Previous finals- scatches of solutions

Question 1 [Final 2020]

1. Prove that a finite extensive form game of perfect information has an equilibrium in pure strategies.
2. Present an example of an extensive form game that does not have an equilibrium in pure strategies. but where each player has at least one information set which is a singleton.

Solution

1. For game of Perfect recall we can use Backward Induction which gives us strategy in pure actions. What we need to show is that result of BI is NE

Lemma 0.1. *Result of BI is NE in pure strategies*

Proof. (This proof I found in internet). Let $s^* = (s_1^*, \ldots, s_n^*)$ be the outcome of Backward Induction. Consider any player i and any strategy s_i. To show that s^* is a Nash equilibrium, we need to show that

$$u_i\left(s^*\right) \geq u_i\left(s_i, s_{-i}^*\right)$$

Take any node at which player i moves, and s_i^* and s_i prescribe the same moves for player i at every node that comes after this node.

(There is always such a node; for example the last node player i moves.) Considera new strategy s_i^1 according to which i plays everywhere according to s_i except for the above node, where he plays according to s_i^*.

According to $\left(s_i^1, s_{-i}^*\right)$ or $\left(s_i, s_{-i}^*\right)$, after this node, the play is as in $\left(s_i^*, s_{-i}^*\right)$, the outcome of the backward induction.

Moreover, in the construction of s^*, we have had selected the best move for player i given this continuation play. Therefore, the change from s_i to s_i^1, can only increase the payoff of i :

$$u_i\left(s_i^1, s_{-i}^*\right) \geq u_i\left(s_i, s_{-i}^*\right)$$

Applying the same procedure to s_i^1, now construct a new strategy s_i^2 that differs from s_i^1 only at one node, where player i plays according to s_i^*, and

$$u_i\left(s_i^2, s_{-i}^*\right) \geq u_i\left(s_i^1, s_{-i}^*\right)$$

Repeat this procedure, producing a sequence of strategies $s_i \neq s_i^1 \neq s_i^2 \neq \cdots \neq s_i^m \neq \cdots$. Since the game has finitely many nodes, and we are always changing the moves to those of s_i^*, there is some M such that $s_i^M = s_i^*$. By construction, we have

$$u_i\left(s^*\right) = u_i\left(s_i^M, s_{-i}^*\right) \geq u_i\left(s_i^{M-1}, s_{-i}^*\right) \geq \cdots \geq u_i\left(s_i^1, s_{-i}^*\right) \geq u_i\left(s_i, s_{-i}^*\right)$$

completing the proof. It is tempting to conclude that backward induction results in Nash equilibrium be- cause one plays a best response at every node, finding the above proof unnecessarily long. □

2) vide Q4 2018

Question 2 [Final 2020]

1. Define subgame perfect equilibria and sequential equilibria.

2. State whether any of the following is true or false, and provide either a proof or a counterexample to prove your / claim:

   (a) The behavioral strategy profile of a sequential equilibrium is a subgame perfect equilibrium.

   (b) For any subgame perfect equilibrium there is a sequential equilibrium with that behavioral strategy profile

Solution

1.

Definition 0.2 (Subgame perfect equilibrium (SPE)). *For a finite EFG, a behavioral strategy profile $\beta \in B$ is a subgame perfect equilibrium if and only if $\forall x \in \mathcal{X}$ for which G_x is a well defined EFG the restriction of β to G_x is a* NE *of G_x. Intuitively, a SPE is a NE that is also a NE in every subgame.*

2.a)

Theorem 0.3. *Every SE is a SPE.*

Proof. Suppose not, i.e. there exists some $\beta \in B$ that's a part of an SE of some finite game G but that's not a SPE for G. Then we know there exists some subgame G_x such that β_x, the restriction of β to G_x, is not a NE. Thus there exists some $b_x^i \in B_x^i$ for some player i such that:

$$\sum_{z \in Z_x} u^i(z) \prod_{x \in \text{Path}(z) \cap \mathcal{I}_x^i} b_x^i\left(x, c_z\right) \prod_{x \in \text{Path}(z) \setminus \mathcal{I}_x^i} \beta_x^{-i}\left(x, c_z\right) > \sum_{z \in Z_x} u^i(z) \prod_{x \in \text{Path}(z)} \beta_x\left(x, c_z\right)$$

Note that the l.h.s. of the above inequality can be expressed as some system of beliefs μ induced by β_x. This implies b_x^i is optimal with respect to μ given β_x^{-i}, but since β is a SE, β_x^i is optimal with respect to μ given β_x^{-i}.
But this is of the above inequality. So it must be that for $\beta \in B$ that's a part of an SE it is also a SPE. $\square$

2.b) In Selten Horse example from last class Type2 equilibria : $p_1 = 0$,$p_2 \in [\frac{1}{3}, 1]$ and $p_3 = 1$, are SPE but not SE.

Question 3 [Final 2020]

In the Race (m, k) game, where m and k are two positive integers with $m > k$, two players alternate in subtracting coins from a pile of coins with m coins initially. Player 1 starts, and can take away 1 or 2 or $, \ldots k$ coins (note he must take away at least 1). Then the move goes to player 2 who can do the same from the remaining pile. The player who removes the last coin wins.

1. Write the extensive form game for Race$(5, 3)$.
2. Write the set of pure strategies for player 1 in Race$(5, 3)$.
3. Find the optimal strategy profiles (that is, find out (1) when a player can win and (2) how he can win when he can) for the Race (5,3) game.
4. Find the optimal strategy profiles for the Race (m, k) game.

Solution

Race(5,3)

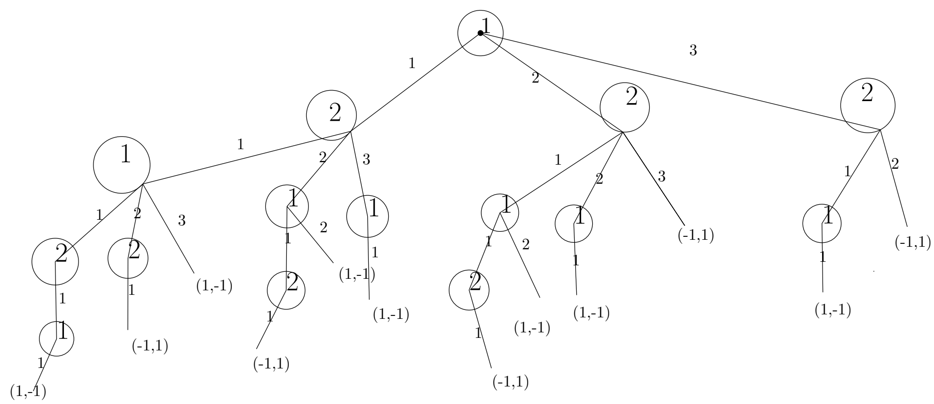

Figure 1

Question 4 [Final 2020]

The Backward Induction procedure is based on replacing the nodes that have all immediate successors in the set of final nodes, in finite games that are also of perfect information. We consider here whether one can extend this procedure the other games

1. Consider first the case in which some of the information sets are not singletons

   a) Can you identify a "last information set" (that is an information set such that all immediate successors of all the nodes in the set are in the set of final nodes?

   b) When such a set exists, what can you do?

2. Consider next the case in which the game has countably many nodes.

   a) Can you define a backward induction procedure?

   b) Can you modify the definition in some way to implement the idea of backward induction in this case?

Solution

Definition 0.4 (The backward induction procedure)**.** *in a game with perfect information is as follows:*

1. *For each node $x \in X$ such that $IS(x) \subseteq Z$, choose an action $s^i(x) \in C_x^i$ of the player $i \in I$ with $x \in P^i$ such that $s^i(x)$ leads to a node $z \in IS(x)$ with $u^i(z) \geq u^i(z') \forall z' \in IS(x)$*

2. *Replace x with a final node with utilities u_x, where u_x is the vector of utilities resulting from $s^i(x)$*

3. *Repeat $(1)-(2)$ until an action $s^i(x)$ has been assigned to every $x \in P^i$ for all $i \in I$.*

The resulting pure strategy profile $(s^i(x))_{x \in P^i, i \in I}$ is called a solution to the backwards induction procedure.

Question 1 [Final 2019]

1. Prove that for any finite EFG of perfect information, there is a last move node, that is a move node x such that $IS(x) \subseteq Z$.

2. Prove, or disprove by showing a counter-example to the statement: In any finite EFG of perfect recall, there is a last information set I^i for some player i, that is, an information set such that for any node $x \in I^i, IS(x) \subseteq Z$.

Solution

1) Suppose not.
Then $\exists$ node $y_1 \in IS(x), y_1 \notin Z$ and
Then $\exists$ node $y_2 \in IS(y_1), y_1 \notin Z$
...
$\forall n \quad \exists$ node $y_n \in IS(y_1), y_{n-1} \notin Z$ violates EFG beign finite.

2) Suppose not . Thn every info set I^i_F for each player i has a moving node

- all these info sets belong to different player $\rightarrow$ infinite number of players violates finite EFG
- $\exists$ some player i his info set repeated shows up on some path, rename the info set as $I^i_{k_1}, \ldots I^i_{k_n}, \ldots$ by order of the path : $\exists y_1 \in I^i_{k_1}, c_1 \in C_{I^i_{k_1}}$, $y_2 \in I^i_{k_2}, c_2 \in C_{I^i_{k_2}}$ and $y_2 \succeq y_1$. By perfect recall (eliminate the possibility of having redundant structure): $\forall x \in I^i_{k_2}$ $x \succeq y_1$ i.e. all the nodes of $I^i_{k_2}$ should come after info set $I^i_{k_1}$.
  Some $I^i_{k_3}, \ldots I^i_{k_n}, \ldots$ means that i has infinite number of info sets which violates finite EFG.

Question 2 [Final 2019]

In this question we will consider only finite Extensive Form Games (EFG) of perfect recall. Take a behavioral strategy profile $\hat{\beta}$ of the EFG. A one-stage deviation from $\hat{\beta}^i$ for player i is a behavioral strategy γ^i of i that differes from $\hat{\beta}^i$ at a single information set.

1. Define EFG of perfect information.
2. The One Stage Deviation Principle for finite EFG of perfect information is the following statement:

   "A behavioral strategy profile is a subgame perfect equilibrium of the EFG of perfect information if and only if for no player and no subgame there is a one-stage deviation in the subgame that gives to the deviating player a strictly larger payoff in that subgame, than the original strategy."

   Prove the One Stage Deviation Principle for games of perfect information.

Solution

Question 3 [Final 2019]

Consider again the One Stage Deviation Principle.

1. State the principle for games for general extensive form games with perfect recall, but not necessarily of perfect information. Do you need some additional condition in your statement, as compared to the state of the principle in the case of games of perfect information?
2. Prove your claim.

Solution

Question 4 [Final 2019]

1. The Three players matching pennies is, in word, the following extensive form game.

   a) Player 1 moves first, and chooses head (H^1) or tails (T^1) . This choice is not communicated to anyone else.

   b) After the choice of player 1 , player 2 moves second, and chooses heads (H^2) or tails (T^2).This choice is not communicated to anyone else.

   c) Player 3 is told whether the choices of player 1 and 2 matched (that is, they were either H^1 and H^2, or T^1 and T^2) , or they did not, but not the specific choices. Below, "match" and "not match" will be defined analogously.

   d) Player 3 then moves, and chooses heads (H^3) or tails (T^3) ; then the game ends.

   e) Payoffs are as follow:

   - If the choice of player 3 matches that of player 1, then player 1 pays two units each to player 2 and 3, so payoffs are (-4,2,2)

- If the choice of player 3 does not match that of player 1, then player 2 and 3 pay two units each to player 1, so payoffs are (4,-2,-2)

Please address the following questions:

(a) Write the extensive form of the game;

(b) Find the Nash equilibria of the game;

(c) Find the sequential equilibria of the game.

2. Now consider again the Three players matching pennies above, but now allow players 1 and 2 to exit at their information set, in the order. Payoffs at these additional end notes are as follows:

   - If player 1 exits, the game ends and payoffs are (-2,1,1);
   - If player 2 exits, the game ends and payoffs are (2,-1,-1); The payoffs at other end nodes are as in the previous game. Please address the following questions

   (a) Write the extensive form of the game

   (b) Find the Nash equilibria of the game

   (c) Find the sequential equilibria of the game.

Solution

Question 1 [Final 2018]

Consider the following game:

- $I = \{1, 2\}$. Nature moves, and decides a quantity M which can take values 2 and 5 with probability $p \in [0, 1]$.M is told to the player 1 but not told to the player 2 .
- Player 1 has to decide how much out of 10 units he transfers to the player 2. This amount, denoted by a, must be an integer. a is multiplied by M and the total amount Ma is then communicated to player 2.
- Player 2 can then decide an amount b, non-negative integer less or equal than Ma to be transferred back to player 1.
- Final payoffs are $(10 - a + b, Ma - b)$. Then the game ends.

a) Write it as a EFG.

b) Find the Nash equilibria of this game.

c) Find the subgame perfect equilibria for this game.

Solution

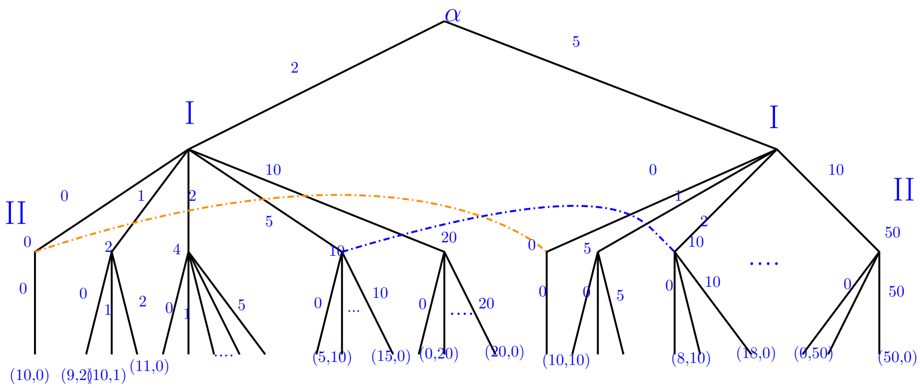

Figure 2

Information sets : MR1 $J^1 = \{I^1_1, I^1_2\}$
Mr2 $J^2 = \{I^2_{Ma} : \quad M_a \in \{0, 2, 4, 5, 6, 8, 10, 12, 14, 15, 16, 18, 20, 25, 30, 35, 40, 45, 50\}\}$

$G :\succeq$ partial order
$U(a, b) = (10 - a + b, Ma - b)$
$C^1_k = \{0, 1 \ldots 10\} \quad k = 1, 2$
$C^1_{Ma} = \{0, 1 \ldots Ma\}$

Definition 0.5 (Extensive Form Game)**.** *consists of*

- *The set of all nodes is $\mathcal{X}$.*
- *The set of all final nodes is $Z = (z^1, z^2, \ldots)$, where z are the consequences of the EFG.*
- *The initial node is α. Nature, sometimes denoted player 0 , chooses α with $p \in \Delta(IS(\alpha))$*
- *The set of move nodes for player i is X^i; also called player i 's partition. Note $\forall i \neq j$, $X^i \cap X^j = \emptyset$ and $\cup_{i \in I} X^i \equiv X = \mathcal{X} \backslash \{\alpha, Z\}$*
- *Let $\succeq$ be an asymmetric partial order over $\mathcal{X}$, where for $x, y \in \mathcal{X}, x \succeq y$ means x comes after y. Note that $\forall x \in \mathcal{X}, x \succeq \alpha$*
- *$\forall x, y \in \mathcal{X}$, let $x \succeq_c y$ mean x follows action c played at y.*
- *$\forall x \in \mathcal{X}$, the set of predecessor nodes is $P(x) \equiv \{y \in \mathcal{X} \mid x \neq y, x \succeq y\}$.*
- *$\forall x \in \mathcal{X}$, the set of immediate predecessor nodes is*

$$IP(x) \equiv \{z \in P(x) \mid \nexists y \neq z, y \neq x, z \preceq y \preceq x\}$$

- *$\forall x \in \mathcal{X}$, the set of successor nodes is $S(x) \equiv \{y \in \mathcal{X} \mid x \neq y, x \preceq y\}$.*
- *$\forall x \in \mathcal{X}$, the set of immediate successor nodes is*

$$IS(x) \equiv \{z \in S(x) \mid \nexists y \neq z, y \neq x, z \succeq y \succeq x\}$$

Observe that $Z = \{x \in \mathcal{X} \mid S(x) = \emptyset\}$.

- *$\forall i, u^i : Z \to \mathbb{R}$ is a vNM utility function.*

- *An information set for player i is I^i_k, where $k = 1, \dots, K^i$. Note $\forall k \neq j, I^i_k \cap I^i_j = \emptyset$ and $\cup_{k=1}^{K^i} I^i_k = X^i$. Player i 's set of information sets is $\mathcal{I}^i \equiv X_{k=1}^{K^i} I^i_k$*
- *For each $I^i_k \in \mathcal{I}^i$, an action for player i is $c^i_{I^i_k}$, or equivalently c^i_k. The set of actions for player i is $C^i_{I^i_i}$, or equivalently C^i_k.*

b) Mr 1 may hae intention to give sth to Mr2 as it multiplies and can get back ($b = Ma$). However Mr2 has no incentive to give back anything and Mr1 knows it. Therefore NE:

$$a(2) = a(5) = 0$$

$$BR(Ma) = \begin{cases} \{0, 1, \dots Ma/2\} & \text{if } Ma \in \{0, 2, 4, \dots 18\} \\ \{0, 1, \dots Ma/5\} & \text{if } Ma \in \{0, 10, 20, \dots 50\} \end{cases}$$

To ensure payoffs of Mr1 are ≤ 10 then $a(2) = a(5) = 0$ belongs to $BR(b(Ma)$ so Mr1 can not get more than 10.
Any strategy of Mr2 belongs to $BR(0)$

c)

$$SPE = \{(a, b) = (0, 0)\}$$

Mr 2 won't transfer back anything so Mr1 plays 0.

Question 2 [Final 2018]

In the following, fix the set of players I and for each player $i \in I$ fix the set of actions A^i.

1. The Nash equilibrium existence theorem shows that one can associate a set of mixed strategy profiles to every profile of utilities $(u^i)_{i \in I}$. This defines the Nash equilibrium correspondence. Prove that this correspondence is u.h.c. Define carefully all objects you define.

2. Characterize the most general form of transformations on utilities that leaves the best response unchanged.

Solution

Question 3 [Final 2018]

Consider a game: Find all the sets that survive IEWDS.

	L	C	R
T	(1,2)	(2,3)	(0,3)
M	(2,2)	(2,1)	(3,2)
B	(2,1)	(0,0)	(1,0)

Solution

IEWDS - three different solutions 1)

$C_1^0 = \{T, M, B\}$	$C_2^0 = \{L, C, R\}$
$C_1^1 = \{T, M, B\}$	$C_2^1 = \{L, R\}$
$C_1^2 = \{M, B\}$	$C_2^2 = \{L, R\}$
$C_1^3 = \{M, B\}$	$C_2^3 = \{L\}$
$\ldots$	$\ldots$
$C_1^\infty = \{M, B\}$	$C_2^\infty = \{L\}$

2)

$C_1^0 = \{T, M, B\}$	$C_2^0 = \{L, C, R\}$
$C_1^1 = \{T, M, B\}$	$C_2^1 = \{R\}$
$C_1^2 = \{M\}$	$C_2^2 = \{R\}$
$\ldots$	$\ldots$
$C_1^\infty = \{M\}$	$C_2^\infty = \{R\}$

3)

$C_1^0 = \{T, M, B\}$	$C_2^0 = \{L, C, R\}$
$C_1^1 = \{T, M, B\}$	$C_2^1 = \{L, R\}$
$C_1^2 = \{M\}$	$C_2^2 = \{L, R\}$
$\ldots$	$\ldots$
$C_1^\infty = \{M\}$	$C_2^\infty = \{L, R\}$

Question 4 [Final 2018]

1. Present an examples of an EFG that does not have equilibrium in pure strategies but where each player has at least one information set which is a singleton.
2. In a finite EFG let Y be non-empty subset of the set of nodes of the tree X, with the partial order induced by the restriction of the order of the tree. Prove that there is an element in Y with no immediate successors.
3. Prove that in finite EFG there is a node x which has all the immediate successors in the set of final nodes.
4. Prove that in an EFG of perfect information every node defines a subgame.

Solution 1) uniqe NE ($\frac{1}{2}, \frac{1}{2})(\frac{1}{2}, \frac{1}{2})$. it survives Backward induction

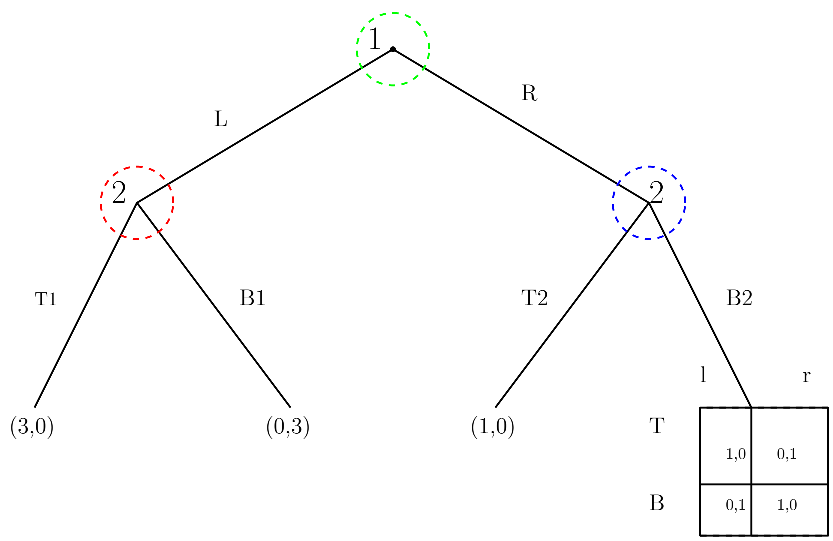

Figure 3

2) Let $Y = \{x_1, \dots x_k\}$. WTS: $\exists x \in Y : IS(x) = \emptyset$

Consider following cases

- If any final node belongs to Y then the result holds trivially
- Suppose no find nodes belongs to Y.
  Suppose for a contradiction that each element of Y has an immediate successor:

$$\forall x \in Y \exists y \in Y : \quad y \in IS(x)$$

  Divide $x \in Y$ into subsets defined over paths to final nodes of the game, i.e. $P(z) : z \in Z$. If there is no more than one node that belongs to each path we get a contradiction.

Now suppose there is a path with at least two nodes in the restriction. By assumption they are linearly ordered . Reorder them as: $z^1, z^2 \dots z^k$ s.t.

$$z^1 \succeq z^2 \succeq \dots \succeq z^k$$

Then z^1 does not have an immediate succesor which is a contradiction

3) Suppose not. Then:

$$\forall x \exists y : y \succeq x \quad y \notin Z \quad y \neq x$$

But this contradicts the game being finite 4)

$$G = (\alpha, \mathcal{X}, \succeq, P, (X^i, I^i_k, C^i_{I^i_k}, u^i)_{i \in I})$$

defines EFG.
Note that trivial to start with a final node or with nature node. Define G_x as:

$$I_x = I, \quad \mathcal{X}_x = \{y \in \mathcal{X} : y \succeq x\}, \quad X^i_x = \{y \in X^i : y \in \mathcal{X}_x\}$$

etc. Subgame has to satisfy:

$$\begin{aligned}
&\mathcal{X}_x = S(x) \cup \{x\} \\
&\succeq_x = \succeq \mid \mathcal{X}_x \\
&X^i_x = X^i \cap \mathcal{X}_x \\
&Z_x = Z \cap S(x) \\
&u^i_x : Z_x \to \mathbb{R} \\
&\mathcal{I}^i_x = \{I^i_k \in \mathcal{I}^i \mid I^i_k \subseteq S(x)\}
\end{aligned}$$

Subgame has to satisfy

$$\forall i \in I \forall I^i_k \quad I^i_k \cap G_x = \emptyset \quad \text{or} \quad I^i_k \subseteq G_x$$

Since G is of perfect info

$$\forall i \in I \forall I^i_k \quad ||I^i_k|| = 1$$

thus information set is or is not in subgame

$$\forall i \in I \forall I^i_k \quad I^i_k \cap G_x = \emptyset \quad \text{or} \quad I^i_k \subseteq G_x$$

Question 1 [Final 2017]

A normal form game has I players, action sets A^i and utility functions u^i, for $i \in I$. The vector $u^i : i = 1, \ldots, n$ is a utility profile. Fix the set of players and the action sets. The Nash equilibrium correspondence (NEC) associates to every utility profile the set of Nash equilibria of the normal form game with that profile.

1. Prove that one can restrict the domain of the correspondence to a compact set of ntility profiles.
2. Prove that the NEC is closed valued.
3. Prove that the NEC is upper-herni-continuous, or find an example to show it is not

Solution

Question 2 [Final 2017]

Define a Perfect Equilibrium of a normal form game as the limit of sequences of Nash equilibria of a perturbed game.

1. Prove that a perfect equilibrium is a Nash equilibrium.
2. A strategy is fully mixed if it gives positive probability to all the actions in the action set of the player, and a strategy profile is fully mixed if the strategies of all players are fully mixed. Prove that a fully mixed strategy profile is perfect.
3. Give as simple sufficient condition for a Nash equilibrium to be perfect.

Solution

Theorem 0.6. *If $s \in S$ is a PE, then it is also a NE.*

Proof. Let $s \in S$ be a PE. Then $\exists \{\epsilon_m\}_{m \in \mathbb{N}}, \{s_m\}_{m \in \mathbb{N}}$ such that $\epsilon_m \to 0, s_m \to s$, and $\forall m \in \mathbb{N}, s_m$ is a NE of the ϵ_m -perturbed game. Take any $i \in I$ and any $t^i \in S^i$. Since $\epsilon_m \to 0$, it follows that $\epsilon_m^i \to 0$, and thus there exists a sequence $t_m^i \in S_{\epsilon_i^i}^i$ such that $t_m^i \to t^i$. Take such a sequence. Then, since s_m is a NE of the ϵ_m -perturbed game, it follows that

$$u^i\left(s_m^i, s_m^{-i}\right) \geq u^i\left(t_m^i, s_m^{-i}\right) \quad \forall m \in \mathbb{N}$$

Since $u^i(\cdot)$ is continuous $\forall i \in I$, then

$$\begin{aligned} & \lim u^i\left(s_m^i, s_m^{-i}\right) \geq \lim u^i\left(t_m^i, s_m^{-i}\right) \\ & \Longrightarrow u^i\left(s^i, s^{-i}\right) \geq u^i\left(t^i, s^{-i}\right) \end{aligned}$$

Since $t^i \in S^i$ was taken arbitrarily, $s^i \in BR^i\left(s^{-i}\right)$. Since $i \in I$ was taken arbitrarily, $s \in BR(s)$, so s is a NE □

Theorem 0.7. *If $s \in S$ is a fully mixed* NE, *then it is also a* PE.

Proof. Let $s \in S$ be a fully mixed NE for some finite NFG, i.e. $\forall i \in I, \forall a^i \in A^i$, $s^i\left(a^i\right) > 0$. From this, note there exists

$$\bar{s}^i \equiv \min_{a^i \in A^i} s^i\left(a^i\right) \forall i \in I \text{ and } \bar{s} \equiv \min_{i \in I} \bar{s}^i$$

and that $\bar{s} > 0$. It follows that, for any sequence of perturbations $\{\epsilon_n\}_{n \in \mathbb{N}}$ such that $\epsilon_n \rightarrow 0, \exists N \in \mathbb{N}$ such that, $\forall m \geq N$

$$\forall i \in I \forall a^i \in A^i, \quad e_m^i\left(a^i\right) < \bar{s}$$

so $\forall m \geq N, s \in S_{\epsilon_m}$. Now recall that since s is a NE of the original game,

$$\forall i \in I, \forall t^i \in S^i, \quad u^i\left(s^i, s^{-i}\right) \geq u^i\left(t^i, s^{-i}\right)$$

Note that $S^i_{\epsilon^i_m} \subseteq S^i$, so since $\forall m \geq N, s \in S_{\epsilon_m}$, we know that $\forall m \geq N, s$ is a NE of the ϵ_m -perturbed game. Now take a sequence $\{s_m\}$ such that $s_m = s \forall m \in \mathbb{N}$ and construct a new sequence of perturbations $\{\hat{\epsilon}_m\} = \{\epsilon_m\}_{m \geq N}$. Then s is a PE by definition. □

3) $s \in NE(\Gamma) \iff$
$\exists\left(s_n\right) \in S^{\infty}$ s.t. (i) s_n is fully mixed, (ii) $s_n \rightarrow s$ and (iii)$\forall i, n \quad s^i \in BR^i_{s^i}\left(s_n^{-i}\right)$

Question 3 [Final 2017]

As in question 1, fix the set of players and the action sets.

1. Identify the set of all transformations of the utility function of a player that leave the Best Response correspondence unchanged. Please note you will state and prove statement like "The set of transformations T (put the definition here) of the utility functions of player 1 is such that for all normal formal games with that player set and action sets the Best Response for the utility function is u^1 and for its transformation $T(u^1)$ are the same."

2. Prove your answer in detail.

Solution

Question 4 [Final 2017]

1. Consider the utility function of player 1 :

	L	C	R
T	7	5	1
M	1	4	3
B	4	1	7

   What are the correlated strategies for which player 1 will want to follow the recommended action for each of his actions?

2. What are the set of correlated equilibria of a zero-sum game?

Solution