

1. Induction

Prove that $2^n < n!$ for all integers $n \geq 4$.

2. GCD

Find $\gcd(2n + 1, 3n + 2)$, where n is a positive integer.

3. Marbles

Box A contains 1 black and 3 white marbles, and box B contains 2 black and 4 white marbles. A box is selected at random, and a marble is drawn at random from the selected box.

- What is the probability that the marble is black?
- Given that the marble is white, what is the probability that it came from box A?

4. Hospital

In a hospital unit there are 8 nurses and 5 physicians; 7 nurses and 3 physicians are females. If a staff person is selected, find the probability that the subject is a nurse or a male.

5. Palindromes

How many 5-digit palindromes are there? (A palindrome is a number that reads the same way forwards and backwards. For example, 27872 and 48484 are palindromes, but 28389 and 12541 are not.)

6. Bit String

How many bit strings of length 10 contain at least five consecutive 0's.

7. Combinatorial Proof

Prove $\binom{2n}{n} = 2 \binom{2n-1}{n-1}$.

Hint: If you want to pick n out of $2n$ items, you can either pick the first item or not pick it.

8. Balls and Bins

We throw n balls into n bins randomly.

1. What is the probability that the first bin is empty?
2. What is the probability that the first k bins are empty?
3. What is the probability that the second bin is empty given that the first one is empty?
4. Are the events that “the first bin is empty” and “the first two bins are empty” independent?

9. Random Variables

You have a die which has one side with a 0, one side with a 2, and four sides with 1s. (So the six sides are 0,1,1,1,1,2.) You roll the die twice. Let X be the product of the two rolls.

- a. What is $E[X]$?
- b. What is $Var[X]$?

10. Accidents

The number of accidents (per month) at a certain factory has a Poisson distribution. If the probability that there is at least one accident is $1/2$, what is the probability that there are exactly two accidents?