A Method to Distribute Mortality in Diameter Distribution Models

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ABSTRACT. Bailey (1980) derived tree diameter growth models by transforming variables which preserved the functional form of the probability density function that approximated diameter distributions. The necessary assumption was either no mortality, or that mortality was proportionally distributed among diameter classes. The latter assumption might not be realistic since smaller trees should suffer more from competition than large trees and are more likely to die. This paper deals with the case when mortality is not proportionally distributed. If diameters in a stand originally follow a Weibull distribution and mortality for a growing period can be assumed to occur at the beginning of that period, then a Weibull was found to successfully approximate the diameter distribution immediately after mortality. The stand then grows without further mortality and reaches the end of the period with its diameters remaining Weibull. A similar method to derive a diameter distribution after mortality was developed for the beta distribution. The Weibull technique was applied to data from loblolly plantations. Results showed that the new approach worked reasonably well and was comparable with a diameter distribution model in approximating diameter distribution of a stand at the end of the growth period. FOR. SCI. 43(3):435–442.

Additional Key Words: Weibull, beta, probability density function, survival.

BAILEY (1980) DEVELOPED diameter growth functions in cases when the same form of probability density function (pdf) was applied to the diameter distribution at the beginning and at the end of a growing period. He assumed that either no mortality occurred or mortality was proportionally distributed over the diameter distribution.

This paper will examine the case when mortality is not proportionally distributed among diameter classes. A method will be introduced to maintain the same form of pdf at the beginning and the end of the growth period. Specific details involving the Weibull and beta distributions will be discussed. An application of this method in modeling diameter distribution of loblolly pine (*Pinus taeda* L.) plantations will be presented.

Diameter Distribution Models

This type of yield model was based on the approximation of a diameter distribution by a probability function. The beta distribution, first used by Clutter and Bennett (1965), was later incorporated into growth and yield models (Bennett and Clutter 1968, Lenhart and Clutter 1971, Burkhart and Strub 1974, Bennett et al. 1978). Bailey and Dell (1973) showed that the Weibull function was flexible and easy to use. Numerous growth and yield systems based on the Weibull distribution have since been developed (Smalley and Bailey 1974, Feduccia et al. 1979, Matney and Sullivan 1982, Baldwin and Feduccia 1987, Brooks et al. 1992). The S_B distribution was found by Hafley and Schreuder (1977) to be appropriate for both diameter and height, and was the driving force in a yield prediction model (Hafley et al. 1982). The bivariate S_{BB} was used to model both diameter and height distributions (Schreuder and Hafley 1977, Hafley and Buford 1985) and diameter distributions at two points in time (Knoebel and Burkhart 1991).

Mortality

Prediction of mortality is an important component in growth and yield models. Modeling individual tree mortality has been mostly done with logistic regression (Hamilton and Edwards 1976, Monserud 1976) with some exceptions (e.g., Glover and Hool 1979, Amateis et al. 1989, Guan and Gertner 1991). Stand-level mortality has been modeled with

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regression equations (Lohrey and Bailey 1976, Clutter and Jones 1980, Matney et al. 1987) or with probability distributions (Somers et al. 1980, Buford and Hafley 1985). A method to produce compatible estimates between individual tree mortality and stand-level mortality was introduced by Harrison and Daniels (1988) and modified by McTague and Stansfield (1994).

Mortality occurs throughout the growth period due to competition among trees. However, the process of modeling stand-level mortality can be greatly simplified by assuming that mortality for an entire growing period occurs only at the beginning of that period. After that, the stand will grow without further mortality. The above assumption will be used throughout this paper. Three different situations concerning mortality will be discussed below.

Case 1

If no mortality occurred during the growing period, and if diameter variable was transformed to preserve the functional form of the distribution, tree diameter growth models were derived by Bailey (1980). He showed that for the Weibull, lognormal, and generalized gamma distributions, a nonlinear diameter growth function was implied. A linear growth function was derived for the exponential, normal, beta and Johnson's S_B distributions.

Case 2

When mortality is assumed to be proportionally distributed over the diameter distribution, Bailey's system still works in the following manner. First, stand density is reduced across diameter classes by the same proportion, i.e., the diameter distribution remains unchanged but with fewer number of trees. The stand then will grow without further mortality as in case 1.

Case 3

The assumption that mortality is proportionally distributed over the diameter distribution might not be realistic. Smaller trees should suffer more from competition than large trees and are more likely to die. Case 3 deals with the situation when mortality is not proportionally distributed across diameter classes. This case was not covered by Bailey (1980). As an example, diameters in a stand originally follow a Weibull distribution. It is expected that a higher proportion of small trees will die as compared with large trees. Let us suppose that mortality for this entire growing period occurs at the beginning of the period. If a Weibull function can be used to approximate diameter distribution immediately after mortality, then we return to case 1, when the stand is assumed to grow without mortality to the end of the growth period, and its diameter distribution remains Weibull. This paper attempts to approximate the diameter distribution of a stand immediately after mortality with the same form of pdf as used for that stand before mortality. Specific details for the Weibull and beta distributions will be discussed in the following sections.

It should be mentioned here that the end result of natural mortality is actually very similar to that of low thinning in terms of the distribution of trees being eliminated. A higher proportion of small trees is usually removed in a low thinning operation, and most of the large trees are left to grow. Therefore low thinning can be treated as natural mortality in this regard; and the proposed approach can also be applied to modeling a stand immediately after low thinning to preserve the functional form of the distribution before and after thinning.

The Weibull Distribution

Suppose a Weibull pdf is used to describe the diameter distribution, with location parameter a, scale parameter b, and shape parameter c. Let $X = D - a$, where D is diameter at breast height (dbh). Now the location parameter drops out and X follows a two-parameter Weibull pdf as follows:

$$f(x) = (c/b)\,(x/b)^{c-1}\,exp\,[-(x/b)^c] \quad x > 0 \tag{1}$$

Let $g(x)$ be survival ratio or the ratio of number of trees after vs. before mortality. This ratio should decrease for small diameters and increase for large diameters. The function $g(x)$ can be modeled using a Weibull cumulative distribution function (cdf) as follows:

$$g(x) = 1 - \exp(-r_1 x^{r_2}) \quad x > 0, \tag{2}$$

where r_1 and r_2 are coefficients.

When $r_2 = 1$, $g(x)$ becomes an exponential cdf. Equation (2) ensures that $g(x)$ has a value of near 0 for small x and near 1 for large x (Figure 1). Number of trees per acre after mortality, R, can be calculated from number of trees before mortality, N, as follows:

$$R = N\int_0^\infty f(x)g(x)dx \tag{3}$$

Good results were obtained when parameter r_2 was approximately 2 and therefore r_2 was fixed at 2. The remaining parameter, r_1, of $g(x)$ can be numerically solved from equation (3), since f(x) has already been defined.

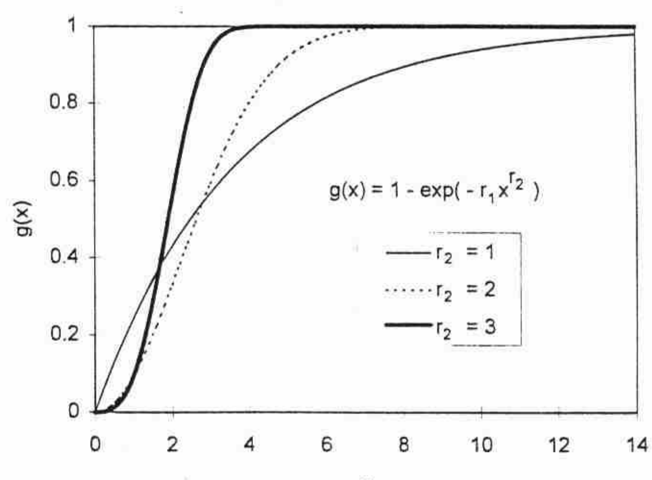

Figure 1. Shape of function *g(x)* for survival ratio at diameter *x*.

Let $h(x)$ be the pdf after mortality at the beginning of the growth period, $h(x)$ is computed from $f(x)$ and $g(x)$ as

$$h(x) = (N/R)\, f(x)\, g(x) \quad x > 0. \tag{4}$$

Although $h(x)$ is not a Weibull pdf, it can be closely approximated by $w(x)$, which is a Weibull pdf. The two parameters, b_w and c_w, of $w(x)$ can be solved by equating the first two moments of $h(x)$ and $w(x)$ as follows:

$$\int_0^{\infty} xh(x)dx = b_w \Gamma(1 + 1/c_w) \tag{5}$$

$$\int_0^{\infty} x^2 h(x)dx = b_w^2 \Gamma(1 + 2/c_w) \tag{6}$$

where $\Gamma(x)$ denotes the complete gamma function evaluated at x.

This method transforms a Weibull pdf before mortality, $f(x)$, to a Weibull pdf after mortality, $w(x)$. Now the situation resembles that in case 1, when the stand is allowed to grow to the end of the period without further mortality. According to Bailey (1980), diameters at the end of the growing period also follows a Weibull distribution, $k(x)$, if diameter growth is a power function:

$$D_2 = \beta_0 + \beta_1 (D - a)^{\beta_2} \tag{7}$$

where D_i is tree diameter at time i, a is the Weibull location parameter at the beginning of the growing period, and the β's are coefficients. Bailey (1980) also discussed the relationship between parameters of $w(x)$ and $k(x)$ and coefficients of the implied diameter growth function.

A simulation study was conducted to evaluate the feasibility of using a Weibull $w(x)$ to approximate $h(x)$, the pdf immediately after mortality. Since the numbers of rejections at the 5% level were close to 5 out of 100 samples for most combinations of parameters and residual ratios, the approximation of the pdf after mortality by a Weibull distribution could be considered reliable. Figure 2 shows an example of $f(x)$, $h(x)$, and $w(x)$.

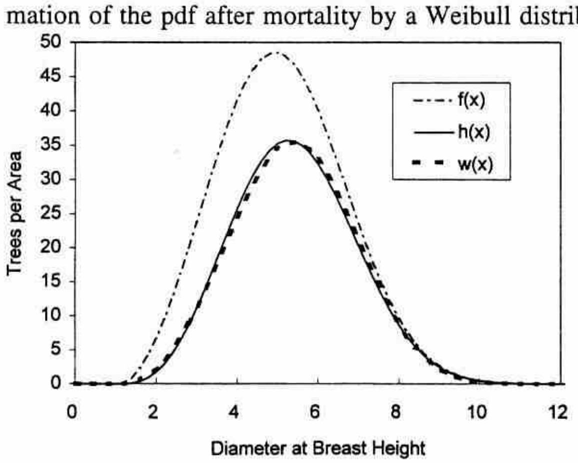

Figure 2. An example of $f(x)$ that is a Weibull distribution for tree diameters before mortality, $h(x)$ that describes diameter distribution after mortality, and a Weibull $w(x)$ that approximates $h(x)$.

The Beta Distribution

Let $Y = (D - Dmin)/(Dmax - Dmin)$, where $Dmin$ and $Dmax$ are minimum and maximum diameters, respectively. The beta pdf in terms of Y is as follows:

$$f(y) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} y^{\alpha - 1}(1 - y)^{\beta - 1} \quad 0 < y < 1 \tag{8}$$

The survival ratio function in this case was selected such that it approaches 0 for y near 0, and 1 for y near 1:

$$g(y) = exp\,[-r_1\,(1 - y)/y] \quad 0 < y < 1 \tag{9}$$

Even though $g(y)$ has a form different from $g(x)$ in Equation (2) (because the ranges of y and x are different), the shape of $g(y)$ is very similar to that presented in Figure 1.

The pdf after mortality at the beginning of the growth period is:

$$h(y) = (N/R)\, f(y)\, g(y) \quad 0 < y < 1. \tag{10}$$

This function, $h(y)$, can be approximated by a beta pdf, $b(y)$. Similar to the method presented above for the Weibull distribution, the two parameters, α_b and β_b, of $b(y)$ can be solved by equating the first two moments of $h(y)$ and $b(y)$ as follows:

$$\int_0^1 yh(y)dy = \frac{\alpha_b}{\alpha_b + \beta_b} \tag{11}$$

$$\int_0^1 y^2 h(y)dy = \frac{\alpha_b(\alpha_b + 1)}{(\alpha_b + \beta_b)(\alpha_b + \beta_b + 1)} \tag{12}$$

This technique transforms a beta pdf before mortality, $f(y)$, to a beta pdf after mortality, $b(y)$. The stand will grow to the end of the growth period without suffering from mortality, as described above in case 1. If the diameter growth function is linear, $D_2 = \beta_0 + \beta_1 D_1$, the diameter distribution of the stand at the end of the growth period is also beta (Bailey 1980).

As with the Weibull case, a simulation trial was conducted to determine how well the beta $b(y)$ approximated $h(y)$. Again, the numbers of rejections at the 5% level were near 5 out of 100 samples in most cases, indicating that the beta distribution $b(y)$ could successfully approximate $h(y)$, the pdf immediately after mortality.

A Numerical Example

Data

Data available for this study were from the Southwide Seed Source Study, which involved 15 loblolly pine seed sources planted at 13 locations from Texas to Maryland.

Detail about this research project was presented by Wells and Wakeley (1966). A total of 522 plots from 12 locations having diameter and height measurements from age 10 to age 25 or 27 were selected for this study. Most of the plots were measured four times, with intervals between measurements ranging from 4 to 7 yr. Table 1 shows plot distribution by site index, stand age, and density.

The data were divided into a fit data set and a validation data set. Five plots were randomly selected from each location to form a validation data set. The fit data set, which consisted of the remaining 462 plots was used to develop predicting equations. The validation data set was used to evaluate the performance of the following three methods of modeling diameter distributions. A description of stand attributes for the fit and validation data sets is presented in Table 2.

Methods of Modeling Diameter Distribution

The evaluation involved different methods of using the Weibull distribution to model diameters of trees survived at the end of a growth period, given information at the beginning of that period.

Method 1

This method served as a "control" to the rest of the methods, by demonstrating how well the Weibull pdf fit diameter distributions of trees at the end of growth periods *when stand attributes were known*. Information at the end of the growth period (age A_2) included surviving trees per acres (N_2), minimum diameter ($Dmin_2$), quadratic mean diameter (Dq_2), and the 93rd diameter percentile ($D93_2$). Method 1 is a modified percentile method that computes Weibull parameters at age A_2 as follows:

$$a = 0.60\,(Dmin_2),$$

$$b = (D93_2 - a)\,/\,(2.65926)^{1/c}$$

c is solved from

$$a^2 + 2ab\Gamma(1 + 1/c) + b^2\Gamma(1 + 2/c) - Dq_2^{\,2} = 0$$

Method 2

Developed by Baldwin and Feduccia (1987), this method employed the Weibull to model diameter distributions and involved the following steps:

Table 1. Plot distribution by site index, stand age, and density.

Site index[a] (ft)	Stand age (yr)	No. of trees/ac[a] ≤100	200	400	600	800	1,000	>1,100	Total
					No. of plots				
40	10						2	2	4
	15					2	1	1	4
	20			1					1
	22			3					3
	25			1					1
	27			3					3
50	10		3	2	2	3	37	12	59
	15	2	1	2	4	7	34	9	59
	20	2	2	5	23	4			36
	22		6	17					23
	25	3	2	8	21	2			36
	27		7	16					23
60	10	7	8	15	23	38	64	26	181
	15	7	12	18	47	32	39	13	168
	16			4	7				11
	20	7	25	57	76	5			170
	22		2	6					8
	25	10	29	30	54	2			125
	27	1	26	25					52
70	10	14	35	48	38	61	36	6	238
	15	15	39	53	55	14			176
	16			26	23				49
	20	14	66	100	39	5			224
	22	1							1
	25	11	75	58	12	1			157
	27	1	35	15	1				52
80	10	1	2	7	6	11	10	3	40
	15	2	3	9	9	6			29
	16			5	3				8
	20	1	8	20	6	2			37
	25	3	13	4	6				26
	27		5	2					7
	Total	102	404	560	455	195	223	72	2,011

[a] Conversion factors: 1 ft = 0.30 m; 1 tree/ac = 2.47 trees/ha.

Table 2. Description of stand attributes for the fit and validation data sets

	Fit data set (n = 1,782)			Validation data set (n = 229)		
Stand attribute[a]	Minimum	Mean	Maximum	Minimum	Mean	Maximum
A (yr)	10	18	27	10	17	27
N (trees/ac)	25	522	1,210	25	544	1,185
B (ft²/ac)	46	117	269	2	120	234
Hd (ft)	14	49	82	18	47	79
$Dmin$ (in.)	0.0	4.2	16.6	0.3	4.0	10.4
$\bar{D}$ (in.)	1.6	6.8	16.6	2.2	6.7	11.9
Dq (in.)	1.8	7.0	16.6	2.6	6.8	12.0
$D93$ (in.)	2.5	8.8	16.8	3.9	8.7	14.4

[a] Notation:
- A = Stand age,
- N = Number of trees/ac (1 tree/ac = 2.47 trees/ha),
- B = Basal area per acre (1 ft²/ac = 0.23 m²/ha),
- Hd = Average height of the dominants and codominants (1 ft = 0.30 m),
- $Dmin$ = Minimum diameter in the plot (1 in.= 2.54 cm),
- $\bar{D}$ = Arithmetic mean diameter,
- Dq = Quadratic mean diameter, and
- $D93$ = The 93rd diameter percentile.

1. Information at the beginning of the growth period (age A_1) was obtained: trees per acre (N_1), minimum diameter ($Dmin_1$), quadratic mean diameters (Dq_1), and the 93rd diameter percentile ($D93_1$).
2. Stand attributes at age A_2 (N_2, $Dmin_2$, Dq_2, and $D93_2$) were predicted using the following equations:

$$N_2 = [N_1^{b_1} + (b_2 + b_3 / S)(A_2^{b_4} - A_1^{b_4}]^{1/b_1} \quad (13)$$

$$Y_2 = [(1 - A_2 / A_1)(b_1 + b_2 / N_1 + (A_2 / A_1)Y_1^{b_3})]^{1/b_3} \quad (14)$$

where

N_i = Trees per acre at age A_i,

S = Site index (base age 25 yr) in ft,

Y_i = Diameter ($Dmin$, Dq, or $D93$) in inches at age A_i, and

b_i = Regression coefficients calculated from the fit data set (Table 3).

3. Weibull parameters at age A_2 were computed using the modified percentile method as in method 1.

Method 3

This method utilized the new procedures introduced in this paper. The following steps were taken:

1. Information at the beginning of the growth period (age A_1) was obtained: trees and basal area per acre (N_1 and B_1, respectively), minimum diameter ($Dmin_1$), and arithmetic and quadratic mean diameters ($\bar{D}_1$ and Dq_1, respectively).
2. Weibull parameters before mortality at age A_1 were computed as follows:

$$a = 0.60\,(Dmin_1),$$

$$b = (\bar{D}_1 - a) / \Gamma(1 + 1/c),$$

c is solved from

$$a^2 + 2ab\Gamma(1 + 1/c) + b^2\Gamma(1 + 2/c) - Dq_1^2 = 0$$

3. Stand density in terms of trees per acre (N_2) at the end of the growing period (age A_2) was predicted from Equation (13). This is the same survival equation used in method 2 above.
4. Parameter r_2 of $g(x)$ was set to 2. Parameter r_1 was solved from Equation (3).
5. The Weibull distribution after mortality at age A_1, $w(x)$, was defined by solving for its parameters b_w, and c_w.
6. The stand then would grow to age A_2 without mortality. The individual tree diameter growth equation used in this study was

Table 3. Equations to project stand attributes from age A_1 to age A_2. Stand attributes as defined in Table 2.

Stand attribute	Eq.	b_1	b_2	b_3	b_4	n	Fit index[a]	$s_{y.x}$
N	13	−1.35199	0.01364	−0.35242	2.87452	1,320	0.79	109.21
$Dmin$	14	−133.86226	0.10302	2.81386		1,320	0.64	1.17
Dq	14	−85.18064	0.06594	2.33188		1,320	0.93	0.48
$D93$	14	−171.14194	0.12864	2.39072		1,320	0.89	0.68
D	15	0.07327	−0.00071	0.53494		23,977	0.93	0.56

[a] Fit Index = $1 - [\Sigma(y_i - \hat{y}_i)^2 / \Sigma(y_i - \bar{y}_i)^2]$.

$$D_2 = D_1 \left(\frac{1-\exp[(b_1+b_2B_1)A_2]}{1-\exp[(b_1+b_2B_1)A_1]} \right)^{b_3} \quad (15)$$

where

D_i = tree diamter in inches at age A_i

B_1 = basal area (ft^2/ac) at age A_1, and

b_i = regression coefficients calculated from the fit data set (Table 3).

Equation (15) was based on the Chapman-Richards growth function (Chapman 1961, Richards, 1959). Bailey (1980) stated that the appropriate transformation from A_1 to A_2 to maintain the Weibull pdf is

$$D_2 = \beta_0 + \beta_1 (D_1 - a)^{\beta_2} \quad (16)$$

where D_i is tree diameter at time i, a is the Weibull location parameter at A_1 and the β's are coefficients. Equation (15) can be written in the form of Equation (16) with

$$\beta_2 = 1,$$

$$\beta_1 = \left(\frac{1-\exp[b_1+b_2B_1)A_2]}{1-\exp[b_1+b_2B_1)A_1]} \right)^{b_3}, \text{ and}$$

$$\beta_0 = \beta_1 a$$

where B_1 is basal area per acre at age A_1.

7. Parameters of $k(x)$, Weibull distribution at the end of the growth period were calculated according to Bailey (1980):

$$a_k = \beta_0$$

$$b_k = \beta_1 b_w^{\beta_2}$$

$$c_k = c_w / \beta_2$$

Method 4

This method is very similar to method 3, except for the following modifications:

1. Observed values of surviving trees per acre (N_2) at the end of the growth periods were used instead of predicted values from Equation (13).
2. Diameter growth function of the form $D_2 = b_0 + b_1 D_1$ was fitted to data from each individual plot for each growth period. This equation was used instead of Equation (15) which was fitted to diameter data of all plots.

The modifications should prevent additional errors due to predicting total survival and projecting diameter growth.

Evaluation

The three methods were evaluated using three forms of an error index recommended by Reynolds et al. (1988). For each observation (plot measurement at age A_2), the following error indices were calculated for each of the three methods:

$$EI_N = \sum_i |n_i - \hat{n}_i|$$

$$EI_B = \sum_i |b_i - \hat{b}_i|, \text{ and}$$

$$EI_V = \sum_i |v_i - \hat{v}_i|.$$

where

$n_i - \hat{n}_i$ = observed minus predicted trees per acre in the ith diameter class (1 in. class),

$b_i - \hat{b}_i$ = observed minus predicted basal area (ft^2/ac) in the ith diameter class,

$v_i - \hat{v}_i$ = observed minus predicted volume (ft^3/ac) in the ith diameter class.

Total outside-bark volume for each tree was computed using Baldwin and Feduccia's (1987) individual tree volume equation. The overall error index for each method based on trees per acre was computed as the mean of all EI_N's. Mean error indices based on basal area and volume per acre were also calculated in the same manner. The mean error indices measured how well the pdf's fit the empirical diameter distributions in terms of number of trees, basal area, and volume per acre; the smaller the mean error index value, the better the fit.

Results and Discussion

Mean error indices based on trees, basal area, and volume per acre for each of the three methods are presented in Table 4. A low mean error index indicated little departure between observed and predicted attributes (number of trees, basal area, or volume per acre, respectively) in each diameter class.

Method 1 involved fitting a Weibull function to the diameter data using observed stand attributes (diameter percentiles and surviving number of trees) at the *end* of the growth period. In this method, a smooth Weibull function was derived from three observed diameter values ($Dmin_2$, Dq_2, and $D93_2$) to represent the diameter data. Difference between observed and predicted values for each diameter class should be minimum compared to the other three methods. The resulting error index values for number of trees, basal area, and volume per acre (Table 4) were indeed lowest among all methods as expected. These values should be used

Table 4. Mean error indices for four methods based on 166 observations.

Method	Error index based on					
	Trees/ac		Basal area/ac		Volume/ac	
	Mean error index value [a]					
1	197.93	a	61.05	a	1,659	a
4	199.75	a	62.63	ab	1,694	a
3	233.57	b	68.93	b	1,852	a
2	235.08	b	69.83	b	1,871	a

[a] The smaller the error index value, the better the fit. For each column, values with the same letter are not significantly different at the 5% level.

as the standards against which to evaluate the rest of the methods.

Baldwin and Feduccia's (1987) model was chosen to represent Weibull-based growth and yield models because this system was relatively straightforward and also produced reasonable results. Both Baldwin and Feduccia's (1987) procedure (method 2) and the new technique (method 3) required the same amount of information at the beginning of the growth period at age A_1: trees per acre (N_1), minimum diameter ($Dmin_1$), quadratic mean diameters (Dq_1). An additional variable was the 93rd diameter percentile ($D93_1$) for method 2 and the average diameter ($\overline{D}_1$) for method 3. Both methods employed Equation (13) to predict total number of surviving trees per acre (N_2) at age A_2, the end of the growing period. That was where the similarities ended. In method 2, $Dmin_2$, Dq_2, and $D93_2$ at age A_2 were predicted, and Weibull parameters at age A_2 were then recovered from these diameter attributes. On the other hand, method 3 involved the computations of Weibull parameters based on $Dmin_1$, Dq_1, and $\overline{D}_1$ at age A_1. These Weibull parameters were then modified to account for mortality, and finally "grown" to age A_2.

Method 3 produced consistently lower error index values than Baldwin and Feduccia's (1987) model (method 2) in all categories (Table 4). However, differences between the two methods were not significant at the 5% level. These results demonstrated that comparable practical results were obtained for method 2, a purely empirical model, and method 3, which was based on an extension of Bailey's (1980) theoretical relationship between distributions at two time periods. Error indices from methods 2 and 3 were significantly different at the 5% level from those produced by method 1, in terms of trees and basal area per acre. Methods 2 and 3 were based on information at the *beginning* of the growth period, and on the *projection* of both survival and diameter distribution to the end of the growth period. That explains the higher error indices for these two methods.

How well would method 3 perform if total number of surviving trees at the end of the growth period was known? Method 4 was developed to alleviate the additional prediction problems incurred by method 3 in predicting total tree survival and diameter growth. In method 4, observed rather than predicted values were used for total surviving trees per acre at the end of the growth period. Furthermore, better individual diameter growth prediction was also obtained by fitting a separate diameter growth function to data from each plot. Therefore, difference between results of methods 1 and 4 was due primarily to whether or not the proposed procedure of distributing mortality was successful. Values of error index were still 0.9% to 2.6% higher for method 4 as compared to method 1, even though they were not significantly different at the 5% level (Table 4). Since method 1 involved fitting Weibull distributions to observed data, the above findings showed that the Weibull functions produced by method 4 was almost as good as those from method 1 in characterizing diameter distribution data. It is apparent that the proposed technique to distribute mortality among diameter classes provided reasonable results for this data set.

It should be noted that if diameter distribution of the initial stand follows a Weibull, then the future stand is expected to also resemble a Weibull. The proposed technique should be suitable in this case. On the other hand, when multimodal diameter distributions are involved, other methods such as parameter-free diameter-distribution recovery (Matney et al. 1990) and stand table projection (Pienaar and Harrison 1988, Nepal and Somers 1992) are more appropriate.

Conclusion

That mortality is not proportionally distributed among diameter classes is a reasonable assumption and the technique suggested here should broaden Bailey's (1980) approach to describe the relationship between diameter distributions at the beginning and the end of a growing period. Although this study focuses on the theoretical extension of Bailey's (1980) work, the new approach also provided reasonable approximation for diameter distributions as applied to real data. In the example, performance of the new method was comparable to that of Baldwin and Feduccia's (1987) empirical approach of modeling diameter distributions. Similar error index values from models 1 and 4 further demonstrated that the concept of variable transformation to preserve the functional form of the pdf that approximates diameter distributions had merits.

Even though only the Weibull and beta distributions were discussed in this paper, a similar approach with minor modifications should work with other distributions as well. This technique can also be applied to modeling a stand immediately after low thinning. Low thinning is similar to natural mortality in that a higher proportion of small trees is usually removed in a low thinning operation. The proposed approach preserves the functional form of the distribution before and after thinning. However, the residual ratio function $g(x)$ might have a different shape and thus might need to be respecified.

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