Incorporating Whole-Stand and Individual-Tree Models in a Stand-Table Projection System

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Abstract: A stand table provides number of trees per unit area for each diameter class. This article presents three methods to project a current stand table into the future by predicting mortality and diameter growth for each diameter class by use of an individual-tree model. The stand table was then adjusted to produce the same total number of trees and basal area per hectare as predicted from the whole-stand model. The three methods evaluated in this study were stand table adjustment, constrained least squares (LS), and modified constrained LS. Data from the Southwide Seed Source Study of loblolly pine (*Pinus taeda* L.) showed that incorporating the individual-tree model helped improve the projection of stand tables, as compared to a previous approach. The three methods produced comparable results; error indices from these methods were within 5% of one another. The constrained LS method consistently provided the best fit (lowest error indices) as compared to the other methods. FOR. SCI. 53(1):45–49.

Keywords: *pinus taeda*, loblolly pine, constrained least squares, tree survival, diameter growth

A STAND TABLE, usually developed from a conventional forest inventory, shows number of trees in each diameter class. By characterizing the distribution of diameters in a stand, it allows calculations of product volumes and, finally, financial value of that particular stand. Growth modeling based on projection of stand tables started with simple assumptions on tree diameter growth and mortality (Chapman and Meyer 1949, Avery and Burkhart 2002). Predictions from regression equations for stand survival and basal area were used to compute future stand tables by Clutter and Jones (1980) and Pienaar and Harrison (1988). These approaches deviated from simple stand table projection systems in the past, even though the assumption that trees in each class were uniformly distributed still remained. Nepal and Somers (1992) developed a stand table projection algorithm in which trees in each class were assumed to follow a truncated Weibull distribution. The algorithm was later modified by Cao and Baldwin (1999a, b).

The objective of this study was to incorporate predictions from whole-stand and individual-tree growth models into a reliable system for projecting a current stand table into the future.

Data

Data used in this study were from unthinned plantations in the Southwide Seed Source Study, which included 15 seed sources of loblolly pine (*Pinus taeda* L.) planted at 13 locations across 10 southern states (Wells and Wakeley 1966). Each plot of size 0.0164 ha consisted of 49 trees planted at a 1.8 m × 1.8 m spacing. Tree diameters were measured at ages 10, 15 (or 16), 20 (or 22), and 25 (or 27) years. A subset of the original data (100 plots) was randomly selected, and one growth period (ranging from 4 to 7 years) was randomly chosen for each plot to avoid correlation errors among multiple growth periods from the same plot. These observations constituted the fit data set, to be used for developing the whole-stand and individual-tree growth models. Another 100 plots were randomly selected from the rest of the original data to form the evaluation data set. All possible growth periods from these plots were included in the evaluation data set, totaling 300 short-projection (4–7 yr) periods, 200 medium-projection (9–12 yr) periods, and 100 long-projection (15–17 yr) periods. Summary statistics for the fit and evaluation data sets appear in Table 1.

Methods

The system proposed in this article for stand table projection requires a current stand table, a tree survival and diameter growth model, and a stand-density projection model. The current stand table is projected into the future by use of the following steps.

Step 1: Compute Survival.—All mortality is assumed to occur at the beginning of the growing period. The tree survival equation is used to predict surviving number of trees in each diameter class.

Step 2: Grow Diameters.—All trees in each class increase in diameter during the growing period. This process is simplified by projecting the lower and upper limits of each diameter class into the future, using the tree diameter growth equation.

Step 3: Reclassify Trees.—After diameter growth in step 2, trees in each class either stay in that class or move up to higher classes. Assuming that trees in each diameter class follow a truncated Weibull distribution (Nepal and Somers 1992), these trees are then reclassified into new diameter classes.

Step 4: Adjust Number of Trees.—The resulting future stand table does not produce the same stand density in

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Table 1. Means (and standard deviations) of stand variables, by data type and age

Age	Number of plots	Dominant height (m)	Number of trees (no./ha)	Basal area (m^2/ha)	Minimum diameter (cm)	Maximum diameter (cm)
Fit data set[1]						
10	25	9.00 (1.44)	2,238 (388)	24.48 (6.34)	5.9 (1.8)	16.4 (2.4)
15	38	12.89 (1.46)	1,773 (556)	31.96 (7.67)	8.1 (2.3)	21.7 (3.1)
16	8	16.38 (1.06)	1,289 (194)	30.92 (3.11)	10.4 (1.7)	23.7 (1.5)
20	45	16.96 (1.96)	1,159 (360)	31.28 (8.50)	11.7 (2.6)	25.4 (3.7)
22	2	15.32 (0.15)	580 (216)	12.96 (6.51)	10.9 (2.2)	20.3 (1.1)
25	17	18.46 (1.64)	1,105 (426)	31.85 (11.06)	12.6 (2.6)	26.1 (3.0)
27	9	20.59 (2.68)	739 (227)	30.64 (10.27)	14.8 (3.4)	30.2 (4.6)
Evaluation data set[2]						
10	100	9.04 (1.61)	2,071 (437)	22.13 (6.12)	5.2 (2.0)	16.2 (2.7)
15	86	13.04 (1.96)	1,735 (463)	30.66 (6.20)	8.2 (2.7)	20.9 (3.2)
16	14	16.16 (1.13)	1,273 (194)	29.89 (3.25)	9.0 (1.8)	23.8 (2.0)
20	95	17.05 (2.21)	1,301 (323)	35.23 (7.70)	11.6 (2.1)	25.5 (3.5)
22	5	14.00 (2.11)	952 (119)	19.11 (8.23)	8.3 (1.1)	22.0 (4.3)
25	72	19.66 (2.46)	1,208 (328)	40.35 (9.31)	13.4 (2.5)	28.5 (4.0)
27	28	20.47 (3.10)	795 (201)	32.35 (6.82)	15.1 (3.3)	30.7 (4.0)

[1] The fit data set consisted of 100 plots. Growth periods (one randomly selected per plot) spanned from age 10 to 15 yr (30 plots), 10 to 16 yr (4 plots), 15 to 20 yr (22 plots), 16 to 20 yr (11 plots), 20 to 25 yr (20 plots), 20 to 27 yr (7 plots), and 22 to 27 yr (6 plots).
[2] The validation data set consisted of 100 plots. All possible growth periods from each plot resulted in a total of 600 observations: 4 to 7 yr (300), 10 to 12 yr (200), and 15 to 17 yr (100).

terms of number of trees and basal area per hectare as predicted from the whole-stand model. In this step, the number of trees in each diameter class is adjusted to match the predicted number of trees and basal area per hectare. Three adjustment methods were investigated in this study.

Stand Table Adjustment Method

Number of trees per hectare in the ith diameter class is adjusted using Nepal and Somers' (1992) procedure as follows:

$$n_i^* = n_i \alpha \exp(\beta D_i), \quad (1)$$

where n_i^* and n_i are adjusted and unadjusted number of trees per hectare in the ith class, respectively, D_i is the midpoint of the ith class, and α and β are parameters to be solved such that total number of trees and basal area match those predicted from the whole-stand model.

Constrained Least-Squares (LS) Method:

As the name implies, the method involves minimizing

$$\sum (n_i^* - n_i)^2, \quad (2)$$

subject to two constraints that total number of trees and basal area are the same as predicted from the stand-density projection model. The constrained LS method was originally developed by Matney et al. (1990) for allocating mortality and diameter growth to the tree list, and was modified by Cao and Baldwin (1999a) to adjust the future stand table. The approach used in this study is slightly different from Cao and Baldwin's (1999a) method in that a third constraint involving average diameter is omitted. The adjusted number of trees per hectare in the ith diameter class is given as

$$n_i^* = n_i + \lambda_1 + \lambda_2 D_i^2, \quad (3)$$

Table 2. Regression equations to predict stand and tree attributes

Attribute	Equation[1]
Dominant height	$H_{t+1} = \exp[3.9128 + (\ln(H_t - 3.9128)(A_t/A_{t+1})^{0.6609}] + \varepsilon$ $R^2 = 0.92$; RMSE = 0.90 m
Number of trees/ha	$N_{t+1} = \exp[(A_t/A_{t+1})\ln(N_t) + (1 - A_t/A_{t+1})(3.8705 + 16.3614/A_t + 0.0008\,N_t)] + \varepsilon$ $R^2 = 0.82$; RMSE = 268 trees/ha
Basal area/ha	$B_{t+1} = \exp[(A_t/A_{t+1})\ln(B_t) + (1 - A_t/A_{t+1})\{3.1769 + 11.3393/H_t - 0.0120\,B_t\}] + \varepsilon$ $R^2 = 0.70$; RMSE = 5.34 m^2/ha
Tree diameter	$d_{i,t+1} = d_{i,t} + (5.6043\,Dq_t^{-0.4350})/[1 + \exp(2.2280 - 14.6569/A_t + 0.0760\,H_t + 0.03972\,B_t - 0.1530\,d_{i,t})] + \varepsilon$ $R^2 = 0.94$; RMSE = 1.28 cm
Tree survival probability	$p_{i,t+1} = p_{i,t}/[1 + \exp(3.7112 - 0.0254\,H_t - 13.0205\,RS_t - 4.2899\,d_{i,t}/Dq_t)]$ MD = 0.0015; MAD = 0.2862

[1] H = dominant height; A = stand age in years; N = number of trees/ha; B = basal area/ha; Dq = quadratic mean diameter in cm; $RS = (10{,}000/N)^{0.5}/H$ = relative spacing; subscripts t and $(t + 1)$ denote measurements at age A_t and A_{t+1}, respectively, where $A_{t+1} = A_t + 1$. $p_{i,t}$ and $d_{i,t}$ = survival probability and diameter of tree i at age A_t, respectively; ε = error term; RMSE = root-mean-squared error; MD = mean difference; and MAD = mean absolute difference.

where λ_j values are Lagrangian multipliers computed from

$$\lambda_2 = \frac{(\Sigma D_i^2)(\Sigma n_i - \hat{N}_2) + m(\hat{B}_2/K - \Sigma n_i D_i^2)}{m\Sigma D_i^4 - (\Sigma D_i^2)^2} \quad (4)$$

and

$$\lambda_1 = \frac{\hat{N}_2 - \Sigma n_i - \lambda_2 \Sigma D_i^2}{m}, \quad (5)$$

where $\hat{N}_2$ and $\hat{B}_2$ are predicted number of trees and basal area per hectare, respectively, at the end of the growth period, $K = \pi/40{,}000$ = a constant to convert diameter in cm to basal area in m^2, m = number of diameter classes, and the summation sign denotes the sum overall diameter classes.

Modified Constrained LS Method

The surviving number of trees in each diameter class computed from step 1 was proportionally adjusted to match total stand survival predicted from the stand survival model. Then the diameter growth from step 2 was also proportionally adjusted such that the sum over all diameter classes produced the same basal area per hectare as projected by the stand basal area prediction model. The next two steps—reclassifying trees and adjusting number of trees—were carried out as in the constrained LS method.

Evaluation

The three methods of incorporating the individual-tree model were evaluated on their abilities to project future stand tables. Also included in the evaluation was Nepal and Somers' (1992) method, in which the diameter growth function was derived from the Weibull distribution.

Two forms of error index proposed by Reynolds et al. (1988) were used to determine how well the three methods performed at the diameter-class level:

$$\mathrm{EI}_{\mathrm{N}j} = \sum_{i=1}^{m_j} |\mathrm{n}_{ij} - \hat{n}_{ij}|, \quad (6)$$

$$\mathrm{EI}_{\mathrm{B}j} = \sum_{i=1}^{m_j} |b_{ij} - \hat{b}_{ij}|, \quad (7)$$

where $\mathrm{EI}_{\mathrm{N}j}$ and $\mathrm{EI}_{\mathrm{B}j}$ are error indices based on number of trees/ha and basal area/ha for the jth plot, respectively; n_{ij} and $\hat{n}_{ij}$ are observed and predicted number of trees per hectare of the ith diameter class in the jth plot, respectively; b_{ij} and $\hat{b}_{ij}$ are observed and predicted basal area in m^2/ha of the ith diameter class in the jth plot, respectively; and m_j is the number of diameter classes in the jth plot.

Results and Discussion

Equations comprising the whole-stand and individual-tree models are listed in Table 2. These equations were carefully selected after evaluating many alternatives, based on R^2, mean difference, and mean absolute difference. Annual prediction equations were used in this study because they had been demonstrated to work well in projecting both stand-level and tree-level attributes (Cao 2000, Ochi and Cao 2003, Cao 2004). Since cross-equation correlations likely existed among error components (Borders 1989), parameters of the equations to predict number of trees and basal area per hectare were simultaneously estimated with option SUR (seemingly unrelated regression) of SAS procedure MODEL (SAS Institute Inc. 1993). Individual-tree survival and diameter growth were expressed as functions of tree diameter and stand-level variables (age, dominant height, stand density, and quadratic mean diameter).

The means and standard deviations of Reynolds et al.'s (1988) error indices for the three adjusting methods based on the evaluation data set were presented in Table 3. A low value of error index indicated little difference between the observed and predicted numbers of trees in each diameter class.

Without exception, the constrained LS method consistently produced the lowest mean error index values, based either on number of trees or basal area, among the three methods for all projection lengths. However, the differences among the three methods were small. For EI_{N}, they were less than 1% for short and medium projections and 2.5% for long projections. The differences increased in magnitude for EI_{B}, from 2.1% for short projections to 4.4% for long

Table 3. Means (and standard deviations) of Reynolds et al.'s (1988) error indices based on number of trees/ha and basal area/ha for four methods

	Projection length		
Method[1]	Short (4–7 yr)	Medium (9–12 yr)	Long (15–17 yr)
Error index based on number of trees/ha[2]			
Stand table adjustment	627 (230)	692 (209)	729 (207)
Constrained least squares	**623** (226)	**686** (204)	**711** (195)
Modified constrained LS	626 (225)	689 (202)	718 (184)
Nepal and Somers (1992)	<u>688</u> (225)	<u>778</u> (186)	<u>821</u> (171)
Error Index based on basal area/ha[2]			
Stand table adjustment	15.88 (6.38)	20.66 (6.76)	24.67 (7.08)
Constrained least squares	**15.72** (6.33)	**20.28** (6.83)	**23.90** (7.15)
Modified constrained LS	16.05 (6.32)	20.76 (6.64)	24.74 (6.74)
Nepal and Somers (1992)	<u>17.47</u> (6.99)	<u>23.32</u> (7.27)	<u>28.43</u> (7.71)

[1] The first three methods are adjusting methods proposed in this study. The last one was developed by Nepal and Somers (1992).
[2] For each type of error index and each projection length, numbers in bold denote the smallest mean (best), and underlined numbers denote the largest mean (worst) among the four methods.

projections. A summary for error index values was also included in Table 3 for the Nepal and Somers (1992) method, which consistently provided the highest mean error index values for all growth periods.

Table 4 shows mean Reynolds et al.'s (1988) error index based on number of trees per ha (EI_N) by diameter class for each of the four methods. This table confirms findings from Table 3 that the three new methods produced similar results, with the constrained LS slightly better. The Nepal and Somers (1992) method, however, did not perform well for small-diameter classes (below 14 cm) and over-extended the future stand table to beyond 40 cm. Similar results were also obtained when basal area was substituted for number of trees as basis for error index for each diameter class.

Effects of Incorporating the Individual-Tree Model

The benefits of incorporating the individual-tree model into the stand table projection system were evident when contrasting the stand table adjustment method to the one introduced by Nepal and Somers (1992). These two approaches were similar, except for two major differences. First, number of trees in each class was not modified before the adjustment by Nepal and Somers (1992), but was reduced by use of the tree survival equation in the stand table adjustment method. Second, Nepal and Somers (1992) used an implied diameter growth function derived from the Weibull distribution (Bailey 1980), whereas the stand table adjustment method projects diameters from the tree diameter growth function. These changes should result in a more realistic distribution of trees before the adjustment step, and did provide better predictions for future stand tables. Compared to the stand table adjustment method, mean values of error index for Nepal and Somers' (1992) approach (based on data used in this study) increased between 9 and 11% for EI_N and between 8 and 15% for EI_B (Table 3). Similar findings were also obtained when the implied diameter growth function (Bailey 1980) was substituted for the individual-tree model in the constrained LS method.

Furthermore, Table 4 shows that the implied diameter growth function in the Nepal and Somers (1992) method over-predicted future diameters for large diameter classes (over 40 cm). These results confirmed that incorporating the individual-tree model did improve the projection of stand tables.

Stand Table Adjustment versus Constrained LS Method

Cao and Baldwin (1999a) used *observed* stand attributes to evaluate the stand table adjustment approach against the constrained LS method; the latter used number of trees, basal area per hectare, and average diameter as constraints. In this study, *predicted* number of trees and basal area per hectare were used instead. In addition, the average diameter constraint was dropped because it seemed to cause difficulty in convergence for many plots. Results from Cao and Baldwin (1999a) and this study consistently showed that the

Table 4. Mean Reynolds et al.'s (1988) error index based on number of trees/ha by diameter class for four methods. The four methods are stand table adjustment (STP), constrained least squares (CLC), modified constrained least squares (MCLS), and Nepal and Somers (NP)

DBH (cm)	Short (4–7 yr)				Medium (9–12 yr)				Long (15–17 yr)			
	STP	CLS	MCLS	NP	STP	CLS	MCLS	NP	STP	CLS	MCLS	NP
2	21	36	**19**	60	**0**	24	3	32	**0**	13	1	21
4	21	30	**16**	51	**2**	26	2	29	**0**	15	**0**	17
6	24	33	**21**	50	**4**	17	5	28	**1**	10	2	17
8	**45**	**45**	**45**	58	13	17	**11**	31	**3**	8	**3**	20
10	69	67	**66**	79	34	34	**32**	46	20	20	**15**	35
12	**73**	73	**73**	74	58	**55**	56	59	46	**44**	**44**	48
14	88	**80**	87	83	84	**78**	84	87	84	**78**	79	80
16	85	**82**	90	91	94	**86**	91	89	81	**73**	81	85
18	**81**	**81**	82	86	96	**88**	99	97	81	**74**	81	79
20	74	**73**	74	76	97	**93**	95	102	108	**100**	107	110
22	**55**	**55**	**55**	59	**68**	**68**	69	71	85	**75**	81	**75**
24	**54**	**54**	**54**	52	63	63	64	**62**	79	**74**	79	75
26	45	**44**	**44**	45	57	56	57	**54**	66	**65**	**65**	67
28	39	39	40	**38**	53	56	**48**	50	**54**	60	58	59
30	41	40	40	**34**	44	44	45	**38**	**51**	55	44	50
32	**31**	**31**	32	**31**	29	29	**28**	30	31	31	**30**	32
34	38	38	36	**24**	26	28	30	**22**	34	31	34	**23**
36	25	25	26	**21**	**24**	23	**14**	19	26	25	20	**17**
38	39	36	61	**33**	4	**1**	34	20	30	31	40	**17**
40	18	16	**15**	36	15	**11**	33	15	28	**26**	30	29
42				11				2				4
44				7				6				2
46				6				4				2
48								2				2
50												1
52												1

For each diameter class and each projection length, numbers in bold denote the smallest mean (best) and underlined numbers denote the largest mean (worst) among the four methods.

constrained LS method outperformed the stand table adjustment method in projecting a stand table into the future.

Constrained LS versus Modified Constrained LS Method

The "least squares" strategy of the constrained least squares method aimed to make "minimum" changes to the future stand tables to satisfy the two stand density constraints. One might think intuitively that if the future stand table was first adjusted to produce totals close to the target stand density, then the subsequent least squares adjustment would require only minor changes in the stand table and might lead to better results. Table 3 shows that this extra adjustment included in the modified constrained LS method was not necessary, and in this case even undesirable. This method consistently produced higher means of error index than did the constrained LS, and even ranked last based on EI_B. From Table 4, the modified constrained LS method performed better for the smaller diameter classes, but lost ground for the middle and larger diameter classes.

Summary

The general stand table projection system presented in this article involved (1) computing survival of trees in each diameter class from the tree survival equation, (2) projecting diameters using the tree diameter growth equation, (3) reclassifying trees into new diameter classes, and (4) adjusting number of trees in each class to match total number of trees and basal area per hectare as predicted from the whole-stand model. Results indicated that incorporating the individual-tree model helped improve the projection of stand tables, as compared to the previous method suggested by Nepal and Somers (1992). The three adjusting methods (stand table adjustment, constrained LS, and modified constrained LS) produced comparable error indices, but the constrained LS consistently provided the best results as compared to the other methods.

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