CS 4110 – Programming Languages and Logics
Homework #1

Due. Wednesday, September 10, 2014 at 11:59pm.

Instructions. This assignment may be completed with one partner. You and your partner should submit a single solution on CMS. Please do not offer or accept any other assistance on this assignment. Late submissions will not be accepted.

Exercise 1. Suppose we extend our language of arithmetic expressions with an "Elvis" operator that evaluates to its first sub-expression if it is non-zero, and its second sub-expression otherwise:

$$e ::= x \mid n \mid e_1 + e_2 \mid e_1 * e_2 \mid x := e_1 \,;\, e_2 \mid e_1 \;\texttt{?:}\; e_2$$

The following inference rules extend the operational semantics to handle these expressions:

$$\frac{\langle \sigma, e_1 \rangle \to \langle \sigma', e_1' \rangle}{\langle \sigma, e_1 \;\texttt{?:}\; n_2 \rangle \to \langle \sigma', e_1' \;\texttt{?:}\; n_2 \rangle}\ \textsc{Elvis1} \qquad \frac{\langle \sigma, e_2 \rangle \to \langle \sigma', e_2' \rangle}{\langle \sigma, e_1 \;\texttt{?:}\; e_2 \rangle \to \langle \sigma', e_1 \;\texttt{?:}\; e_2' \rangle}\ \textsc{Elvis2}$$

$$\frac{}{\langle \sigma, 0 \;\texttt{?:}\; n \rangle \to \langle \sigma, n \rangle}\ \textsc{Elvis3} \qquad \frac{m \neq 0}{\langle \sigma, m \;\texttt{?:}\; n \rangle \to \langle \sigma, m \rangle}\ \textsc{Elvis4}$$

Unfortunately, they do not implement the intended semantics. For example, evaluating

$$x := 1 + 1 \,;\, (x \;\texttt{?:}\; x := 1 + x \,;\, 0)$$

yields 3, not 2.

- **(a)** Give the derivation tree for the first step of evaluation for the above expression using σ.
- **(b)** Give the rest of the sequence of configurations encountered during the evaluation of the above expression. Note that you do not need to write out the derivation trees. Just give the sequence of configurations encountered during evaluation.
- **(c)** Revise the rules so they implement the intended semantics. Using your rules, evaluating the above expression with the empty store should evaluate to 2 (you do not need to show this).

Exercise 2. Now suppose instead we extend our language with truncating integer division:

$$e ::= x \mid n \mid e_1 + e_2 \mid e_1 * e_2 \mid e_1 \;/\; e_2 \mid x := e_1 \,;\, e_2$$

We add new evaluation rules to the operational semantics to handle these expressions:

$$\frac{\langle \sigma, e_1 \rangle \to \langle \sigma', e_1' \rangle}{\langle \sigma, e_1 \;/\; e_2 \rangle \to \langle \sigma', e_1' \;/\; e_2 \rangle}\ \textsc{LDiv} \qquad \frac{\langle \sigma, e_2 \rangle \to \langle \sigma', e_2' \rangle}{\langle \sigma, e_1 \;/\; e_2 \rangle \to \langle \sigma', e_1 \;/\; e_2' \rangle}\ \textsc{RDiv} \qquad \frac{n \neq 0 \qquad p = m/n}{\langle \sigma, m \;/\; n \rangle \to \langle \sigma, p \rangle}\ \textsc{Div}$$

- **(a)** Is the semantics deterministic? If not, give a counter example.
- **(b)** Does the semantics terminate? If not, give a counterexample.
- **(c)** Does the progress theorem still hold? If not, give a counterexample.

Exercise 3. Let c range over characters drawn from a fixed alphabet. The following grammar describes the set of strings over the same alphabet.

$$s ::= \epsilon \mid c \cdot s$$

There are two cases: ϵ represents the empty string and $c \cdot s$ represents the string obtained by concatenating c and s.

(a) Give an inductive definition of the set of strings s using inference rules.
(b) Give the cases we would need to establish to prove that a property P holds of all strings s by structural induction—e.g., $P(\epsilon)$, etc.
(c) Complete the definition of the *length* relation which associates a string to the number of characters contained in it, by giving the cases for single characters and concantenations.

$$\frac{}{(\epsilon, 0) \in \mathit{length}}$$

Exercise 4. Explain the flaw in the following "proof."

Theorem. If n is an even number greater than or equal to 2, then there exists i such that $n = 2^i$.

Proof. We will show that $P(n)$ holds for all n by mathematical induction.

- **Base case:** $P(0)$ holds since 0 is not greater than or equal to 2.
- **Induction case:** We must show that $P(n)$ implies $P(n+1)$. We analyze several sub-cases. If n is 1 then $P(1)$ holds since $1 = 2^0$. If n is 2 then $P(2)$ holds since $2 = 2^1$. Otherwise, if n is odd, then the assumption that n is even is false and $P(n)$ vacuously holds. Otherwise, n is even, so there exists an m such that $n = 2 \times m$. By induction hypothesis, there exists j such that $m = 2^j$. Hence, we have,

$$\begin{array}{rcll} n & = & 2 \times m & \text{As } n \text{ is even} \\ & = & 2 \times 2^j & \text{By induction hypothesis on } m \\ & = & 2^{j+1} & \text{By substitution} \end{array}$$

which finishes the sub-case and the inductive proof. □

Debriefing

(a) How many hours did you spend on this assignment?
(b) Would you rate it as easy, moderate, or difficult?
(c) How deeply do you feel you understand the material it covers (0%100%)?
(d) If you have any other comments, we would like to hear them! Please write them here or send email to `jnfoster@cs.cornell.edu`.