



Recitations 9

[Definitions used today]

- Arrow problem, Negishi problem, Social Planner Problem, Pareto efficient allocation, Competitive Equilibrium, Junge Economy

Question 1 [Midterm 2018]

Consider a pure exchange economy with two agents, $i \in \{1, 2\}$ and two goods $m \in \{1, 2\}$. There exists a total of 1 unit of each good. Agent 1's preference is represented by utility function $u_1(c_{1,1}, c_{1,2}) = \sqrt{c_{1,1}} + \sqrt{c_{1,2}}$. Agent 2's preference is represented by the utility function $u_2(c_{2,1}, c_{2,2}) = 0$. Both agents have an unbounded ability to eat any non-negative amount of either good.

- Sketch the utility possibilities frontier for this economy.
- Set up the two "Arrow Problems" for this economy.
- Are all Pareto efficient allocations a solution to each Arrow problem?
- Are all solutions to each Arrow problem Pareto efficient? If so, prove why so. If not, argue why not.
- Set up the class of "Negishi Problems" for this economy.
- Are all Pareto efficient allocations a solution to a Negishi problem? (If so, which ones.)
- Are all solutions to a Negishi problem Pareto efficient?

Question 2 [Piccione & Rubinstein, 2006]

Some results on jungle equilibrium.

- Define setting.
- Define feasible allocation and jungle equilibrium.
- Show that a jungle equilibrium exists.
- Let $a = (a_1, \dots, a_n)$ and $b = (b_1, \dots, b_n)$ be strictly positive vectors, and suppose that $a \cdot x > 0$ and $b \cdot x < 0$, for some vector $x = (x_1, \dots, x_n)$. Show that there exists a vector $y = (y_1, \dots, y_n)$ such that:
 - $y_k > 0$ for some k for which $x_k > 0$
 - $y_l < 0$ for some l for which $x_l < 0$
 - $y_h = 0$ for $h \neq k, h \neq l$
 - $a \cdot y > 0$ and $b \cdot y < 0$
- Show that if a jungle is smooth then \hat{z} is the unique jungle equilibrium.
- Show that the allocation \hat{z} is efficient.
- Jungle equilibrium vs. competitive equilibrium: trading houses (with and without gold).
- Can a jungle allocation be supported by a vector of prices in competitive equilibrium?
- Suppose that the jungle is smooth. Show that in the exchange economy in which $w_i = \hat{z}_i, i \in \{1, \dots, N\}$, there exists a sequence of price vectors p_n such that, for every agent i , the sequence of demands of agent i given p_n converges to z_i .
- Think about jungle equilibrium with production.

Question 3 [Homework 1]

Consider the following pure exchange economies with two agents (both of the agent consume nonnegative amount of goods):

1. 1 good world with total endowment $e = 4$. Person 1's utility function is $u_1(c_1) = c_1$, Person 2's utility function is $u_2(c_2) = [c_2]$.
2. 2 goods world with total endowment $e_1 = e_2 = 4$. Each person has the utility function $u_i(c_{i1}, c_{i2}) = c_{i1}^2 + c_{i2}^2$
3. 2 goods world with total endowment $e_1 = e_2 = 4$. Each person has the utility function $u_i(c_{i1}, c_{i2}) = \sqrt{c_{i1}} + \sqrt{c_{i2}}$. Agent 1 can eat any non-negative amount of both goods. Agent 1 however, cannot eat more than 1 unit of each good.

Answer following questions:

- a) What is the utility possible set / frontier?
- b) Show whether every solution to the Pareto Problem is Pareto efficient.
- c) Show whether every Pareto efficient allocation is a solution to the Pareto problem.
- d) Show whether every solution to the Negishi Problem is Pareto efficient.
- e) Show whether every Pareto efficient allocation is a solution to the Negishi problem for some specification of Pareto weights λ_i

Question 4 [Prelim QII Fall 2020]

Consider the following pure exchange economies with two agents $i \in \{1, 2\}$ and two goods $m \in \{1, 2\}$. There exists a total of 25 units of each good in the economy. Agent 1 has preferences represented by utility $u_1(c_{11}, c_{12}) = \sqrt{c_{11}} + \sqrt{c_{12}}$. Agent 2 has preferences represented by utility $u_2(c_{21}, c_{22}) = \sqrt{\min\{c_{21}, 16\}} + \sqrt{\min\{c_{22}, 16\}}$. Both agents have an unbounded ability to eat any non-negative amount of either good.

- a) Carefully characterize the set of Pareto Efficient allocations for this economy and sketch utility possibility frontier for this economy
- b) Set up Social Planner's (or Negishi Problem) for this economy
- c) Are all Pareto Efficient allocations solutions to the Social Planner's problem? Are all solutions to the Social Planner's problem Pareto Efficient? Explain
- d) Define a Competitive Equilibrium and give the set of Competitive Equilibria for this economy for all possible endowment specifications subject to the aggregate endowment being 25 for each good/ are they all Pareto Efficient? Are any Pareto Efficient. If not why not?
- e) Can all Pareto Efficient Allocations in this environment be supported as a Competitive Equilibrium for some set of endowments (again where the aggregate endowment is 25 for each good)? If not, what assumption of the 2nd Welfare Theorem is violated?