Recitations 11

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MINI II

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RECITATION 11

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GOOD LUCK!

Q 2 M 2019

$$P(p) = \begin{cases} \{\bar{q} \in \bar{\Delta} : \bar{q} \in \arg\max_q \sum_{m=1}^{M} q_m \cdot z_m(p) \quad p \in \Delta\} \\ \{\bar{q} \in \bar{\Delta} : p \cdot \bar{q} = 0 \quad p \in \partial\Delta\} \end{cases}$$

Δ - int of $\mathbb{R}^m$ subset $\quad p_m > 0$

$\bar{\Delta}$ - closure $\quad \sum p_m = 1 \quad p_m \geq 0$

$M = 2$ $\quad$ (P) - p_1 $\quad p_2 = 1 - p_1$

$p \in [0,1]$

$$z_m(p) = \sum_{i \in I} x_{im}(p, e_i) - \sum_{i \in I} e_{i,m}$$

(a) $p_1 > 0 \quad p_2 = 0$

(b) $p_2 > 0 \quad p_1 = 0$

(c) $p \in \Delta \quad z_1(p) > z_2(p)$

(d) $p \in \Delta \quad \cancel{z_2}(p) < z_2(p)$

(e) $p \in \Delta \quad z_1(p) = z_2(p)$

(a) $p_1 > 0 \quad p_2 = 0$

$$q_1 \cdot \underset{>0}{p_1} + q_2 \, p_2 = 0 \qquad q_1 \cdot \cancel{p_1} = 0 \qquad q_1 = 0$$

$q_1 = 0 \Rightarrow q_2 = 1 \qquad q \in \bar{\Delta}$

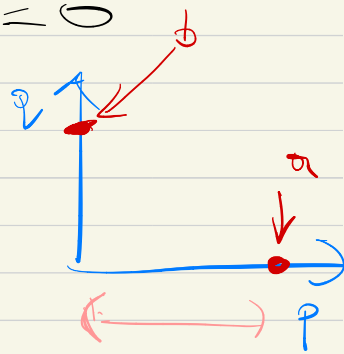
$P(p) = \{0\}$

(b) $p_2 > 0 \quad p_1 = 0$

$$q_1 \cdot \underset{=0}{p_1} + q_2 \cdot \underset{>0}{p_2} = 0$$

$q_2 = 0 \Rightarrow q_1 = 1$

$P(p) = \{1\}$



(c) $p_m > 0 \qquad z_1(p) > z_2(p)$

$$\bar{q} \in \underset{q \in \bar{\Delta}}{\arg\max} \; q_1 z_1(p) + q_2 z_2(p)$$

$q_1 = 1 \qquad q_2 = 0 \qquad \underset{x \in [0,1]}{\max} \; x \cdot a + (1-x) \cdot b \qquad a > b$

(d) $p_m > 0$

$$\bar{q} \in \underset{q \in \bar{\Delta}}{\arg\max} \; q_1 \cdot z_1(p) + q_2 \, z_2(p), \qquad z_1(p) < z_2(p)$$

$q_2 = 1 \Rightarrow q_1 = 0$

(e) $z_1(p) = z_2(p) \qquad p_m > 0$

$$\bar{q} \in \arg\max \; q_1 z_1(p) + q_2 \, z_2(p) = \underbrace{(q_1 + q_2)}_{=1} \cdot z_1(p)$$

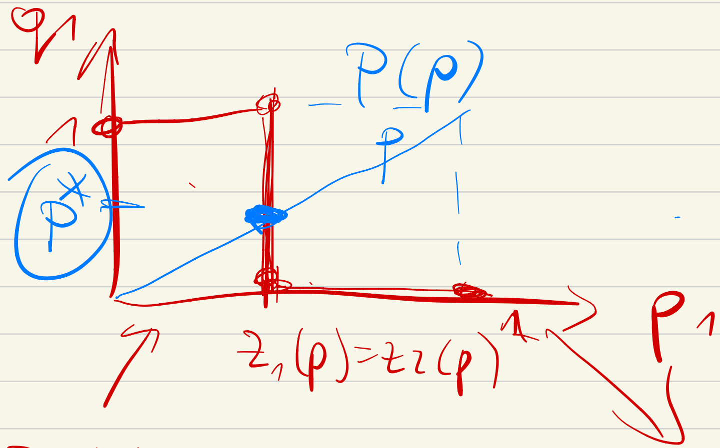
$$\bar{q} \in \arg\max \; q_1 \cdot z_1(p) + q_2 z_2(p)$$

$$= \underbrace{(q_1 + q_2)}_{1} \cdot z_1(p)$$

$\bar{q} \in \bar{\Delta}$ $\quad q_1 \in [0,1]$

$P(p) = [0,1]$ $\quad q_2 = 1 - q_1$

① Draw a correspondence P



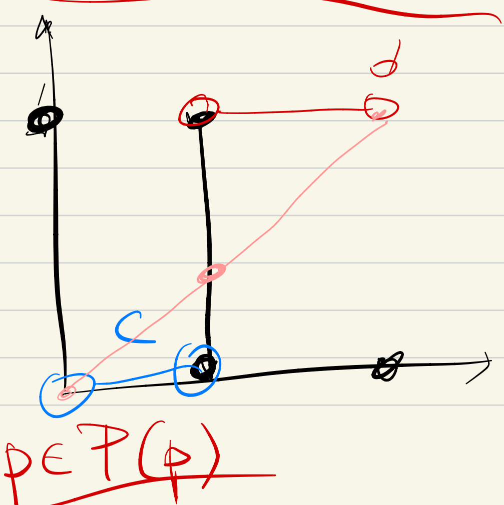
uhc

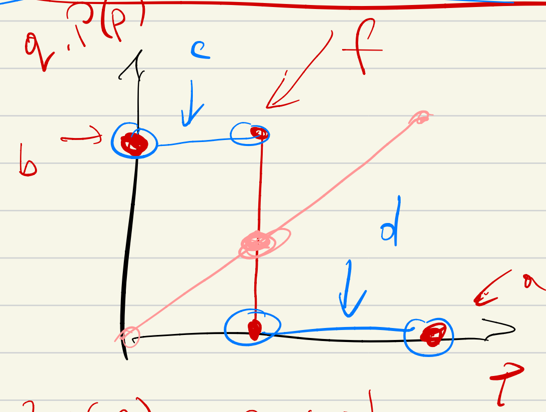
closed

$p \in \partial\Delta$

$p \in \Delta$

$z_1(p) > 0 > z_2(p)$ $\qquad z_1(p) < 0 < z_2(p)$





$z_1(p) > z_2(p)$

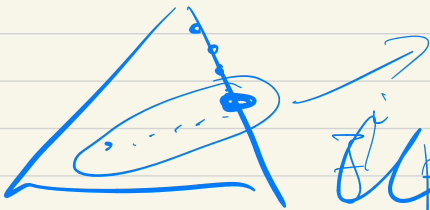
$\Downarrow$

$z_1(p) > 0 > z_2(p)$

closed graph of $P(p)$

- $p \in \Delta \quad p_n \to p$ $\quad p_n \in \Delta$

∘ $p \in \partial\Delta \quad p_n \to p$: $\circledast$



$\mathcal{P}(p)$

$\exists\, i, n$ $\quad$ ~~p~~ p^n int

$$z^i(p^n) < \max_j \left(z^j(p^n) \right)$$

∘ boundary

Welves

Q3 M 2018

$i=1,2.$ $\max\ c_{i1}^2 + c_{i2}^2$

$p_1 \cdot c_{i1} + p_2\, c_{i2} \leq p_1 e_{i,1} + p_2 e_{i,2}$

KKT F assumption

- BC is affine ✓

KKT S
- BC QC ✓
- f is concave

or f is QC $\nabla f(x) \neq 0$

f QC $\Rightarrow x \in \ker Df(x)$

$x D^2 f(x)\, x^T \leq 0$

$\forall x \in \ker Df(x) \quad x \neq 0$

$x \cdot D^2 f(x)\, x^T < 0 \Rightarrow$ QC

- $f(x) = \cancel{x^2}\ x_1^2 + x_2^2$

$Df(x) = 2 \cdot x = 2 \cdot (x_1, x_2)$

$(x_2, -x_1) \in \ker Df(x) \qquad (x_2, -x_1) \cdot (2x_1, 2x_2)$

$= 0$

~~$(1,-1) \in \ker$~~ $Df(1,-1$

$$Df(x) = (2x_1, 2x_2)$$

$$v \in \ker Df(x)$$

$$(v_1, v_2) \cdot (2x_1, 2x_2) = 0$$

$$v_1 = x_2 \qquad v_2 = -x_1$$

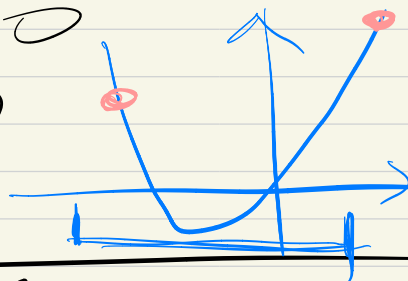
$$(x_2, -x_1) \cdot (2x_1, 2x_2) = 0$$

$$D^2 f(x) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$(x_2, -x_1) \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} (x_2, -x_1)^T$$

$$= 2x_2^2 + 2x_1^2 \not> 0$$

$$< 0$$



It is not QC

$$\max \quad c_{i1}^2 + c_{i2}^2$$

$$p_1 c_{i1} + p_2 c_{i2} = w_i$$

(1) $p_1 > p_2 \qquad c_i = \left(0, \frac{w_i}{p_2}\right)$

(2) $p_1 < p_2 \qquad c_i = \left(\frac{w_i}{p_2}, 0\right)$

(3) $p_1 = p_2 \qquad c_1 (0, e_{11} + e_{12}) \quad c\, e_{21}$

$p_1 = p_2$ BC:

$$p_1 c_{i1} + p_2 c_{i2} \leq p_1 e_{i1} + p_2 e_{i2}$$ (1)

$$w_i = p_1 \cdot e_{i1} + p_2 \cdot e_{i2}$$

MCC:

$$c_{11} + c_{21} = 1 = e_{11} + e_{21}$$

$$c_{12} + c_{22} = 1 = e_{12} + e_{22}$$

Go (1) $p_1 = p = p_2$ (= from mon)

$$p \cdot (c_{i1} + c_{i2}) = p(e_{i1} + e_{i2})$$

$$c_{11} + c_{12} = e_{11} + e_{12}$$

$$c_{21} + c_{22} = e_{21} + e_{22}$$

$$\max c_{i1}^2 + c_{i2}^2$$

$$= \max c_{i1}^2 + (e_{i1} + e_{i2} - c_{i1})^2$$

$$= 2 c_{i1}^2 + (e_{i1} + e_{i2})^2 - 2 c_{i1}(e_{i1} + e_{i2})$$

$$= 2\left(c_{i1} - \frac{e_{i1} + e_{i2}}{2}\right)^2 + \left(\frac{e_{i1} + e_{i2}}{2}\right)^2$$

$c_{i1} = 0$ $c_{i1} = e_{i1} + e_{i2}$

$c_{i1} = 0$ ou $c_{i1} = e_{i1} + e_{i2}$

~~Ch~~

$c_{11} + c_{21} = 1$

$c_{12} + c_{22} = 1$

$($ $c_{11} = 0$ ou $c_{11} = e_{11} + e_{12}$ $)$

and

$($ $c_{21} = 0$ ou $c_{21} = e_{21} + e_{22}$ $)$

1. $c_{11} = 0 = c_{21} = 0$ ↯

2. $c_{11} = e_{11} + e_{12}$
   $c_{12} = e_{21} + e_{22}$ ↯
   $c_{11} + c_{12} = 2$ ↯

3. $c_{11} = 0$ $\quad c_{12} = e_{21} + e_{22}$

4. $c_{21} = 0$ $\quad c_{11} = e_{11} + e_{12}$

CE

$p_1 > p_2 \quad \to c_i = \left(0, \frac{w_i}{p_2}\right)$

$p_1 < p_2 \quad c_i = \left(\frac{w_i}{p_1}, 0\right)$

~~$p_1 = p_2 \quad c_1 = (0, e_{21} + e_{12})$~~

$p_1 = p_2$

$\tilde{c}$: $\begin{cases} c_1 = (0, e_{11} + e_{12}) \\ c_2 = (e_{21} + e_{22}, 0) \end{cases}$

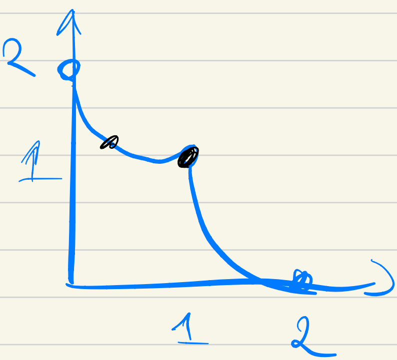
or

$\bar{c}$: $\begin{cases} c_1 = (e_{11} + e_{12}, 0) \\ c_2 = (0, e_{21} + e_{22}) \end{cases}$

$w_i = e_{i1} + e_{i2}$

utility is monotone so ~~$\forall i$~~
1st W Thm $\Rightarrow$ $\tilde{c}, \bar{c}$ are PO

D) 2



~~$\left(\frac{1}{2}, 0\right)$~~ $\left(1, \frac{1}{2}\right)$

~~$\left(\frac{1}{2}, 1\right)$~~ $\left(0, \frac{1}{2}\right)$

WTS

$(1, \frac{1}{2}), (0, \frac{1}{2})$

- To have C.E $p_1 = p_2 = 1$

$w_i = p_1 e_{i1} + p_2 e_{i2} =$
$= e_{i1} + e_{i2}$

For i $i = 1$

$p_1 \cdot c_{11} + p_2 c_{12} = \frac{3}{2}$

CE $\to$ $(0, \frac{3}{2})$ or $(\frac{3}{2}, 0)$

this is when we have > u

For $i = 2$ $(0, \frac{1}{2})$

$p_1 \cdot c_{21} + p_2 c_{22} = w_i$

$w_i = p_1 e_{21} + p_2 e_{22} = e_{21} + e_{22}$

$\cancel{p_1 \cdot e_2} \cdot 0 + 1 \cdot \frac{1}{2} = w_2 = \frac{1}{2}$

CE $\Rightarrow$ $(0, \frac{1}{2})$ $(\frac{1}{2}, 0)$

to get utility >

can we find such endowments

$c_1 = (1, \frac{1}{2})$ $c_2 = (0, \frac{1}{2})$

$$\max \; (c_{i1})^{\alpha} + (c_{i2})^{\alpha}$$

$\cancel{\alpha\alpha}$ $\alpha = 2 \Leftarrow$ Q3 2018

$\alpha = 1$ Q1 2019

$\alpha = \frac{1}{2}$ { Q2 2018, { Q1 2018, Q2 2017 } }

Q2 M 2018

- $\max \; (c_{11})^{1/2} + (c_{12})^{1/2}$

$$p_1 c_{11} + p_{12} \leq 4 \cdot p_1$$

- $\max \; (c_{21})^{1/2} + (c_{22})^{1/2}$

$$p_1 c_{21} + p_2 \leq 4 \cdot p_2$$

FOCs:

$$\frac{c_{12}}{c_{11}} = \left(\frac{p_1}{p_2}\right)^2$$

$$\frac{c_{22}}{c_{21}} = \left(\frac{p_1}{p_2}\right)^2$$

$$c_{11} = \frac{4p_2}{p_1 + p_2} \qquad c_{21} = \frac{4\left(\frac{p_2}{p_1}\right)^2}{p_1 + p_2}$$

MCC:

$$4 = \frac{4p_2}{p_1+p_2} + \frac{4p_2}{p_1+p_2}\left(\frac{p_2}{p_1}\right) \Rightarrow$$

$$4 = \frac{p_1+p_2}{p_1} \cdot \frac{4p_2}{p_1+p_2} \Rightarrow p_1 = p_2$$

$c_{11} = 2 \qquad c_{12} = 2$

$$c_{12} = c_{11} \cdot \left(\frac{p_1}{p_2}\right)^2$$

$c_{11} = 2 = c_{22} = c_{22} = c_{12} = 2$

$(2, 2, 2, 2) = c \qquad p = (1, 1)$

- There is restriction on agent 2

this is PO ~~ex~~

Proof: p' not. $\tilde{c}$ – FEAS

$$u_1(\tilde{c}_1) \geq u_1(c_1)$$

$$u_2(\tilde{c}_2) \geq u_2(c_2)$$

at least one $>$

$$p_1 \cdot \tilde{c}_{11} + p_2 \cdot \tilde{c}_{12} \geq 4p_1$$

$$p_2 \tilde{c}_{21} + p_2 \tilde{c}_{22} \geq 4p_2$$

at least one $>$

$$p_1 (\underbrace{\tilde{c}_{11} + \tilde{c}_{21} - 4}_{0}) + p_2 (\underbrace{\tilde{c}_{21} + \tilde{c}_{22} - 4}_{0}) > 0$$

$0 > 0$ ↯

(b) max

$p_1 c_{11} + p_1 c_{12} \leq 0$

$c_{11} = c_{12} = 0$

max $(c_{21})^{1/2} + (c_{22})^{1/2}$

$c_{21} \leq 3 \quad c_{22} \leq 3$

$p_1 c_{21} + p_2 c_{22} \leq 4(p_1 + p_2)$

$c_{21} = 3 = c_{22}$

$c = (0, 0, 3, 3)$

How to get prices?

$3(p_1 + p_2) \leq 4(p_1 + p_2)$ ✓

any prices works

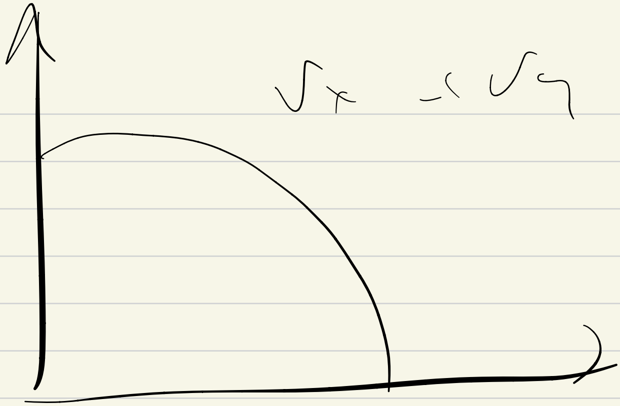
$p_1 \geq 0 \quad p_2 \geq 0$

It is not PO

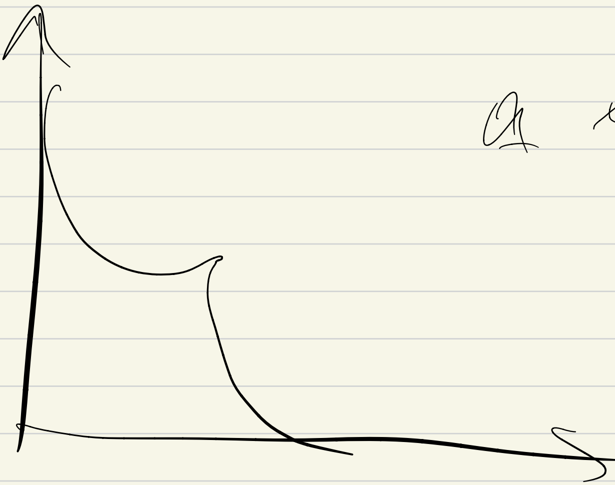
$\hat{c} = (1, 1, 3, 3)$

$u_1(\hat{c}_1) > u_1(c_1)$

$u_2(\hat{c}_2) = u_2(c_2)$



$\sqrt{x} + \sqrt{y}$



$a x^2 + y^2$

$x + y$

$$
\zeta(p) = \begin{cases} \{\, \bar q \in \bar\Delta \;\; \bar q \in \arg\max \sum_{m=1}^{M} q_m \cdot z_m(p) \\ \qquad \text{if } p \in \Delta \\ \{\, q \in \bar\Delta \;\; q \cdot p = 0 \,\} \\ \qquad \text{if } p \in \partial\Delta \end{cases}
$$

$p \in \partial\Delta \longrightarrow$

$p \in \Delta \qquad q_1 z_1(p) + (1 - q_1) z_2(p)$

$= \cancel{p_n}\, q_1 \underline{(z_1(p) - z_2(p))} + z_2(p)$

Convex valued

① $p \in \cancel{\partial\Delta}\; \partial\Delta \qquad q_1, q_2 \in P(p)$

$q_1 \cdot p = 0$

$q_2 \cdot p = 0$

$\forall \lambda \in [0,1] \qquad (\lambda q_1 + (1-\lambda) q_2) \cdot p = 0$

$\lambda q_1 + (1-\lambda) q_2 \in P(p)$ ✓

②

② $p \in \Delta$ $\quad q_1 . q_2 \in P(p)$

$$q_1 \cdot z(p) \geq q \cdot z(p) \quad \forall q \in \Delta$$

$$q_2 \cdot z(p) \geq q \cdot z(p) \quad \forall q \in \Delta$$

$$(\lambda q_1 + (1-\lambda) q_2) \cdot z(p) \geq q \, z(p)$$

$$\lambda q_1 + (1-\lambda) q_2 \in P(p)$$

non empty

$\circ p \in \Delta$: $\max_{q \in \Delta} q \cdot z(p)$

$\downarrow$ fixed

Δ compact

$q \cdot z(p) = \sum q_m \cdot z_m(p)$ is cont.

$f \cdot (q_1, q_2, \ldots q_m)$

Weierstrass thm $\Rightarrow \exists \, q \in \arg\max$

$\circ p \in \partial\Delta$ $\quad q \cdot p = 0$ $\quad \exists q$. non empty uhc: $\Gamma\ \Theta \rightrightarrows X$ X compact

Γ has closed graph $\Rightarrow \Gamma$ uhc

WTS:

$(p, \Gamma(p))$ is closed

$p_n \to p$

More

on

my

webpage