CS 70 Discrete Mathematics and Probability Theory
Spring 2015 Vazirani Discussion 11M

1. Balls in Bins

You have n bins and you throw balls into them one by one randomly. A collision is when a ball is thrown into a bin which already has another ball.

a. What is the probability that the first ball thrown will cause the first collision?

b. What is the probability that the second ball thrown will cause the first collision?

c. What is the probability that, given the first two balls are not in collision, the third ball thrown will cause the first collision?

d. What is the probability that the third ball thrown will cause the first collision?

e. What is the probability that, given the first $m-1$ balls are not in collision, the m^{th} ball thrown will cause the first collision?

f. What is the probability that the m^{th} ball thrown will cause the first collision?

2. Birthdays

Suppose you record the birthdays of a large group of people, one at a time until you have found a match, i.e., a birthday that has already been recorded. (Assume there are 365 days in a year.)

a. What is the probability that it takes more than 20 people for this to occur?

b. What is the probability that it takes exactly 20 people for this to occur?

c. Suppose instead that you record the birthdays of a large group of people, one at a time, until you have found a person whose birthday matches your own birthday. What is the probability that it takes exactly 20 people for this to occur?

3. Independence in balls and bins

You have k balls and n bins labelled $1, 2, \ldots, n$, where $n \geq 2$. You drop each ball uniformly at random into the bins.

a. What is the probability that bin n is empty?

b. What is the probability that bin 1 is non-empty?

c. What is the probability that both bin 1 and bin n are empty?

d. What is the probability that bin 1 is non-empty and bin n is empty?

e. What is the probability that bin 1 is non-empty given that bin n is empty?

f. What does this tell us about the independence of the two events, A: bin 1 is non-empty and B: bin n is non-empty?