Recitations 13

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MIN II

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12/10/20 RECITATION 13

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ZOOM

Today:
- cnt
- HW
- exam

MONDAY
- practice
practice
practice

TUESDAY 11:15
EXAM

Recursive ~~re~~ version of SPP

Economy $i = 1, 2$

$e_t^1 = s_t \qquad e_t^2 = 1 - s_t$

$s_t \sim IID \qquad \pi(s^t) = \pi(s_t) \cdot \ldots \cdot \pi(s_0)$

$[\bar{s}_1 \ldots \bar{s}_S] \qquad \Pi_i = \mathbb{P}(s_t = \bar{s}_i)$

- v – $\mathbb{E}\, \beta^t u$ of HH 1
- $P(v)$ – (maximal) $E \beta^t u$ of HH2 when v is promised to HH 1

Pareto PB $\rightarrow \{ c_{it}(s^t) \}\ i = 1, 2,\ t,\ s^t$

$$\max P(v)$$

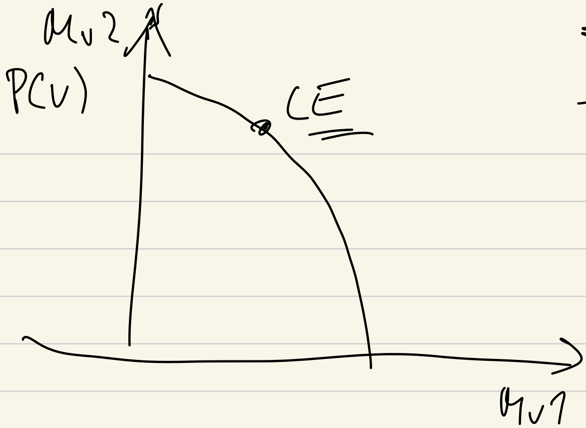
$$\text{s.t.} \quad \sum \beta^t u(c_{1t}(s^t)) \geq v$$

$$\sum_{i=1,2} c_{it}(s^t) = 1$$

CE $\Rightarrow$ PO

$$c_{1t}(s^t) = \bar{c}$$

$$c_{2t}(s^t) = 1 - \bar{c}$$



$$\sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 u(c_{it}) = A$$

$$c_{it} = \bar{c}$$

A for 1

$$\frac{u(\bar{c})}{1-\beta} = V$$

B for drv 2 $\quad c_{2t} = 1 - \bar{c}$

$$\sum \beta^t \mathbb{E}_0 u(c_{2t}) = P(v) = \frac{u(1-\bar{c})}{1-\beta}$$

$$V = \sum_{i=1}^{S} \left[u(c_i) + \beta w_i \right] \pi_i$$

$$P(v) = \sum_{i=1}^{S} \left[u(1-c_i) + \beta P(w_i) \right] \pi_i$$

$$V(k, \varepsilon) = \max \{ u(c) + \beta \underline{\mathbb{E}_{\pi} v(k', \varepsilon') | \varepsilon} \}$$

w_i - continuation value at state i

c_i - consumption level of Mv 1 at state i

$$P(v) = \max_{c_i, w_i} \left\{ \sum_{i=1}^{S} \left[u(1-c_i) + \beta P(w_i) \right] \pi_i \right.$$

$$\text{s.t.} \quad \sum_{i=1}^{S} \left[u(c_i) + \beta w_i \right] \pi_i \geq v$$

$$0 \leq c_i \leq 1$$

$$\left. w_i \in \left[\frac{u(0)}{1-\beta}, \frac{u(1)}{1-\beta} \right] \right\}$$

$$\mathcal{L} = \sum \pi_i \left[u(1-c_i) + \beta P(w_i) + \Theta \left(u(c_i) + \beta w_i - v \right) \right]$$

$$\left(v = \sum \pi_i v \right)$$

FOC:

$$\frac{\partial \mathcal{L}}{\partial c_i}: \quad -u'(1-c_i) + \Theta u'(c_i) = 0 \qquad (1)$$

$$\frac{\partial \mathcal{L}}{\partial w_i}: \quad P'(w_i) + \Theta = 0 \qquad (2)$$

+Envelope $\quad P'(v) = -\Theta \qquad (3)$

P is str. concave function. P' str decr.

(2) & (3) $\quad v = w_i \Rightarrow w_i$ is state independent

(1) & (2) $\quad \dfrac{u'(1-c_i)}{u'(c_i)} = \Theta = \underline{-P'(w_i)} = f_1\left(\dfrac{\mu_1}{\mu_2}\right)$

RHS does not depend on state

CE

HH $\rightarrow \max \sum_{t=0}^{\infty} \beta^t \mathbb{E}_0 u(c_t^i)$

s.t. (μ_i) $\sum_t \sum_{s^t} p_t^0(s^t) \cdot (c_t^i(s^t) - e_t^i(s^t)) \leq 0$

Non negative

MCCs ≥ 0 p_t^0

FOCs: $\frac{u'(c_t^i(s^t))}{u'(c_t^j(s^t))} = \frac{\mu_i}{\mu_j} \quad \forall t, s^t$

$c_t^i(s^t) = \underbrace{(u')^{-1}\left(\frac{\mu_i}{\mu_1} \cdot u'(c_t^1(s^t))\right)}$

Plug it into MCC

$\sum_i c_t^i(s^T) = \sum_i \quad = \sum_i y_t^i(s^t) = \bar{y}$

$f(c_t^i(s^t)) = \bar{y}$

$c_t^i(s^t) = $ fct. of aggr. endl. only

HW 4 Ex. 1

$c_t^1(s^t) = c_t^2(s^t) = 1$

$\lambda_1 = \lambda_2$

$\frac{u'(c_i)}{u'(1-c_i)} = \gamma$

$\underline{\underline{\mathcal{S}_0}}$ $\quad$ $\underline{c_{it}(s^t) = c_{i0}(\bar{s}^0)}$

DP^2 $\quad$ $\dfrac{u'(c_i)}{u'(1-c_i)} = -\underline{p'(v)}$

$\downarrow$ i, s^t, t ; $s^t \to s_i$

$\dfrac{u'(x)}{u'(1-x)} = \text{const} \Rightarrow c_{it}(s^t) = \underline{\text{const}}$

$c_{1t}(s^t) + c_{2t}(s^t) = 2 =$

$= c_{10}(\bar{s}^0) + c_{20}(\bar{s}^0)$

$\Rightarrow$ $\underline{c_{1t}(s^t) = c_{10}(\bar{s}^0)} = \underline{1}$

$\underline{\left(p_t = \left(\frac{\beta}{2}\right)^t\right)}$ $\quad$ $\sum_{s^t} e_{1t}(s^t) = 1$

CE $\quad \sum_{t, s^t} p_t \left(c_{it}(s^t) - e_{it}(s^t)\right) = 0$

$\sum_i \sum_t \sum_{s^t} \left(\frac{\beta}{2}\right)^t \left(c_{10}(\bar{s}^0) - e_{it}(s^t)\right) = 0$ $\quad$ 1

$N \to \infty$

$2^t \to 2$

$2^t \to \infty$

$c_{it}(s^t) = \dfrac{1}{N} \sum_i e_{it}(s^t) \to 1$

$(\beta/2^N)^t = p(s^t)$ $\quad$ $\cancel{\sum (\beta^t)}^i (c_{i0}(\bar{s}^0) - 1) = 0$