CS 70 Discrete Mathematics and Probability Theory
Spring 2015 Vazirani Discussion 12W

1. Sanity Check!

a. Prove or give a counterexample: for any random variables X and Y, $\text{Var}[X+Y] = \text{Var}[X] + \text{Var}[Y]$.

b. Derive Chebyshev's inequality using Markov's inequality for some random variable X.

2. Balls and Bins Again

For this problem we toss m balls into n bins.

a. What is the expected number of collisions?

b. Now, let's define X to be the number of collisions. At what threshold of collisions c can we ensure that the probability of having more than c collisions is less than $1/2n$?

3. Coin flips

a. Suppose we flip a fair coin n times and we wish to understand the probability that we get at least $3n/4$ heads. Use Markov's inequality to come up with an upper bound for this probability.

b. Use Markov's inequality to come up with a similar upper bound on the probability that the number of heads is at least n.

c. Find the true probability that there are at least n heads in a sequence of n fair coin flips. Is the bound you derived in the previous part tight?

4. More coin flips

a. Suppose we flip a biased coin 100 times and X is the number of heads we get. We know that $\text{Var}[X] = 16$. What are the possible values for the expected vaue of X?

b. Now suppose $\text{E}[X] = 20$. Use Chebyshev's inequality to derive an upper bound on $\Pr[X \geq 40]$.