CS 70   Discrete Mathematics and Probability Theory
Spring 2015   Vazirani   Discussion 11W

1. **Sanity Check!**

   Define a random variable X to be the result of rolling two standard dice and summing the results.

   (a) What is the distribution of X?

   (b) Directly compute the expectation of X.

   (c) Compute the expectation of X using linearity of expectation.

2. **Parking Meter**

   Eve drives to school everyday. Tired of always paying for parking, she decides one day not to pay her parking fees. Assume that there is a probability of 0.05 that she gets caught. The parking fee is $0.25 and if she is caught, her parking ticket is $10.

   (a) How does the expected cost of parking 10 times without paying the meter compare with the cost of paying the meter each time?

(b) If she parks at the meter 10 times, what is the probability that she will have to pay more than the total amount she could end up saving by not paying the meter?

3. Quadruply-repeated ones

We say that a string of bits has *k quadruply-repeated ones* if there are *k* positions where four consecutive 1's appear in a row. For example, the string 0100111110 has two quadruply-repeated ones.

What is the expected number of quadruply-repeated ones in a random *n*-bit string, when $n \geq 3$ and all *n*-bit strings are equally likely?

4. Inversions

Consider a random permutation $a_1, a_2, \ldots, a_n$ of the numbers $1, 2, \ldots, n$. A pair (i, j) is an inversion if $i < j$ and $a_i > a_j$. What is the expected number of inversions in the permutation?

5. Coin flip

What is the expected number of times that a person must toss a fair coin to get a head for the first time?