Recitations 10

From the time of Adam Smith's Wealth of Nations in 1776, one recurrent theme of economic analysis has been the remarkable degree of coherence among the vast numbers of individual and seemingly separate decisions about the buying and selling of commodities. In everyday, normal experience, there is something of a balance between the amounts of goods and services that some individuals want to supply and the amounts that other, differerent individuals want to sell [sic]. Would-be buyers ordinarily count correctly on being able to carry out their intentions, and would-be sellers do not ordinarily find themselves producing great amounts of goods that they cannot sell. This experience of balance is indeed so widespread that it raises no intellectual disquiet among laymen; they take it so much for granted that they are not disposed to understand the mechanism by which it occurs.

Kenneth Arrow (1973)

[Definitions used today]

- Existence of Equilibrium, Excess Aggregate Demand, Boundary Condition, Welfare Theorems,

Question 1 [Very Very Easy Existence Theorem]

Let $Z : \bar{\Delta} \to \mathbb{R}^l$ is a differentiable function that satisfies Walras' law ($\forall p \in \bar{\Delta} : p \cdot Z(p) = 0$), z is homogenous of degree 1 in p, $\det Dz(p) \neq 0$, the boundary condition: $p_n \to p \in \partial\Delta \Rightarrow \|Z(p_n)\| \to \infty$ and $\sum_{i=1}^{n-1} p_i z_i(p)$ is nondecreasing function of p_n. Then $\exists p^* \in \bar{\Delta}$ such that $Z(p^*) \leq 0$. Further, $Z(p^*) = 0$ only if $p^* \in \Delta$.

Question 2 [Very Easy Existence Theorem]

Let $Z : \bar{\Delta} \to \mathbb{R}^l$ is a continuous function that satisfies Walras' law ($\forall p \in \bar{\Delta} : p \cdot Z(p) = 0$), then $\exists p^* \in \bar{\Delta}$ such that $Z(p^*) \leq 0$. Further, $Z(p^*) = 0$ only if $p^* \in \Delta$.

Question 3 [Easy Existence Theorem]

Let $Z : \Delta \to \mathbb{R}^l$ be a continuous function that is bouded from below, satisfying Walras' Law and the boundary condition: $p_n \to p \in \partial\Delta \Rightarrow \|Z(p_n)\| \to \infty$. Then $\exists p^* \in \Delta$ such that $Z(p^*) = 0$.

Question 4 [Final 2017] Suppose in an I agent,2 good world, prices are normalized such that $p_1 + p_2 = 1$ (with p_1 and p_2 non-negative) and excess demand for good 1,$x_i(pi) : (0, 1) \to \mathbb{R}$ is continuous.Let$P_1(p_1) \subset [0, 1]$ specify Debreu's correspondence for proving existence.

a) What is the set $P_1(0)$? What is the set $P_1(l)$?

b) What is the set $P_1(p_i)$ if $x_1(p_1) > 0$? What is the set $P_1(p_1)$ if $x_1(p_1) < 0$?

c) What is the set $P_1(p_1)$ if $x_1(p_1) = 0$?

d) Graph the correspondence assuming $x_1(p_1) > 0$ for some $p_1 \in (0, 1)$, $x_1(p_1) < 0$ for some $p_1 \in (0, 1)$ and is decreasing.Is this enough for the existence of a fixed point