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Evaluation of Two Methods for Cubic-Volume Prediction of Loblolly Pine to Any Merchantable Limit

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ABSTRACT. This study compares two methods of estimating merchantable volumes to specified top diameter or height limits. The prediction models evaluated were (1) volume ratio models that give the ratios of merchantable to total volume, and (2) taper equations that when integrated provide volume estimates of any segment of the bole. Data from plantations and natural stands of loblolly pine were used to compare the models for ability to predict merchantable volumes. Additional evaluations were made to compare taper equations. Results showed that the choice of the model depends on the objective. Two sets of volume ratio models are recommended for predicting merchantable volumes, whereas a new taper equation is found to estimate consistently well both volumes and diameters. FOREST SCI. 26:71–80.

ADDITIONAL KEY WORDS. *Pinus taeda*, taper equation, mensuration.

TOTAL TREE VOLUMES are usually estimated using volume equations. These equations customarily predict tree volumes from diameter at breast height (dbh) and either total or merchantable height. However, foresters are often more interested in estimating merchantable volumes, that is, the content of tree boles from stump height to some fixed top diameter or height limit. When equations for different merchantable volumes are fitted independently, they often have the undesirable characteristic of producing volume surfaces that cross illogically within the range of the data. Consequently, inconsistent estimates are produced for different merchantable volumes of a single stem. Recent studies have attempted to define the mathematical relationship between dbh, total height, top diameter or height limit, and merchantable and total volumes. Volume ratio models have been developed to predict the ratio of merchantable to total volume. Stem profile greatly affects merchantable volumes, and studies on tree taper have produced many taper equations that, when integrated, can be used to predict merchantable volume to any point on the tree bole.

This study compares several previously developed models and some new models. The objective was to determine the models that predict most accurately and precisely merchantable volume to any desired top diameter limit and to any specified height or proportion of total height.

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Notation

The following notation will be used.

a_i, b_i, b_i' = regression coefficients estimated from sample data,

D = diameter at breast height in centimeters,

d = top diameter outside bark (ob) or inside bark (ib) in centimeters at height h,

H = total height in meters,

h = height above the ground to top diameter d, or height to limit of utilization, in meters,

h_s = stump height in meters,

$K = \pi/[4(100)^2] = \pi/40{,}000$, a constant which when multiplied by D^2 equals basal area in square meters,

V = total cubic-meter volume above the ground (ib or ob),

V' = total cubic-meter volume above the stump (ib or ob),

v = cubic-meter volume from the ground to some top diameter or height limit,

v' = merchantable cubic-meter volume (above the stump) to some top diameter or height limit,

$z = (H - h)/H$, relative tree height from the tip to top diameter d, or the proportion of tree height that is unmerchantable.

Models

All of the models compared should provide logically related estimates of merchantable volumes to various top height or diameter limits; that is, merchantable volume curves do not cross total volume curves within the range of the data. In these models, independent variables measured on standing trees were restricted to dbh and total height; limits of utilization included stump height and either top diameter or height to the top bolt.

In this study, two approaches for predicting merchantable volumes were considered. The first, integrating taper equations, is a logical way to estimate merchantable volumes of bole segments. The integration limits are either the utilization limits or the ends of the bole segment. In the second method, volume ratio models are used to provide estimates of the ratios of merchantable to total volume. Merchantable volumes are obtained as products of these ratios and total volume, where total volume is estimated from a total volume equation. After evaluating a number of models for predicting total volume of loblolly pine, Burkhart (1977) concluded that the simple combined variable equation ($V = b_1' + b_2'D^2H$) performed as well as, if not better than, any of the alternatives compared. Thus the combined variable equation was used to estimate total stem volumes for this evaluation. Two types of merchantable volume prediction situations were considered: (1) prediction of merchantable volume to any point on the tree bole specified by either distance from the tip or height above the ground, and (2) prediction of merchantable volumes to predetermined top diameter limits.

Models from Past Work.—Eight models, each representing a different type of either volume ratio function or taper equation, were chosen from the literature for this study. Table 1 lists the models selected; the taper equations are divided into two categories—compatible and noncompatible. Compatible taper equations, as defined by Demaerschalk (1971, 1972, 1973), when integrated produce an identical estimate of total volume to that given by existing volume equations. Noncompatible taper equations are defined here as those that are not compatible. Several taper models, such as those by Demaerschalk and Kozak (1977), Bennett

TABLE 1. *Models from past work included in the analysis.*

Model	Reference	Equation[a]
		Volume Ratio Models
(1)	Honer (1967)	$(v/V - 1) = b_1(h/H - 1) + b_2(h^2/H^2 - 1)$.
(2)	Honer (1967)	$v'/V = b_1 + b_2(d^2/D^2)(1 + h_s/H) + b_3[(d^2/D^2)(1 + h_s/H)]^2$.
(3)	Burkhart (1977)	$v'/V' = 1 + b_1(d^{b_2}/D^{b_3})$.
		Noncompatible Taper Equations
(4)	Kozak and others (1969)	$d^2/D^2 = b_1(h/H - 1) + b_2(h^2/H^2 - 1)$.
(5)	Ormerod (1973)	$d = D_{1.37}\left[\dfrac{H - h}{H - 1.37}\right]^{b_1}$; where $D_{1.37} = D =$ dbh, if d is diameter ob, $= D_{ib} =$ dbh ib, if d is diameter ib, and $D_{ib} = b_3' + b_4'D$, b_3' and b_4' are regression coefficients.
(6)	Max and Burkhart (1976)	$d^2/D^2 = b_1(h/H - 1) + b_2(h^2/H^2 - 1) + b_3(a_1 - h/H)^2 I_1 + b_4(a_2 - h/H)^2 I_2$; where $I_i = 1,\ h/H \leq a_i;\ i = 1, 2,$ $= 0,\ h/H > a_i$.
		Compatible Taper Equations
(7)	Demaerschalk (1973)	$\dfrac{d^2}{D^2} = \dfrac{b_1'(b_1 + 1)}{KD^2H} z^{b_1} + \dfrac{b_2'(b_2 + 1)}{K} z^{b_2}$; where b_1' and b_2' are from $V = b_1' + b_2' D^2 H$.
(8)	Goulding and Murray (1976)	$(d^2KH/V - 2z) = b_1(3z^2 - 2z) + b_2(4z^3 - 2z) + b_3(5z^4 - 2z) + b_4(6z^5 - 2z)$.

[a] Symbols are defined in text.

and others (1978), Liu and Keister (1978), were published after this study was completed and therefore they were not included in this analysis.

New Models.—Model (3) (Table 1) can predict only merchantable volumes to various top diameters. A similar nonlinear model was developed to estimate v'/V' from h and H, instead of d and D (Table 2).

$$v'/V' = 1 + b_1(H - h)^{b_2}/H^{b_3}. \tag{9}$$

Munro (1968) achieved good results with a taper equation where (d^2/D^2) was predicted with a fifth-degree polynomial in (h/H). Also, Goulding and Murray's (1976) compatible taper equation was a fifth-degree polynomial in $(H-h)/H$. Integration of either model yields a sixth-degree polynomial volume function in either (h/H) or $(H-h)/H$. Based on these results the following model for $R = v/V$, the ratio of volume below a specified merchantable limit to total tree volume, was developed:

$$R = \sum_{i=0}^{6} (b_i z^i).$$

TABLE 2. *List of new models evaluated.*

Model	Reference	Equation[a]
		Volume Ratio Models
(9)	Modified Burkhart	$v'/V' = 1 + b_1(H - h)^{b_2}/H^{b_3}$.
(10)	Polynomial model	$(v/V + z - 1) = b_2(z^2 - z) + b_3(z^3 - z) + b_4(z^4 - z) + b_5(z^5 - z) + b_6(z^6 - z)$.
(11)	Polynomial model	$(v/V - 1) = b_1(d/D) + b_2(d/D)^2 + b_3(d/D)^3 + b_4(d/D)^4 + b_5(d/D)^5 + b_6(d/D)^6$.
		Taper Equation
(12)	Segmented model	$(d^2KH/V - 2z) = b_1(3z^2 - 2z) + b_2(z - a_1)^2I_1 + b_3(z - a_2)^2I_2$; where $I_i = 1,\ z \geq a_i;\ i = 1, 2,$ $= 0,\ z < a_i$.

[a] Symbols are defined in text.

The equation was conditioned so that $R = 1$ when $h = H$, and $R = 0$ when $h = 0$. Imposing these conditions, i.e.,

$$\sum_{i=0}^{6} b_i = 0 \quad \text{and} \quad b_0 = 1,$$

this volume ratio model becomes

$$(R + z - 1) = b_2(z^2 - z) + b_3(z^3 - z) + b_4(z^4 - z) + b_5(z^5 - z) + b_6(z^6 - z). \tag{10}$$

Another sixth-degree polynomial volume ratio model was developed, using (d/D) instead of $(H-h)/H$:

$$R = \sum_{i=0}^{6} (b_i X^i),$$

where $X = d/D$. The constraint that R equal 1 when d is 0 results in $b_0 = 1$. The final model is

$$(R-1) = b_1X + b_2X^2 + b_3X^3 + b_4X^4 + b_5X^5 + b_6X^6. \tag{11}$$

A segmented polynomial model, similar to Max and Burkhart's (1976) taper equation, was defined by grafting three submodels at two join points. Each section of the bole was described by a modified Goulding and Murray (1976) taper equation (using a quadratic function instead of a fifth-degree polynomial in $(H-h)/H$). The model, written in reparameterized form, is

$$(d^2KH/V - 2z) = b_1(3z^2-2z) + b_2(z-a_1)^2I_1 + b_3(z-a_2)^2I_2, \tag{12}$$

where a_1 and a_2 are two join points and

$$\begin{aligned} I_i &= 1 \quad \text{if} \quad z \geq a_i; \quad i = 1, 2, \\ &= 0 \quad \text{if} \quad z < a_i. \end{aligned}$$

A list of the new models is shown in Table 2.

DATA

Data from trees felled on plots in loblolly pine plantations and natural stands were used in this study. Sample tree data from plantations were obtained in the Virginia Piedmont and Coastal Plain and the Coastal Plain of Delaware, Maryland, and North Carolina. Natural-stand sample trees were from the Virginia Piedmont and Coastal Plain and the Coastal Plain of North Carolina.

These single-stemmed trees were felled and cut into 4-foot (1.22-m) sections. Stump heights were not measured; but all were about 0.5 foot (0.15 m), thus a constant stump height of 0.5 foot (0.15 m) was assumed. Tree dbh and total height were measured to the nearest 0.1 inch (0.25 cm) and 0.1 foot (0.03 m), respectively; diameters ib and ob were measured at the stump and at 4-foot (1.22-m) intervals up the stem to an approximate 2-inch (5.1-cm) top diameter ob. Individual tree volumes were determined by using Smalian's formula for each 4-foot (1.22-m) section and summing the volumes of the appropriate sections. The top was assumed to be a cone and the stump a cylinder for computing their volumes.

There were 427 trees from plantations and 209 trees from natural stands. Twenty-five percent of the sample trees were selected at random from each 1-inch (2.5-cm) dbh class. This 25 percent sample was considered as an independent data set representing the population and was withheld for testing purposes. The remaining trees, used for fitting of the models, are referred to as the sample data.

EVALUATING ACCURACY AND PRECISION

The models were evaluated to determine the best choice for predicting merchantable volume to various top diameter limits and for predicting merchantable volume to various height limits. In addition, the taper equations were compared to determine which models described well tree taper.

Merchantable volumes outside bark were computed at 10-percent intervals of total height. These actual volumes were compared to predicted volumes from each of the models. Predicted diameters outside bark at these points on tree boles were also compared to actual diameters. Identical procedures were carried out for volumes and diameters inside bark.

The following three criteria were employed to evaluate the models in terms of merchantable volumes: (1) bias (the mean of the differences between the actual and predicted volumes), (2) mean absolute difference (the mean of the absolute differences), and (3) standard deviation of the differences.

The same criteria were applied to the comparison of taper equations by computing the actual tree diameter minus the predicted diameter.

Various methods of ranking used to evaluate the models provided similar results. The procedures of one method are illustrated as follows. First, the models were compared based on their ability to fit the *sample data*. Second, comparisons were based on the ability of the models to predict for the *population* represented by the independent data. Therefore, four data sets were used in the comparison: (1) plantations sample data, (2) natural-stands sample data, (3) plantations independent data, and (4) natural-stands independent data.

For each data set, biases of all models were computed and compared. A rank was then assigned to each model; rank number one corresponded to the model which had the smallest absolute value of bias. The same procedures were repeated for the other two criteria—mean absolute difference and standard deviation of the differences. The smaller these values, the better the rank. The sum of the three ranks for each model demonstrates its performance compared with other models.

TABLE 3. Regression coefficients of outside and inside bark models fitted to sample data from plantations and natural stands.

Model[a]	Parameter estimates					
	b_1	b_2	b_3	b_4	b_5 (a_1)	b_6 (a_2)
	Plantations—Outside Bark Models					
(1)	2.0122	−1.1150				
(2)	1.0444	−0.6517	−0.0378			
(3)	−0.2415	2.9974	2.5985			
(4)	−2.9517	1.4742				
(5)	0.7821					
(6)	−3.5119	1.6446	−1.1513	49.6843	0.8035	0.1207
(7)	0.0231	2.3600				
(8)	−7.8474	21.5766	−23.7167	9.3581		
(9)	−0.7908	2.4667	2.3892			
(10)		−0.3940	0.5102	−5.2258	7.8606	−3.7643
(11)	0.0658	−1.1767	5.5220	−11.9273	9.0608	−2.2319
(12)	0.5732	−0.4225	149.1100		0.2629	0.9026
	Plantations—Inside Bark Models					
(1)	2.0629	−1.1325				
(2)	1.0315	−0.6847	−0.3953			
(3)	−0.6996	3.2581	3.0411			
(4)	−1.8228	0.8133				
(5)	0.7143					
(6)	−2.5660	1.1933	−1.2047	40.3962	0.7075	0.1096
(7)	−0.2602	1.8875				
(8)	−7.6936	21.8428	−24.2497	9.5149		
(9)	−0.8085	2.3920	2.3139			
(10)		−0.3828	0.4006	−5.4047	8.1950	−3.8225
(11)	0.8952	−10.0187	39.5232	−72.0578	56.2896	−15.5767
(12)	0.4451	−1.3844	141.0483		0.5278	0.9030
	Natural Stands—Outside Bark Models					
(1)	1.9630	−1.0510				
(2)	1.0962	−0.8817	0.0937			
(3)	−0.3282	2.7219	2.4472			
(4)	−2.6447	1.2453				
(5)	0.6861					
(6)	−2.4602	0.9751	−0.7044	131.4842	0.8026	0.0867
(7)	18.8363	1.3284				
(8)	−11.2313	30.6853	−34.1392	13.5623		
(9)	−0.6461	2.2689	2.1337			
(10)		−0.4193	−0.0738	−4.4367	8.0969	−4.1857
(11)	−0.3108	1.1441	1.3018	−9.5979	9.2735	−2.5687
(12)	0.3883	−0.5929	226.5123		0.3046	0.9070
	Natural Stands—Inside Bark Models					
(1)	2.0147	−1.0663				
(2)	1.0166	−0.4404	−0.8243			
(3)	−1.5625	3.6967	3.7088			
(4)	−1.6435	0.6543				
(5)	0.6332					
(6)	−4.3412	2.1196	−2.3682	87.4820	0.7974	0.0883
(7)	−1.1033	1.5136				
(8)	−9.5378	27.5400	−31.3995	12.5176		
(9)	−0.7338	2.2009	2.0996			

TABLE 3. *Continued.*

	Parameter estimates					
Model[a]	b_1	b_2	b_3	b_4	b_5 (a_1)	b_6 (a_2)
(10)		−0.3351	−0.6960	−3.3269	7.0204	−3.6788
(11)	0.9645	−11.5922	47.9574	−88.7671	69.9356	−19.4968
(12)	0.2403	−1.3614	250.2568		0.5087	0.9159

[a] Models are defined in Tables 1 and 2.

Results and Discussion

Regression techniques were employed to fit the outside and inside bark models to the sample data from plantations and from natural stands. Parameter estimates for these models are displayed in Table 3. These parameter estimates may differ from previously published values for this data set because they are based on a subsample of the data (referred to here as the "sample" data). Table 4 summarizes the comparisons by showing the overall ranks of the models for various objectives.[1]

In general, the results are similar for different data sets. The ranks of the models differ only slightly from plantations to natural stands for both sample and independent data sets.

Model (1) is a volume ratio model and model (4) is a taper equation. However, both are simple quadratic models and neither estimates merchantable volumes as well as other models. Model (4) also fails to accurately estimate tree diameters. The results indicate that a simple quadratic equation apparently cannot adequately describe tree taper or predict volumes.

Models (7) and (8) are both compatible taper equations. Model (7) is not a good predictor of either merchantable volumes or diameters. Model (8) adequately estimates merchantable volumes to various height limits (ranked third among nine models) but poorly predicts tree diameters (ranked fifth among six taper equations). Based on these results, a compatible taper equation does not appear to be a good choice if the sole purpose is to describe tree taper.

Model (5) is a good nonlinear taper equation (ranked second) and seems to predict diameters ob better than diameters ib. In case of diameters inside bark, dbh inside bark in the model must be estimated from dbh outside bark, causing some loss in both accuracy and precision. Integration of both outside and inside bark taper equations, however, did not provide good estimates of merchantable volumes.

Model (6) ranked first in ability to estimate tree diameters at various points on the bole. The superior predictive ability of this taper equation is probably due to its flexibility. Using quadratic submodels to describe stem taper of three bole sections accounts for butt swell which is often underestimated by other taper equations. One disadvantage is that the model is not compatible. Despite its excellent performance as a taper equation, integrating model (6) over the entire bole does not give a total volume value that corresponds to that from the total volume equation. This inconsistency probably accounts for the poorer ranking of the model for predicting merchantable volumes to various top limits.

[1] For taper equations, predicted volumes were obtained by integration. In addition, an alternative method was used which involved predicting tree diameters at 4-foot (1.22-m) intervals, computing volume of each 4-foot (1.22-m) section using Smalian's formula, and summing the volumes of the appropriate sections. Both methods yielded similar ranking results.

TABLE 4. *Overall rank of the models for various objectives.*

Overall rank	Objective: Volumes to various heights	Volumes to top diameters	Diameters
1	(9) Modified Burkhart	(12) Segmented model	(6) Max and Burkhart (1976)
2	(10) Polynomial model	(11) Polynomial model	(5) Ormerod (1973)
3	(8) Goulding and Murray (1976)	(3) Burkhart (1977)	(12) Segmented model
4	(12) Segmented model	(6) Max and Burkhart (1976)	(4) Kozak and others (1969)
5	(6) Max and Burkhart (1976)	(5) Ormerod (1973)	(8) Goulding and Murray (1976)
6	(5) Ormerod (1973)	(2) Honer (1967)	(7) Demaerschalk (1973)
7	(7) Demaerschalk (1973)	(4) Kozak and others (1969)	
8	(4) Kozak and others (1969)		
9	(1) Honer (1967)		

Model (12) is a segmented polynomial taper equation consisting of three submodels. Each submodel is in the form of a modified Goulding and Murray (1976) model, which is itself a compatible quadratic taper equation. It is not obvious, however, whether or not model (12) is a compatible taper model. Integrating model (12) over the entire bole gives an estimate of total volume:

$$V = \hat{V}\,[1 + b_2(1-a_1)^3/3 + b_3(1-a_2)^3/3]$$

or

$$V = (\hat{V})(C)$$

where $\hat{V}$ = estimated total volume from $\hat{V} = b_1' + b_2'D^2H$, and

$$C = 1 + b_2(1 - a_1)^3/3 + b_3(1 - a_2)^3/3.$$

The values obtained for C were: plantation trees, outside bark 0.9896; plantation trees, inside bark 0.9943; natural-stand trees, outside bark 0.9942; natural-stand trees, inside bark 0.9959. The relationship between regression coefficients suggests that model (12) is essentially a compatible taper equation. Compatibility is the main difference between models (6) and (12), since both are segmented taper equations. Model (12) consistently provides better volume estimates than does model (6). In fact, it was ranked first in predicting merchantable volumes to top diameter limits.

Some precision in estimating diameters is apparently sacrificed to ensure the taper equation is compatible; this character is shared by model (12) as well as other compatible taper equations. However, using three submodels definitely improved taper prediction. Model (12) was ranked only after models (6) and (5) in predicting diameters.

All volume ratio models considered, except models (1) and (2), reliably predicted merchantable volumes. These models were fitted directly to volume data whereas taper equations were fitted to diameter data and then integrated to provide volume estimates.

Models (3) and (9) only have three coefficients and still fit the data well owing

TABLE 5. Values of bias, mean absolute difference, and standard deviation of the differences of the recommended models for independent data.

Type of data	Outside bark $\bar{D}$	Outside bark $\lceil D \rceil$	Outside bark s_D	Inside bark $\bar{D}$	Inside bark $\lceil D \rceil$	Inside bark s_D[a]
			VOLUMES TO VARIOUS HEIGHTS (1 unit = 1 dm^3)			
			Model (9): Modified Burkhart			
Plantations	0.24	5.99	9.25	−0.29	5.10	9.31
Natural stands	0.33	16.77	23.52	0.94	15.54	21.99
			Model (10): Polynomial model			
Plantations	0.15	6.01	9.25	−0.49	5.02	9.11
Natural stands	−1.59	17.18	23.98	−0.75	15.60	22.27
			Model (12): Segmented model			
Plantations	2.67	6.55	9.74	1.37	5.03	9.39
Natural stands	3.97	17.28	23.33	4.04	15.61	21.41
			VOLUMES TO TOP DIAMETERS (1 unit = 1 dm^3)			
			Model (3): Burkhart (1977)			
Plantations	0.99	9.21	14.55	−0.21	8.09	14.37
Natural stands	2.73	27.13	36.40	1.26	26.22	34.90
			Model (11): Polynomial model			
Plantations	−0.42	8.88	13.40	−1.06	8.04	13.65
Natural stands	−2.31	27.08	36.98	−0.12	25.87	34.92
			Model (12): Segmented model			
Plantations	1.21	8.99	13.90	0.39	7.73	14.02
Natural stands	2.47	25.80	34.60	4.22	25.57	34.27
			DIAMETER (1 unit = 1 cm)			
			Model (6): Max and Burkhart (1976)			
Plantations	−0.16	0.63	0.82	−0.18	0.53	0.69
Natural stands	−0.24	0.84	1.12	−0.09	0.70	0.95
			Model (12): Segmented model			
Plantations	−0.26	0.66	0.81	−0.30	0.63	0.78
Natural stands	−0.25	0.84	1.12	−0.21	0.86	1.16

[a] $\bar{D}$ = Bias,
$\lceil D \rceil$ = Mean absolute difference,
s_D = Standard deviation of the differences.

to the flexibility of the nonlinear model form.[2] Models (10) and (11), on the other hand, are sixth-degree polynomial volume ratio models having five (10) or six (11) parameters to be estimated. These models estimate the volume ratios which are used to convert total stem volumes into volumes to various top limits; all volumes are above ground. Merchantable volume from stump height to some top limit is calculated by subtraction. Models (10) and (11) gave consistently good results; they both ranked second. The main advantage models (10) and (11) have over models (3) and (9) is that the former are flexible enough to deal with various stump heights.

[2] In addition to the results reported here, statistical weights were applied when fitting volume ratio models (3) and (9). Although some improvement in predictive ability was realized when estimating volumes to fixed top diameters (model 3), coefficients from the unweighted solution gave better results for predicting volumes for various heights (model 9).

Conclusions and Recommendations

These results demonstrate that, for the models considered, one did not consistently perform best for several related prediction objectives. If a single equation is desired, we recommend a reliable taper equation that, when integrated, also provides reasonable prediction of merchantable volumes to either top height or top diameter limits.

The two sets of volume ratio models (3,9) and (10,11) yield about equally good volume estimates and are recommended for predicting merchantable volumes to various heights and/or top diameters. If stump heights are approximately the same and may be assumed constant, models (3) and (9) with fewer parameters are more appropriate. On the other hand, models (10) and (11) are more flexible for data with large variation among stump heights.

If the sole objective is to describe tree taper, model (6) appears to be a good choice. Model (12), a reasonably good multipurpose taper equation, provides consistently good estimates of both diameters and volumes. A summary of how these recommended models behave for the two independent data sets is presented in Table 5.

Although this evaluation was made on two loblolly pine data sets, we feel that the recommended models will perform reasonably well for other single-stemmed coniferous trees.

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