Tutorial 8

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Tree Based Methods

In this week's lecture, you are introduced to the concept of classification and regression trees (CART). In the lecture, we focused on the theory and algorithm. In this tutorial, we will apply CART to a real dataset. There are a lot of different ways that the tree can be built and we will consider a few approaches. One major draw back of CART is the poor predictive power. Many methods have been proposed to improve the predictive power of CARTS. Next week, we will discuss bagging, random forest and boosted trees, which are algorithm developed to improve the predictability of tree.

Discuss the following topics

- What is NP problem?
- Discuss morphological differences between classification and regression trees
- How does CART grow? How do you find the optimal splits?
- Explain what is the greedy algorithm
- Explain what is the cost-complexity criteria and the objective of the criteria
- How does CART build a regression tree?
- What is surrogate split?

Independendent Learning

8.3.1 Fitting Classification Trees

Loading Required Libraries

The `tree` library is used to construct classification and regression trees.

```
library(tree)
```

```
## Warning: package 'tree' was built under R version 4.3.3
```

We first use classification trees to analyze the `Carseats` data set. In these data, `Sales` is a continuous variable, and so we begin by recoding it as a binary variable. We use the `ifelse()` function to create a variable, `High`, which takes on a value of `"Yes"` if the `Sales` variable exceeds 8, and `"No"` otherwise.

```
library(ISLR2)
```

```
## Warning: package 'ISLR2' was built under R version 4.3.3
```

```
attach(Carseats)
High <- factor(ifelse(Sales <= 8, "No", "Yes"))
```

Finally, we use the `data.frame()` function to merge `High` with the rest of the `Carseats` data.

```
Carseats <- data.frame(Carseats, High)
```

We now use the `tree()` function to fit a classification tree in order to predict `High` using all variables except `Sales`.

```
tree.carseats <- tree(High ~ . - Sales, Carseats)
```

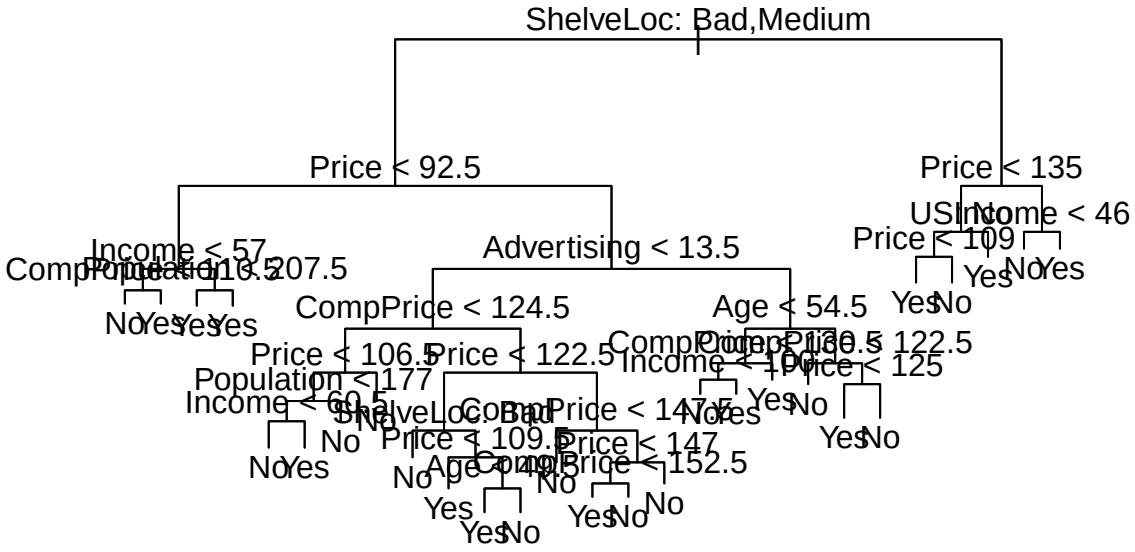
The `summary()` function lists the variables used in the tree, the number of terminal nodes, and the (training) error rate.

```
summary(tree.carseats)
```

```
##
## Classification tree:
## tree(formula = High ~ . - Sales, data = Carseats)
## Variables actually used in tree construction:
## [1] "ShelveLoc"   "Price"       "Income"      "CompPrice"   "Population"
## [6] "Advertising" "Age"         "US"
## Number of terminal nodes:  27
## Residual mean deviance:  0.4575 = 170.7 / 373
## Misclassification error rate: 0.09 = 36 / 400
```

One of the most attractive properties of trees is that they can be graphically displayed. We use the `plot()` function to display the tree structure and the `text()` function to display node labels.

```
plot(tree.carseats)
text(tree.carseats, pretty = 0)
```



If we type the name of the tree object, R prints output corresponding to each branch of the tree.

```
tree.carseats
```

```
## node), split, n, deviance, yval, (yprob)
##       * denotes terminal node
##
##   1) root 400 541.500 No ( 0.59000 0.41000 )
##     2) ShelveLoc: Bad,Medium 315 390.600 No ( 0.68889 0.31111 )
##       4) Price < 92.5 46  56.530 Yes ( 0.30435 0.69565 )
##         8) Income < 57 10  12.220 No ( 0.70000 0.30000 )
##          16) CompPrice < 110.5 5   0.000 No ( 1.00000 0.00000 ) *
##          17) CompPrice > 110.5 5   6.730 Yes ( 0.40000 0.60000 ) *
##         9) Income > 57 36  35.470 Yes ( 0.19444 0.80556 )
##          18) Population < 207.5 16  21.170 Yes ( 0.37500 0.62500 ) *
##          19) Population > 207.5 20   7.941 Yes ( 0.05000 0.95000 ) *
##       5) Price > 92.5 269 299.800 No ( 0.75465 0.24535 )
##        10) Advertising < 13.5 224 213.200 No ( 0.81696 0.18304 )
##          20) CompPrice < 124.5 96  44.890 No ( 0.93750 0.06250 )
##            40) Price < 106.5 38  33.150 No ( 0.84211 0.15789 )
##              80) Population < 177 12  16.300 No ( 0.58333 0.41667 )
##               160) Income < 60.5 6   0.000 No ( 1.00000 0.00000 ) *
##               161) Income > 60.5 6   5.407 Yes ( 0.16667 0.83333 ) *
##              81) Population > 177 26   8.477 No ( 0.96154 0.03846 ) *
##            41) Price > 106.5 58   0.000 No ( 1.00000 0.00000 ) *
##          21) CompPrice > 124.5 128 150.200 No ( 0.72656 0.27344 )
```

```
##            42) Price < 122.5 51  70.680 Yes ( 0.49020 0.50980 )
##              84) ShelveLoc: Bad 11   6.702 No ( 0.90909 0.09091 ) *
##              85) ShelveLoc: Medium 40  52.930 Yes ( 0.37500 0.62500 )
##               170) Price < 109.5 16   7.481 Yes ( 0.06250 0.93750 ) *
##               171) Price > 109.5 24  32.600 No ( 0.58333 0.41667 )
##                 342) Age < 49.5 13  16.050 Yes ( 0.30769 0.69231 ) *
##                 343) Age > 49.5 11   6.702 No ( 0.90909 0.09091 ) *
##            43) Price > 122.5 77  55.540 No ( 0.88312 0.11688 )
##              86) CompPrice < 147.5 58  17.400 No ( 0.96552 0.03448 ) *
##              87) CompPrice > 147.5 19  25.010 No ( 0.63158 0.36842 )
##               174) Price < 147 12  16.300 Yes ( 0.41667 0.58333 )
##                 348) CompPrice < 152.5 7   5.742 Yes ( 0.14286 0.85714 ) *
##                 349) CompPrice > 152.5 5   5.004 No ( 0.80000 0.20000 ) *
##               175) Price > 147 7   0.000 No ( 1.00000 0.00000 ) *
##         11) Advertising > 13.5 45  61.830 Yes ( 0.44444 0.55556 )
##           22) Age < 54.5 25  25.020 Yes ( 0.20000 0.80000 )
##             44) CompPrice < 130.5 14  18.250 Yes ( 0.35714 0.64286 )
##               88) Income < 100 9  12.370 No ( 0.55556 0.44444 ) *
##               89) Income > 100 5   0.000 Yes ( 0.00000 1.00000 ) *
##             45) CompPrice > 130.5 11   0.000 Yes ( 0.00000 1.00000 ) *
##           23) Age > 54.5 20  22.490 No ( 0.75000 0.25000 )
##             46) CompPrice < 122.5 10   0.000 No ( 1.00000 0.00000 ) *
##             47) CompPrice > 122.5 10  13.860 No ( 0.50000 0.50000 )
##               94) Price < 125 5   0.000 Yes ( 0.00000 1.00000 ) *
##               95) Price > 125 5   0.000 No ( 1.00000 0.00000 ) *
##      3) ShelveLoc: Good 85  90.330 Yes ( 0.22353 0.77647 )
##        6) Price < 135 68  49.260 Yes ( 0.11765 0.88235 )
##         12) US: No 17  22.070 Yes ( 0.35294 0.64706 )
##           24) Price < 109 8   0.000 Yes ( 0.00000 1.00000 ) *
##           25) Price > 109 9  11.460 No ( 0.66667 0.33333 ) *
##         13) US: Yes 51  16.880 Yes ( 0.03922 0.96078 ) *
##        7) Price > 135 17  22.070 No ( 0.64706 0.35294 )
##         14) Income < 46 6   0.000 No ( 1.00000 0.00000 ) *
##         15) Income > 46 11  15.160 Yes ( 0.45455 0.54545 ) *
```

Splitting Data into Training and Test Sets

In order to evaluate the performance of a classification tree, we split the data into training and test sets.

```
set.seed(2)
train <- sample(1:nrow(Carseats), 200)
Carseats.test <- Carseats[-train, ]
High.test <- High[-train]
tree.carseats <- tree(High ~ . - Sales, Carseats, subset = train)
```

We use the `predict()` function to predict the class labels for the test data.

```
tree.pred <- predict(tree.carseats, Carseats.test, type = "class")
table(tree.pred, High.test)
```

```
##          High.test
## tree.pred  No Yes
```

```
##       No  104  33
##       Yes  13  50
```

```
(104 + 50) / 200
```

```
## [1] 0.77
```

Pruning the Tree

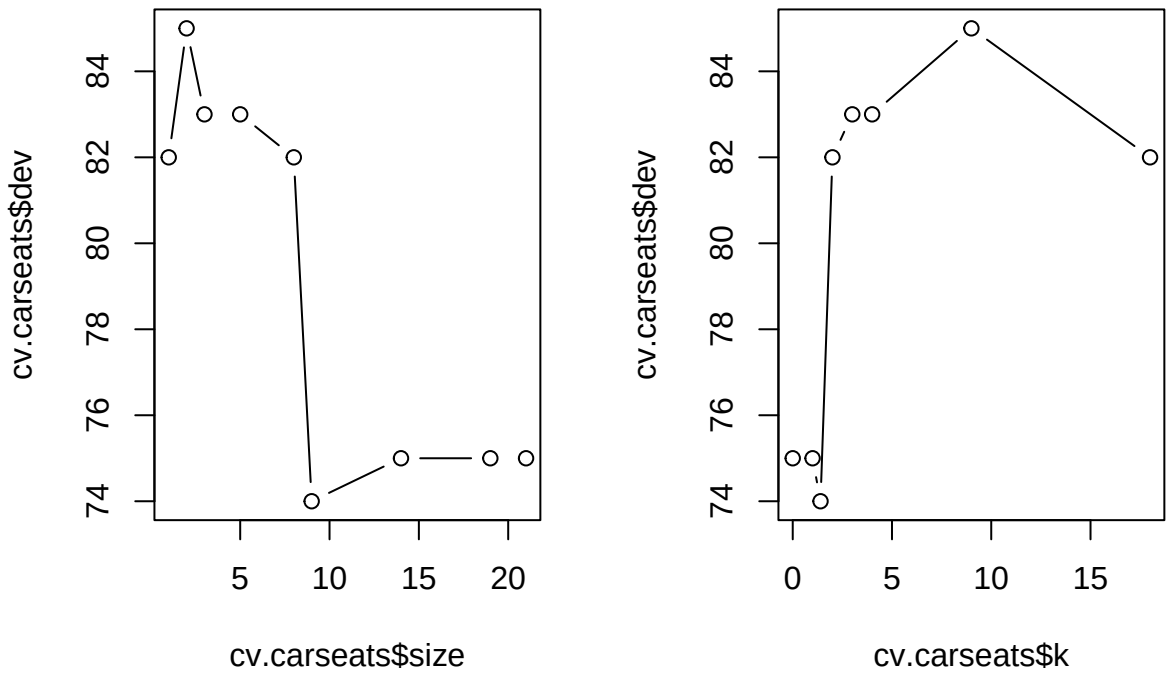
Next, we perform cross-validation using the `cv.tree()` function to determine the optimal level of tree complexity.

```
set.seed(7)
cv.carseats <- cv.tree(tree.carseats, FUN = prune.misclass)
cv.carseats
```

```
## $size
## [1] 21 19 14  9  8  5  3  2  1
##
## $dev
## [1] 75 75 75 74 82 83 83 85 82
##
## $k
## [1] -Inf  0.0  1.0  1.4  2.0  3.0  4.0  9.0 18.0
##
## $method
## [1] "misclass"
##
## attr(,"class")
## [1] "prune"         "tree.sequence"
```

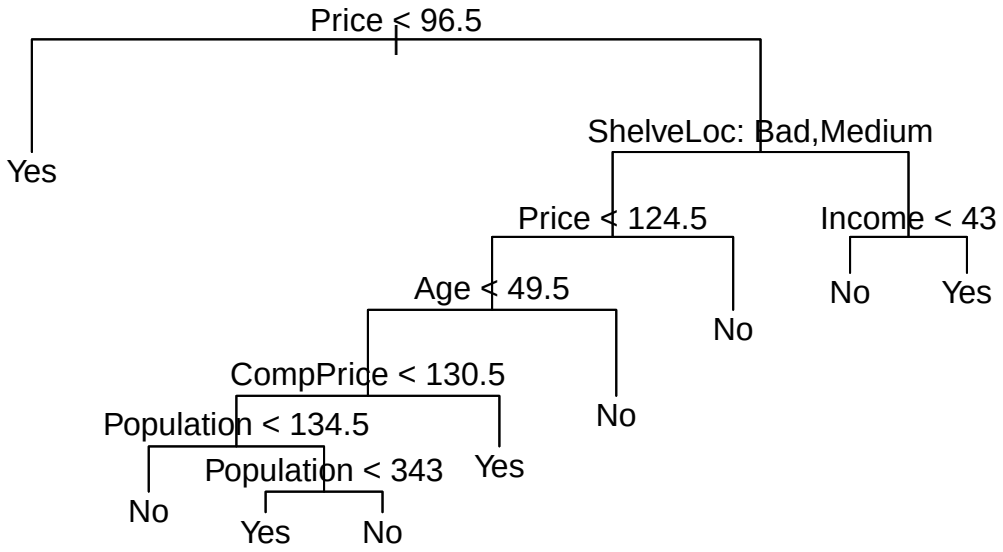
We plot the error rate as a function of both size and the cost-complexity parameter `k`.

```
par(mfrow = c(1, 2))
plot(cv.carseats$size, cv.carseats$dev, type = "b")
plot(cv.carseats$k, cv.carseats$dev, type = "b")
```



We now prune the tree to obtain the nine-node tree.

```
prune.carseats <- prune.misclass(tree.carseats, best = 9)
plot(prune.carseats)
text(prune.carseats, pretty = 0)
```



We check how the pruned tree performs on the test data.

```
tree.pred <- predict(prune.carseats, Carseats.test, type = "class")
table(tree.pred, High.test)
```

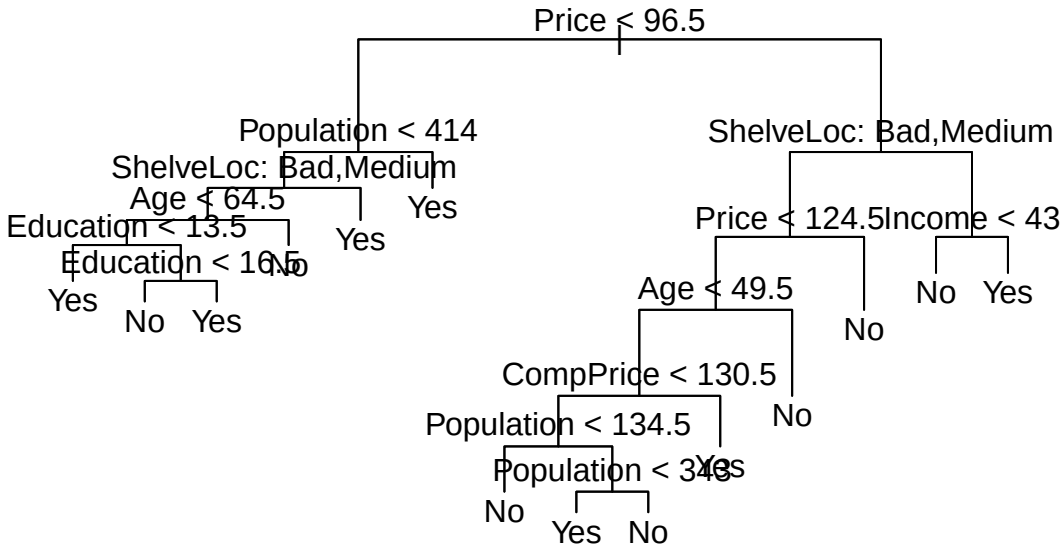
```
##          High.test
## tree.pred No Yes
##       No  97  25
##       Yes 20  58
```

```
(97 + 58) / 200
```

```
## [1] 0.775
```

If we increase the value of `best`, we obtain a larger pruned tree with lower classification accuracy.

```
prune.carseats <- prune.misclass(tree.carseats, best = 14)
plot(prune.carseats)
text(prune.carseats, pretty = 0)
```




```
tree.pred <- predict(prune.carseats, Carseats.test, type = "class")
table(tree.pred, High.test)
```

```
##          High.test
## tree.pred  No Yes
##       No  102  31
##       Yes  15  52
```

```
(102 + 52) / 200
```

```
## [1] 0.77
```

Exercises