

1. Sanity Check!

- Define X to be the sum of n standard six-sided dice. What is $E[X]$?
- Suppose we have a biased coin that comes up heads with probability p . After n tosses, what is the expected number of occurrences of the subsequence HTH ? (For example, the sequence $HTHHTHTTH$ has two occurrences of HTH .)

2. Bernoulli and Binomial Distribution

A random variable X is called a Bernoulli random variable with parameter p if $X = 1$ with probability p and $X = 0$ with probability $1 - p$.

- Calculate $E[X]$ and $\text{Var}[X]$.
- A Binomial random variable with parameters n and p is defined to be the sum of n independent, identically distributed Bernoulli random variables with parameter p . If Z is a Binomial random variable with parameters n and p , what are $E[Z]$ and $\text{Var}[Z]$?

3. Chopping up DNA

In a certain biological experiment, a piece of DNA consisting of a linear sequence (or string) of 4000 nucleotides is subjected to bombardment by various enzymes. The effect of the bombardment is to randomly cut the string between pairs of adjacent nucleotides: each of the 3999 possible cuts occurs independently and with probability $1/500$.

- What is the expected number of pieces into which the string is cut?

- b. What is the variance of the above quantity? (Hint: use problem 2.)
- c. Suppose that the cuts are no longer independent, but highly correlated: when a cut occurs in a particular location, nearby locations are much more likely to be cut as well. The probability of each individual cut remains $1/500$. Does the expected number of pieces increase, decrease, or stay the same?

4. Will I Get My Package?

A sneaky delivery guy of some company is out delivering n packages to n customers. Not only does he hand a random package to each customer, he tends to open a package before delivering with probability $\frac{1}{2}$ (independently of the choice of the package). Let X be the number of customers who receive their own packages unopened.

- a. Compute the expectation $E(X)$.
- b. What is the probability that customers i and j both receive their own packages unopened?
- c. Compute the variance $\text{Var}(X)$.