Previous finals

Question 1 [Final 2017]

Suppose in an I agent, 2 good world, prices are normalized such that $p_1+p_2=1$, where both p_1 and p_2 are non-negative, and excess demand for good $1 x_1(p_1):(0,1) \rightarrow \mathbb{R}$ is continuous. Let $P_1(p_1) \subset [0,1]$ specify Debreu's correspondence for proving existence.

a) What is the set of $P_1(0)$? What is the set of $P_1(1)$?

b) What is the set $P_1(p_1)$ if $x_1(p_1)>0$? What is the set $P_1(p_1)$ if $x_1(p_1)<0$?

c) What is the set $P_1(p_1)$ if $x_1(p_1)=0$?

d) Graph the correspondence assuming $x_1(p_1)>0$ for some $p_1 \in (0,1), x_1(p_1)<0$ for some $p_1 \in (0,1)$ and $x_1(\cdot)$ is decreasing. Is this enough for the existence of a fixed point?

Question 2 [Final 2017]

Consider a 2 good, 2 agent world. Good one, x denotes oranges. Good two, y, denotes orange juice. Agent $i=1$ has utility function $u_1(x,y)$ and agent $i=1$ has a utility function $u_2(x,y)$. Suppose each agent is endowed with 2 orange and no orange juice. Further assume there exist two identical firms which can turn oranges into orange juice according to the production function $f(x)=x$.

a) Define a competitive equilibrium.

b) Assuming u_1 and u_2 satisfy the usual properties (strictly increasing in both arguments, concave, differentiable), derive a set of necessary equations for equilibrium (do not try to solve the system).

c) Discuss which object s will be determined in equilibrium and which will not.

Question 3 [Final 2017]

Consider a complete markets economy with I agents, $T+1$ dates (form $t=0$ to $t=T$), where at each date, a publicly observable random variable $s \in S$ is realized. Each agent i 's endowment of the single consumption good at date t depends only on the realization of s at date t. If $c_{it}(s^t)$ denotes agent's i consumption at date t after history $s^t=(s_0,\ldots,s_t)$, his preferences are represented by $\sum_{t=1}^{T} \beta^t \sum_{s^t} \pi(s^t) u_i(c_{it}(s^t))$

a) Define a feasible allocation.

b) Sketch out what is necessary for the FWT to hold.

c) Assuming the FWT holds and that the utility possibilities set is strictly convex, show that in any equilibrium if two agents have the same preferences, if agent i consumes more than agent j for any date t and history s^t then agent i consumes more than agent j at every date t and history s^t.

Question 4 [Final 2018]

Consider an I agent, M good world, with prices are normalized such that $\sum_m p_m=1$ (with all p_m non-negative). Debreu's existence proof showed, under certain conditions, that a particular correspondence, $P(p)$, mapping $p \in \Delta^{M-1}$ to subsets of Δ^{M-1} was guaranteed to have a fixed point p^* such that $p^* \in P(p^*)$ and that $x_m(p^*)=0$ for all goods m, where $x_m(p^*)$ is the excess demand for good m.

a) Suppose $p_m>0$ for all m except good 1, with $p_1=0$. What is $P(p)$?

b) Suppose $p_m>0$ for all m and that $x_1(p) \neq x_m(p)$ for all $m \neq 1$. What is $P(p)$?

c) Debreu assumes preferences are such that if one considers a sequence of price vectors such that the price of one good goes to zero (say good 1), while the prices of all other goods stay positive, $x_1(p)$ goes to infinity (while $x_m(p)$ does not for all $m>1$). What role does this assumption play in the proof?

Question 5 [Final 2018]

Consider a 3 good, I agent, 4 firm world. Firm 1 can turn good 1 into good 2 such that if it destroys x units of good 1, it produces x units of good 2. Firm 2 can turn good 1 into good 2 such that if it destroys x units of good 1, it produces $2x$ units of good 2. Firms 3 and 4 can turn good 1 into good 3 such that if they destroy x units of good 1, they produce $\sqrt{x}$ units of good 3. Each agent is endowed with 1 unit of good 1, no units of good 2 or 3, and owns an equal share of each firm.

a) Carefully define a Competitive Equilibrium.

b) Discuss which objects will be determined in equilibrium and which won't.

Question 6 [Final 2018]

Consider a complete markets economy with 2 agents $i \in \{1, 2\}$, a single consumption good, and $T + 1$ dates ($t = 0$ to $t = T$). Agent 1's preferences are represented by $\sum_{t=0}^{T} \beta^t \sum_{s^t} \pi_t (s^t) c_{1,t} (s^t)$. Agent 2 's preferences are represented by $\sum_{t=0}^{T} \delta^t \sum_{s^t} \pi_t (s^t) \log (c_{2,t} (s^t))$, where $\delta < \beta < 1$ and $c_{i,t} (s^t)$ represents agent $i's$ consumption at date t after history s^t.

a) Suppose agent 1's endowment $e_{1,t} = 2$ for all t, while agent 2's endowment $e_{2,t} = 0$ with probability .5 and $e_{2,t} = 4$ with probability .5 at each date, i.i.d. Assume endowments are observable and there is no ability to store the good over time.

    a) Define a feasible allocation in terms of transfers.
    b) Characterize the set of efficient allocations.

b) Now suppose agent 2's endowment $e_{2,t} = 2$ for all t, while agent 1's endowment $e_{1,t} = 0$ with probability .5 and $e_{1,t} = 4$ with probability .5 at each date, i.i.d. (and still assume endowments are observable and there is no ability to store the good over time.).

    a) Define a feasible allocation in terms of transfers.
    b) Characterize the set of efficient allocations.

c) Now again suppose agent 2's endowment $e_{2,t} = 2$ for all t, while agent 1's endowment $e_{1,t} = 0$ with probability .5 and $e_{1,t} = 4$ with probability .5 at each date, i.i.d., and still assume there is no ability to store the good over time, but that agent 1's endowment realization is private.

    a) Define an incentive compatible allocation in terms of transfers.
    b) Characterize the set of efficient allocations. (hint: what does incentive compatibility at period T imply? What then does incentive compatibility in period $T - 1$ imply?

Question 7 [Final 2019]

Suppose (c, y, p) is a Competitive Equilibrium with production given initial endowments $\left\{ \{e_{i,m}\}_{m=1}^{\lambda i} \right\}_{i=1}^{l}$ and ownership of firms $\left\{ \{\theta_{i,h}\}_{h=1}^{H} \right\}_{i=1}^{I}$. Further $\sum_m p_m e_{i,m} + \sum_h \theta_{i,h} \sum_m p_m y_{h,m}$ for all i. Define a Competitive Equilibrium and a Pareto Efficient Allocation with production. Show that (c, y) must be a Pareto efficient allocation.

Question 8 [Final 2019]

(25 points) Consider a 2 good, 2 agent world, Good one, x, denotes oranges and good two, y denotes orange juice. Each agent has utility function $u(x, y) = \ln(x) + \ln(y)$. Suppose each agent is endowed with 1 orange and no orange juice. Further assume there exist two identical firms which can turn oranges into orange juice according to the production function $f(x) = x$

a) Draw the production sets for each firm.

b) Define and find ALL competitive equilibrium over all possible specifications of firm ownership.

Question 9 [Final 2019]

Consider an economy with a finite I number of agents, each of whom receives an endowment of 1 with probability $\frac{1}{2}$ and an endowment of 2 with probability $\frac{1}{2}$, independent of the realizations of the other agents. (That is, each flips his own coin.) Let the state of the world m specify the endowment realization of each agent and let $c_{i,m}$ and $e_{i,m}$ denote the consumption and endowment of agent i in state of the world m, respectively, with $c_i = (c_i, 1, \ldots, c_{i,M})$ and $e_i = (e_{i,1}, \ldots, e_{i,M})$. Suppose each person i ranks consumption vectors c_i such that $c_i \succeq_i \hat{c}_i$ if and only if $\sum_m \pi_m \frac{c^{1-c}}{1-\sigma} \geq \sum_m \pi_m \frac{\hat{c}_{i,m}^{1-\sigma}}{1-\sigma}$, where π_m denotes the probability of the state of the world m.

a) Define an allocation, being careful about the number of states of the world. When is such an allocation feasible?

b) Characterize the set of Pareto efficient allocations. In particular, prove that each agent's consumption is a constant fraction of the aggregate endowment in any Pareto efficient allocation.

c) Finally assume every agent's endowment of potatoes is private to him. Under what conditions (if any) is a Pareto efficient allocation assuming full information (or full observability) incentive compatible if endownents are, in fact, not answer). observable? (Justify your