# Flipping Houses in a Decentralized Market

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**Abstract**

**CHANGE IT AT THE END** In this paper I show that house flipping is important for understanding housing market patterns. How large are price effects and what are the implications of heterogeneity of tenure composition in presence of search friction? What are policies affecting important margins and how important they are for welfare gains? To answer this question I build search theoretic model of over the counter trade in which heterogeneous agents choose trading partner - flipper. I document housing market patterns and calibrate model using full transaction universe data for Ireland. I identify effects of flipping activity on local prices. Finally I find quantitative effects of welfare improving policies.

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# 1 Introduction

# 2 Model

Equations first, later add text around it : environment ( I am describing whole problem but showing HJB equations-should i add sequential or Bellman optimality ?), players, strategies,payoffs, equilibrium, frictionless limit, distributions, value functions, prices

**Environment.** Economy is populated by measure 1 of households and mass $f$ of flippers. Time is continuous and agents are infinitely lived. There are : nonstorable consumption good $c$, indivisible housing asset and dividends from the asset $\delta$. Houses are identical and their supply is fixed at $s$.

Houses belong to households or flippers in quantity $q \in \{0, 1\}$. There is neither production of houses, nor deterioration of housing supply.

Both households and flippers have access to risk-free savings account with common rate $r$. They use consumption and trade houses to maximize their utilities. All agents have risk neutral preferences.

Households are heterogenous in how much they value owning a house $\delta$. Dividends $\delta$ are non-tradable and evolve stochastically.

Trade in houses is decentralized, meetings are random. For now trade is restricted to happen between flipper and household and household and flipper only. One-on-one meetings between interested parties arrive with Poisson intensity $\lambda$. Once a meeting happens flipper (acting as buyer or seller) proposes a price. Household accepts or rejects the offer. If offer is accepted asset changes owners, price is paid and subperiod ends.

**Households.** Non-owner household has zero flow utility. Household-owner of a house enjoys dividends $\delta$. Valuations come from fixed distribution with cumulative distribution function $G(\cdot)$. Assume that $G(\cdot)$ has compact support $[0, \bar{\delta}]$.[1] Distribution $G(\cdot)$ is public knowledge. Valuations are private to household, in particular flippers don't know individual household's valuation. With Poisson intensity $\gamma$ valuation changes and it is drawn from distribution $G(\cdot)$.

[1]Assume uniform distribution on $[0, 1]$: $G(\delta) = \delta$

**Flippers.** Flippers have zero flow utility from owning (or not owning) an asset. Their only role is to facilitate trade[2]. They do it in a way that they are the only way households can buy or sell houses. By private information assumption about types they can not condition terms of trade on type $\delta$ of household they trade with.

Ignore this box. Fix this and think where to position this page: Strategies:

Model parameters can be summarized by $\theta = \{r, \lambda, \gamma, f, s, u(x) = x, G, \bar{\delta}\}$
value functions $V(q;\delta,\theta), W(q;\theta)$; distributions $H(q,\delta;\theta), F(q;\theta)$;cutoffs $\delta^*_q$, prices $P_q(\delta^*_{-q};\theta)$ are functions of $\theta$ but I will drop it.
Additionally symmetric actions by agents with the same state variables.

**Strategies.** We focus on history independent (no dependence on history of past realizations of $\lambda, \gamma$) and stationary equilibrium with cutoffs.

Shocks $\gamma, \lambda$ are realized and if the meeting happens prices $P_q$ are proposed by a flipper(with $q = 0, 1$ respectively).
Household choice is to accept or reject offer. Their decision is contingent on successful meeting and on price offer. Meetings between one specific household and individual flipper have a.s. zero chances to repeat flipper can extract all surplus. Agents decision are characterized by cutoff $\delta^*_q$ and will guide tell them when to buy or sell. Agent will buy asset if he does not have one and his $\delta \geq \delta^*_0$ and sell asset if he has one and $\delta \leq \delta^*_0$. We break ties by making agents at cutoffs to trade in equilibrium. Payments follows and house changes hands.

Jumping ahead to equilibrium we will focus on stationary equilibrium with cuttoffs: there will be cumulative distribution of households $H(q, \delta)$ and fraction of flippers $F(q)$ for each $q \in \{0, 1\}$, two prices : $P_0$ proposed by flipper when he buys a house and $P_1$ proposed by flipper when he sells a house, two cuttofs $\delta^*_0(P_1)$ for households buying a house and $\delta^*_1(P_0)$ for households selling a house, value functions for flippers $W(q)$ and for households $V(q, \delta)$.

**Accountings** Households and flippers who own a house hold all of $s$ houses:

$$\int_0^{\bar{\delta}} dH(1, \delta) + F(1) = s \tag{1}$$

For any $\delta$ sum of all households without a house and below $\delta$ and households with a house

[2]For now they don't improve quality

and below $\delta$ has to be equal corresponding level of cdf of type $G(\delta)$:

$$\int_0^\delta dH(0,\delta) + \int_0^\delta dH(1,\delta) = G(\delta) \quad \forall \delta \in [0, \bar{\delta}] \tag{2}$$

Sum of fraction of flippers without a house $F(0)$ and wit $F(1)$ is equal $f$

$$F(0) + F(1) = f \tag{3}$$

**Law of Motions** In stationary equilibrium inflow and outflows to both homeownership and non-ownership both for households and flippers has to balance. Trade and change in evolution of types generate those flows. Let's focus on inflows and outflows to $[0, \delta]$ taking into account position of $\delta$ vs cutoffs $\delta_0^*, \delta_1^*$.
Homeownership (inflow and outflow to $[0, \delta], q = 1$)

$$\underbrace{\mathbb{1}\{\delta \geq \delta_0^*\}[\lambda F(1) \int_{\delta_0^*}^{\delta} dH(0,\delta)]}_{\text{inflow from trade}} + \underbrace{\gamma G(\delta) \int_{\delta}^{\bar{\delta}} dH(1,\delta)}_{\text{inflow from change of type from}[\delta,\bar{\delta}]} = \tag{4}$$

$$= \underbrace{\mathbb{1}\{\delta \leq \delta_1^*\}[\lambda F(0) \int_0^{\delta} dH(1,\delta)]}_{\text{outflow from trade}} + \underbrace{\gamma(1 - G(\delta)) \int_0^{\delta} dH(1,\delta)}_{\text{outflow from change of type to}[\delta,\bar{\delta}]} \tag{5}$$

Inflows to homeownership comes from buying houses by households and change in valuations. If first term is positive (trade happens) if $\delta$ is high enough such that household who don't own a house are willing to trade (their valuation is between $\delta_0^*$ and $\delta$) and trade will happen with intensity $\lambda F(1)$. Second inflow to $[0, \delta]$ is proportional to mass of household who are owners and are above $\delta$ and with intensity $\gamma$ are hit with taste shock and redraw valuation to be below $\delta$ which happens with probability $G(\delta)$.
Outflows from homeownership comes from selling houses by households and change in valuation. Trade happens for low enough valuations (below or at $\delta_1^*$), mass of interested households equal to integral and rate at which trade happens is equal to $\lambda F(0)$. Second outflow from $[0, \delta]$ is proportional to mass of household who are non-owners and are below $\delta$ and with intensity $\gamma$ are hit with taste shock and redraw valuation to be above $\delta$ which happens with probability $1 - G(\delta)$

In similar way we can derive flows to and from non ownership by households. Not owning (inflow and outflow to $[0,\delta], q=0$ )

$$\underbrace{\mathbb{1}\{\delta \geq \delta_0^*\}[\lambda F(1)\int_{\delta_0^*}^{\delta} dH(0,\delta))]}_{\text{inflow from trade}} + \underbrace{\gamma(1-G(\delta))\int_0^{\delta} dH(0,\delta)}_{\text{inflow from change of type from}[0,\delta]} =$$
$$= \underbrace{\mathbb{1}\{\delta \leq \delta_1^*\}[\lambda F(0)\int_0^{\delta} dH(1,\delta)]}_{\text{outflow from trade}} + \underbrace{\gamma G(\delta)\int_{\delta}^{\bar{\delta}} dH(0,\bar{\delta})}_{\text{outflow from change of type to}[\delta,\bar{\delta}]}$$

In appendix we derive from conditions above balance of trade for flippers. It equal trade of flippers who buy houses with trade of flippers who sell houses.

$$F(1)\int_{\delta_0^*}^{\bar{\delta}} dH(0,\delta) = F(0)\int_0^{\delta_1^*} dH(1,\delta) \tag{6}$$

By simple algebra (look at appendix) we get characterization of distribution using probability distribution function which given $\delta_0^*, \delta_1^*$ fully characterizes $H, F$:

$$\lambda F(1)\mathbb{1}\{\delta \geq \delta_0^*\}+\gamma(s-F(1)) = dH(1,\delta)\cdot[\lambda(f-F(1))\mathbb{1}\{\delta \leq \delta_1^*\}+\gamma+\lambda F(1)\mathbb{1}\{\delta \geq \delta_0^*\}] \tag{7}$$

**Claim 1.** *In equilibrium* $\delta_1^* < \delta_0^*$

Moreover $dH(q,\delta)$ is piecewise linear function of $\delta$.

**Proposition 1.** *For all parameters there exists a pair :* $\{F(1), F(0)\}, \{dH(0,\delta), dH(1,\delta)\} \in [0,1]^2$ *such that* $F(1)+F(0)=f$ *and* $\int_0^{\hat{\delta}} dH(0,\delta)+dH(1,\delta) = G(\hat{\delta})$

I showed that for $f < s$ there is a $F(1), F(0) \in (0, f)$. Investigate case $f > s$. It is important for taking limits (but i can go around it)

Proposition with proof in appendix: value functions can be transformed to use blackwell conditions and it is contraction (second algorithm: it can be also rewritten as system of linear equations to solve ala ben moll too-may be useful if i need to do transitions). Make comments about shapes and slopes of value functions and non differentiability at cutoffs. Derive using perturbation method condition for flippers ala monopolist problem's logic. Derive equations for $P_1 - P_0 = \frac{\delta_0 - \delta_1}{r+\gamma} = \frac{1}{2(r+\gamma)}$ and talk about not convergence of prices to frictionless limit (for $\lambda \to \infty$) and for (for $f \to \infty$) distinguish what two frictions are here at play and how monopolist problem intersects with $f$ and not with $\lambda$ cite DGP to make it more credible (it is one of propositions so i am doing sth right). only when $\gamma \to \infty$ which has to imply bc of assumption that $\lambda \to \infty$ prices converge (check proof of existance of $F(1)$ if that is really necesarry condition). Argue that I am doing right limit if i want to answer question: "what is effect on prices if exchange with dealer is immediate?", unlike literature which does which is $\lambda$ limit!

**Value functions** Household's problem

$$rV(0,\delta) = \gamma \int_0^{\bar{\delta}} [V(0,\delta') - V(0,\delta)]dG(\delta') + \lambda F(1) \max\{-P_1 + V(1,\delta) - V(0,\delta), 0\}$$

$$rV(1,\delta) = \delta + \gamma \int_0^{\bar{\delta}} [V(1,\delta') - V(1,\delta)]dG(\delta') + \lambda F(0) \cdot \max\{P_0 + V(0,\delta) - V(1,\delta), 0\}$$

Flipper's problem

$$rW(0) = \max_{P_0} \lambda \int_0^{\delta_1^*(P_0)} dH(1,\delta)[-P_0 + W(1) - W(0)]$$

$$rW(1) = \max_{P_1} \lambda \int_{\delta_0^*(P_1)}^{\bar{\delta}} dH(0,\delta)[P_1 + W(0) - W(1)]$$

Flipper without a house who meets a seller has a total contact rate of $\lambda \int_0^{\delta_1^*(P_0)} dH(1,\delta)$ this is his quantity (fraction) of trades as function of his price offer $P_0$. Secondly he pays $P_0$ and he changes state so he has reservation value of $W(1) - W(0)$. This problem resembles problem of monopolist who by choosing price affects quantity. Suppose that equilibrium price $P_0$ has been perturbated and increased by infinitesimal amount- there is a new gain to flipper without a house- higher rate of meeting interested seller but there is a additional cost-namely that he has to pay a bit more. In equilibrium marginal changes of costs and benefits equalize which allows us derive it as first order condition using this perturbation:

$$\underbrace{\int_0^{\delta_1^*(P_0)} dH(1,\delta)}_{\text{MB to } F(1)\text{from charging less}} = \underbrace{[-P_0 + W(1) - W(0)] \cdot \delta_1'^*(P_0) \cdot dH(1, \delta_1^*(P_0))}_{\text{MC to } F(1)\text{from decreasing prices}}$$

Likewise for flipper who is selling a house perturbation of form decreasing a price around equilibrium price $P_1$ allows us to get:

$$\underbrace{\int_{\delta_0^*(P_1)}^{\bar{\delta}} dH(0,\delta)}_{\text{MB to } F(1)\text{from charging more}} = \underbrace{[P_1 + W(0) - W(1)] \cdot \delta_0'^*(P_1) \cdot dH(0, \delta_0^*(P_1))}_{\text{MC to } F(1)\text{from increasing prices}}$$

In appendix we are able to derive formulas for $W(1), W(0)$:

$$W(0) = \frac{\lambda(\delta_1^*)^2}{r(r+\gamma)} dH(, \delta_1^*)$$

$$W(1) = \frac{\lambda(1-\delta_0^*)^2}{r(r+\gamma)} dH(0, \delta_0^*)$$

**Prices** are such that agent at the cutoff is indifferent between trading and not trading, i.e. in equilibrium:

$$P_0 = V(1, \delta_1^*(P_0)) - V(0, \delta_1^*(P_0))$$

$$P_1 = V(1, \delta_0^*(P_1)) - V(0, \delta_0^*(P_1))$$

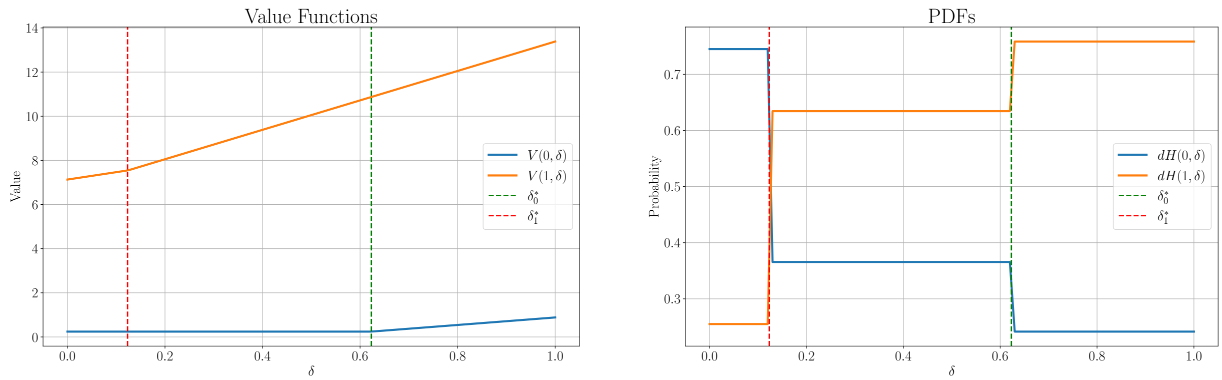
In appendix we express price spread as function of cuttoffs

$$P_1 - P_0 = \frac{\delta_0^* - \delta_1^*}{r+\gamma} = \frac{1}{2(r+\gamma)} \tag{8}$$

Notice that neither $f$ or $\lambda$ enter this equation. This result confirms case of DGP in case of monopolistic dealer.

Figure 1: Solution- household side: $V, H, \delta_i^*$

(a) $V(q,\delta)$ (b) $dH(q,\delta)$




*Note:* $\gamma = 0.1, \lambda = 2.0, s = 0.66, f = 0.10, r = 0.05$

**Definition 1.** *Stationary equilibrium is:*

1. *distributions :* $H : (q, \delta; \theta) \to \mathbb{R}$, $F : (q; \theta) \to \mathbb{R}$
2. *value functions* $V : (q, \delta; \theta) \to \mathbb{R}$, $W : (q; \theta) \to \mathbb{R}$
3. *decision rules (cuttoffs)* $\delta_q^* : \theta \to \mathbb{R}, \quad q \in \{0, 1\}$
4. *prices* $P_q : (\delta_{-q}^*; \theta) \to \mathbb{R}_+, \quad q \in \{0, 1\}$

- *Given prices* $P_.$ *and masses* $F$*: value functions* $V$ *and* $\delta_.^*$ *solve household problem (given by HJB equation)*
- *Given cutoffs* $\delta_.^*$ *and distributions* $H$*: value functions* $W$ *and prices* $P_.$ *solve flipper problem (given by HJB equations)*
- *Low of motions hold*
- *Accounting hold*

Numerical example table or full blown calibration?

## 2.1 Frictionless limit

As reference for rest what are distributions, allocations and prices in frictionless limit ($\lambda \to \infty$). $\gamma$ shocks are such that don't need any LOMs. Top $s$ households will hold house, rest won't and since flippers have flow of 0, none will own. The thing is my experiment is really $f \uparrow$ and not $\lambda \to \infty$

Top $s$ households will hold house, rest won't and since flippers have flow of 0, none will own.
In frictionless equilibrium it will be that $P^* = \frac{\delta^*}{r} = \frac{1-s}{r}$.
Volume of trade is $\lambda s G(\delta^*) = \lambda s(1-s)$ Turnover is $\lambda(1-s)$

Can I do sth like below, I need to define functions and at the end write definition of equilibrium and proposition about existence. How to define cutoffs if I can't show LoMs before saying sth about them?!

### Misallocation and role of flippers

$$M(\delta) = \int_0^\delta \mathbb{1}\{\delta' < \delta^*\} dH(1, \delta') + \int_0^\delta \mathbb{1}\{\delta' > \delta^*\} dH(0, \delta')$$

Households enjoy flow utility $\delta$ from owning an asset and that evolves stochastically, 0 flow for flippers

Figure 2: CDFss for various $f$

(a) $dH(0, \delta)$ (b) $dH(1, \delta)$

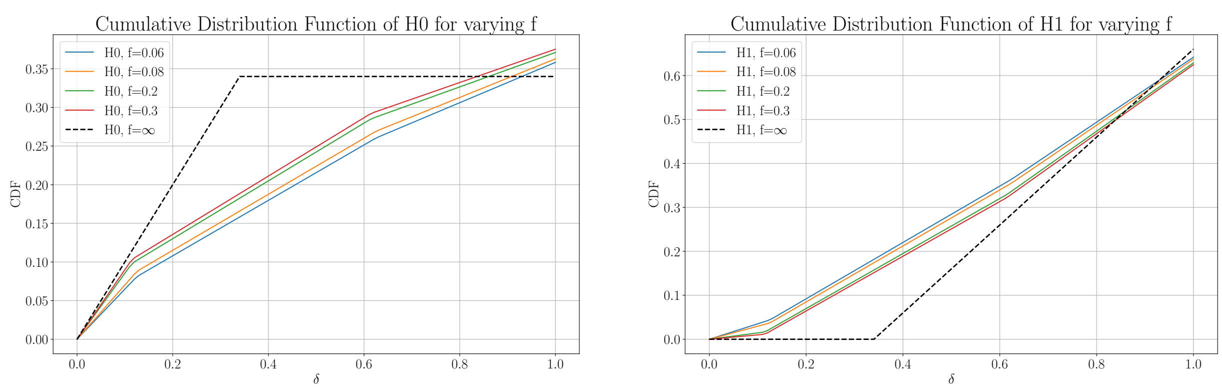
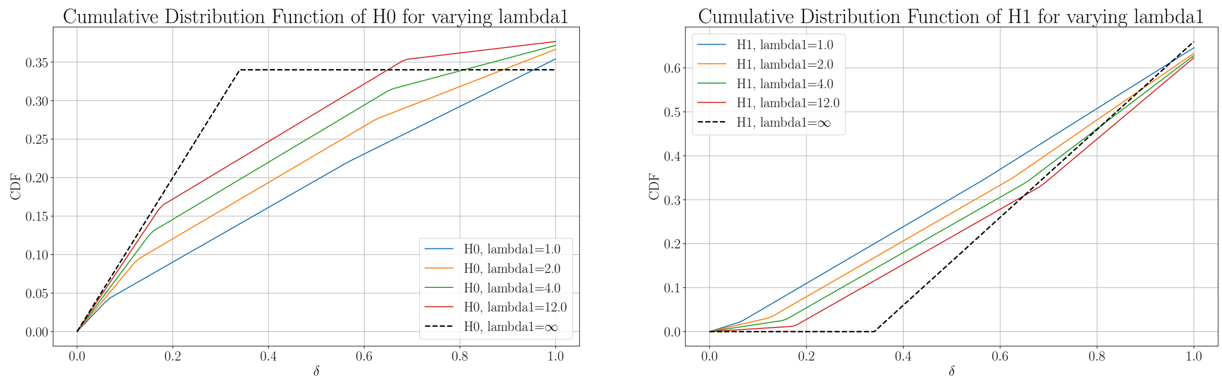


Figure 3: CDFss for various $\lambda$

(a) $dH(0, \delta)$ (b) $dH(1, \delta)$




FOSD. Use Weil paper to comment on position and shape of misallocation vs $\delta^*$. Can it be at all that distribution converges to frictionless limit?

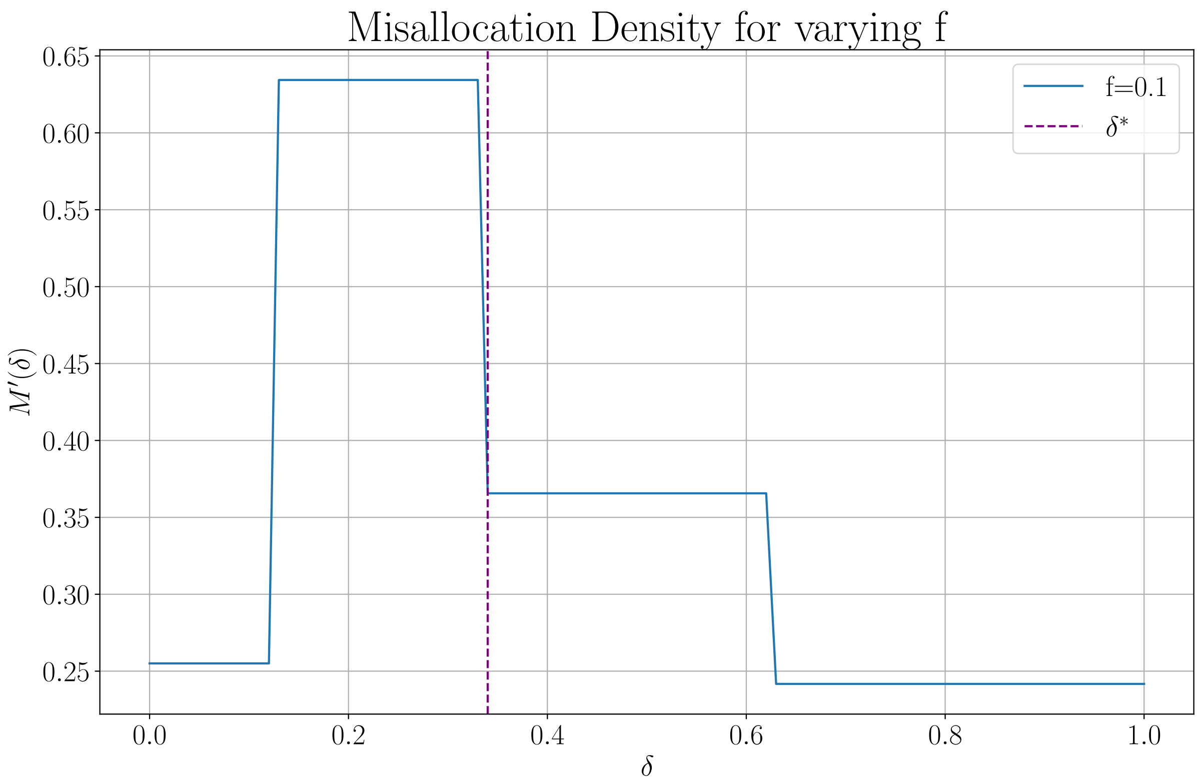
then

$$M'(\delta) = \mathbb{1}\{\delta < \delta^*\}dH(1, \delta) + \mathbb{1}\{\delta > \delta^*\}dH(0, \delta)$$

## 2.2 Comparative statics

Properties of $P(1) - P(0)$, closed for expressions for $W(0), W(1)$, slopes of $V$ and comparative statics goes here (or appendix at the end). Also think if making grid more dense at the ends of $[0, 1]$ makes sense (for sure makes sense in Full model case).

Figure 4: Misallocation density

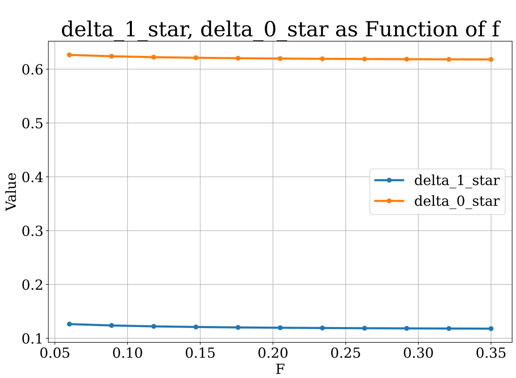


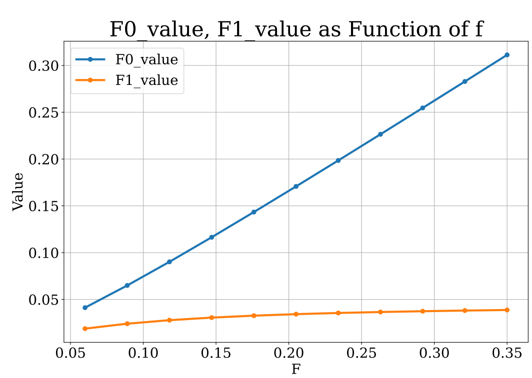

*Notes:* XXX.

Figure 5: Solutions when i vary $f \in [0.06, 0.35]$

(a) $\delta_q^*$

(b) $F(q)$



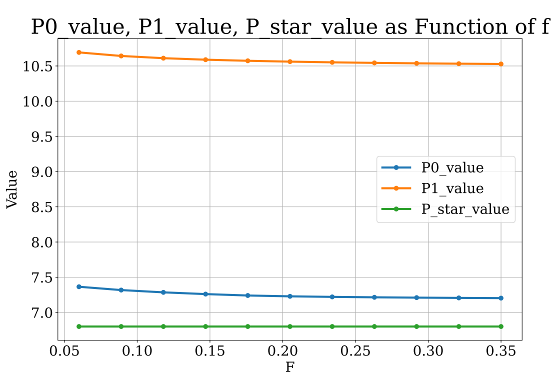

*Note* $\gamma = 0.1, \lambda = 2.0, s = 0.66, f = 0.10, r = 0.05$

| Variable | Relative Change |
|---|---|
| $\delta_0^*$ | -0.0028 |
| $\delta_1^*$ | -0.0138 |
| $F(1)$ | 0.2193 |
| $F(0)$ | 1.3560 |
| $P_0$ | -0.0045 |
| $P_1$ | -0.0032 |
| $dH0_a$ | 0.0652 |
| $dH0_b$ | 0.0847 |
| $dH0_c$ | -0.0337 |
| $dH1_a$ | -0.1597 |
| $dH1_b$ | -0.1563 |
| $dH1_c$ | 0.0182 |
| trade | 0.1435 |
| profit | 1.8456 |
| $\int V(0,\delta)dH(0,\delta)$ | 0.1581 |
| $\int V(1,\delta)dH(1,\delta)$ | -0.0015 |
| Households welfare | 0.0007 |
| $W(0)F(0)$ | -0.1947 |
| $W(1)F(1)$ | -0.1315 |
| Flippers welfare | -0.0667 |

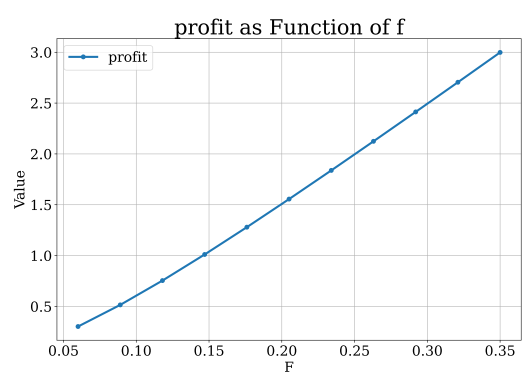
Table 1: Relative Changes with respect to $f$. Change of $f$ from 0.06 to 0.35

Figure 6: Solutions when i vary $f \in [0.06, 0.35]$

(a) $P(q)$



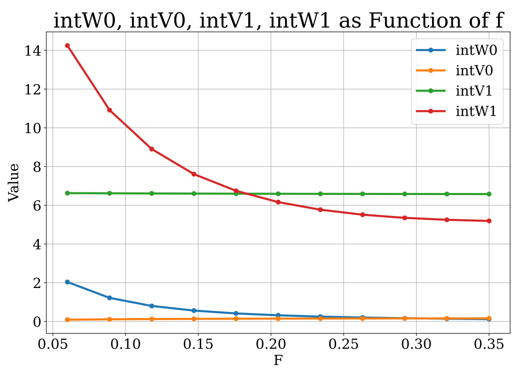

(b) Profit



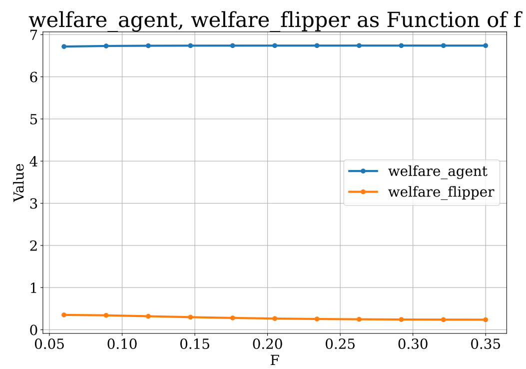

*Note* $\gamma = 0.1, \lambda = 2.0, s = 0.66, f = 0.10, r = 0.05$

Figure 7: Solutions when i vary $f \in [0.06, 0.35]$

(a) welfare of each group (per group)




(b) Aggregate welfare hh vs f




*Note* $\gamma = 0.1, \lambda = 2.0, s = 0.66, f = 0.10, r = 0.05$

## 2.3 Calibrated model (or numerical example?)

Explain calibration to 2012 data, matched moments for $\gamma, \lambda$. Get as validation estimation of $\frac{dP(0)}{df}\frac{f}{P} = -0.021$ and compare it with empirical counter part $\beta = -0.022$ and argue that model works. Fix in code so I generate tables with latex names and get rid of $dH0_i$ from table maybe? Change profit to profit per transaction or return? Am I fine with calibration of $\lambda, f$ or should I do MSM (wchich i have coded for one parameter)- ask Larry,Chari, Chris?!

**Change in $F(1)$ equivalent to change in data**

| Variable | Relative Change |
|---|---|
| $\delta_0^*$ | -0.0127 |
| $\delta_1^*$ | -0.0630 |
| $F(0)$ | 6.1819 |
| $P_0$ | -0.0207 |
| $P_1$ | -0.0145 |
| $dH0_a$ | 0.2974 |
| $dH0_b$ | 0.3862 |
| $dH0_c$ | -0.1537 |
| $dH1_a$ | -0.7279 |
| $dH1_b$ | -0.7125 |
| $dH1_c$ | 0.0831 |
| trade | 0.6544 |
| profit | 8.4143 |
| $\int V(0,\delta)dH(0,\delta)$ | 0.7208 |
| $\int V(1,\delta)dH(1,\delta)$ | -0.0067 |
| Households welfare | 0.0032 |
| $W(0)F(0)$ | -0.8879 |
| $W(1)F(1)$ | -0.5995 |
| Flippers welfare | -0.3043 |

Table 2: Relative Changes with respect to $F(1)$. Change of $f$ from 0.06 to 0.35 that changed $F(1)$ from 0.018 to 0.038

## 2.4 Counterfactuals?

## 2.5 Decomposition- $r$ vs $f$

Take changes in data over 2012-2022 of $r$ and $f$(implied by trade) and changes in prices in data $P_0, P_1$. then see what each change of $r$ and $f$ separately will change for prices, is there interaction term or transition? Can i argue sth about endogeneity of $f$ when changing $r$-ask Chris, Larry and Chari! I think Diego had sth like that (I need to make sure that i look at effect of $r$ only on $P$ and compensate any changes in $H$ so I have real decomposition-talked with Mauz and Braulio about it).

# 3 Full model

Allow for trade between households with them meeting at individual rate $\rho << \lambda$ as well-derive conditions for $dH$ being quadratic in $\delta$, value functions being differentiable at cutoffs and characterize prices (or show their derivatives). Prices between agents now will depend on type (allowing them to know only their own valuation) $P(\delta)$ that will generate distribution of prices- write proposition about regular equilibria and argue for them. Calibrate to the whole data. Or maybe I don't need big model?

# 4 Data

In estimating $\lambda$ I am making supply demand endogeneity mistake I think. I drop casuality link, just stick to correlation. moreover there is first selection to cheap location (regress flippers on prices or leave study event) and maybe later decrease in prices bc of composition of demand (they buy houses no one would buy bc they are rubbish and then actually creating a attractive housing asset but boy oh boy how to show it with my data?). Talk with empirical people!

# 5 Conclusion

# A Balance of trade

In order to get balance of trade condition from law of motions for households to the following: Differentiate (2) (and abuse $d\delta$ notation):

$$dH(0,\delta) + dH(1,\delta) = G'(\delta)d\delta = 1$$

Rearange and differentiate (4) at the interior of intervals (holds for $\delta \neq \delta_1^*, \delta_0^*$ to get

$$\mathbb{1}\{\delta \geq \delta_0^*\}[\lambda F(1)dH(0,\delta)] + \gamma \underbrace{G'(\delta)}_{1} \int_0^{\bar{\delta}} dH(1,\delta) = \tag{9}$$

$$= \mathbb{1}\{\delta \leq \delta_1^*\}[\lambda F(0)dH(1,\delta)] + \gamma dH(1,\delta) \tag{10}$$

Integrate over $\delta$ on $[0,\bar{\delta}]$ to get flippers (inflow= outflow) condition:

$$F(1)\int_{\delta_0^*}^{\bar{\delta}} dH(0,\delta) = F(0)\int_0^{\delta_1^*} dH(1,\delta) \tag{11}$$

# B Characterization of $H, F$

Now let's look at (9) and let's rearrange it

$$\mathbb{1}\{\delta \geq \delta_0^*\}[\lambda F(1)\underbrace{dH(0,\delta)}_{1-dH(1,\delta)}] + \gamma \underbrace{\int_0^{\bar{\delta}} dH(1,\delta)}_{s-F(1)} =$$

$$= \mathbb{1}\{\delta \leq \delta_1^*\}[\lambda \underbrace{F(0)}_{f-F(1)} dH(1,\delta)] + \gamma dH(1,\delta)$$

$$\lambda F(1)\mathbb{1}\{\delta \geq \delta_0^*\} + \gamma(s-F(1)) = dH(1,\delta)\cdot[\lambda(f-F(1))\mathbb{1}\{\delta \leq \delta_1^*\} + \gamma + \lambda F(1)\mathbb{1}\{\delta \geq \delta_0^*\}] \tag{12}$$

which holds for $\delta \neq \delta_1^*, \delta_0^*$.

# C Solving for $H, F$

1. $\delta < \delta_1^* < \delta_0^*$

$$\gamma(s - F(1)) = dH(1,\delta)\cdot[\lambda(f - F(1)) + \gamma]$$

$$dH^1(1,\delta) = \frac{\gamma(s - F(1))}{\lambda(f - F(1)) + \gamma}$$

2. $\delta_1^* < \delta < \delta_0^*$

$$\gamma(s - F(1)) = dH(1,\delta) \cdot [\gamma]$$

$$dH^2(1,\delta) = s - F(1)$$

3. $\delta_1^* < \delta_0^* < \delta$

$$\lambda F(1) + \gamma(s - F(1)) = dH(1,\delta) \cdot [\gamma + \lambda F(1)]$$

$$dH^3(1,\delta) = \frac{\lambda F(1) + \gamma(s - F(1))}{\gamma + \lambda F(1)}$$

$$F(1) \int_{\delta_0^*}^{\bar{\delta}} (1 - dH^3(1,\delta)) = (f - F(1)) \int_0^{\delta_1^*} dH^1(1,\delta)$$

**Claim 2.** *For all parameters there exists a pair : $\{F(1), F(0)\}, \{dH(0,\delta), dH(1,\delta)\} \in [0,1]^2$ such that $F(1) + F(0) = f$ and $\int_0^{\hat{\delta}} dH(0,\delta) + dH(1,\delta) = G(\hat{\delta})$*

# D Proof of existence $F(0), F(1) \in [0, \min\{s, f\}]$

Define polinominal of degree 3, argue that under $\lambda > \gamma$ it's third order term enters with negative number, $!\exists F(1) \in [0, s]$ and show that the same holds for $F(0)$

Because $dH(0,\delta)$ is constant on $[\delta_0^*, \bar{\delta}]$ and $dH(1,\delta)$ is constant on $[0, \delta_1^*]$interval we have

$$F(1)(\bar{\delta} - \delta_0^*)dH(0,\delta_0^*) = F(0)\delta_1^* dH(1,\delta_1^*)$$

From 12 and keeping in mind that trade happens at the cuttoffs as well

$$dH(1,\delta_1^*) = \frac{\gamma(s - F(1))}{\lambda(f - F(1)) + \gamma}$$

$$dH(0,\delta_0^*) = \frac{\gamma(1 - s + F(1))}{\gamma + \lambda F(1)}$$

define

$$g(x) = x(\bar{\delta} - \delta_0^*)(1 - s - x)(\lambda(f - x) + \gamma) - (f - x)\delta_1^*(s - x)(\lambda x + \gamma)$$

$$h(x) = (f - x)(\bar{\delta} - \delta_0^*)(1 - s - f + x)(\lambda x + \gamma) - x\delta_1^*(s - f + x)(\lambda f - \lambda x + \gamma)$$

$$g(0) = -f\delta_1^* s\gamma < 0$$

Assume that $f < s$ , $s + f < 1$

$$g(f) = f(\bar{\delta} - \delta_0^*)(1 - s - f)\gamma$$

So there is a root $(F(1))$ on $(0, f)$ from IVThm. Now for $F(0)$:

$$h(0) = f(\bar{\delta} - \delta_0^*)(1 - s - f)(\gamma) > 0$$

$$h(f) = 0 - f\delta_1^* s\gamma < 0$$

so there is root $F(0)$ in $(0, f)$ as well.

# E Value functions

Fix intervals

Household's problem

$$rV(0,\delta) = \gamma \int_0^{\bar{\delta}} [V(0,\delta') - V(0,\delta)]dG(\delta') + \lambda F(1) \max\{-P_1 + V(1,\delta) - V(0,\delta), 0\}$$

$$rV(1,\delta) = \delta + \gamma \int_0^{\bar{\delta}} [V(1,\delta') - V(1,\delta)]dG(\delta') + \lambda F(0) \cdot \max\{P_0 + V(0,\delta) - V(1,\delta), 0\}$$

Consider three cases

1. $\delta < \delta_1^* < \delta_0^*$ In this case non owner buys $V(0,\delta)$ and owner does not participate in trade $V(1,\delta)$

$$V(0,\delta) = \frac{\gamma}{r+\gamma} \int_0^{\bar{\delta}} V(0,\delta')dG(\delta')$$

$$V(1,\delta) = \frac{\delta}{r+\gamma+\lambda F(0)} + \gamma \int_0^{\bar{\delta}} V(1,\delta')dG(\delta') + \frac{\lambda F(0)}{r+\gamma+\lambda F(0)}(P_0 + V(0,\delta))$$

2. $\delta_1^* < \delta < \delta_0^*$ This is inaction region neither household homeowner nor non owner trades when facing trade opportunity

$$V(0,\delta) = \frac{\gamma}{r+\gamma} \int_0^{\bar{\delta}} V(0,\delta')dG(\delta')$$

$$V(1,\delta) = \frac{\delta}{r+\gamma} + \frac{\gamma}{r+\gamma} \int_0^{\bar{\delta}} V(1,\delta')dG(\delta')$$

3. $\delta_1^* < \delta_0^* < \delta$ In this case owner sells

$$V(0,\delta) = \frac{\gamma}{r+\gamma+\lambda F(1)} \int_0^{\bar{\delta}} V(0,\delta')dG(\delta') + \frac{\lambda F(1)}{r+\gamma+\lambda F(1)}(-P_1 + V(1,\delta))$$

$$V(1,\delta) = \frac{\delta}{r+\gamma} + \frac{\gamma}{r+\gamma} \int_0^{\bar{\delta}} V(1,\delta')dG(\delta')$$

Value functions have kinks at cutoffs ($V(1,\delta_1^*)$ and $V(1,\delta_1^*)$) but they are continuous functions. Calculate reservation value of each of three cases

$$\Delta V(\delta) = V(1,\delta) - V(0,\delta)$$

1. $\delta < \delta_1^* < \delta_0^*$

$$\Delta V(\delta) = \frac{\delta + \gamma \int_0^{\bar{\delta}} \Delta V(\delta')dG(\delta') + \lambda F(0)P_0}{r+\gamma+\lambda F(0)}$$

2. $\delta_1^* < \delta < \delta_0^*$

$$\Delta V(\delta) = \frac{\delta + \gamma \int_0^{\bar{\delta}} \Delta V(\delta')dG(\delta')}{r+\gamma}$$

3. $\delta_1^* < \delta_0^* < \delta$

$$\Delta V(\delta) = \frac{\delta + \gamma \int_0^{\bar{\delta}} \Delta V(\delta')dG(\delta') + \lambda F(1)P_1}{r+\gamma+\lambda F(1)}$$

Notice that value functions are piecewice linear on relevant intervals with following slopes (notice that value functions are non differentiable at kinks) :

$$\frac{dV(0,\delta)}{d\delta} = \begin{cases} 0 & \text{if} \delta \in [0,\delta_0) \\ \frac{\lambda F(1)}{(r+\gamma)(r+\gamma+\lambda F(1))} & \text{if} \delta \in (\delta_0, \bar{\delta}] \end{cases}$$

$$\frac{dV(1,\delta)}{d\delta} = \begin{cases} \frac{1}{r+\gamma+\lambda F(1)} & \text{if} \delta \in [0,\delta_1) \\ \frac{1}{r+\gamma} & \text{if} \delta \in (\delta_1, \bar{\delta}] \end{cases}$$

$$\frac{d\Delta V(\delta)}{d\delta} = \begin{cases} \frac{\lambda F(1)}{(r+\gamma)(r+\gamma+\lambda F(1))} & \text{if} \delta \in (\delta_1, \bar{\delta}) \\ \frac{1}{r+\gamma} & \text{if} \delta \in (\delta_1, \delta_0) \\ \frac{\lambda F(1)}{(r+\gamma)(r+\gamma+\lambda F(1))} & \text{if} \delta \in (\delta_1, \bar{\delta}] \end{cases}$$

Set $\mathbb{E}\Delta V := \int_0^{\bar{\delta}} \Delta V(\delta')dG(\delta')$.

Prices are such that agent a cutoff has zero reservation value then:

$$P_1 = \Delta V(\delta_0^*) = \frac{\delta_0^* + \gamma \mathbb{E}\Delta V}{r+\gamma}$$

$$P_0 = \Delta V(\delta_1^*) = \frac{\delta_1^* + \gamma \mathbb{E}\Delta V}{r+\gamma}$$

then

$$P_1 - P_0 = \frac{\delta_0^* - \delta_1^*}{r+\gamma} \tag{13}$$

keep in mind that cutoffs are functions of prices (because price offers are observed by agents first when deciding about trade cutoff) so $\delta_0^*(P_1)$ ,$\delta_1^*(P_0)$. Assuming differentiability of cuttoffs with respect in equation 13 to prices we get

$$\delta_1^{*\prime}(P_0) = \delta_0^{*\prime}(P_1) = r+\gamma$$

Flipper's problem

$$rW(0) = \max_{P_0} \lambda \int_0^{\delta_1^*(P_0)} dH(1,\delta)[-P_0 + W(1) - W(0)]$$

$$rW(1) = \max_{P_1} \lambda \int_{\delta_0^*(P_1)}^{\bar{\delta}} dH(0,\delta)[P_1 + W(0) - W(1)]$$

**Optimal price choice- perturbation** differentiate wrt $P(0)$ and $P(1)$ respectively to get:

$$0 = \underbrace{\int_{\delta_0^*(P_1)}^{\bar{\delta}} dH(0,\delta)}_{\text{MB to } F(1)\text{from charging more}} - \underbrace{[P_1 + W(0) - W(1)] \cdot \delta_0^{\prime *}(P(1)) \cdot dH(0, \delta_0^*(P_1))}_{\text{MC to } F(1)\text{from changing cuttoff today}}$$

$$0 = \underbrace{\int_0^{\delta_1^*(P_0)} dH(1,\delta)}_{\text{MB to } F(1)\text{from charging less}} + \underbrace{[-P_0 + W(1) - W(0)] \cdot \delta_1^{\prime *}(P_0) \cdot dH(1, \delta_1^*(P_0))}_{\text{MC to } F(1)\text{from changing cuttoff today}}$$

Because $dH(0,\delta)$ is constant on $[\delta_0^*, \bar{\delta}]$ and $dH(1,\delta)$ is constant on $[0, \delta_1^*]$interval we have

$$\int_0^{\delta_1^*(P_0)} dH(1,\delta) = \delta_1^*(P_0) dH(1, \delta_1^*(P_0)) = \delta_1^{*\prime}(P_0) dH(1, \delta_1^*(P_0))(-P_0 + W(1) - W(0))$$

$$\int_{\delta_0^*(P_1)}^{\bar{\delta}} dH(0,\delta) = (\delta_0^*(P_1) - \bar{\delta})dH(0,\delta_0^*(P_1)) = \delta_0^{*\prime}(P_1)dH(0,\delta_0^*(P_1))(-P_1 + W(0) - W(1))$$

$$\frac{\delta_1^*(P_0)}{\delta_1^{*\prime}(P_0)} = -P_0 + W(1) - W(0)$$

$$\frac{\bar{\delta} - \delta_0^*(P_1)}{\delta_0^{*\prime}(P_1)} = P_1 + W(0) - W(1)$$

sum those two to get

$$\frac{\bar{\delta} - \delta_0^*(P_1) + \delta_1^*(P_0)}{r+f} = \frac{P_1 - P_0}{r+f} = \delta_0^*(P_1) - \delta_1^*(P_0)$$

$\bar{\delta} = 1$ so:

$$\frac{1}{2} = \delta_0^*(P_1) - \delta_1^*(P_0)$$

Now plug stuff back to original problem to get $W(1), W(0)$:

$$W(0) = \frac{\lambda(\delta_1^*)^2}{r(r+\gamma)} dH(,\delta_1^*)$$

$$W(1) = \frac{\lambda(1-\delta_0^*)^2}{r(r+\gamma)} dH(0,\delta_0^*)$$

One more step using flipper probelm to get iterative (monotone ) sequence - type the proof idea :constant is negative, contraction and $dH < 1$

$$\delta_1^* = -\frac{\gamma}{2}\mathbb{E}\Delta V + \frac{\lambda}{2r}[(1-\delta_0^*)^2 dH(0,\delta_0^*) - (\delta_1^*)^2 dH(1,\delta_1^*)]$$
$$\delta_0^* = \frac{1}{2} - \frac{\gamma}{2}\mathbb{E}\Delta V + \frac{\lambda}{2r}[(1-\delta_0^*)^2 dH(0,\delta_0^*) - (\delta_1^*)^2 dH(1,\delta_1^*)]$$

# F Code

Clean this up

All functions in code are have as last arguments $\gamma, \lambda\_, s, f, r$ and some other variables.

For fixed $\delta_0^*, \delta_1^*, F(1)$ we define function $dH1\_\text{comp}$ as:

$$dH(1,\delta) = \frac{\lambda F(1)\mathbb{1}\{\delta \geq \delta_0^*\} + \gamma(s - F(1))}{\lambda(f - F(1))\mathbb{1}\{\delta \leq \delta_1^*\} + \gamma + \lambda F(1)\mathbb{1}\{\delta \geq \delta_0^*\}}$$

we define function solve_$F1$ which finds $F(1)$ given $\delta_0^*, \delta_1^*$ using:

$$F(1)\int_{\delta_0^*}^{\bar{\delta}}(1 - dH(1,\delta) = (f - F(1))\int_0^{\delta_1^*} dH(1,\delta)$$

we define function $F1$ which has values of solve_$F1$.
then we define function $dH1$ and plug back $dH(1,\delta)$ and define function $dH0$ and calculate $dH(0,\delta)$as:

$$dH(0,\delta) = 1 - dH(1,\delta)$$

and we define function $F0$ which calculates $F(0)$ as

$$F(0) = f - F(1)$$

Now for fixed $\delta_0^*, \delta_1^*$ we define function value_function that solves for $V(0,\delta), V(1,\delta)$ using value function iterations (expression on right hand side is n-th iteration and expression on right defines n+1 iteration). Interpolate value functions on grid of $\delta \in [0,\bar{\delta}]$ as well to find prices, when you calculate iterations

$$\delta \leq \delta_0^* \quad V(0,\delta) = \frac{\gamma}{r+\gamma}\int_0^{\bar{\delta}} V(0,\delta')dG(\delta')$$

$$\delta > \delta_0^* \quad V(0,\delta) = \frac{\gamma}{r+\gamma+\lambda F(1)}\int_0^{\bar{\delta}} V(0,\delta')dG(\delta') + \frac{\lambda F(1)}{r+\gamma+\lambda F(1)}[-P(1) + V(1,\delta)]$$

$$\delta < \delta_1^* \quad V(1,\delta) = \frac{\delta}{r+\gamma+\lambda(f-F(1))} + \frac{\gamma}{r+\gamma+\lambda(f-F(1))}\int_0^{\bar{\delta}} V(1,\delta')dG(\delta') +$$
$$+\frac{\lambda(f-F(1))}{r+\gamma+\lambda(f-F(1))}[P(0) + V(0,\delta) - V(1,\delta)]$$

$$\delta \geq \delta_1^* \quad V(1,\delta) = \frac{\delta}{r+\gamma} + \frac{\gamma}{r+\gamma}\int_0^{\bar{\delta}} V(1,\delta')dG(\delta')$$

we define function $P0$:

$$P(0) = V(1,\delta_1^*) - V(0,\delta_1^*)$$

we define function $P1$:

$$P(1) = V(1, \delta_0^*) - V(0, \delta_0^*)$$

Update cuttofs by using following reqursion

$$\delta_1^{new} = -\frac{\gamma}{2}\mathbb{E}\Delta V^n + \frac{\lambda}{2r}[(1-\delta_0^n)^2 dH(0,\delta_0^n) - (\delta_1^n)^2 dH(1,\delta_1^n)]$$

$$\delta_0^{new} = \frac{1}{2} - \frac{\gamma}{2}\mathbb{E}\Delta V^n + \frac{\lambda}{2r}[(1-\delta_0^n)^2 dH(0,\delta_0^n) - (\delta_1^n)^2 dH(1,\delta_1^n)]$$

And get $n+1$ iteration using relaxation parameter $\chi = \min\{\frac{1}{\gamma}, \frac{r}{\lambda}\}$:

$$\delta_i^{n+1} = \chi\delta_i^{new} + (1-\chi)\delta_i^n$$

Type it up. $\delta_1 = -\frac{\gamma}{2}\mathbb{E}\Delta V(\cdot) + \dots$

# G Algorithm

Rewrite it by looking at proof and code (more detail)

1. For $n$-th iteration of cutoffs $\delta_0^n, \delta_1^n$
2. Solve for distributions $H^n(q,\cdot), F^n(q)$ using accountings and LOMs
3. Solve for $V^n(q)$ using $\delta_q^n, F^n(q)$ from value functions . Find prices $P^n(q)$
4. Solve for $W^n(q)$ using $\delta_q^n, H^n(q,\cdot)$
5. Update to $n+1$ iteration of $\delta_0^{n+1}, \delta_1^{n+1}$

# References