Inherent Correlations Between Stand **Biomass Variables Calculated from Tree** Measurements

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ABSTRACT. Correlating stand-level variables is an important component of forest production ecology; however, correlations among variables calculated with equations having common independent variables are potentially spurious. Monte Carlo simulation techniques were used to determine the inherent or null correlation coefficients among stand-level biomass variables calculated with published, individual-tree equations using loblolly pine (Pinus taeda L.) data. Null correlations of foliage mass/ha with branch mass/ha, stem mass/ha, and periodic annual increments of biomass were high with similar equation forms and exponents in the equations. Most, but not all, correlation coefficients of foliage mass/ha with other biomass components and periodic annual increments of biomass were significantly different from the corresponding, null correlation coefficients. Stating the probability of a greater difference between the observed and the null correlation coefficients proved crucial in distinguishing between potentially meaningful and spurious correlations because in many cases, the observed correlation coefficients were close to the null values. Interpretation of the correlations among stand variables varied with the equations used to predict the variables. Consequently, in addition to comparing correlation coefficients to appropriate null values, conclusions drawn from the correlation among stand-level variables depend on the accuracy and precision of the equations used to calculate them. For. Sci. 49(2):279-284.

Key Words: Monte Carlo simulation, simple correlation, spurious correlation, production ecology, prediction equations.

ALCULATING STAND-LEVEL BIOMASS variables from measurements of individual trees is a consequence of tree size and the desire to nondestructively determine stand growth. Typically, equations predicting biomass variables from assessable ground-based measurements are developed with destructively sampled trees. The quality of the resulting equations is a function of adequate statistical sampling (e.g., adequate replication across the measured range of size classes and of stand and tree conditions) and model development. The accuracy of the stand variables predicted with equations depends on the similarities between the trees in the stand and the trees used in developing the equations. For biomass components such as foliage that can

vary widely throughout the year, prediction accuracy also depends on when measurements are made during the year.

Well constructed, appropriately applied regression equations are good predictors of stand-level estimates of biomass components, but good correlations between biomass components may actually be self-correlations of the tree dimensions used to predict them. The numeric value of such a spurious correlation coefficient for calculated stand variables is the correlation coefficient resulting solely from the mathematical relationships between the pair of tree-level biomass equations used to calculate the variables. If equations for different biomass components are of the same form, a pair of stand-level variables may be perfectly correlated. As the

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functional form of the equations diverge, the inherent linear correlation between calculated stand-level variables should diminish and become much more difficult to predict, if not impossible.

Predetermined relationships among variables predicted from a common set of measurements is one of several potential sources of spurious correlations. Autocorrelation among data collected over time on an experimental unit is a classic source of spurious correlations (Granger and Newbold 1974), and Loehle (1988) describes a potentially insidious source of spurious correlations where covariance results from unrelated but coincidental temporal or spatial changes in one or both of the variables.

The possibility of inherent correlations among variables predicted from a common set of measurements has created considerable difficulties in evaluating even simple stand-level relationships. For example, Yoda et al. (1963) proposed a simple equation relating the average biomass per plant (B/N) to the number of plants per unit ground area (N) raised to the negative 3/2 power for even-aged monocultures experiencing mortality from overcrowding: B/N = $a \cdot N^{3/2}$, where a is a constant. Based on a wide range of empirical evidence, this relationship was eventually called the -3/2 law of self-thinning (White and Harper 1970, Westoby 1984). Controversy on whether the correlation between plant density and average biomass per plant is inherently spurious fractured confidence in the law, however (Weller 1987, Zeide 1987, Prairie and Bird 1989, Kenny 1991, Hamilton et al. 1995, Petraitis 1995).

The reliance on prediction equations makes correlation coefficients found in the study of forest production ecology potentially spurious. For example, Webb et al. (1983) compiled data collected mostly during the International Biological Program (1964–1974) and among the many relationships analyzed in their study, determined correlations between forest production and stand foliage. A common means for estimating stand-level variables during this program was the use of prediction equations based on stem diameter (e.g., Grier and Logan 1977, Whittaker et al. 1974). Consequently, the correlation coefficients calculated by Webb et al. (1983) for deciduous and coniferous forests could be logical consequences of predicting both forest production and stand foliage from stem diameter.

One means of quantifying whether a correlation coefficient is spurious is determining the probability that a given correlation coefficient r is not equal to r_n , where r_n is the null value of r expected solely due to chance. The null value of r for a pair of stand-level variables that are calculated from tree dimensions should be the inherent correlation for a specific combination of prediction equations and sampling conditions: it is likely not equal to zero because the prediction equations are usually based on common dimensions. With the distribution of a null value of r, the probability that a given value of r is greater or less than r_n can be determined, and the likelihood that a correlation coefficient is not a logical consequence of the prediction equations can be set to a threshold probability for $|r-r_n|$.

The principal objective of this study was to create the distributions of null correlation coefficients to make probability statements that observed correlation coefficients between foliage mass/ha and other stand-level biomass components are the result of the inherent mathematical relationships among the equations. A secondary objective of this study was to relate the inherent correlations among stand-level variables to similarities in the functional equation forms used in calculating these variables. The study was conducted with data collected from loblolly pine (Pinus taeda L.) plantations because several sets of prediction equations representing a range of functional forms have been published for this species.

Methods

Data

The data used in this analysis are described in detail by Dean and Baldwin (1996) and were collected from a variety of long-term, growth-and-yield studies on loblolly pine established in Louisiana and surrounding states. Data used in this analysis were collected in the unthinned plots and included diameter at breast height (1.37 m), total height, and height to the base of the live crown. Trees ranged in age from 11 to 45 yr, quadratic mean diameter ranged from 14.5 to 38.0 cm, and density ranged from 120 to 2,400 trees/ha. In total, 204 plots were used in this study. Seventy-four of these plots were measured at least twice at a 5 yr interval.

The oven-dry biomass values of foliage, the live branches, and stems of individual trees were computed from various combinations and transformations of diameter at breast height (D), total height (H_t) , and height to the base of the live crown (H_{blc}) (Table 1). The equations are grouped in sets of common equation forms (sets A-C) or by source (set **D**).

Individual-tree values of foliage mass, branch mass, and stem mass were summed for each combination of equation set, plot, and measurement period and expressed on a per hectare basis. Simple correlation coefficients were calculated for the various calculations of foliage mass/ha paired with the various per hectare calculations of branch mass, stem mass, and the net, periodic annual increments of foliage mass, branch mass, and stem mass. These correlation coefficients are the observed correlations to be tested against the null correlation coefficients of specific pairings of foliage mass/ha with branch mass/ ha, stem mass/ha, and periodic annual increments of foliage mass, branch mass, and stem mass.

Inherent Correlations

Monte Carlo simulation techniques were used to create the distribution of null correlations of foliage mass/ha with branch mass/ha, stem mass/ha and the net, periodic annual increments in biomass calculated with the various equation sets and the loblolly pine data. The fundamental assumption used in this study is that the principal source of scientifically relevant correlations is the morphological relationships among the various tree dimensions that reflect a tree's response to its environment. When these relationships are eliminated by

Table 1. Biomass equations used to calculate aboveground biomass components of individual trees. All equations were developed for planted loblolly pines (Pinus taeda L.), except for one equation from planted slash pine (Pinus elliottii var. elliottii Engelm.) data (Parresol and Thomas 1989).

Set	Component	Equation*	Source
A	Foliage	$M_{\rm f} = 0.01865 \ D^{2.020}$	Jokela and Martin (2000)
	Branch	$M_b = 0.00658 D^{2.714}$	Jokela and Martin (2000)
	Stem	$M_s = 0.02597 D^{2.684}$	Jokela and Martin (2000)
В	Foliage	$M_{\rm f} = -0.26 + 0.0146 D^2 H_{\rm f}$	Shelton et al. (1984)
	Branch	$M_b = 1.96 + 0.0005 D^2 H_t$	Shelton et al. (1984)
	Stem	$M_s = 1.25 + 0.00326 D^2 H_t$	Shelton et al. (1984)
C	Foliage	$M_{\rm f} = 0.13497 \ D^{2.912819} \ H_t^{-1.474651}$	Baldwin (1987) [†]
	Branch	$M_b = 0.00726 D^{3.454388} H_t^{-1.088445}$	Baldwin (1987) [†]
	Stem	$M_s = 0.00558416 D^{2.07788} H_t^{1.21556}$	Parresol and Thomas (1989)
D	Foliage	$M_{\rm f} = 0.14791 (D^2 H_{\rm f})^{-0.11} (H_{\rm f} - H_{\rm blc})^{1.16} Age^{0.55}$	Albaugh et al. (1998)††
	Branch	$M_b = 0.01629 (D^2 H_t)^{-0.123} (H_t - H_{blc})^{1.90} Age^{0.607}$	Albaugh et al. (1998) ^{††}
	Stem	$M_s = 0.73 + (0.019367 - 0.000452 Age) D^2 H_t$	Albaugh et al. (1998) ^{††}

Variables: D = diameter at breast height in cm; $H_t =$ total height in m; $H_{blc} =$ height in m to the base of the live crown; M_f , M_h , and M_s = foliage, branch, and stem dry weight in kg, respectively.

making the tree dimensions independent of each other, the correlation coefficient between a pair of stand-level variables calculated from these dimensions reflects the inherent, mathematical relationship between the equations. Repeated random recombinations of tree dimensions create the distribution of null correlation coefficients that is the basis for comparing an observed correlation coefficient against the null condition.

A simulation data set containing 100 replications of each plot was created. Each replication consisted of the same number of sample trees as in the original plot. D and H_t of each tree were randomly drawn from two-parameter Weibull functions parameterized with plot-specific means and standard deviations of the variables. The Weibull distribution is a single function that can describe the broad range of shapes in diameter and height distributions (Bailey and Dell 1973, Hafley and Schreuder 1977). The two-parameter form of the Weibull distribution was used to produce positive random values. The live-crown ratio, $C_r = (H_t - H_{blc})/H_t$, was randomly drawn from a uniform distribution with plot-specific minimum and maximum values. The value of H_{blc} was then calculated as $H_t(1-C_r)$.

A single correlation coefficient (r_n) was determined for each replicate for the various estimates of foliage mass/ha paired with the various per hectare estimates of branch mass, stem mass, and net, periodic annual increments of foliage mass, branch mass, and stem mass. The inherent correlation coefficient for each pairwise combination was defined as the null value of correlation coefficient, which was estimated by \bar{r}_n , the mean value of r_n calculated from the 100 replications for that combination. The probability of a greater difference between the null value \bar{r}_n and the observed correlation coefficient r for specific variable pairs was determined with the Student's t test with t calculated as $(r - \bar{r}_n)/s$ with 99 degrees of freedom, where s is the standard deviation based on 100 estimates of r_n . The null hypothesis that $r = \bar{r}_n$ was rejected if the probability of a greater absolute value of t was less than 0.05.

Results and Discussion

Inherent Correlation Coefficients

As expected, the inherent correlation coefficients for the various biomass components are generally highest when the functional form of the prediction equations and the fitted coefficients in the equations are similar. In equation-set A. the prediction equations are all simple power functions of D, and the exponents for D vary from 2.020 to 2.714. The values of \overline{r}_n resulting from this equation set are greater than 0.87 (Table 2). In equation-set **D**, the prediction equations for foliage mass and branch mass have the same functional form, and the inherent correlation between stand-level variables calculated with these two equations is correspondingly high: $\overline{r}_n = 0.937$. The prediction equations for foliage mass and stem mass in equation-set D are much different, and the inherent correlation between foliage mass/ha and stem mass/ ha calculated with these equations is much lower: $\bar{r}_n = 0.373$.

Regardless of the equation set used to calculate the various biomass components, the null values for correlations between foliage mass/ha and either branch or stem mass/ha in these loblolly pine plots are relatively high and are all positive. While the minimum value of \bar{r}_n for these variable pairs is 0.220, the average is 0.648. These results suggest that any experimental value of r that is significantly greater than 0 must be regarded as potentially spurious until tested against the appropriate null value of r. Although not considered explicitly in this analysis, logarithmically transforming data will increase the null value of the correlation coefficient because of its tendency to linearize relationships.

The null value of correlation coefficients between foliage mass/ha and periodic annual increment of foliage and stem mass of loblolly pine are inherently negative (Table 2). The sign of the null correlation coefficients between foliage mass/ ha and periodic annual increment in branch mass varies with the equation used to calculate branch mass. When branch mass is based on the equations in sets A, B, or C, the sign is negative. When branch mass is based on the equation in set **D**, the sign is positive.

Converted from original coefficients in English units.

^{††} Equations for the fertilized and irrigation treatments were used here because the site had such low natural fertility that the equations developed from trees in the unfertilized and unirrigated plots did not produce biomass estimates comparable to those calculated from other equation sets.

Table 2. Summary statistics for inherent correlation coefficients between variable X_1 (foliage mass/ha, M_i), and variable X_2 (branch mass/ha, M_b , stem mass/ha, M_s , and net, periodic annual increments in foliage mass, $\Delta M_{\rm fr}$ branch mass, $\Delta M_{\rm br}$ or stem mass, $\Delta M_{\rm s}$). Biomass and periodic annual increments calculated using the various sets of equations as grouped in Table 1 with loblolly pine data where individual tree values of diameter at breast height, total height, and live-crown ratio are randomly reassigned and

independent of one another.	(n = 100 repetitions)	of correlations.)
$X_1 = M_f$ from set A	$X_1 = M_f$ from set B	$X_1 = M_f$ from set C

		$X_1 = M_f$ from	om set A	$X_1 = M_f$ from set B		$X_1 = M_f$ from	om set C	$X_1 = M_f$ from set D		
Set	X_2	Mean	SD*	Mean	SD	Mean	SD	Mean	SD	
A	M_b	0.873	0.005	0.766	0.009	0.650	0.012	0.429	0.020	
	M_s	0.883	0.005	0.777	0.008	0.660	0.012	0.442	0.020	
	$\Delta M_{ m f}$	-0.436	0.038	-0.432	0.043	-0.281	0.054	-0.342	0.057	
	ΔM_b	-0.405	0.041	-0.387	0.046	-0.295	0.055	-0.268	0.058	
	ΔM_s	-0.406	0.041	-0.389	0.046	-0.294	0.055	-0.270	0.058	
В	M_b	0.880	0.003	0.854	0.003	0.525	0.016	0.616	0.012	
	M_s	0.844	0.003	0.806	0.004	0.480	0.016	0.561	0.012	
	$\Delta M_{ m f}$	-0.408	0.031	-0.455	0.030	-0.259	0.049	-0.451	0.041	
	ΔM_b	-0.324	0.038	-0.322	0.040	-0.199	0.055	-0.238	0.049	
	ΔM_s	-0.310	0.039	-0.301	0.041	-0.190	0.056	-0.207	0.050	
\mathbf{C}	M_b	0.784	0.013	0.607	0.018	0.789	0.010	0.220	0.029	
	M_s	0.768	0.005	0.732	0.004	0.383	0.018	0.491	0.013	
	$\Delta M_{ m f}$	-0.392	0.049	-0.370	0.056	-0.395	0.062	-0.288	0.069	
	ΔM_b	-0.410	0.046	-0.383	0.051	-0.359	0.058	-0.264	0.062	
	ΔM_s	-0.274	0.039	-0.263	0.041	-0.182	0.056	-0.173	0.048	
D	M_b	0.609	0.013	0.763	0.008	0.227	0.023	0.937	0.001	
	M_s	0.694	0.012	0.618	0.011	0.700	0.017	0.373	0.010	
	$\Delta M_{ m f}$	-0.041	0.029	-0.102	0.024	0.087	0.047	-0.268	0.020	
	ΔM_b	0.155	0.029	0.159	0.026	0.113	0.044	0.074	0.025	
	ΔM_s	-0.604	0.021	-0.619	0.021	-0.240	0.043	-0.514	0.028	

^{*} Standard deviation.

Observed Correlation Coefficients

Most of the observed correlation coefficients of foliage mass/ha with branch mass/ha, stem mass/ha, and the periodic biomass increments calculated with the various equation sets are significantly different from 0 at the 5% level (Table 3). According to this standard statistical analysis, the majority of these correlation coefficients would be judged as potentially meaningful. However, many values of r that are significantly different from 0 are not significantly different than the corresponding value of \bar{r}_n . For the biomass and biomass increments calculated with the simple equations in set A, a Student's t test indicates that the probabilities of greater differences between the observed values of r and their corresponding null values are all

Table 3. Summary statistics for observed correlation coefficients (r) between variable X_1 (foliage mass/ha, M_1), and variable X_2 (branch mass/ha, M_b , stem mass/ha, M_s , and net, periodic annual increments in foliage mass, ΔM_t , branch mass, ΔM_b , or stem mass, ΔM_s). Biomass and periodic annual increments calculated using the various sets of equations as grouped in Table 1 with loblolly pine data

		$X_1 = M_f$ from set A			$X_1 = M_f$ from set B			$X_1 = M_f$ from set C			$X_1 = M_f$ from set D		
Set	X_2	r	$P > r ^*$	$P > t ^{\dagger}$	r	P > r	P > t	r	P > r	P > t	r	P > r	P > t
A	M_b	0.880	< 0.001	0.165	0.796	< 0.001	0.001	0.668	< 0.001	0.137	0.409	< 0.001	0.320
	M_s	0.889	< 0.001	0.233	0.807	< 0.001	< 0.001	0.677	< 0.001	0.160	0.425	< 0.001	0.397
	$\Delta M_{ m f}$	-0.441	< 0.001	0.896	-0.491	< 0.001	0.173	-0.114	0.244	0.003	-0.372	< 0.001	0.600
	ΔM_b	-0.444	< 0.001	0.344	-0.483	< 0.001	0.039	-0.127	0.195	0.003	-0.335	0.001	0.251
	ΔM_s	-0.443	< 0.001	0.369	-0.483	< 0.001	0.043	-0.126	0.199	0.003	-0.335	0.001	0.265
В	M_b	0.915	< 0.001	< 0.001	0.897	< 0.001	< 0.001	0.574	< 0.001	0.003	0.541	< 0.001	< 0.001
	M_s	0.890	< 0.001	< 0.001	0.864	< 0.001	< 0.001	0.549	< 0.001	< 0.001	0.486	< 0.001	< 0.001
	$\Delta M_{ m f}$	0.057	0.560	< 0.001	-0.121	0.217	< 0.001	0.431	< 0.001	< 0.001	-0.139	0.155	< 0.001
	ΔM_b	-0.515	< 0.001	< 0.001	-0.637	< 0.001	< 0.001	-0.048	0.625	0.007	-0.520	< 0.001	< 0.001
	ΔM_s	-0.643	< 0.001	< 0.001	-0.721	< 0.001	< 0.001	-0.161	0.100	0.606	-0.474	< 0.001	< 0.001
C	M_b	0.835	< 0.001	0.000	0.596	< 0.001	0.543	0.766	< 0.001	0.024	0.240	0.013	0.489
	M_s	0.755	< 0.001	0.011	0.809	< 0.001	< 0.001	0.480	< 0.001	< 0.001	0.396	< 0.001	< 0.001
	$\Delta M_{ m f}$	0.172	0.078	< 0.001	0.297	0.002	< 0.001	-0.245	0.011	0.017	0.117	0.231	< 0.001
	ΔM_b	0.645	< 0.001	< 0.001	0.581	< 0.001	< 0.001	0.519	< 0.001	0.001	0.275	0.004	< 0.001
	ΔM_s	-0.545	< 0.001	< 0.001	-0.622	< 0.001	< 0.001	-0.046	0.639	0.017	-0.323	0.001	0.002
D	M_b	0.768	< 0.001	< 0.001	0.797	< 0.001	< 0.001	0.605	< 0.001	< 0.001	0.950	< 0.001	< 0.001
	M_s	0.682	< 0.001	0.320	0.655	< 0.001	0.001	0.639	< 0.001	0.001	0.703	< 0.001	< 0.001
	$\Delta M_{ m f}$	0.691	< 0.001	< 0.001	0.562	< 0.001	< 0.001	0.726	< 0.001	< 0.001	0.252	0.009	< 0.001
	ΔM_b	0.728	< 0.001	< 0.001	0.609	< 0.001	< 0.001	0.665	< 0.001	< 0.001	0.235	0.016	< 0.001
	ΔM_s	0.184	0.057	< 0.001	0.097	0.321	< 0.001	0.151	0.122	< 0.001	-0.273	0.005	< 0.001

[†] H_0 : $r = \bar{r}_n$, where \bar{r}_n is mean value of the inherent correlation coefficient for the corresponding variable pair in Table 2.

greater than 0.165, substantially greater than the critical maximum set for this study of 0.05. Consequently, when stand-level biomass estimates are calculated with equation-set A, correlations among these loblolly pine plots cannot be distinguished from correlations due solely to chance. In contrast to the results using equation-set A, when biomass values are calculated with the more complex equations in set **D**, the correlations of foliage mass/ha with the other biomass components and periodic annual increments may be meaningful because the probability of greater differences between the observed correlation coefficients and their corresponding null values are all less than 0.05 (Table 3).

The importance of being able to distinguish between potentially meaningful correlation coefficients and coefficients simply due to chance is illustrated with the correlation between foliage mass/ha and branch mass per acre, both calculated with equation-set D. The null correlation coefficient for this pair of variables averages 0.937, indicating that strong correlations between these two variables are likely spurious. However, quantifying the variation of the null correlation coefficients between these variables with Monte Carlo simulation enables the observed correlation between foliage mass/ha and branch mass/ha, which is only 1.4% greater than the inherent value, to be reliably distinguished from the value representing a spurious correlation.

Whether a specific correlation coefficient is statistically different from its inherent correlation coefficient depends on the equations used to calculate the stand variables. This creates the question of how to judge the various conclusions that may be reached with the different equation sets. Legitimate conclusions concerning the covariance of calculated stand-level variables can only be obtained with equations that accurately and precisely predict individual-tree biomass. The foliage mass and branch mass equations in equation-set C were developed for use with the loblolly pine data set used in this analysis. Therefore, the most legitimate conclusions concerning the correlation between the calculated stand-level variables included in this study are the positive relationships of foliage mass/ha with branch mass/ha and periodic annual increment in branch mass and the weak negative relationship between foliage mass/ha and periodic annual increment in foliage mass/ha. Equation sets A, B, and D were developed for loblolly pine trees that were between 6 and 20 yr old, much younger than the oldest trees included in this data set. Based on the results obtained with equation set A, foliage mass/ha is not significantly related to the branch mass/ha, and based on the results with equation set **D**, foliage mass/ha is negatively related to periodic annual increment in branch mass. Both of these conclusions conflict with the results obtained with the foliage and branch mass equations in equation set C.

Conclusions

Based on the widespread use of the functional forms in equations set A, B, and C, the correlations of foliage mass/ ha with either branch mass/ha or stem mass/ha are inherently positive and moderately strong, averaging 0.648. The inherent correlations of foliage mass/ha with periodic annual increments of the various aboveground biomass components based on equation sets A, B, and C are mostly negative for these loblolly pine data and are generally weaker than the correlations of foliage mass with branch and stem mass. Low inherent correlations between calculated stand-level variables occurred most often when the variables were calculated with equations that differed in functional form and exponents within the equations.

Most, but not all, of the correlation coefficients of foliage mass/ha with other biomass components and periodic annual increments of the biomass components were significantly different from their associated null values. Correlation coefficients that were not significantly different from associated null correlation coefficients were generally related to the simple equations in set A. The ability of making a probability statement concerning the difference between the observed and null correlation coefficients was crucial in determining the potential biological significance of the observed correlations in these loblolly pine plots because in many cases, the observed correlation coefficients were close to the inherent values. Interpretation of the correlations among stand variables varied with the equations used to predict the variables. Consequently, in addition to comparing correlation coefficients to an appropriate null value, this study indicates that valid conclusions also require that the most appropriate equation set be used in predicting stand-level variables.

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