

## biometrics

# Modeling Dominant Height Growth of Cedar (*Cedrus libani* A. Rich) Stands in Turkey

Ramazan Özçelik, Quang V. Cao, Esteban Gómez-García, Felipe Crecente-Campo, and Ünal Eler

Sustainable forest management requires accurate prediction from a growth and yield system. Such a system relies heavily on some measure of site productivity, which is often the site index. A model was developed for predicting dominant height growth and site index of even-aged cedar (*Cedrus libani* A. Rich.) stands in Turkey. Stem-analysis data from 148 trees were used for model development and validation. Six dynamic height–age equations were derived using the generalized algebraic difference approach (GADA). Autocorrelation was modeled by expanding the error term as an autoregressive process. Based on numerical and graphical analysis, a GADA formulation derived from the Chapman–Richards model was selected. Based on relative error in dominant height prediction, 80 years was selected as the best reference age. The resulting equation provided the best compromise between biological and statistical aspects and, therefore, is recommended for height growth prediction and site classification of cedar stands in Turkey.

**Keywords:** site index model, height–age model, algebraic difference equations

Lebanon cedar (*Cedrus libani* A. Rich.) is among the most economically important conifer species in Turkey, because of its physical properties (such as resistance to weathering and decay) and aesthetic properties (such as fragrance and color) (Bozkurt et al. 1990). Furthermore, Lebanon cedar (hereafter “cedar”) forests are vital for maintaining biological diversity and preserving soil and water resources in the Taurus Mountains of Turkey, where they are primarily found. The Taurus Mountains separate the Mediterranean coastal region of southern Turkey from the central Anatolian Plateau. Presently, cedar forests span an area of 482,391 hectares, and the total standing volume of cedar is approximately 27.4 million m<sup>3</sup> (GDF 2015).

Information on growth and yield of cedar forests is necessary for improving future management and planning strategies for timber resources. Site productivity is among the most important components in a forest growth and yield model. Unlike the criterion used in agriculture, site quality should not be assessed in terms of volume yield, which can be affected by factors such as past cutting, site, weather, stand density, and the length of rotation (Davis et al. 2001). Good site quality is generally associated with higher height growth rates; therefore, site quality is often assessed based

on stand height (Clutter et al. 1983). Since average stand heights can be affected by silvicultural practices such as thinning, heights of the dominant and codominant trees are commonly used for determining site productivity. Site index, used to measure productivity, is defined as the dominant height of a stand at a specified base age. A key assumption in the use of site index is that dominant height is not affected by management activities; therefore, site index can provide an assessment of the underlying productivity of the site, regardless of the management of timber on that site.

As indicated by Vargas-Larreta et al. (2013), determining potential productivity by use of site index models is a practical and convenient method that helps in implementing suitable forest-management practices (Tewari et al. 2014) and facilitating inventory updates and projections (López-Sánchez et al. 2015).

Recently in Turkey, site index models have been constructed for Scots pine (*Pinus sylvestris* L.) (Ercanlı et al. 2014), black pine (*Pinus nigra* Arnold.) (Seki and Sakıcı 2017), and Brutian pine (*Pinus brutia* Ten.) (Kahriman et al. 2018). As preservation of biodiversity and sustainable forest management have become important areas of research in recent years, management of cedar forests has received increased attention. Long-term strategies require growth and yield

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models, of which site index models form an essential component. Despite this thrust, the set of site index curves for cedar stands in Turkey (Evcimen 1963) has not been updated until recently. Aydin (2008) published a site index model for Antalya Province, Turkey. However, he neither evaluated various well-known height–age functions nor considered different fitting techniques such as algebraic difference approach (ADA) or the generalized algebraic difference approach (GADA).

Several methods have been developed for site index estimation because of its importance in forestry (Sánchez-González et al. 2010, Tahar et al. 2012). The models that predict dominant height growth must possess some desirable properties (Bailey and Clutter 1974, Goelz and Burk 1992, McDill and Amateis 1992, Cieszewski and Bailey 2000), the most important of which is path invariance, i.e., projecting dominant height first from time  $t_1$  to  $t_2$  and then from  $t_2$  to  $t_3$  should yield an identical result as projecting directly from  $t_1$  to  $t_3$ . Two approaches were used to formulate height–age models: the algebraic difference approach (ADA) (Bailey and Clutter 1974) and the GADA (Cieszewski and Bailey 2000). The ADA approach essentially makes a parameter site-specific by replacing it with an initial condition. The disadvantage of ADA models is that they either are anamorphic (same curve shape for different sites) or have a single asymptote. The GADA approach can, however, expand the base equation according to various theories about growth characteristics, allowing multiple parameters to be site-specific (Cieszewski 2002, Diéguez-Aranda et al. 2006). In addition, the GADA system produces curves that have desirable characteristics such as path invariance and polymorphism (different curve shapes for different sites) (Bailey and Clutter, 1974, Cieszewski and Bailey 2000). This is the reason why GADA has been widely used for modeling height–age relation in forestry applications (Vargas-Larreta et al. 2013, López-Sánchez et al. 2015).

Previous site index curves for natural cedar stands in Turkey (Evcimen 1963) were from anamorphic models that produce height curves of the same shape. Therefore, a new polymorphic site index model is needed to accurately predict dominant heights and stand productivity of cedar forests. In this study, a height–age system for natural even-aged cedar stands in Turkey was developed to predict height growth and site index using the GADA.

## Materials and Methods

### Data

Cedar generally occurs at elevations of between 800 and 2,100 m in the Taurus Mountains. However, cedar populations are highly fragmented, consisting of small groups and individuals, which have been observed at lower and higher elevations (Boydak 2003). Although the forests are distributed among varying geological foundations, they are primarily situated on calcareous material. As indicated by Boydak (2003), the largest and most important karst region in Turkey is the Taurus Mountains. Soil depths generally range from shallow to medium deep (Kantarci 1991). The mean annual temperature in the distribution areas varies between 6 and 12° C, whereas the mean annual precipitation ranges from 120 to 600 mm. Snow cover persists between 1 and 5 months. During the growing season, relative humidity varies from 40 to 60 percent. The climate of the studied areas is the typical Mediterranean climate.

A stem analysis of 148 felled dominant or codominant trees from 148 research sample plots was used in this study. These plots

were from natural, even-aged stands. More than 90 percent of the trees were cedar. The plots were located all over the distribution area of cedar in the Taurus Mountains of Turkey and represented well the range of sites, stand densities, and ages for cedar. For this purpose, a nonrandom selection method was used. The distribution area of Lebanon cedar was systematically divided into squares (1 km square grid). Two stands were selected in each square, one in a high-quality site and the other in a low-quality site. The same method was applied to stand density and age, whenever possible. A new plot was randomly located inside each of the two stands. Depending on stand density, plot size varied from 200 to 4,000 m<sup>2</sup> in order to contain at least 30 trees per plot. This minimum number of trees was required because plots are intended to develop stand dynamic growth models. Moreover, a simple random location scheme is not useful for this purpose, since it does not guarantee that plots in a large combination of different sites, stand densities, and ages are available for modeling purposes. Description statistics of the selected plots are provided in Table 1. Dominant trees were defined as the 100 thickest trees per hectare, with no deformation or broken tops. The number of dominant trees in a plot is, therefore, related to the plot size. One dominant tree was destructively sampled in each plot. This tree was selected with a diameter at breast height value close to the mean diameter of the dominant trees and with a measured height within 5 percent of mean height for dominant trees (Barrio-Anta and Diéguez-Aranda 2005). On felled trees, total bole length was measured to the nearest 0.01 m. The boles were then cut at 0.3 m and 1.3 m, and approximately 2 m intervals subsequently. The number of rings was visually counted in the field, taking into account the possibility of existence of false or double rings in this species. Ring numbers were then used to compute stump age at each measurement point. A total of 1,715 height–age pairs were used. Stem-analysis data were converted into height–age data using Carmean's (1972) method and the modification described by Newberry (1991), based on recommendations from previous studies (Dyer and Bailey 1987, Fabbio et al. 1994).

### Model Fitting and Selection

The twofold evaluation scheme was used in this study. This scheme involved (1) randomly dividing the data into two groups with equal number of trees (Table 2 and Figure 1), (2) estimating

### Management and Policy Implications

Lebanon cedar (*Cedrus libani* A. Rich.) is commercially one of the most important pine species in Turkey. Cedar forests have a key role in providing important indirect benefits and environmental services. Cedar is presently found primarily in the Taurus Mountains of Turkey in the world. Therefore, forest managers need detailed information supplied by growth and yield prediction models for sustainable forest management of cedar forests. Site index models provide a convenient and effective tool for determining potential productivity, thus allowing implementation of appropriate forest-management practices and facilitating inventory updates and projections. In this study, a system for both height growth prediction and site index estimation was developed for even-aged natural cedar stands in Turkey, using the generalized algebraic difference approach. The growth and yield system that uses the resulting site index model should produce reliable predictions that enable forest managers to make appropriate decisions in efforts to manage cedar forests.

parameters of the dominant height growth model separately for each group, (3) predicting dominant height in one group using parameters estimated from the other group, and (4) computing evaluation statistics based on height predictions from both groups. Bohora and Cao (2014) employed a similar evaluation scheme using five groups.

Based on results from prior studies (Corral-Rivas et al. 2004, Diéguez-Aranda et al. 2005, 2006, Vargas-Larreta et al. 2013, Seki and Sakıcı 2017, Kahriman et al. 2018), three well-known growth functions for modeling dominant height were evaluated in the analysis: the Chapman–Richards model (Richards 1959, Chapman 1961), the Bailey and Clutter (1974) model, and the log-logistic model, which is equivalent to the Hossfeld model (Cieszewski 2000). Table 3 shows six dynamic models based on these functions. The GADA method was applied by considering two parameters to be site-specific (models 1a, 2a, and 3a), or only one parameter to be site-specific (models 1b, 2b, and 3b). In the latter case, the GADA method was actually the same as the ADA method.

The SAS/ETS® MODEL procedure (SAS Institute 2008) was used to estimate parameters in the above models using the base-age invariant method (Bailey and Clutter 1974) that employed dummy variables (Cieszewski et al. 2000). In this method, the dummy variables ensure that the model is “individual-specific” by assigning the initial height for a particular tree to be site-specific.

To account for possible correlation among residuals from the same individual, we used a continuous-time autoregressive error structure applied for unequal period lengths. This error structure is valid for data that are unbalanced or irregularly spaced (Diéguez-Aranda et al. 2006).

The six models were evaluated by comparing error statistics associated with the height estimations from these models. The following global error statistics were computed for height estimations:

$$\text{Mean difference (MD)} = \frac{\sum_{i=1}^n (Y_i - \hat{Y}_i)}{n}, \quad (1)$$

**Table 1. Descriptive statistics for plots.**

| Variables              | Mean  | Min. | Max.  | SD     |
|------------------------|-------|------|-------|--------|
| Size (m <sup>2</sup> ) | 1,512 | 200  | 4,000 | 837    |
| n (stem/plot)          | 100   | 52   | 149   | 16.26  |
| N (stem/hectare)       | 1,003 | 192  | 5,600 | 868.43 |
| Age (years)            | 94    | 26   | 190   | 35.98  |

**Table 2. Summary statistics of the felled trees and of stem analysis by group.**

|                               | Variable                    | Mean | Minimum | Maximum | SD   |
|-------------------------------|-----------------------------|------|---------|---------|------|
| <i>Group 1</i>                |                             |      |         |         |      |
| Felled trees ( <i>n</i> = 74) | dbh (cm)                    | 35.0 | 7.9     | 60.6    | 10.2 |
|                               | Total height (m)            | 19.6 | 5.7     | 32.4    | 5.2  |
|                               | Age (years)                 | 93.3 | 26.0    | 190.0   | 37.3 |
|                               | No. of cross-sectional cuts | 11.7 | 5.0     | 18.0    | 2.5  |
| Stem analysis                 | Height (m)                  | 10.6 | 0.3     | 32.4    | 7.3  |
|                               | Age (years)                 | 50.0 | 1.0     | 190.0   | 37.2 |
| <i>Group 2</i>                |                             |      |         |         |      |
| Felled trees ( <i>n</i> = 74) | dbh (cm)                    | 32.7 | 9.0     | 51.4    | 8.9  |
|                               | Total height (m)            | 19.2 | 6.1     | 29.0    | 4.9  |
|                               | Age (years)                 | 91.6 | 29.0    | 174.0   | 33.4 |
|                               | No. of cross-sectional cuts | 11.5 | 5.0     | 16.0    | 2.5  |
| Stem analysis                 | Height (m)                  | 10.5 | 0.3     | 29.0    | 7.2  |
|                               | Age (years)                 | 49.0 | 1.0     | 174.0   | 35.0 |

Note: dbh, diameter at breast height.

$$\text{Mean absolute difference (MAD)} = \frac{\sum_{i=1}^n |Y_i - \hat{Y}_i|}{n}, \quad (2)$$

$$\text{Fit index (FI)} = 1 - \frac{\sum_{i=1}^n (Y_i - \hat{Y}_i)^2}{\sum_{i=1}^n (Y_i - \bar{Y})^2} \quad (3)$$

where *n* is the number of observations, *Y<sub>i</sub>* and *Ŷ<sub>i</sub>* are observed and predicted height values, respectively, and *Ȳ* is the mean value of *Y<sub>i</sub>*. FI is analogous to *R*<sup>2</sup> in linear regression.

### Selection of Reference Age

Site index at a base age can be obtained from any given height–age pair (Clutter et al. 1983). Conversely, site index can be used to predict dominant height at any age (Bailey and Clutter 1974). Therefore, the selection of an appropriate base age is important (Diéguez-Aranda et al. 2005).

The base age should be determined by considering the following: (1) the reference age should be less than or equal to the youngest rotation age for common silvicultural treatments; and (2) the base age should be selected to provide a reliable predictor of height at other ages (Goelz and Burk 1996). Diéguez-Aranda et al. (2005) suggested as young a base age as possible to facilitate early decisions on silvicultural treatments. In this study, base-age selection was based on evaluation of performance of different base ages, all lower than the common rotation age for the species in the region. Dominant heights at various ages were predicted from observed heights at these base ages for each tree. The relative error in predictions (RE%) was computed from the observed (*Y<sub>i</sub>*) and predicted (*Ŷ<sub>i</sub>*) heights as follows:

$$\text{RE \%} = 100 \frac{\sqrt{\sum_{i=1}^n (Y_i - \hat{Y}_i)^2 / (n - p)}}{\bar{Y}} \quad (4)$$

where *p* is the number of parameters in the model and all other variables as previously defined.

## Results and Discussion

Autocorrelation was detected in fitting the models (Figure 2). To account for this autocorrelation, we used CAR(2), which is a continuous-time autoregressive model of second order, to

describe the error structure. As a result, the trends in residuals disappeared (Figure 2, third row). Similar findings were noted by other researchers (Diéguez-Aranda et al. 2005, Lauer and Kush 2010, Vargas-Larreta et al. 2013). Accounting for autocorrelation prevents underestimation of the covariance matrix of the parameters, thereby making it possible to carry out the usual statistical tests (West et al. 1984). Spatial correlation between individuals in nearby locations could also be expected, but the methodology implemented treated each individual independently, by use of a dummy variable to fit an “individual-specific” model, thus helping to avoid any problems associated with spatial correlation if present. The serial correlation was taken into account by a continuous-time autoregressive model structure that also performed well in previous studies (e.g., Diéguez-Aranda et al. 2005, Nord-Larsen 2006, Lauer and Kush 2010).

Parameters for each model estimated from the entire dataset and their corresponding error statistics from the twofold evaluation scheme are listed in Table 4. The GADA models with two

site-specific parameters (1a, 2a, and 3a) described the data better than did the ADA models with only one site-specific parameter (1b, 2b, and 3b) (Table 4 and Figure 3). This is especially true at old ages on better sites. The results showed that, except for model 3b, approximately 97 percent of the variation in height predictions was explained by all models.

The predicted height curves overlaid by observed stem-analysis data (Figure 3) revealed that the models fit the shape of the general trajectories well, except for model 1b and model 2a. For ages younger than the base age (80 years) on the best site, models 2b and 3b produced a slight underestimation of dominant height. Figure 3 also shows that, for the best site, there was a rapid increase in height growth for models 1b and 2a at young ages, followed by a rapid decrease at older ages. Moreover, although curves from model 3a look similar to those from model 1a at younger ages, they are steeper at older ages, especially for the lowest site.

A box plot of residuals from models 1a, 2a, and 3a is presented for each age class (Figure 4). The residuals appeared to have homogeneous variance and displayed no obvious trends. In general, for age class between 50 and 130 years, all models showed similar behavior. For older age classes, model 1a provided good estimates with no bias. On the other hand, models 2a and 3a showed some problems estimating heights for older age classes as compared to younger and middle age classes. Similar trends have been found in other studies as well (Vargas-Larreta et al. 2013, López-Sánchez et al. 2015).

Among six models, model 1a produced the best values of evaluation statistics from the twofold evaluation (Table 4). The curves from model 1a also adequately described height trajectories from the stem-analysis data (Figure 3). This model is a result of the GADA formulation of the Chapman–Richards model, with two parameters being site-specific. The following model is therefore proposed for height growth prediction and site classification:

$$\hat{Y} = \exp(X_0) [1 - \exp(-0.01298 t)]^{(-9.2762 + 39.1543/X_0)}, \text{ with}$$

$$X_0 = 0.5 [\ln(Y_0) + 9.2762 L_0 + \sqrt{(\ln(Y_0) + 9.2762 L_0)^2 - 4 \times 39.1543 L_0}], \text{ and}$$

$$L_0 = \ln[1 - \exp(-0.01298 t_0)]$$

where  $Y$  is predicted dominant height (m) at age  $t$  (in years),  $Y_0$  is the observed dominant height at age  $t_0$ , and  $\ln(\cdot)$  is the natural logarithm.

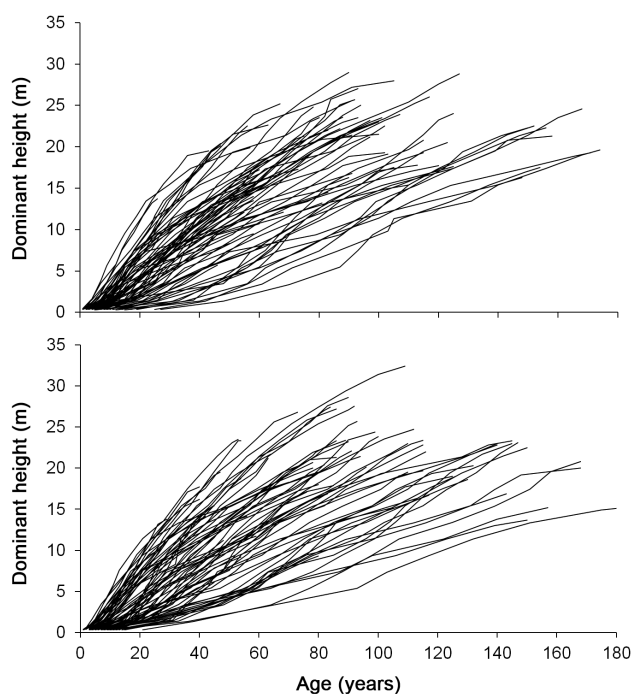


Figure 1. Stem-analysis data, by group.

Table 3. Base models and generalized algebraic difference approach formulations considered.

| Base model  | Site parameter                                      | Solution for $X$ with initial values ( $t_0$ , $Y_0$ )  | Dynamic equation   | Model    |
|---|---|---|--|----------|
| Chapman–Richards:<br>$Y = a_1 [1 - \exp(-a_2 t)]^{a_3}$ | $a_1 = \exp(X)$<br>$a_3 = b_1 + b_2/X$<br>$a_2 = X$ | $X_0 = \frac{1}{2} \left[ (\ln Y_0 - b_1 L_0) + \sqrt{(\ln Y_0 - b_1 L_0)^2 - 4 b_2 L_0} \right]$<br>$L_0 = \ln[1 - \exp(-b_3 t_0)]$<br>$X_0 = \frac{-\ln\left(1 - \left(\frac{Y_0}{a_1}\right)^{\frac{1}{a_3}}\right)}{t_0}$ | $Y = \exp(X_0) [1 - \exp(-b_3 t)]^{(b_1 + b_2/X_0)}$<br>$Y = b_1 [1 - \exp(-X_0 t)]^{b_3}$ | 1a<br>1b |
| Log-logistic:<br>$Y = \frac{a_1}{1 + a_2 t^{-a_3}}$     | $a_1 = b_1 + X$<br>$a_2 = b_2 X$<br>$a_2 = X$       | $X_0 = \frac{Y_0 - b_1}{1 - b_2 Y_0 t_0^{-b_3}}$<br>$X_0 = \left(\frac{b_1}{Y_0} - 1\right) t_0^{b_3}$  | $Y = \frac{b_1 + X_0}{1 + b_2 X_0 t^{-b_3}}$<br>$Y = \frac{b_1}{1 + X_0 t^{-b_3}}$         | 2a<br>2b |
| Bailey–Clutter:<br>$Y = \exp(a_1 + a_2 t^{a_3})$        | $a_1 = b_1 + X$<br>$a_2 = b_2 X$<br>$a_2 = X$       | $X_0 = \frac{\ln(Y_0) - b_1}{1 + b_2 t_0^{b_3}}$<br>$X_0 = [\ln(Y_0) - b_1] t_0^{-b_3}$   | $Y = \exp(b_1 + X_0 + b_2 X_0 t^{b_3})$<br>$Y = \exp(b_1 + X_0 t^{b_3})$                   | 3a<br>3b |



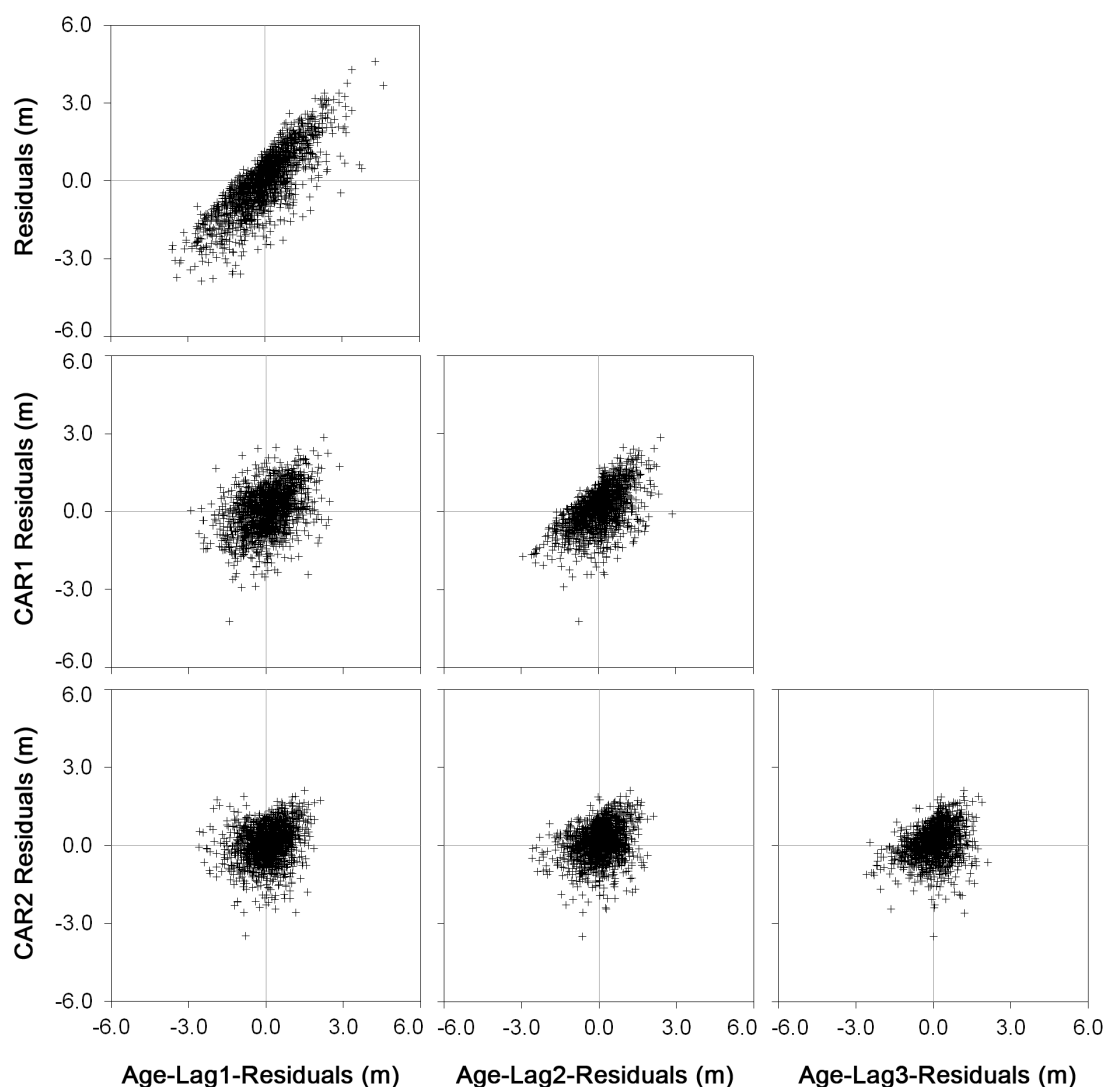
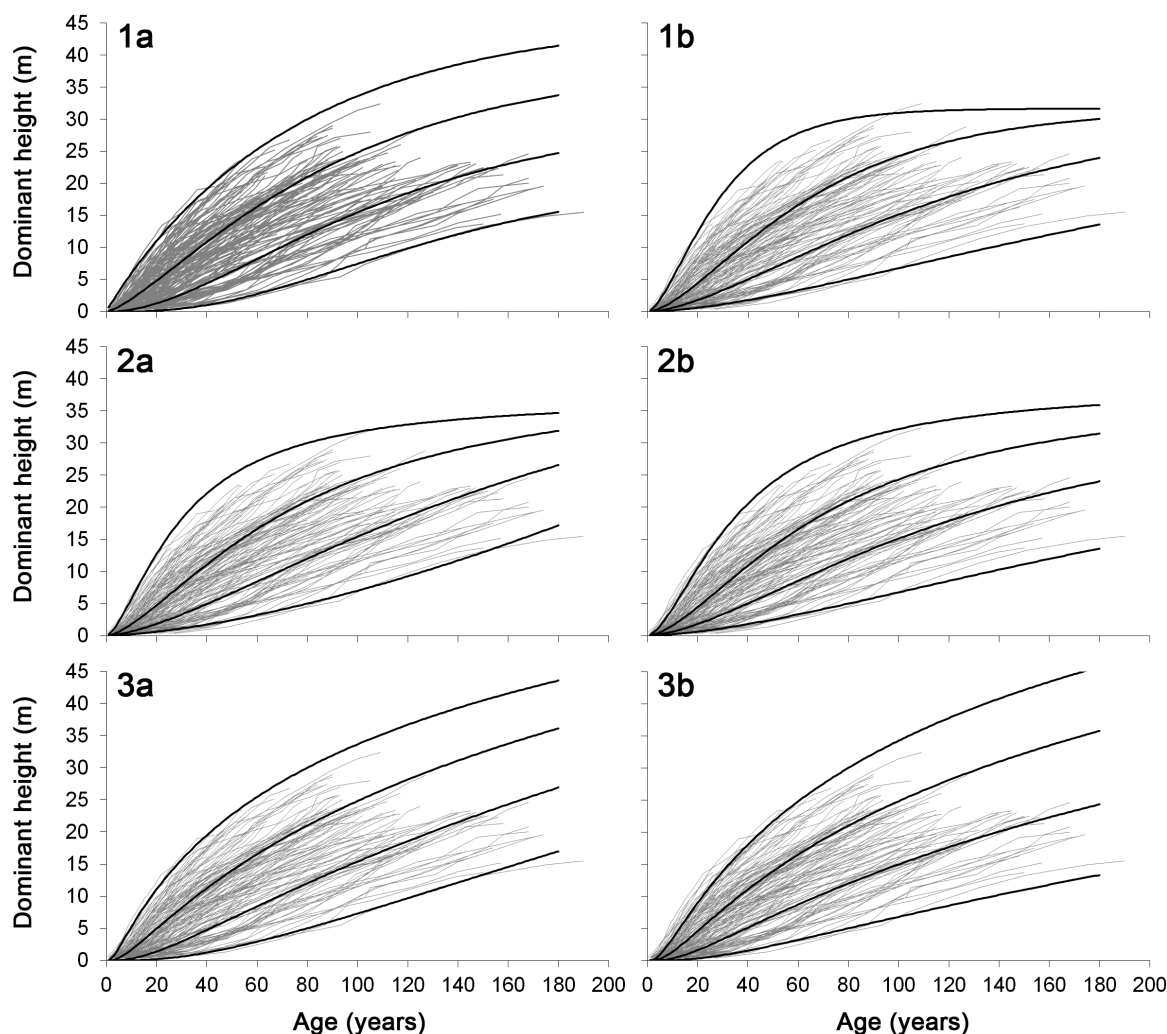


Figure 2. Residuals were plotted against the following: Age-Lag1-Residuals (left column), Age-Lag2-Residuals (middle column), and Age-Lag3-Residuals (right column) for model 1a, fitted without considering the autocorrelation parameters (first row), and using continuous-time autoregressive error structures of first and second order (second and third rows, respectively).

Table 4. Parameter estimated from the entire dataset and their corresponding goodness-of-fit statistics from the twofold evaluation scheme, by model.

| Model | Parameter | Estimate | Approx. standard error | Twofold evaluation |                          |               |
|-------|-----------|----------|------------------------|--------------------|--------------------------|---------------|
|       |           |          |                        | Mean difference    | Mean absolute difference | FI            |
| 1a    | $b_1$     | -9.276   | 1.0575                 | <b>0.0145</b>      | <b>0.6097</b>            | <b>0.9850</b> |
|       | $b_2$     | 39.15    | 3.7483                 |                    |                          |               |
|       | $b_3$     | 0.0129   | 0.0007                 |                    |                          |               |
| 1b    | $b_1$     | 31.63    | 0.6808                 | -0.0600            | 0.6916                   | 0.9749        |
|       | $b_2$     | 1.729    | 0.0443                 |                    |                          |               |
|       | $b_3$     | 35.88    | 0.8575                 |                    |                          |               |
| 2a    | $b_1$     | 143.4    | 29.5816                | -0.0177            | 0.6538                   | 0.9800        |
|       | $b_2$     | 1.507    | 0.0303                 |                    |                          |               |
|       | $b_3$     | 38.84    | 1.1368                 |                    |                          |               |
| 2b    | $b_1$     | 1.592    | 0.0302                 | -0.0588            | 0.6877                   | 0.9754        |
|       | $b_2$     | 4.189    | 0.0623                 |                    |                          |               |
|       | $b_3$     | -13.75   | 1.9640                 |                    |                          |               |
| 3a    | $b_1$     | -0.3988  | 0.0214                 | -0.1679            | 0.7043                   | 0.9712        |
|       | $b_2$     | 4.756    | 0.1073                 |                    |                          |               |
|       | $b_3$     | -0.4595  | 0.0224                 |                    |                          |               |
| 3b    | $b_1$     | 4.756    | 0.1073                 | -0.2598            | 0.8047                   | 0.9514        |
|       | $b_2$     | -0.4595  | 0.0224                 |                    |                          |               |
|       | $b_3$     | -0.4595  | 0.0224                 |                    |                          |               |

For each evaluation statistic, a bold number denotes the best model.



**Figure 3.** Cedar height–age curves for site indices of 5, 12, 21, and 30 m at a reference age of 80 years, overlaid by observed heights over time.

Figure 5 shows that, for model 1a, the lowest relative error value in the range of 20–150 years occurred at approximately 80 years. Generally, the rotation age of 120 years is used for stands that are managed for production in Turkey. As indicated by previous studies (Diéguez-Aranda et al. 2006, Rojo-Alboreca et al. 2017), the reference age should be selected to be as young as possible, in order to help in earlier decisionmaking of the silvicultural treatments to be applied to the stands. Therefore, 80 years seems to be an appropriate base age to reference site index. However, in this dataset, after 80 years the number of observations decreases. As indicated by Diéguez-Aranda et al. (2005), low representation in data from an insufficient number of observations can lead to inaccurate decisions. Model 1a can be used to either predict dominant height growth or determine site index.

Model 1a was then compared with an existing model (Aydin 2008), which is a polymorphic model based on the Chapman–Richards function, developed with data from a small area in the Taurus Mountains. Aydin’s model tends to underestimate growth for older ages by reaching the asymptote too soon (Figure 6). This model therefore does not appear to be adequate for site index estimation in the region.

A limitation of the data used in this study is the lack of observations at older ages in high-quality sites (Figure 1). Therefore, predictions for ages over 150 years in the lower-quality sites and over 100 years on the higher-quality sites should be taken with caution,

since there are fewer observations beyond these ages. Stem-analysis data are crucial for developing accurate height–age models because the GADA methodology (Cieszewski et al. 2000) takes advantage of the individual trends in model fitting. Convergence in model fitting was easily achieved, and no fitting problems were encountered. Many advantages in using stem-analysis data instead of remeasured sample plots are noted by Palahi et al. (2004), including accurate prediction of dominant height development at young ages.

Model selection is viewed as a compromise between biological and statistical considerations, rather than as a pure exercise in statistical inference. Polymorphic site index curves from this study, derived from the Chapman–Richards model using the GADA approach, yielded the best biological and statistical results. Satisfactory results for both considerations are not common because sometimes the best statistical models show some biological inconsistency, either inside or outside the range of data used in model fitting.

More recently, nonlinear mixed-effects models have received a great deal of attention in the forestry literature because they are flexible models that can handle longitudinal, repeated-measures, and multivariate multilevel data (Fang and Bailey 2001, Xu 2004, Wang et al. 2008). This modeling approach can be used as an alternative to develop site index models. Xu (2004) found similar height growth predictions from different model-fitting approaches (nonlinear

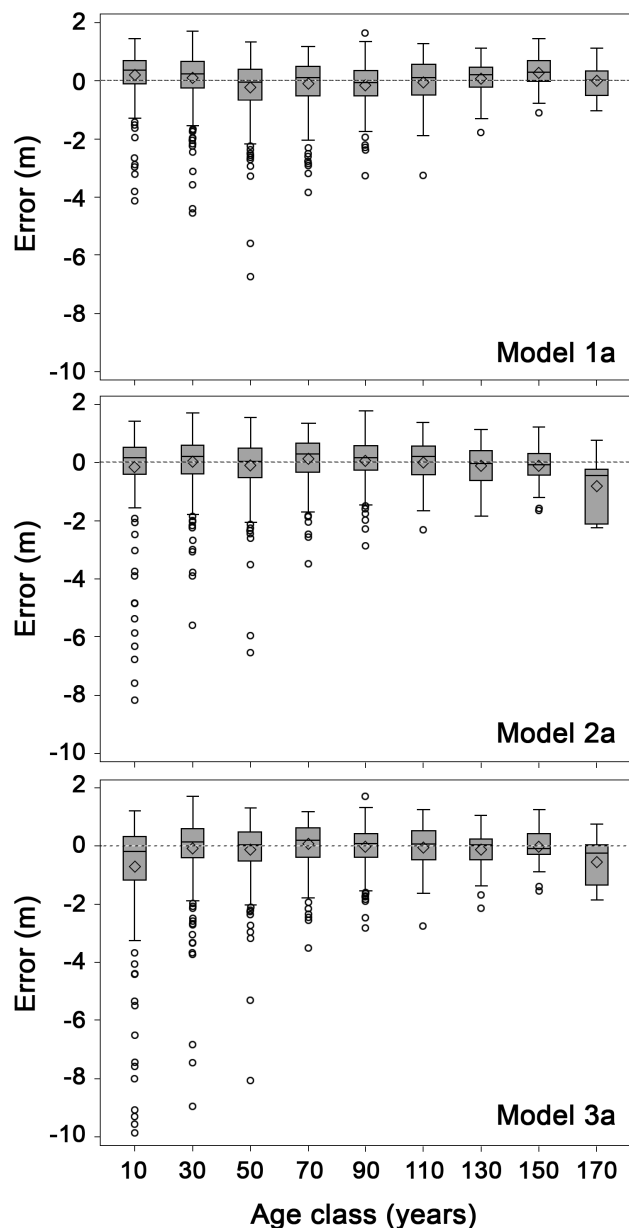


Figure 4. Box plot of residuals for models 1a, 2a, and 3a, by age class.

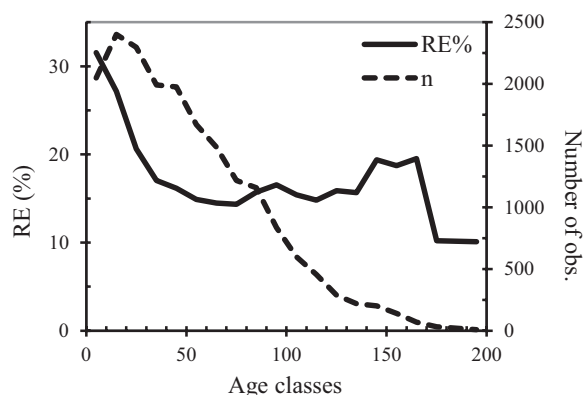


Figure 5. Relative error in height prediction related to choice of reference age for model 1a.

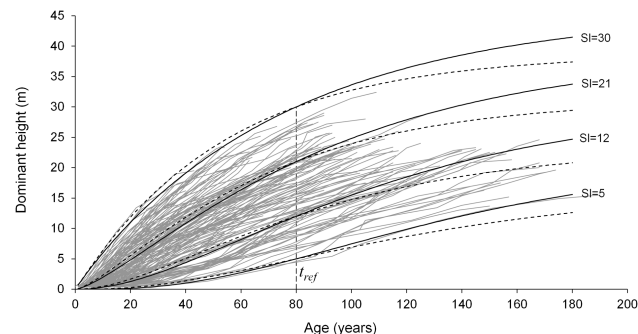


Figure 6. Comparison of the Chapman-Richards curves (solid line) for site indices of 5, 12, 21, and 30 m at a reference age of 80 years (model 1a), against the curves developed by Aydin (2008, dotted line) for cedar stands, overlaid on the profile plots of data set used.

least-squares, generalized nonlinear least-squares, and nonlinear mixed-effects modeling). Wang et al. (2008) suggested that there was no big difference in empirical site index prediction between the traditional method (ordinary least squares) and the mixed model. The one main advantage of the mixed model approach is the ability to “localize” its predictions when one or more height–age pairs are available.

There are two major applications of the site index model developed in this study. The first application is in forest growth and yield forecasting, where the site index model is used to predict the growth of dominant and codominant trees, which plays an important role in projecting merchantable timber yield to some point in the future. The second application is in forest stand productivity classification, the aim of which is to improve the accuracy and precision of forest inventory projections in order to ensure a reliable and sustainable forest management for future generations.

In applying the developed equation, two assumptions should be met: (a) dominant height is not altered by silvicultural treatments, and (b) plots are suitable (in terms of plot size, number of plots, and their locations) to define dominant height. As indicated by García and Batho (2005), a range in plot size can cause bias in estimating dominant height. We thus recommend using plots with at least 30 trees, following suggestions from previous studies (Diéguez-Aranda et al. 2006, Castedo-Dorado et al. 2007, Arias-Rodil et al. 2015). We recommend the use of large plots (at least 0.05–0.08 hectares) to avoid any bias in site index estimation, as it has been shown that top heights calculated by the usual method of the average height of the 100 largest trees per hectare vary somewhat with plot size (García 1998, Magnussen 1999, García and Batho 2005, Mason 2019). Moreover, we suggest using some kind of correction, such as the U-estimator (García 1998, García and Batho 2005), which is not sensitive to sample plot size, to correctly calculate dominant height at plot level. This approach has shown good results in previous studies (e.g., García 1998, García and Batho 2005, Ochal et al. 2017, Mason 2019).

## Conclusion

The height–age model developed in this study is recommended for prediction of dominant height growth and also for site classification of cedar stands in Turkey. This model can replace the model developed by Evcimen (1963) because it improves the prediction

capabilities, has biological meaning, is more flexible, and is path-invariant. The Chapman–Richards model resulted from this study was developed using the GADA and has desirable properties such as polymorphism and multiple asymptotes for different site index values. The results show that predictions for ages over 150 years in the lower-quality sites and over 100 years on the higher-quality sites should be taken with caution, since there are fewer observations beyond these ages. A base age of 80 years was chosen for site quality estimation because it gave the smallest relative error. This site index model, which can be employed to classify stands based on dominant heights, has been developed for natural cedar stands and can be implemented when planning future forestation activities in natural cedar stands of the Taurus Mountains in Turkey.

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