Developing a system of annual tree growth equations for the loblolly pine – shortleaf pine type in Louisiana

Quang V. Cao, Shiansong Li, and Marc E. McDill

Abstract: An individual tree growth modeling system was developed for the loblolly pine (*Pinus taeda* L.) – shortleaf pine (*Pinus echinata* Mill.) forest type in Louisiana using USDA Forest Service Forest Inventory and Analysis (FIA) data. In this study, the loblolly pine – shortleaf pine forest type was divided into three species groups: loblolly pine, other pines, and hardwoods. The growth system includes models for individual tree survival, diameter growth, height growth, and change in crown ratio for each of the three species groups. A multivariate extension of a two-step, model-based interpolation method is proposed to estimate parameters of annual tree growth equations based on measurements from 7-year growth periods. Results based on evaluation statistics such as mean difference, mean absolute difference, and fit index show that the two-step interpolation method is clearly superior to the averaging method, which is a typical linear interpolation method. The two-step method produced growth equations that project tree and plot attributes more accurately than the averaging method. Additionally, a Monte Carlo simulation analysis reveals that parameter estimates obtained from the two-step method are closer to the true parameter values than those from the averaging method. The approach described here should be useful for estimating parameters of other systems of annual tree growth equations from periodic measurements. Models such as the one presented here may be useful for projecting the characteristics of undisturbed forest inventory plots, e.g., FIA plots, and providing more up-to-date inventory estimates.

Résumé : Un modèle de croissance d'arbre individuel a été développé pour les peuplements de pin à encens (*Pinus taeda* L.) et de pin à courtes feuilles (*Pinus echinata* Mill.) en Louisiane avec les données d'analyse et d'inventaire forestier du Service forestier des États-Unis. Dans cette étude, le type forestier pin à encens – pin à courtes feuilles a été divisé en trois groupes d'espèces: le pin à encens, les autres pins et les feuillus. Les modules suivants forment le modèle de croissance : survie de l'arbre, croissance en diamètre, croissance en hauteur et variation du rapport de cime pour chacun des trois groupes d'espèces. Une expansion multivariée d'une méthode d'interpolation en deux étapes est proposée pour estimer les paramètres des équations de croissance annuelle de l'arbre. Elle est basée sur des mesures d'accroissement périodique sur 7 ans. Les résultats basés sur des données statistiques telles que la différence moyenne, la différence moyenne absolue et l'indice d'ajustement, montrent que la méthode d'interpolation en deux étapes est clairement supérieure à la méthode d'interpolation linéaire typique basée sur la moyenne. Les équations de croissance établies par la méthode en deux étapes extrapolent les caractéristiques de l'arbre et du peuplement de façon plus exacte que la méthode basée sur la moyenne. En outre, l'analyse par simulation Monte Carlo révèle que la valeur des paramètres estimée par la méthode en deux étapes est plus proche de la vraie valeur que celle obtenue par la méthode basée sur la moyenne. L'approche décrite dans cet article devrait être utile pour estimer les paramètres d'autres modèles de croissance annuelle d'arbre à partir de mesures périodiques. Les modèles tels que celui présenté dans cet article peuvent être utiles pour extrapoler les caractéristiques des placettes d'inventaire forestier non perturbées, comme celles du Service forestier américain et ainsi fournir des résultats d'inventaire plus à jour.

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Introduction

Individual tree growth and yield models use a tree as the basic unit and can provide detailed information about stand composition and dynamics (Avery and Burkhart 2002). Distance-independent models usually project tree growth as a function of current tree size and stand variables, without information on individual tree locations. Predicted future

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tree dimensions are then summarized to project yield per unit area. Individual tree growth and yield models are especially useful for management planning, because they are capable of simulating a wide variety of management activities, particularly thinning.

Forest Inventory and Analysis (FIA) data are publicly available for each of the 50 United States (see http://fia.fs.fed.us). Because FIA data are collected from permanent plots using well-documented, standardized methods and formats, growth and yield models can potentially be developed from these data for any region of the United States. Furthermore, FIA plots are located to provide a representative sample of forest conditions across all ownerships. Thus, models that are developed with FIA data may more accurately reflect management conditions on nonindustrial private forest land than models developed from the research data sets typically used for developing growth and yield models that are collected primarily on industrial or public forests.

The FIA data collection system is currently being converted to a continuous, annual process (Gillespie 1999) that occurs in all regions simultaneously. Under this new system, data for a given region will be collected over a longer period of time than under the old state-by-state approach, where all data collection efforts were concentrated in a single state at a time. One potential use for growth and yield models such as the ones developed in this study will be to project plot characteristics to a common base year.

Whether a growth and yield model is used for management planning purposes or for projecting inventory plot characteristics, it is generally desirable to have the capability to project stand or plot characteristics 1 year at a time. However, most inventory data, including FIA data, are not collected annually. When the desired time interval of a growth equation (usually 1 year) is not the same as the interval at which the data were collected, some kind of interpolation method is necessary before fitting data to model equations. The simplest interpolation assumes that growth is linear for the growing period. In this case, the mean growth rate over the measurement interval is assumed to be a good estimate of the growth in the first year. This approach, called the averaging method, was employed by Bruce (1981) and Amateis et al. (1989). McDill and Amateis (1993) proposed two interpolation methods that use the shape of the specified growth function to estimate the interpolation proportion of first-year growth to periodic growth. In their one-step method, estimation of parameters and interpolation proportions is implemented simultaneously. Their two-step method involves (*i*) fitting the growth function using an initial guess of the interpolation proportion and (*ii*) computing the interpolation proportions. This two-step process is repeated until the estimated proportions stabilize. Their Monte Carlo analysis showed that the averaging method produced biased parameter estimates, but the one-step and two-step methods failed to show any bias in parameter estimates. McDill and Amateis (1993) concluded that the one-step and two-step methods produced little difference in parameter estimates, and that ease of use was the only reason for selecting one over the other.

The objectives of this study were (*i*) to extend McDill and Amateis' (1993) two-step interpolation method from fitting a single growth equation to fitting a system of three annual tree growth equations and (*ii*) to apply the new method in developing an individual tree growth model for the loblolly pine (*Pinus taeda* L.) – shortleaf pine (*Pinus echinata* Mill.) forest type in Louisiana using FIA data.

Two interpolation methods for predicting annual tree growth

In this study, we consider two interpolation methods for predicting annual tree growth when growth data are collected at intervals of more than 1 year. The commonly used averaging method assumes that growth is linear for the growing period. On the other hand, the two-step method, which is a model-based interpolation method, makes effective use of the growth model in estimating the interpolation proportion of first-year growth to periodic growth.

Consider a general stochastic difference equation describing the development of the variable $Y_{i,t}$:

$$[1] \qquad Y_{i,t+1} - Y_{i,t} = f(Y_{i,t}, t) + \varepsilon_{i,t}$$

where $Y_{i,t}$ is the state of the ith individual at time t, $f(Y_{i,t}, t)$ is some function of $Y_{i,t}$ and t, and $\varepsilon_{i,t}$ is an error term.

Equation 1 can be rewritten as

$$[2] \qquad z_i(Y_{i,t+q} - Y_{i,t}) = f(Y_{i,t}, t) + \varepsilon_{i,t}$$

where q is the length of the growth period in years and z_i is an interpolation proportion. The interpolation proportion estimates the ratio of first-year growth and q-year periodic growth.

The averaging method

The averaging method assumes that $z_i = 1/q$ or that tree growth is a linear function in a short projection period:

$$[3] \qquad \Delta Y_{i,t} = \frac{Y_{i,t+q} - Y_{i,t}}{q} = f(Y_{i,t}, t) + \varepsilon_{i,t}$$

where $\Delta Y_{i,t}$ is the mean annual change in $Y_{i,t}$ for the q-year growth period.

The model-based interpolation method

The basic principle of the model-based interpolation method is that the interpolation should be consistent with the growth function to be estimated. In the two-step interpolation method (McDill and Amateis 1993), the interpolation proportion, z_i, is estimated by

$$[4] \qquad \hat{z}_i = \frac{\hat{Y}_{i,t+1} - Y_{i,t}}{\hat{Y}_{i,t+q} - Y_{i,t}}$$

where $\hat{Y}_{i,t+1}$ and $\hat{Y}_{i,t+q}$ are predicted values of $Y_{i,t+1}$ and $Y_{i,t+q}$, respectively.

An iterative procedure was employed to compute the interpolation proportion. First, the parameters in eq. 2 are estimated using $\hat{z}_i = 1/q$ (averaging method). Then, the values of $Y_{i,t}$ can be projected q years into the future, 1 year at a time, for each observation. New estimates of $\hat{z}_i$ constructed from eq. 4 are used to refit eq. 2. This process is repeated until either the estimated interpolation proportions or the parameter estimates themselves from two successive iterations do not

Table 1. Summary statistics of stand and tree variables, by species group.

Species group	Variable[a]	Description	Mean	SD
Stand-level attributes				
All (n = 218)	S	Site class[b]	4.5	0.9
	B	Basal area ($m^2 \cdot ha^{-1}$)	17.37	9.55
	T	Trees per hectare	379	220
Tree-level attributes				
Loblolly pine (n = 1333)	D	DBH (cm)	32.31	13.08
	H	Total height (m)	20.9	6.1
	CR	Crown ratio	0.39	0.12
	ΔD	Annual DBH growth	0.58	0.33
	ΔH	Annual height growth	0.5	0.3
	ΔCR	Annual change in crown ratio	–0.01	0.01
Other pines (n = 227)	D	DBH (cm)	30.43	10.80
	H	Total height (m)	20.6	5.0
	CR	Crown ratio	0.34	0.11
	ΔD	Annual DBH growth	0.41	0.28
	ΔH	Annual height growth	0.4	0.3
	ΔCR	Annual change in crown ratio	0.00	0.01
Hardwoods (n = 418)	D	DBH (cm)	29.67	13.64
	H	Total height (m)	18.7	5.2
	CR	Crown ratio	0.48	0.13
	ΔD	Annual DBH growth	0.33	0.25
	ΔH	Annual height growth	0.2	0.4
	ΔCR	Annual change in crown ratio	–0.01	0.01

[a]Variables measured in 1984. Annual growth is average of 7-year growth period (1984–1991).
[b]Values of site class were 1–7, with 7 being best.

change by more than some prespecified tolerance. In other words, the iterations terminate when the parameter estimates stabilize.

The two-step method was generalized to the multivariate problem of predicting annual changes in tree diameter, height, and crown ratio as follows:

(1) Assume $\hat{z}_{D_i} = \hat{z}_{H_i} = \hat{z}_{C_i} = 1/q$, where $\hat{z}_{D_i}$, $\hat{z}_{H_i}$, and $\hat{z}_{C_i}$ are estimates of the interpolation proportions for models to predict annual changes in tree diameter, height, and crown ratio, respectively.
(2) Simultaneously obtain parameter estimates for the diameter growth, height growth, and crown ratio models.
(3) Predict diameter, height, crown ratio, and survival at time $t + 1$, $t + 2$, …, $t + q$.
(4) Calculate new $\hat{z}_i$'s according to eq. 4.
(5) Go back to step 2. Stop when all parameters vary little from one iteration to the next, i.e., when

$$[5] \qquad \max_{1 \le j \le k} [(\hat{\beta}_j^{(r+1)} - \hat{\beta}_j^{(r)})] < \tau$$

where $\hat{\beta}_j$ is the estimate of the jth parameter (j varies from 1 to k, where k is the total number of parameters in the growth system), the superscripts for $\hat{\beta}_j$ denote iteration number, and τ is a prespecified tolerance limit. In this study, τ was set at 0.000 005, ensuring that all parameter estimates remained unchanged up to the fifth decimal place in the last two iterations.

Data

Data for this study consisted of measurements in Louisiana collected in 1984 and 1991 by the Forest Inventory and Analysis (FIA) unit of the USDA Forest Service Southern Research Station. The FIA sample plots were located at the forested intersections of gridlines spaced 4.83 km apart and covering the entire state. Summaries of the 1984 and 1991 FIA data for Louisiana, descriptions of the data collection procedures, and maps showing plot locations can be found in May and Bertelson (1986) and Vissage et al. (1992), respectively. Point sampling techniques with a basal area factor (BAF) of 8.61 $m^2 \cdot ha^{-1}$ were used to determine whether or not a tree was included in the sample. Tree variables include species, diameter at breast height (DBH), total height, and height to base of live crown. Stand variables include plot location, ownership, stand origin, forest cover type, and site class (1–7, with 7 being best).

A total of 218 plots from the loblolly pine – shortleaf pine forest type were used. The loblolly pine – shortleaf pine forest type was classified into three broad species groups in this project: (*i*) loblolly pine; (*ii*) other pines, which include longleaf pine (*Pinus palustris* Mill.), shortleaf pine, slash pine (*Pinus elliottii* Engelm.), and spruce pine (*Pinus glabra* Walt.); and (*iii*) hardwoods, which include oaks (*Quercus* spp.), ashes (*Fraxinus* spp.), hickories (*Carya* spp.), and other hardwood species. Trees in the sample totaled 1333 for loblolly pine, 227 for other pines, and 418 for hardwoods. Summary statistics for stand and tree variables for each species group are listed in Table 1.

Model forms

The process of selecting model forms for predicting individual tree survival and annual changes in diameter, height, and crown ratio involved randomly splitting the data into a

Table 2. Parameter estimates of the survival model for each species group.

Species group	Parameter	Estimate	SE
Loblolly pine	α_0	7.1122	0.3344
(n = 1333)	α_1	–0.0650	0.0067
	α_2	0.0719	0.0118
	α_3	1.3285	0.3169
	α_4	–0.0671	0.0044
	α_5	–2.6848	0.1299
Other pines	α_0	2.9930	0.7350
(n = 227)	α_1	–0.0283	0.0163
	α_2	0.0581	0.0253
	α_3	4.0600	0.7351
	α_4	–0.0235	0.0083
	α_5	–1.2552	0.2798
Hardwoods	α_0	1.4679	0.3864
(n = 418)	α_1	–0.0831	0.0099
	α_2	0.2740	0.0151
	α_3	1.9974	0.2675
	α_5	–1.5707	0.1502

Note: The general survival model is given in eq. 6. Parameter α_4 was not significantly different from zero at the 5% level for the hardwoods group and was omitted.

Table 3. Final equation forms for predicting annual changes in tree diameter, height, and crown ratio by species group.

Species group	Equation[a]
Loblolly pine	$\Delta D = \beta_1 \mathrm{CR}^{\beta_2} H^{\beta_3}\, e^{\beta_4[1-(D_q/D)]}\, S^{\beta_5} B^{\beta_6} + \varepsilon$
	$\Delta H = \beta_7 H^{\beta_8}\, e^{\beta_9[1-(D_q/D)]}\, S^{\beta_{10}} + \varepsilon$
	$\Delta \mathrm{CR} = \beta_{11} + \beta_{12}\mathrm{CR} + \beta_{13}\ln(D) + \beta_{14}\ln(H) + \beta_{15}\ln(B) + \varepsilon$
Other pines	$\Delta D = \beta_{16} H^{\beta_{17}} B^{\beta_{18}} D^{\beta_{19}} + \varepsilon$
	$\Delta H = \beta_{20} H^{\beta_{21}} + \varepsilon$
	$\Delta \mathrm{CR} = \beta_{22} + \beta_{23}\mathrm{CR} + \beta_{24}\ln(D) + \beta_{25}\ln(H) + \varepsilon$
Hardwoods	$\Delta D = \beta_{26} \mathrm{CR}^{\beta_{27}} H^{\beta_{28}}\, e^{\beta_{29}[1-(D_q/D)]}\, B^{\beta_{30}} D^{\beta_{31}} + \varepsilon$
	$\Delta H = H^{\beta_{32}}\, e^{\beta_{33}[1-(D_q/D)]}\, S^{\beta_{34}} + \varepsilon$
	$\Delta \mathrm{CR} = \beta_{35} + \beta_{36}\mathrm{CR} + \beta_{37}\ln(D) + \beta_{38}\ln(H) + \varepsilon$

[a]ε is a random error, and the other variables are defined in Table 1.

fit data set (50%) for model development and a validation data set (50%) for evaluating model performance. Evaluation statistics included the mean difference between observed and predicted values, the mean absolute difference, and either fit index (computationally similar to R^2 of linear regression) for annual growth or log-likelihood value for tree survival.

Logistic regression has been successfully used to model tree mortality in many species (e.g., Monserud 1976; Hamilton 1986; Avila and Burkhart 1992; Monserud and Sterba 1999; Eid and Tuhus 2001; Shen et al. 2001). Variables used in the logistic model in this study included tree variables such as tree diameter (D), total height (H), and crown ratio (CR), and stand variables such as basal area (B) and quadratic mean diameter (D_q). The final survival model was as follows:

$$[6] \qquad P_{\mathrm{live},7} = \left(1 + e^{-[\alpha_0 + \alpha_1 D + \alpha_2 H + \alpha_3 \mathrm{CR} + \alpha_4 B + \alpha_5 (D_q/D)]}\right)^{-1}$$

where $P_{\mathrm{live},7}$ is the probability that the tree survived the 7-year growth period (1984–1991). The data were then pooled and parameters of the final survival model for each species group were estimated (Table 2) using the LOGISTIC procedure (SAS Institute Inc. 1989). Assuming that the annual survival probability (P_{live}) remained constant during the growing period, P_{live} can be computed from

$$[7] \qquad P_{\mathrm{live}} = \left(1 + e^{-[\alpha_0 + \alpha_1 D + \alpha_2 H + \alpha_3 \mathrm{CR} + \alpha_4 B + \alpha_5 (D_q/D)]}\right)^{-1/7}$$

The algorithm for using the annual survival equation is as follows. For each year in the growing period, the number of trees per unit area represented by each tree (or the expansion factor) is multiplying by P_{live}. Since $P_{\mathrm{live}} < 1$, the expansion factor for each tree is reduced over time to reflect tree mortality.

Numerous models for predicting annual changes in diameter, height, and crown ratio were considered. The selected general model forms for predicting annual diameter and height growth were modified from Amateis et al.'s (1989) diameter growth equation:

$$[8] \qquad \Delta D = \beta_1 \mathrm{CR}^{\beta_2} H^{\beta_3}\, e^{\beta_4[1-(D_q/D)]}\, S^{\beta_5} B^{\beta_6} D^{\beta_7} + \varepsilon$$

$$[9] \qquad \Delta H = \beta_1 \mathrm{CR}^{\beta_2} H^{\beta_3}\, e^{\beta_4[1-(D_q/D)]}\, S^{\beta_5} + \varepsilon$$

where variables are as defined in Table 1. The selected general model form for change in crown ratio was a linear function of D, H, CR, and B:

$$[10] \qquad \Delta \mathrm{CR} = \beta_0 + \beta_1\ \mathrm{CR} + \beta_2\ \ln(D) + \beta_3\ \ln(H) + \beta_4\ \ln(B) + \varepsilon$$

Table 3 presents the final model forms for predicting annual changes in tree diameter, height, and crown ratio for each of the three species groups. The two interpolation methods were then employed to obtain parameter estimates for the growth models based on all available data. The ordinary least squares method is usually used to estimate parameters for each model separately. However, diameter growth, height growth, and change in crown ratio are probably correlated with each other. The nine equations in Table 3, with 38 parameters, were fitted simultaneously using SAS procedure MODEL, option SUR (SAS Institute Inc. 1993). The fitting method followed the procedure recommended by Borders (1989).

Results and discussion

The iterative procedure of the two-step method failed to converge because $\hat{z}_{C_i}$, the interpolation proportion for crown ratio, varied greatly (from –2.96 to 6.05 in the first iteration). On the other hand, the ranges of $\hat{z}_{D_i}$ and $\hat{z}_{H_i}$ were much narrower, from 0.05 to 0.25. To eliminate this problem, the range of $\hat{z}_{C_i}$ was truncated, whereas $\hat{z}_{D_i}$ and $\hat{z}_{H_i}$ were untouched. The value of $\hat{z}_{C_i}$ was set to $z_{C,5}$ if it was less than $z_{C,5}$, and to $z_{C,95}$ if it was greater than $z_{C,95}$, where

Table 4. Parameter estimates for predicting annual changes in tree diameter, height, and crown ratio for the FIA data, by estimating method.

	Averaging		Two-step	
Parameter[a]	Estimate	SE	Estimate	SE
β_1	2.6108	0.3195	5.7925	0.6778
β_2	0.5449	0.0526	0.7126	0.0517
β_3	–0.3290	0.0500	–0.5567	0.0473
β_4	0.4820	0.0601	0.4878	0.0576
β_5	0.3935	0.0583	0.4619	0.0554
β_6	–0.2216	0.0228	–0.2099	0.0209
β_7	3.4540	0.5137	8.2347	1.1083
β_8	–0.9736	0.0440	–1.2841	0.0378
β_9	0.5119	0.0662	0.6304	0.0618
β_{10}	0.5557	0.0778	0.6497	0.0718
β_{11}	0.0121	0.0048	0.0251	0.0071
β_{12}	–0.0952	0.0042	–0.1496	0.0061
β_{13}	0.0205	0.0019	0.0348	0.0028
β_{14}	–0.0133	0.0027	–0.0261	0.0040
β_{15}	–0.0039	0.0009	–0.0056	0.0013
β_{16}	2.6566	1.0031	5.6099	2.1122
β_{17}	–1.0721	0.1594	–1.4055	0.1480
β_{18}	–0.2729	0.0615	–0.2679	0.0602
β_{19}	0.6198	0.1306	0.7055	0.1275
β_{20}	2.7744	1.2107	4.3714	1.8693
β_{21}	–0.6675	0.1506	–0.8050	0.1490
β_{22}	0.0289	0.0113	0.0492	0.0165
β_{23}	–0.0872	0.0093	–0.1299	0.0136
β_{24}	0.0216	0.0039	0.0324	0.0057
β_{25}	–0.0244	0.0054	–0.0382	0.0080
β_{26}	0.6946	0.3294	0.8138	0.3963
β_{27}	0.6946	0.1436	0.8462	0.1486
β_{28}	0.7144	0.1952	0.8954	0.2019
β_{29}	0.7301	0.1833	0.7388	0.1874
β_{30}	–0.1615	0.0714	–0.1818	0.0720
β_{31}	–0.5304	0.1787	–0.6616	0.1844
β_{32}	–0.9162	0.1435	–0.9601	0.1460
β_{33}	0.7049	0.1784	0.7384	0.1818
β_{34}	0.7279	0.2538	0.8440	0.2565
β_{35}	0.0338	0.0070	0.0544	0.0103
β_{36}	–0.0788	0.0049	–0.1114	0.0072
β_{37}	0.0171	0.0021	0.0250	0.0031
β_{38}	–0.0216	0.0032	–0.0334	0.0046

[a]The equations are given in Table 3.

$z_{C,5}$ and $z_{C,95}$ are the 5th and 95th percentiles of $\hat{z}_{C_i}$ at each iteration, respectively. With this modification, the procedure converged after 18 iterations, a fairly small number for a system containing 38 parameters. At the last iteration, the range of $\hat{z}_{C_i}$ was –3.32 to 1.79 before truncation and 0.15 to 0.26 after truncation.

Parameter estimates for predicting annual changes in tree diameter, height, and crown ratio for the FIA data are shown in Table 4 for both the averaging and two-step interpolation methods. Figure 1 presents the residual plots of changes in diameter, height, and crown ratio for the 7-year growth period for models fitted using each interpolation method. Table 5 presents evaluation statistics for the models fitted using the two interpolation methods. The statistics in Table 5 were computed using the system of equations to project the plot data for the 218 FIA plots from the initial measurement in 1984–1991, the year the plots were remeasured. The evaluation statistics are based on comparing the projected tree and plot data to the observed values from the remeasurement. This analysis confirms that the two-step method provided less bias in predicting future tree-level and stand-level attributes (significantly lower mean difference with $p < 0.0001$). The bias from the two-step method was in fact very close to zero in predicting periodic changes in tree diameter, height, and crown ratio. The two-step method also outperformed the averaging method in producing significantly ($p < 0.05$) lower mean absolute differences (MAD), except for diameter growth where the MAD was lower for the two-step method but not significantly lower ($p = 0.48$) and number of trees per hectare growth where the MAD was lower for the averaging method ($p = 0.18$). The fit index values, which are analogous to R^2, were always higher for the two-step method, indicating better predictions for all tree-level and stand-level attributes.

Monte Carlo analysis

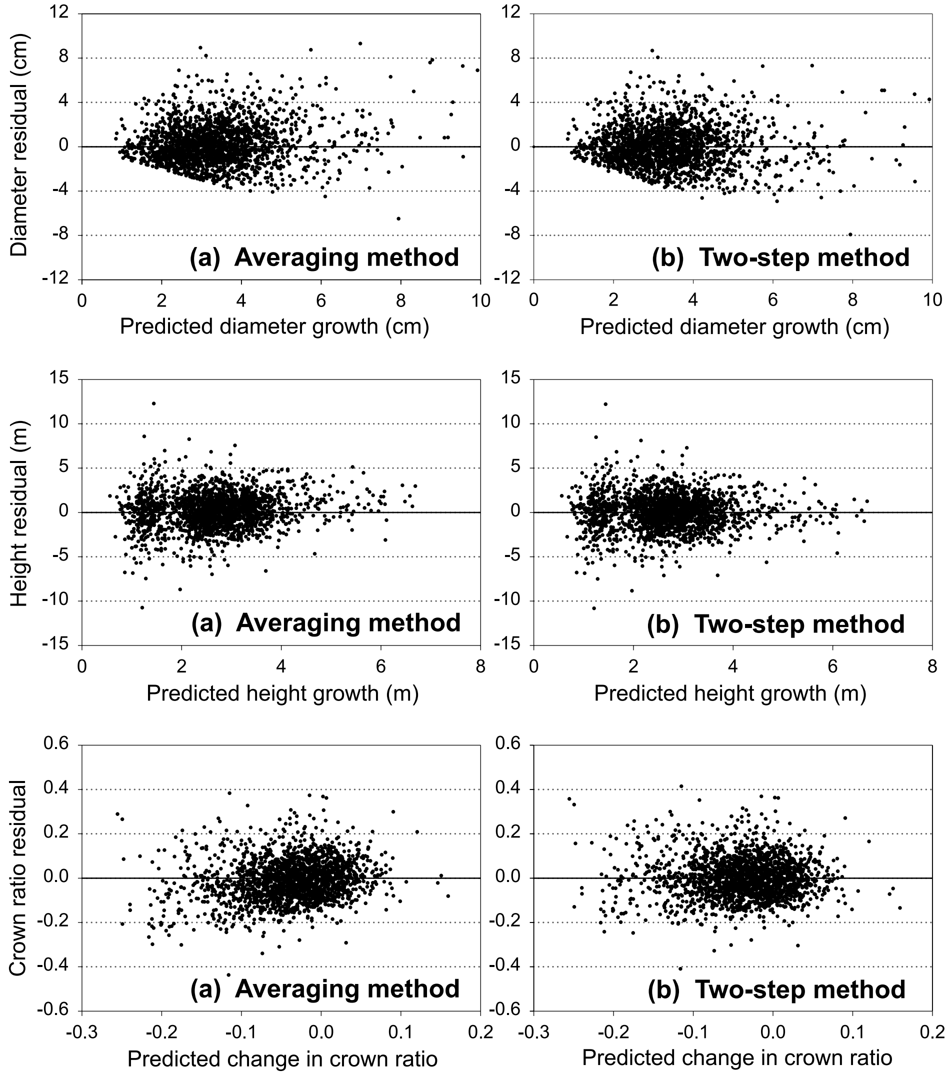
When a modeling system is evaluated using empirical data, its performance is affected by (*i*) whether or not the model was sufficiently specified, (*ii*) the method used to estimate its parameters, (*iii*) both, or (*iv*) eccentricities in the data. Monte Carlo analysis involves generating data from equations with known coefficients and, therefore, ensuring that the model is correctly specified. In this study, equations for predicting annual changes in tree diameter, height, and crown ratio are employed to generate simulation data, which are then used to estimate parameters of the growth equations. Because the models are correctly specified, the two interpolation methods can be accurately evaluated; the method that provides estimates closer to the true (known) parameters is the better method.

Generating data

Twenty simulated data sets were generated. Each data set contained 218 plots, which was the total number of plots in the FIA data. The following tasks were performed for each plot.

(1) The actual FIA data measured in 1984 were used as the initial data.
(2) Which trees survived the 7-year growing period were determined based on the survival model (eq. 7). At the end of each year in the period, a tree survived if a uniform (0, 1) random number was less than the survival probability computed for that tree.
(3) Each surviving tree was "grown" every year for 7 years, using the regression equations that predict annual changes in tree diameter, height, and crown ratio (Ta-

Fig. 1. Plots of residuals from the averaging method (*a*) and the two-step interpolation method (*b*) of periodic changes in diameter, height, and crown ratio versus predicted growth for the 7-year period.



ble 3). Coefficients from the averaging method (Table 4) were used for this procedure. A random term was added to each predicted annual growth from a normal distribution with mean zero and variance equal to the mean squared error from the appropriate regression equation. If predicted annual diameter or height growth was negative, the process was repeated until positive growth was attained.

Table 6 shows that the means and standard deviations of tree variables in the 20 simulation data sets matched those in the actual FIA data set.

Analysis

Parameters for predicting annual changes in tree diameter, height, and crown ratio were predicted for each of the 20 simulation data sets using the averaging and two-step interpolation methods. In most cases, the two-step method converged in 11 or 12 iterations. Table 7 shows the evaluation statistics for each of the 38 parameters computed using each method based on the 20 simulation data sets. Except for one case, the two-step method consistently produced better evaluation statistics for all parameters. The ratios of corresponding statistics from the averaging method and the two-step method averaged 7.30 for mean difference, 3.24 for mean absolute difference, and 13.33 for mean squared error. The lower values of these evaluation statistics from the two-step interpolation method indicate that it provided parameter estimates that were closer to the true parameter values, based on the simulation results.

Conclusion

The results in this paper confirm that McDill and Amateis' (1993) concerns about biased estimators from the averaging method were valid. The results also indicate that their two-step interpolation method can be successfully extended to fit a system of multiple equations with data that are collected at multiple-year intervals. The two-step interpolation method was clearly superior to the averaging method when used to estimate parameters of a system of three growth equations. The parameter values obtained using the two-step method were shown to provide more accurate

Table 5. Evaluation statistics for two interpolation methods based on tree-level and stand-level attributes from the FIA data.

Attribute	Evaluation statistic[a]	Averaging method	Two-step method	P^b
Tree-level attributes				
Diameter growth (cm)	MD	0.24	–0.01	<0.0001
	MAD	1.43	1.42	0.4802
	FI	0.3791	0.4061	<0.0001
Total height growth (m)	MD	0.2	0.0	<0.0001
	MAD	1.5	1.5	<0.0001
	FI	0.2470	0.2679	<0.0001
Change in crown ratio	MD	–0.01	0.00	<0.0001
	MAD	0.07	0.07	<0.0001
	FI	0.3648	0.3941	<0.0001
Stand-level attributes				
Trees/ha	MD	–28.1	–5.6	<0.0001
	MAD	31.5*	34.1	0.1804
	FI	0.8755	0.9103	<0.0001
Basal area (m^2/ha)	MD	0.23	–0.19	<0.0001
	MAD	2.02	1.94	0.0468
	FI	0.9094	0.9139	<0.0001
Volume (m^3/ha)	MD	4.24	–0.80	<0.0001
	MAD	21.77	20.67	0.0109
	FI	0.9301	0.9351	<0.0001

[a]The statistics are as follows: MD (mean difference) = $\sum_{i=1}^{n}(y_i - \hat{y}_i/n)$, MAD (mean absolute difference) = $\sum_{i=1}^{n}\left|y_i - \hat{y}_i\right|/n$, and FI (fit index) = $1 - \sum_{i=1}^{n}(y_i - \hat{y}_i)^2/\sum_{i=1}^{n}(y_i - \bar{y})^2$, where y_i and $\hat{y}_i$ are, respectively, observed and predicted values of the ith observation of either tree-level or stand-level attribute, and n is the number of observations.

[b]The P value resulting from testing the hypothesis that statistic from the two-step method is better (lower for MD and MAD and higher for FI) than the averaging method. In only one case (marked with an asterisk) was the averaging method superior to the two-step method.

Table 6. Summary statistics of tree variables measured in 1991 from the 20 simulation data sets and the FIA data, by species group.

		FIA data		Simulation data	
Species group	Variable	Mean	SD	Mean	SD
Loblolly pine	DBH (cm)	36.46	13.23	36.24	13.09
	Total height (m)	24.2	5.7	24.3	5.7
	Crown ratio	0.34	0.11	0.35	0.08
Other pines	DBH (cm)	33.31	11.03	33.09	11.32
	Total height (m)	23.3	4.9	23.5	4.8
	Crown ratio	0.33	0.11	0.33	0.08
Hardwoods	DBH (cm)	32.11	14.13	32.45	14.03
	Total height (m)	20.2	5.4	20.9	5.2
	Crown ratio	0.41	0.14	0.42	0.10

Note: The actual FIA data measured in 1984 was used as the initial data in generating the simulation data sets.

projections of tree and plot attributes than parameter values obtained using the averaging method. Additionally, the Monte Carlo simulation study presented here confirms that the two-step method produces parameter estimates that are closer to the true parameter values. As a result, predictions from the growth and yield model with parameters estimated from the two-step method should be more reliable.

Together with the survival model (Table 2), the equations presented here form an individual tree growth modeling system for the loblolly pine – shortleaf forest type in Louisiana. Combined with volume equations for individual trees, the modeling system will be useful for simulating a variety of management alternatives for individual stands, although additional work to validate how well the model performs for

Table 7. Evaluation statistics for each parameter based on 20 simulation data sets by estimation method.

Parameter	Mean difference		Mean absolute difference		Mean squared error ($\times 10^3$)	
	Averaging	Two-step	Averaging	Two-step	Averaging	Two-Step
β_1	0.9816	0.1921	0.9816	0.2090	970.524	59.771
β_2	0.1585	0.0728	0.1585	0.0728	25.560	5.923
β_3	–0.1153	–0.0155	0.1153	0.0254	13.788	0.929
β_4	0.0650	0.0582	0.0650	0.0586	5.020	4.375
β_5	0.0545	0.0237	0.0545	0.0278	3.441	1.154
β_6	–0.0152	–0.0145	0.0156	0.0155	0.311*	0.317
β_7	1.7745	0.9366	1.7745	0.9366	3160.963	918.168
β_8	–0.3319	–0.1887	0.3319	0.1887	110.492	36.232
β_9	0.1773	0.1260	0.1773	0.1260	31.921	16.525
β_{10}	0.1473	0.0961	0.1473	0.0961	22.621	10.325
β_{11}	0.0049	0.0003	0.0049	0.0017	0.026	0.005
β_{12}	–0.0244	–0.0004	0.0244	0.0014	0.598	0.003
β_{13}	0.0062	0.0003	0.0062	0.0007	0.038	0.001
β_{14}	–0.0051	–0.0003	0.0051	0.0009	0.026	0.002
β_{15}	–0.0009	0.0000	0.0009	0.0004	0.001	0.000
β_{16}	1.0089	0.3923	1.0089	0.4498	1063.351	266.054
β_{17}	–0.2982	–0.1787	0.2982	0.1787	93.744	37.302
β_{18}	–0.0463	–0.0412	0.0463	0.0412	2.733	2.326
β_{19}	0.1407	0.1085	0.1430	0.1135	22.693	14.823
β_{20}	1.3369	1.0306	1.3369	1.0306	1845.006	1191.186
β_{21}	–0.2590	–0.2067	0.2590	0.2067	70.156	47.228
β_{22}	0.0101	0.0026	0.0101	0.0046	0.114	0.034
β_{23}	–0.0205	–0.0010	0.0205	0.0039	0.430	0.024
β_{24}	0.0057	0.0006	0.0057	0.0013	0.033	0.003
β_{25}	–0.0074	–0.0014	0.0074	0.0018	0.056	0.007
β_{26}	0.1801	0.1336	0.1801	0.1399	38.148	26.273
β_{27}	0.3512	0.2737	0.3512	0.2737	124.949	77.207
β_{28}	0.3774	0.2892	0.3774	0.2892	144.563	86.676
β_{29}	0.3577	0.3283	0.3577	0.3283	129.777	109.872
β_{30}	–0.0724	–0.0650	0.0724	0.0650	5.569	4.596
β_{31}	–0.3188	–0.2469	0.3188	0.2469	103.303	63.020
β_{32}	–0.4478	–0.4306	0.4478	0.4306	201.379	186.410
β_{33}	0.4064	0.3982	0.4064	0.3982	165.922	159.445
β_{34}	0.5853	0.5335	0.5853	0.5335	345.200	287.948
β_{35}	0.0110	0.0020	0.0110	0.0022	0.122	0.007
β_{36}	–0.0173	–0.0010	0.0173	0.0015	0.300	0.004
β_{37}	0.0046	0.0005	0.0046	0.0007	0.022	0.001
β_{38}	–0.0068	–0.0011	0.0068	0.0012	0.047	0.002

Note: The statistics are as follows: MD (mean difference) = $\sum_{i=1}^{k}(\beta_i - \hat{\beta}_i)/k$, MAD (mean absolute difference) = $\sum_{i=1}^{k}\left|\beta_i - \hat{\beta}_i\right|/k$, and MSE (mean squared error) = $\sum_{i=1}^{k}(\beta_i - \hat{\beta}_i)^2/k$, where β_i and $\hat{\beta}_i$ are, respectively, true and predicted values of the ith parameter, and k = 38 is the number of parameters. In only one case (marked with an asterisk) was the averaging method superior to the two-step method.

thinned versus unthinned stands would be desirable. As discussed in the introduction, models such as the one presented here may be useful for projecting the characteristics of undisturbed forest inventory plots, e.g., FIA plots. Such techniques would provide more up-to-date inventory estimates and could potentially reduce inventory costs by allowing a reduction in the sampling intensity of undisturbed plots (Gillespie 1999). This approach would also require the development of remote sensing techniques to identify disturbed plots and alternative projection methods, such as imputation, for projecting the states of disturbed plots. Additional research would also be needed to assess the impact of these methods on the variance of the resultant inventory estimators (Gillespie 1999).

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