4) Rules:

Good - Shot by bad and ugly

Bad - Shot by Good & by ugly if no good present

Ugly - Shot by good in bad's absence (or) by bad in good's absence

Good: P(Good survives) = 1 - P(Good only dies)

→ In order for good to survive, it has ~~to~~ to kill bad first & then ugly.

→ In a round, P(all surviving) = $(1-x)(1-y)(1-z)$

→ In a round, P(killing bad) = $x(1-y)(1-z)$

→ Theoretically, it could take an indefinite amount of rounds ~~there~~ before good kills bad, therefore the probability of good killing bad is their sum of all possibilities.

$$\therefore P(\text{Good kills Bad}) = x(1-y)(1-z) + [(1-x)(1-y)(1-z)]\, x(1-y)(1-z) + [(1-x)(1-y)(1-z)]^2 x(1-y)(1-z) \ldots\ldots$$

$$= x(1-y)(1-z)\,[\underbrace{1+(1-x)(1-y)(1-z) + ((1-x)(1-y)(1-z))^2 + \ldots}_{\text{Sum of infinite GP}}]$$

$$\therefore = x(1-y)(1-z)\cdot\left[\frac{1}{1-(1-x)(1-y)(1-z)}\right]$$

After killing bad, good kills ugly without dying. In the similar steps to the previous case,

$$P(\text{Good kills ugly}) = \frac{x(1-z)}{1-(1-x)(1-z)}$$

$\therefore$ Probability of good surviving,

$$P(\text{Good survives}) = P(\text{Good kills bad}) \times P(\text{Good kills ugly})$$

$$= \frac{x^2(1-y)(1-z)^2}{[1-(1-x)(1-y)(1-z)]\,[1-(1-x)(1-z)]}$$

<u>Bad</u>:

- Bad has three ways of surviving:

(i) Bad kills good and then ugly

(ii) Ugly kills good and bad kills ugly

(iii) Bad & ugly kill good at the same time & bad kills ugly

The total probability of bad surviving is the sum of above possibilities

(i) Bad kills good:

$$P(\text{bad kills good}) = (1-x)y(1-z) + (1-x)y(1-z)[(1-x)(1-y)(1-z)] + (1-x)y(1-z)[(1-x)(1-y)(1-z)]^2 \ldots$$

$$= \frac{(1-x)y(1-z)}{1-(1-x)(1-y)(1-z)}$$

4 (ii) ~~B~~ Ugly Kills good :

Applying the earlier method, P (Ugly kills good)

$$= \frac{(1-x)(1-y)z}{1-(1-x)(1-y)(1-z)}$$

(iii) Both bad & ugly hit good :

$$P(\text{Bad \& ugly kill good}) = \frac{(1-x)yz}{1-(1-x)(1-y)(1-z)}$$

Bad kills ugly :

$$P(\text{bad kills ugly}) = y(1-z) + y(1-z)[(1-y)(1-z)] + y(1-z)[(1-y)(1-z)]^2 \ldots\ldots$$

$$= \frac{y(1-z)}{1-(1-y)(1-z)}$$

Total probability of bad surviving :

P (bad kills ~~ugly~~ good) x P (bad kills ugly) + P (ugly kills good) x P (bad kills ugly) + P (bad & ugly hit good) x P (bad kills ugly)

$$\therefore \quad P(\text{bad survives}) = \left[\frac{(y+z+yz)(1-x)}{1-(1-x)(1-y)(1-z)}\right]\left[\frac{y(1-z)}{1-(1-y)(1-z)}\right]$$

Ugly: There are more possibilities by which ugly survives,

(i) Ugly kills good then bad. (ii) Bad ~~Ugly~~ kills good then ugly kills bad

(iii) Bad & ugly kill good then ugly kills bad

(iv) Good kills bad then ugly kills good

(v) Good and bad shoot each other simultaneously

(vi) Good kills bad while ugly kills good (Note that this ~~happens~~ could happen in ~~the~~ same round)

(vii) Good kills bad & ugly & bad kill good simultaneously.

Probability of ugly surviving is the sum of above possibilities.

Ugly kills bad (after good dies):

$$P(\text{ugly kills bad}) = \frac{(1-y)\cancel{x}z}{1-(1-y)(1-z)}$$

$$P(\text{ugly kills good}) = \frac{(1-x)z}{1-(1-x)(1-z)}$$

$$P(\text{good \& bad kill each other}) = \frac{xy(1-z)}{1-(1-x)(1-y)(1-z)}$$

$$P(\text{all three hit}) = \frac{xyz}{1-(1-x)(1-y)(1-z)}$$

$\therefore$ P(ugly survives) =

$$\frac{1}{1-(1-x)(1-y)(1-z)}\left[x(y+z-yz) + \frac{(1-x)(y+z-yz)(1-y)z}{1-(1-y)(1-z)} + \frac{x(1-y)(1-z)\cdot(1-x)z}{1-(1-x)(1-z)}\right]$$