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Q.6. For an estimator $\hat{\theta}$ estimating θ to be unbiased

$$E(\hat{\theta}) = \theta$$

a) Estimator 1 $= \hat{\theta}_1 = \frac{X_1 + X_2 + \ldots + X_7}{7}$

$\Rightarrow E(\hat{\theta}_1) = \frac{1}{7}\left(E(X_1) + E(X_2) + \ldots + E(X_7)\right)$

$\Rightarrow E(\hat{\theta}_1) = \frac{1}{7}\left(E(X_1) + E(X_2) + \ldots + E(X_7)\right)$

$= \frac{1}{7}(\mu + \mu + \ldots + \mu)$

$= \frac{7\mu}{7}$

$= \mu \rightarrow 1$

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Estimator 2 : $\hat{\theta}_2 = \dfrac{2X_1 - X_6 + X_4}{2}$

$$E(\hat{\theta}_2) = \frac{1}{2}\left(2E(X_1) - E(X_6) + E(X_4)\right)$$

$$= \frac{1}{2}(2\mu - \mu + \mu)$$

$$= \mu \rightarrow \text{(ii)}$$

From (i) & (ii) we get that both the estimators are unbiased.

b) We know that,

error = variance + bias^2

Since bias for both estimators is 0 so

error = variance

$\therefore$ We will answer the question of better estimator using variance

$$var(\hat{\theta_1}) = var\left(\frac{X_1 + X_2 + \dots + X_7}{7}\right)$$

Using property

$$Var\left(\sum_{i=1}^{N} a_i X_i\right) = \sum_{i=1}^{N} a_i^2 \, var(X_i)$$ for uncorrelated random variables we get.

$$var(\hat{\theta_1}) = \frac{1}{49} var(X_1) + \frac{1}{49} var(X_2) + \dots + \frac{1}{49} var(X_7)$$

$$= \frac{1}{49}\left(var(X_1) + var(X_2) + \dots + var(X_7)\right)$$

$$= \frac{1}{49} \times (\sigma^2 + \sigma^2 + \dots + \sigma^2)$$

$$= \frac{7\sigma^2}{49} = \frac{\sigma^2}{7} \rightarrow (1)$$

Similarly

$$var(\hat{\theta_2}) = var\left(\frac{2x_1 - x_6 + x_4}{2}\right)$$

$$= var(x_1) + \frac{1}{4} var(x_6) + \frac{1}{4} var(x_4)$$

$$= \sigma^2 + \frac{1}{4}\sigma^2 + \frac{1}{4}\sigma^2$$

$$= \frac{3\sigma^2}{2} \rightarrow (2)$$

from (1) & (2) we get

$$error(\hat{\theta_1}) = \frac{\sigma^2}{7} \quad \& \quad error(\hat{\theta_2}) = \frac{3\sigma^2}{2}$$

$\therefore$ $\hat{\theta_1}$ is the better estimation.