第七章答案

7.1B-4

7.1B-5

$$N=\begin{array}{c|cc} & K & J \\ \hline K & \frac{2}{3} & \frac{2}{3} \\ J & \frac{1}{3} & \frac{1}{3} \end{array} \quad N_K=\begin{array}{c|cc} & Y & W \\ \hline Y & \frac{1}{4} & \frac{1}{4} \\ W & \frac{3}{4} & \frac{3}{4} \end{array} \quad N_J=\begin{array}{c|cc} & Y & W \\ \hline Y & \frac{4}{5} & \frac{4}{5} \\ W & \frac{1}{5} & \frac{1}{5} \end{array}$$

$$N_{KY}=\begin{array}{c|ccc} & A & B & C \\ \hline A & \frac{6}{11} & \frac{4}{7} & \frac{1}{2} \\ B & \frac{3}{11} & \frac{2}{7} & \frac{1}{3} \\ C & \frac{2}{11} & \frac{1}{7} & \frac{1}{6} \end{array} \qquad \begin{aligned} W_{KYA}&=0.5390 \\ W_{KYB}&=0.2972 \\ W_{KYC}&=0.1638 \end{aligned}$$

$$N_{KW}=\begin{array}{c|ccc} & A & B & C \\ \hline A & \frac{2}{7} & \frac{1}{3} & \frac{3}{11} \\ B & \frac{1}{7} & \frac{1}{6} & \frac{2}{11} \\ C & \frac{4}{7} & \frac{1}{2} & \frac{6}{11} \end{array} \qquad \begin{aligned} W_{KWA}&=0.2972 \\ W_{KWB}&=0.1638 \\ W_{KWC}&=0.5390 \end{aligned}$$

$$N_{JY}=\begin{array}{c|ccc} & A & B & C \\ \hline A & \frac{4}{7} & \frac{3}{4} & \frac{1}{3} \\ B & \frac{1}{7} & \frac{3}{16} & \frac{1}{2} \\ C & \frac{2}{7} & \frac{1}{16} & \frac{1}{6} \end{array} \qquad \begin{aligned} W_{JYA}&=0.5516 \\ W_{JYB}&=0.2768 \\ W_{JYC}&=0.1716 \end{aligned}$$

$$N_{JW}=\begin{array}{c|ccc} & A & B & C \\ \hline A & \frac{4}{7} & \frac{6}{10} & \frac{1}{2} \\ B & \frac{2}{7} & \frac{3}{10} & \frac{3}{8} \\ C & \frac{1}{7} & \frac{1}{10} & \frac{1}{8} \end{array} \qquad \begin{aligned} W_{JWA}&=0.5572 \\ W_{JWB}&=0.3202 \\ W_{JWC}&=0.1226 \end{aligned}$$

则 $A: \frac{2}{3}\times\left[\frac{1}{4}\times0.5390+\frac{3}{4}\times0.2972\right]+\frac{1}{3}\times\left[\frac{4}{5}\times0.5516+\frac{1}{5}\times0.5572\right]=0.4227$

$B: \frac{2}{3}\times\left[\frac{1}{4}\times0.2972+\frac{3}{4}\times0.1638\right]+\frac{1}{3}\times\left[\frac{4}{5}\times0.2768+\frac{1}{5}\times0.3702\right]=0.2266$

$C: \frac{2}{3}\times\left[\frac{1}{4}\times0.1638+\frac{3}{4}\times0.5390\right]+\frac{1}{3}\times\left[\frac{4}{5}\times0.1716+\frac{1}{5}\times0.1226\right]=0.3507$

故选A房子

A、A_K、A_J 均为二阶矩阵，有一致性，C_R 为0

$A_{KY}W_{KY}=(1.6248, 0.8943, 0.4921)^T$，$(\lambda_{KY})_{max}=1.6248+0.8943+0.4921=3.0112$

$C_I=\frac{\lambda_{max}-3}{n-1}=5.6\times10^{-3}$，$C_R=\frac{C_I}{R_I}=9.6\times10^{-3}$

$A_{KW}W_{KW}=(0.8943, 0.4921, 1.6248)^T$，$(\lambda_{KW})_{max}=3.0112$

$C_I=0.0056$，$C_R=9.6\times10^{-3}$

$A_{JY}W_{JY}=(2.002, 0.9295, 0.5397)^T$，$(\lambda_{JY})_{max}=3.4712$

$C_I=0.2356$，$C_R=0.4062$

$A_{JW}W_{JW}=(1.6880, 0.9666, 0.3686)^T$

$(\lambda_{JW})_{max}=3.0230$，$C_I=0.0116$，$C_R=0.02$

7.1B-5

7.1B-5

$$N = \begin{array}{c} \\ R \\ M \\ A \end{array}\begin{array}{c} \begin{array}{ccc} R & M & A \end{array} \\ \begin{bmatrix} \frac{1}{6} & \frac{1}{7} & \frac{5}{29} \\ \frac{1}{6} & \frac{1}{7} & \frac{4}{29} \\ \frac{2}{3} & \frac{5}{7} & \frac{20}{29} \end{bmatrix} \end{array} \qquad \begin{array}{l} W_R = 0.161 \\ W_M = 0.149 \\ W_A = 0.690 \end{array}$$

$$N_R = \begin{array}{c} \\ H \\ P \end{array}\begin{array}{c} \begin{array}{cc} H & P \end{array} \\ \begin{bmatrix} \frac{2}{3} & \frac{2}{3} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix} \end{array} \quad \begin{array}{l} W_{RH} = \frac{2}{3} \\ W_{RP} = \frac{1}{3} \end{array} \qquad N_M = \begin{array}{c} \\ H \\ P \end{array}\begin{array}{c} \begin{array}{cc} H & P \end{array} \\ \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{2}{3} \end{bmatrix} \end{array} \quad \begin{array}{l} W_{MH} = \frac{1}{3} \\ W_{MP} = \frac{2}{3} \end{array} \qquad N_A = \begin{array}{c} \\ H \\ P \end{array}\begin{array}{c} \begin{array}{cc} H & P \end{array} \\ \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \end{array} \quad \begin{array}{l} W_{AH} = \frac{1}{2} \\ W_{AP} = \frac{1}{2} \end{array}$$

$H = 0.161 \times \frac{2}{3} + 0.149 \times \frac{1}{3} + 0.69 \times \frac{1}{2} = 0.502$ $\quad\therefore H > P$，优先选择 H

$P = 0.161 \times \frac{1}{3} + 0.149 \times \frac{2}{3} + 0.69 \times \frac{1}{2} = 0.498$

$\because A_R, A_M, A_A$ 为 2 阶矩阵 $\quad \therefore$ 一致，$CR = 0$

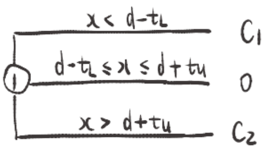
$$AW = \begin{bmatrix} 1 & 1 & \frac{1}{4} \\ 1 & 1 & \frac{1}{5} \\ 4 & 5 & 1 \end{bmatrix} \begin{bmatrix} 0.161 \\ 0.149 \\ 0.69 \end{bmatrix} = \begin{bmatrix} 0.4825 \\ 0.448 \\ 2.079 \end{bmatrix}$$

$\lambda_{max} \approx 0.4825 + 0.448 + 2.079 = 3.0095$

$CI \approx \frac{3.0095-3}{3-1} = 0.00475$ ， $CR \approx \frac{0.00475}{0.58} = 0.0082 < 0.1$ 一致性程度可接受.

7.2A-5

7.2A-5



期望损失：$C_1 \cdot Pr(x < d-t_L) + C_2 \cdot Pr(x > d+t_u) = C_1 [1 - Pr(d-t_L \le x \le d+t_u)]$

$\because x \sim N(\mu, \sigma^2)$

$\therefore$ 当区间中心 $\frac{d-t_L+d+t_u}{2} = \mu$ 时，$Pr(d-t_L \le x \le d+t_u)$ 最大，期望损失最小

即 $d = \mu + \frac{1}{2}(t_L - t_u)$

7.2B-4

7.2B-5(a)

$Pr\{Y=y_1\}=0.95\times0.96^2+0.05\times0.85^2=0.911625$, $Pr\{Y=y_2\}=0.95\times0.96\times0.04+0.05\times0.95\times0.15=0.08571$

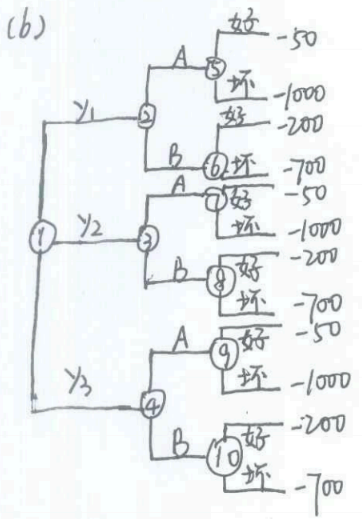
$Pr\{Y=y_3\}=0.95\times0.04^2+0.05\times0.15^2=0.002645$

$Pr\{X=1|Y=1\}=\dfrac{Pr\{Y=1|X=1\}Pr\{X=1\}}{Pr\{Y=1|X=1\}Pr\{X=1\}+Pr\{Y=1|X=2\}Pr\{X=2\}}=0.9604$ （此处改为 $Pr\{X=x_1|Y=y_1\}$）

同理得 $Pr\{X=x_1|Y=y_2\}=0.8512$，$Pr\{X=x_1|Y=y_3\}=0.5747$

$Pr\{X=x_2|Y=y_1\}=0.0396$，$Pr\{X=x_2|Y=y_2\}=0.1488$，$Pr\{X=x_2|Y=y_3\}=0.4253$

(b)



Y_1选A：$-50\times0.96-1000\times0.04=-88$

Y_1选B：$-200\times0.96-700\times0.04=-220$

Y_2选A：$-50\times0.851-1000\times0.149=-192$

Y_2选B：$-200\times0.851-700\times0.149=-275$

Y_3选A：$-50\times0.575-1000\times0.425=-453$

Y_3选B：$-200\times0.575-700\times0.425=-413$

检验结果为 y_1/y_2 时，给A优先供货

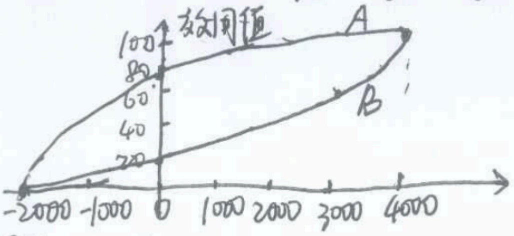
为 y_3 时，给B优先供货

7.2C-1

7.2C-1(a) $(-2000)p+(1-p)4000=0$，$p=\frac{2}{3}\Rightarrow U(0)=\frac{100}{3}$

(b)

	-2000	-1000	0	1000	2000	3000	4000	
A	0	70	80	85	90	95	100	(保)
B	0	10	20	30	50	60	100	(冒)



(c) 项目Ⅰ：$0.4\times95+0.6\times70=80$

项目Ⅱ：$0.6\times90+0.4\times80=86$

选项目Ⅱ，期望收益1200元

(d) 项目Ⅰ：$0.4\times60+0.6\times10=30$

项目Ⅱ：$0.6\times50+0.4\times20=38$

选项目Ⅱ，期望收益1200元

7.3A-1

7.3A-1:

(a) ① 等可能性准则:

$E\{v(a_1,S)\}=\frac{1}{3}(85+60+40)=61.7$

$E\{v(a_2,S)\}=\frac{1}{3}(92+85+81)=86$

$E\{v(a_3,S)\}=\frac{1}{3}(100+88+82)=90$,应选择 a_3=整晚学习

② 悲观主义准则:

$a_1:40, a_2:81, a_3:82$,应选 a_3.

③ 最小遗憾准则:

	S_1	S_2	S_3	
a_1	15	28	42	→ 42
a_2	8	3	1	→ 8
a_3	0	0	0	→ 0

应选 a_3.

④ 折中主义准则:

$a_1: \alpha 85+(1-\alpha)40=45\alpha+40$

$a_2: \alpha 92+(1-\alpha)81=11\alpha+81$

$a_3: \alpha 100+(1-\alpha)82=18\alpha+82$,应选 a_3

(b)

	S_1	S_2	S_3
a_1	B	D	F
a_2	A	B	B
a_3	A	B	B

4种准则下,a_1 均非最优,a_2,a_3 相同,故会改变Hank的选择,更偏向于 a_2:一半学习一半聚会

7.4A-2

7.4A-2

(a)

	B_1	B_2	B_3	行极小
A_1	1	q	6	1/q
A_2	p	5	10	5/p
A_3	6	2	3	2
列极大	6/p	5/q	10	

若(2,2)为鞍点.则 $p\geq 5$, $q\leq 5$

(b)

	B_1	B_2	B_3	行极小
A_1	2	4	5	2
A_2	10	7	q	7/q
A_3	4	p	6	4/p
列极大	10	7/p	6/q	

若(2,2)为鞍点.则 $p\leq 7$, $q\geq 7$

7.4A-3

7.4A-3

(a) 令 $V^- = \max_i\{\min_j\{a_{ij}\}\} \triangleq \min_j\{a_{pj}\}$
$V^+ = \min_j\{\max_i\{a_{ij}\}\} \triangleq \max_i\{a_{iq}\}$
$\Rightarrow V^- \le a_{pq}$，$V^+ \geqslant a_{pq}$ $\Rightarrow V^- \le V^+$

(b) ($\Leftarrow$) 已知 $V^- = V^+$，

则 $V^- = V^+ = a_{pq}$ $\Rightarrow$ $\min_j\{a_{pj}\} = \max_i\{a_{iq}\} = a_{pj}$

即对 $\forall i,j$，有 $a_{iq} \le a_{pq} \le a_{pj}$ $\Rightarrow$ a_{pq} 为纯策略鞍点.

($\Rightarrow$) 已知存在纯策略鞍点，设为 a_{i_0,j_0}

则 $\max_i a_{i,j_0} = a_{i_0,j_0} = \min_j a_{i_0,j}$

$V^- = \max_i\{\min_j\{a_{ij}\}\} \geqslant \min_j a_{i_0,j} = a_{i_0,j_0}$
$V^+ = \min_j\{\max_i\{a_{ij}\}\} \le \max_i a_{i,j_0} = a_{i_0,j_0}$
$\Rightarrow V^- \geqslant V^+$

又由(a)：$V^- \le V^+$，得 $V^- = V^+$

7.4B-2

7.4B-2

由定理 7.1(3)，易知 $\sum_j a_{ij} y_j^* \le V$，$\sum_i a_{ij} x_i^* \geqslant V$

(a) ① 反证法，假设 $\exists i$，s.t. $x_i^* > 0$，且 $\sum_j a_{ij} y_j^* < V$，又 $\sum_i x_i^* = 1$

则 $x^{*T}Ay^* = \sum_i x_i^*(a_{ij} y_j^*) < V$，与 V 是对策值矛盾.

∴ 若 $x_i^* > 0$，则 $\sum_j a_{ij} y_j^* = V$

② 反证法，假设 $\exists i$，s.t. $\sum_j a_{ij} y_j^* < V$，且 $x_i^* > 0$，又 $\sum_i x_i^* = 1$

则 $x^{*T}Ay^* = \sum_i x_i^*(a_{ij} y_j^*) < V$，与 V 是对策值矛盾.

∴ 若 $\sum_j a_{ij} y_j^* < V$，则 $x_i^* = 0$

(b) 与(a)类似

7.4B-5

7.4B-5

设 $V = x^{*T}Ay^*$，由定理 7.1(3)，$\max_i e_i^T A y^* \le V \le \min_j x^{*T} A e_j$
又 $V^- = \max_i \min_l\{a_{il}\} = \max_i \min_l e_i^T A e_l \le \max_i e_i^T A y^*$
$V^+ = \min_j \max_k\{a_{kj}\} = \min_j \max_k e_k^T A e_j \geqslant \min_j x^{*T} A e_j$
$\Rightarrow V^- \le V \le V^+$

7.4B-7

7.4B-7

$Ay^* = [\frac{31}{9}\ \ -\frac{26}{9}\ \ \frac{25}{9}\ \ -\frac{17}{9}]^T$，$x^*A = [-\frac{14}{3}\ \ 3\ \ \frac{7}{3}\ \ -\frac{10}{9}]$

$\max_i\{e_i^T A y^*\} = \frac{31}{9}$，$\min_j\{x^{*T} A e_j\} = -\frac{14}{3}$

$\Rightarrow \max_i\{e_i^T A y^*\} > \min_j\{x^{*T} A e_j\}$，

由定理 7.1(3)，x^*, y^* 不是最优解

7.4C-1a

7.4C-1(a)

∵ [1 1 0.1] < [10 1 10]　∴去除第2行，得 $\begin{bmatrix} 5 & 50 & 50 \\ 10 & 1 & 10 \end{bmatrix}$

又∵ $\begin{bmatrix} 5 \\ 10 \end{bmatrix} < \begin{bmatrix} 50 \\ 10 \end{bmatrix}$　∴去除第3列，得 $\begin{array}{c|cc} & y_1 & 1-y_1 \\ \hline x_1 & 5 & 50 \\ 1-x_1 & 10 & 1 \end{array}$

B的纯策略	A的期望收益
B_1	$5x_1+10(1-x_1)=10-5x_1$
B_2	$50x_1+(1-x_1)=49x_1+1$

$\Rightarrow$ 最优解 $x_1^*=\frac{1}{6}$，$V_A(x_1^*)=\frac{55}{6}$

A的纯策略	B的期望损失
A_1	$5y_1+50(1-y_1)=50-45y_1$
A_2	$10y_1+(1-y_1)=1+9y_1$

$\Rightarrow y_1^*=\frac{49}{54}$，$V_B(y_1^*)=\frac{55}{6}$

∴ $x^*=(\frac{1}{6},0,\frac{5}{6})$，$y^*=(\frac{49}{54},\frac{5}{54},0)$，$V=\frac{55}{6}$

7.4C-2

7.4C-2

(a) 设m,n为2个要地，失去阵地-1分，均势0分，胜利1分

A、B策略为

	m	n
A1	2	0
A2	1	1
A3	0	2

	m	n
B1	3	0
B2	2	1
B3	1	2
B4	0	3

A的赢得矩阵为 $\begin{bmatrix} -1 & -1 & 0 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix}$，去除第1、4列，第2行得 $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

(b) 求解略，$x^*=(0.5,0,0.5)$，$y^*=(0,0.5,0.5,0)$，$V=-0.5$，敌人会赢.

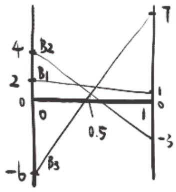
7.4D-1ad

7.4D-1

(a)

$$\begin{array}{c|cc} & y_2 & 1-y_2 \\ x_1 & \begin{bmatrix} 1 & -3 & 7 \\ 2 & 4 & -6 \end{bmatrix} \\ 1-x_1 & \end{array}$$

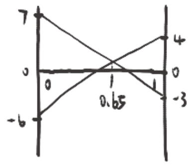
B纯策略	A期望收益
B1	$x_1+2(1-x_1)=2-x_1$
B2	$4-7x_1$
B3	$-6+13x_1$



$x_1^*=0.5$，$V_A(x_1^*)=0.5$

B采取混合策略 B2、B3

A纯策略	B期望损失
A1	$7-10y_2$
A2	$-6+10y_2$



$y_2^*=0.65$，$V_B(y_2^*)=0.5$

综上：$x^*=(0.5, 0.5)$，$y^*=(0, 0.65, 0.35)$，$V=0.5$

(d)

$$\begin{array}{c|cc} & y_1 & 1-y_1 \\ & -1 & -2 \\ x_2 & 3 & -4 \\ 1-x_2 & -7 & 6 \end{array}$$

B纯策略	A期望收益
B1	$-2+y_1$
B2	$-4+7y_1$
B3	$6-13y_1$

图略，$y_1^*=0.5$，$V_B(y_1^*)=-0.5$

A采取混合策略 A2、A3

A纯策略	B期望损失
A1	$-7+10x_2$
A2	$6-10x_2$

$x_2^*=0.65$，$V_A(x_2^*)=-0.5$

综上：$x^*=(0, 0.65, 0.35)$，$y^*=(0.5, 0.5)$，$V=-0.5$

7.4D-2

7.4D-2:

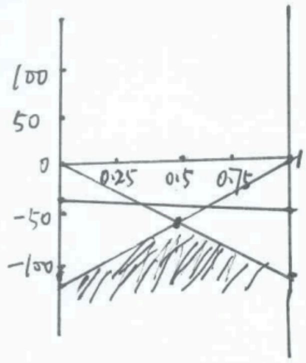
A_1: Robin选A　A_2: Robin选B

B_1: 警力全为A　B_2: 警力全在B　B_3: 警力对半分

	B_1	B_2	B_3
A_1	-100	0	-50
A_2	0	-100	-30

无纯策略解

B的纯策略	A的期望收益
B_1	$-100x_1$
B_2	$100x_1-100$
B_3	$-20x_1-30$



$f(x_1)=\min\{-100x_1, -100+100x_1, -20x_1-30\}$

$f(x_1)$在$0\le x_1\le 1$上的极大值在$x_1^*=0.5$处

取得$f(x^*)=-50$，为B_1、B_2的交点

A的纯策略	B的期望损失
A_1	$-100y_1$
A_2	$100y_1-100$

$-100y_1=100y_1-100 \Rightarrow y_1^*=0.5,\ f(x^*)=-50$

故对于Robin而言，应等概率选A·B；对警察而言应等概率选择全部在A或B

7.4E-1a

7.4E-1a

取$\alpha=\frac{1}{2}$，有 $[-3\ \ 1\ \ 0\ \ -4] < \alpha[4\ \ 4\ \ -4\ \ 1] + (1-\alpha)[-4\ \ -2\ \ 4\ \ 4]$

去掉第4行，得 $\begin{bmatrix} 4 & 4 & -4 & 1 \\ -4 & -2 & 4 & 4 \\ 2 & -4 & -1 & -5 \end{bmatrix}$． 无法进一步化简．

A: $\max\ z=v$

$$\text{s.t.}\ \begin{cases} v-4x_1+4x_2-2x_3\le 0 \\ v-4x_1+2x_2+4x_3\le 0 \\ v+4x_1-4x_2+x_3\le 0 \\ v-x_1-5x_2+5x_3\le 0 \\ x_1+x_2+x_3=1 \\ x_1, x_2, x_3\ge 0 \end{cases}$$

B: $\min\ w=u$

$$\text{s.t.}\ \begin{cases} u-4y_1-4y_2+4y_3+y_4\ge 0 \\ u+4y_1+2y_2-4y_3-4y_4\ge 0 \\ u-2y_1+4y_2+y_3+5y_4\ge 0 \\ y_1+y_2+y_3+y_4=1 \\ y_1, y_2, y_3, y_4\ge 0 \end{cases}$$

7.4E-5

7.4 E-5

(a) 设策略为 (m,n)，其中 m 为伸出手指数，n 为猜对手伸出手指数

A的赢得矩阵：

$$\begin{array}{c} \\ (1,1) \\ (1,2) \\ (2,1) \\ (2,2) \end{array} \begin{array}{c} \begin{array}{cccc} (1,1) & (1,2) & (2,1) & (2,2) \end{array} \\ \begin{bmatrix} 0 & 2 & -3 & 0 \\ -2 & 0 & 0 & 3 \\ 3 & 0 & 0 & -4 \\ 0 & -3 & 4 & 0 \end{bmatrix} \end{array}$$

A：$\max z = V$

s.t.
$$\begin{cases} V + 2x_2 - 3x_3 \le 0 \\ V - 2x_1 + 3x_4 \le 0 \\ V + 3x_1 - 4x_4 \le 0 \\ V - 3x_2 + 4x_3 \le 0 \\ x_1 + x_2 + x_3 + x_4 = 1 \\ x_1, x_2, x_3, x_4 \ge 0 \end{cases}$$

B：$\min w = u$

s.t.
$$\begin{cases} u - 2y_2 + 3y_3 \ge 0 \\ u + 2y_1 - 3y_4 \ge 0 \\ u - 3y_1 + 4y_4 \ge 0 \\ u + 3y_2 - 4y_3 \ge 0 \\ y_1 + y_2 + y_3 + y_4 = 1 \\ y_1, y_2, y_3, y_4 \ge 0 \end{cases}$$

解得 $x^* = (0, 0.6, 0.4, 0)$，$y^* = (0, 0.6, 0.4, 0)$，$V = 0$

(b) A的赢得矩阵：
$$\begin{bmatrix} 0 & 1 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

类似地，解得 $x^* = (0, 0.5, 0.5, 0)$，$y^* = (0, 0.5, 0.5, 0)$，$V = 0$

(c) 由于双方对称，$V = 0$