Recitation 3

[Definitions used today]

- (weakly/strongly) convex, continuous, monotone preferences, locally non-satiated utility function
- utility maximization, Debreu theorem, lexicographic preferences
- WARP, GWARP, GARP, Topkis theorem, Afriat theorem

Question 1 [Weak vs strong continuity] 182 [Question I.1 Fall 2014 majors]

Let $\succeq$ be a transitive and complete preference relation on (connected) set $X \subseteq \mathbb{R}^N_+$:
Prove that the following statements are equivalent

- $\succeq$ on X is **weakly continuous** if $\forall x \in X$ the preferred-to-x set $U(x) = \{y \in X : y \succeq x\}$ and lower countur set $L(x) = \{y \in X : x \succeq y\}$ are closed.
- $\succeq$ on X is **strongly continuous** if for all sequences $\{x_n\}\{y_n\} \in X$ such that $x_n \to x$, $y_n \to y$, if $\forall n$, $x_n \succeq y_n$, then $x \succeq y$.

Question 2 [Properties of preferences]

Prove following statements

1. If a preorder $\succeq$ is monotone in $\mathbb{R}^l$, then it is locally nonsatiated.
2. If a preorder $\succeq$ is transitive, weakly monotone, and locally nonsatiated then it is monotone
3. A preorder $\succeq$ is weakly convex $\iff$ the upper contour sets $U(x) = \{y \in X : y \succeq x\}$ are convex for all $x \in X$
4. If a preorder $\succeq$ is continuous and strictly convex then it is convex

Question 3 Consider the following preference relations on $\mathbb{R}^2_+$

1. $x \succeq y \iff \min\{x_1, x_2\} \geq \min\{y_1, y_2\}$
2. $x \succeq y \iff \max\{x_1, x_2\} \geq \max\{y_1, y_2\}$

are they convex? Are they strictly convex?

Question 4 Give an example of preferences/utility function such that :

1. satisfy non-satiation, but not weak monotonicity
2. satisfy non-satiation, but not local non-satiation
3. satisfy local non-satiation, strict monotonicity, but not quasi-concave
4. does not satisfy continuous but it is representable by a utility function

Question 5 [Utility representation] 157 [I.1 Fall 2013 majors]

Consider preference relation $\succeq$ on the consumption set $\mathbb{R}^L_+$. Suppose that $\succeq$ is reflexive and complete.

1. State a definition of $\succeq$ having a utility representation. Is utility representation, if it exists, unique?
2. State a theorem providing sufficient conditions on $\succeq$ to have a utility representation. Be as general as you can and clearly define any extra properties of $\succeq$ that you use
3. [**Debreu Theorem**] Let $\succeq$ be a complete, transitive and continuous, strictly increasing (i.e. strongly monotone) preference relation on $\mathbb{R}^L_+$, show that it has a continuous utility representation

Question 6 [Lexicographic preference]

Consider the following lexicographic preferences on the consumption set $\mathbb{R}^2_+$: the value $x_1 + x_2$ has the first priority, the value of x_2 has the second priority.

1. Is this preference relation continuous? Prove of give a counter example.
2. Does this preference relation have the utility representation? Prove of give a counter example.
3. Consider the lexicographic preferences on $\mathbb{R}^N_{++}$ such that the first priority is described by an increasing and continuous utility function $u_1(x)$ and the second priority is described by another increasing and continuous utility function $u_2(x)$. Show that, if u_1 is strictly concave, then the Walrasian demand of the lexicographic preference coincides with the Walrasian demand of u_1 for every $p \in \mathbb{R}^N_+$, $p \neq 0$ and $w > 0$.

Question 7 [Midterm 2018]

Consider a list of observations $\{(p_1, x_1), \ldots, (p_T, x_T)\}$ where $p_t \in \mathbb{R}^N_+$ and $x_t \in \mathbb{R}^N_+$ are price vector and a corresponding consumption plan of a consumer respectively, for every $t \in \{1, \ldots, T\}$.

1. State the Generalized Weak Axiom of Revealed Preference (GWARP) and Generalized (strong) Axiom of Revealed Preference (GARP) for these observations.
2. Show that if a locally non-satiated utility function rationalized observations then GARP holds.
3. Suppose that the observations are generated by a demand function $d(p, w)$ that is $x_t = d(p_t, w_t)$ for every t. Function d is given as
$$d(p,w) = \begin{cases} (\frac{w}{p_1}, 0) & \text{if } p_1 \geq p_2 \\ (\frac{w}{p_1+p_2}, \frac{w}{p_1+p_2}) & \text{if } p_2 > p_1 \end{cases}$$
Does GWARP hold for arbitrary observations generated by d? Can demand d be rationalized by a locally non-satiated utility function?
4. Show that if a locally non-satiated utility function rationalized observations then GWARP holds.
5. Show that the assumption of local non-satiation in the previous point cannot be dispensed with - i.e. give an example of a utility function that rationalizes a set of pairs of prices and consumption bundles that violates GWARP

Question 8 [Properties of Walrasian Demand]

Prove following claims

1. **[Walras Law]** Show that if a preference relation $\succeq$ is continuous and locally non-satiated then $p \cdot x^*(p, w) = w$, for all $x^*(p, w)$ that belong to the Walrasian Demand correspondence.
2. **[GWARP]** Show that if a preference relation $\succeq$ is continuous and locally non-satiated then for all $w > 0$
$$w' > 0, p >> 0 \text{ and } p' >> 0: \quad p \cdot x^*(p', w') \leq w \Rightarrow p' \cdot x^*(p, w) \geq w'$$

Question 9 230 [I.1 Fall 2016 minors]

Let d : be a demand function of prices and income satisfying budget equation $pd(p, w) = w$ for every p and w

1. Show that if d is a Walrasian demand function of a consumer with strictly increasing utility function, then the Generalized Weak Axiom of Revealed Preference (GWARP) holds for every T -tuple of price-quantity pairs $\{p^t, x^t\}_{t=1}^T$, where $x^t = d(p^t, w^t)$ $p^t \in \mathbb{R}^L_{++}$ and $w^t \in \mathcal{R}_+$ for every $t = 1, \ldots, T$. State GWARP
2. onsider the following demand function for $L = 2$ and show that GWARP does not hold for $\hat{d}$.:
$$\hat{d}(p,w) = \begin{cases} \left(\frac{w}{p_1}, 0\right) & \text{if } p_1 \geq p_2 \\ \left(0, \frac{w}{p_2}\right) & \text{if } p_2 > p_1 \end{cases}$$
3. State the Afriat's Theorem. The proof is not required
4. Prove the necessity of an axiom for rationalizability