Q4) a) $X \sim N(10, 2)$

$Y \sim N(15, 5)$ [Approximated to nearest integers as obtained from the notebook "Q4.ipynb"]

$$E[W] + E[V] = E[W+V]$$

$$= E[X+Y] \quad [\because W+V = X+Y]$$

$$= E[X] + E[Y]$$

$$= 10 + 15$$

$$= 25$$

b) If X and Y are normally distributed random variables, then linear combination of them is also a ~~random~~ normally distributed random variable.

$\Rightarrow$ X-Y is a Normally distributed RV.

$\mu_{X-Y} = \mu_X - \mu_Y = 10 - 15 = -5$

$$\boxed{\mu_{X-Y} = -5}$$

$\sigma^2_{X-Y} = \text{Var}(X) + \text{var}(Y) = 4 + 25 = 29$

$$\boxed{\sigma^2_{X-Y} = 29}$$

If X is a normally distributed random variable, then $|X|$ is folded normally distributed RV.

$$\mu_Z = \sigma\sqrt{\frac{2}{\pi}}\exp\left(\frac{-\mu^2}{2\sigma^2}\right) + \mu\,\text{erf}\left(\frac{\mu}{\sqrt{2\sigma^2}}\right) \quad [\text{Here } Z = |X|]$$

(1)

$$E[|X-Y|] = 6.0264 \quad [\text{Calculated using } ①]$$

Similarly,

$$E[|X|-|Y|] = E[|X|] - E[|Y|]$$

$$\doteq -5.0038 \quad [\text{Calculated using } ①]$$