Recitations 13

[Definitions used today]

- Incentive Compatibility, State dependent allocations, Truthtelling outcome, Lying outcome

Question 1 [HW 4]

Suppose $t \in \{0, \ldots, T\}$. At each date t, nature flips a coin. With 50% probability, agent 1 has an endowment of 2 bananas and agent 2 has an endowment of zero bananas, and with 50% probability, agent 1 has an endowment of 0 bananas and agent 2 has an endowment of 2 bananas. There is no production and all endowments are observable. Let s_t be the joint endowment realization at date t, and $s^t = \{s_0, \ldots, s_t\}$ Assume preferences are characterized by $\sum_{t=0}^{T} \beta^t \sum_{s^t} \pi_t(s^t) u(c_t(s^t))$ where $\pi(s^t)$ is the (obvious) probability of sequence s^t and u is some strictly concave function.

a) Characterize the set of feasible allocations.

b) Characterize the set of Pareto efficient allocations.

c) Characterize the competitive equilibrium from these endowments.

d) Suppose instead there are N agents each of whom at each date flips a fair coin and if heads, has an endowment of 2 bananas, and if tails has an endowment of zero bananas. Redo the previous parts to this question. What happens as $N \to \infty$?

Question 2 [HW4 4]

Consider a two period world, $t \in \{0, 1\}$, where each of I agents is endowed with 1 apple each period. In each period, an I length vector $\theta_t = \{\theta_{1,t}, \ldots, \theta_{I,t}\}$ is drawn where each $\theta_{i,t} \in \left\{\frac{1}{2}, \frac{3}{2}\right\}$. Every possible θ_t is drawn with equal probability at each date.

a) What is a history? What is an allocation? What is a feasible allocation?

b) Suppose an agent $i$before date zero ranks allocations according to $\sum_t \sum_{s^t} \pi(s^t) \theta_{i,t} \ln(c_{i,t}(s^t))$. Find the competitive equilibrium assuming θ_t is observable at each date $t \in \{0, 1\}$.

c) Discuss to what extent the equilibrium you derive depends on the observability of θ_t

Question 3 Additional

Consider a two period world, $t \in \{0, 1\}$, where each agent is endowed with 1 apple each period. In each period, an I length vector $\theta_t = \{\theta_{1,t}, \ldots, \theta_{I,t}\}$ is drawn where each $\theta_{i,t} \in \left\{\frac{1}{2}, \frac{3}{2}\right\}$. Every possible θ_t is drawn with equal probability at each date.

1. after the realization of the date zero's θ_t, what is the natural (or timeconsistent) way each agent would rank allocations? Do again after the realization date one's θ_t

2. Given these ex-post preference orderings, what can you say about incentive compatibility if $\theta_{i,1}$ is private to agent i. In particular, what is the appropriate incentive constraint and what restrictions does this put on allocation?

3. Finally, what can you say about incentive compatibility if $\theta_{i,0}$ is also private to agent i. In particular, what is the appropriate incentive constraint and what restrictions does this put on allocation?