Recitations 12

[Definitions used today]

- Incentive Compatibility, State dependent allocations, Truthtelling outcome, Lying outcome

Question 1 [Ex2 Midterm 2020] Consider the following 2 agent, 2 good, endowment economy. Both agents, $i \in \{1,2\}$ have utility function $u_i(c_{i,1}, c_{i,2}) = 2\min(c_{i,1}, c_{i,2})$., where $c_{i,m}$ is the amount of good $m \in \{1,2\}$ agent i consumes. There is 1 divisible unit of each good in the world, and each agent is able to consume any non-negative amount of either good.

1. What is the set of Pareto efficient allocation for this economy?
2. Derive the utility possibilities set for this economy.
3. Specify the Arrow problem here, carefully specifying what is taken as a parameter. Is every solution Pareto efficient? Is every Pareto efficient allocation the solution to some Arrow problem?
4. Set up the Negishi problem here, again carefully specifying what is taken as a parameter. Is every solution Pareto efficient? Is every Pareto efficient allocation the solution to some Negishi problem?
5. Derive the set of Competitive Equilibria. Are they all Pareto Efficient?
6. Are all Pareto efficient allocations achievable for some set of initial endowments?
7. Suppose an allocation c is Pareto efficient. Describe the set $B(c)$.

Question 2

Consider a 2 good, 2 agent world. Good one x, denotes oranges and good two, y denotes orange juice. Agent $i = 1$ has utility function $u_1(x,y) = 2\log(x) + \log(y)$ and agent $i = 2$ has utility function $u_2(x,y) = \log(x) + 2\log(y)$. Suppose each agent is endowed with 1 orange and no orange juice. Further assume there exist two identical firms which can turn oranges into orange juice according to the production function $f(x) = \sqrt{x}$

1. Define and find the Competitive Equilibrium. For what planner weights (if any) does this solve the Negishi problem (with production)?
2. Now suppose agent 1 is endowed with 1 orange (and no orange juice) and agent2 is endowed with 0 oranges. Each agent owns half of each firm. Find the competitive equilibrium and the weights (if any) for which this is a solution to the Negishi Problem.
3. Do again but assume agent 1 is endowed with 0 oranges (and no orange juice)and agent 2 is endowed with 1 orange (and zero juice) (with again each owning half of each firm).

Question 3

Suppose $t \in \{0, \ldots, T\}$. At each date t, nature flips a coin. With 50% probability, agent 1 has an endowment of 2 bananas and agent 2 has an endowment of zero bananas, and with 50% probability, agent 1 has an endowment of 0 bananas and agent 2 has an endowment of 2 bananas. There is no production and all endowments are observable. Let s_t be the joint endowment realization at date t, and $s^t = \{s_0, \ldots, s_t\}$ Assume preferences are characterized by $\sum_{t=0}^{T} \beta^t \sum_{s^t} \pi_t(s^t) u(c_t(s^t))$ where $\pi(s^t)$ is the (obvious) probability of sequence s^t and u is some strictly concave function. 0

1. Characterize the set of feasible allocations.
2. Characterize the set of Pareto efficient allocations.
3. Characterize the competitive equilibrium from these endowments. instead there are N agents each of whom at each date flips a fair coin and if heads, has an endowment of 2 bananas, and if tails has an endowment of zero bananas. Redo the previous parts to this question. What happens as $N \to \infty$?

Question 4

Consider a two period world, $t \in \{0, 1\}$, where each agent is endowed with 1 apple each period. In each period, an I length vector $\theta_t = \{\theta_{1,t}, \ldots, \theta_{I,t}\}$ is drawn where each $\theta_{i,t} \in \left\{\frac{1}{2}, \frac{3}{2}\right\}$. Every possible θ_t is drawn with equal probability at each date.

1. What is a history? What is an allocation? What is a feasible allocation?
2. Suppose an agent i before date zero ranks allocations according to $\sum_t \sum_{s^t} \pi\left(s^t\right) \theta_{i,t} \ln\left(c_{i,t}\left(s^t\right)\right.$ (note this does not fit into the class of preferences discussed in class).
3. Find the competitive equilibrium assuming θ_t is observable at each date $t \in \{1,2\}$
4. Next, after the realization of the date zero's θ_t, what is the natural (or timeconsistent) way each agent would rank allocations? Do again after the realization date one's θ_t
5. Given these ex-post preference orderings, what can you say about incentive compatibility if $\theta_{i,1}$ is private to agent i. In particular, what is the appropriate incentive constraint and what restrictions does this put on allocation?
6. Finally, what can you say about incentive compatibility if $\theta_{i,0}$ is also private to agent i. In particular, what is the appropriate incentive constraint and what restrictions does this put on allocation?