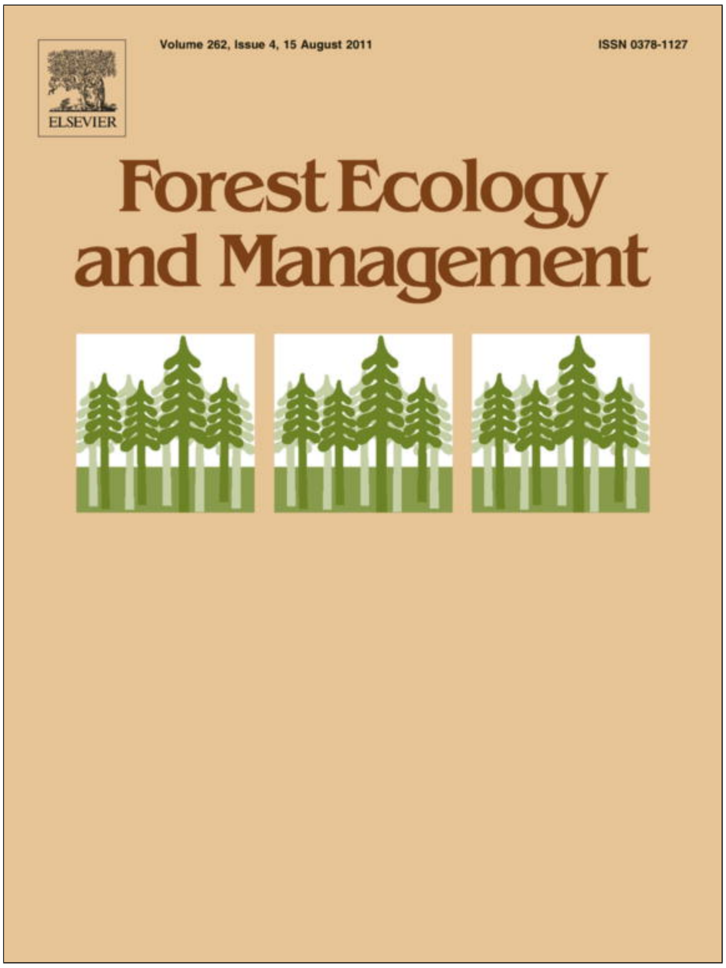
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Calibrating fixed- and mixed-effects taper equations

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ABSTRACT

Accurate and affordable measurements of upper-stem diameters are now possible thanks to recent advances in laser technology. Measurement of the midpoint upper-stem diameter can be employed to improve the accuracy of diameter predictions along the tree bole. Felled-tree data from a loblolly pine (*Pinus taeda* L.) plantation was used to evaluate two approaches: (1) calibrating a segmented taper equation by constraining a parameter, and (2) localizing the taper model by predicting the random effects for each tree. The calibration technique is much simpler and produced less-biased prediction of diameters and is therefore recommended. Calibration results were similar for both fixed- and mixed-effects taper models, even though a slight gain in accuracy and precision was attained with the mixed-effects model.

1. Introduction

Upper-stem diameter measurements can now be obtained by use of better and more affordable laser dendrometers in a manner that was not previously possible. The laser dendrometers are portable and produce a mean bias of 0.13 cm for measuring diameters (Parker and Matney, 1999; Williams et al., 1999; Kalliovirta et al., 2004). This kind of bias is comparable to that obtained from the Barr & Stroud dendrometer (Williams et al., 1999), long considered to be the standard tool in measuring diameters.

When a taper equation is constrained such that its curve passes through a measured upper-stem diameter, the result is improved prediction of tree taper anywhere on the bole. Czaplewski and McClure (1988) conditioned the Max and Burkhart (1976) taper equation for diameter at breast height (*dbh*) and an upper-stem diameter (at 5.27 m or 17.3 feet) to obtain a 10% to 25% reduction in root mean squared error as compared to the unconditioned model. Cao (2009) calibrated the original Max and Burkhart (1976) taper equation by constraining the curve to go through *dbh* and an upper-stem diameter for his data set, increasing R^2 from 0.953 to 0.975.

There has been a growing interest in applying mixed-effects models to solve forestry regression problems. Specifically, mixed-effects models have been applied to predict tree taper because they take into account the correlation among multiple diameter measurements on an individual stem (Garber and Maguire, 2003; Leites and Robinson, 2004; Trincado and Burkhart, 2006; Lejeune et al., 2009; Yang et al., 2009). These models contain both fixed-effects parameters that are common to all trees in the sample, and random effects that are specific to each individual tree.

The existence of one or more upper-stem diameter measurements for a particular tree allows for calibration of a mixed-effects taper model by predicting the random effects for this tree. The objective of this study was to evaluate the use of the midpoint upper-stem diameter in calibrating fixed-effects and mixed-effects taper models.

2. Data

Data used in this study were from a loblolly pine (*Pinus taeda* L.) plantation at the Hill Farm Research Station, Homer, Louisiana. Site index ranged from 18.6 to 23.2 m (base age 25 years) for the study area. Measurements from trees felled in a thinning at age 21 were randomly divided into a fit data set and a validation data set. The fit data consisted of 133 trees, with *dbh* ranging from 12.1 to 49.5 cm and total height from 9.1 to 23.4 m. Outside-bark diameters were taken at 64-cm intervals, starting from the stump, up to the tree tip, totaling 3233 observations. The validation data consisted of 3418 observations from 146 trees, with *dbh* ranging from 13.0 to 41.7 cm and total height from 8.3 to 22.8 m.

3. Methods

The Max and Burkhart (1976) taper equation can be written in the following modified form:

$$y_{ij} = b_1^* x + b_2 x_{ij}^2 + b_3 I_1 (x_{ij} - a_1)^2 + b_4 I_2 (x_{ij} - a_2)^2 + \varepsilon_{ij} \quad (1)$$

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where $y_{ij} = d_{ij}^2/D_i^2$, D_i = *dbh* of tree i, ($i = 1, 2, \ldots, N$), N = number of trees in the sample, d_{ij} = the ith upper-stem diameter outside bark measured at height h_{ij} on tree i, ($j = 1, 2, \ldots, n_i$), n_i = number of diameter measurements for tree i, $x_{ij} = (H_i - h_{ij})/(H_i - 1.37)$ = relative height from the tree tip, H_i = total height, a_1 and a_2 = join points to be estimated from the data, $I_k = 1$ if $z > a_i$, and 0 otherwise, k = 1, 2, and b_m's = regression coefficients, m = 2, 3, 4.

Note that b_1^* is not a regression parameter, but is instead computed from

$$b_1^* = 1 - b_2 - b_3(1 - a_1)^2 - b_4 I_2'(1 - a_2)^2 \tag{2}$$

where $I_2' = 1$ if $a_2 < 1$, and 0 otherwise. This constraint makes sure that $\hat{y} = 1$ at $x = 1$, or $\hat{d} = D$ when $h = 1.37$ m, meaning that the model yields *dbh* at breast height.Parameter estimates from Eq. (1) were obtained by use of procedure NLIN from SAS (SAS Institute Inc., 2004).

3.1. Mixed-effects model

In the mixed-effects framework, all parameters of Eq. (1) can be expressed as fixed-effects parameters (common to all trees), with certain parameters containing additional random components, which are specific to individual trees. Candidates for random parameters are various combinations of the parameters (b_2, b_3, b_4, a_1, and a_2). In matrix form, Eq. (1) becomes:

$$\mathbf{y_i} = \mathbf{f}(\mathbf{b}, \mathbf{u_i}, \mathbf{x_i}) + \boldsymbol{\varepsilon}_\mathbf{i} \tag{3}$$

where $\mathbf{y_i} = [y_{i1}, y_{i2}, \ldots, y_{i,n_i}]^\mathrm{T}$, $\mathbf{x_i} = [x_{i1}, x_{i2}, \ldots, x_{i,n_i}]^\mathrm{T}$, $\boldsymbol{\varepsilon}_\mathbf{i} = [\varepsilon_{i1}, \varepsilon_{i2}, \ldots, \varepsilon_{i,n_i}]^\mathrm{T}$, and $\mathbf{b}$ and $\mathbf{u_i}$ are column vectors of fixed- and random-effects parameters, respectively. The assumptions are:

$$\boldsymbol{\varepsilon}_\mathbf{i} \sim \mathrm{N}(\mathbf{0}, \mathbf{R}),$$

$$\mathbf{u_i} \sim \mathrm{N}(\mathbf{0}, \mathbf{D}),$$

where $\mathbf{R}$ and $\mathbf{D}$ are diagonal matrices, because the $\boldsymbol{\epsilon}_\mathbf{i}$ and $\mathbf{u_i}$ are assumed to be independent.

Procedure NLMIXED from SAS was used to obtain fixed- and random-effects parameters of Eq. (7).

3.2. Calibration for the midpoint upper-stem diameter

Cao (2009) found that best results were obtained when the diameter for calibration (d_{i0}) was measured at $x_{i0} = 0.5$, the midpoint between the tree tip and breast height. The predicted diameter at relative height x_{i0} is constrained to equal d_{i0}, resulting in:

$$\hat{y}_{i0}^* = y_{i0} = d_{i0}^2/D_i^2. \tag{4}$$

The calibrated taper equation is as follows:

$$y_{ij}^* = b_1^* x + b_2^* x_{ij}^2 + b_3 I_1 (x_{ij} - a_1)^2 + b_4 I_2 (x_{ij} - a_2)^2 + \varepsilon_{ij} \tag{5}$$

where

$$\begin{aligned} b_2^* &= 2 - 4y_0 - b_3\left[2(1 - a_1)^2 - J_1(1 - 2a_1)^2\right] \\ &\quad - b_4\left[2(1 - a_2)^2 - J_2(1 - 2a_2)^2\right], \text{ and} \end{aligned} \tag{6}$$

$$b_1^* = 1 - b_2^* - b_3(1 - a_1)^2 - b_4 I_2'(1 - a_2)^2 \tag{7}$$

The calibration procedure was carried out for both fixed-effects and mixed-effects models. In the latter, only fixed-effects parameters were used in Eqs. (6) and (7).

3.3. Localizing the taper equation

The random parameters $\mathbf{u_i}$ for tree i can be computed by use of the first-order Taylor series expansion (Fitzmaurice et al., 2004; Meng and Huang, 2009):

$$\hat{\mathbf{u}}_\mathbf{i}^{\mathbf{k}+1} = \hat{\mathbf{D}}\mathbf{Z}_\mathbf{i}^\mathbf{T}(\mathbf{Z_i}\hat{\mathbf{D}}\mathbf{Z}_\mathbf{i}^\mathbf{T} + \hat{\mathbf{R}}_\mathbf{i})^{-1}[\mathbf{y_i} - \mathbf{f}(\hat{\mathbf{b}}, \hat{\mathbf{u}}_\mathbf{i}^\mathbf{k}, \mathbf{x_i})] \tag{8}$$

where $\hat{\mathbf{u}}_\mathbf{i}^\mathbf{k}$ = estimate of the random parameters at the kth iteration, $\hat{\mathbf{D}}$ = estimate of $\mathbf{D}$, the variance–covariance matrix for $\mathbf{u_i}$, $\mathbf{Z_i} = \frac{\partial \mathbf{f}(\mathbf{b}, \mathbf{u_i}, \mathbf{x_i})}{\partial \mathbf{u_i}}\Big|_{\hat{\mathbf{b}}, \mathbf{u_i}}$, $\hat{\mathbf{R}}_\mathbf{i}$ = estimate of $\mathbf{R_i}$, the variance–covariance matrix for $\boldsymbol{\varepsilon}_\mathbf{i}$, y_i = the $m \times 1$ vector of observed diameters, and m = number of diameter measurements used in localizing the taper model (m = 1 in this case).

An iterative procedure was needed to estimate $\mathbf{u_i}$. A null value for $\mathbf{u_i}$ was used as the starting value ($\hat{\mathbf{u}}_\mathbf{i}^0 = \mathbf{0}$). The value for $\hat{\mathbf{u}}_\mathbf{i}$ was then repeatedly updated from Eq. (8) until the absolute difference between two successive iterations was smaller than a predetermined tolerance limit. The end result would be the Empirical Best Linear Unbiased Predictor (EBLUP) for random effects.

3.4. Evaluation

The performance of the unadjusted, calibrated, and localized models was evaluated by use of three statistics: R^2, mean difference (MD) between observed and predicted diameters, and mean absolute difference (MAD).

4. Results and discussion

Different combinations of mixed parameters from Eq. (1) were implemented (Table 1). The mixed-effects combination of b_3 and b_4 produced the lowest values of Akaike's information criterion (AIC) and Bayesian information criterion (BIC), resulting in the following mixed model:

$$\begin{aligned} y_{ij} &= b_1^* x + b_2 x_{ij}^2 + (b_3 + u_3) I_1 (x_{ij} - a_1)^2 \\ &\quad + (b_4 + u_4) I_2 (x_{ij} - a_2)^2 + \varepsilon_{ij} \end{aligned} \tag{9}$$

where $b_1^* = 1 - b_2^* - (b_3 + u_3)(1 - a_1)^2 - (b_4 + u_4) I_2'(1 - a_2)^2$.

Table 2 shows parameter estimates for fixed- and mixed-effects taper models, based on the fit data set. Evaluation results based on the validation data are presented in Table 3 for both types of models.

4.1. Calibrated model versus unadjusted model

The calibration of the fixed-effects model improves all evaluation statistics: MD decreased from 0.5817 to 0.1689 (or 71%),

Table 1
Model selection statistics for evaluating the inclusion of random parameters.

Random parameters	AIC[a]	BIC[b]
None	−9109	−9073
b_2	−9449	−9429
b_3	−10,402	−10,382
b_4	−10,157	−10,382
a_1	No convergence	
a_2	No convergence	
b_2 and b_3	No convergence	
b_2 and b_4	−10,997	−10,973
b_3 and b_4	**−11,001**	**−10,978**[c]
b_2, b_3, and b_4	No convergence	

[a] Akaike's information criterion (smaller is better).
[b] Bayesian information criterion (smaller is better).
[c] Underlined numbers denotes the minimum statistic among all models.

Table 2
Estimates of parameters (and standard errors) for fixed-effects and mixed-effects taper models, based on the fit data set.

Parameters	Fixed-effects model	Mixed-effects model
b_2	2.0112 (0.2440)	2.0952 (0.1819)
b_3	−1.8319 (0.2482)	−1.8972 (0.1849)
b_4	5.7746 (0.3455)	7.3620 (0.4853)
a_1	0.1954 (0.0166)	0.1901 (0.0115)
a_2	0.8553 (0.0068)	0.8755 (0.0041)
var (ε)		0.0015 (0.0001)
var (u_3)		0.0703 (0.0097)
var (u_4)		16.4421 (2.7890)

Table 3
Evaluation statistics[a] based on the validation data for fixed-effects and mixed-effects taper models.

Model	Status	MD	MAD	R^2
Fixed-effects	Not calibrated	0.5817	1.3119	0.9390
Mixed-effects	Not calibrated	0.5839	1.3078	0.9392
Fixed-effects	Calibrated at x_0 = 0.5	0.1689	0.9479	0.9604
Mixed-effects	Calibrated at x_0 = 0.5	0.1621	0.9427	0.9609
Mixed-effects	Localized using EBLUP	0.3079	0.9416	0.9603

[a] MD = mean difference, MAD = mean absolute difference, and $R^2 = 1 - \Sigma(y_i - \hat{y}_i)^2/\Sigma(y_i - \bar{y})^2$, where y_i = upper-stem diameter.

MAD decreased from 1.3119 to 0.9479 (28%), and root mean squared error decreased from 103.75 to 83.55 (19%). Similar results were obtained in the case of mixed-effects models (Table 3).

4.2. Calibrated model versus localized model

Given a diameter measurement at the midpoint (x_0 = 0.5) of each tree in the validation data, the mixed-effects model can be either calibrated by adjusting b_1^* and b_2^*, or localized by computing the EBLUP of random effects for that tree. The difference between the two methods is that the calibrated taper curve passes through the midpoint diameter (d_0), but the localized curve does not. Although values for MAD and R^2 were comparable for the two methods, MD value was lower for the calibrated model (0.1621) than for the localized model (0.3079). Fig. 1 shows that the calibrated and localized taper curves were similar for two example trees; both were superior to the unadjusted model.

4.3. Fixed-effects model versus mixed-effects model

The estimates for the fixed parameters of both models were close (Table 2). Consequently, when the fixed-effects parameters of the mixed model were adjusted (or calibrated), the results from calibration of both models were similar.

5. Summary and conclusions

Measurement of the midpoint upper-stem diameter can be employed to improve the accuracy of diameter predictions along the tree bole. Felled-tree data from a loblolly pine (*P. taeda* L.) plantation was used to evaluate two approaches: (1) calibrating a segmented taper equation by constraining a parameter, and (2) localizing the taper model by predicting the random effects for each tree. The calibration technique is much simpler and produced less-biased prediction of diameters and is therefore recommended. Results from calibration were similar for both fixed- and mixed-effects taper models, even though a slight gain in accuracy and precision was attained with the mixed-effects model.

This study confirmed findings from Czaplewski and McClure (1988) and Cao (2009) that an extra upper-stem diameter measurement clearly helped improve the prediction of a taper equation. Regardless of the approach taken (fixed- or mixed-effects model), the increase in accuracy and precision of the predictions was substantial, and would allow forest managers to better predict tree volumes to any utilization standard. Methodology from this planted loblolly pine data set can be applied to other conifer species and similar results might be expected as well.

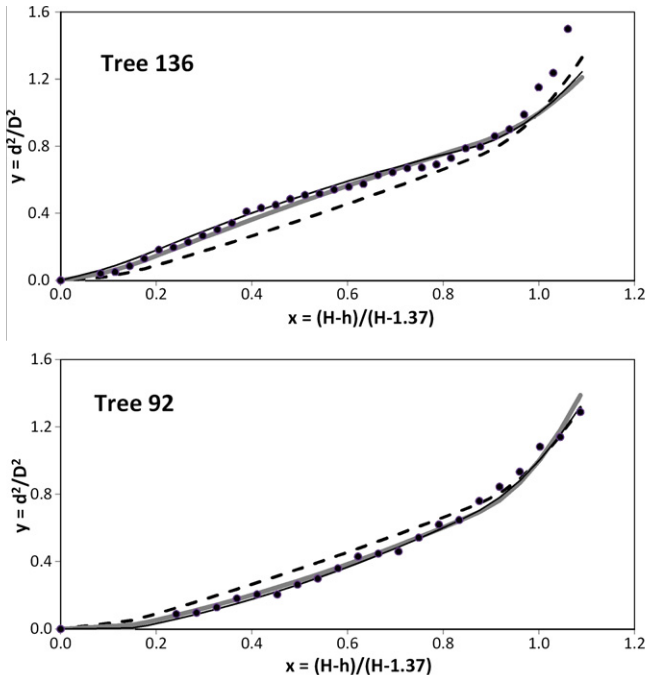

Fig. 1. Observed tree diameters (circular dots) and predicted taper curves from the fixed-effects model (dashed lines) for two example trees. The calibrated curves (thin lines) and localized curves (thick gray lines) improve taper prediction for these two trees as compared to the unadjusted curve.

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