Economics 8101 **Midterm Exam**
Fall Semester 2018.

1. (30 points) Let $f\colon I\!R^n_+ \to I\!R_+$ be a production function that is strictly increasing and satisfies $f(0) = 0$. Let $C^*(w, z)$ be the (minimum) cost function, where $w \in I\!R^n_+$ is a vector of input prices and $z > 0$ is an output level.

   (a) Show that cost function C^* is a non-decreasing function of output level z, for every $w >> 0$.

   (b) Show that, if production function f is concave, then cost function C^* is a convex function of output level z, for every $w >> 0$.

2. (30 points) Consider Walrasian demand function $x^*(p, w)$ - assumed single valued - for $p \gg 0$ and $w \geq 0$ generated by a continuous and locally nonsatiated utility function $u\colon I\!R^L_+ \to I\!R$. State and prove the Law of Compensated Demand for x^*.

3. (40 points) Consider a list of observations $(p^1, x^1), \ldots, (p^T, x^T)$ where $p^t \in R^n_+$ and $x^t \in R^n_+$ are a price vector and a corresponding consumption plan of a consumer, respectively, for every $t = 1, \ldots, T$.

   (i) State the the Generalized Weak Axiom of Revealed Preference (GWARP) and the Generalized (Strong) Axiom of Revealed Preference for these observations. Prove that the strong axiom holds if observations can be rationalized by a locally non-satiated utility function.

   (ii) Suppose that the observations are generated by a demand function $d(p, w)$, that is, $x^t = d(p^t, w^t)$ for every t. Function d is given as

   $$(\frac{w}{p_1}, 0) \ \text{ if } \quad p_1 \geq p_2,$$

   and

   $$(\frac{w}{p_1 + p_2}, \frac{w}{p_1 + p_2}) \ \text{ if } \quad p_2 > p_1.$$

   for $p_1 > 0, p_2 > 0$, and $w \geq 0$. Does GWARP hold for arbitrary observations generated by d? Can demand d be rationalized by a locally non-satiated utility function?

1. (40 points) Let $f: \mathbb{R}^n_+ \to \mathbb{R}_+$ be a production function that is strictly increasing and satisfies $f(0) = 0$. Let $C^*(w, z)$ be the (minimum) cost function, where $w \in \mathbb{R}^n_+$ is a vector of input prices and $z > 0$ is an output level.

   (a) Show that cost function C^* is a non-decreasing function of output level z, for every $w >> 0$.

   (b) Show that, if production function f is concave, then cost function C^* is a convex function of output level z, for every $w >> 0$.

2. (30 points) Consider a firm with production function $f : \mathbb{R}^n_+ \to \mathbb{R}_+$. The firm maximizes its profit at prices $w \in \mathbb{R}^n_{++}$ for inputs and $q \in \mathbb{R}_{++}$ for output. In addition, the firm is taxed at rate $t \geq 0$ of its total cost. The firm's profit maximization problem is

   $$\max_{x \geq 0} qf(x) - wx(1 + t).$$

   Show that if production function f is supermodular, then the firm's input demand x^* - assumed single valued - is monotone nonincreasing in tax rate t. Production function f is strictly increasing but need not be differentiable.

3. (30 points) Consider Walrasian demand function $x^*(p, w)$ - assumed single valued - for $p \gg 0$ and $w \geq 0$ generated by a continuous and locally nonsatiated utility function $u: \mathbb{R}^L_+ \to \mathbb{R}$. State and prove the Law of Compensated Demand for x^*.

Economics 8101
Fall Semester 2015.

Midterm Exam

1. (40 points) Consider the problem of finding a Pareto optimal allocation of aggregate resources $\omega \in I\!R^n_+$ in an economy with two agents:

$$\max_x \ \mu u_1(x) + (1-\mu)u_2(\omega - x)$$

$$\text{subject to } \ x \leq \omega, \ x \geq 0,$$

where $u_i : I\!R^n_+ \to I\!R$, for $i = 1, 2$, are agents' utility functions, and μ is the welfare weight of agent 1. μ lies in the interval $[0, 1]$. Let $x^*(\mu)$ be the set of solutions.

   (i) Show that, if u_1 and u_2 are strictly increasing and supermodular in x (but may not be everywhere differentiable), then $x^*(\mu)$ is non-decreasing in μ. You may assume that $x^*(\mu)$ is single-valued.

   (ii) Under what conditions on u_1 and u_2 is the solution $x^*(\mu)$ unique. Be as general as you can and justify your answer.

2. (30 points) Consider a demand function for two goods specified as follows: Demand $d(p_1, p_2, w)$ equals

$$\left(\frac{w}{p_1}, 0\right) \ \text{if} \ \ p_1 \geq p_2,$$

and

$$\left(0, \frac{w}{p_2}\right) \ \text{if} \ \ p_2 > p_1.$$

for $p_1 > 0, p_2 > 0$, and $w \geq 0$.

Show that the Generalized Weak Axiom of Revealed Preference (GWARP) does not hold for d. State GWARP.

3. (30 points) Let $f \colon I\!R^n_+ \to I\!R_+$ be a production function that is strictly increasing and satisfies $f(0) = 0$. Let $C^*(w, z)$ be the (minimum) cost function, where $w \in I\!R^n_+$ is a vector of input prices and $z > 0$ is an output level.

   (a) Show that cost function C^* is a non-decreasing function of output level z, for every $w >> 0$.

   (b) Show that, if production function f is concave, then cost function C^* is a convex function of output level z, for every $w >> 0$.

Economics 8101 **Midterm Exam**
Fall Semester 2013, Session I

1. (25 points) Consider a consumer with a locally non-satiated utility function $u: \mathbb{R}^L_+ \to \mathbb{R}$. State a definition of locally non-satiated. Let $x^*(p,w)$ be the Walrasian demand at (p,w). Consider p and w such that $p \gg 0$ and $w > 0$.

   Show that if $x \in x^*(p,w)$ and $u(x') \geq u(x)$, then $px' \geq w$.

2. (25 points) Consider a continuous and strictly increasing production function $f: \mathbb{R}^n_+ \to \mathbb{R}$. Let $x^*(w,z)$ be the cost-minimizing (that is, conditional) factor demand for a vector of factor prices w and an output level z, where $w \gg 0$ and $z > 0$.

   State a definition of production function f being strictly quasi-concave. Show that if f is strictly quasi-concave, then the conditional factor demand is single valued (i.e., unique). [The proof that the demand is non-empty is not required.]

   $f(x) \geq \bar{z}$ convex

3. (25 points) Consider the following lexicographic preferences on the consumption set $\mathbb{R}^2_+$: the value of $x_1 + x_2$ has the first priority; the value of x_2 has the second priority.

   (i) Show that this preference is not continuous.

   (ii) Derive the Walrasian demand function $x^*(p,w)$ for this lexicographic preference for arbitrary price vector $p >> 0$ and income $w > 0$.

4. (25 points) State the Weak Axiom of Profit Maximization for a finite collection (observations) of pairs of price vectors and production plans of L goods, $\{p^t, y^t\}_{t=1}^T$. Show that if the Weak Axiom holds and $p^t >> 0$ for every t, then the production set with free-disposal defined by

$$Y = \mathrm{co}\{y^t : t = 1, \ldots, T\} + \mathbb{R}^L_-$$

   rationalizes these observations. Here, "co" denotes the convex hull, i.e., the set of all convex combinations of elements of the set.