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Evaluation of Fitting and Adjustment Methods for Taper and Volume Prediction of Black Pine in Turkey

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A modified form (Cao, Q.V. *South. J. Appl. For.* 33[2]:58–61, 2009) of the taper model of T.A. Max and H.E. Burkhart (*For. Sci.* 22[3]:283–289, 1976) was used in this study to predict taper and volume of black pine. A total of 16 methods were evaluated, including 4 fitting methods (optimized for taper, cumulative volume, taper and cumulative volume, and taper and total volume) and 4 adjustment strategies (unadjusted, adjusted to match dbh, predicted total volume, and dbh and predicted total volume). Results showed that adjustment did not improve the performance of the taper model. Also for this data set, the model with parameters optimized for taper and either total or cumulative volume performed better than the other fitting methods in predicting both taper and cumulative volume.

Keywords: taper, volume, segmented regression model, optimization, black pine

The estimates of total and merchantable volume of trees in a stand are essential tools for forest management and planning. Volume prediction to any merchantable limit is obtained from either volume-ratio equations that predict merchantable volume as a percentage of total tree volume (Burkhart 1977, Cao et al. 1980, Reed and Green 1984) or integration of stem taper equations (Jordan et al. 2005, Diéguez-Aranda et al. 2006, Burkhart and Tomé 2012). Numerous taper equations have been developed to describe tree taper for different tree species, employing functions varying from simple to complex. Variable exponent taper models (Kozak 1988, Newnham 1992, Sharma and Zhang 2004), although flexible, cannot be integrated analytically and require numerical integration techniques to produce volumes. Segmented taper equations (Max and Burkhart 1976, Cao et al. 1980, Fang and Bailey 1999) can be integrated to calculate volumes and can be algebraically rearranged to directly estimate merchantable height for a given top diameter (Kozak and Smith 1993).

Ideally, a volume estimation system should be compatible; i.e., the volume of the tree bole obtained through the integration of the taper model should be equal to the volume predicted from a volume equation. The basic rationale behind the design of compatible systems of taper and volume equations is that taper and volume should be considered mathematically and be biologically related (Munro and Demaerschalk 1974). A compatible taper equation can be derived either from a total or a merchantable volume equation (Demaerschalk 1972, Clutter 1980) or obtained by imposing conditions on the parameters such that its integration results in stem volume (Goulding and Murray 1976, Cao et al. 1980, Van Deusen et al. 1982, Reed and Green 1984, Lenhart et al. 1987, Van Deusen 1988, Fang and Bailey 1999, Diéguez-Aranda et al. 2006).

Reed (1982) and Reed and Green (1984) presented a method of simultaneously fitting all the equations in the taper and volume system. They observed that this procedure substantially reduced the overall system estimation error, as opposed to fitting the taper equation and then algebraically solving for the coefficients of the total and volume ratio equations. Van Deusen (1988) reformulated the above simultaneous estimation as a seemingly unrelated regressions (SUR) problem that can be solved by use of standard statistical software packages.

Black pine (*Pinus nigra* Arnold.) is commercially one of the most important pine species in Turkey, occupying an area of about 4.24 million ha. According to the latest survey, the standing volume was approximately 296.72 million m^3 (General Directorate of Forests 2015). In addition, black pine forests have a key role in providing important indirect benefits and environmental services such as protection of soil and water resources, conservation of biological diversity, support to agricultural productivity, carbon sequestration, and climate change mitigation and adaptation (Atalay and Efe 2012). Therefore, forest managers need detailed information, such as stand structure and

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volume classified by merchantable products, supplied by growth and yield prediction models for sustainable forest management of black pine forest.

Several studies have been carried out to test the suitability of taper models for describing the stem profile and predicting stem volume of different tree species at the regional level in Turkey (Brooks et al. 2008, Sakıcı et al. 2008, Özçelik and Brooks 2012, Özçelik and Crecente-Campo 2016). For example, Özçelik and Brooks (2012) found that the Clark et al. (1991) taper model performed better than the model of Max and Burkhart (1976) for predicting taper and volume of some species in Turkey, including black pine. However, in this study, we decide not to use the Clark et al. (1991) taper model because it is overly complicated, requiring eight parameters when the upper stem diameter measurement at 5.30 m is not available. Furthermore, an expression for prediction of height at a given bole diameter does not exist in closed form. Instead, a modified form (Cao 2009) of the Max and Burkhart (1976) taper equation was used in this study. It consists of three quadratic functions joined together to form a smooth and continuous curve. It is relatively simple for a segmented model and therefore has been used extensively in describing stem taper of many tree species (Byrne and Reed 1986, Muhairwe 1999, Diéguez-Aranda et al. 2006, Brooks et al. 2008, Jiang and Liu 2011). This taper model has been calibrated such that the taper curves go through diameter at breast height (dbh) (Cao 2009) and an upper stem diameter (Czaplewski and McClure 1988, Cao 2009, Cao and Wang 2011, Sabatia and Burkhart 2015).

The objective of this study was to evaluate different approaches for estimating parameters of a modified form (Cao 2009) of the original Max and Burkhart (1976) taper equation for black pine, coupled with methods for calibrating the taper equation for dbh and/or predicted total volume.

Data

The data used in this study consisted of taper measurements from trees felled in natural stands located throughout the area of distribution of black pine in southern Turkey. A total of 535 black pine trees were felled: 172 in the Lakes ecoregion, 240 in the Maritime Ecoregion, and 123 in the Interior of Mediterranean. Based on information from a previous inventory, sample trees were selected to ensure a representative distribution by diameter and height classes. dbh (*D*, cm, at 1.3 m aboveground level) was measured to the nearest 0.1 cm for each tree. The trees were later felled, leaving stumps of average height of 0.30 m. Total bole length (*H*, m), was measured to the nearest 0.01 m. Diameter outside bark (*d*) was measured at heights (*h*, m) of 0.3 m (stump height) and then at intervals of 1 m along the remainder of the stem. At each measurement point, two perpendicular outside-bark diameters were measured and then arithmetically averaged. Section volumes were calculated in cubic meters by use of Smalian's formula. The top section (from the last diameter measurement to the tree tip) was treated as a cone. Outside-bark total stem volume (above stump) was obtained by summing the overbark section volumes and the conical volume of the top of the tree. Summary statistics of the main variables are presented in Table 1.

Table 1. Summary statistics of data used in this study.

Ecoregion	Variable	Minimum	Mean	Maximum
1: Lakes (n = 172)	dbh (cm)	9.0	29.5	56.6
	Total height (m)	7.2	14.8	22.6
	Total volume (m^3)	0.0	0.6	2.2
	Number of sections	7.0	14.1	21.0
2: Maritime (n = 240)	dbh (cm)	10.5	31.1	59.0
	Total height (m)	7.9	18.2	30.3
	Total volume (m^3)	0.0	0.8	3.4
	Number of sections	8.0	17.7	30.0
3: Interior of Mediterranean (n = 123)	dbh (cm)	8.5	38.4	82.0
	Total height (m)	7.7	19.4	34.6
	Total volume (m^3)	0.0	1.4	6.5
	Number of sections	7.0	18.6	33.0

Methods

Total Volume Equation

A number of different types of volume equations have been developed to predict tree volume. Schumacher and Hall (1933) introduced a tree volume equation that has been successfully applied to many tree species. The Schumacher and Hall (1933) model was used to estimate total stem volume:

$$V_i = aD_i^b H_i^c + \varepsilon_i \tag{1}$$

where V_i is total volume of tree i in m^3, D_i is dbh of tree i in cm, H_i is total height of tree i in m, a, b, and c are regression coefficients, and ε_i is error.

Taper Equation

The Max and Burkhart (1976) taper model is preferred because volume can be easily integrated, and a height prediction from bole diameter can be obtained directly, without the need for numerical techniques. A modified form (Cao 2009) of the original Max and Burkhart (1976) taper equation was used in this study:

$$\hat{y}(z_{ij}) = b_1 + b_2 z_{ij}^2 + b_3(z_{ij} - a_1)^2 I_1 + b_4(z_{ij} - a_2)^2 I_2 \tag{2}$$

where $y(z_{ij}) = d_{ij}^2/D_i^2$, $\hat{y}$ is predicted value of y, D_i is dbh in cm of tree i, H_i is total height of tree i, d_{ij} is bole diameter in cm at height h_{ij} of location j on tree i, h_{ij} is height from the ground in m, $z_{ij} = 1 - h_{ij}/H_i$ is relative height from the tree tip, $I_k = \begin{cases} 1, \textit{ if } z_{ij} > a_k \\ 0, \textit{ otherwise} \end{cases}$, $k = 1, 2$, and a_h and b_h are regression coefficients. An increase in performance was obtained when d_{ij} was used as the dependent variable rather than d_{ij}^2/D_i^2. The regression model is

$$d_{ij} = D_i \sqrt{b_1 z_{ij} + b_2 z_{ij}^2 + b_3(z_{ij} - a_1)^2 I_1 + b_4(z_{ij} - a_2)^2 I_2} + \varepsilon_{ij} \tag{3}$$

Volume from height h_{i1} to height h_{i2} is obtained by integration as follows:

$$\hat{v}_i = KD_i^2 H_i \left\{ \left(\frac{b_1}{2} z_{i2}^2 + \frac{b_2}{3} z_{i2}^3 + \frac{b_3}{3}(z_{i2} - a_1)^3 I_{12} + \frac{b_4}{3}(z_{i2} - a_2)^3 I_{22} \right) - \left(\frac{b_1}{2} z_{i1}^2 + \frac{b_2}{3} z_{i1}^3 + \frac{b_3}{3}(z_{i1} - a_1)^3 I_{12} + \frac{b_4}{3}(z_{i1} - a_2)^3 I_{21} \right) \right\} \tag{4}$$

where $z_{im} = 1 - h_{im}/H_i$, $m = 1, 2$, $K = 0.00007854$, a constant to convert diameter in cm to area in m^2, $I_{km} = \begin{cases} 1, \textit{ if } z_{im} > a_k \\ 0, \textit{ otherwise} \end{cases}$, $k = 1, 2$; $m = 1, 2$, and ε_{ij} is error.

In this article, cumulative volume refers to volume from the stump to where diameter is measured, and total volume is volume from the stump to the tree tip.

Fitting Methods

Four different approaches were used to estimate parameters (b_1–b_4 and a_1–a_2) of the taper equation.

Fitting Method 1—Optimized for Taper

This is the normal least-squares method commonly used in fitting taper equations. The goal was to minimize $\sum_{i=1}^{N}\sum_{j=1}^{n_i}(d_{ij} - \hat{d}_{ij})^2$, where n_i is number of diameter measurements for tree i, N is number of trees, and $\hat{d}_{ij}$ is predicted bole diameter at location j on tree i.

Fitting Method 2—Optimized for Cumulative Volume

The parameters were selected such that, when integrated, the taper model would produce a good prediction for cumulative volume. This was done by minimizing $\sum_{i=1}^{N}\sum_{j=1}^{n_i}(v_{ij} - \hat{v}_{ij})^2$, where v_{ij} and $\hat{v}_{ij}$ are observed and predicted cumulative volume of tree i from the stump to the jth diameter measurement, respectively.

Fitting Method 3—Optimized for Both Taper and Cumulative Volume

In this approach, both $\sum_{i=1}^{N}\sum_{j=1}^{n_i}(d_{ij} - \hat{d}_{ij})^2$ and $\sum_{i=1}^{N}\sum_{j=1}^{n_i}(v_{ij} - \hat{v}_{ij})^2$ were simultaneously minimized by the use of seemingly unrelated regression (SAS proc MODEL, option SUR). The resulting taper model should produce reliable predictions for both diameter and cumulative volume.

Fitting Method 4—Optimized for Both Taper and Total Volume

Similar to the previous approach, the objective was to simultaneously minimize $\sum_{i=1}^{N}\sum_{j=1}^{n_i}(d_{ij} - \hat{d}_{ij})^2$ and $\sum_{i=1}^{N}\sum_{j=1}^{n_i}(V_i - \hat{V}_i)^2$, where V_i and $\hat{V}_i$ are observed and predicted total volume of tree i, respectively. Seemingly unrelated regression was also used for this approach, which is geared toward optimizing both diameter and total volume.

Autoregressive models, AR(1) and AR(2), were considered in this study to deal with correlation between successive measurements on the same tree. The AR(1) model produced slightly better results and therefore was used to obtain parameter estimates of all regression models.

Adjustment Methods

Some parameters obtained from each of the four above-mentioned approaches were adjusted so that the resulting taper model produced predictions that match dbh and/or total volume predicted separately from the total volume model (Equation 1).

Adjustment Method 1—Unadjusted

Parameter estimates of the taper equation remained unchanged.

Adjustment Method 2—Adjusted to Match dbh

Predicted diameter from the Max and Burkhart (1976) taper equation does not equal dbh ($h = 1.30$ m). Cao (2009) proposed an adjustment procedure in which parameter b_1 was replaced with b_1^* such that predicted dbh is D_i:

$$b_1^* = b_1 + \frac{1 - \hat{y}(z_{iBH})}{z_{iBH}} \tag{5}$$

where $z_{iBH} = 1 - 1.3/H_i$. Details are given in the Appendix.

Adjustment Method 3—Adjusted to Match Total Volume

In this method, b_1 was replaced with b_1^* such that the resulting total volume matches $\tilde{V}_i$, which is predicted from the total volume Equation 1:

$$b_1^* = b_1 + \frac{2(\tilde{V}_i - \hat{V}_i)}{KD_i^2 H_i z_{iS}^2} \tag{6}$$

where $\tilde{V}_i$ is total volume of tree i by integration of the taper Equation 3, $K = 0.00007854$, a constant to convert diameter in cm to area in m^2, $z_{iS} = 1 - h_{iS}/H_i$, and h_{iS} is stump height for tree i.

Details are given in the Appendix.

Adjustment Method 4—Adjusted to Match Both dbh and Total Volume

Parameters b_2 and b_1 were replaced with b_2^* and b_1^*, respectively, so that predicted dbh matches D_i and the resulting total volume matches $\tilde{V}_i$:

$$b_2^* = b_2 + \frac{2z_{iBH}(\hat{V}_i - \hat{V}_i) - KD_i^2 H_i z_{iS}^2[1 - \hat{y}(z_{iBH})]}{KD_i^2 H_i z_{iS}^2(2z_{iS}/3 - z_{iBH})} \tag{7}$$

$$b_1^* = b_1 + \frac{1 - \hat{y}(z_{iBH})}{z_{iBH}} - (b_2^* - b_2)z_{iBH}. \tag{8}$$

Details are given in the Appendix.

Model Evaluation

A total of 16 methods (4 fitting methods × 4 adjustment methods) were evaluated in this study. The leave-one-out validation scheme was applied at the region level. Coefficients obtained from two ecoregions were used to predict for the remaining region. Predicted values from all ecoregions were used to compute evaluation statistics, separately for diameters and volumes. The evaluation statistics included mean difference (MD) between observed and predicted values, the mean of absolute difference (MAD), and fit index (FI). MD measures the average bias of the prediction, MAD measures the magnitude of the bias, and FI is similar to R^2 in linear regression. The expressions of these statistics are summarized as follows:

$$\text{Mean difference: MD} = \frac{\sum_{i=1}^{N}\sum_{j=1}^{n_i}(x_{ij} - \hat{x}_{ij})}{\sum_{i=1}^{N} n_i}$$

$$\text{Mean absolute difference: MAD} = \frac{\sum_{i=1}^{N}\sum_{j=1}^{n_i}|x_{ij} - \hat{x}_{ij}|}{\sum_{i=1}^{n_i}}$$

$$\text{Fit index: FI} = 1 - \frac{\sum_{i=1}^{N}\sum_{j=1}^{n_i}(x_{ij} - \hat{x}_{ij})^2}{\sum_{i=1}^{N}\sum_{j=1}^{n_i}(x_{ij} - \bar{x}_i)^2}$$

where x_{ij} is either diameter (d_{ij}) or volume (v_{ij}) and $\hat{x}_{ij}$ and $\bar{x}_i$ are predicted and average values of x_{ij}, respectively.

Results and Discussion

Total Volume Prediction

Table 2 shows that the total volume model clearly provided best prediction for total volume, based on all three statistics. This was expected because parameters of the total volume equation were optimized for total tree volume. On the other hand, when total volume

Table 2. Evaluation statistics for total volume.

Method	MD	MAD	FI
Total volume equation	−0.0019*	0.0682*	0.9814*
Taper equation: taper	0.0212†	0.0728†	0.9774
Taper equation: cumulative volume	0.0132	0.0717	0.9773
Taper equation: taper and cumulative volume	0.0151	0.0721	0.9772†
Taper equation: taper and total volume	0.0152	0.0721	0.9772†

* Best method for each criterion.
† Worst method for each criterion.

was obtained by integrating the taper equation, results varied, depending on the fitting method (Table 2). Whereas similar FI values were obtained from all fitting methods, MD and MAD values were best when the taper coefficients were optimized for cumulative volume and were worst when they were optimized for taper. The method of optimizing for both taper and either total or cumulative volume did not produce the best nor the worst MD and MAD evaluation statistics.

Overall, all four fitting methods showed very good results, explaining more than 97% of the variation, with the worst MAD being 0.0728 m^3.

Diameter and Cumulative Volume Prediction

Tables 3 and 4 show evaluation statistics for taper and cumulative volume, respectively. Overall, all the methods showed very good results, explaining more than 95% of the taper variation and 83% of the volume variation.

A relative rank (Poudel and Cao 2013) for each evaluation statistic was computed for each combination of fitting method and adjustment method. In this ranking system, the best and the worst methods have relative ranks of 1 and 16, respectively, whereas the ranks of the remaining methods are expressed as real numbers between 1 and 16. Because not only the magnitude but also the order of the evaluation statistic is taken into consideration, the relative ranking system should provide more information than the traditional ordinal ranks. The sum of three ranks for each method was computed. These sums were then ranked again to give a relative rank for each method (Tables 3 and 4), which shows that the two best methods for both taper and volume prediction were the ones that optimized both taper and either total or cumulative volume with no adjustment. Additional details about the ranking system can be found in studies reported by Poudel and Cao (2013) and Özçelik and Crecente-Campo (2016).

Adjustment Method

The ranks of each adjustment method for taper prediction from Table 3 were summed over the four optimization methods. These sums were then ranked to yield a relative rank for each

Table 3. Evaluation statistics for taper.

Optimization	Adjustment	MD	MAD	FI	Relative rank
Taper	Unadjusted	0.4552	1.6032	0.9706	2.22
	dbh	−0.6670	1.7886	0.9614	10.89
	Predicted total volume	−0.6755	1.7261	0.9658	7.93
	dbh and predicted TV	−0.6673	1.7607	0.9637	9.42
Cumulative volume	Unadjusted	0.1812	1.6348	0.9692	1.51
	dbh	−0.9768*	1.8906*	0.9586*	16.00
	Predicted total volume	−0.7946	1.7788	0.9641	10.42
	dbh and predicted TV	−0.7460	1.7814	0.9642	10.09
Taper and cumulative volume	Unadjusted	0.3562	1.5917	0.9710	1.18
	dbh	−0.7504	1.8137	0.9604	12.32
	Predicted total volume	−0.6833	1.7245	0.9659	7.91
	dbh and predicted TV	−0.6525	1.7636	0.9635	9.43
Taper and total volume	Unadjusted	0.3484†	1.5870†	0.9711†	1.00†
	dbh	−0.7411	1.8054	0.9607	11.98
	Predicted total volume	−0.6920	1.7229	0.9660	7.93
	dbh and predicted TV	−0.6669	1.7606	0.9637	9.42

* Worst method for each criterion.
† Best method for each criterion.

Table 4. Evaluation statistics for cumulative volume.

Optimization	Adjustment	MD	MAD	FI	Relative rank
Taper	Unadjusted	0.0096	0.0888	0.8850	1.03
	dbh	0.1537*	0.1780*	0.8401	16.00*
	Predicted TV	−0.0511	0.1087	0.8662	5.66
	dbh and predicted TV	−0.0540	0.1122	0.8626	6.35
Cumulative volume	Unadjusted	0.0132	0.0885	0.8858	1.05
	dbh	0.1468	0.1736	0.8392*	15.61
	Predicted TV	−0.0402	0.1015	0.8725	4.18
	dbh and predicted TV	−0.0461	0.1063	0.8685	5.09
Taper and cumulative volume	Unadjusted	0.0060†	0.0888†	0.8842†	1.00†
	dbh	0.1508	0.1758	0.8417	15.59
	Predicted TV	−0.0498	0.1077	0.8669	5.47
	dbh and predicted TV	−0.0543	0.1123	0.8624	6.38
Taper and total volume	Unadjusted	0.0062	0.0888†	0.8842†	1.00†
	dbh	0.1518	0.1766	0.8412	15.74
	Predicted TV	−0.0498	0.1077	0.8669	5.47
	dbh and predicted TV	−0.0539	0.1121	0.8627	6.33

* Worst method for each criterion.
† Best method for each criterion.

Table 5. Overall relative ranks for predicting taper and cumulative volume by adjustment method.

Adjustment method	Taper	Volume
Unadjusted	1.00	1.00
dbh	4.00	4.00
Predicted total volume	2.87	1.85
dbh and predicted total volume	3.15	2.02

adjustment method. The same procedures were repeated for statistics for volume prediction from Table 4. Table 5 presents the overall relative rank for predicting taper and volume, by adjustment method.

The unadjusted method clearly was the best method for both taper and volume prediction (Table 5). On the other hand, adjustment to match dbh was the worst method for both taper and volume prediction. This was not in accordance with results obtained by Cao (2009), who found that, compared with the unadjusted regression equation, calibration for dbh improved all evaluation statistics. Compared with that for the unadjusted method, calibration for dbh in this study actually worsen taper prediction: MD on average increased by 47% in absolute value, MAD increased by 12%, and FI decreased by 1%.

The adjustment method for predicted total volume produced a compatible taper equation, as defined by Demaerschalk (1972). However, it consistently ranked below the unadjusted method for both taper and volume prediction. Adding the dbh constraint did not improve performance of this method (Table 5).

As indicated by the MD values (Tables 3 and 4), all adjustment methods added bias and therefore hurt rather than helped the accuracy and possibly precision of the taper model. The problem was more pronounced with the adjustment for dbh and might be the reason why this adjustment method ranked lower than the other adjustment methods based on all three evaluation statistics, for both taper and volume prediction.

Optimization Method

Because adjustment did not help improve the performance of the taper model for this data set, the following evaluation of the optimization methods was based solely on the unadjusted method. Table 3 shows that, for taper prediction, optimizing taper and total volume produced slightly better results than optimizing taper and cumulative volume (ranks of 1.00 and 1.18, respectively). The worst method was the one that optimized taper (rank of 2.22). For volume prediction, similar evaluation statistics were obtained from all four optimization methods, with relative ranks varied from 1.00 for the methods of optimizing taper and cumulative volume and optimizing taper and total volume to 1.05 (Table 4).

Cao et al. (1980) found that a taper equation, when optimized for taper, was excellent in predicting taper but did not perform as well as a volume ratio model in predicting cumulative volume. The taper model with parameters optimized for cumulative volume in this study can be considered similar to a volume ratio model, yet did not rank higher in volume prediction than the model optimized for taper (Table 4). On the other hand, optimizing for taper produced the worst rank for taper prediction among the fitting methods (Table 3).

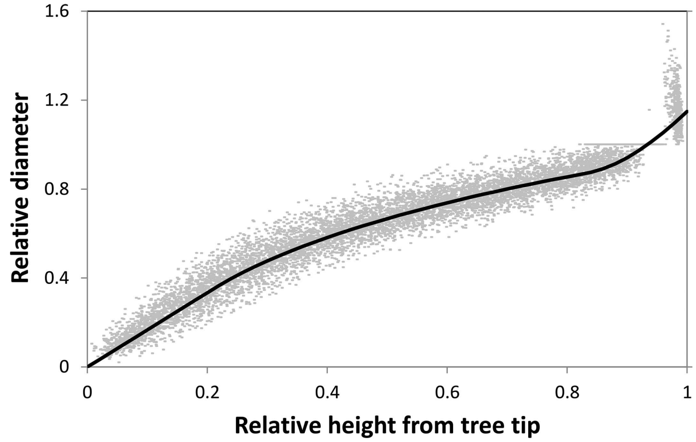

Figure 1. Graph of observed data and predictions from the taper model optimized for taper and total volume.

Simultaneous optimizing for both taper and cumulative volume was found by Reed and Green (1984) to produce smaller total system squared error. They did not, however, show how well the system predicted separately for taper and volume. In our study, the methods of optimizing for taper and either cumulative volume or total volume produced the best results for predicting taper and cumulative volume.

In the last step, the proposed taper and volume models for optimization alternatives were refit to the entire data set (Table 6). Figure 1 shows the observed data with predictions from the taper model with coefficients optimized for both taper and total volume.

Summary and Conclusions

A modified form (Cao 2009) of the Max and Burkhart (1976) taper model was used in this study to predict taper and cumulative volume of black pine. A total of 16 methods were evaluated, including 4 fitting methods (optimized for taper, cumulative volume, taper and cumulative volume, and taper and total volume) and 4 adjustment strategies (unadjusted, adjusted to match dbh, predicted total volume, and dbh and predicted total volume). Results showed that adjustment did not improve the performance of the taper model. In addition, for this data set, the model with parameters

Table 6. Parameter estimates (and standard errors) for taper and volume equations of different optimization alternatives based on all sample data.

Optimization alternative	Parameters					
	b_1	b_2	b_3	b_4	a_1	a_2
Taper	−0.0039 (0.0094)	2.7312 (0.0539)	−2.9670 (0.0616)	15.0767 (0.5001)	0.2156 (0.0048)	0.8333 (0.0031)
Cumulative volume	0.2509 (0.0964)	1.7582 (0.4015)	−2.4046 (0.3615)	5.0218 (0.7929)	0.2884 (0.0435)	0.7313 (0.0267)
Taper and cumulative volume	−0.0116 (0.0082)	2.8046 (0.0517)	−3.0910 (0.0560)	15.0865 (0.4803)	0.2159 (0.0045)	0.8319 (0.0029)
Taper and total volume	−0.0020 (0.0085)	2.7934 (0.0539)	−3.0592 (0.0584)	14.9620 (0.4910)	0.2126 (0.0045)	0.8314 (0.0030)

optimized for taper and either total or cumulative volume performed better than the other fitting methods in predicting both taper and cumulative volume.

Appendix

Adjustment to Match dbh

$$\hat{y}(z_{iBH}) = b_1 z_{iBH} + b_2 z_{iBH}^2 + b_3(z_{iBH} - a_1)^2 I_1 + b_4(z_{iBH} - a_2)^2 I_2 \tag{A1}$$

Replacing b_1 with b_1^* such that $\hat{y}(z_{iBH}) = 1$:

$$1 = b_1^* z_{iBH} + b_2 z_{iBH}^2 + b_3(z_{iBH} - a_1)^2 I_1 + b_4(z_{iBH} - a_2)^2 I_2 \tag{A2}$$

Subtracting A1 from A2 and simplifying:

$$b_1^* = b_1 + \frac{1 - \hat{y}(z_{iBH})}{z_{iBH}} \tag{A3}$$

Adjustment to Match $\tilde{V}_i$

Integrating the taper Equation 3 from the stump ($z_{ij} = z_{iS}$) to the tree tip ($z_{ij} = 0$) gives

$$\hat{V}_i = KD_i^2 H_i \left[\frac{b_1}{2} z_{iS}^2 + \frac{b_2}{3} z_{iS}^3 + \frac{b_3}{3}(z_{iS} - a_1)^3 I_{1S} + \frac{b_4}{3}(z_{iS} - a_2)^3 I_{2S} \right] \tag{A4}$$

Replacing b_1 with b_1^* to integrate to $\tilde{V}_i$:

$$\tilde{V}_i = KD_i^2 H_i \left[\frac{b_1^*}{2} z_{iS}^2 + \frac{b_2}{3} z_{iS}^3 + \frac{b_3}{3}(z_{iS} - a_1)^3 I_{1S} + \frac{b_4}{3}(z_{iS} - a_2)^3 I_{2S} \right] \tag{A5}$$

where $\tilde{V}_i$ is total volume predicted from the total volume Equation 1. Subtracting A4 from A5 and simplifying:

$$b_1^* = b_1 + \frac{2(\tilde{V}_i - \hat{V}_i)}{KD_i^2 H_i z_{iS}^2} \tag{A6}$$

Adjustment to Match dbh and $\tilde{V}_i$

Replacing b_1 with b_1^* and b_2 with b_2^* such that $\hat{y}(z_{iBH}) = 1$ and $\hat{V}_i = \tilde{V}_i$:

$$1 = b_1^* z_{iBH} + b_2^* z_{iBH}^2 + b_3(z_{iBH} - a_1)^2 I_1 + b_4(z_{iBH} - a_2)^2 I_2 \tag{A7}$$

$$\tilde{V}_i = KD_i^2 H_i \left[\frac{b_1^*}{2} z_{iS}^2 + \frac{b_2^*}{3} z_{iS}^3 + \frac{b_3}{3}(z_{iS} - a_1)^3 I_{1S} + \frac{b_4}{3}(z_{iS} - a_2)^3 I_{2S} \right] \tag{A8}$$

Subtracting A1 from A7 yields

$$1 - \hat{y}(z_{iBH}) = (b_1^* - b_1) z_{iBH} + (b_2^* - b_2) z_{iBH}^2 \tag{A9}$$

Subtracting A4 from A8 and simplifying:

$$\tilde{V}_i - \hat{V}_i = KD_i^2 H_i \left[(b_1^* - b_1)\frac{z_{iS}^2}{2} + (b_2^* - b_2)\frac{z_{iS}^3}{3} \right] \tag{A10}$$

Solving for b_1^* and b_2^* in the system of two equations (A9 and A10) yields:

$$b_2^* = b_2 + \frac{2 z_{iBH}(\tilde{V}_i - \hat{V}_i) - KD_i^2 H_i\, z_{iS}^2 [1 - \hat{y}(z_{iBH})]}{KD_i^2 H_i z_{iS}^2 (2 z_{iS}/3 - z_{iBH})} \tag{A11}$$

$$b_1^* = b_1 + \frac{1 - \hat{y}(z_{iBH})}{z_{iBH}} - (b_2^* - b_2) z_{iBH} \tag{A12}$$

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