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ARTICLE

Evaluation of distance-independent competition indices in predicting tree survival and diameter growth

Shuaichao Sun, Quang V. Cao, and Tianjian Cao

Abstract: Competition indices play a significant role in modeling individual-tree growth and survival. In this study, six distance-independent competition indices were evaluated using 200 permanent plots of loblolly pine (*Pinus taeda* L.). The competition indices were classified into three families: (1) size ratios, which include diameter ratio and basal area ratio; (2) relative position indices, which include basal area of larger trees (BAL) and tree relative position based on the cumulative distribution function (CDF); and (3) partitioned stand density index and relative density. Results indicated that different families of competition indices were suitable for different tree survival or diameter growth prediction tasks. The diameter ratio was superior for predicting tree survival, whereas the relative position indices (BAL and CDF) performed best for predicting tree diameter growth, with CDF receiving the highest rank.

Key words: individual-tree model, diameter ratio, cumulative distribution function, stand density index, relative density.

Résumé : Les indices de compétition jouent un rôle important dans la modélisation de la croissance et de la survie d'arbres individuels. Dans cette étude, six indices de compétition indépendants de la distance ont été évalués à l'aide de 200 places échantillons permanentes de pin à encens (*Pinus taeda* L.). Les indices de compétition ont été subdivisés en trois familles : (1) les ratios de taille incluant les ratios basés sur les diamètres ou les surfaces terrières; (2) les indices de position relative qui incluent la surface terrière des plus gros arbres (STG), et la position relative des arbres basée sur la fonction de distribution cumulative (FDC); et (3) l'indice de densité fractionnée du peuplement et la densité relative. Les résultats indiquent que certaines familles d'indices de compétition conviennent mieux pour prédire la croissance en diamètre et d'autres pour prédire la survie des arbres. Le ratio basé sur les diamètres était meilleur pour prédire la survie des arbres tandis que les indices de position relative (STG et FDC) avaient la meilleure performance pour la prédiction de la croissance en diamètre des arbres; des deux indices, FDC était le meilleur. [Traduit par la Rédaction]

Mots-clés : modèle d'arbre individuel, ratio des diamètres, fonction de distribution cumulative, indice de densité du peuplement, densité relative.

Introduction

Having the highest level of resolution in the suite of forest growth and yield models, individual-tree models have inherent flexibility that project tree and stand development through time for combinations of species mixtures and stand structures. Typically, for even-aged forests, individual-tree models consist of three basic components: a diameter growth model, a height growth model, and a survival model (Burkhart and Tomé 2012). The height growth model can sometimes be replaced by a height–diameter model that provides height prediction for a specified tree diameter.

The effect of competition on growth of individual trees plays a key role in individual-tree models, as it reflects the individual tree's resource supply (Stage 1973; Daniels and Burkhart 1975; Amateis et al. 1989; Huang and Titus 1995). Competition in forests may be defined as interactions among individual trees that lead to a reduction in the survival, growth, and regeneration of the tree concerned (Begon et al. 1986). Competition indices are therefore mathematical formulations that express how much each tree is affected by its neighbors (Burkhart and Tomé 2012).

Competition indices can be classified as distance dependent and distance independent, depending on whether tree coordinates are required (Munro 1974). Because distance-independent indices are less demanding in data collection and easy to calculate, they are preferable in many applications. Moreover, most comparisons revealed that distance-dependent indices showed little if any advantage over distance-independent counterparts (Martin and Ek 1984; Daniels et al. 1986; Cole and Lorimer 1994; Biging and Dobbertin 1995; Wimberly and Bare 1996; Corral Rivas et al. 2005; Kahriman et al. 2018).

Competition indices have long been studied in an attempt to more accurately predict tree survival and growth. Even though a number of studies have involved comparisons of various competition indices (e.g., Biging and Dobbertin 1995; Corral Rivas et al. 2005; Maleki et al. 2015), no clear trends have emerged from these analyses. Different competition indices have often been compared either by computing simple correlation coefficients with tree survival and growth (Lorimer 1983; Pukkala and Kolström 1987; Holmes and Reed 1991) or by determining significance of the competition index when added to an individual-tree model in which tree size and stand variables already exist in the base model

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Table 1. Means (and standard deviation) of stand and tree attributes by group and age.

	Stand age, years			
Attribute	10	15	20	25
Group 1 (100 plots)				
Tree diameter (cm)	11.7 (3.1)	14.5 (3.8)	17.3 (4.4)	19.8 (4.9)
Basal area ($m^2 \cdot ha^{-1}$)	21.5 (6.2)	31.6 (7.2)	33.6 (7.6)	38.1 (8.5)
No. of trees per hectare	1987 (642)	1750 (613)	1303 (358)	1162 (322)
Dominant height (m)	9.0 (1.2)	13.2 (1.6)	16.4 (1.9)	19.2 (2.3)
Group 2 (100 plots)				
Tree diameter (cm)	11.6 (2.9)	14.8 (3.9)	17.6 (4.6)	20.0 (5.1)
Basal area ($m^2 \cdot ha^{-1}$)	22.1 (6.0)	32.0 (7.2)	33.2 (7.2)	37.2 (8.4)
No. of trees per hectare	1976 (629)	1702 (602)	1243 (348)	1112 (342)
Dominant height (m)	9.2 (1.2)	13.4 (1.6)	16.7 (2.0)	19.5 (2.4)

(Daniels et al. 1986; Biging and Dobbertin 1995; Wimberly and Bare 1996; Corral Rivas et al. 2005; Kahriman et al. 2018).

In addition, most studies employed the same competition index for different components of an individual-tree model (e.g., Stage 1973; Amateis et al. 1989; Pukkala et al. 2009; de-Miguel et al. 2012). However, a single competition index cannot holistically represent all components of competition. Response to competition from each prediction goal could also be different. It is not clear from previous studies whether the same competition index can be equally effective in predicting both future tree survival and diameter growth or whether different indices should be used for different objectives.

Thus, the objective of this study was to evaluate the inclusion of various distance-independent competition indices in models that predict tree survival and diameter growth.

Data

Data available for this study were from 200 permanent plots randomly selected from the Southwide Seed Source Study, which include 15 loblolly pine (*Pinus taeda* L.) seed sources planted at 13 locations across 10 southern states in the USA. A detailed description of the data can be found in Wells and Wakeley (1966). Each plot of size 0.0164 ha comprised 49 trees planted at a 1.8 m × 1.8 m spacing. Measurements including tree survival and diameters were taken at the beginning and end of three growth periods at ages 10–15, 15–20, and 20–25. Therefore, a total of 600 growth periods was obtained.

The leave one out evaluation scheme (Cao 2017) was applied in this study by randomly dividing the data into two groups of 100 plots each. Parameters of the tree survival and diameter growth models were estimated from the data of one group. Then the parameters were used to predict tree survival and diameter growth for the other group. Evaluation statistics were finally computed by pooling predictions from both groups. Table 1 shows summary statistics for stand and tree attributes by group and age.

Methods

Tree survival and diameter growth models

The following set of individual-tree models were used as the base models for predicting tree survival and diameter growth:

$$(1)\qquad p_{ij} = \frac{1}{1 + \exp(\alpha_0 + \alpha_1/A_{1,i} + \alpha_2\mathrm{RS}_{1,i} + \alpha_3 B_{1,i} + \alpha_4 d_{1,ij})}$$

$$(2)\qquad \hat{d}_{2,ij} = d_{1,ij} + \exp(\beta_0 + \beta_1/A_{1,i} + \beta_2\mathrm{RS}_{1,i} + \beta_3 B_{1,i} + \beta_4 d_{1,ij})$$

where p_{ij} is survival probability of tree j in plot i during the 5-year growth period; $A_{1,i}$ is stand age (in years) for plot i at the beginning of the growth period; $\mathrm{RS}_{1,i} = (\sqrt{10000/N_{1,i}})/H_{1,i}$ is relative spacing for plot i at age $A_{1,i}$; $N_{1,i}$ is number of trees per ha in plot i at age $A_{1,i}$; $H_{1,i}$ is dominant height (in m; calculated as mean height of dominant and codominant trees) of plot i at age $A_{1,i}$; $B_{1,i}$ is stand basal area (in $m^2 \cdot ha^{-1}$) of plot i at age $A_{1,i}$; $d_{1,ij}$ is the diameter (in cm) of tree j in plot i at age $A_{1,i}$; $\hat{d}_{2,ij}$ is the predicted diameter (in cm) of tree j in plot i at age $A_{2,i}$ (age at the end of the growth period); and α and β variables are regression parameters. The models were modified from the ones previously developed by Cao (2006).

For the purpose of evaluation, each distance-independent competition index ($\mathrm{CI}_{1,ij}$) was included as an additional term in eqs. 1 and 2 as follows:

$$(3)\qquad p_{ij} = \frac{1}{1 + \exp(\alpha_0 + \alpha_1/A_{1,i} + \alpha_2\mathrm{RS}_{1,i} + \alpha_3 B_{1,i} + \alpha_4 d_{1,ij} + \alpha_5\mathrm{CI}_{1,ij})}$$

$$(4)\qquad \hat{d}_{2,ij} = d_{1,ij} + \exp(\beta_0 + \beta_1/A_{1,i} + \beta_2\mathrm{RS}_{1,i} + \beta_3 B_{1,i} + \beta_4 d_{1,ij} + \beta_5\mathrm{CI}_{1,ij})$$

As commonly seen in individual-tree models, the models in this study contained both stand-level variables ($A_{1,i}$, $\mathrm{RS}_{1,i}$, and $B_{1,i}$) and tree-level variables ($d_{1,ij}$ and $\mathrm{CI}_{1,ij}$). The stand-level variables (including the competition indices such as $\mathrm{RS}_{1,i}$ and $B_{1,i}$) describe the common behavior of all trees in the same plot, and the tree-level variables (including $\mathrm{CI}_{1,ij}$) add the variations among trees.

Competition indices

Six distance-independent competition indices described below were employed to predict tree survival and diameter growth.

Diameter ratio (DR)

Glover and Hool (1979) introduced a distance-independent index based on the ratio of diameter of the subject tree to the mean diameter. As a measure of central tendency, quadratic mean diameter has been considered more appropriate than arithmetic mean for characterizing measured trees (Curtis and Marshall 2000). Therefore, quadratic mean diameter was applied to replace the mean diameter in this study:

$$(5)\qquad \mathrm{DR}_{1,ij} = \frac{d_{1,ij}}{D_{q_{1,i}}}$$

where $\mathrm{DR}_{1,ij}$ is diameter ratio of tree j in plot i; and $D_{q_{1,i}}$ is quadratic mean diameter of plot i at age $A_{1,i}$.

Basal area ratio (BR)

Daniels et al. (1986) employed a competition index that considers the ratio of the tree basal area to the mean tree basal area on the plot as follows:

$$(6)\qquad \mathrm{BR}_{1,ij} = \frac{B_{1,ij}}{\bar{B}_{1,i}} = \frac{d_{1,ij}^2}{D_{q_{1,i}}^2}$$

where $\mathrm{BR}_{1,ij}$ is basal area ratio of tree j in plot i; and $\bar{B}_{1,i}$ is the mean tree basal area in plot i at age $A_{1,i}$. This index is the square of the diameter ratio as defined in eq. 5.

Basal area in larger trees (BAL)

Total basal area of trees larger than the subject tree is a commonly used distance-independent competition index in individual-tree growth models (Wykoff et al. 1982; Wykoff 1990). The assumption is that larger neighbors place the subject tree at a competitive disadvantage, whereas smaller neighbors do not affect the subject tree.

Cumulative distribution function (CDF)

Stand diameter distribution is generally characterized by CDF. The CDF of a particular tree diameter describes the relative position of that tree compared with others in the stand. Clutter et al. (1984) used the CDF of the subject tree as a competition index to predict individual tree heights. It was also employed in this study to predict tree survival and diameter growth. If j is the rank (from smallest to largest) of the subject tree in plot i in terms of diameter, the CDF of that tree ($F_{1,ij}$) was computed by use of Poudel and Cao's (2013) method as follows:

$$(7) \qquad F_{1,ij} = \frac{j - 0.5}{n_i}$$

where n_i is the number of trees in plot i. Similar to Clutter et al. (1984), $\ln(F_{1,ij})$ rather than $F_{1,ij}$ was used as competition index in this study, where $\ln(\cdot)$ is the natural logarithm.

Partitioned stand density index

Since Reineke (1933) proposed a stand density index (SDI) to quantify stocking in even-aged stands, it has been widely used as a measure of competition and site occupancy. Stage (1968) partitioned Reineke's SDI to determine the contribution of each individual tree to the overall stand SDI. The theoretical foundation of Stage's formulation was presented by Lu et al. (2017). The derivation of the method for partitioning SDI from stand level to individual-tree level can be found in Appendix A. The resulting partitioned SDI was used as an individual-tree competition index in this study.

Partitioned relative density

Curtis (1982) developed an index of relative density (RD), which is a simple and convenient diameter-based stand density measure. In this study, we partitioned RD from stand level to individual-tree level using a similar approach as the one used in partitioning SDI (details are also shown in Appendix A).

By comparing the derivation for the partitioned SDI and RD, it can be found that the two indices are similar except for a constant term. Consequently, regressions involving these indices will produce identical predictions, leading to identical evaluation statistics.

Families of competition indices

The six distance-independent competition indices evaluated in this study could be categorized into three general families based on their properties: size ratios (diameter and basal area ratios), which quantify the relative size position of an individual tree in a stand; relative position indices (BAL and CDF), which are calculated based on information from trees either larger or smaller than the subject tree; and partitioned density indices, which divide SDI and RD into components for individual trees.

Based on Tomé and Burkhart's (1989) interpretation for three primary components of competition, the size ratios represent microenvironmental and genetic influences, the relative position indices signify the influence of local neighbors from distance-independent perspective, and finally, the partitioned density indices characterize the general environment surrounding the tree.

Evaluation

Parameters of eqs. 3 and 4 for each competition index were estimated using data from one group and then evaluated using data from the other group. Predicted values from both groups were then pooled to calculate evaluation statistics.

Besides, to make a comprehensive comparison, we also computed the Pearson correlation coefficients between competition indices and tree survival and diameter growth.

Table 2. Evaluation statistics for tree survival prediction.

Competition index	MD	MAD	AUC	Rank
None	0.0001	0.2290	0.7907	5.52
$DR_{1,ij}$	−0.0007	**0.2272**	**0.7925**	**1.00**
$BR_{1,ij}$	−0.0007	0.2279	0.7918	4.29
$BAL_{1,ij}$	−0.0001	0.2289	0.7904	6.00
$\ln(F_{1,ij})$	−0.0004	0.2283	0.7909	5.40
$SDI_{1,ij}$ or $RD_{1,ij}$	**0.0000**	0.2282	0.7899	4.08

Note: Bold numbers denote the best competition index for each criterion; underlined numbers denote the worst competition index.

Evaluation statistics

For evaluating both tree survival and diameter growth models, the mean difference (MD) and mean absolute difference (MAD) were used:

$$(8) \qquad MD = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n_i}(y_{ij} - \hat{y}_{ij})}{\sum_{i=1}^{m} n_i}$$

$$(9) \qquad MAD = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n_i}|y_{ij} - \hat{y}_{ij}|}{\sum_{i=1}^{m} n_i}$$

where m denotes the number of plots. For tree survival, y_{ij} equals 1 if tree j in plot i was alive at age $A_{2,i}$ and 0 if it was dead, and $\hat{y}_{ij}$ equals p_{ij} in eq. 1. For diameter growth, y_{ij} and $\hat{y}_{ij}$ are observed and predicted diameter of tree j in plot i at age $A_{2,i}$, respectively.

In addition to MD and MAD, tree survival and diameter growth were evaluated by use of the AUC and fit index, respectively. AUC is the area under the receiving operating characteristic (ROC) curve. The ROC curve shows the ability of a system to classify binary data at various threshold settings (Swets 1986). The AUC ranges from 0.5 (poor fit) to 1 (perfect fit). Similar to Cao (2017), the fit index (R^2) was defined as follows:

$$(10) \qquad R^2 = 1 - \frac{\sum_{i=1}^{m}\sum_{j=1}^{n_i}(d_{2,ij} - \hat{d}_{2,ij})^2}{\sum_{i=1}^{m}\sum_{j=1}^{n_i}(d_{2,ij} - \bar{d}_2)^2}$$

where $d_{2,ij}$ is the observed diameter of tree j in plot i at age $A_{2,i}$, and $\bar{d}_2$ is the mean diameter for all trees at the end of the growing period.

A good model should produce smaller absolute value of MD, smaller MAD, and larger AUC and R^2.

Ranking of evaluation statistics

To describe the relative position of each competition index, we employed Poudel and Cao's (2013) relative rank, which is defined as follows:

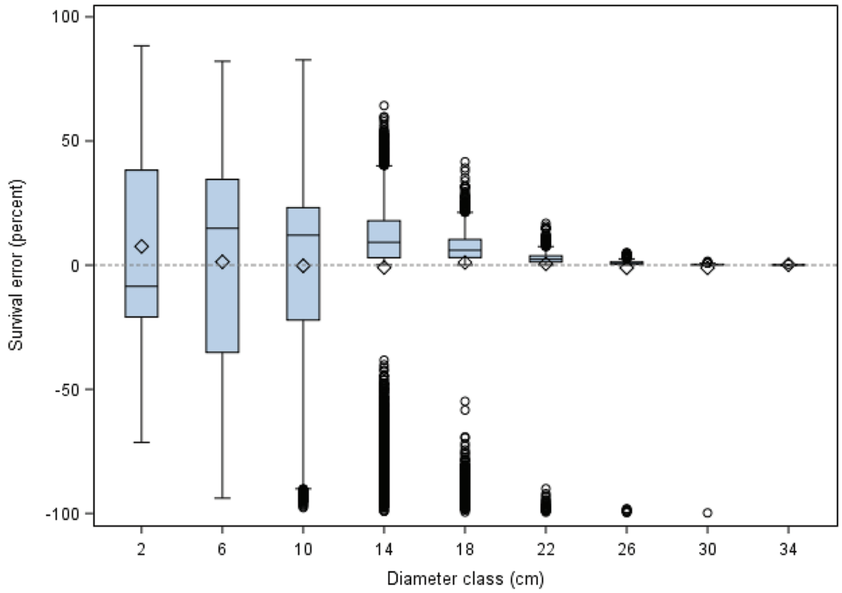
$$(11) \qquad R_i = 1 + \frac{(k - 1)(S_i - S_{min})}{S_{max} - S_{min}}$$

for minimization objective (|MD| and MAD) and

$$(12) \qquad R_i = k - \frac{(k - 1)(S_i - S_{min})}{S_{max} - S_{min}}$$

for maximization objective (AUC and R^2); where R_i is relative rank of the ith competition index (i = 1, 2, …, k); k is the number of competition indices evaluated; S_i is value of each evaluation statistic for the ith competition index; S_{min} and S_{max} are minimum

Fig. 1. Predicted tree survival errors against diameter class, with $DR_{1,ij}$ as competition index in eq. 3. The box plots display the whiskers, percentiles, and mean of the predicted survival errors for each diameter class, as well as outliers. [Colour online.]



and maximum values of S_i, respectively. As a result, the best competition index should consistently receive a rank of 1, whereas the worst one receives a rank of k.

A two-step approach was carried out: a relative rank was first calculated for each statistic from each competition index, and then, a final rank was computed based on the sum of all three ranks for each competition index.

Results

Table 2 presents the evaluation statistics of each competition index for predicting tree survival. Diameter ratio ($DR_{1,ij}$) was overall the best performer in predicting tree survival, ranking far behind the diameter ratio were the two partitioned competition indices (rank of 4.08) and the basal area ratio (4.29). The basal area in larger trees ($BAL_{1,ij}$) ranked last (6.00) among competition indices. The tree survival model with $BAL_{1,ij}$ even performed worse than the control model with no competition index (rank of 5.52). The other relative position index, $\ln(F_{1,ij})$, did not fare much better with a ranking of 5.40.

Figure 1 shows the tree survival prediction errors by 4 cm diameter class, with $DR_{1,ij}$ as competition index. Most errors occurred on smaller diameter classes. The error rate for larger trees was low because most of them survived and the survival model correctly predicted a higher probability for these trees to survive.

The statistics for evaluating different competition indices in predicting diameter growth are displayed in Table 3. The two relative position indices performed well in predicting tree diameter growth, each ranked either first or second in all three evaluation statistics. Although $\ln(F_{1,ij})$ was the best ranked model, $BAL_{1,ij}$ was a close second with a rank of 1.36. The two partitioned competition indices ranked last (6.00) in diameter growth prediction, even behind the control model without competition index (rank of 5.97). On the other hand, the diameter and basal area ratios ranked in the middle, ranking 3.66 and 4.92, respectively.

Figure 2 shows the diameter growth prediction errors by 4 cm diameter class, with the inclusion of $\ln(F_{1,ij})$ as competition index. Except for the outliers, the errors seemed to have homogenous variance. The growth model performed well for most diameter classes, with the exception at the 34-cm class where diameter growth was overestimated.

Table 3. Evaluation statistics for tree diameter growth prediction.

Competition index	MD	MAD	R^2	Rank
None	−0.0148	<u>0.9080</u>	<u>0.9382</u>	5.97
$DR_{1,ij}$	−0.0181	0.8870	0.9399	3.66
$BR_{1,ij}$	<u>−0.0192</u>	0.8932	0.9394	4.92
$BAL_{1,ij}$	**−0.0118**	0.8837	0.9401	1.36
$\ln(F_{1,ij})$	−0.0124	**0.8798**	**0.9404**	**1.00**
$SDI_{1,ij}$ or $RD_{1,ij}$	−0.0169	0.9039	0.9385	<u>6.00</u>

Note: Bold numbers denote the best competition index for each criterion; underlined numbers denote the worst competition index.

Competition indices within a family produced similar results (Tables 2 and 3). The only exception was the gap in ranks between $DR_{1,ij}$ (1.00) and $BR_{1,ij}$ (4.95) in predicting tree survival (Table 2).

Table 4 presents the Pearson correlation coefficient between each competition index and tree survival or diameter growth. The diameter ratio ($DR_{1,ij}$) produced the strongest correlation coefficient with tree survival, corroborating earlier results. On the other hand, the second strongest correlation with tree survival was $\ln(F_{1,ij})$ (rank of 1.08), which had a low ranking of 5.40 in terms of evaluation statistics (Table 2). For diameter growth, the top correlation coefficient was from $BAL_{1,ij}$, whereas the other relative position index $\ln(F_{1,ij})$ placed fourth with a ranking of 2.44. These two competition indices held the top spots in the ranking of evaluation statistics, with $\ln(F_{1,ij})$ edging out $BAL_{1,ij}$ (Table 3).

The estimated parameters and statistics for tree survival and diameter growth models (eqs. 3 and 4) with the best competition indices, i.e., $DR_{1,ij}$ for survival prediction and $\ln(F_{1,ij})$ for diameter growth prediction, are shown in Table 5. These parameter estimates were computed from the entire data set.

Discussion

Simple correlation coefficients

Results from the correlation coefficients (Table 4) and the evaluation statistics (Tables 2 and 3) were similar for some competition indices and different for others, maybe because the relationship between each competition index and either tree survival or diameter growth was assumed to be linear in the case of correlation but nonlinear for regression models.

Fig. 2. Predicted diameter growth errors against diameter class, with $\ln(F_{1,ij})$ as competition index in eq. 4. The box plots display the whiskers, percentiles, and mean of the predicted diameter growth errors for each diameter class, as well as outliers. [Colour online.]

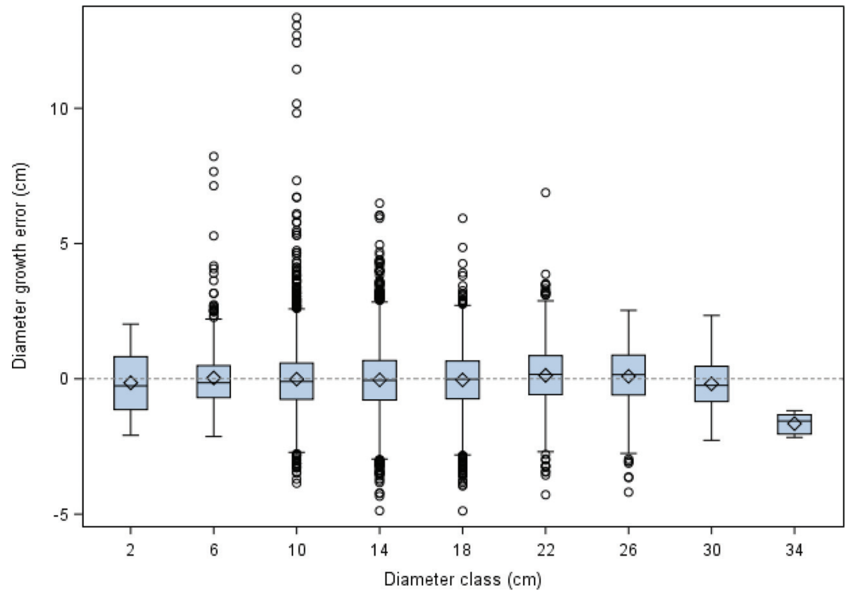


Table 4. Correlation coefficients (and relative ranks) between competition indices and tree survival and diameter growth.

Competition index	Survival	Diameter growth
$DR_{1,ij}$	**0.3523 (1.00)**	0.4297 (1.97)
$BR_{1,ij}$	0.3266 (1.94)	0.4309 (1.96)
$BAL_{1,ij}$	−0.3107 (2.52)	**−0.5315 (1.00)**
$\ln(F_{1,ij})$	0.3501 (1.08)	0.3805 (2.44)
$SDI_{1,ij}$ or $RD_{1,ij}$	<u>0.2431</u> (<u>5.00</u>)	<u>0.1113</u> (<u>5.00</u>)

Note: Bold numbers denote the strongest correlation with either tree survival or diameter growth; underlined numbers denote the weakest correlation. All correlation coefficients are highly significant (p < 0.0001).

Table 5. Parameter estimates and statistics of the best tree survival and diameter growth models.

	Survival model		Diameter growth model	
Parameter	Estimate	SE	Estimate	SE
α_0	−5.1337	0.3547	—	—
α_1	8.6740	2.1347	—	—
α_2	10.1137	1.1799	—	—
α_3	−0.0344	0.0058	—	—
α_4	0.2618	0.0158	—	—
α_5	1.7943	0.2350	—	—
β_0	—	—	−1.0197	0.0619
β_1	—	—	13.7545	0.3340
β_2	—	—	1.8015	0.1231
β_3	—	—	−0.0129	0.0011
β_4	—	—	0.0609	0.0015
β_5	—	—	0.1918	0.0090
−2lnL	11990.6		—	
RMSE	—		1.1710	

Note: SE, standard error; −2lnL, −2 × log-likelihood; RMSE, root mean square error. All parameters are highly significant (p < 0.0001).

Competition indices families

Except for the partitioned density indices, which produced identical evaluation statistics, competition indices in the same family yielded different results. For the size ratios, the diameter ratio outperformed the basal area ratio for predicting either tree survival or diameter growth (Tables 2 and 3). On the other hand, for the relative position indices, $\ln(F_{1,ij})$ was consistently better than $BAL_{1,ij}$ (Tables 2 and 3). It is worth noting that $\ln(F_{1,ij})$ is also easier to compute.

Although the basal area ratio belongs to the same size ratio family, it did not perform as well as the diameter ratio in predicting tree survival (Table 2). Squaring the diameter ratio to obtain the basal area ratio produces a competition index that might overestimate the relative size relationship between the subject tree and the "average" tree in terms of survival.

Best competition indices for different models

For tree survival prediction, the best overall competition index was the diameter ratio ($DR_{1,ij}$) (Table 2). This index ranked first in both MAD and AUC but last in terms of MD. The last place in MD ranking might be misleading, because its MD value of −0.0007 was actually very close to zero. On the other hand, CDF ($\ln(F_{1,ij})$) in the family of relative position indices performed best in predicting diameter growth (Table 3).

Although CDF seemed stable in describing the individual tree's relative competitive environment for our even-aged stands of loblolly pine, it has important limitations as an index of competition. With the range between 0 and 1, CDF only depicts a tree's competitive position within the stand while ignoring the variation among different stands. Trees with the same CDF from different stands may face different competitive pressures. Therefore, for heterogeneous stands such as uneven-aged or mixed stands, CDF might not perform as well.

In addition, the computation of CDF (eq. 7) depends on the order of tree diameters and therefore does not explicitly take into account the size difference between each individual tree. This size difference might play an important role in tree survival prediction, especially in young stands. In contrast to CDF, the value of diameter ratio has no upper bound, allowing it to adequately characterize the relative size difference within a stand. Besides, size ratios (diameter and basal area ratios) have also been shown by a few studies (Lorimer 1983; Daniels et al. 1986; Holmes and Reed 1991) to provide equal or better prediction of tree diameter growth compared with many distance-dependent competition indices.

The performance of competition indices may also change as stands develop over time, and the best index for tree survival or diameter growth may differ at different stages of stand development. For example, Filipescu and Comeau (2007) found that the effects of competition indices changed between younger and older stands. Meanwhile, as a shade-intolerant species, loblolly pine may be sensitive to shading, which means that the effect of each competition index might be different before and after the canopy closes and shading begins.

Effects of underlying thinning

As the most important management tool, thinning with different types, times, and intensities can notably affect the performance of competition indices. In general, size ratios (diameter and basal area ratios) do not perform logically when a stand is thinned (Weiskittel et al. 2011). By the same token, CDF may also not be good in such stands as the amount of competitive pressure reduced by residual trees cannot be depicted appropriately. Consequently, an absolute measure of a competition index like BAL might be a better choice in this situation. It has also been demonstrated that BAL is a reliable competition index for individual-tree growth prediction in both even-aged and uneven-aged stands (Wykoff et al. 1982; Wimberly and Bare 1996; Monserud et al. 1997; Pukkala et al. 2009; Shater et al. 2011; de-Miguel et al. 2012).

Conclusions

In this study, we evaluated models containing no competition index versus those with each of the six distance-independent competition indices for predicting tree survival and diameter growth in loblolly pine stands. Results indicated that different competition indices were suitable for different tree survival or diameter growth prediction tasks. The model involving diameter ratio was the best to predict tree survival. For predicting diameter growth, both of the relative position indices performed well, with $\ln(F_{1,ij})$ being the best.

In summary, the inclusion of a competition index helped improve the performance of the individual-tree model. We recommend diameter ratio, $DR_{1,ij}$, for tree survival and cumulative distribution function, $\ln(F_{1,ij})$, for diameter growth prediction.

Inference from results of this study is limited because it was conducted on even-aged loblolly pine stands. The findings in this study need to be tested at other stands with different tree species and stand structures. Moreover, future studies should consider other competition indices involving tree heights and crown dimensions, as well as distance-dependent competition indices.

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Appendix A

Partitioning the SDI

Reineke's (1933) stand density index (SDI) is expressed as follows:

$$\text{(A1)} \qquad \mathrm{SDI}_i = N_i\left(\frac{D_{q_i}}{25}\right)^c$$

where N_i = number of trees per ha in plot i; D_{q_i} = quadratic mean diameter (in cm) of plot i; c = 1.605 = self-thinning constant from Reineke (1933); and 25 cm is approximately 10 inches (used in Reineke's (1933) original formula).

Let $W_i = \sum_{j=1}^{n_i} \mathrm{EF}_{ij} d_{ij}^2$ = sum of d_{ij}^2 for all trees in 1 ha, where n_i = number of trees in plot i; EF_{ij} = expansion factor, or number of trees per ha represented by tree j in plot i; and d_{ij} = DBH of tree j in plot i.

Then

$$\text{(A2)} \qquad N_i = \sum_{j=1}^{n_i} \mathrm{EF}_{ij}$$

$$\text{(A3)} \qquad D_{q_i} = \left(\frac{W_i}{N_i}\right)^{1/2}$$

$$\text{(A4)} \qquad \mathrm{SDI}_i = 25^{-c} N_i^{1-(c/2)} W_i^{c/2}$$

According to Kudryavtsev (1994), a real-valued function is called a homogeneous function of degree r if and only if

$$\text{(A5)} \qquad f(tx_1, tx_2, \cdots, tx_n) = t^r f(x_1, x_2, \cdots, x_n)$$

where t can be any value if $f(tx_1, tx_2, \ldots, tx_n)$ lies within its domain.

For t = 1 and r = 1 with two variables (x_1, x_2), Euler's theorem (Kudryavtsev 1994) gives

$$\text{(A6)} \qquad f(x_1, x_2) = \frac{\partial f}{\partial x_1}x_1 + \frac{\partial f}{\partial x_2}x_2$$

Because SDI_i is a homogeneous function of degree 1 (r = 1), eq. A6 becomes

$$\text{(A7)} \qquad \mathrm{SDI}_i = \frac{\partial \mathrm{SDI}_i}{\partial N_i}N_i + \frac{\partial \mathrm{SDI}_i}{\partial W_i}W_i = aN_i + bW_i$$

where

$$\text{(A8)} \qquad a = 25^{-c}\left(1 - \frac{c}{2}\right)D_{q_i}^c$$

and

$$\text{(A9)} \qquad b = 25^{-c}\frac{c}{2}D_{q_i}^{c-2}$$

From the definition of W_i and eq. A2,

$$\text{(A10)} \qquad \mathrm{SDI}_i = \sum_{j=1}^{n_i} \mathrm{EF}_{ij}\left(a + bd_{ij}^2\right) = \sum_{j=1}^{n_i} \mathrm{SDI}_{ij}$$

where SDI_{ij} is the partitioned SDI for tree j in plot i.

Therefore

$$\text{(A11)} \qquad \mathrm{SDI}_{ij} = \mathrm{EF}_{ij}\left(a + bd_{ij}^2\right)$$

which is similar to Stage's (1968) partitioned SDI expression.

Partitioning the RD

Curtis's (1982) relative density (RD) is defined as follows:

$$\text{(A12)} \qquad \mathrm{RD}_i = \frac{B_i}{D_{q_i}^{2-c}}$$

where $B_i = KN_iD_{q_i}^2$ = stand basal area of plot i, and $K = \frac{\pi}{40000}$.

Equations A3 and A12 give

$$\text{(A13)} \qquad \mathrm{RD}_i = KN_iD_{q_i}^c = KN_i^{1-(c/2)}W_i^{c/2}$$

Note that RD_i is similar to SDI_i, with the constant 25^{-c} from eq. A4 replaced with K in eq. A13. The partition for RD_i therefore is similar to eq. A11:

$$\text{(A14)} \qquad \mathrm{RD}_{ij} = \mathrm{EF}_{ij}\left(a' + b'd_{ij}^2\right)$$

where

$$\text{(A15)} \qquad a' = K\left(1 - \frac{c}{2}\right)D_{q_i}^c$$

and

$$\text{(A16)} \qquad b' = K\frac{c}{2}D_{q_i}^{c-2}$$

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