

1. Continuous Probability

Consider the following game: you spin a spinner and wait until it comes to rest at some angle $\theta \in [0, 360)$. The amount of money you win is $\theta/36 - 6$ dollars (if this number is negative, you lose money). Let X be the random variable for your winnings.

a. What is the probability density function for X ?

b. What is the expectation?

c. What is the variance?

2. Exponential Distribution

A brand new lightbulb has just been installed in our classroom, and you know the life span of a lightbulb is exponentially distributed with a mean of 50 days.

a. Suppose an electrician is scheduled to check on the lightbulb in 30 days and replace it if it is broken. What is the probability that the electrician will find the bulb broken?

- b. Suppose the electrician finds the bulb broken and replaces it with a new one. What is the probability that the new bulb will last at least 30 days?
- c. Suppose the electrician finds the bulb in working condition and leaves. What is the probability that the bulb will last at least another 30 days?

3. The Central Limit Theorem

We have a game in which I have a bag full of 1 dollar and 5 dollar bills, and each round you draw a single bill. I advertise that the average profit of a single round is 3 dollars and the variance is 4.

Note: if $\phi(x)$ is a standard normal density function,

$$\int_{-\infty}^{-5} \phi(x) dx \approx 0.00000029$$

$$\int_{-\infty}^{-4} \phi(x) dx \approx 0.00003167$$

$$\int_{-\infty}^{-3} \phi(x) dx \approx 0.00134990$$

$$\int_{-\infty}^{-4.5} \phi(x) dx \approx 0.00000340$$

$$\int_{-\infty}^{-3.5} \phi(x) dx \approx 0.00023263$$

$$\int_{-\infty}^{-2.5} \phi(x) dx \approx 0.00620967$$

- a. Suppose you play 100 rounds of this game. Using the central limit theorem, what is the (approximate) probability that your average profit is less than 2.5?
- b. Now, suppose you play 400 rounds. What is the (approximate) probability that your average profit is less than 2.5?