# Programming Assignment I

CS5691: PATTERN RECOGNITION AND MACHINE LEARNING

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### I Polynomial Curve Fitting

#### I.1 Python Code

```
import pandas as pd
  import numpy as np
   from mpl_toolkits.mplot3d import Axes3D
   import matplotlib.pyplot as plt
   import random
   from matplotlib import cm
   def process_dataset_1(degree, lamb, batch):
       train = int(0.7*batch)
                                  #70% of data
10
       valid = int(0.2*batch)
                                  #20 % of data
       test = int(batch-(train+valid)) #10 % of data
12
       #generating polynomial combinations
14
       def polybasisfun(vec,deg):
15
           def degcomb(vec,deg):
16
               if deg == 0:
                    return [1]
18
               if deg == 1:
19
                    return vec
20
               if len(vec) == 1:
21
                    return [vec[0]**deg]
22
               u = []
23
               for i in range(deg+1):
                   u+=([(vec[0]**(i))*x for x in (degcomb(vec[1:],deg-i))])
               return u
           u = np.array([1])
27
           for i in range(1,deg+1):
               u=np.append(u,degcomb(vec,i))
           return u
       # Read data from csv file
       f0 = pd.read csv('datasets/function1.csv')
       odata = f0.to_numpy()
       np.random.shuffle(odata)
       # split into train, validate, test
       data = odata[:train,1:2]
       validata = odata[-(valid):,1:2]
39
       testdata = odata[-(valid+test):-(valid),1:2]
       yd = odata[:train,2]
41
       yv = odata[-(valid):,2]
42
       yt = odata[-(valid+test):-(valid),2]
43
44
       # formulate X matrix
45
       X = np.apply_along_axis(polybasisfun,1,data,degree)
46
```

```
47
       # calculate weights
48
       w = ((np.linalg.inv((lamb*np.eye(np.shape(X)[1]))+((np.transpose(X))@X)))@((np.transpose(X))@X)))
49
50
51
       # Get predictions
52
       def output(v,deg,w):
53
           return (w@(np.transpose(polybasisfun(v,deg))))
54
       yp = np.apply_along_axis(output,1,data,deg=degree,w=w)
55
       yvp = np.apply along axis(output,1,validata,deg=degree,w=w)
56
       ytp = np.apply_along_axis(output,1,testdata,deg=degree,w=w)
57
58
59
       print('ERMS training = %f'%(np.linalg.norm((yp-yd),2)/np.sqrt(train)))
60
       print('ERMS test = %f'%(np.linalg.norm((ytp-yt),2)/np.sqrt(test)))
61
       print('ERMS_valid = %f'%(np.linalg.norm((yvp-yv),2)/np.sqrt(valid)))
62
       x = np.linspace(min(data),max(data),50)
       x = np.reshape(x,(x.shape[0],1))
       y = np.apply_along_axis(lambda x:output(x,degree,w),1,x)
       fig0 = plt.figure(figsize=(9,9))
70
       plt.plot(x,y,color='red')
       plt.scatter(data,yd,color='blue')
       plt.title("degree = {} | lambda = {}".format(degree,lamb),fontsize = 20)
       plt.xlabel('X-axis(x)')
74
       plt.ylabel('Y-axis(y)')
76
       return np.linalg.norm((yp-yd),2)/np.sqrt(train), np.linalg.norm((ytp-yt),2)/np.sq
78
   80
81
   # # Call function to experiment dataset 1
82
   Degree = [2,3,6,9]
83
   lamb = [0,10e-2,10e-1,10,10e2]
84
   batch = [10,200]
85
   Erms = []
87
   # Experiment with Degree
88
   for 1 in lamb:
89
       erms = process_dataset_1(25, 1, 350)
90
       Erms.append(erms)
91
```

### II Linear Model for Regression using Polynomial Basis Functions

#### II.1 Python Code

```
import pandas as pd
   import numpy as np
   from mpl_toolkits.mplot3d import Axes3D
   import matplotlib.pyplot as plt
   import random
   from matplotlib import cm
   #polynomial basis function regression
   degree = 6
   lamb = 0
  batch = 500
  train = int(0.7*batch)
                             #70% of data
   test = int(0.2*batch)
                            #20 % of data
   valid = batch-(train+test) #10 % of data
   #generating polynomial combinations
   def polybasisfun(vec,deg):
       def degcomb(vec,deg):
           if deg == 0:
               return [1]
20
           if deg == 1:
               return vec
22
           if len(vec) == 1:
               return [vec[0]**deg]
24
           u = []
25
           for i in range(deg+1):
26
               u+=([(vec[0]**(i))*x for x in (degcomb(vec[1:],deg-i))])
27
           return u
28
       u = np.array([1])
29
       for i in range(1,deg+1):
30
           u=np.append(u,degcomb(vec,i))
31
       return u
32
   f0 = pd.read csv('function1 2d.csv')
33
   odata = f0.to numpy()
34
   np.random.shuffle(odata)
35
   data = odata[:train,1:3]
36
   validata = odata[-(valid):,1:3]
37
   testdata = odata[-(valid+test):-(valid),1:3]
38
   yv = odata[-(valid):,3]
   yt = odata[-(valid+test):-(valid),3]
   X = np.apply_along_axis(polybasisfun,1,data,degree)
   yd = odata[:train,3]
42
   w = ((np.linalg.inv((lamb*np.eye(np.shape(X)[1]))+((np.transpose(X))@X)))
                                                      @((np.transpose(X))@yd))
```

```
def output(v,deg,w):
46
       return (w@(np.transpose(polybasisfun(v,deg))))
47
48
   yp = np.apply along axis(output,1,data,deg=degree,w=w)
49
   yvp = np.apply along axis(output,1,validata,deg=degree,w=w)
50
   ytp = np.apply along axis(output,1,testdata,deg=degree,w=w)
51
   print('ERMS training = %f'%(np.linalg.norm((yp-yd),2)/np.sqrt(train)))
52
   print('ERMS test = %f'%(np.linalg.norm((ytp-yt),2)/np.sqrt(test)))
53
   print('ERMS valid = %f'%(np.linalg.norm((yvp-yv),2)/np.sqrt(valid)))
54
   x = np.linspace(min(data[:,0]), max(data[:,0]), 200)
55
   y = np.linspace(min(data[:,1]), max(data[:,1]), 200)
56
   X,Y=np.meshgrid(x,y)
   Z = (np.array([output([i,j],degree,w) for i,j in
58
        zip(np.ravel(X),np.ravel(Y))])).reshape(Y.shape)
   fig = plt.figure(figsize=(16,12))
60
   ax = plt.axes(projection='3d')
61
   ax.plot surface(X,Y,Z,color='red', alpha=0.2,)
   ax.scatter(data[:,0],data[:,1],yd,alpha=1,color='black',linewidths=1)
   ax.set xlabel('X-axis(x1)',fontsize = 15)
   ax.set ylabel('Y-axis(x2)',fontsize = 15)
   ax.set zlabel('Z-axis(y)',fontsize = 15)
   ax.set title("batch size = {} | degree = {} | lambda = {}".format(batch, degree,
                lamb), fontsize = 20)
   plt.show()
69
70
   fig1 = plt.figure(figsize=(9,9))
   plt.scatter(yd,yp,marker='x')
72
   plt.title('training',fontsize=20)
73
   plt.xlabel('target',fontsize=20)
   plt.ylabel('predicted',fontsize=20)
   fig2 = plt.figure(figsize=(9,9))
76
   plt.scatter(yt,ytp,marker='x')
77
   plt.title('test',fontsize=20)
78
   plt.xlabel('target',fontsize=20)
   plt.ylabel('predicted',fontsize=20)
80
   fig3 = plt.figure(figsize=(9,9))
81
   plt.scatter(yv,yvp,marker='x')
82
   plt.title('validation',fontsize=20)
83
   plt.xlabel('target',fontsize=20)
84
   plt.ylabel('predicted',fontsize=20)
```

## III Linear Model for Regression using Gaussian Basis Functions

#### III.1 Python Code

46

```
# -*- coding: utf-8 -*-
   import numpy as np
   import pandas as pd
   import matplotlib.pyplot as plt
   def process dataset 3(D, var, r):
10
11
       11 11 11
12
       D: No of gauss functions for fit
13
       var: variance for fit
14
       r: regularisation term
15
       11 11 11
16
       # Read txt file
17
18
      f = open('datasets/2_music.txt', 'r')
19
20
      length = 0
21
      for line in f:
22
23
          d = [float(i) for i in line.split(',')]
          if length == 0:
              data = np.array(d)
27
              data = data[np.newaxis,:]
          else:
              d = np.array(d)
              d = d[np.newaxis,:]
              data = np.append(data,d,axis=0)
          length = length+1
      f.close()
35
      data = pd.DataFrame(data)
39
       # shuffle dataset
40
      data = data.sample(frac=1)
41
42
       # get dependent and independent varaible
43
      data = np.array(data)
44
45
```

```
X = data[:,0:-2]
47
       y = data[:,-2:]
48
49
       # get unique values of y
50
       y_unique = np.unique(y,axis=0)
51
52
       # length of data for fit
53
       train_len = int(np.shape(X)[0]*0.7)
54
       val_len = int(np.shape(X)[0]*0.2)
55
       test len = int(np.shape(X)[0]) - val len - train len
56
57
       # test train split
58
       X_train = X[0:train_len]
59
       X test = X[train len:train len+test len]
       X val = X[train len+test len:train len+test len+val len]
61
62
       y train = y[0:train len]
       y_test = y[train_len:train_len+test_len]
       y_val = y[train_len+test_len:train_len+test_len+val_len]
       # K-means clustering
       loop = 1
       prev_zni = np.zeros((train_len,D-1))
70
       while(loop>0):
           zni = np.zeros((train_len,D-1))
           if loop == 1:
74
                # randomly choose k points
                random_index = np.random.randint(0,train_len,D-1)
76
                # Initialize MUi
78
                MUi = X_train[random_index,:]
80
            # Determine points belonging to clusters
           for j in range(0,train_len):
82
                i = np.argmin(np.linalg.norm(X_train[j,:]-MUi,axis=1),axis=0)
83
                zni[j,i] = 1
84
85
            # Determine number of datapoints in the clusters
86
           Ni = np.sum(zni,axis=0)
87
88
            # Update MUi
89
           for j in range(0,D-1):
90
                if Ni[j] == 0:
91
                    continue
92
93
                z = zni[:,j]
94
                MUi[j,:] = (np.sum(X_train*z[:,np.newaxis],axis=0))/Ni[j]
95
```

```
# compare previous and current Zni
97
           comp = prev_zni - zni
98
99
           prev zni = zni
100
           loop = loop+1
101
102
           # if no change break the loop
103
           if np.max(comp) == 0 and np.min(comp) == 0:
104
                break
105
106
        # Assign Mui
107
       mu = MUi
108
109
        # gaussian function
110
       phii = lambda x,mu : np.e**(-(np.linalg.norm(x-mu))/var**2)
111
112
        # Formulate phi matrix for training data
       phin = np.zeros((train_len,D))
114
       for i in range(0,train_len):
           for j in range(0,D):
                if j==0:
                    phin[i,j] = 1
                    \#phin[i,j] = phii(X_train[i,:],mu[j,:])
120
               else:
                   phin[i,j] = phii(X_train[i,:],mu[j,:])
122
123
        # weights with quadratic regularisation
124
       W = np.linalg.inv(phin.T@phin + r*np.identity(D))@phin.T@y_train
125
126
        # weights with tikhanov regularisation
127
        # phin_bar = np.zeros((D,D))
128
        # for i in range(0,D):
129
              for j in range(0,D):
130
                  phin_bar[i,j] = np.exp(np.linalg.norm(mu[i,:]-mu[j,:])/var**2)
131
132
        \# W = np.linalq.inv(phin.T@phin + r*phin_bar)@phin.T@y_train
133
134
        135
136
        # Test on training dataset
137
       y_pred = np.zeros((train_len,2))
138
139
        # Formulate phi matrix for training data
140
       phin = np.zeros((train_len,D))
141
142
       for i in range(0,train len):
143
           for j in range(0,D):
144
                if j==0:
145
                   phin[i,j] = 1
```

```
# phin[i,j] = phii(X_train[i,:],mu[j,:])
147
                else:
148
                    phin[i,j] = phii(X train[i,:],mu[j])
149
150
       for i in range(0,train len):
151
           pred = (phin[i,:])@W
152
           y pred[i,:] = pred
153
154
            \# term1 = np.deg2rad((pred[0] - y_unique[:,0])/2)
155
            # term2 = np.deg2rad((pred[1] - y unique[:,1])/2)
156
            \# a = (np.sin(term1)**2) + \# np.cos(np.deg2rad(pred[0]))*
157
                   np.cos(np.deg2rad(y\_unique[:,0]))*(np.sin(term2#)**2)
158
            # dist = 2*6373*np.arctan2(np.sqrt(a),np.sqrt(1-a))
159
160
            # index = np.argmin(dist)
161
            \# y\_pred[i,:] = y\_unique[index,:]
162
       Erms_train1 = ((np.linalg.norm(y_pred[:,0]-y_train[:,0])))/np.sqrt(train_len)
164
       Erms_train2 = ((np.linalg.norm(y_pred[:,1]-y_train[:,1])))/np.sqrt(train_len)
        # # scatter plot
        # fig1 = plt.figure(figsize=(9,9))
        # plt.scatter(y_train[:,0],y_pred[:,0],marker='x')
        # plt.scatter(y_train[:,1],y_pred[:,1],marker='x')
170
        # plt.xlabel('target', fontsize=20)
        # plt.ylabel('predicted', fontsize=20)
172
        # plt.title("Gaussian Basis Function (For Dataset 3 of # best performing
173
        #
                      model with training data) | degree = {} | # lambda = {}".format(D
174
                      fontsize = 20)
        # plt.legend(["y0","y1"])
176
        177
178
        # Test on validation dataset
179
       y_pred = np.zeros((val_len,2))
180
181
        # Formulate phi matrix for validation data
182
       phin = np.zeros((val len,D))
183
184
       for i in range(0,val len):
185
           for j in range(0,D):
186
                if j==0:
187
                    phin[i,j] = 1
188
                    \# phin[i,j] = phii(X val[i,:],mu[j,:])
189
                else:
190
                   phin[i,j] = phii(X_val[i,:],mu[j])
191
192
       for i in range(0,val len):
193
           pred = (phin[i,:])@W
194
           y_pred[i,:] = pred
195
```

```
197
       Erms_val1 = ((np.linalg.norm(y_pred[:,0]-y_val[:,0])))/np.sqrt(val_len)
198
       Erms_val2 = ((np.linalg.norm(y_pred[:,1]-y_val[:,1])))/np.sqrt(val_len)
199
200
       # scatter plot for validation data
201
       # fig1 = plt.figure(figsize=(9,9))
202
       # plt.scatter(y_val,y_pred,marker='x')
203
       # plt.xlabel('target', fontsize=20)
204
       # plt.ylabel('predicted',fontsize=20)
205
       # plt.title("Gaussian Basis Function (For Dataset 2 of best performing model
206
                   with validation data) | degree = \{\} | lambda # = \{\}".format(D,r),
207
       #
                    fontsize = 20)
208
209
       210
211
       # Test on test dataset
212
       y pred = np.zeros((test len,2))
214
       # Formulate phi matrix for test data
       phin = np.zeros((test len,D))
       for i in range(0,test len):
          for j in range(0,D):
              if j==0:
220
                  phin[i,j] = 1
                  \#phin[i,j] = phii(X_test[i,:],mu[j,:])
222
223
                  phin[i,j] = phii(X_test[i,:],mu[j])
224
       for i in range(0,test len):
226
          pred = (phin[i,:])@W
227
          y pred[i,:] = pred
228
229
       Erms_test1 = ((np.linalg.norm(y_pred[:,0]-y_test[:,0])))/np.sqrt(test_len)
230
       Erms test2 = ((np.linalg.norm(y pred[:,1]-y test[:,1])))/np.sqrt(test len)
231
232
       # scatter plot for test data
233
       fig1 = plt.figure(figsize=(9,9))
234
       plt.scatter(y test[:,0],y pred[:,0],marker='x')
235
       plt.scatter(y_test[:,1],y_pred[:,1],marker='x')
236
       plt.xlabel('target',fontsize=20)
237
       plt.ylabel('predicted',fontsize=20)
238
       plt.title("Gaussian Basis Function (For Dataset 3 of best performing model
239
                 with test data) | degree = {} | lambda = {}".format(D,r),
240
                  fontsize = 20)
241
       plt.legend(["y0","y1"])
242
       243
       return Erms train1, Erms train2, Erms val1, Erms val2, Erms test1, Erms test2
244
245
```

```
247
    def process_dataset_2(D, var, r):
248
249
        11 11 11
250
        D: No of gauss functions for fit
251
        var: variance for fit
252
        r: regularisation term
253
         11 11 11
254
255
        # Read csv file
256
        data = pd.read_csv('datasets/function1_2d.csv')
257
258
        # shuffle dataset
259
        \# data = data.sample(frac=1)
260
261
        # get dependent and independent varaible
262
        X = data[['x1', 'x2']]
        y = data['y']
264
        # convert into numpy
        X = np.array(X)
        y = np.array(y)
268
        # length of data for fit
270
        train len = int(np.shape(X)[0]*0.7)
271
        test_len = int(np.shape(X)[0]*0.2)
272
        val_len = int(np.shape(X)[0]*0.1)
273
274
        # test train split
        X_train = X[0:train_len]
276
        X_test = X[train_len:train_len+test_len]
277
        X_val = X[train_len+test_len:train_len+test_len+val_len]
278
279
        y_train = y[0:train_len]
280
        y test = y[train len:train len+test len]
281
        y_val = y[train_len+test_len: train_len+test_len+val_len]
282
283
        # K-means clustering
284
        loop = 1
285
        prev_zni = np.zeros((train_len,D-1))
286
287
        while(loop>0):
288
             zni = np.zeros((train_len,D-1))
289
290
             if loop == 1:
291
                 # randomly choose k points
292
                 random index = np.random.randint(0,train len,D-1)
293
294
                 # Initialize MUi
295
                 MUi = X train[random index,:]
```

```
297
            # Determine points belonging to clusters
298
            for j in range(0,train_len):
299
                i = np.argmin(np.linalg.norm(X_train[j,:]-MUi,axis=1),axis=0)
300
                zni[j,i] = 1
301
302
            # Determine number of datapoints in the clusters
303
            Ni = np.sum(zni,axis=0)
304
305
            # Update MUi
306
            for j in range(0,D-1):
307
                if Ni[j] == 0:
308
                    continue
309
310
                z = zni[:,j]
311
                MUi[j,:] = (np.sum(X_train*z[:,np.newaxis],axis=0))/Ni[j]
312
            # compare previous and current Zni
314
            comp = prev_zni - zni
            prev zni = zni
            loop = loop+1
318
            # if no change break the loop
320
            if np.max(comp) == 0 and np.min(comp) == 0:
321
                break
322
323
        # Assign Mui
324
       mu = MUi
325
326
        # gaussian function
327
       phii = lambda x,mu : np.e**(-(np.linalg.norm(x-mu))/var**2)
328
329
        # Formulate phi matrix for training data
330
       phin = np.zeros((train len,D))
331
332
       for i in range(0,train_len):
333
            for j in range(0,D):
334
                if j==0:
335
                    phin[i,j] = 1
336
                else:
337
                    phin[i,j] = phii(X_train[i,:],mu[j-1,:])
338
339
340
        # weights with quadratic regularisation
341
       W = np.linalg.inv(phin.T@phin + r*np.identity(D))@phin.T@y_train
342
343
        344
345
        # Test on training dataset
```

```
y_pred = np.zeros((train_len,1))
347
348
        # Formulate phi matrix for training data
349
       phin = np.zeros((train len,D))
350
351
       for i in range(0,train len):
352
            for j in range(0,D):
353
                if j==0:
354
                    phin[i,j] = 1
355
                else:
356
                    phin[i,j] = phii(X_train[i,:],mu[j-1])
357
358
       for i in range(0,train_len):
359
            y pred[i,:] = (phin[i,:])@W
360
361
       Erms_train = ((np.linalg.norm(y_pred-y_train[:,np.newaxis])))/
362
                       np.sqrt(train len)
364
        # # scatter plot for training data
        # fig1 = plt.figure(figsize=(9,9))
        # plt.scatter(y_train,y_pred,marker='x')
368
        # plt.xlabel('target', fontsize=20)
        # plt.ylabel('predicted', fontsize=20)
370
        # plt.title("Gaussian Basis Function (For Dataset 2 of best performing model
                      with training data) | degree = \{\} | lambda = \# \{\}".format(D,r),
        #
372
        #
                       fontsize = 20)
373
374
        376
377
        # Test on validation dataset
378
       y_pred = np.zeros((val_len,1))
379
380
        # Formulate phi matrix for validation data
381
       phin = np.zeros((val len,D))
382
383
       for i in range(0,val_len):
384
            for j in range(0,D):
385
                if j==0:
386
                    phin[i,j] = 1
387
                else:
388
                    phin[i,j] = phii(X_val[i,:],mu[j-1])
389
390
       for i in range(0,val len):
391
            y_pred[i,:] = (phin[i,:])@W
392
393
       Erms val = ((np.linalg.norm(y pred-y val[:,np.newaxis])))/np.sqrt(val len)
394
395
        # scatter plot for validation data
```

```
# fig1 = plt.figure(figsize=(9,9))
397
       # plt.scatter(y_val,y_pred,marker='x')
398
       # plt.xlabel('target', fontsize=20)
399
       # plt.ylabel('predicted',fontsize=20)
400
       # plt.title("Gaussian Basis Function (For Dataset 2 of best performing model
401
                    with validation data) | degree = \{\} | lambda #= \{\}". format(D,r),
402
       #
                     fontsize = 20)
403
404
       405
406
       # Test on test dataset
407
       y pred = np.zeros((test len,1))
408
409
       # Formulate phi matrix for test data
410
       phin = np.zeros((test len,D))
411
412
       for i in range(0,test len):
          for j in range(0,D):
414
              if j==0:
                  phin[i,j] = 1
              else:
                  phin[i,j] = phii(X_test[i,:],mu[j-1])
418
       for i in range(0,test len):
420
          y pred[i,:] = (phin[i,:])@W
421
422
       Erms_test = ((np.linalg.norm(y_pred-y_test[:,np.newaxis])))/np.sqrt(test_len)
423
424
       # # scatter plot for validation data
425
       # fig1 = plt.figure(figsize=(9,9))
426
       # plt.scatter(y_test,y_pred,marker='x')
427
       # plt.xlabel('target', fontsize=20)
428
       # plt.ylabel('predicted', fontsize=20)
429
       # plt.title("Gaussian Basis Function (For Dataset 2 of best performing
430
                    model\ with\ test\ data)\ /\ degree = \{\}\ /\ lambda\ \# = \{\}''.format(D,r),
431
       #
                     fontsize = 20)
432
433
       434
       return Erms train, Erms val, Erms test
435
436
437
   438
439
   # Call function to experiment dataset 2/3
440
   D = [2,3,6,9,20,33,40,100]
441
   var = [1,3,5,10,50,100]
442
   reg = [0,10e-5,10e-4,10e-3,10e-2,10e-1,10,10e2,10e3,10e4,10e5]
443
   Erms = []
445
   # Experiment with variance
```

```
for v in var:
447
        erms = process dataset 3(100, v, 0)
448
         Erms.append(erms)
449
450
   # Plot table
451
   fig = plt.figure(figsize=(9,9))
452
   row_labels=["var = 1", "var = 3", "var = 5", "var = 10", "var = 50", "var = 100"]
453
   column_labels=["Erms_train", "Erms_val", "Erms_test"]
454
   column labels=["Erms train y0","Erms_train_y1", "Erms_val0","Erms_val1",
455
                      "Erms test0", "Erms test1"]
456
   plt.axis('tight')
457
   plt.axis('off')
458
   plt.table(cellText=Erms,rowLabels=row_labels,colLabels=column_labels, loc="center")
459
460
   Erms = []
461
   # Experiment with Dimensions
462
   for d in D:
        erms = process_dataset_3(d, 3, 0)
464
        Erms.append(erms)
   # Plot table
   fig = plt.figure(figsize=(9,9))
   row_labels=["D = 2", "D = 3", "D = 6", "D = 9", "D = 20", "D = 30",
469
                   "D = 40", "D = 100"
470
   column labels=["Erms train", "Erms val", "Erms test"]
   column_labels=["Erms_train_y0", "Erms_train_y1", "Erms_val0", "Erms_val1",
                      "Erms_test0", "Erms_test1"]
473
   plt.axis('tight')
474
   plt.axis('off')
   plt.table(cellText=Erms,rowLabels=row labels,colLabels=column labels,
476
                loc="center")
477
478
   Erms = []
479
   # Experiment with regularisation
480
   for r in reg:
481
        erms = process_dataset_3(100, 3, r)
482
        Erms.append(erms)
483
484
   # Plot table
485
   fig = plt.figure(figsize=(9,9))
486
   row_labels=["lambda = 0", "lambda = 10e-5", "lambda = 10e-4", "lambda = 10e-3",
487
                   "lambda = 10e-2", "lambda = 10e-1", "lambda = 10", "lambda = 10e2",
488
                   "lambda = 10e3", "lambda = 10e4", "lambda = 10e5"]
489
   column_labels=["Erms_train", "Erms_val", "Erms_test"]
490
   column labels=["Erms train y0", "Erms train y1", "Erms val0", "Erms val1",
491
                      "Erms_test0", "Erms_test1"]
492
   plt.axis('tight')
493
   plt.axis('off')
494
   plt.table(cellText=Erms,rowLabels=row_labels,colLabels=column_labels,
495
                loc="center")
496
```

# IV Direct links For Python Codes

- Polynomial Curve Fitting
- Linear Model for Regression using Polynomial Basis Functions
- Linear Model for Regression using Gaussian Basis Functions