Recitations 15

[Definitions used today]

- players, actions, action profiles, consequences
- game on consequences, game in normal form
- lotteries: simple and compound
- vNM axioms: weak order, continuity, monotonicity, reduction, substitution

Question 1

Suppose [WO, C, M] hold. Let $\mathcal{L} \equiv \Delta(C)$ and $C = \{c_1, \dots, c_m\}$. Show that:

$$\forall_{F \in \mathcal{L}} \delta_{c^1} \succeq F \succeq \delta_{c^n}$$

where δ_{c^i} gives probability 1 to the consequence i

Question 2 [von Neumann-Morgenstern]

1. (existence) $\succeq$ on $\mathcal{G}$ satisfies WO, Cty, M, R and S if and only if there exists a linear $u : \mathcal{G} \to \mathbb{R}$ that represents $\succeq$
2. (uniqueness) If u, v are linear representations of $\succeq$, then $\exists A > 0, B \in \mathbb{R}$ such that $u(\cdot) = Av(\cdot) + B$

Show $\Rightarrow$ part of existence and uniqueness

Question 3 III.1 Fall 2016 majors

Consider a preference order $\succeq$, and assume that it satisfies the von Neumann-Morgenstern (vNM) axioms. Let, for any two lotteries L and M, and any $\alpha \in [0,1], (L, \alpha, M)$ be the compound lottery that gives the lottery L with probability α and the lottery M with probability $1 - \alpha$

(a) State what a vNM representation is, and then state the vNM axioms in the form you prefer: the axioms you state must characterize preferences with the vNM representation.

(b) Prove that $\succeq$ satisfies the Sure Thing Principle (STP), namely that for any lotteries L, M, N and R and any $\alpha \in [0,1]$

$$(L, \alpha, M) \succ (N, \alpha, M) \quad \text{if and only if} \quad (L, \alpha, R) \succ (N, \alpha, R)$$

If you assume the STP among your axioms, then prove that your axioms imply that the preference order has a vNM representation.

(c) Suppose that $L \succ M$; prove that for any $\alpha \in (0,1]$

$$(L, \alpha, M) \succ M$$

(d) Prove that if u and v are two linear utility functions representing $\succeq$, then u is a positive affine transformation of v

Question 4 [Marshak Machina Triangle] Consider a set C of three consequences 1,2,3 and a set of lotteries over C.

- Draw a 2D diagram that represents a three dimensional simplex.
- Draw two simple lotteries L_1 and L_2. Consider a compound lottery $L_3 = (L_1, p; L_2, 1-p)$. How to represent it on the diagram?
- Suppose preferences are given by a Bernoulli function $u : C \to \mathbb{R}$. Write an equation for an indifference curve. Show that indifference curves are parallel. Draw some indifference curves on the diagram.