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Predicting tree height from tree diameter and dominant height using mixed-effects and quantile regression models for two species in Turkey

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ABSTRACT

Height-diameter models were developed for Brutian pine (*Pinus brutia* Ten.) and Taurus cedar (*Cedrus libani* A. Rich.) in Turkey. A modified Chapman-Richards model that includes dominant height was used to predict tree height from diameter. Using the twofold evaluation scheme, five alternative modeling approaches were evaluated: (1) fixed-effects model, (2) calibrated fixed-effects model, (3) calibrated mixed-effects model, (4) three-quantile regression method, and (5) five-quantile regression method. Parameters of fixed-effects, mixed-effects and quantile regression models were calibrated by use of a subset of height measurements, ranging from 1 to 10 sample trees per plot. Evaluation statistics show that both quantile regression models produced similar results, and that the mixed-effects model approach yielded the best results in predicting tree heights. Model performance improved with increasing sample size; but gains in performance generally increased at a decreasing rate. A sample size of four trees per plot appears to be a good compromise between sampling cost and predictive accuracy and precision.

1. Introduction

Brutian pine (*Pinus brutia* Ten.) and Taurus cedar (*Cedrus libani* A. Rich.) are two of the economically and ecologically most important tree species in Turkey and presently found primarily in the Mediterranean Region of Turkey. The forests of brutian pine and Taurus cedar are predominantly composed of mature stands and provide important global and national benefits related to carbon storage and biodiversity, along with the other ecosystem services in Turkey. Brutian pine forms extensive forests, especially in regions where the Mediterranean climate prevails (Boydak, 2004). This tree species grows in pure stands and is commonly found in fire-related ecosystems. Brutian pine grows on many soil types, but primarily on rendzina soils on soft limestones and marl deposits (Atalay et al., 1998). Taurus cedar forests are located primarily in the Taurus Mountains in the Mediterranean Region of Turkey at elevations between 800 m and 2100 m on calcareous formations. However, this species can also be found at lower and higher elevation as small populations.

In recent years, Turkey has adopted the approach of multipurpose and ecologically based forest management. The General Directorate of Forests (GDF) therefore needs to develop and evaluate growth and yield prediction models for sustainable management of forest resources. Many growth and yield models require total tree height (h) and breast height diameter (d) as basic input variables (Temesgen et al., 2007), therefore, the height-diameter (h–d) models are considered one of the most important components of growth and yield models. *H-d* models are very useful for yield estimation (Curtis, 1967; Parresol, 1992), site index and dominant height estimation (Curtis, 1967; Calama and Montero, 2004), stand structural analysis (Spies and Cohen, 1992; Morrison et al., 2012; Von Gadow et al., 2001), damage appraisal and stand stability (Parresol, 1992; Vospernik et al., 2010), stand growth dynamics (Curtis, 1967; Burkhart and Strub, 1974; Wykoff et al., 1982), individual tree and stand volume prediction (Peng, 2001a; Gómez-García et al., 2014), and product recovery and carbon budgeting models (Newton and Amponsah, 2007). Furthermore, h–d models are needed to better understand the nature of various relationships that characterize, differentiate, and influence the development of forest ecosystems (Peng, 2001b). However, so far, the available information about *h-d* relationships concerning these tree species is very limited.

The cost of measuring tree height in forest inventory is greater than that of measuring tree diameter, leading to the need for equations to predict tree height from measured diameter at breast height. Most h–d

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models have been applied to pure even-aged stands or plantations (e.g. Soares and Tomé, 2002; Sánchez et al., 2003; Diéguez-Aranda et al., 2005). Despite the homogeneous characteristics of these types of forest, a single *h–d* model is not usually adequate for all possible situations, since the *h–d* relationship varies from stand to stand, and is not even constant within the same stand over time (Curtis, 1967). Some *h-d* models also have been developed to mixed, uneven-aged stands in which different species, age, structure, and levels of competition (Temesgen and Gadow, 2004; Corral-Rivas et al., 2014; Crecente-Campo et al., 2014).

The most widely used method for minimizing this level of variance is to fit a local *h–d* equation for each plot and measurement occasion. The main problem with this approach is that it requires a high sampling effort. Recently, several studies have shown that the use of plot-level information in addition to dbh could provide more accurate prediction of height than using only dbh. Stand variables that have been added to the base model include stand-density measures (Temesgen and Gadow, 2004), stand density, dominant height, and diameter distribution percentiles (Calama and Montero, 2004; Dorado et al., 2006; Temesgen et al., 2008), stand density and relative position (Temesgen et al., 2007), stand density, species composition, and top height (Huang et al., 2009), relative spacing index and quadratic mean diameter (Ducey, 2009; Saud et al., 2016). The inclusion of these variables improved the precision of height estimates. In contrast, Sharma and Zhang (2004) and Huang et al. (2009) found that site index and species composition did not improve the precision of height prediction in their studies, respectively. Huang et al. (2009) found that top height is the most significant contributor among different stand-level variables. Moreover, dominant height is usually measured for use in site index equations or other submodels of growth simulators (Gómez-García et al., 2014).

H-d relationships are often modelled by use of ordinary least squares (*OLS*) regression technique. However, this method is not reliable because the assumption of random and independent observations is often violated and the presence of autocorrelation is not accounted for. These problems can be addressed with nonlinear mixed-effects models (NMEM), which allow for both *population-averaged* and *subject-specific* models. The first considers only fixed-parameters, common to the population, while the second considers both fixed- and random-effects parameters, common to each subject. The inclusion of random parameters, specific for each plot, allows for modeling the variability of the *h-d* relationship among different locations, after defining a common fixed functional structure (Lindstrom and Bates, 1990). If prediction for a new stand is required and prior information from a small sample of trees measured for *h* and *d* is available, the *h-d* curve can be calibrated for that particular stand. Many studies have used mixed-models to describe *h-d* relationship (e.g., Lappi, 1997; Calama and Montero, 2004; Trincado et al., 2007; Sharma and Parton, 2007; Huang et al., 2009; Crecente-Campo et al., 2010; Özçelik et al., 2013; VanderSchaaf, 2014; Gómez-García et al., 2015; Saud et al., 2016; Zang et al., 2016). Temesgen et al. (2008) used a correction factor to calibrate a nonlinear fixed-effects model to local conditions, and found that generally, although the calibrated mixed-effects model performed better than the calibrated fixed-effects model, the differences were greatly dampened when additional tree- and stand-level factors were included.

Some authors have also tested the performance of NMEM for predicting tree height when a subsample of tree heights from a new stand is available. However, there is no unified agreement on what should be the sample size to localize *h-d* curves. Calama and Montero (2004) recommended using four tree heights for calibration and Trincado et al. (2007) observed that increases in sample sizes from one tree to two and three trees provided successively smaller gains. Similar findings have been reported by Temesgen et al. (2008) and Huang et al. (2009) for sample sizes ranging from 1 to 15 trees and from 1 to 9 trees, respectively.

Quantile regression, introduced by Koenker and Bassett (1978), has been gaining popularity in forestry research in recent years. Quantile regression is a method of estimating the complete conditional distribution of dependent variables and assessing the effects of predictors at different quantiles, whereas the mean regression estimator addresses only the conditional mean or the central effects of the covariates. As a result, quantile regression is a flexible method to depict varying patterns of the relationship between *y* and *x*. Quantile regression has been used in research dealing with error assessment in forest inventory (Mäkinen et al., 2008), self-thinning boundary lines (Zhang et al., 2005; Ducey and Knapp, 2010), spreading rates of insects (Evans and Gregoire, 2007) and diseases (Evans and Finkral, 2010), diameter percentiles (Mehtätalo et al., 2008), diameter growth (Bohora and Cao, 2014), and tree taper (Cao and Wang, 2015).

Zang et al. (2016) used quantile regression to describe various patterns of the *h-d* relationship, but did not show how to use curves from different quantiles to predict tree height from diameter. Calibration of quantile regression models has so far been based on only one observation, either dbh measurement at a certain age (Bohora and Cao, 2014) or bole diameter measurement at a relative tree height (Cao and Wang, 2015). New procedures were therefore necessary to deal with calibration data consisting of multiple observations, i.e., more than one sampled height per plot.

The goals of this study were to (a) evaluate the predictive capability of *h-d* models developed using fixed-effects, mixed-effects, and quantile regression, (b) evaluate the impact of including stand-level variables into the model, (c) compare different approaches for calibrating *h-d* curves to local conditions, and (d) determine the required sample size of trees for calibration.

2. Material and methods

2.1. Data

The Mediterranean region starts from Gelibolu peninsula, follows the Aegean and Mediterranean coasts and ends up in the Amanos Mountains. While the taxa of Brutian pine, Black pine, Taurus cedar and Cilician fir constitute the coniferous forests in this region; juniper and oaks species also accompany them. Data for this study were obtained from natural, even-aged, and pure stands (more than 90% of trees) throughout the area of distribution of these species in Mediterranean Region of Turkey. A total of 58 temporary sample plots for Brutian pine and 88 temporary sample plots for Taurus cedar were measured. The distribution of sample plots represent the existing range of ages, stand densities and sites with altitude ranging from 50 to 1025 m for Brutian pine and 1210 to 1750 m for Taurus cedar. Plot size ranged from 120 to 3000 m^2, depending on stand density, in order to achieve a minimum of 30 trees per plot. For each tree, two perpendicular diameters (outside-bark 1.3 m above ground level) were measured to the nearest 0.1 cm and were then averaged to obtain diameter at breast height (*d*, cm). In each plot, approximately one-third of the trees were selected to ensure a representative distribution by diameter and height classes. Total heights of these trees were measured to the nearest 0.5 m with a Blume-Leiss hypsometer. The dominant height was defined as the average height of the 100 largest-diameter trees per hectare, depending on plot size.

The data were randomly divided into two groups; each contained the same number of plots (Table 1). We used the two-fold evaluation scheme (Bohora and Cao, 2014), in which parameters of the *h-d* model fitted to one group was applied to predict for the other group. The predictions from both groups were then used to compute evaluation statistics for different methods.

2.2. Nonlinear fixed-effects model

A large number of nonlinear model forms were evaluated for both tree species; including those reported by Huang et al. (2000) and Peng (2001b). The nonlinear models were fitted by use of *OLS* to the whole

Table 1
Summary statistics by group and tree species.

Variable[a]	Group 1				Group 2			
	Mean	Min.	Max.	S.D.	Mean	Min.	Max.	S.D.
	Brutian pine (2370 trees in 29 plots)				*Brutian pine (2362 trees in 29 plots)*			
d (cm)	21.3	4.0	66.0	12.3	20.8	4.0	71.0	12.6
h (m)	13.5	2.0	30.0	5.2	13.36	2.3	34.0	5.4
N (treesha^{-1})	1129	371	3311	756.6	1075	200	2157	574.4
BA ($\text{m}^2\,\text{ha}^{-1}$)	37.3	20.1	61.2	10.5	35.6	19.1	62.9	10.3
Hd (m)	19.4	13.0	26.6	4.4	20.2	13.5	31.9	4.7
	Taurus cedar (1483 trees in 44 plots)				*Taurus cedar (1462 trees in 44 plots)*			
d (cm)	32.5	4.1	71.8	12.7	32.3	7.6	68.0	11.5
h (m)	16.9	2.6	32.1	5.4	17.0	5.4	32.0	4.6
N (trees ha^{-1})	653	170	1597	335.4	565	168	1319	297.7
BA ($\text{m}^2\,\text{ha}^{-1}$)	47.6	21.3	85.1	18.6	44.3	14.2	80.6	15.9
Hd (m)	21.8	7.6	30.4	4.8	21.5	13.0	30.7	3.6

[a] *d*, diameter at breast height (1.3 m above ground level); *h*, total tree height; *N*, number of trees per hectare; *BA*, basal area per hectare and *Hd*, dominant height.

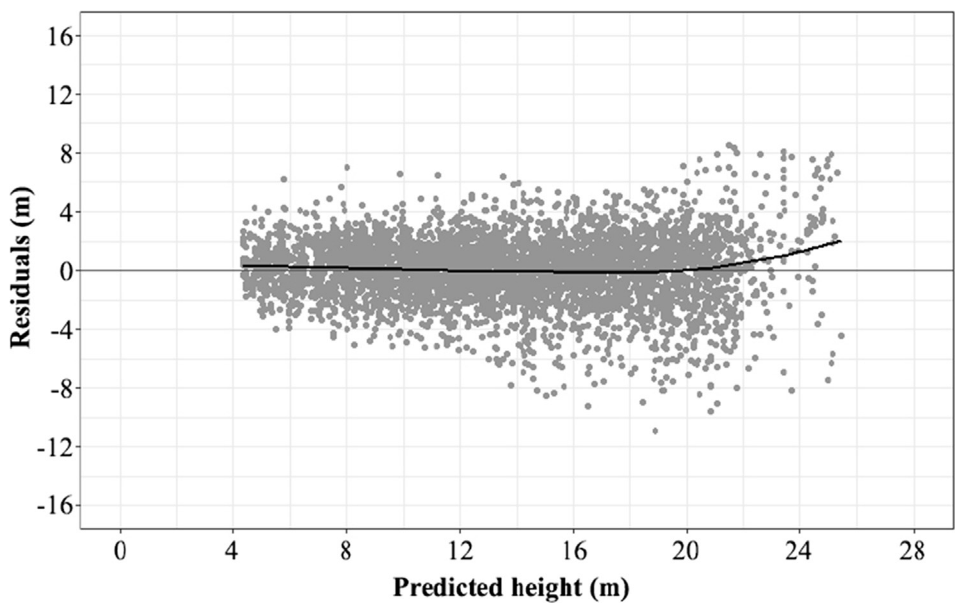

Fig. 1. Residual plot from the modified Chapman-Richards model for Brutian pine.

data set. Among the candidate models, the Chapman-Richards delivered the best performance to model the *h-d* relationship for both tree species.

$$h_{ij} = 1.3 + \beta_1[1-exp(-\beta_2 d_{ij})]^{\beta_3} + \varepsilon_{ij}, \tag{1}$$

where h_{ij} and d_{ij} are, respectively, total height (m) and diameter at breast height (cm) of the *j*th tree in the *i*th plot, β_1–β_3 are model parameters, and ε_{ij} is random error.

In order to account for the effects of stand density and dominant height on height prediction, the following stand-level variables were added to the above base model: basal area per ha, number of trees per ha, dominant height, and quadratic mean diameter. The final model used in this study, with dominant height as the extra stand variable, has the following form:

$$h_{ij} = 1.3 + (\beta_1 + \beta_2 Hd_i)[1-exp(-\beta_3 d_{ij})]^{\beta_4} + \varepsilon_{ij}, \tag{2}$$

where Hd_i is dominant height of the *i*th plot, β_1–β_4 are model parameters, and all other variables are as previously defined.

2.3. Calibration of the fixed-effects model

Temesgen et al. (2008) showed that when the heights of a subsample of n_{im} trees from the i^{th} stand are known, the predicted heights of the remaining trees from the same stand can be calibrated by use of the following correction factor:

$$k^* = \frac{\sum_{j=1}^{n_{im}}[(\hat{h}_{hi}-1.30)(h_{ij}-1.30)]}{\sum_{j=1}^{n_{im}}(\hat{h}_{ij}-1.30)}, \tag{3}$$

where k^* is correction factor, $\hat{h}_{hi}$ is predicted height from Eq. (2), and h_{ij} is observed height. Then the predicted height for a tree from the same stand can be calibrated as follows:

$$\tilde{h}_{ij} = 1.3 + k^*(\beta_1 + \beta_2 Hd_i)[1-exp(-\beta_3 d_{ij})]^{\beta_4}. \tag{4}$$

2.4. Nonlinear mixed-effects model

In the mixed-effects framework, all parameters of Eq. (2) can be expressed as fixed-effects parameters (common to all trees), and some or all parameters can contain additional random components, which are specific to individual plots. Eq. (2) can be written in matrix form as follows;

$$\mathbf{y_i} = \mathbf{f}(\mathbf{b},\mathbf{u_i},\mathbf{d_i}) + \boldsymbol{\varepsilon}_\mathbf{i}, \tag{5}$$

where $\mathbf{y_i} = [h_{i1},h_{i2},....,h_{in_i}]^T$, $\mathbf{d_i} = [d_{i1},d_{i2},d_{i3},....,d_{in_i}]^T$, $\boldsymbol{\varepsilon}_\mathbf{i} = [\varepsilon_{i1},\varepsilon_{i2},.....,\varepsilon_{in_i}]^T$, n_i is number of height measurements for plot *i*, and $\mathbf{b}$ and $\mathbf{u_i}$ are column vectors of fixed- and random effects parameters, respectively. The

Table 2
Fit statistics[a] for different combinations of random parameters for the modified.

Random Parameters	*AIC* (smaller is better)	ΔAIC	*BIC* (smaller is better)	ΔBIC
Taurus cedar				
None	13,225	418	13,255	429
β_1	12,937	130	12,952	126
β_2	12,947	140	12,962	136
β_3	12,844	37	12,859	33
β_4	12,835	28	12,849	23
β_1 and β_2	12,939	132	12,958	32
β_1 and β_3	**12,807**	**0**	**12,826**	**0**
β_1 and β_4	12,812	5	12,832	6
β_2 and β_3	13,002	195	13,022	196
β_2 and β_4	12,812	5	12,832	6
β_3 and β_4	12,817	10	12,837	11
β_1, β_2 and β_3	12,812	5	12,839	13
β_1, β_2 and β_4	12,819	12	12,846	20
β_2, β_3 and β_4	12,809	2	12,836	10
Brutian pine				
None	20,983	1209	21,015	1219
β_1	20,199	425	20,211	415
β_2	20,197	423	20,209	413
β_3	19,842	68	19,833	37
β_4	19,868	94	19,889	93
β_1 and β_2	20,201	427	20,217	421
β_1 and β_4	19,784	10	19,816	20
β_2 and β_4	19,794	20	19,810	14
β_3 and β_4	19,779	5	19,796	0
β_1, β_2 and β_4	19,790	16	19,813	17
β_1, β_3 and β_4	**19,774**	**0**	**19,796**	**0**

[a] A bold, underlined number denotes the combination that results in the best statistic for each tree species. ΔAIC and ΔBIC are, respectively, differences in AIC and BIC values with respect to those from the best candidate model.

assumptions are:

$$\boldsymbol{\varepsilon}_\mathbf{i} \sim N(\mathbf{0},\mathbf{R}), \quad \text{and} \quad \mathbf{u_i} \sim N(\mathbf{0},\mathbf{D}),$$

where **R** and **D** are diagonal matrices, assuming that the $\boldsymbol{\varepsilon}_\mathbf{i}$ and $\mathbf{u_i}$ are independent. Procedure NLMIXED from SAS (SAS Institute Inc., 2008) was used to obtain fixed- and random-effects parameters of Eq. (5). When a subsample of trees in plot *i* are measured, the random parameters $\mathbf{u_i}$ for that plot can be computed from the first-order Taylor series expansion (Meng and Huang, 2009):

$$\hat{\mathbf{u}}_\mathbf{i}^{\mathbf{k+1}} = \hat{\mathbf{D}}\mathbf{Z}_\mathbf{i}^\mathbf{T}(\mathbf{Z_i}\hat{\mathbf{D}}\mathbf{Z}_\mathbf{i}^\mathbf{T} + \hat{\mathbf{R}})^{-1}[\mathbf{y_i} - \mathbf{f}(\hat{\mathbf{b}},\hat{\mathbf{u}}_\mathbf{i}^\mathbf{k},\mathbf{d_i}) + \mathbf{Z_i}\hat{\mathbf{u}}_\mathbf{i}^\mathbf{k}], \tag{6}$$

where $\hat{\mathbf{u}}_\mathbf{i}^\mathbf{k}$ is estimate of the random parameters for plot *i* at the *k*th iteration, $\hat{\mathbf{D}}$ is estimate of **D**, the variance-covariance matrix for $\mathbf{u_i}$, $\mathbf{Z_i} = \frac{\partial \mathbf{f}(\mathbf{b},\mathbf{u_i},\mathbf{d_i})}{\partial \mathbf{u_i}}\Big|_{\hat{\mathbf{b}},\hat{\mathbf{u}}_\mathbf{i}}$, **R** the variance-covariance matrix for $\boldsymbol{\varepsilon}_\mathbf{i}$, $\mathbf{y_i}$ is the $m \times 1$ vector of observed heights, and *m* is number of tree measurements used in localizing the *h-d* curve. An iterative procedure was needed to estimate $\mathbf{u_i}$. Using a null starting value $(\hat{\mathbf{u}}_\mathbf{i}^0 = \mathbf{0})$, Eq. (6) was repeatedly updated until the absolute difference between two successive iterations was smaller than a predetermined tolerance limit. The result was an approximation of the empirical best linear unbiased predictor (EBLUP) for random effects.

2.5. Quantile regression

The same function form in Eq. (2) was used to predict the τth height quantile:

$$\hat{y}_\tau(\mathrm{d_{ij}}) = 1.3 + (\beta_1 + \beta_2 Hd_i)[1 - exp(-\beta_3 d_{ij})]^{\beta_4} \tag{7}$$

where $\hat{y}_\tau(d_{ij})$ is predicted value of the τth quantile of tree height at diameter d_{ij}. In contrast to the mean regression technique, which employs the least-squares procedure, parameters from the quantile regression are obtained by minimizing

Table 3
Parameter estimates (and standard error) for the fixed- and mixed-effects regression models and quantile regression models at five quantiles (τ).

Type[a]	β_1	β_2	β_3	β_4	σ^2	$\sigma^2_{u_1}$	$\sigma^2_{u_2}$	$\sigma^2_{u_3}$	$\sigma_{u_1u_2}$	$\sigma_{u_1u_3}$	$\sigma_{u_2u_3}$
Brutian pine											
FE	9.6754 (0.3219)	0.4848 (0.0117)	0.0564 (0.0023)	1.0422 (0.0302)	4.9246 (0.1012)						
ME	5.0897 (1.5707)	0.7135 (0.0640)	0.0526 (0.0069)	1.0101 (0.0613)	3.5498 (0.0742)	4.8809 (2.7182)	0.0005 (0.0001)	0.0881 (0.0398)	−0.0436 (0.0148)	−0.2847 (0.2668)	0.0041 (0.0022)
QR (τ)											
τ = 0.1	12.126 (0.674)	0.1919 (0.0227)	0.0641 (0.0052)	1.4027 (0.0767)							
τ = 0.3	9.8345 (0.5077)	0.3940 (0.0193)	0.0666 (0.0037)	1.2801 (0.0459)							
τ = 0.5	9.6631 (0.3740)	0.4929 (0.0131)	0.0570 (0.0028)	1.0503 (0.0335)							
τ = 0.7	8.7195 (0.4823)	0.5847 (0.0151)	0.0548 (0.0034)	0.9151 (0.0295)							
τ = 0.9	7.5689 (0.5610)	0.7229 (0.0257)	0.0465 (0.0041)	0.7060 (0.0343)							
Taurus cedar											
FE	8.1368 (0.4168)	0.6219 (0.0173)	0.0551 (0.0029)	1.5113 (0.0862)	5.2044 (0.1356)						
ME	5.8724 (0.8954)	0.7076 (0.0370)	0.0616 (0.0031)	1.7321 (0.1007)	4.1295 (0.1105)	2.5106 (0.7363)	0.0002 (0.0001)		−0.0174 (0.0031)		
QR (τ)											
τ = 0.1	7.7396 (0.7608)	0.4890 (0.0310)	0.0619 (0.0070)	2.1073 (0.2651)							
τ = 0.3	8.3668 (0.6100)	0.5661 (0.0274)	0.0580 (0.0044)	1.7833 (0.1453)							
τ = 0.5	8.2217 (0.5840)	0.6240 (0.0278)	0.0537 (0.0032)	1.4444 (0.0967)							
τ = 0.7	7.8260 (0.4272)	0.7045 (0.0190)	0.0487 (0.0030)	1.1901 (0.0670)							
τ = 0.9	7.4940 (0.6594)	0.7751 (0.0255)	0.0451 (0.0049)	0.9286 (0.0944)							

[a] *FE*, fixed-effects model; *ME*, mixed-effects model; *QR*, quantile models.

Table 4
Evaluation statistics[a] for the fixed-effects, adjusted fixed-effects, mixed effects, three quantile (QR3), and five quantile (QR5) regression models, by species.

Number of trees sampled	Brutian pine					Taurus cedar				
	Fixed-effects	Calibrated fixed	Mixed-effects	Quantile regression		Fixed-effects	Calibrated fixed	Mixed-effects	Quantile regression	
				QR3	QR5				QR3	QR5
Mean Difference (MD)										
0	0.2199				−0.0237					
1		−0.1977	0.1891	**0.1789**	−0.1874		0.0759	0.1036	0.0727	**0.0718**
2		**0.0084**	0.2848	0.0101	0.0187		**0.0293**	0.1637	0.0438	0.0523
3		−0.0330	0.1977	−0.0232	**−0.0169**		−0.0188	0.1537	−0.0045	**0.0031**
4		−0.0392	0.1776	−0.0319	**−0.0293**		−0.0536	0.1407	−0.0401	**−0.0333**
5		−0.0401	0.1607	−0.0340	**−0.0275**		−0.0592	0.1408	−0.0459	**−0.0391**
6		−0.0119	0.1601	−0.0092	**−0.0080**		−0.0394	0.1383	−0.0310	**−0.0290**
7		**−0.0164**	0.1330	−0.0190	−0.0220		−0.0320	0.1357	−0.0205	**−0.0157**
8		**−0.0077**	0.1269	−0.0109	−0.0111		−0.0357	0.1251	−0.0253	**−0.0186**
9		**−0.0164**	0.1144	−0.0198	−0.0270		−0.0366	0.1141	−0.0268	**−0.0173**
10		**−0.0095**	0.1076	−0.0147	−0.0229		−0.0327	0.1065	−0.0257	**−0.0175**
Mean Absolute Difference (MAD)										
0	1.7565				1.7918					
1		2.2324	**1.7176**	2.0786	2.0787		2.3522	**1.7848**	2.2576	2.2627
2		1.8736	**1.6557**	1.7956	1.7913		1.9749	**1.7575**	1.9468	1.9417
3		1.7574	**1.5940**	1.7031	1.6941		1.8317	**1.7239**	1.8239	1.8183
4		1.6675	**1.5566**	1.6349	1.6290		1.7774	**1.6964**	1.7682	1.7631
5		1.6340	**1.5342**	1.6060	1.6012		1.7421	**1.6797**	1.7367	1.7319
6		1.6097	**1.5184**	1.5854	1.5804		1.7164	**1.6655**	1.7107	1.7054
7		1.5953	**1.5037**	1.5719	1.5663		1.6943	**1.6541**	1.6888	1.6833
8		1.5854	**1.4918**	1.5603	1.5557		1.6810	**1.6422**	1.6755	1.6700
9		1.5747	**1.4846**	1.5521	1.5468		1.6714	**1.6316**	1.6658	1.6604
10		1.5687	**1.4754**	1.5456	1.5394		1.6602	**1.6238**	1.6552	1.6491
Fit Index (FI)										
0	0.8139					0.7909				
1		0.6692	**0.8148**	0.7226	0.7180		0.6164	**0.7915**	0.6556	0.6502
2		0.7617	**0.8202**	0.7884	0.7885		0.7350	**0.7952**	0.7437	0.7434
3		0.8009	**0.8392**	0.8138	0.8157		0.7758	**0.8037**	0.7768	0.7772
4		0.8224	**0.8457**	0.8272	0.8283		0.7895	**0.8107**	0.7908	0.7913
5		0.8296	**0.8501**	0.8335	0.8345		0.7980	**0.8145**	0.7986	0.7992
6		0.8342	**0.8530**	0.8373	0.8384		0.8047	**0.8177**	0.8050	0.8059
7		0.8366	**0.8553**	0.8396	0.8404		0.8098	**0.8202**	0.8100	0.8109
8		0.8391	**0.8576**	0.8420	0.8429		0.8125	**0.8227**	0.8127	0.8135
9		0.8415	**0.8587**	0.8438	0.8449		0.8148	**0.8249**	0.8150	0.8159
10		0.8426	**0.8603**	0.8450	0.8461		0.8171	**0.8264**	0.8174	0.8184

[a] A bold, underlined number denotes the best method for each species and sampling effort.

$$S = \sum_{h_{ij} \geqslant \hat{y}_\tau(d_{ij})} \tau[h_{ij} - \hat{y}_\tau(t_{ij})] + \sum_{h_{ij} < \hat{y}_\tau(d_{ij})} (1-\tau)[\hat{y}_\tau(d_{ij}) - h_{ij}] \tag{8}$$

A set of five quantile regressions (based on the 0.1, 0.3, 0.5, 0.7, and 0.9 quantiles) was developed by use of SAS procedure NLP (SAS Institute Inc., 2010) for Eq. (7).

2.5.1. One sample tree height per plot

When only one tree height ($m = 1$) was measured in each plot, the goal was to identify two quantile regression curves that were closest to that tree height. If h_{ij} was encompassed by the kth and $(k + 1)$st quantile regressions, i.e. $\hat{y}_k(d_{ij}) \leqslant h_{ij} \leqslant \hat{y}_{k+1}(d_{ij})$, a modified height growth curve that passed through this point was generated by interpolation:

$$\hat{h}_{ij} = \alpha\hat{y}_k(d_{ij}) + (1-\alpha)\hat{y}_{k+1}(d_{ij}), \tag{9}$$

where $\alpha = \frac{\hat{y}_{k+1}(d_{ij}) - h_{ij}}{\hat{y}_{k+1}(d_{ij}) - \hat{y}_k(d_{ij})}$ is the interpolation ratio.

If the tree height was above the highest (q^{th}) quantile regression curve, Eq. (9) was still appropriate, with $\hat{y}_k$ defined as $\hat{y}_{q-1}$ and $\hat{y}_{k+1}$ as $\hat{y}_q$. The method became extrapolation in nature. Similarly, if the tree height was below the lowest (1st) quantile regression curve, $\hat{y}_k$ and $\hat{y}_{k+1}$ in Eq. (9) were defined as $\hat{y}_1$ and $\hat{y}_2$, respectively.

2.5.2. Two or more sample tree heights per plot

When more than one tree height ($m \geq 2$) were measured in each plot, the mean difference between observed and predicted heights of all sample trees in the plot was computed for each quantile regression. The two consecutive quantile regressions, kth and $(k + 1)$st, were below and above h_{ij}, respectively, in the case of only one sampled height. For $m \geq 2$, the kth and $(k + 1)$st quantile regressions were where the mean difference changed sign. If the mean difference was positive for all quantile regression curves, then the majority of tree heights were below the lowest (1st) quantile regression curve, and $\hat{y}_k$ and $\hat{y}_{k+1}$ in Eq. (9) were defined as $\hat{y}_1$ and $\hat{y}_2$, respectively. Conversely, if the mean difference was negative for all quantile regression curves, then $\hat{y}_k$ and $\hat{y}_{k+1}$ in Eq. (9) were defined as $\hat{y}_q$ and $\hat{y}_{q-1}$, respectively.

In both of the above cases, the interpolation ratio, α, was obtained so as to minimize $\sum_{j=1}^{m} (h_{ij} - \hat{h}_{ij})^2$, where $\hat{h}_{ij}$ is tree height predicted from Eq. (9).

2.6. Evaluation

Five methods evaluated in this study were: (1) fixed-effects model, (2) calibrated fixed-effects model, (3) calibrated mixed-effects model, (4) three-quantile regressions, which were based on three quantiles (0.1, 0.5, and 0.9), and (5) five-quantile regressions, which were based on five quantiles (0.1, 0.3, 0.5, 0.7, and 0.9).

In the last four methods, parameters of the adjusted fixed-effects, mixed-effects and quantile regression models were "localized" by use of sampled heights in each plot, and then applied to predict all tree

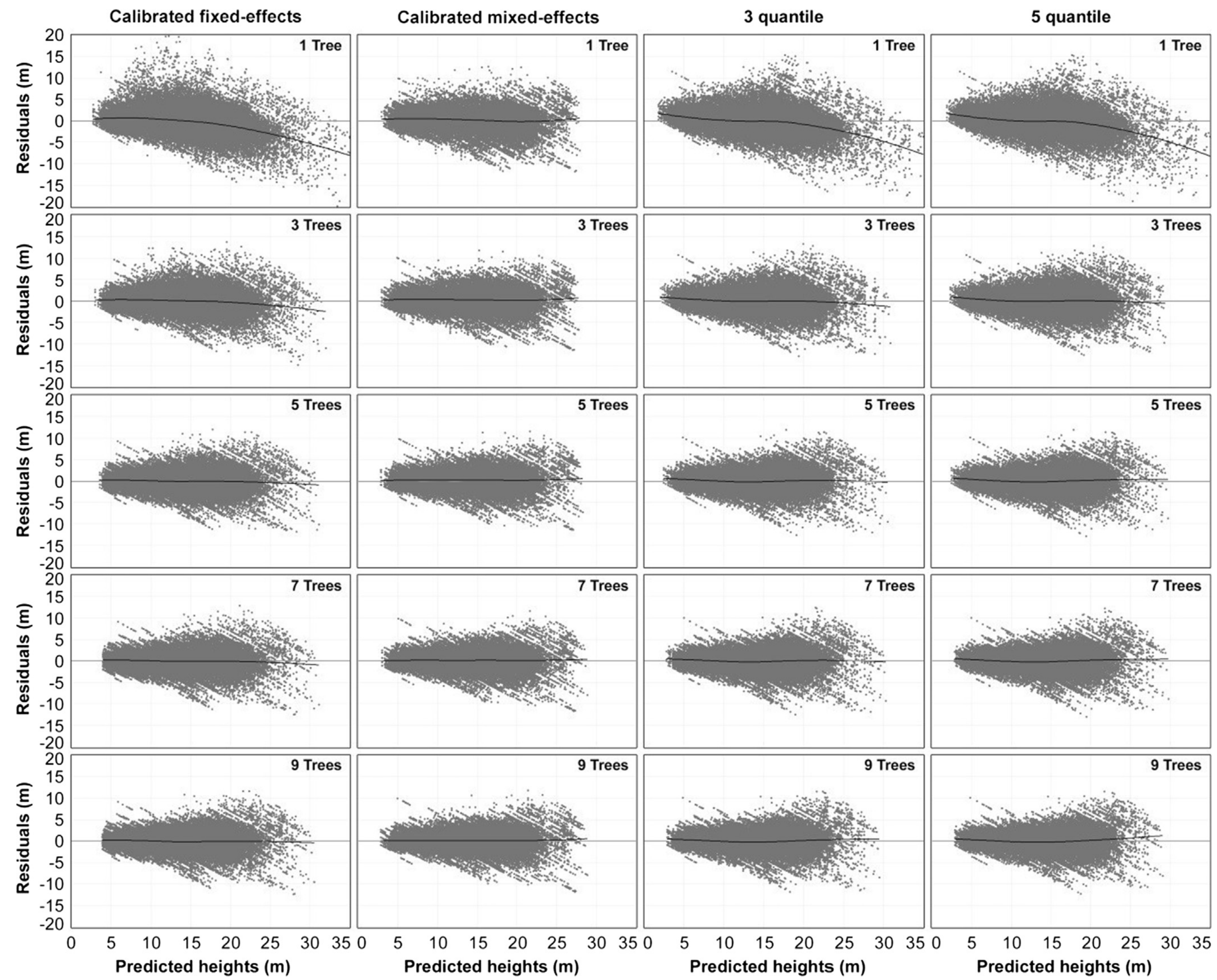

Fig. 2. Plots of residuals against predicted heights from the calibrated fixed-effects and mixed-effects models, and the 3-quantile and 5-quantile regression methods based on different height samples of 1, 3, 5, 7, and 9 trees for Brutian pine. Solid lines are from loess regression.

heights in the plots. Ten sampling scenarios were considered, corresponding to number of tree heights (ranging from one to ten) measured in each plot. Ten repetitions were used for each sampling scenario. The evaluation statistics computed for the validation data set are as follows.

$$\textit{Mean difference}:\ MD = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n_i}(h_{ij}-\widehat{h}_{ij})}{\sum_{i=1}^{n} n_i} \quad (10)$$

$$\textit{Mean absolute difference}:\ MAD = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n_i}|h_{ij}-\widehat{h}_{ij}|}{\sum_{i=1}^{n} n_i} \quad (11)$$

$$\textit{Fit index}:\ FI = 1-\frac{\sum_{i=1}^{n}\sum_{j=1}^{n_i}(h_{ij}-\widehat{h}_{ij})^2}{\sum_{i=1}^{n}\sum_{j=1}^{n_i}(h_{ij}-\widehat{h}_{i})^2} \quad (12)$$

where n is number of plots, n_i is number of tree measurements for plot i, h_{ij} and $\widehat{h}_{ij}$ are observed and predicted values of tree height, respectively, and $\overline{h}_i$ is average value of h_{ij} for plot i.

The predicted tree height $\widehat{h}_{ij}$, was computed from Eq. (9) for the quantile regression models, and from Eq. (2) with random parameters calculated from Eq. (6) for the mixed-effect models. The evaluation statistics for all methods were the averages of the ten repetitions.

3. Results and discussion

Residual plot (Fig. 1) from fitting the modified Chapman-Richards model (Eq. (2)) shows homoscedasticity for Brutian pine and confirms that weighting was not necessary.

The *h-d* model with random components of β_1 and β_3 for Taurus cedar, and β_1, β_3, and β_4 for Brutian pine produced the lowest values of Akaike's information criterion (AIC) and Bayesian information criterion (BIC) among various combinations of random parameters (Table 2).

The final mixed-effects model for Taurus cedar was:

$$h_{ij} = 1.3 + [(\beta_1 + u_1) + \beta_2 Hd_i][1-exp(-(\beta_3 + u_2)d_{ij})]^{\beta_4}, \quad (13)$$

and for Brutian pine was:

$$h_{ij} = 1.3 + [(\beta_1 + u_1) + \beta_2 Hd_i][1-exp(-(\beta_3 + u_2)d_{ij})]^{\beta_4 + u_3}, \quad (14)$$

where u_1–u_3 are random parameters.

Table 3 lists estimates of model parameters, by method, for both tree species. Evaluation statistics for the different methods are shown in Table 4 for Brutian pine and Taurus cedar. Plots of residuals from calibration based on various sample sizes (number of tree heights sampled per plot) are shown in Fig. 2 for Brutian pine. Fig. 3 shows changes in the evaluation statistics as number of sample trees per plot increases.

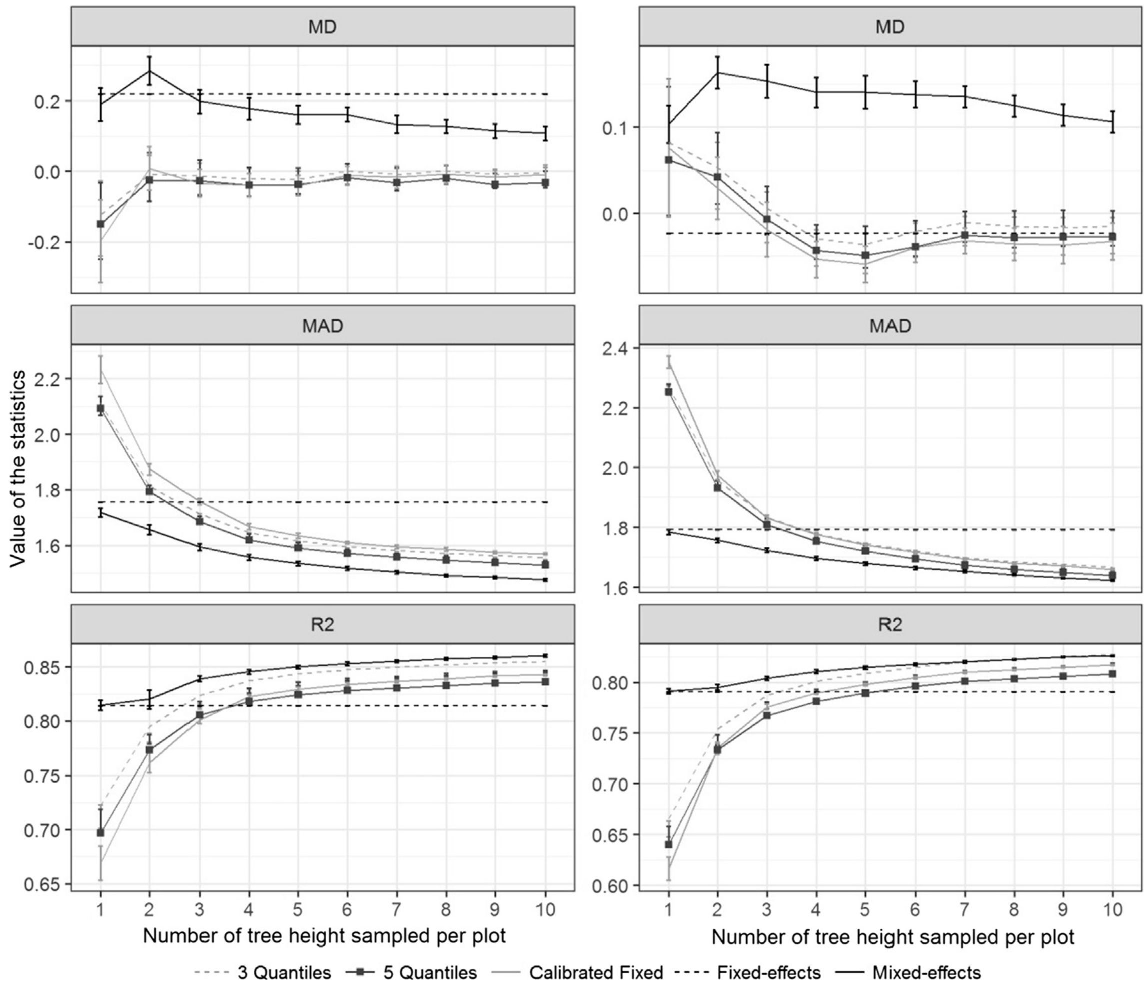

Fig. 3. Plots of evaluation statistics and error bars from five methods against number of tree heights sampled per plot for Brutian pine (left) and Taurus cedar (right).

3.1. Comparison of fixed-effects and calibrated fixed-effects models

Calibrating the fixed-effects models was beneficial when the number of sampled heights per plot exceed a certain level. For both species, both MAD and FI values only improved when height was sampled on four or more trees (Fig. 3). Calibration of the fixed-effects model using four or more sampled heights produced better MD values for Brutian pine, but worse MD values for Taurus cedar.

3.2. Comparison of fixed-effects and calibrated mixed-effects models

Whereas the results for MD values were mixed, the calibrated mixed-effects model consistently outperformed the fixed-effects models based on MAD and FI values (Fig. 3). For both species, the reduction in MAD values ranged from 0.4 to 16.0%, whereas FI values increased by 0.1 to 5.7%.

The findings of improved height prediction from calibration have been previously confirmed by many authors (Calama and Montero, 2004; Trincado et al., 2007; Temesgen et al., 2008; Huang et al., 2009; Gómez-García et al., 2014).

3.3. Comparison of calibrated fixed-effects and calibrated mixed-effects models

For many of the sample sizes, the MD values from the calibrated fixed-effects models were better than those from the calibrated mixed-effects models. Conversely, based on MAD and FI values, the mixed models were consistently superior to the fixed-effects models after calibration, with a reduction in MAD of approximately 20% for one sample tree height per plot, to about 2% when four or more tree heights are sampled per plot. The increase in FI was also of a similar magnitude. These results were similar for both species (Fig. 3). Temesgen et al. (2008) also concluded that calibration generally worked better for the mixed-effects model than for the fixed-effects model.

3.4. Quantile regression models

The difference between the three- and five-quantile regression methods was small and can be considered negligible. The curves depicting the evaluation statistics for the two quantile regression methods are similar and sometimes indistinguishable from each other (Fig. 3). Compared to the uncalibrated fixed-effects model, the quantile regression methods improved the MAD and FI values only when four or more tree heights were sampled per plot. This held true for both Taurus cedar and Brutian pine (Table 4 and Fig. 3).

Based on the MAD and FI values, the two quantile regression methods were better than the calibrated fixed models, but were consistently outranked by the calibrated mixed models (Fig. 3). For both species, the calibrated mixed models produced lower MAD and higher fit index values.

Zang et al. (2016) used the 50th quantile for their quantile regression model without any calibration. Calibration of quantile regression models in other applications was based on only one observation, which was either dbh measurement at a certain age (Bohora and Cao, 2014) or bole diameter measurement at a relative tree height (Cao and Wang, 2015). Because this study dealt with calibration data consisting of more

Table 5
Testing of contrasts for Brutian pine and Taurus cedar. A bold, underlined number denotes a significant difference ($p < 0.05$) in the test of contrasts.

Contrasts	*p*-level	
	Brutian pine	Taurus cedar
1 tree vs. 2 and more trees	**< .0001**	**< .0001**
2 tree vs. 3 and more trees	**< .0001**	**< .0001**
3 tree vs. 4 and more trees	**0.0080**	**0.0012**
4 tree vs. 5 and more trees	0.2130	0.1422
5 tree vs. 6 and more trees	0.4826	0.4061
6 tree vs. 7 and more trees	0.3636	0.4384
7 tree vs. 8 and more trees	0.7063	0.4754
8 tree vs. 9 and 10 trees	0.6441	0.6961
9 tree vs. 10 trees	0.9449	0.8445

than one observation, new procedures were successfully developed to (1) identify the two consecutive quantile curves that encompass the majority of the observations, and (2) compute the ratio that allows interpolation between the two curves.

3.5. Comparison of modeling approaches

For both tree species, the calibrated mixed-effects model approach consistently outperformed all of the methods evaluated in this study, including the fixed-effects models, calibrated fixed-effects models, and two quantile regression methods. This approach produced the best MAD and FI values for all height sampling intensities (Fig. 3).

The quantile regression method should be flexible (Cao and Wang, 2015), because parameters of the regressions based on various quantiles are different (Table 2) and therefore produce *h-d* curves of different shapes. However, this method assumes that curves from different plots generally follow the basic shapes dictated by the quantile regressions, and disallows the fact that some of these curves might cross one another. On the other hand, calibration of random parameters in the mixed models makes crossover possible, and in turn, produces more realistic solutions.

3.6. Sample size for calibration

Model calibration by use of a subset of tree height measurements for each plot considerably increased the predictive capability of the *h-d* model. As sampling effort increased, performance of the methods also improved, as found by Temesgen et al. (2008) and Huang et al. (2009).

Whereas the calibrated mixed-effects models produced better MAD and FI values than did the fixed-effects models for all sample sizes, the calibrated fixed-effects models and the two quantile regression methods outperformed the fixed-effects models only when more than three heights were sampled.

The curves depicting the change in MAD and FI values as number of sampled tree heights per plot increases (Fig. 3) show a general trend of a curve with a steep slope, followed by a line having a gradual slope. For both species, the switchover occurs at a sample of four tree heights per plot, which appears to strike a balance between sampling cost of measuring tree heights and improvement in predictive accuracy and precision.

The above conclusion was confirmed statistically by a series of test of contrasts: sample of 1 tree per plot versus 2 or more trees; 2 trees versus 3 or more trees, etc. Results indicated that sampling 4 trees per plot yielded non-significant differences among the evaluation statistics as compared to sampling 5 or more trees (Table 5).

4. Summary and conclusion

Height-diameter models were developed for Brutian pine and Taurus cedar in Turkey. Previous studies showed that the predictive ability of the base *h-d* model can be improved if additional stand-level variables are added into the model. In this study, we found that the final model, formed by incorporating dominant height into the Chapman-Richards model, increased the predictive accuracy and precision. Five alternative methods were evaluated: (1) fixed-effects model, (2) calibrated fixed-effects model, (3) calibrated mixed-effects model, (4) three-quantile regression model, and (5) five-quantile regression model. Parameters of the fixed-effects, mixed-effects, and quantile regression models for both model forms were then calibrated by use of a subset of height measurements, ranging from 1 to 10 sample trees per plot. The calibrated mixed-effects approach was consistently superior to all other methods for both tree species and for all calibration sample sizes. The results suggest that the use of the calibrated mixed-effects model approach is an advisable option for tree height estimation. A sample of four trees per plot seemed to be a good compromise between sampling cost and predictive accuracy and precision.

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