CS 70 Spring 2015 — Discrete Mathematics and Probability Theory — Vazirani — Discussion 14M

1. Normal distribution

A set of grades on a Discrete Math examination in an inferior school (not UC!) are approximately normally distributed with a mean of 64 and a standard deviation of 7.1.

(Note: you may assume that if X is normal with mean 0 and variance 1, then $\Pr[X \leq 1.3] \approx 0.9$ and $\Pr[X \leq 1.65] \approx 0.95$.)

a. Find the lowest passing grade if the bottom 5% of the students fail the class.

b. Find the grade of the highest B if the top 10% of the students are given A's.

2. Guessing Age

You meet someone at a party, and judging by how he looks, you guess that he is either 19, 20, or 21 with probability $1/2$, $1/3$, and $1/6$ respectively. He then tells you that he is a 3rd-year, and you know that 30% of 19-year-olds, 60% of 20-year-olds, and 40% of 21-year-olds are 3rd-years. What is the MAP estimate for his age?

3. MAP Estimation with Coins

a. Suppose you have a coin that you suspect to be biased, i.e. there is some p such that $\Pr(\text{Heads}) = p$. However, you have no information about how biased the coin is, meaning that your belief about the bias is that it is uniform on the interval $[0,1]$. You flip the coin and it comes up heads. What is the MAP estimate for p? Recall that in the continuous case,

$$\hat{p}_{\text{MAP}} = \operatorname*{argmax}_{p \in [0,1]} \Pr(X \mid p) \cdot f(p)$$

where $f(p)$ is the probabilty density function of your belief about p.

b. Your friend tells you that the likelihood of the bias decreases linearly to 0 as p moves away from $\frac{1}{2}$. In other words,

$$f(p) = 1 - 2\left|p - \frac{1}{2}\right|$$

What is the MAP estimate for p?

4. Vegas

On the planet Vegas, everyone carries a coin. Many people are honest and carry a fair coin (heads on one side and tails on the other), but a fraction p of them cheat and carry a trick coin with heads on both sides. You want to estimate p with the following experiment: you pick a random sample of n people and ask each one to flip his or her coin. Assume that each person is independently likely to carry a fair or a trick coin.

a. Suppose you find through your experiment that a fraction q of n coin flip are heads, how should you use this to estimate p?

b. How many people do you need to ask to be 95% sure that your answer is off by at most 0.05? Compute this using two different methods, i.e. Central Limit Theorem and Chebyshev's inequality, and then compare the answers.

Note: If Z is a standard normal random variable, $\Pr[Z \leq 1.96] \approx 0.975$.