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biometrics

Deriving a Diameter Distribution from Stand Table Data

Quang V. Cao and Dean W. Coble

Stand tables list numbers of trees per unit area for each diameter class. A stand table projection model, which projects a stand table through time, requires knowledge of the distribution of tree diameters in each class so that it can predict the movements of trees from one diameter class into another. In this study, we evaluated different statistical distributions for modeling stand table diameter data. The distributions were uniform, truncated and segmented Weibull, and truncated and segmented S_B. Results indicated that the truncated Weibull with parameters obtained from the least-squares method was overall the best method.

Keywords: truncated distribution, segmented distribution, uniform distribution, Weibull, S_B

Forest management decisions are based on information provided by growth-and-yield models, which ranged from simple whole-stand models, to size-class distribution models, and finally to complicated individual-tree simulation models. Size-class distribution models can be based on a probability density function such as the beta (Clutter and Bennett 1965), the Weibull (Bailey and Dell 1973), Johnson's S_B distribution (Hafley and Schreuder 1977), or a mixture of more than one distribution (Zhang et al. 2003, Podlaski 2010). Qin et al. (2007) discussed different approaches to project a diameter distribution through time. Because inventory data are typically summarized into stand tables, which list the number of trees per unit area for each diameter class, another type of size-class distribution method based on stand tables is often used. This method, called the stand table projection method, predicts a future stand table based on the current stand table.

When projected into the future, trees in a diameter class will increase in size and, as a result, span more than one diameter class. Some trees in the current diameter class will stay in that class, whereas others will move up one or more diameter classes. Knowledge of the distribution of tree diameters in each class should enable us to model these movements of trees from one class into another.

Traditional stand table projection methods assume a uniform distribution within each diameter class (Pienaar and Harrison 1988, Avery and Burkhart 2002, Husch et al. 2003). Nepal and Somers (1992) introduced a method in which a truncated Weibull was used to describe the distribution of trees in each diameter class. This approach was later adopted by other researchers (Cao and Baldwin 1999a, 1999b, Cao 2007, Allen et al. 2011). To date, no study has been conducted to determine whether the uniform, truncated Weibull, or other distributions would be most appropriate for fitting the stand table data.

The objectives of this study were to investigate the use of other statistical functions for characterizing diameter distribution, given number of trees in each diameter class, to evaluate the different methods for characterizing diameter distributions from stand table data containing 2- and 4-cm diameter classes, and to evaluate the different methods in projecting the stand tables into the future.

Data

Loblolly pine (*Pinus taeda* L.) data from the Southwide Seed Source Study (Wells and Wakeley 1966) were used in this study. A sample of 100 plots was randomly selected from the original data. Tree diameter at breast height (dbh) in each plot was measured at ages 10, 15 (or 16), 20 (or 22), and 25 (or 27) years (Table 1). Numbers of trees in each 2-cm class were totaled for the beginning and end of each of the 300 growth periods. The process was repeated for 4-cm diameter classes.

Slash pine (*Pinus elliottii* Engelm.) data from the East Texas Pine Plantation Research Project (Allen et al. 2011) were also used in this study. Dbh was measured on 78 plots that ranged in age from 9 to 40 years (Table 1). Similar to the loblolly pine data, the numbers of trees in each 2- and 4-cm class were totaled for the beginning and end of each of the 387 growth periods.

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Table 1. Range of tree variables, by age.

Age	No. of plots	No. of trees/plot	Dbh (cm)	Total height (m)
Loblolly pine				
10 yr	100	12–48	2.0–23.1	2.3–13.4
15 yr	79	12–47	3.0–30.2	3.6–18.3
16 yr	21	16–27	5.8–27.4	7.0–19.8
20 yr	96	12–35	4.0–34.3	4.9–22.6
22 yr	4	12–17	7.9–25.9	10.1–18.0
25 yr	66	11–30	8.6–37.6	9.1–26.6
27 yr	34	11–21	8.9–38.1	8.9–25.6
Slash pine				
7 yr	22	38–158	1.0–17.0	0.9–11.6
8 yr	21	26–227	0.8–18.3	0.6–11.6
9 yr	24	41–178	1.0–21.6	1.5–14.9
10 yr	16	37–158	1.5–22.6	2.7–13.4
11 yr	27	26–132	1.0–26.4	1.5–15.2
12 yr	29	41–227	2.0–24.9	2.7–17.7
13 yr	17	26–125	1.8–28.2	2.1–16.5
14 yr	30	35–157	1.8–29.2	2.4–18.9
15 yr	27	47–212	2.5–29.0	2.1–22.9
16 yr	17	21–113	2.5–32.5	2.4–22.3
17 yr	19	34–206	4.3–32.0	4.0–25.3
18 yr	35	37–159	2.0–33.3	2.1–22.9
19 yr	14	18–118	5.1–35.1	6.4–23.2
20 yr	23	33–153	3.0–34.5	4.3–28.4
21 yr	28	29–152	2.8–37.1	2.7–25.0
22 yr	15	16–147	6.1–37.6	7.9–25.6
23 yr	20	29–111	3.8–38.4	3.4–29.3
24 yr	14	30–76	7.9–38.6	10.1–26.8
25 yr	12	16–140	6.9–37.6	7.0–25.6
26 yr	11	21–76	3.8–46.5	4.3–27.4
27 yr	9	34–119	6.9–36.8	7.0–29.6
28 yr	8	16–83	5.1–42.7	7.3–25.9
29 yr	9	19–49	7.6–37.3	8.5–29.6
30 yr	5	16–112	7.1–42.2	6.4–29.6
31 yr	3	13–76	8.9–34.3	11.9–26.2
32 yr	2	20–38	12.2–38.9	13.7–30.2
33 yr	2	19–33	11.7–39.1	12.8–28.7
34 yr	1	18–18	18.3–40.4	18.9–27.7
35 yr	2	30–66	8.9–40.9	12.8–28.7
37 yr	1	53–53	9.1–37.1	11.9–29.3
40 yr	1	46–46	9.1–38.9	10.7–29.3

Method

Suppose that x_i and x_{i-1} are the upper and lower limits, respectively, of the ith diameter class and F_i and F_{i-1} are corresponding empirical cumulative distribution functions (cdfs). The goal was to connect the F_i values by use of a smooth monotonic nondecreasing function. Cdfs are a natural choice for this task. The statistical distributions investigated in this study include the uniform, truncated and segmented Weibull, and truncated and segmented S_B distributions.

Uniform Distribution

When a histogram is used to display a diameter distribution, the implicit assumption is that tree diameters in each class follow a uniform (or rectangular) distribution. In other words, all trees in the same diameter class have the same probability of occurrence. The cumulative distribution function of X, a diameter random variable, for the ith diameter class is

$$\hat{F}_i(x) = F_{i-1} + \left(\frac{F_i - F_{i-1}}{x_i - x_{i-1}}\right)(x - x_{i-1}), \qquad x_{i-1} \le x \le x_i, \quad (1)$$

where x_{i-1} and x_i are the lower and upper limits of the ith diameter class, respectively. The corresponding probability density function (pdf) of X for the ith diameter class is

$$\hat{f}_i(x) = \left(\frac{F_i - F_{i-1}}{x_i - x_{i-1}}\right), \ x_{i-1} \le x \le x_i. \quad (2)$$

Truncated Weibull Distribution

Nepal and Somers (1992) assumed that all tree diameters in each plot follow a general Weibull distribution. The Weibull function (Bailey and Dell 1973) has been extensively used to characterize distribution of tree diameters in forest stands (McTague and Bailey 1987, Kangas and Maltamo 2000, Zhang et al. 2003, Newton et al. 2005). The cdf of this Weibull function is

$$G(x) = 1 - \exp\left[-\left(\frac{x-a}{b}\right)^c\right], \qquad x \ge a, \quad (3)$$

and its corresponding pdf is

$$g(x) = \left(\frac{c}{b}\right)\left(\frac{x-a}{b}\right)^{c-1}\exp\left[-\left(\frac{x-a}{b}\right)^c\right], \qquad x \ge a, \quad (4)$$

where $a = x_0$ is the Weibull location parameter, x_0 is the lower limit of the smallest diameter class, and b and c are the scale and shape parameters, respectively.

Nepal and Somers (1992) further assumed that tree diameters in each class follow a different truncated version of the general Weibull distribution. The cdf of this truncated Weibull distribution for the ith diameter class is

$$\hat{F}_i(x) = F_{i-1} + \left(\frac{F_i - F_{i-1}}{G(x_i) - G(x_{i-1})}\right)[G(x) - G(x_{i-1})], \qquad x_{i-1} \le x \le x_i, \quad (5)$$

and its corresponding pdf is

$$\hat{f}_i(x) = \left(\frac{F_i - F_{i-1}}{G(x_i) - G(x_{i-1})}\right)g(x), \qquad x_{i-1} \le x \le x_i. \quad (6)$$

The scale and shape parameters (b and c, respectively) of the general Weibull distribution are unknown and can be estimated by means of the method of moments, i.e., recovered from the arithmetic and quadratic mean diameters (Nepal and Somers 1992). These parameters can also be estimated by use of the maximum likelihood estimation (MLE) method to maximize the likelihood function or the least-squares (LS) method to minimize the squared difference between the observed and predicted cdfs. The LS method is similar to the cdf regression technique proposed by Cao (2004). All three methods are included in the evaluation.

Truncated S_B Distribution

This approach is similar to the one above, except that the S_B distribution replaces the Weibull as the general diameter distribution. Johnson's S_B distribution (Johnson 1949) has recently been used to model diameter distributions (Siipilehto 1999, Zhang et al. 2003, Rennolls and Wang 2005, Palahí et al. 2007). The S_B cdf is

$$G(x) = \Phi\left[\gamma + \delta\ln\left(\frac{x - \lambda_1}{\lambda_2 - x}\right)\right], \qquad \lambda_1 \le x \le \lambda_2, \quad (7)$$

where $\Phi(\cdot)$ is the standard normal cdf, $\lambda_1 = x_0$ is the minimum value of x, $\lambda_2 = x_m$ is the maximum value of x, x_m is the upper limit of the largest diameter class, m is the number of diameter classes in the

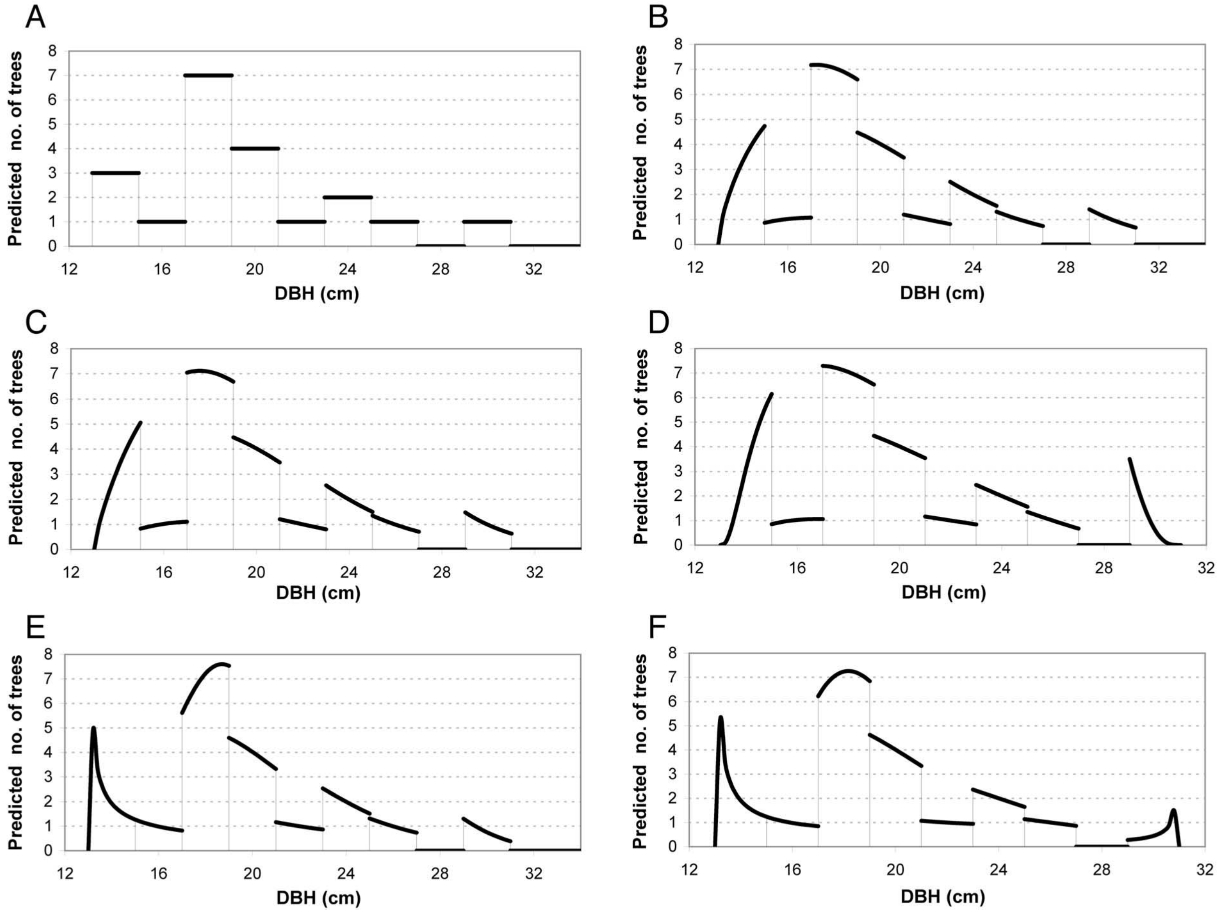

Figure 1. Probability density functions derived from tree diameters from a sample plot. The distributions are uniform (A), truncated Weibull (moments) (B), truncated Weibull (LS) (C), truncated S_B (D), segmented Weibull (E), and segmented S_B (F). The graph for the truncated Weibull (MLE) is similar to those in B and C.

plot, and γ and δ are unknown shape parameters. The S_B probability density function is

$$g(x) = \frac{\delta(\lambda_2 - \lambda_1)}{(x - \lambda_1)(\lambda_2 - x)\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left[\gamma + \delta ln\left(\frac{x - \lambda_1}{\lambda_2 - x}\right)\right]^2\right\}, \quad \lambda_1 \le x \le \lambda_2. \quad (8)$$

Tree diameters in the ith diameter class follow the following truncated S_B cdf

$$\hat{F}_i(x) = F_{i-1} + \left(\frac{F_i - F_{i-1}}{G(x_i) - G(x_{i-1})}\right)[G(x) - G(x_{i-1})], \quad x_{i-1} \le x \le x_i, \quad (9)$$

and its corresponding pdf is

$$\hat{f}_i(x) = \left(\frac{F_i - F_{i-1}}{G(x_i) - G(x_{i-1})}\right) g(x), \qquad x_{i-1} \le x \le x_i. \quad (10)$$

The two shape parameters, γ and δ, can be obtained from the percentile method that involves the 50th and 95th percentiles (Knoebel and Burkhart 1991), the MLE method, or the LS method, which minimizes the squared difference between the observed and predicted cdfs. Preliminary analysis revealed that the performance of the percentile and MLE methods was much worse than that of the LS method; therefore, these two methods were dropped from the evaluation.

Segmented Weibull Distribution

Cao and Burkhart (1984) proposed the joining of different functions, each in the form of a modified Weibull cdf, to form a segmented cdf to approximate diameter distributions of forest stands, especially those that were irregular as in thinned or mixed stands. A similar approach was applied in this study to use a segment of different Weibull cdfs for each diameter class and then join them together to form a segmented Weibull distribution. The cdf for the ith diameter class is

$$\hat{F}_i(x) = 1 - \exp\left[-\left(\frac{x - a}{b_i}\right)^{c_i}\right]; \qquad x_{i-1} \le x \le x_i, \quad (11)$$

and its corresponding pdf is

$$\hat{f}_i(x) = \left(\frac{c_i}{b_i}\right)\left(\frac{x - a}{b_i}\right)^{c_i - 1} \exp\left[-\left(\frac{x - a}{b_i}\right)^{c_i}\right], \qquad x_{i-1} \le x \le x_i. \quad (12)$$

Note that the Weibull location parameter, a, is kept constant at x_0 (the lower limit of the smallest diameter class), whereas the scale and shape parameters for the ith diameter class, b_i and c_i, vary from

Table 2. Evaluation statistics of the seven methods for the current stands and their relative ranks (in parentheses) for 2- and 4-cm diameter classes, by species.

Method	Kolmogorov-Smirnov	Anderson-Darling	R^2	$-\ln(L)$
Loblolly pine				
2-cm diameter classes ($n = 300$)				
Uniform	0.1340 (3.74)	0.2910 (1.00)	0.9773 (2.01)	13,902 (1.81)
Truncated Weibull (moments)	0.1362 (5.51)	0.3067 (1.42)	0.9776 (1.34)	13,816 (1.00)
Truncated Weibull (MLE)	0.1306 (1.00)	0.3661 (3.02)	0.9777 (1.00)	13,893 (1.71)
Truncated Weibull (LS)	0.1346 (4.18)	0.3408 (2.34)	0.9777 (1.13)	13,885 (1.65)
Truncated S_B (LS)	0.1354 (4.84)	0.3265 (1.96)	0.9764 (3.99)	13,912 (1.89)
Segmented Weibull	0.1381 (7.00)	0.3727 (3.20)	0.9758 (5.27)	13,912 (1.90)
Segmented S_B	0.1365 (5.79)	0.5135 (7.00)	0.9751 (7.00)	14,461 (7.00)
4-cm diameter classes ($n = 300$)				
Uniform	0.1672 (5.96)	0.5394 (1.88)	0.9500 (3.42)	14,944 (6.69)
Truncated Weibull (moments)	0.1688 (7.00)	0.5048 (1.00)	0.9539 (2.17)	14,670 (1.98)
Truncated Weibull (MLE)	0.1591 (1.00)	0.7400 (7.00)	0.9388 (7.00)	14,820 (4.56)
Truncated Weibull (LS)	0.1615 (2.50)	0.5448 (2.02)	0.9574 (1.06)	14,621 (1.13)
Truncated S_B (LS)	0.1645 (4.34)	0.5498 (2.15)	0.9535 (2.31)	14,683 (2.21)
Segmented Weibull	0.1636 (3.77)	0.5243 (1.50)	0.9576 (1.00)	14,613 (1.00)
Segmented S_B	0.1638 (3.88)	0.6820 (5.52)	0.9554 (1.69)	14,962 (7.00)
Slash pine				
2-cm diameter classes ($n = 387$)				
Uniform	0.0865 (5.33)	0.3320 (2.50)	0.9915 (7.00)	68,767 (4.09)
Truncated Weibull (moments)	0.0870 (7.00)	0.3293 (2.32)	0.9917 (5.30)	68,530 (1.91)
Truncated Weibull (MLE)	0.0850 (1.00)	0.3771 (5.50)	0.9918 (4.52)	68,733 (3.77)
Truncated Weibull (LS)	0.0853 (1.80)	0.3427 (3.21)	0.99234 (1.26)	68,610 (2.64)
Truncated S_B (LS)	0.0851 (1.23)	0.3093 (1.00)	0.9924 (1.00)	68,432 (1.00)
Segmented Weibull	0.0863 (4.76)	0.3492 (3.64)	0.9922 (2.17)	68,511 (1.72)
Segmented S_B	0.0858 (3.26)	0.3997 (7.00)	0.9922 (2.37)	69,083 (7.00)
4-cm diameter classes ($n = 387$)				
Uniform	0.1188 (7.00)	1.0088 (6.22)	0.9725 (6.03)	71,450 (7.00)
Truncated Weibull (moments)	0.1159 (5.59)	0.8057 (3.92)	0.9774 (4.21)	70,459 (3.71)
Truncated Weibull (MLE)	0.1163 (5.80)	1.0777 (7.00)	0.9699 (7.00)	70,703 (4.52)
Truncated Weibull (LS)	0.1061 (1.00)	0.5689 (1.24)	0.9860 (1.00)	69,793 (1.50)
Truncated S_B (LS)	0.1067 (1.25)	0.5480 (1.00)	0.9855 (1.17)	69,642 (1.00)
Segmented Weibull	0.1074 (1.58)	0.5866 (1.44)	0.9856 (1.13)	69,778 (1.45)
Segmented S_B	0.1074 (1.61)	0.6641 (2.31)	0.9850 (1.38)	70,522 (3.92)

one diameter class to the next. The constraint $\hat{F}_{i-1}(x_{i-1}) = \hat{F}_i(x_{i-1}) = F_{i-1}$ is imposed so that the resulting cdf is continuous at all join points. Consequently, b_i and c_i are computed as follows

$$c_i = \frac{\ln\left[\dfrac{\ln(1 - F_i)}{\ln(1 - F_{i-1})}\right]}{\ln(x_i - a) - \ln(x_{i-1} - a)}, \tag{13}$$

and

$$b_i = \frac{x_i - a}{[-\ln(1 - F_i)]^{1/c_i}}. \tag{14}$$

A drawback of this method is that a minimum of three diameter classes is required.

Segmented S_B Distribution

Similar to the segmented Weibull distribution, a segment of different S_B cdfs is used to describe a diameter distribution for each diameter class. These segments are then joined together to form a segmented S_B distribution. The cdf for the ith diameter class is

$$\hat{F}_i(x) = \Phi\left[\gamma_i + \delta_i \ln\left(\frac{x - \lambda_1}{\lambda_2 - x}\right)\right], \qquad x_{i-1} \le x \le x_i, \tag{15}$$

and its corresponding pdf is

$$\hat{f}_i(x) = \frac{\delta_i(\lambda_2 - \lambda_1)}{(x - \lambda_1)(\lambda_2 - x)\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left[\gamma_i + \delta_i \ln\left(\frac{x - \lambda_1}{\lambda_2 - x}\right)\right]^2\right\},$$

$$x_{i-1} \le x \le x_i. \tag{16}$$

The parameters λ_1 and λ_2 are set at the minimum and maximum diameters (x_0 and x_m, respectively) and kept constant over the segments. The shape parameters for the ith diameter class, γ_i and δ_i, are computed as follows

$$\delta_i = \frac{z_i - z_{i-1}}{\ln\left(\dfrac{x_i - \lambda_1}{\lambda_2 - x_i}\right) - \ln\left(\dfrac{x_{i-1} - \lambda_1}{\lambda_2 - x_{i-1}}\right)}, \tag{17}$$

and

$$\gamma_i = z_i - \delta_i \ln\left(\frac{x_i - \lambda_1}{\lambda_2 - x_i}\right), \tag{18}$$

where $z_i = \gamma_i + \delta_i \ln[(x_i - \lambda_i)/(\lambda_2 - x_i)]$ and $z_{i-1} = \gamma_i + \delta_i \ln[(x_{i-1} - \lambda_1)/(\lambda_2 - x_{i-1})]$.

Similar to the segmented Weibull, the segmented S_B requires a minimum of three diameter classes.

Evaluation

The seven methods were fitted to the stand table (2- and 4-cm classes) of the current stands but were evaluated on the original diameter measurements. Four goodness-of-fit statistics were used;

the lower the value of these statistics, the better the fit. The four evaluation statistics were the following

1. The average one-sample Kolmogorov-Smirnov (KS) statistic (Massey 1951)

$$\mathrm{KS} = \frac{1}{N}\sum_{i=1}^{N} \mathrm{KS}_i, \tag{19}$$

where $\mathrm{KS}_i = \max\{\max_{1\le j\le n_i}(j/n_i - u_j), \max_{1\le j\le n_i}[u_j - (j-1)/n_i]$, n_i is the number of trees in the ith plot-age combination, $u_j = \hat{F}_i(x_j)$, and N is the total number of plot-age combinations.

2. The average Anderson-Darling (AD) statistic (Anderson and Darling 1954)

$$\mathrm{AD} = \frac{1}{N}\sum_{i=1}^{N} \mathrm{AD}_i, \tag{20}$$

where $\mathrm{AD}_i = -n_i - \sum_{j=1}^{n_i} (2j-1)[\ln(u_j) + \ln(1-u_{n_i-j+1}]/n_i$.

3. The R^2 value

$$R^2 = 1 - \frac{\sum_{i=1}^{N}\sum_{j=1}^{n_i}[F_i(x_j) - \hat{F}_i(x_j)]^2}{\sum_{i=1}^{N}\sum_{j=1}^{n_i}[F_i(x_j) - \bar{F}]^2}, \tag{21}$$

where $F_i(x_j)$ and $\hat{F}_i(x_j)$ are the observed and predicted cdf, respectively, of the jth diameter in the ith class, and

$$\bar{F} = \frac{\sum_{i=1}^{N}\sum_{j=1}^{n_i} F_i(x_j)}{\sum_{i=1}^{N} n_i}.$$

4. The negative log-likelihood statistic

$$-\ln L = -\sum_{i=1}^{N}\sum_{j=1}^{n_i}\ln[\hat{f}_i(x_j)]. \tag{22}$$

In addition, the seven methods were incorporated into the stand table projection system developed by Nepal and Somers (1992). The methods were then evaluated on the basis of their ability to predict diameters of the future stands. Two forms of error index (EI) proposed by Reynolds et al. (1988) were used to quantify goodness of fit

$$\mathrm{EI}_{\mathrm{Ni}} = \sum_{k=1}^{m_i}|n_{ik} - \hat{n}_{ik}|, \tag{23}$$

and

$$\mathrm{EI}_{\mathrm{Bi}} = \sum_{k=1}^{m_i}|b_{ik} - \hat{b}_{ik}|, \tag{24}$$

where $\mathrm{EI}_{\mathrm{Ni}}$ and $\mathrm{EI}_{\mathrm{Bi}}$ are error indices based on number of trees/ha and basal area/ha for the ith plot, respectively, n_{ik} and $\hat{n}_{ik}$ are the observed and predicted numbers of trees/ha of the kth diameter class in the ith plot, respectively, b_{ik} and $\hat{b}_{ik}$ are the observed and predicted numbers of trees/ha of the kth diameter class in the ith plot, respectively, and m_i is the number of diameter classes in the ith plot.

Poudel and Cao (2013) stated that the traditional ordinal ranks for m methods (1, 2, …, m) show the order of the methods but fail to depict the exact positions of the methods compared with one another. They proposed a new method of ranking, in which method i is given the following relative rank

$$R_i = 1 + \frac{(m-1)(S_i - S_{\min})}{S_{\max} - S_{\min}}, \tag{25}$$

where R_i is the relative rank of method i (i = 1, 2, …, m), m is the number of methods (equal to 7 for this study), S_i is the goodness-of-fit statistic produced by method i, $S_{\min}$ is the minimum value of S_i, and $S_{\max}$ is the maximum value of S_i.

In this relative ranking system, the best and the worst methods have relative ranks of 1 and m, respectively. Ranks of the remaining methods are expressed as real numbers between 1 and m, and thus the magnitude and not only the order of the S_i values is taken into consideration.

Table 3. Mean error indices of the seven methods for the future stands and their relative ranks (in parentheses) for 2- and 4-cm diameter classes, by species.

Method	Mean error index based on: No. trees/ha	Mean error index based on: Basal area (m^2/ha)
Loblolly pine		
2-cm diameter classes (n = 300)		
Uniform	499 (2.67)	15.62 (4.09)
Truncated Weibull (moments)	490 (1.00)	15.36 (1.41)
Truncated Weibull (MLE)	492 (1.39)	15.32 (1.00)
Truncated Weibull (LS)	491 (1.31)	15.33 (1.05)
Truncated S_B (LS)	516 (5.53)	15.79 (5.88)
Segmented Weibull	524 (7.00)	15.90 (7.00)
Segmented S_B	506 (3.78)	15.82 (6.13)
4-cm diameter classes (n = 300)		
Uniform	336 (3.10)	10.38 (3.33)
Truncated Weibull (moments)	309 (1.00)	9.73 (1.00)
Truncated Weibull (MLE)	324 (2.18)	10.01 (1.99)
Truncated Weibull (LS)	317 (1.61)	9.78 (1.19)
Truncated S_B (LS)	388 (7.00)	11.41 (7.00)
Segmented Weibull	350 (4.14)	10.54 (3.90)
Segmented S_B	330 (2.60)	10.17 (2.56)
Slash pine		
2-cm diameter classes (n = 387)		
Uniform	171 (3.25)	4.82 (5.72)
Truncated Weibull (moments)	167 (1.84)	4.74 (3.22)
Truncated Weibull (MLE)	167 (1.67)	4.70 (1.99)
Truncated Weibull (LS)	165 (1.00)	4.66 (1.00)
Truncated S_B (LS)	179 (6.18)	4.87 (7.00)
Segmented Weibull	182 (7.00)	4.83 (5.88)
Segmented S_B	167 (1.80)	4.70 (1.98)
4-cm diameter classes (n = 300)		
Uniform	149 (5.50)	4.03 (5.85)
Truncated Weibull (moments)	129 (3.35)	3.59 (3.58)
Truncated Weibull (MLE)	114 (1.67)	3.19 (1.52)
Truncated Weibull (LS)	108 (1.00)	3.09 (1.00)
Truncated S_B (LS)	162 (7.00)	4.25 (7.00)
Segmented Weibull	161 (6.85)	3.85 (4.96)
Segmented S_B	111 (1.30)	3.16 (1.37)

Results and Discussion

Figure 1 shows different probability density functions derived from a stand table from a sample plot. For this example, the shapes of the truncated Weibull and S_B are similar (except for the largest diameter class), whereas the shapes of the segmented Weibull and S_B are similar (except for the largest diameter class).

Evaluation statistics of the seven methods and their relative ranks for 2- and 4-cm diameter classes are shown for the loblolly and slash

Table 4. Overall ranks for the seven methods.

	Loblolly pine				Slash pine					
	2-cm classes		4-cm classes		2-cm classes		4-cm classes			
	Current	Future	Current	Future	Current	Future	Current	Future	Total	Overall rank
Uniform	1.54	3.27	6.25	3.21	6.72	4.74	7.00	5.67	38.42	7.00
Truncated Weibull (moments)	1.76	1.02	3.53	1.00	5.79	2.64	4.58	3.46	23.79	3.79
Truncated Weibull (MLE)	1.00	1.01	7.00	2.08	5.12	1.89	6.47	1.60	26.17	4.31
Truncated Weibull (LS)	1.77	1.00	1.00	1.40	2.83	1.00	1.09	1.00	11.08	1.00
Truncated S_B	2.78	5.67	3.00	7.00	1.00	7.00	1.00	7.00	34.45	6.13
Segmented Weibull	4.18	7.00	1.25	4.02	4.15	6.84	1.32	5.90	34.67	6.18
Segmented S_B	7.00	4.89	6.31	2.58	7.00	1.96	2.32	1.33	33.39	5.90

pine data sets, based on diameters of the current stands (Table 2) and future stands (Table 3). The sum of the relative ranks for each method was used to determine the overall relative rank for that method (Table 4).

Loblolly Pine Data

Table 4 shows that the truncated Weibull (MLE) was the best performer for the 2-cm class, with ranks of 1.00 (current stands) and 1.01 (future stands). It was closely followed by the truncated Weibull (LS), which ranked 1.77 and 1.00 for the current and future stands, respectively. The uniform distribution did relatively well, ranking 1.54 and 3.27. The bottom two methods were the segmented Weibull and S_B.

For the 4-cm diameter classes, the truncated Weibull (MLE) was last in predicting current diameters, whereas the truncated S_B was last in predicting future diameters. The best overall for this group was the truncated Weibull (LS), with relative ranks of 1.00 and 1.40 for the current and future stands, respectively. The uniform distribution did not perform well for the current stands (ranking 6.25) but was better for predicting diameters of the future stands (ranking 3.21).

Slash Pine Data

For the 2-cm classes, the truncated Weibull (LS) was the most consistent method with relative ranks of 2.83 and 1.00 for the current and futures stands, respectively (Table 4). The uniform distribution did poorly and ranked last for this diameter width.

The truncated Weibull (LS) was the clear winner for the 4-cm diameter classes, ranking 1.09 and 1.00 in predicting diameters of the current and future stands, respectively. The segmented S_B was second, and the uniform distribution again was last.

Predicting Diameters of Current and Future Stands

Common sense dictates that if a method does well in representing diameter distributions of the current stands, it should do equally well in predicting diameter distributions of the future stands. Results from Table 4 shows that the rankings for the current and future stands did not necessarily go hand-in-hand. The truncated S_B and the segmented Weibull predicted diameters of the current stands better than those of the future stands. The reverse was true for the truncated Weibull (moments) and the truncated Weibull (MLE). Results were mixed for the rest of the distributions. Even though the same stand table projection system (Nepal and Somers 1992) was used to project diameters into the future, the difference in the relative rankings before and after the growth period indicates that the projection system is a complicated process and that its outcome can be unpredictable.

Diameter Class Width

When the class width increases from 2 to 4 cm, the change in relative rankings was generally negligible (Table 4). The exceptions for the loblolly pine data were worse rankings produced by distributions such as the uniform, truncated Weibull (MLE), and truncated S_B. For the slash pine data, the two segmented distributions (Weibull and S_B) improved their rankings as the class width increases from 2 to 4 cm.

Different Results from Data Sets

For the most part, results from the loblolly and the slash pine data sets were similar. The difference in relative rankings for some methods might be caused by the variations among data sets rather than by the difference in species. Overall, the truncated Weibull (LS) was the best performer for both loblolly and slash pine data. The segmented S_B and the uniform distribution were the worst methods for the loblolly pine and slash pine data, respectively.

Which Distribution to Use?

The *uniform* distribution is the simplest method and the implicit distribution used in straightforward stand table projection models (Avery and Burkhart 2002, Husch et al. 2003). However, its overall performance ranked last compared with those for the rest of the methods, indicating that this simple distribution was not adequate to model diameter distributions from stand table data.

The *S_B* distribution has never been used in a stand table projection model. In this study, the truncated and segmented S_B received overall rankings of 6.13 and 5.90, respectively, which were near the bottom of the seven methods (Table 4). The poor performance of the S_B distribution might be because its lower and upper limits were arbitrarily set at the smallest and largest diameter classes, respectively.

The *segmented Weibull* distribution received an overall relative rank of 6.18, next to last among the seven methods. Because a different set of Weibull parameters are used for each segment (diameter class), the segmented Weibull should be flexible to handle distributions with variations among diameter classes. However, its projections of diameters of the future stands were poorer than those from the remaining methods, even though it predicted the current stands relatively well.

The version of the *truncated Weibull* distribution used in many recent stand projection models (Nepal and Somers 1992, Cao and Baldwin 1999a, 1999b, Cao 2007, Allen et al. 2011) was based on the method of moments. It was found to be just a mediocre performer, ranking 3.79 overall (Table 4). The MLE version received an overall relative rank of 4.31. The LS version of the truncated Weibull distribution was the best and most consistent among the seven methods. Whereas the moments approach is based on point estimates, i.e., the diameter moments, the aim of the LS approach is to optimize the entire distribution by minimizing the squared difference between the observed and predicted cumulative probabilities.

Table 4 shows three distinct groups of methods, based on their overall relative ranks. The truncated Weibull (LS) was clearly the winner, performing well in predicting diameters of both current and future stands for both data sets. Group 2 included the moments and MLE versions of the truncated Weibull distribution. The last group, made up by the segmented Weibull and S_B, truncated S_B, and uniform distributions, performed poorly and should not be used for stand projection.

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Appendix

Below are the derivations for estimating parameters of the segmented Weibull and S_B distributions.

Segmented Weibull Distribution

The cdf for the ith diameter class passes through F_i, resulting in

$$F_i = \hat{F}_i(x_i) = 1 - \exp\left[-\left(\frac{x_i - a}{b_i}\right)^{c_i}\right], \tag{A1}$$

which can be rewritten as

$$\left(\frac{x_i - a}{b_i}\right)^{c_i} = -\ln(1 - F_i). \tag{A2}$$

Similarly, the following equation shows that the cdf for the ith diameter class also passes through F_{i-1}

$$\left(\frac{x_{i-1} - a}{b_i}\right)^{c_i} = -\ln(1 - F_{i-1}). \tag{A3}$$

Dividing A2 by A3 gives

$$\left(\frac{x_i - a}{x_{i-1} - a}\right)^{c_i} = \frac{\ln(1 - F_i)}{\ln(1 - F_{i-1})}. \tag{A4}$$

The parameters b_i and c_i are then computed as follows

$$c_i = \frac{\ln\left[\dfrac{\ln(1 - F_i)}{\ln(1 - F_{i-1})}\right]}{\ln(x_i - a) - \ln(x_{i-1} - a)}, \tag{A5}$$

and

$$b_i = \frac{x_i - a}{[-\ln(1 - F_i)]^{1/c_i}}. \quad \text{(A6)}$$

Segmented S_B Distribution

The S_B cdf for the ith diameter class (Equation 15) is based on a segment of the standard normal cdf. Let $z = \gamma_i + \delta_i \ln[(x - \lambda_1)/(\lambda_2 - x)]$ be the corresponding standard normal random variable.

The segmented S_B cdf for the ith diameter class passes through F_i, resulting in

$$F_i = \hat{F}_i(x_i) = \Phi\left[\gamma_i + \delta_i \ln\left(\frac{x_i - \lambda_1}{\lambda_2 - x_i}\right)\right] = \Phi(z_i). \quad \text{(A7)}$$

Therefore,

$$z_i = \Phi^{-1}(F_i) = \gamma_i + \delta_i \ln\left(\frac{x_i - \lambda_1}{\lambda_2 - x_i}\right). \quad \text{(A8)}$$

Similarly, the same cdf for the ith diameter class also passes through F_{i-1}, leading to

$$z_{i-1} = \Phi^{-1}(F_{i-1}) = \gamma_i + \delta_i \ln\left(\frac{x_{i-1} - \lambda_1}{\lambda_2 - x_{i-1}}\right). \quad \text{(A9)}$$

The shape parameters for the ith diameter class, γ_i and δ_i, are obtained from A8 and A9 as follows

$$\delta_i = \frac{z_i - z_{i-1}}{\ln\left(\frac{x_i - \lambda_1}{\lambda_2 - x_i}\right) - \ln\left(\frac{x_{i-1} - \lambda_1}{\lambda_2 - x_{i-1}}\right)}, \quad \text{(A10)}$$

and

$$\gamma_i = z_i - \delta_i \ln\left(\frac{x_i - \lambda_1}{\lambda_2 - x_i}\right). \quad \text{(A11)}$$