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CS 4110
HW 1

1. (a)

$$\dfrac{\dfrac{2 = 1+1}{\langle \sigma, \texttt{1+1} \rangle \to \langle \sigma, 2 \rangle}\ (\text{Add})}{\langle \sigma, \texttt{x:=1+1; (x ?: x:=1+x; 0)} \rangle \to \langle \sigma, \texttt{x:=2; (x ?: x:=1+x; 0)} \rangle}\ (\text{Assgn1})$$

   (b)

$$\begin{array}{ll} & \langle \sigma, \texttt{x:=2; (x ?: x:=1+x;0)} \rangle \\ \to & \langle \sigma', \texttt{(x ?: x:=1+x; 0)} \rangle, \text{ where } \sigma' = \sigma[x \mapsto 2] \\ \to & \langle \sigma', \texttt{(x ?: x:= 1 + 2; 0)} \rangle \\ \to & \langle \sigma', \texttt{(x ?: x:=3; 0)} \rangle \\ \to & \langle \sigma'', \texttt{x ?: 0} \rangle, \text{ where } \sigma'' = \sigma'[x \mapsto 3] \\ \to & \langle \sigma'', \texttt{3 ?: 0} \rangle \\ \to & \langle \sigma'', \texttt{3} \rangle \end{array}$$

   (c) new rules:

$$\dfrac{\langle \sigma, e_1 \rangle \to \langle \sigma', e_1' \rangle}{\langle \sigma, e_1 \texttt{?:} e_2 \rangle \to \langle \sigma', e_1' \texttt{?:} e_2 \rangle}\ \text{Elvis1}$$

$$\dfrac{\langle \sigma, e_2 \rangle \to \langle \sigma', e_2' \rangle}{\langle \sigma, n \texttt{?:} e_2 \rangle \to \langle \sigma', n \texttt{?:} e_2' \rangle}\ \text{Elvis2}$$

   Elvis3 and Elvis4 retain their old definitions.

2. (a) No, $\langle \sigma, \texttt{(5+2)/(3+2)} \rangle$ could step to $\langle \sigma, \texttt{7/(3+2)} \rangle$ or $\langle \sigma, \texttt{(5+2)/5} \rangle$.
   (b) Yes.
   (c) No, there aren't any rules that allow $\langle \sigma, \texttt{7/0} \rangle$ to progress any further, yet `7/0` is not an integer.

3. (a) note: I'm going to call the set of strings, "**Str.**" to help differentiate a string, s, in the set of strings, from the set **Str.**

$$\dfrac{}{\epsilon \in \textbf{Str.}}$$

$$\dfrac{s \in \textbf{Str.}}{c :: s \in \textbf{Str.}}$$

   (b) We'd need to prove that the base case ($P(\epsilon)$) holds, and that if $P(s)$ holds for some $s \in \textbf{Str.}$, then $P(c :: s)$ for some c in the fixed alphabet.
   (c)

$$\frac{(s, n) \in length}{(c :: s, n+1) \in length}$$

4. To answer this question, I think its helpful to consider a particular example–specifically, $n = 6$. Clearly, 6 can't be expressed as 2^i for some $i \in \mathbb{N}$, so what went wrong? The proof is correct in arguing that n can be expressed as $2 \times m$ (it's, true by definition of "evenness"), and its also correct in arguing that the inductive hypothesis holds for m (because $m = 3$, and therefore the preposition–which can roughly be restated as even$(n) \implies \exists i \in \mathbb{N}$ such that $n = 2^i$–is also true, albeit vacuously. What goes wrong is asserting that m can be expressed as 2^j simply because the inductive hypothesis held. The inductive hypothesis only says that $m = 2^j$ if m is even. And in this case m is odd, so the inductive hypothesis can't be used to conclude $m = 2^j$.