



Recitations 16

[Definitions used today]

- Best correspondence, Nash Equilibrium, Minimax Theorem

Question 1

1/2	L	R
T	3,1	0,0
B	0,0	1,3

- Define: pure actions, mixed actions, best correspondences
- Find all Nash Equilibria

Question 2 [153 III.1 Spring 2013 majors]

A two players finite action normal form game is zero sum if the sum of the utilities of the two players is equal to 0 for any action profile, so $u^1 = -u^2$. **The Minimax Theorem** states that in this case

$$\min_{\alpha^2 \in \Delta(A^2)} \max_{\alpha^1 \in \Delta(A^1)} u(\alpha^1, \alpha^2) = \max_{\alpha^1 \in \Delta(A^1)} \min_{\alpha^2 \in \Delta(A^2)} u(\alpha^1, \alpha^2) \equiv v$$

Prove the minimax theorem. You can use Nash equilibrium existence theorem.

Question 3

For a zero sum game of two players define the value of the game as $V : \mathbb{R}^{nm} \rightarrow \mathbb{R}$ (where $n = \#A^1$ and $m = \#A^2$) :

$$V(u) = \max_{s^1 \in \Delta(A^1)} \min_{s^2 \in \Delta(A^2)} U(s^1, s^2 | u)$$

where for a given strategy profile $s^1 = (p_1, \dots, p_n)$, $s^2 = (q_1, \dots, q_m)$ and payoffs $u \in \mathbb{R}^{nm}$ we define

$$U(s^1, s^2 | u) = \sum_{i=1}^n \sum_{j=1}^m p_i q_j u_{ij}$$

Show that **The value of a game** is

- continuous
- non-decreasing
- homogenous of degree one in payoffs.

Question 4

Under standard assumptions, prove the following properties of best response in mixed $BR_i(s)$:

- non-empty valued,
- compact valued,
- upper hemi continuous.
- convex-valued

Question 5

Show that $BR_i(s) = \text{co}(\{\delta_{b^i} : b^i \in BR_{A^i}^i(s)\})$