n) \to \psi$$

$$s = (s_1, \dots s_n) \in S$$

$$V_\delta(\pi) = \sup_{s \in S} \mathbb{E}[(1-\delta) \sum_{n \geq 1} \delta^{n-1} u_w(s_n)]$$

BHW: $\delta = 0$

Lemma V_S is bounded, continuous and convex in public beleifs π with extremem slopes:

$$V_\delta'(0+) \geq u_H(1) - u_L(1)$$

$$V_\delta'(1-) \leq u_H(A) - u_L(A)$$

Gittins(1979- old literature on indexes

Theorem 1. ***Optimal Behaviour For any public belief π, an agent with posterior belief r takes the action a with maximal welfare index***

$$w(a, \pi, r) = (1-\delta)\bar{u}(a, r) + \delta\tau(p(a, \pi, \xi), r)$$

for some supporting subtangent $\tau(p, r)$ to v at public belief p, when evaluated at posterior r.

Recall that the optimal strategy in the selfish herding model of SS was a simple interval rule: Choose action a if one's posterior belief r lies in an interval I_a, where $\{I_a\}$partitions [0,1] .

Actions with empty intervals are not taken. The rule may randomize at the threshold (boundary) θ_a between adjacent intervals I_a and I_{a+1}.

Since the welfare index $w(a, \pi, r)$ is affine in r, interval rules remain socially optimal.

Theorem 2. ***Interval Rules. An interval rule $\{I_a\}$ is optimal at any public belief π.***