Projection of a diameter distribution through time

Jianhua Qin, Quang V. Cao, and David C. Blouin

Abstract: Three approaches to characterizing the diameter distribution of a future stand are presented. The first approach is the "parameter-recovery" method, which links a whole-stand model to a diameter-distribution model. The next two approaches provide linkages between an individual-tree model and a diameter-distribution model. Tree-survival and diameter-growth equations were applied to the tree list (the "tree-projection" method) or to the diameter distribution (the "distribution-projection" method) at the beginning of the growth period. A numerical example of Weibull distributions that characterized diameter data from the Southwide Seed Source Study of loblolly pine (*Pinus taeda* L.) is presented. All three methods produced similar results in terms of Reynolds et al.'s (1988) error indices, whereas the distribution-projection method outperformed the other two methods in predicting total and merchantable volumes per hectare. This study demonstrated that the diameter-distribution model could be linked to either a whole-stand model or an individual-tree model with comparable success.

Résumé : Trois approches sont présentées pour prédire la distribution diamétrale d'un peuplement futur. La première est la méthode de la récupération de paramètres qui établit un lien entre un modèle de peuplement et un modèle de distribution diamétrale. Les deux autres approches établissent des liens entre un modèle d'arbre individuel et un modèle de distribution diamétrale. Les équations de survie et de croissance diamétrale sont appliquées à la liste d'arbres (méthode par projection d'arbre) ou à la distribution diamétrale (méthode par projection de distribution) au début de la période de croissance. Un exemple numérique illustre l'application de la distribution de Weibull pour caractériser les données diamétrales provenant de l'étude des provenances de graines de pin à encens (*Pinus taeda* L.) dans le sud des États-Unis. Les trois approches produisent des résultats similaires en termes d'indices d'erreurs proposés par Reynolds et al. (1988) alors que la méthode de projection de la distribution est meilleure que les deux autres méthodes pour prédire le volume marchand et le volume total à l'hectare. Cette étude démontre que le modèle de distribution diamétrale peut être lié à un modèle de peuplement ou à un modèle d'arbre avec autant de succès.

[Traduit par la Rédaction]

Introduction

Growth and yield models can be classified into three broad categories: whole-stand models, diameter-distribution models, and individual-tree models. Whole-stand models use the entire stand as the modeling unit (Murphy and Farrar 1982; Lloyd and Harms 1986; Pienaar and Harrison 1989; Ochi and Cao 2003), whereas the focus of individual-tree models is each tree in the stand (Belcher et al. 1982; Burkhart et al. 1987; Zhang et al. 1997; Cao et al. 2002). Diameter-distribution models are based on statistical distributions such as the beta function (Clutter and Bennett 1965; Lenhart and Clutter 1971; Bennett et al. 1978), Weibull function (Smalley and Bailey 1974; Matney and Sullivan 1982; Bailey et al. 1989), or S_B function (Hafley et al. 1982) to describe stand structure. Rather than being completely separate, these three types of models can be considered related to one another. A linkage between whole-stand models and diameter-distribution models has been established via the parameter-recovery approach (Hyink and Moser 1983). This approach ensures that the resulting distribution matches diameter moments (Cao et al. 1982; Lynch and Moser 1986) and diameter percentiles (Baldwin and Feduccia 1987; Bailey et al. 1989). Bailey (1980) was the first researcher to attempt to link a diameter-distribution model to an individual-tree model. He derived implied diameter-growth functions in cases where the same form of probability density function (pdf) was used to describe the diameter distributions throughout the growing period. This approach assumed no mortality or that mortality was proportionally distributed over the diameter distribution. The case when tree mortality was not proportionally distributed was later covered by Cao (1997), who modeled survival ratio as a function of diameter. The diameter distribution after mortality was then approximated with a Weibull or beta distribution (depending on the original distribution). Finally, the implied diameter-growth function (Bailey 1980) was employed to grow the stand to the end of the growth period.

The objective of this study was to develop a generalized framework to describe the linkages between a diameter-distribution model and either an individual-tree model or a whole-stand model.

Received 19 May 2006. Accepted 3 August 2006. Published on the NRC Research Press Web site at cjfr.nrc.ca on 4 April 2007.

J. Qin. Pennington Biomedical Research Center, Louisiana State University System, 6400 Perkins Road, Baton Rouge, LA 70808, USA.
Q.V. Cao.[1] School of Renewable Natural Resources, Louisiana State University Agricultural Center, Baton Rouge, LA 70803, USA.
D.C. Blouin. Department of Experimental Statistics, Louisiana State University Agricultural Center, Baton Rouge, LA 70803, USA.

[1]Corresponding author (e-mail: jqin1@lsu.edu).

 doi:10.1139/X06-216

A generalized framework

In a diameter-distribution system, a diameter random variable, X, at age A_1, the beginning of a growth period, is characterized by $f(x)$, a pdf. At the end of the growth period, at age A_2, the diameter random variable is Y and its pdf is $w(y)$. Diameter-distribution models generally require that the two pdf's, $f(x)$ and $w(y)$, belong to the same family of distributions, such as Weibull, beta, or S_B.

We will now consider two scenarios (Fig. 1), depending on whether inputs obtainable at age A_1 consist of a list of tree diameters or just the first two diameter moments.

Scenario 1

In the first scenario, a list of trees at age A_1 is available and the goal is to predict the future diameter distribution at age A_2. We will next consider three general paths that one could take to link a diameter-distribution model and either a whole-stand model or an individual-tree model.

Path A

From the tree list at age A_1, the first two diameter moments, arithmetic diameter ($\overline{D}_1$) and quadratic mean diameter (Dq_1), are computed. If the diameter distribution at A_1 is desired, parameters of $f(x)$ can be recovered with the moment-estimation method, in which the resulting distribution produces the first two moments that match $\overline{D}_1$ and Dq_1, respectively. The two diameter moments are then projected to $\overline{D}_2$ and Dq_2, respectively, at age A_2 by using regression equations. Parameters of the future diameter distribution, $w(y)$, are obtained through moment estimation (Cao et al. 1982; Lynch and Moser 1986). This path represents a straightforward *parameter-recovery method*, which links a whole-stand model (to project to $\overline{D}_2$ and Dq_2) and a diameter-distribution model. Variations of this approach that include diameter percentiles have also been employed (Baldwin and Feduccia 1987; Bailey et al. 1989).

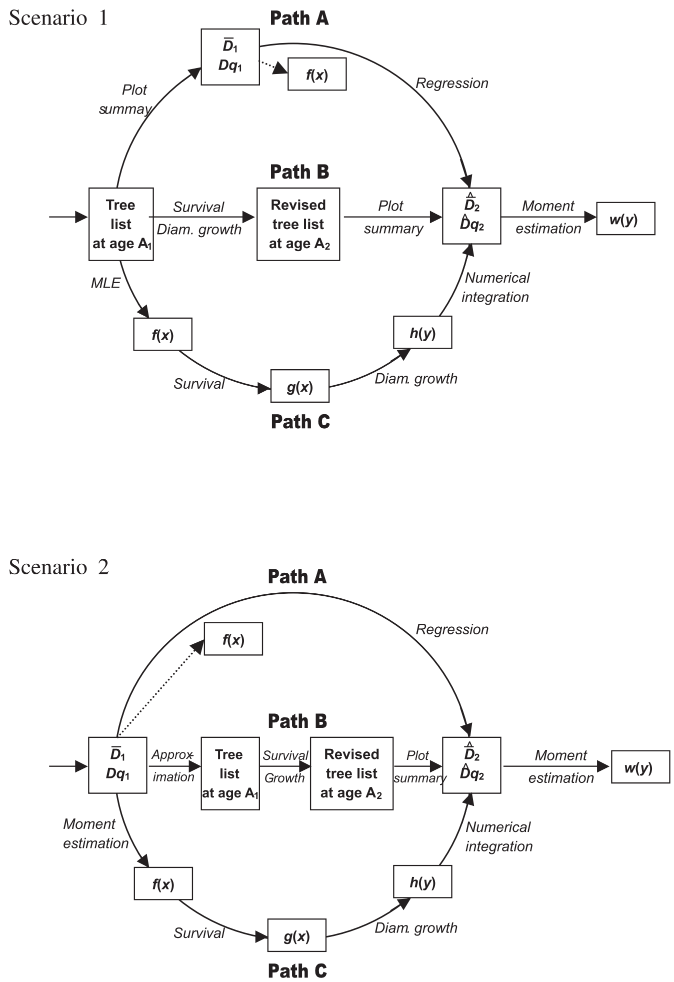
Path B

The trees in the tree list are first subjected to the survival component of an individual-tree model to determine number of surviving trees at age A_2. These trees then grow in diameter according to the tree diameter growth function. At the end of the growth period, the tree list is revised and then summarized to yield arithmetic and quadratic mean diameters, which are again used to compute parameters of the future diameter distribution, $w(y)$. This path utilizes the *tree-projection method*, which joins the individual-tree modeling approach in the first part with the moment-estimation approach in the second part.

Path C

Parameters of the diameter pdf at age A_1, $f(x)$, are obtained from the tree list either by moment estimation or maximum likelihood (the preferred method). Normally, no information is known regarding the survival status of trees during the growth period. We assume here that all mortality occurs at the beginning of the growth period. Suppose $p(x)$ is the probability that a tree of diameter x is alive at age A_2, given that it was alive at age A_1. The pdf for tree diameter at age A_1 after mortality, $g(x)$, is computed from $p(x)$ and $f(x)$ as follows:

Fig. 1. Diagrams illustrating the use of individual-tree and diameter-distribution models in projecting diameter distributions. A tree list is available in scenario 1, whereas only the first two diameter moments $\overline{D}_1$ (and Dq_1) are available in scenario 2. Paths A, B, and C refer to the parameter-recovery, tree-projection, and distribution-projection method, respectively.



$$[1] \qquad g(x) = \frac{p(x)f(x)}{\int_{x\,\mathrm{min.}}^{x\,\mathrm{max.}} p(x)f(x)\,\mathrm{d}x}$$

where $x_{\mathrm{min.}}$ and $x_{\mathrm{max.}}$ are the minimum and maximum of variable x in $f(x)$, respectively. Note that $p(x)$ acts as a weight function in eq. 1.

Tree diameter (Y) at age A_2 can be predicted from tree diameter (X) at age A_1 using the following regression equation:

$$[2] \qquad y = q(x) + \varepsilon$$

where $q(x)$ is a function involving diameter x and stand variables such as stand height and stand density, and ε is random error.

The above diameter-growth model is then changed into a deterministic model by dropping the error term. The pdf of tree diameter Y at age A_2 is

$$[3] \qquad h(y) = g(x)(\mathrm{d}x/\mathrm{d}y)$$

where $y = q(x)$ and $\mathrm{d}x/\mathrm{d}y = \mathrm{d}[q^{-1}(x)]/\mathrm{d}y$.

The kth noncentral moment from $h(y)$, $E(Y^k)$, is calculated as follows:

$$[4] \qquad E(Y^k) = \int_{y_{\min.}}^{y_{\max.}} y^k h(y)\,\mathrm{d}y$$

or

$$[5] \qquad E(Y^k) = \int_{y_{\min.}}^{y_{\max.}} [q(x)]^k g(x)\,\mathrm{d}x$$

where $y_{\min.}$ and $y_{\max.}$ are the minimum and maximum of variable y, respectively.

Equation 5 negates the need to calculate $h(y)$ and allows the computation of $E(Y^k)$ directly from $g(x)$. This property is particularly useful when tree diameter X at age A_1 cannot be expressed as a function of diameter Y at age A_2, i.e., $q^{-1}(y)$ and consequently $h(y)$ do not exist in closed forms. Numerical-integration techniques such as the Gaussian quadrature method (Hornbeck 1975; Press et al. 1992) can be used to evaluate eqs. 1 and 5.

After mortality and diameter growth, the diameter pdf at age A_2, $h(y)$, does not have the same functional form as the starting pdf, $f(x)$. However, $h(y)$ can be approximated by $w(y)$, which has the same functional form as $f(x)$. Parameters of $w(y)$ can be recovered from the first two diameter moments from eq. 5. The approach used in this path is called the *distribution-projection method*, which blends features of the individual-tree and diameter-distribution models in projecting a diameter distribution into the future.

Scenario 2

In the second scenario, information available at age A_1 includes arithmetic and quadratic mean diameters $\overline{D}_1$ (and Dq_1). The steps for path A remain the same as in scenario 1. Path C requires the moment-estimation method rather than the maximum-likelihood technique for deriving $f(x)$ at age A_1. For path B, parameters of $f(x)$ are also estimated from the two diameter moments by using moment estimation. A tree list then has to be generated from $f(x)$. This can be done (*i*) randomly by Monte Carlo simulation or (*ii*) deterministically by dividing the diameter range into small intervals, each containing trees with diameter as the midpoint of that interval. The rest of the steps in path B follow those in the first scenario.

A numerical example

Data

Data used in this study were obtained from the Southside Seed Source Study, which included 15 seed sources of loblolly pine (*Pinus taeda* L.) planted at 13 locations across 10 southern US states (Wells and Wakeley 1966). Seedlings were planted at a 1.8 m × 1.8 m spacing. Each plot, 0.0164 ha in size, consisted of 49 trees. Tree diameters and heights were measured at 10, 15, 20, and 25 years of age. A subset of the original data was randomly selected as the fit data set, to be used for fitting the models. The fit data set consisted of 120 five year growth periods from 100 plots. The validation data set, which comprised another 120 growth periods (from 103 plots), was randomly selected from the rest of the data. Summary statistics for the fit and validation data sets appear in Table 1.

Projecting the diameter distribution

The Weibull pdf (Bailey and Dell 1973) was selected to model the diameter distribution of a stand at the beginning of the growth period

$$[6] \qquad f(x) = \left(\frac{c}{b}\right)\left(\frac{x-a}{b}\right)^{c-1} \exp\left[-\left(\frac{x-a}{b}\right)^{c}\right]$$

where $a > 0$, $b > 0$, and $c > 0$ are the location, scale, and shape parameters, respectively.

Each of the three paths leading from either the current tree list (scenario 1) or the current diameter moments (scenario 2) to the future Weibull pdf will be covered in depth in this section.

Path A

A stand-level system of regression equations for predicting the future number of trees per hectare, minimum diameter, quadratic mean diameter, and diameter standard deviation are shown in Table 2. A method suggested by Borders (1989) was used to simultaneously estimate parameters of the regression system; this fitting procedure involved the use of option SUR (seemingly unrelated regression) of SAS® procedure MODEL (SAS Institute Inc. 1993). Arithmetic mean diameter was computed from quadratic mean diameter and standard deviation. The location parameter (a_y) of the Weibul pdf at age A_2 was computed as half the value of the predicted minimum diameter. The scale and shape parameters (b_y and c_y, respectively) were recovered from the first two diameter moments (Cao et al. 1982; Cao 2004)

$$[7] \qquad b_y(\hat{\overline{D}}_2 - a_y)/\Gamma_1$$

and

$$[8] \qquad 0 = a_y{}^2 - \hat{D}q_2^2 + 2a_y b_y \Gamma_1 + b_y{}^2\Gamma_2$$

where

$\hat{\overline{D}}_2$ is the predicted average diameter at age A_2
$\hat{D}q_2$ is the predicted quadratic mean diameter at age A_2
$\Gamma_k = \Gamma(1 + k/c_y)$, $k = 1, 2$
$\Gamma(x)$ denotes the complete gamma function evaluated at x.

Note that even though c_y does not appear in eqs. 7 and 8, it is included implicitly in Γ_1 and Γ_2.

Path B

The survival probability, $p(x)$, of each tree in the tree list was computed from a logistic function (Hamilton and Edwards 1976; Monserud 1976)

$$[9] \qquad p(x) = [1 + \exp(3.3444 - 0.0010N_1 + 0.1273B_1 - 0.5188x)]^{-1} \qquad (n = 3587;\ \text{all coefficients were significant } (P < 0.0001))$$

where x, N_1, and B_1 are tree diameter, number of trees, and basal area per hectare (m^2), respectively, at age A_1.

Table 1. Summary statistics for stand- and tree-level attributes by data type and growth period.

	Growth period		
Attribute	Age 10–15 years	Age 15–20 years	Age 20–25 years
Fit data set			
No. of growth periods[a]	40	40	40
Beginning (age A_1)			
No. of trees/ha	2254 (402)	1801 (446)	1416 (328)
Basal area (m^2/ha)	23.60 (5.03)	30.61 (4.12)	33.41 (6.75)
No. of trees/plot	37 (7)	30 (7)	23 (5)
Tree diameter (cm)	11.2 (2.6)	14.3 (3.5)	16.9 (3.7)
End (age A_2)			
No. of trees/ha	1797 (472)	1359 (266)	1249 (327)
Basal area (m^2/ha)	31.31 (5.34)	33.90 (6.28)	37.39 (7.96)
No. of trees/plot	29 (8)	22 (4)	20 (5)
Tree diameter (cm)	14.5 (3.5)	17.4 (4.0)	19.0 (4.3)
Validation data set			
No. of growth periods[b]	40	40	40
Beginning (age A_1)			
No. of trees/ha	2152 (504)	1666 (427)	1274 (332)
Basal area (m^2/ha)	22.41 (6.13)	31.23 (5.17)	34.64 (7.84)
No. of trees/plot	35 (8)	27 (7)	22 (6)
Tree diameter (cm)	11.2 (2.8)	15.0 (3.7)	17.3 (4.0)
End (age A_2)			
No. of trees/ha	1823 (608)	1341 (357)	1213 (342)
Basal area (m^2/ha)	31.93 (8.18)	33.01 (8.91)	37.26 (9.72)
No. of trees/plot	30 (10)	21 (5)	20 (6)
Tree diameter (cm)	14.5 (3.7)	18.1 (4.3)	19.2 (4.6)

Note: In each pair of values, the first is the mean and the second (in parentheses) is the standard deviation.
[a]From a total of 100 plots; more than one growth period per plot was selected in some of these plots.
[b]From a total of 103 plots; more than one growth period per plot was selected in some of these plots.

Table 2. Regression equations (based on 120 observations) used to predict stand attributes used in the parameter-recovery method (Baldwin and Feduccia 1987).

Stand attribute at age A_2	Equation
No. of trees/ha	$N_2 = N_1/[1 + \exp(19.8928\ 53.7130\ RS_1\ 0.9259\ H_1\ 0.0288\ N_1/A_1 + 63.6957/A_1)] + \varepsilon$; $R^2 = 0.81$; RMSE = 193 trees/ha
Quadratic mean diameter	$\mathrm{Dq}_1 = \exp\{(A_1/A_2)\ln(\mathrm{Dq}_1) + (1 - A_1/A_2)[3.8444 - 0.2013\ln(N_1) + 0.3934\ln(D\ln(\mathrm{Dq}_1)]\} + \varepsilon$; $R^2 = 0.93$; RMSE = 0.74 cm
Minimum diameter	$D_{\mathrm{min.2}} = \exp\{(A_1/A_2)\ln(D_{\mathrm{min.1}}) + (1 - A_1/A_2)[9.0640 + 0.0607A_1 - 0.6289\ln(N_1) - 1.0336 \times \ln(D_{\mathrm{min.1}})]\} + \varepsilon$; $R^2 = 0.61$; RMSE = 1.67 cm
Diameter standard deviation	$D_{\mathrm{SD2}} = \exp\{(A_1/A_2)\ln(D_{\mathrm{SD1}}) + (1 - A_1/A_2)[7.2933 - 0.7279\ln(N_1)]\} + \varepsilon$; $R^2 = 0.77$; RMSE = 0.40 cm

Note: N is the number of trees per hectare; RS is the relative spacing; H is the dominant height; Dq is the quadratic mean diameter; $D_{\mathrm{min.}}$ is the minimum diameter; D_{SD} is the standard deviation of diameters; ε is the random error. Subscript 1 and 2 denote measurements at age A_1 and A_2, respectively.

The following equation was used to predict y (tree diameter at age A_2) from x:

$$[10] \quad y = \alpha x^{\beta} + \varepsilon$$

where parameters α and β were expressed as functions of stand variables as follows:

$$[11] \quad \alpha = 1.0328 - 0.0056H_1 - 0.0021B_1$$

$$[12] \quad \beta = 1.2739 - 0.2428A_1/A_2 \qquad (n = 2888;$$
$R^2 = 0.9264$; RMSE = 1.19 cm; all coefficients were significant ($P < 0.0001$))

where H_1 is the average height of the dominants and codominants in the stand at age A_1.

In scenario 2, the value of the location parameter (a) of $f(x)$ was set as half the minimum diameter in the plot. The scale and shape parameters (b and c, respectively) were recovered from the two diameter moments $\overline{D}_1$ (and Dq_1). A tree list at age A_1 was generated from $f(x)$ by dividing the diameter range into 0.1 cm intervals, each containing trees with diameter as the midpoint of that interval.

For both scenarios, all trees in the tree list were subjected to the survival and diameter-growth steps. A revised tree list was computed, resulting in the first two diameter moments for age A_2 ($\hat{\bar{D}}_2$ and $\hat{D}q_2$, respectively). The location parameter (a_y) of $w(y)$, the Weibull pdf at age A_2, was computed from

$$[13] \quad a_y = \alpha a^{\beta}$$

where a is the location parameter of $f(x)$ and α and β were predicted from eqs. 11 and 12, respectively.

The scale parameter (b_y) and shape parameter (c_y) were solved from eqs. 7 and 8.

Path C

The parameters of $f(x)$, the Weibull pdf that approximated the diameter distribution of each plot at age A_1, were estimated by using the maximum-likelihood technique (scenario 1) or the moment-estimation method (scenario 2). Applying the tree-survival equation [9] resulted in $g(x)$, which was the diameter pdf after mortality at age A_1. It is optional to compute $h(y)$, the diameter distribution at age A_2. However, eq. 5 can be used to directly calculate $\hat{\bar{D}}_2$ and $\hat{D}q_2$, the first two diameter moments, respectively, thereby bypassing the calculation of $h(y)$. The last step involved the use of a Weibull pdf, $w(y)$, to approximate $h(y)$ by matching the two diameter moments. The location parameter (a_y) of $w(y)$ was computed from eq. 13. The scale parameter (b_y) and shape parameter (c_y) were solved from the system of eqs. 7 and 8.

The diameter pdf at age A_2, $h(y)$, was not a Weibull pdf but looked very much like one and could be approximated very well by $w(y)$, which was a Weibull pdf (Fig. 2).

Evaluation

The Weibull distribution predicted from each of the three paths was evaluated against observed diameters at age A_2. Two forms of error index (EI) proposed by Reynolds et al. (1988) were used to determine how well the methods performed at the diameter-class level

$$[15] \quad \mathrm{EI}_{N,i} = \sum_{j=1}^{k_i} \left| n_{ij} - \hat{n}_{ij} \right|$$

and

$$[16] \quad \mathrm{EI}_{\mathrm{B},i} = \sum_{j=1}^{k_i} \left| b_{ij} - \hat{b}_{ij} \right|$$

where

$\mathrm{EI}_{N,i}$ is the EI in terms of number of trees per hectare for the ith plot

$\mathrm{EI}_{\mathrm{B},i}$ is the EI in terms of basal area per hectare for the ith plot

n_{ij} and $\hat{n}_{ij}$ are the observed and predicted number of trees per hectare of the jth diameter class in the ith plot, respectively

b_{ij} and $\hat{b}_{ij}$ are the observed and predicted stand basal area (m^2/ha) of the jth diameter class in the ith plot, respectively

k_i is the number of 2 cm diameter classes in the ith plot

A low EI value indicates little difference between the observed and predicted numbers of trees or basal areas in each diameter class.

Table 3 presents the means and standard deviations of Reynolds et al.'s (1988) EI values from different methods for both scenarios, based on the validation data set. The three methods produced similar mean EI values for the two scenarios, varying from 621 to 637 trees per hectare and from 16.3 to 16.6 m^2/ha. When diameter moments were available at age A_1 (scenario 2) instead of a tree list (scenario 1), the parameter-recovery method produced results identical with those in scenario 1, but the tree-projection and distribution-projection methods both provided slightly higher EI values than scenario 1.

The total volume and merchantable volume per hectare at age A_2 predicted using the three methods were further evaluated (Table 4). The distribution-projection method clearly outperformed the other two methods in both scenarios, producing the smallest values of the mean difference (in magnitude) and mean absolute difference and the highest values of fit index (analogous to R^2).

Overall, the results demonstrated that comparable results were obtained when the Weibull distribution was either recovered from predicted stand attributes or projected using a set of individual-tree survival and diameter-growth equations.

Fig. 2. Diameter distributions from plot 1530313: $f(x)$ is the starting Weibull pdf at age 10, $g(x)$ is the Weibull pdf after mortality at age 10, $h(y)$ is the ending pdf at age 15, and $w(y)$ is the Weibull pdf that approximates $h(y)$.

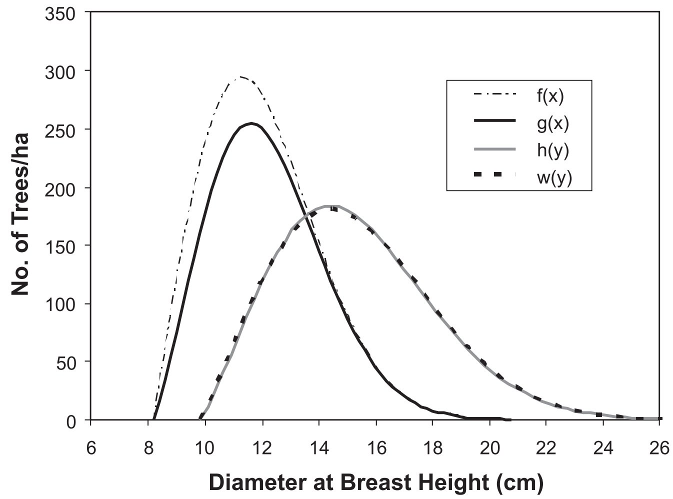


Table 3. Reynolds et al.'s (1988) error indices based on number of trees per hectare (EI_N) and basal area per hectare (EI_B) from the three methods for the validation data.

Path	Method	EI_N	EI_B
Scenario 1			
A	Parameter recovery	637 (175)	16.6 (6.3)
B	Tree rojection	**621** (176)	16.4 (6.6)
C	Distribution projection	**621** (**174**)	**16.3** (**6.2**)
Scenario 2			
A	Parameter recovery	637 (175)	16.6 (6.3)
B	Tree projection	631 (180)	16.7 (6.8)
C	Distribution projection	**628** (**170**)	**16.4** (**6.2**)

Note: Scenario 1: a list of tree diameters is available at the beginning of the growth period; scenario 2: only the arithmetic and quadratic mean diameters are available at the beginning of the period. In each pair of values, the first value is the mean and the second (in parentheses) is the standard deviation. The smaller the EI value, the better the fit. Values in boldface type are the lowest among the three methods.

Table 4. Evaluation statistics for total and merchantable volumes per hectare from the three methods for the validation data.

		Total volume (m^3/ha)			Merchantable volume (m^3/ha)		
Path	Method	MD	MAD	FI	MD	MAD	FI
Scenario 1							
A	Parameter recovery	–5.87	22.67	0.8775	–6.42	22.27	0.8903
B	Tree projection	–3.22	22.58	0.8820	–3.70	21.93	0.8965
C	Distribution projection	0.97	22.48	0.8953	0.39	21.12	0.9107
Scenario 2							
A	Parameter recovery	–5.87	22.67	0.8775	–6.42	22.27	0.8903
B	Tree projection	–4.63	22.86	0.8824	–5.32	22.58	0.8948
C	Distribution projection	0.80	22.19	0.8959	0.56	20.82	0.9114

Note: Scenario 1: a list of tree diameters is available at the beginning of the growth period; scenario 2: only the arithmetic and quadratic mean diameters are available at the beginning of the period. Total volume is total volume inside bark of all trees; merchantable volume is inside-bark volume to a 7.6 cm top of trees greater than 11.0 cm in diameter. Individual-tree volume equations are from Baldwin and Feduccia (1987). The following equation was used to predict tree heights from diameters: $h = 1.14619\ H[1 - 0.98419 \exp(-1.72900\ d/\mathrm{Dq})]$; $n = 6475$; $R^2 = 0.94$; RMSE = 0.90, where h is the total height (m) of a tree with diameter d (cm), H is the height of the dominant tree in the stand (m), and Dq is the quadratic mean diameter (cm). MD is the mean difference (between observed and predicted values); MAD is the mean absolute difference; FI is the fit index ($\mathrm{FI} = 1 - \sum (y_i - \hat{y}_i)^2 / \sum (y_i - \bar{y})^2$).

Discussion

The parameter-recovery method versus the tree-projection method

The main difference between the parameter-recovery method and the tree-projection method is the method for predicting diameter moments at age A_2. A stand-level model is used for this purpose in the parameter-recovery method, whereas an individual-tree model is involved in the tree-projection method. The Weibull pdf, $w(y)$, is then recovered from the two diameter moments in both methods.

The distribution-projection method versus the tree-projection method

Both methods employ the individual-tree model to predict tree survival and diameter growth, and therefore provide a link between an individual-tree model and a diameter-distribution model. In the tree-projection method, however, the tree model is applied to the tree list, whereas the distribution-projection method uses the individual-tree model to "grow" the pdf from $f(x)$ at age A_1 to $h(y)$ at age A_2.

Scenario 2 requires that $f(x)$ be established to generate a tree list in the tree-projection method. Growing trees generated from $f(x)$ in this method can be considered equivalent to the numerical technique for projecting $f(x)$ from age A_1 to $h(y)$ at age A_2 used in the distribution-projection method. In this particular scenario, these two methods are similar from this point of view.

The distribution-projection method versus existing linking methods

Bailey (1980) broke the ground by recognizing that the functional form of the pdf is preserved through time if an appropriate diameter-growth function is implied — a linear diameter-growth function for the normal, exponential, beta, and S_B distributions, and a power diameter-growth function for the Weibull, log normal, and generalized gamma distributions. Cao (1997) expanded Bailey's (1980) work by requiring that survival ratio has to be a certain function of tree diameter. The distribution-projection method presented in this study is basically a more general approach for projection of diameter distributions because it imposes no constraints on the individual-tree survival and diameter-growth equations. Any pair of well-behaved functions for predicting tree survival and diameter growth, regardless of form, can be used to "grow" a diameter distribution.

Conclusion

In this study, three approaches to characterize the diameter distribution of a future stand were considered. The first approach is the parameter-recovery method, which links a whole-stand model to a diameter-distribution model. The next two approaches, the tree-projection and distribution-projection methods, provide linkages between an individual-tree model and a diameter-distribution model. All three methods produced similar results in terms of Reynolds et al.'s (1988) EI values, whereas the distribution-projection method outperformed the other two methods in predicting total and merchantable volumes per hectare. This study demonstrates that linking individual-tree models and diameter-distribution models is indeed possible, and that the resulting performance is comparable to that of a parameter-recovery model. Furthermore, the distribution-projection method is the latest in a series of efforts to project a pdf into the future, following the work pioneered by Bailey (1980) and expanded by Cao (1997). The main improvement over the previous methods is that the distribution-projection method is a more general approach that does not put any constraints on the individual-tree survival and diameter-grown equations.

References

Bailey, R.L. 1980. Individual tree growth derived from diameter distribution models. For. Sci. **26**: 626–632.

Bailey, R.L., and Dell, T.R. 1973. Qualifying diameter distributions with the Weibull function. For. Sci. **19**: 97–104.

Bailey, R.L., Burgan, T.M., and Jokela, E.J. 1989. Fertilized midrotation-aged slash pine plantations stand structure and yield prediction models. South. J. Appl. For. **13**: 76–80.

Baldwin, V.C., Jr., and Feduccia, D.P. 1987. Loblolly pine growth and yield prediction for managed West Gulf plantations. USDA For. Ser. Res. Pap. SO-236.

Belcher, D.M., Holdway, M.R., and Brand, G.J. 1982. A description of STEMS – the Stand and Tree Evaluation and Modeling System. USDA For. Serv. Gen. Tech. Rep. NC-79.

Bennett, F.A., Lloyd, F.T., Swindel, B.F., and Whitehorne, E.W. 1978. Yields of veneer and associated products from unthinned, old-field plantations of slash pine in the north Florida and south Georgia flatwoods. USDA For. Serv. Res. Pap. SE-176.

Borders, B.E. 1989. Systems of equations in forest stand modeling. For. Sci. **35**: 548–556.

Burkhart, H.E., Farrar, K.D., Amateis, R.L., and Daniels, R.F. 1987. Simulation of individual tree growth and stand development in loblolly pine plantations on cutover, site-prepared areas. Publ. FWS-1-87, School of Forestry and Wildlife Resources, Virginia Polytechnic Institute and State University, Blacksburg, Va.

Cao, Q.V. 1997. A method to distribute mortality in diameter distribution models. For. Sci. **43**: 435–442.

Cao, Q.V. 2004. Predicting parameters of a Weibull function for modeling diameter distribution. For. Sci. **50**: 682–685.

Cao, Q.V., Burkhart, H.E., and Lemin, R.C., Jr. 1982. Diameter distributions and yields of thinned loblolly pine plantations. Publ. FWS-1-82, School of Forestry and Wildlife Resources, Virginia Polytechnic Institute and State University, Blacksburg, Va.

Cao, Q.V., Li, S., and McDill, M.E. 2002. Developing a system of annual tree growth equations for the loblolly pine – shortleaf pine type in Louisiana. Can. J. For. Res. **32**: 2051–2059.

Clutter, J.L., and Bennett, F.A. 1965. Diameter distribution in old-field slash pine plantations. Ga. For. Res. Counc. Rep. 13.

Hafley, W.L., Smith, W.D., and Buford, M.A. 1982. A new yield prediction model for unthinned loblolly pine plantations. Tech. Rep. 1, Southern Forest Research Center, School of Forest Research, North Carolina State University, Raleigh, N.C.

Hamilton, D.A., and Edwards, B.M. 1976. Modeling the probability of individual tree mortality. USDA For. Serv. Res. Pap. INT-185.

Hornbeck, R.W. 1975. Numerical methods. Quantum Publishers, Inc., New York.

Hyink, D.M., and Moser, J.W., Jr. 1983. A generalized framework for projecting forest yield and stand structure using diameter distributions. For. Sci. **29**: 85–95.

Lenhart, J.D., and Clutter, J.L. 1971. Cubic-foot yield tables for old-field loblolly pine plantations in the Georgia Piedmont. Ga. For. Res. Counc. Rep. Ser. 3 No. 22.

Lloyd, F.T., and Harms, W.R. 1986. An individual stand growth model for mean plant size based on the rule of self-thinning. Ann. Bot. **57**: 681–688.

Lynch, T.B., and Moser, J.W., Jr. 1986. A growth model for mixed species stands. For. Sci. **32**: 697–706.

Matney, T.G., and Sullivan, A.D. 1982. Compatible stand and stock tables for thinned loblolly pine stands. For. Sci. **28**: 161–171.

Monserud, R.A. 1976. Simulation of forest tree mortality. For. Sci. **22**: 438–444.

Murphy, P.A., and Farrar, R.M., Jr. 1982. Interim models for based area and volume projection of uneven-aged loblolly-shortleaf pine stands. South. J. Appl. For. **6**: 115–119.

Ochi, N., and Cao, Q.V. 2003. A comparison of compatible and annual growth models. For. Sci. **49**: 285–290.

Pienaar, L.V., and Harrison, W.M. 1989. Simultaneous growth and yield prediction equations for *Pinus elliottii* plantations in Zululand. S. Afr. For. J. **149**: 48–53.

Press, W.H., Teukolsky, S.A., Vetterling, W.T., and Flannery, B.P. 1992. Numerical recipes in Fortran 77: the art of scientific computing. 2nd ed. Cambridge University Press, Cambridge, UK.

Reynolds, M.R., Burk, T.E., and Huang, W.C. 1988. Goodness-of-fit tests and model selection procedures for diameter distribution models. For. Sci. **34**: 373–399.

SAS Institute Inc. 1993. SAS/ETS® user's guide. Version 6. 2nd ed. SAS Institute Inc., Cary, N.C.

Smalley, G.W., and Bailey, R.L. 1974. Yield tables and stand structure for loblolly pine plantations in Tennessee, Alabama, and Georgia highlands. USDA For. Serv. Res. Pap. SO-96.

Wells, O.O., and Wakeley, P.C. 1966. Geographic variation in survival, growth, and fusiform rust infection of planted loblolly pine. For. Sci. Monogr. 11.

Zhang, S., Amateis, R.L., and Burkhart, H.E. 1997. Constraining individual tree diameter increment and survival models for loblolly pine plantations. For. Sci. **43**: 414–423.