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Evaluation of Methods for Modeling Individual Tree Survival

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In this study, three methods of estimating parameters of the logistic regression model for predicting tree survival probabilities were evaluated: maximum likelihood; optimized for stand survival (stand optimization); and optimized for both stand and tree survival (combined likelihood). Four methods of disaggregation were considered: no disaggregation; disaggregation based on predictions from a stand-level model; disaggregation based on predictions from a composite estimator; and disaggregation based on observed stand survival. For stand survival prediction, surprisingly, the tree survival model with parameters optimized for stand survival scored better than the stand-level survival model, based on all three evaluation statistics. This method performed even better when it was combined with the stand-level survival model to form a composite estimator. For tree survival prediction without disaggregation, the stand optimization method was edged out by the maximum likelihood method. The disaggregation method slightly degraded the performance of the unadjusted model, based on all evaluation statistics. When adjusted using stand survival values, either observed or predicted, the stand optimization method was clearly the best among the three parameter estimation methods. These results showed that the stand optimization method should be used to estimate parameters of the tree survival model. The decision of using disaggregation should depend on whether or not the stand survival prediction passes a certain minimum threshold. Disaggregation did not improve prediction of tree survival for this data set, maybe because the stand-level prediction did not reach that reliability threshold.

Keywords: disaggregation; stand optimization; maximum likelihood

Growth and yield models provide predictions for future forest stands, given their current conditions. Among many different types of growth and yield models, individual-tree simulation models provide the most flexible outputs because this type of model is based on growth of individual trees. Predicting tree survival is an important component of tree-level models. The probability that a tree survives a growing period has been modeled by use of logistic regressions (Hamilton 1974, Hamilton and Edwards 1976, Monserud 1976, Buchman 1979, 1983, Zhang et al. 1997, Monserud and Sterba 1999) and other methods (Glover and Hool 1979, Amateis et al. 1989, Guan and Gertner 1991a, 1991b).

Stand-level survival can be predicted directly from a stand survival model or indirectly by summing individual tree survival probabilities. According to Weiskittel et al. (2011), three approaches have been used in the past to link tree- and stand-level models: disaggregation; constrained parameters; and combined. In the disaggregation approach, survival is projected at the stand level and then adjusted to individual trees (e.g., Campbell et al. 1979, Harrison and Daniels 1988, Matney et al. 1990, Qin and Cao 2006). The constrained approach uses a multiresponse parameter estimation technique developed by Bates and Granger (1969) to optimize tree-level predictions at multiple levels (e.g., Zhang et al. 1997, Cao 2006). The combined approach uses a composite estimator to link estimates of tree- and stand-level equations to improve both predictions (e.g., Yue et al. 2008, Zhang and Lei 2010, Zhang et al. 2010, 2011, Hevia et al. 2015). Further revision of the methods used to link models with different levels of resolution using disaggregation can be found in Ritchie and Hann (1997), and the background of the three different approaches described above is presented in Weiskittel et al. (2011, Chapter 10) and Cao (2014).

Parameters of a logistic regression model that predicts tree survival can be obtained from different methods. The most common method is to optimize tree survival by use of the maximum likelihood technique. Alternatively, the parameter estimates can be selected to optimize stand survival or both stand and tree survival.

The objective of this study was to evaluate three methods of estimating parameters of the logistic regression model for predicting tree survival probabilities (maximum likelihood, stand optimization, and combined likelihood), coupled with four methods of disaggregation (no disaggregation and disaggregation based on observed stand survival and predictions from a stand-level model and from a composite estimator).

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Table 1. Stand and tree attributes by group and age.

Group	Attribute	Stand age			
		10 yr	15 yr	20 yr	25 yr
1 (150 plots)	Dominant height (m)	9.2 (1.3)	13.4 (1.6)	16.8 (2.0)	19.6 (2.3)
	No. of trees/ha	1,713 (710)	1,631 (632)	1,129 (425)	981 (399)
	Basal area (m^2/ha)	19.6 (7.1)	29.9 (7.0)	29.8 (7.7)	32.9 (9.6)
	Tree diameter (cm)	11.7 (3.1)	14.8 (3.8)	17.8 (4.4)	20.1 (4.8)
2 (150 plots)	Dominant height (m)	9.3 (1.2)	13.4 (1.6)	17.0 (1.9)	20.0 (2.3)
	No. of trees/ha	1,887 (607)	1,607 (698)	1,142 (423)	1,001 (436)
	Basal area (m^2/ha)	21.3 (5.7)	29.7 (8.6)	32.6 (9.1)	36.0 (11.5)
	Tree diameter (cm)	11.6 (2.9)	14.9 (3.8)	18.5 (4.6)	20.7 (5.3)

Means (SD).

Data

Data used in this study were from 300 plots randomly selected from the Southwide Seed Source Study, which include 15 loblolly pine (*Pinus taeda* L.) seed sources planted at 13 locations across 10 southern states (Wells and Wakeley 1966). Each 0.0164-ha plot consisted of 49 trees, planted at 1.8-m × 1.8-m spacing. Included in the data set were measurements of tree diameters and survival at ages 10 and 15 for the first 100 plots, ages 15 and 20 for the second 100 plots, and ages 20 and 25 for the third 100 plots. There were a total of 300 growth periods.

The data were randomly divided into two groups of 150 plots each, with 50 plots of each age class in each group (Table 1). The leave-one-out evaluation scheme was applied in this study. Parameters of the stand and tree survival models were estimated from group 1 (considered the fit data) and then used to predict group 2 (considered the validation data). The same procedure was repeated with group 2 being the fit data and group 1 the validation data. Finally, predictions from both groups were pooled to compute evaluation statistics for the different methods.

Methods

Stand Survival Equation

The model developed by Cao (2006, 2016) was used in this study to predict stand-level survival:

$$\hat{N}_{2i} = \frac{N_{1i}}{1 + \exp(b_0 + b_1 RS_{1i} + b_2 H_{1i} + b_3 N_{1i}/A_{1i} + b_4/A_{1i})} \tag{1}$$

where $\hat{N}_{2i}$ is the predicted number of trees per hectare for plot i at the end of the 5-year growth period, A_{1i} is the stand age in years for plot i at the beginning of the growth period, N_{1i} is the number of trees per hectare at age A_{1i}, H_{1i} is the dominant height (average height of the dominant and codominant trees) in meters at age A_{1i}, and $RS_{1i} = (\sqrt{10{,}000/N_{1i}})/(H_{1i})$ is the relative spacing at age A_{1i}.

The ordinary least-squares method was used to obtain the coefficients of Equation 1.

Tree Survival Equation

The following logistic regression model (Cao 2006, 2016) was used to predict tree survival probability (p_{ij}) of tree j in plot i during the 5-year growth period:

$$p_{ij} = \frac{1}{1 + \exp(b_0 + b_1 H_{1i} + b_2 RS_{1i} + b_3 d_{1ij}/Dq_{1i})} \tag{2}$$

where Dq_{1i} is the quadratic mean diameter of plot i at age A_{1i} and d_{1ij} is the dbh of tree j in plot i at age A_{1i}.

Fitting Methods

Three different approaches were used to estimate parameters b_0–b_3 of Equation 2.

Fitting method 1—maximum likelihood

This is the commonly used method for estimating parameters of a logistic equation. The goal was to maximize the log-likelihood function:

$$\ln L_1 = \sum_i \sum_j \ln(z_{ij}) \tag{3}$$

where $z_{ij} = p_{ij}$ if tree j in plot i was alive and $(1 - p_{ij})$ if it was dead.

Fitting method 2—stand optimization

The sum of predictions of tree survival in each plot can be used to predict stand survival for that plot. This is similar to the following regression model:

$$N_{2i} = \frac{1}{s} \sum_j p_{ij} + \varepsilon_i \tag{4}$$

where N_{2i} is the observed number of trees per hectare for plot i at the end of the 5-year growth period, s is plot size in hectares, and ε_i is error, assumed to be normally distributed with mean 0 and variance σ^2. In this method, the parameters of Equation 2 were selected to minimize $\Sigma_i (N_{2i} - (1/s)\Sigma_j p_{ij})^2$.

Fitting method 3—combined likelihood

The aim of this method was to optimize for both tree-level and stand-level survival by maximizing the following combined log-likelihood function:

$$\ln L = \frac{\ln L_1}{\ln L_{1\max}} + \frac{\ln L_2}{\ln L_{2\max}} \tag{5}$$

where $\ln L_1$ is as previously defined, $\ln L_{1\max}$ is the maximum value of $\ln L_1$ (obtained from fitting method 1), $\ln L_2 = -½\ln(2\pi\sigma^2) - (1/2\sigma^2)\Sigma i\, (N_{2i} - (1/s)\Sigma_j p_{ij})^2$, and $\ln L_{2\max}$ is the maximum value of $\ln L_2$ (obtained from fitting method 2). The above expression for $\ln L_2$ is the log-likelihood for Equation 4. Because $\ln L_1$ and $\ln L_2$ have different magnitudes, they are scaled by $\ln L_{1\max}$ and $\ln L_{2\max}$, respectively, before being combined in Equation 5.

Disaggregation Methods

In these methods, outputs from the individual-tree survival model are adjusted such that the resulting stand summary matches the prediction from the stand survival model.

Disaggregation method 1—unadjusted (no disaggregation)

Outputs from the tree survival model were not adjusted.

Disaggregation method 2—adjusted to match outputs from stand survival model

The adjusted tree survival probability ($\tilde{p}_{ij}$) was calculated from the survival probability (p_{ij}) predicted from Equation 2 by use of the following method (Cao 2010, 2014):

$$\tilde{p}_{ij} = p_{ij}^{\alpha} \tag{6}$$

where α is the adjustment coefficient used to match the sum of adjusted tree survival probabilities to predictions from the stand survival model (Equation 1).

Disaggregation method 3—adjusted to match composite estimator

The method was carried out in two steps. First, the composite estimator of stand survival was computed as the weighted average of stand-level predictions from the stand survival model (Equation 1) and the tree survival model (Equation 2). The weights were computed according to a method described by Tang (1992, 1994) and applied by Zhang et al. (2010). In the second step, Equation 6 was used to adjust predictions from the tree survival model using the composite estimator for stand survival.

Disaggregation method 4—adjusted to match observed stand survival

This method was similar to the previous two methods. Instead of predicted stand survival, observed stand survival was used to adjust the tree-level survival model. This method serves to illustrate the best-case scenario of perfect prediction of stand survival.

Model Evaluation

A total of 12 methods (3 fitting methods $\times$ 4 disaggregation methods) were evaluated in this study. The tree survival model was fitted and adjusted using data from one group and then evaluated using data from the other group. The predictions from both groups were pooled to compute evaluation statistics for the different methods.

Evaluation of Stand Survival Prediction

The following statistics were computed for stand-level evaluation:

$$\text{Mean difference: MD} = \frac{1}{m}\sum_i\left(N_{2i} - \frac{1}{s}\sum_j p_{ij}\right) \tag{7}$$

$$\text{Mean absolute difference: MAD} = \frac{1}{m}\sum_i\left|N_{2i} - \frac{1}{s}\sum_j p_{ij}\right| \tag{8}$$

$$\text{Fit index: FI} = 1 - \frac{\sum_i\left(N_{2i} - \frac{1}{s}\sum_j p_{ij}\right)^2}{\sum_i(N_{2i} - \bar{N}_2)^2} \tag{9}$$

where m is the number of plots and $\bar{N}_2$ is the average number of trees per hectare at the end of the growth period.

Evaluation of Tree Survival Prediction

Tree-level survival predictions were evaluated from

$$\text{Mean difference: MD} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n_i}(y_{ij} - p_{ij})}{\sum_{i=1}^{n} n_i} \tag{10}$$

where $y_{ij} = 1$ if tree j in plot i was alive and 0 if it was dead, and

$$\text{Mean absolute difference: MAD} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n_i}|y_{ij} - p_{ij}|}{\sum_{i=1}^{n} n_i} \tag{11}$$

The range for the area under the receiver operating characteristic curve (AUC) is between 0.5 and 1. The higher the AUC value is, the better the fit.

The relative rank, introduced by Poudel and Cao (2013), was used in this study to display the relative position of each method. The relative rank of method i is defined as

$$R_i = 1 + \frac{(k-1)(S_i - S_{\min})}{S_{\max} - S_{\min}} \tag{12}$$

for minimization objective, and

$$R_i = k - \frac{(k-1)(S_i - S_{\min})}{S_{\max} - S_{\min}} \tag{13}$$

for maximization objective, where R_i is the relative rank of method i ($i = 1, 2, \ldots, k$), k is the number of methods evaluated, S_i is the evaluation statistic produced by method i, $S_{\min}$ is the minimum value of S_i, and $S_{\max}$ is the maximum value of S_i. Note that R_i is a real number rather than an integer, and for either the minimization or maximization objective, the best method receives a rank of 1, whereas the worst method receives a rank of k. After a relative rank was computed separately for each statistic of each method, a final rank was calculated based on the sum of all ranks for each method.

Results and Discussion

Stand Survival Prediction

Table 2 shows the evaluation statistics for stand survival prediction. The overall best performer was the composite estimator from the stand survival model (Equation 1) and the tree survival model (Equation 2) with parameters obtained from the stand optimization method. Surprisingly, the stand survival model ranked last overall among seven methods. Even though it provided decent MAD and FI values, it also produced the worst MD value compared with that for other methods. This contrasts with findings from Qin and Cao (2006), Cao (2006, 2014), and Zhang et al. (2010), who obtained better values for all three evaluations statistics from the stand survival model compared with the tree survival model.

Table 2. Evaluation statistics for stand survival prediction.

Type	Parameter estimation	MD	MAD	FI	Rank
Stand-survival model		−21.63[b]	192.84	0.7739	7.00[b]
Individual-tree model	Maximum likelihood	−1.22[a]	194.78[b]	0.7702[b]	5.13
	Stand optimization	−11.47	184.87	0.7781	1.77
	Combined likelihood	−9.32	192.76	0.7729	5.19
Composite estimator	Maximum likelihood	−16.61	191.18	0.7791	4.52
	Stand optimization	−16.76	184.85[a]	0.7857[a]	1.00[a]
	Combined likelihood	−18.09	190.87	0.7789	4.70

[a] Best method for each criterion.
[b] Worst method for each criterion.

Table 3. Evaluation statistics for tree survival prediction.

Parameter estimation	Disaggregated from	MD	MAD	AUC	Rank
Maximum likelihood	No disaggregation	−0.0009	0.2663[b]	0.7671	8.17
	Stand-level model	−0.0143[b]	0.2584	0.7539	12.00[b]
	Composite estimator	−0.0111	0.2601	0.7635	10.88
	Observed stand survival	0.0000[a]	0.2098	0.8870	1.84
Stand optimization	No disaggregation	−0.0077	0.2221	0.7571	8.43
	Stand-level model	−0.0143[b]	0.2272	0.7509[b]	10.71
	Composite estimator	−0.0112	0.2237	0.7572	9.49
	Observed stand survival	0.0000[a]	0.1745[a]	0.8632	1.00[a]
Combined likelihood	No disaggregation	−0.0063	0.2619	0.7649	9.56
	Stand-level model	−0.0143[b]	0.2575	0.7539	11.96
	Composite estimator	−0.0121	0.2586	0.7623	11.13
	Observed stand survival	0.0000[a]	0.2086	0.8871[a]	1.79

[a] Best method for each criterion.
[b] Worst method for each criterion.

Parameter Estimation Method

The stand optimization method of parameter estimation aimed to optimize for stand survival prediction. It achieved this goal, ranking near the top (1.77). It outperformed both the stand survival model and the tree survival model (with maximum likelihood estimation) in terms of all three evaluation statistics.

The combined likelihood method should be a compromise between tree-level and stand-level optimization. The results for this method confirmed this principle: the values for the evaluation statistics for this method were between those from the maximum likelihood and stand optimization methods. The maximum likelihood method was the least biased method for stand survival prediction and slightly edged out the combined likelihood method with an overall ranking of 5.13 versus 5.19.

Composite Estimator

When stand-level predictions from the stand survival model were combined with those from the tree survival model, the resulting composite estimator ranked better than the tree survival model for predicting stand survival. This was true regardless of which parameter estimation method was used. In general, the composite estimator provided lower MAD and higher FI values then either the stand or tree survival model. It was less biased than the stand survival model but more biased than the tree survival model, based on MD values.

When combined with the stand survival model, the best parameter estimation method (stand optimization) for the tree survival model produced the best results, with an overall ranking of 1.00 for stand-level prediction.

Tree Survival Prediction

The stand optimization method with adjustments from the observed stand survival was the best performer in predicting tree-level survival (Table 3). It produced the best values of MD, MAD, and AUC among the 12 methods. The worst method was the maximum likelihood disaggregated from the stand survival model.

Parameter Estimation Method with No Disaggregation

As expected, the maximum likelihood method was best in predicting tree survival without disaggregation with an overall rank of 8.17. Even though they were not optimized for individual trees, the stand optimization and combined likelihood methods were not far behind (ranks of 8.43 and 9.56, respectively).

Disaggregation Method Based on Predicted Stand Survival

Regardless of the parameter estimation method, disaggregation from either the stand survival model or the composite estimator did not perform as well as that from the unadjusted method. The worst disaggregation method was the one based on the stand survival model, with ranks of 12.00, 10.71, and 11.96 from different parameter estimation methods (Table 3).

Previous studies have reported improvements of the disaggregated tree survival model over the unadjusted model (Cao 2006, 2014, Qin and Cao 2006, Zhang et al. 2011). Cao (2010) evaluated adjustment methods from observed and predicted stand survival and concluded that the success of disaggregation depends largely on the quality of stand prediction. The stand survival model did not give relatively good predictions for these data, resulting in an FI value of 0.77 (Table 2), compared with 0.90 (Cao 2006), 0.91 (Zhang et al. 2011), or 0.83 (Cao 2014). As a result, disaggregation from the stand survival model appeared to hurt rather than help the performance of the tree survival model. Disaggregation from the composite estimator suffered for the same reason: the composite estimator did not perform much better than the stand-level or the tree-level survival models in predicting stand survival.

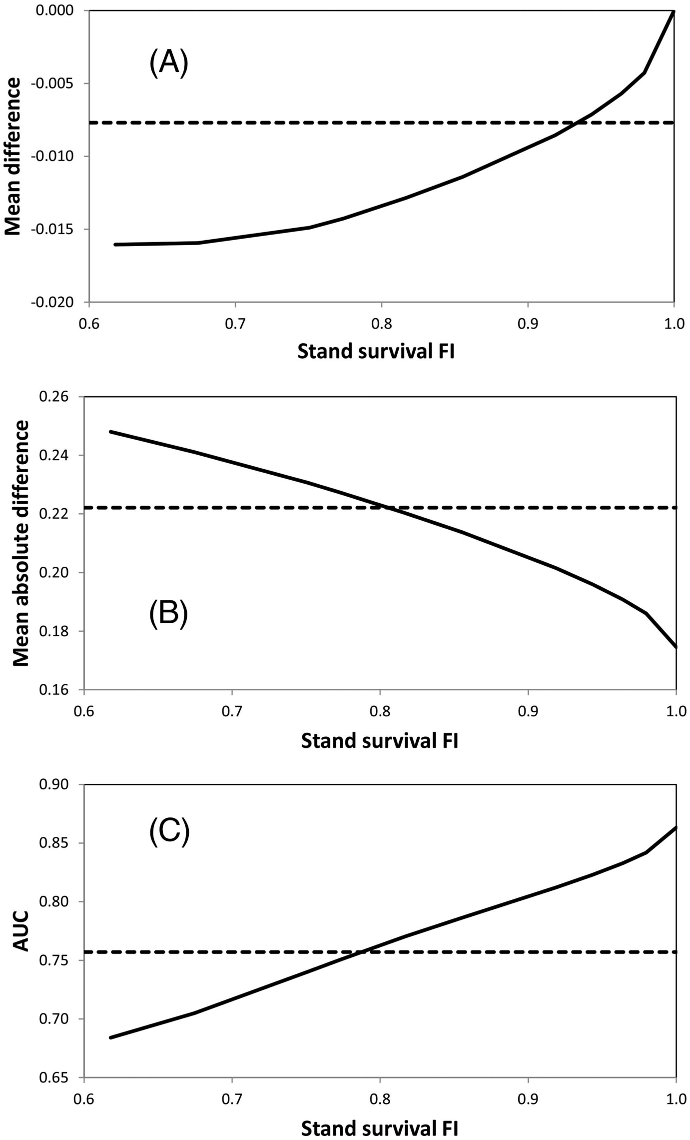

Figure 1. Changes in tree-level evaluation statistics when the tree survival model (stand optimization method) was adjusted from predicted stand survivals having different FI values. Horizontal lines, which represent the statistics of the unadjusted stand optimization method from Table 3, cross the curves at FI values of 0.93, 0.81, and 0.79 based on MD (A), MAD (B), and AUC (C), respectively.

Disaggregation Method Based on Observed Stand Survival

As also found by Cao (2010), disaggregation from the observed stand survival improved tree survival prediction compared that from the unadjusted model for this data set. Disaggregation based on the stand optimization method received a rank of 1.00, whereas the other two parameter estimation methods lagged slightly behind with ranks of 1.79 and 1.84 (Table 3). Compared with the unadjusted stand optimization method (rank of 8.43), the improvement due to disaggregation was compelling: MD from −0.0077 to 0.0000 (a 100% reduction), MAD from 0.2221 to 0.1745 (a 21% reduction), and AUC from 0.7571 to 0.8632 (a 14% increase).

Figure 1 shows the changes in the three tree-level evaluation statistics when the tree survival model (with parameters estimated from the stand optimization method) was adjusted from predicted stand survival having different FI values. These predicted values of stand survival were computed as a weighted average of the observed stand survival and prediction from Equation 1. Varying the weight moved the new predicted values closer to or further away from the observed stand survival, allowing the simulation of stand survival predictions with different FI values. From Figure 1, it is clear that disaggregation could improve the tree-level survival prediction for the stand optimization method only when the FI value of predicted stand survival exceeded a threshold of 0.93, 0.81, and 0.79 based on MD, MAD, and AUC, respectively. The FI values for the stand-level model and the composite estimator ranged from 0.7739 to 0.7857, explaining why disaggregation did not improve prediction of the tree survival model for this data set.

Summary and Conclusions

For stand survival prediction, the stand optimization method scored better than the stand survival model and the other two parameter estimation methods. This method performed even better when it was combined with the stand-level survival model to form a composite estimator.

For tree survival prediction without disaggregation, the stand optimization method was edged out by the maximum likelihood method. The disaggregation method slightly degraded the performance of the unadjusted model, based on all three evaluation statistics. When adjusted using stand survival values, either observed or predicted, the stand optimization method was clearly the best among the three parameter estimation methods.

These results showed that the stand optimization method should be used to estimate parameters of the tree survival model. The decision to use disaggregation should depend on whether or not the stand survival prediction passes a certain minimum threshold. Disaggregation did not improve prediction of tree survival for this data set, maybe because the stand-level prediction did not reach that reliability threshold.

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