Compatibility of Stand Basal Area Predictions Based on Forecast Combination

Xiongqing Zhang, Yuancai Lei, Quang V. Cao

Abstract: Stand growth and yield models include whole-stand models, individual-tree models, and diameter-distribution models. Based on the growth data of Chinese pine (*Pinus tabulaeformis* Carr.) in Beijing, forecast combination was used to adjust predicted stand basal areas from these three types of models. The forecast combination method combines information and disperses errors from different models to improve forecast performance. In this study, weights of the three model estimates used in the forecast combination estimator were determined by the optimal weight method. Results showed that the forecast combination method provided overall better predictions of stand basal area than the three types of models. It also improved the compatibility of stand basal area growth predicted from models of different resolutions and provided a method for integration of stand basal area. FOR. SCI. 56(6):552–557.

Keywords: compatibility, forecast combination, optimal weight, *Pinus tabulaeformis*, stand basal area model

I N FOREST MANAGEMENT, growth and yield models play a very important role in predicting forest growth and studying its processes. Forest growth and yield models can be broken into three broad categories: whole-stand models, individual-tree models, and diameter-distribution models (Munro 1974). Whole-stand models are models that use a stand as a modeling unit (Curtis et al. 1981, Li et al. 1988, Tang et al. 1993, Wei 2006), whereas individual-tree models consider an individual tree as a study object (Meng and Zhang 1996, Zhang et al. 1997, Cao 2000, Cao et al. 2002). Diameter-distribution models, in contrast, use statistical probability functions, such as the Weibull function (Bailey and Dell 1973, Meng 1988, Liu et al. 2004, Newton et al. 2005), beta function (Gorgoso-Várela et al. 2008), or S_B function (Wang and Rennolls 2005), to characterize stand structure.

There are strengths and weaknesses with each type of model. Whole-stand models can predict stand variables directly and provide accurate predictions of stand growth and yield but lack detailed tree-level information. On the other hand, individual-tree models provide more detailed information, and diameter-distribution models offer stand diameter structure, but stand-level outputs from these two types of models often suffer from an accumulation of errors and subsequently poor accuracy and precision (Garcia 2001, Qin and Cao 2006).

From a commonsense point of view, we expect stand-level outputs from different types of models to be similar. However, these models may still provide numerically inconsistent predictions due to model errors (Meng 1996). Daniels and Burkhart (1988) proposed a framework for an integrated system of forest stand models, in which constraints were applied so that stand outputs from models of different levels of resolution are similar. Recently, researchers have tried to solve problems involving compatibility between different types of models. The disaggregation method was used to maintain compatibility by linking stand-level and tree-level models (Zhang et al. 1993, Ritchie and Hann 1997, Qin and Cao 2006). However, this method assumes that predictions from the stand-level model are superior, and it attempts to adjust outputs from the tree-level model to match those from the stand-level model.

Forecast combination, introduced by Bates and Granger (1969), is a good method for improving forecast accuracy (Newbold et al. 1987). This method combines information generated from different models and disperses errors from these models, thus ensuring consistency for outputs from different models. Yue et al. (2008) used this method to deal with the problem of compatibility between tree-level and stand-level models. The objective of this study was to apply the forecast combination method in linking whole-stand, individual-tree, and diameter-distribution models so that the resulting stand basal area prediction is consistent at various levels of resolution.

Data

The data, provided by the Inventory Institute of Beijing Forestry, consisted of a systematic sample of permanent plots with a 5-year re-measurement interval. The plots, 0.067 ha each, were in Chinese pine (*Pinus tabulaeformis* Carr.) plantations situated on upland sites throughout northwestern Beijing. The data consisted of 156 measurements, with a 5-year re-measurement interval, obtained in the following years: 1986, 1991, 1996, and 2001. In this study, 106 plots were used in model development and another 50 plots

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were used for validation. Table 1 shows the distribution of plots. Summary statistics for both data sets are presented in Table 2.

Methods
Stand-Level and Tree-Level Models

Cao (2002) developed a variable-rate method to predict annual diameter growth and survival for an individual tree. This method accounts for the fact that rates of survival and diameter growth vary from year to year. Stand-level growth and survival were also treated in a similar manner (Ochi and Cao 2003). The variable-rate method was used in this study. Annual changes in stand variables (dominant height, stand survival, arithmetic mean diameter, stand basal area, diameter SD, and minimum diameter), and tree variables (dbh and survival probability) were described in a recursive manner (Ochi and Cao 2003, Nord-Larsen 2006, Qin et al. 2007, Cao and Strub 2008). Table 3 lists the stand-level and tree-level growth equations.

Estimates of stand basal area from the tree-level model at age $t + q$ were obtained by summing up the basal area per ha represented by each tree from the individual-tree diameter growth model (Equation 1g) and survival model (Equation 1h). Because cross-equation correlations existed among error components of the above models, a method suggested by Borders (1989) was used to simultaneously estimate parameters of the regression system (Equations 1a–1g). The fitting procedure involved the use of option SUR (seemingly unrelated regression) of the SAS Procedure Model. Parameters of the tree survival equation were separately estimated by use of the NLIN procedure.

Diameter Distribution Model

The Weibull function has been extensively applied in forestry because of its flexibility in describing a wide range of unimodal distributions and the relative simplicity of parameter estimation (Bailey and Dell 1973, Kangas and Maltamo 2000, Mabvurira et al. 2002, Lei 2008). The Weibull probability density function is expressed as

$$f(x; a, b, c) = \left(\frac{c}{b}\right)\left(\frac{x - a}{b}\right)^{c-1} \exp\left[-\left(\frac{x - a}{b}\right)^c\right]$$
$$(a \leq x \leq \infty) \quad (2)$$

where x is dbh, a is the location parameter, b is the scale parameter, and c is the shape parameter.

Moment estimation is one of the widely used parameter recovery methods for estimating Weibull parameters (Liu et al. 2004, Lei 2008). Considering that the location parameter (a) must be smaller than the predicted minimum diameter

Table 1. Distribution of plots

Measurement time	Fit data	Validation data	Total
1986–1991	27	12	39
1991–1996	37	17	54
1996–2001	42	21	63
Total	106	50	156

($\hat{D}_{min}$) in the stand, we set $a = \hat{D}_{min}$ because Frazier (1981) found that this value produced minimal errors in terms of goodness of fit. A straightforward method to recover b and c involves predicted values of arithmetic and quadratic mean diameters ($\hat{D}m$ and $\hat{D}g$, respectively). A possible problem is that $\hat{D}g$ might be too close to or too far from $\hat{D}m$ and can even be smaller than $\hat{D}m$ if not properly constrained. Based on our experience with this data set and other data sets, we found that the resulting Weibull parameters are sensitive to the difference between $\hat{D}m$ and $\hat{D}g$ and therefore might result in unstable estimation of b and c. An alternative method is to use the predicted arithmetic mean diameter ($\hat{D}m$) and diameter variance ($\hat{D}var$ instead of $\hat{D}m$ and $\hat{D}g$ (Diéguez-Aranda et al. 2006, Qin et al. 2007). The Weibull parameters b and c are solutions of the system

$$b = (\hat{D}m_{t+q} - a)/\Gamma_1 \quad (3)$$

$$(\hat{D}var_{t+q}) - b^2(\Gamma_2 - \Gamma_1^2) = 0, \quad (4)$$

where $\Gamma_1 = \Gamma(1 + 1/c)$ and $\Gamma_2 = \Gamma(1 + 2/c)$. Quadratic mean diameter was first predicted from the parameters of the diameter-distribution model and then used to compute an estimate of stand basal area:

$$\hat{D}g_{t+q}^2 = b^2\Gamma_2 + 2ab\Gamma_1 + a^2 \quad (5)$$

$$\hat{B}_{t+q} = (\pi/40000)(\hat{N}_{t+q})(\hat{D}g_{t+q}^2). \quad (6)$$

Forecast Combination

Yue et al. (2008) applied the forecast combination method to combine stand-level and tree-level models. Likewise, predicted stand basal area for our study can be combined from three different models,

$$B^C = w_1 B^T + w_2 B^S + w_3 B^D, \quad (7)$$

where B^C is the combined estimator of stand basal area, B^T is the estimate of stand basal area from the individual-tree model, B^S is the estimate of stand basal area from the stand-level model, B^D is the estimate of stand basal area from the diameter-distribution model, and w_1, w_2, and w_3 are weight coefficients, with $w_1 + w_2 + w_3 = 1$.

Yue et al. (2008) used the variance-covariance method to calculate weight coefficients for the stand basal area combined model. Theoretically, the best weight coefficients can be obtained on the basis of this approach. However, the weights are often unstable in practice, and thus this method has limitations (Zhang et al. 2006). The optimal weight method, which is the ordinary least-squares estimate of the weights, is an alternative method for solving quadratic programming problems and it yields unbiased combining forecasts (Tang 1992, 1994). We used the optimal weight method in this study to calculate the weight coefficients of the combined stand basal area. The objective was to minimize

$$Z = \sum_{i=1}^{n}[B_i - (w_1 B_i^T + w_2 B_i^S + w_3 B_i^D)]^2, \quad (8)$$

Table 2. Summary statistics of stand-level and tree-level variables, by data set

	Fit data ($n = 106$)				Validation data ($n = 50$)			
Variable	Minimum	Maximum	Mean	SD	Minimum	Maximum	Mean	SD
Age (years)	11	55	31	8	13	60	30	8
Dominant height (m)	0.4	17.4	6.8	2.5	2.7	17.4	7.3	3.0
Stand survival (trees ha^{-1})	254	2,284	1,172	509	239	2,090	1,238	502
Stand basal area (m^2 ha^{-1})	0.80	33.10	10.97	6.08	0.61	28.06	11.41	6.54
dbh (cm)	5	36.8	10.48	3.90	5	29.4	10.39	4.00

Table 3. List of recursive stand-level and tree-level growth equations at year $(t + j), j = 1, 2, \ldots, q$, where q is length of growth period in years

Stand-level

$$H_{t+q} = \exp\{(A_{t+q-1}/A_{t+q}) \ln(H_{t+q-1}) + (1 - A_{t+q-1}/A_{t+q})[\alpha_1 + \alpha_2/A_{t+q-1} + \alpha_3 H_{t+q-1}]\} \tag{1a}$$

$$N_{t+q} = \exp\{(A_{t+q-1}/A_{t+q}) \ln(N_{t+q-1}) + (1 - A_{t+q-1}/A_{t+q})[\beta_1 + \beta_2/A_{t+q-1} + \beta_3 N_{t+q-1}]\} \tag{1b}$$

$$Dm_{t+q} = \exp\{(A_{t+q-1}/A_{t+q}) \ln(Dm_{t+q-1}) + (1 - A_{t+q-1}/A_{t+q})[\delta_1 + \delta_2/A_{t+q-1} + \delta_3/\ln(H_{t+q-1}) + \delta_4 \ln(H_{t+q-1})]\} \tag{1c}$$

$$B_{t+q} = \exp\{(A_{t+q-1}/A_{t+q}) \ln(B_{t+q-1}) + (1 - A_{t+q-1}/A_{t+q})[\phi_1 + \phi_2/H_{t+q-1} + \phi_3/\ln(N_{t+q-1})]\} \tag{1d}$$

$$Dsd_{t+q} = \exp\{(A_{t+q-1}/A_{t+q}) \ln(Dsd_{t+q-1}) + (1 - A_{t+q-1}/A_{t+q})[\gamma_1 + \gamma_2 \ln(H_{t+q-1}) + \gamma_3 \ln(N_{t+q-1})]\} \tag{1e}$$

$$Dmin_{t+q} = \exp\{(A_{t+q-1}/A_{t+q}) \ln(Dmin_{t+q-1}) + (1 - A_{t+q-1}/A_{t+q})[\kappa_1 + \kappa_2/A_{t+q-1} + \kappa_3/\ln(N_{t+q-1})]\} \tag{1f}$$

Tree-level:

$$D_{i,t+q} = D_{i,t+q-1} + exp\{\lambda_1 + \lambda_2 A_{t+q-1}/A_{t+q} + \lambda_3 B_{t+q-1} + \lambda_4/Rs_{t+q-1} + \lambda_5/\ln(D_{i,t+q-1})\} \tag{1g}$$

$$P_{i,t+q} = \{1 + \exp[\mu_1 + \mu_2 A_{t+q-1} + \mu_3 D_{t+q-1}/\ln(Dm_{t+q-1}) + \mu_4 \ln(N_{t+q-1})]\} \tag{1h}$$

where $Rs_{t+q} = ((10{,}000/N_{t+q})^{0.5}/H_{t+q}) =$ the relative spacing at age A_{t+q}, $H_{t+q} =$ dominant height in m at age A_{t+q}, $N_{t+q} =$ number of trees ha^{-1} at age A_{t+q}, $Dm_{t+q} =$ arithmetic mean diameter in cm at age A_{t+q}, $B_{t+q} =$ stand basal area in m^2 ha^{-1} at age A_{t+q}, $Dsd_{t+q} =$ diameter standard deviation in cm at age A_{t+q}, $Dmin_{t+q} =$ minimum diameter in cm at age A_{t+q}, $D_{i,t+q} =$ diameter of tree i at age A_{t+q}, $A_{i,t+q} =$ probability that tree i is survived the period from age A_{t+q-1} to A_{t+q}, and $\alpha_1, \alpha_2, \ldots, \mu_4 =$ parameters to be estimated.

subject to the constraint,

$$w_1 + w_2 + w_3 = 1, \tag{9}$$

where B_i is stand basal area of plot i and B_i^k is stand basal area of plot i predicted from method k.

Equations 8 and 9 can be written in matrix notation as

$$\text{minimize } Z = W^TEW \quad \text{subject to } R^TW = 1, \tag{10}$$

where $W = (w_1 + w_2 + w_3)^T$, $R = (1, 1, 1)^T$,

$$E = \begin{pmatrix} e_1^T \\ e_2^T \\ e_3^T \end{pmatrix} (e_1 \quad e_2 \quad e_3) = \begin{pmatrix} e_1^T e_1 & e_1^T e_2 & e_1^T e_3 \\ e_2^T e_1 & e_2^T e_2 & e_2^T e_3 \\ e_3^T e_1 & e_3^T e_2 & e_3^T e_3 \end{pmatrix},$$

$e_k = (e_{k1}, e_{k2}, \ldots, e_{kn})$, e_{ki} is the forecast error of observation i from method k, and n is numbers of plots.

Function 10 can be written as

$$Z = W^TEW + \lambda(R^TW - 1), \tag{11}$$

where λ is a Lagrangian multiplier. By taking the first partial derivative of Z with respect to W and setting it equal to 0, the optimum weight is obtained,

$$W = \frac{E^{-1}R}{R^TE^{-1}R}. \tag{12}$$

The optimum weights can be computed by use of the Forstat statistical package (Tang et al. 2009) or a spreadsheet with matrix capabilities such as Microsoft Excel.

Model Evaluation

Performance of the three growth models and the combined estimator was evaluated based on the fit and validation data sets. The following evaluation statistics were calculated:

Mean Difference:

$$MD = \sum(y_i - \hat{y}_i)/n \tag{13}$$

Mean Absolute Difference:

$$MAD = \sum(y_i - \hat{y}_i)/n \tag{14}$$

R-square:

$$R^2 = 1 - \sum(y_i - \hat{y}_i)^2 \big/ \sum(y_i - \bar{y})^2 \tag{15}$$

where y_i is observed value at age A_{t+q} of stand variables (stand basal area, arithmetic mean diameter, diameter SD, minimum diameter, and stand survival) or tree variables (diameter and survival probability of tree i), $\hat{y}_i$ and $\bar{y}$ are predicted value and average, respectively, of y_i, and n is number of observations.

Results and Discussion

The estimates and standard errors of parameters of the different growth models are presented in Table 4. All of the parameters were significant ($P < 0.0001$). Equation 12 was used to calculate the weight coefficients of the basal area

Table 4. Parameter estimates and model evaluation

Attribute	Parameter	Estimate	SE	R^2
Dominant height (m)	α_1	4.0944	0.0889	0.7591
	α_2	−19.6596	1.4176	
	α_3	−0.0955	0.0061	
Stand basal area (m^2 ha^{-1})	ϕ_1	7.3429	0.1844	0.9282
	ϕ_2	0.0303	0.0033	
	ϕ_3	−25.0852	1.3085	
Arithmetic mean diameter (cm)	δ_1	3.1948	0.0567	0.9144
	δ_2	−12.2948	0.3531	
	δ_3	1.6756	0.3796	
	δ_4	0.0340	0.0081	
Diameter SD (cm)	γ_1	1.4832	0.0989	0.8833
	γ_2	0.5026	0.0186	
	γ_3	−0.0858	0.0140	
Minimum diameter (cm)	κ_1	1.6716	0.1110	0.6221
	κ_2	−10.6887	0.6792	
	κ_3	5.4953	0.7982	
Stand survival (trees ha^{-1})	β_1	2.0224	0.1740	0.8719
	β_2	21.0962	0.6869	
	β_3	0.6388	0.0230	
Diameter growth (cm)	λ_1	15.5015	0.8853	0.9130
	λ_2	−16.6289	0.9134	
	λ_3	−0.0351	0.0030	
	λ_4	0.1522	0.0165	
	λ_5	−1.4820	0.1437	
Tree survival	μ_1	−10.2973	1.2509	957.5840 (−2 ln L)*
	μ_2	0.0582	0.0089	
	μ_3	−0.3783	0.0558	
	μ_4	0.9450	0.1604	

* Log likelihood: $-2 \ln L = -2 \{ \sum P_i \ln(P_i) + \sum(1 - P_i) \ln(1 - P_i) \}$,

combined estimator from the fit data and yielded the following weight values: $w_1 = 0.4369$, $w_2 = 0.1784$, and $w_3 = 0.3848$. These weight values were then used in Equation 7 to compute the combined basal area estimators for both fit and validation data sets.

Table 5 shows evaluation statistics from the combined estimator and the three different types of models. For the validation data, the mean difference for the combined estimator was −0.0478, compared with 0.0811, −0.2885, and −0.0827 for the tree-level, stand-level, and diameter-distribution models, respectively, the mean absolute difference was 1.0698 versus 1.0312, 1.1213, and 1.1539, and the R^2 value was 0.9507 compared with 0.9486, 0.9494, and 0.9244. Figure 1 displays the relationship between residual errors (observed − predicted basal area) and observed stand basal area for different models for the validation data.

These results showed that the forecast combination method was slightly better than the stand-level and diameter-distribution models in predicting stand basal area, based on both fit and validation data. The combined estimator was the most accurate (mean difference) compared with the other single models but only second best in precision (mean absolute difference) next to the tree-level model. Finally, the combined estimator ranked first in R^2, a common statistic for measuring goodness-of-fit in regression models.

The optimal weights of the combined estimator were determined in the study by use of the method employed by other researchers (Tang 1994, Li et al. 2008). The objective of this method is to minimize the sum of squares of errors. That explains why the combined estimator exhibited the highest R^2 values for the fit data, because R^2 is a function of the sum of squares of errors. Even though the weights were computed from the fit data, the combined estimator still produced the best R^2 value for the validation data compared with the three single models. We need to note that because mean square error is expressed as the sum of variance and the square of bias, a high R^2 value can be interpreted as corresponding to good overall performance of a model in terms of bias and variance.

Forest growth and yield models can be broadly classified

Table 5. Evaluation statistics from the forecast combination method and from models having three levels of resolution

	Tree-level model		Stand-level model		Diameter-distribution model		Forecast combine method	
Evaluation statistic	Fit data	Validation data	Fit data	Validation data	Fit data	Validation data	Fit data	Validation data
Mean difference	0.2661	0.0811	−0.1595	−0.2885	−0.2211	−0.0827	**0.0027**	**−0.0478**
Mean absolute difference	**1.1689**	**1.0312**	1.1769	1.1213	1.2693	1.1539	1.1787	1.0698
R^2	0.9255	0.9486	0.9282	0.9494	0.9244	0.9466	**0.9298**	**0.9507**

Value in bold denotes the best statistic among four models for each of the fit and validation data sets.
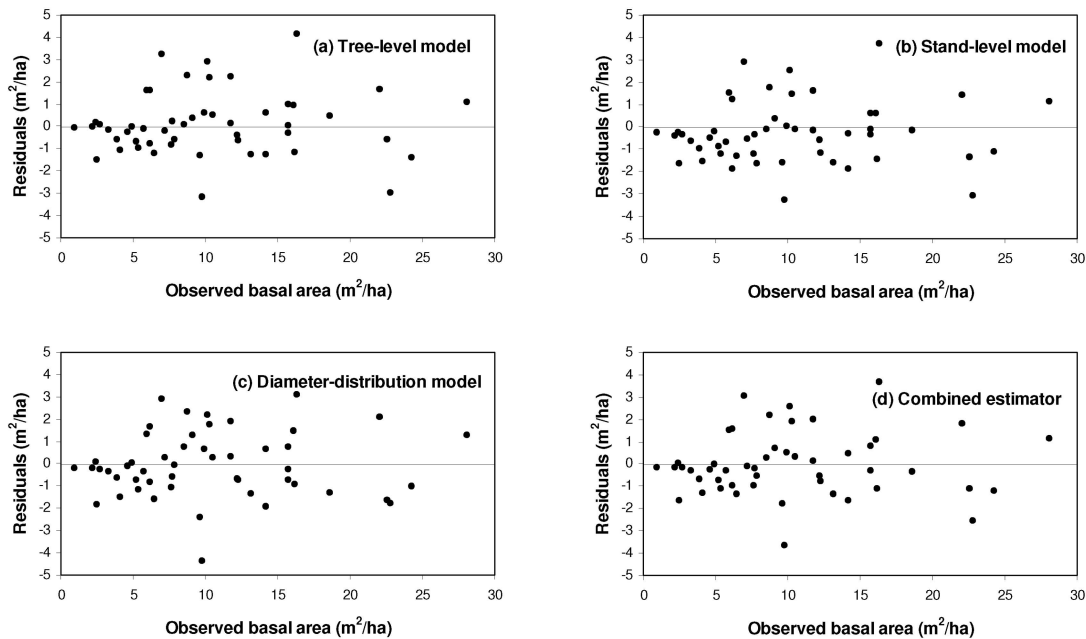
Figure 1. Relationship between residuals (observed-predicted basal area) and observed stand basal areas for different models for the validation data.

into three types: whole-stand, individual-tree, and diameter-distribution models (Munro 1974). Each type has various degrees of success in predicting stand attributes such as basal area per unit area. It makes sense therefore to combine these separate predictions into a single weighted average. The method can be considered an extension of the work by Yue et al. (2008), who combined only tree-level and stand-level models. Another difference was that the work of Yue et al. (2008) minimized the variance of the combined estimator, whereas the sum of squares of error in terms of stand basal area was minimized in this study. The combined estimator provided a consistent prediction of stand basal area at various levels of resolution, thus improving compatibility among these models.

Conclusions

In this study, forecast combination was used to predict stand basal area by combining three different estimators from models having different levels of resolutions. The forecast combination method efficiently uses information generated from different models to improve predictions by reducing errors from a single model. By dealing with inconsistent stand basal area estimates from models with different levels of resolutions, the method provides a feasible integrated system of stand basal area growth models.

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