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Question 1 The Social Value of Public Information [221 IV.2 Spring 2016 majors]

There is a continuum of agents, uniformly distributed on $[0,1]$. Each agent $i \in [0,1]$ chooses $a_i \in R$. Let a be the action profile. Agent i has utility function

$$u_i(a,\theta) = -\left[(1-r)\left(a_i-\theta\right)^2 + r\left(L_i - \bar{L}\right)\right]$$

where $r \in (0,1)$ is a constant, θ represents the state of the economy,

$$L_i = \int_0^1 \left(a_j - a_i\right)^2 dj \quad \text{and} \quad \bar{L} = \int_0^1 L_j dj$$

Intuitively, agent i wants to minimize the distance between his action and the true state θ, and also minimize the distance between his action and the actions of others. The parameter r represents the trade-off between these two objectives. Social welfare (normalized) is

$$W(a,\theta) = \frac{1}{1-r}\int_0^1 u_i(a,\theta)di = -\int_0^1 \left(a_i-\theta\right)^2 di$$

Agent i forms expectations $E_i[\cdot] = E\left[\cdot|\mathcal{I}_i\right]$ conditional on his information $\mathcal{I}_i$ and maximizes expected utility.

1. Show that each agent i 's optimal action is given by

   $$a_i = (1-r)E_i[\theta] + rE_i[\bar{a}]$$

   where $\pi = \int_0^1 a_j dj$ is the average action. Show that if θ is common knowledge then $a_i = \theta$ for every i is an equilibrium.

2. Suppose that θ is drawn heuristically from a uniform prior over the real line. Agents observe a public signal

   $$y = \theta + \eta$$

   where $\eta \sim N\left(0,\sigma^2\right)$. Therefore, $\theta|y \sim N\left(y,\sigma^2\right)$. Now, agents maximize expected utility $E\left[u_i|y\right]$ given the same public information y. Show that $a_i(y) = y$ for every i is an equilibrium. Derive the following expression for welfare given θ :

   $$E[W|\theta] = -\sigma^2$$

3. Assume now that, in addition to the public signal, each agent i observes a private signal

$$x_i = \theta + \epsilon_i$$

where $\epsilon_i \sim N(0, \tau^2)$ is (heuristically) independent across i and of θ and η. Let $\alpha = 1/\sigma^2$ and $\beta = 1/\tau^2$

a) Show that

$$E_i[\theta] = E[\theta | x_i, y] = \frac{\alpha y + \beta x_i}{\alpha + \beta}$$

b) Suppose that there is a number κ such that for every agent j

$$a_j(x_j, y) = \kappa x_j + (1 - \kappa) y$$

Compute the value of $E_i[\bar{a}]$ and show that

$$\kappa = \frac{\beta(1-r)}{\alpha + \beta(1-r)}$$

defines an equilibrium.

4. Show that expected welfare is given by

$$E[W(a,\theta)|\theta] = -\frac{\alpha + \beta(1-r)^2}{[\alpha + \beta(1-r)]^2}$$

Show that

$$\frac{\partial E[W|\theta]}{\partial \beta} > 0$$

and $\frac{\partial E[W|\theta]}{\partial \alpha} \geq 0$ if and only if $\frac{\beta}{\alpha} \leq \frac{1}{(2r-1)(1-r)}$ Interpret and compare with your answer to part (b).