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A unified system for tree- and stand-level predictions

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ABSTRACT

The unified system developed in this study comprises two components, one to predict survival and the other to predict diameter, both at tree and stand levels. The tree survival model defaults to the stand survival model when tree diameter reaches a certain value. Similarly, the tree diameter growth model defaults to the quadratic mean diameter model when tree diameter is equal to quadratic mean diameter. The annual growth approach was used to accommodate growth intervals of different lengths. Parameters of the unified system can be estimated together (Simultaneous Estimation) or separately in two phases (Sequential Estimation). Also investigated was the Disaggregation method in which tree attributes (survival probabilities and diameters) were adjusted to match outputs from the stand-level models (number of trees and basal area per unit area).

Results indicated that, overall, the Disaggregation approach gave a better performance than did the Unadjusted approach. On the other hand, the Sequential Estimation narrowly surpassed the Simultaneous Estimation method (overall rank of 1.00 versus 1.10). Parameters of the unified system should be estimated by use of the Sequential Estimation method because it is a simpler method which involves only a subset of the parameters at each estimation phase.

1. Introduction

Growth and yield models have been an integral part in forest management. These models range from complicated individual-tree simulation models (high resolution), to models that give detailed volume for each tree size class (medium resolution), to relatively simple, whole-stand models (low resolution).

Whole-stand models provide information for the entire stand, such as stand survival (Zhang et al., 1993; Diéguez-Aranda et al., 2005; Zhao et al., 2007; Garcia, 2011; Tewari et al., 2014; Stankova, 2016). They also give predictions for basal area per unit area (Clutter, 1963; Pienaar and Turnbull, 1973; Clutter and Jones, 1980; Bailey and Ware, 1983; Pienaar and Shiver, 1986; Cao and Durand, 1991; Fang et al., 2001; Anta et al., 2006; Naing, 2020), or both stand survival and basal area (Candy, 1989; Somers and Farrar, 1991; Erikäinen, 2002; Garcia, 2011; Dean et al., 2013).

Size-class models classify the trees into diameter classes. Stand table-projection models move the current stand table (that shows number of trees in each diameter class) into the future (Clutter and Jones, 1980; Nepal and Somers, 1992; Cao and Baldwin, 1999; Allen et al., 2011). A different approach is to use a probability density function (pdf), notably the Weibull function, to model the frequency of tree diameters (Smalley and Bailey, 1974; Matney and Sullivan, 1982; Jiang and Brooks, 2009; Carretero and Alvarez, 2013).

Individual-tree models deliver detailed information for each tree such as tree survival (Hamilton, 1974; Monserud, 1976; Buchman, 1983; Amateis et al., 1989; Guan and Gertner, 1991a, 1991b; Zhang et al., 1997; Monserud and Sterba, 1999; Kjell and Lennart, 2005; Cao, 2006, 2017a). Other models are for predicting tree diameter growth (Wensel et al., 1987; Shafii et al., 1990; Quicke et al., 1994; Monserud and Sterba, 1996; Andreassena and Tomter, 2003; Sánchez-Gonzálezet al., 2006; Subedi and Sharma, 2011; Bohora and Cao, 2014), or both tree survival and diameter growth (Amateis et al., 1989; Cao, 1994, 2000; Palahía et al., 2003; Cao and Strub, 2008; Coble et al., 2012; Sun et al., 2019).

Trees are measured not every year but at intervals of either equal or unequal lengths. McDill and Amateis (1993) were the first to develop interpolation methods to fit annual growth models from periodic measurements. Other researchers have since proposed various methods to grow trees every year (Cao, 1994, 2000; Cao et al., 2002; Ochi and Cao, 2003; Nord-Larsen, 2006; Cao and Strub, 2008; Crecente-Campoa et al., 2010; Coble et al., 2012; Dean et al., 2013).

In selecting an appropriate growth and yield model, the users sometimes prefer one type of model for reliability of estimates (e.g. whole-stand models) and another for sufficient detail (e.g. size-class or individual-tree models). Problems arise when one needs to be able to do

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both because outputs from models of different resolutions might be inconsistent with one another.

Linking models having different levels of resolution appears to be the next logical step. The parameter-recovery method (e.g., Matney and Sullivan, 1982; Baldwin and Feduccia, 1987) was developed to link a whole-stand model to a diameter-distribution model. Methods to link a stand table projection model to a whole-stand model were also established (Clutter and Jones, 1980; Pienaar and Harrison, 1989; Nepal and Somers, 1992; Cao and Baldwin, 1999; Cao, 2007; Allen et al., 2011). Ritchie and Hann (1997) reviewed different disaggregation approaches, in which information obtained from an individual-tree model is used to disaggregate stand growth (predicted by a whole-stand model) among trees in the tree list. The result is a bridge between whole-stand and individual-tree models.

Daniels and Burkhart (1988) introduced the concept of an integrated system, which contains a unified mathematical structure for modeling tree and stand growth and can be applied at any level of resolution. Therefore this system can provide consistent growth and yield estimates at various levels of resolution. Cao (2017b, 2019) applied this concept to develop an integrated system to predict survival at both tree and stand levels.

The objective of this study was to develop a unified system that provides consistent estimates for (a) tree and stand survival, and (b) tree diameters and stand basal areas. The system was based on the annual growth approach to handle growth data of unequal intervals.

2. Data

Data used in this study were from 300 plots randomly selected from the Southwide Seed Source Study, which include 15 loblolly pine (*Pinus taeda* L.) seed sources planted at 13 locations across 10 southern states (Wells and Wakeley, 1966). Each 0.0164 ha plot consisted of 49 trees, planted at a 1.8 m × 1.8 m spacing. To avoid correlation problems stemming from repeated measurements, only one growth period (from 4 to 7 years) was randomly selected for each plot. The data, which consisted of measurements from age 10 to age 27, were randomly divided into two groups of 150 plots each. Table 1 shows the distribution of number of plots for each growth period. The means and standard deviations of stand and tree attributes are presented in Table 2.

The two-fold evaluation scheme was applied in this study. Parameters of both stand and tree models were estimated from the fit data (group 1), and then used to predict for the validation data (group 2). The same procedure was repeated with group 2 being the fit data and group 1 the validation data. Predictions from both groups were finally pooled to compute evaluation statistics for the different methods.

3. Methods

3.1. Height-Age model

The height-age model by Bailey and Clutter (1974) was used to project dominant height (H, in m) through time:

Table 1
Distribution of 300 plots, by age and group.

Starting age	Ending age	Group 1	Group 2
		Number of plots	
10	15	45	41
10	16	5	9
15	20	37	34
15	22	4	4
16	20	9	12
20	25	34	30
20	27	12	13
22	27	4	7
Total		150	150

$$\widehat{H}_{i,t+1} = exp\left[a_1 + \left\{\ln\left(\widehat{H}_{it}\right) - a_1\right\}\left(\frac{A_{it}}{A_{i,t+1}}\right)^{a_2}\right] \tag{1}$$

where $\widehat{H}_{it}$ is average height of the dominants and co-dominants for plot i at time t; A_{it} is stand age at time t; and a_1 and a_2 are regression coefficients.

3.2. Stand models

Cao (2006) used a variation of the logistic function to develop a model to predict stand survival (N, number of trees per ha):

$$\widehat{N}_{i,t+1} = \frac{\widehat{N}_{it}}{1 + exp\left\{b_0 + b_1 RS_{it} + b_2\widehat{H}_{it} + b_3\widehat{N}_{it}/A_{it} + b_4/A_{it}\right\}} \tag{2}$$

where $\widehat{N}_{it}$ is number of trees per ha for plot i at time t; $RS_{it} = \frac{\sqrt{10000/\widehat{N}_{it}}}{\widehat{H}_{it}}$ = relative spacing; and b_0–b_4 are regression coefficients.

While the stand survival function is a non-increasing function, the reverse is true for a diameter growth function. The quadratic mean diameter (Q, in cm) growth can be modeled as follows:

$$\widehat{Q}_{i,t+1} = \widehat{Q}_{it}\left\{1 + exp\left[c_0 + c_1\widehat{N}_{it}/A_{it} + c_2/A_{it} + c_3\widehat{Q}_{it}\right]\right\} \tag{3}$$

where $\widehat{Q}_{it}$ is quadratic mean diameter for plot i at time t; and c_0–c_3 are regression coefficients.

Stand basal area (B, in m^2/ha) is obtained from:

$$\widehat{B}_{i,t+1} = K\widehat{N}_{i,t+1}\widehat{Q}^2_{i,t+1} \tag{4}$$

where $\widehat{B}_{i,t+1}$ is stand basal area for plot i at time $t+1$; and $K = \pi/40{,}000$.

3.3. Tree models

In Cao's (2017b) integrated system, the tree survival model was the same as the stand survival model, with an extra term describing tree-level information:

$$p_{ij,t+1} = \frac{p_{ijt}}{1+exp\left\{b_0+b_1RS_{it}+b_2\widehat{H}_{it}+b_3\widehat{N}_{it}/A_{it}+b_4/A_{it}+b_5\left(\widehat{d}_{ijt}-b_6\widehat{Q}_{it}^{\,b_7}\right)\right\}} \tag{5}$$

where p_{ijt} is survival probability for tree j in plot i at time t; and b_0–b_7 are regression coefficients. Note that $b_6\widehat{Q}_{it}^{\,b_7}$ represents Ds_{it}, which is the tree diameter where tree survival probability equals stand survival probability. When $\widehat{d}_{ijt} = Ds_{it}$, the last term of equation (5) drops out, and the tree survival model defaults to the stand survival model (equation (2)).

Tree diameter growth can be derived from quadratic mean diameter growth in a similar manner:

$$\widehat{d}_{ij,t+1} = \widehat{d}_{ijt}\left\{1 + exp\left[c_0 + c_1\widehat{N}_{it}/A_{it} + c_2/A_{it} + c_3\widehat{Q}_{it} + c_4\left(\widehat{d}^2_{ijt} - \widehat{Q}^2_{it}\right)\right]\right\} \tag{6}$$

where $\widehat{d}_{ijt}$ is diameter of tree j in plot i at time t; and $c_0 - c_4$ are regression coefficients. When $\widehat{d}_{ijt} = Q_{it}$, the last term of equation (6) drops out, and the tree diameter growth model defaults to the quadratic mean diameter growth model (equation (3)).

3.4. Parameter estimation

Parameters a_1 and a_2 of the height-age model (Eq. (1)) were obtained separately from the other models by use of the least squares method.

Table 2
Means (and standard deviations) of stand and tree attributes, by group and age.

Group	Age	Dominant height (m)		Number of trees/ha		Basal area (m^2/ha)		Tree diameter (cm)	
1	10	9.1	(1.4)	1911	(628)	21.1	(5.9)	8.9	(1.4)
1	15	12.9	(1.6)	1732	(614)	30.4	(6.6)	12.6	(1.5)
1	16	16.2	(1.1)	1203	(175)	28.8	(3.3)	16.3	(1.1)
1	20	17.0	(2.0)	1220	(345)	33.5	(7.0)	16.6	(2.1)
1	22	15.3	(1.4)	1022	(181)	21.7	(3.9)	15.4	(1.3)
1	25	20.1	(2.4)	1111	(345)	39.0	(7.8)	19.9	(2.4)
1	27	19.5	(2.2)	858	(149)	32.3	(4.2)	19.5	(2.1)
2	10	9.1	(1.4)	1750	(679)	19.3	(7.4)	9.1	(1.5)
2	15	13.4	(1.6)	1546	(601)	28.2	(6.4)	13.2	(1.7)
2	16	16.2	(0.9)	1162	(214)	28.8	(3.5)	16.2	(0.8)
2	20	17.4	(2.3)	1183	(323)	33.5	(7.8)	17.1	(2.4)
2	22	14.4	(1.9)	904	(124)	18.2	(6.7)	14.9	(1.6)
2	25	19.0	(2.3)	1121	(313)	36.4	(9.3)	18.9	(2.3)
2	27	19.3	(3.9)	821	(153)	29.0	(8.9)	19.8	(3.6)

Parameters for the rest of the models can be estimated sequentially or simultaneously.

3.4.1. Sequential Estimation

In the Sequential Estimation approach, the parameters of the stand-level models (Eqs. (2) and (3)) were estimated in the first phase, and the remaining parameters of the tree-level models (Eqs. (5) and (6)) were then estimated in the second phase.

3.4.1.1. Stand level. The Seemingly Unrelated Regressions (SUR) method (SAS proc MODEL, SAS Institute Inc., 2004) was used to estimate parameters of Eqs. (2) and (3) that form a system of equations to simultaneously predict N, Q, and B (stand basal area):

$$N_{2i} = \widehat{N}_{2i} + \varepsilon_N \tag{7a}$$

$$Q_{2i} = \widehat{Q}_{2i} + \varepsilon_Q \tag{7b}$$

$$B_{2i} = \widehat{B}_{2i} + \varepsilon_B \tag{7c}$$

where N_{2i}, Q_{2i}, and B_{2i} are number of trees per ha, quadratic mean diameter, and basal area per ha, respectively, and the ε's are error. The ˆ symbol denotes predicted values, and subscripts $2i$ imply that the attributes, measured at the end of the growth period, belong to plot i.

3.4.1.2. Tree level. In the second phase, parameters b_5–b_7 and c_4 were estimated by use of a maximum likelihood method (Cao and Strub, 2008) in which the combined likelihood (L_T) for both tree survival and tree diameter is maximized:

$$L_T = \sum_i \sum_j lnL_{T_{ij}} \tag{8}$$

$$\text{where } lnL_{T_{ij}} = \begin{cases} ln(1 - p_{ij}), & \text{if tree j is dead,} \\ ln(p_{ij}) - ½\ ln(2\pi\sigma_d^2) - \dfrac{1}{2\sigma_d^2}\sum_i\sum_j \left(d_{2ij} - \widehat{d}_{2ij}\right)^2, & \text{if tree j is alive,} \end{cases}$$

p_{ij} is the probability that tree j in plot i survives the entire growth period, d_{2ij} and $\widehat{d}_{2ij}$ are, respectively, observed and predicted diameter of tree j in plot i at the end of the growth period, σ_d^2 is the tree diameter variance, Σ_j denotes the sum of all trees in plot i, Σ_i denotes the sum for i from 1 to m, and m is number of plots.

3.4.2. Simultaneous Estimation

All parameters (b_0–b_7 and c_0–c_4) in the stand- and tree-level models were simultaneously estimated by maximizing the following combined likelihood function:

$$lnL = \frac{lnL_T}{lnL_{T_{max}}} + lnL_S \tag{9}$$

where lnL_T = log-likelihood at the tree level, as defined earlier in Eq. (8),

$lnL_S = \frac{1}{3}\left(\frac{lnL_N}{lnL_{Nmax}} + \frac{lnL_Q}{lnL_{Qmax}} + \frac{lnL_B}{lnL_{Bmax}}\right)$ = log-likelihood at the stand level,

$$lnL_N = -½ln\left(2\pi\sigma_N^2\right) - \frac{1}{2\sigma_N^2}\sum_i\left(N_{2i} - \widehat{N}_{2i}\right)^2$$

$$lnL_Q = -½ln\left(2\pi\sigma_Q^2\right) - \frac{1}{2\sigma_Q^2}\sum_i\left(Q_{2i} - \widehat{Q}_{2i}\right)^2$$

$$lnL_B = -½ln\left(2\pi\sigma_B^2\right) - \frac{1}{2\sigma_B^2}\sum_i\left(B_{2i} - \widehat{B}_{2i}\right)^2$$

subscript max denotes for each attribute the maximum value of the likelihood function, which is obtained from the Sequential Estimation method, and

σ_N^2, σ_Q^2, and σ_B^2 are variance of N, Q, and B, respectively.

The first term of Eq. (9) is the log-likelihood at the tree level, and the second term is the log-likelihood at the stand level, averaged for N, Q, and B. This modified maximum likelihood procedure aims to optimize equally for the prediction of both tree- and stand-level attributes. This is a result of a progression for defining a modified likelihood function, starting with optimizing for two variables (survival and diameter) at the tree level (Cao and Strub, 2008), to optimizing for one variable (survival) at both tree and stand levels (Cao, 2017b), to the current problem of optimizing for multiple variables (survival, diameter, and basal area) at both tree and stand levels. SAS proc MODEL (SAS Institute Inc., 2004) was also used to estimate the parameters.

3.5. Disaggregation

The tree models (Eqs. (5) and (6)) provide for a future age a new tree list that, when aggregated, would result in values different from those predicted by the stand-level models (Eqs. (2)–(4)). The method suggested by Cao (2010) was applied here to adjust the predicted tree survival probability and diameter at the end of the growth period such that the resulting aggregated values match number of trees per ha ($\widehat{N}_{2i}$) and basal area per ha ($\widehat{B}_{2i}$):

$$p_{ij}^{*} = p_{ij}^{\alpha_i} \text{ such that } \sum_j p_{ij}^{*} = s_i \widehat{N}_{2i} \tag{10}$$

$$d_{2ij}^{*2} = d_{1ij}^2 + \beta_i\left(\widehat{d}_{2ij}^2 - d_{1ij}^2\right), \quad \text{where } \beta_i = \frac{(s_i\widehat{B}_{2i}/K) - \sum_i\sum_j\left(p_{ij}^{*}d_{1ij}^2\right)}{\sum_i\sum_j\left[p_{ij}^{*}\left(\widehat{d}_{2ij}^2 - d_{1ij}^2\right)\right]} \tag{11}$$

where p_{ij} and p_{ij}^{*} are predicted and adjusted tree survival probability for the entire growth period, respectively, d_{1ij} is tree diameter at the beginning of the growth period, d_{2ij} and d_{2ij}^{*} are predicted and adjusted tree diameter at the end of the growth period, respectively, and subscripts ij denote tree j in plot i.

3.6. Evaluation

After the coefficients were obtained from one group, they were used to predict for the other group. Predicted values from both groups were then pooled for the computation of evaluation statistics.

3.6.1. Stand-level prediction

The following statistics were computed for stand-level evaluation:

$$Mean\ \ difference : MD = \frac{1}{m}\sum_i\left(y_{2i} - \widehat{y}_{2i}\right) \tag{12a}$$

$$Mean\ \ absolute\ \ difference : MAD = \frac{1}{m}\sum_i\left|y_{2i} - \widehat{y}_{2i}\right| \tag{12b}$$

$$Fit\ \ index : FI = 1 - \frac{\sum_i\left(y_{2i} - \widehat{y}_{2i}\right)^2}{\sum_i\left(y_{2i} - \overline{y}_2\right)^2} \tag{12c}$$

where m = number of plots; y_{2i} and $\widehat{y}_{2i}$ are, respectively, observed and predicted values of N, Q, or B of plot i at the end of the growth period; and $\overline{y}_2$ = average of y_{2i}.

3.6.2. Tree-level prediction

Evaluation statistics for tree diameter predictions were similar to those presented in equations (12a–12c). Tree-level survival predictions were evaluated from:

$$Mean\ \ difference : MD = \frac{\sum_i\sum_j\left(y_{ij} - p_{ij}\right)}{\sum_i n_{1i}} \tag{13a}$$

where $y_{ij} = 1$ if tree j in plot i was alive and 0 if it was dead; Σ_i denotes the sum for i from 1 to m; Σ_j denotes the sum for j from 1 to n_{1i}; and n_{1i} = number of trees in plot i at the beginning of the growth period.

$$Mean\ \ absolute\ \ difference : MAD = \frac{\sum_i\sum_j\left|y_{ij} - p_{ij}\right|}{\sum_i n_{1i}} \tag{13b}$$

AUC: area under the ROC (Receiver Operating Characteristic) curve. The range for AUC is between 0.5 (poorest fit) and 1 (perfect fit).

The relative rank, which was introduced by Poudel and Cao (2013) to display the relative position of each method, is defined as:

$$R_i = 1 + \frac{(k-1)(S_i - S_{min})}{S_{max} - S_{min}} \text{ for minimization objective, and} \tag{14}$$

Table 4
Evaluation statistics for stand-level prediction, by estimation method.

Variable[1]	Evaluation statistics	Sequential Estimation		Simultaneous Estimation	
		Direct[2]	Summation[3]	Direct	Summation
N	MD	13.89	1.45	−6.37	−3.06
	MAD	164.91	152.87	163.29	158.26
	FI	0.8059	0.8331	0.8097	0.8204
Q	MD	0.0002	−0.5996	0.0263	−0.5735
	MAD	0.7194	0.9149	0.7170	0.9100
	FI	0.9295	0.8986	0.9297	0.8970
B	MD	0.2833	−2.3267	−0.3841	−2.3282
	MAD	3.7483	4.0064	3.7079	4.0998
	FI	0.6536	0.5585	0.6511	0.5435
Sum of the ranks		21.32	23.84	19.41	24.13
Overall Rank		***2.22***	***3.82***	***1.00***	***4.00***

[1] N = number of trees per ha; Q = quadratic mean diameter (cm); B = basal area (m^2/ha).
[2] Direct: outputs are computed directly from the stand-level models.
[3] Summation: outputs are computed from the tree-level models and then aggregated to stand level.

Table 3
Parameter estimates, by group and estimation method.

Parameter estimate	Sequential Estimation			Simultaneous Estimation		
	Group 1	Group 2	All	Group 1	Group 2	All
Eq. (1)						
a_1	4.3237	3.6763	3.9031	4.3237	3.6763	3.9031
a_2	−0.4903	−0.7882	−0.6490	−0.4903	−0.7882	−0.6490
Eqs. (2) and (5)						
b_0	13.6901	11.5091	12.1328	14.4280	12.4655	13.3724
b_1	−45.1106	−45.0738	−43.9576	−44.0339	−46.0137	−45.1171
b_2	−0.5342	−0.4519	−0.4735	−0.5860	−0.5084	−0.5432
b_3	−0.0316	−0.0281	−0.0283	−0.0304	−0.0277	−0.0286
b_4	43.0118	51.8511	44.5730	36.2337	50.3021	43.2080
b_5	−0.3084	−0.2680	−0.2680	−0.3085	−0.2682	−0.2878
b_6	1.3386	1.4190	1.4190	0.9645	0.9524	0.9403
b_7	0.8580	0.8388	0.8388	0.9824	0.9890	0.9924
Eqs. (3) and (6)						
c_0	−2.3421	−2.5600	−2.4773	−2.3999	−2.5025	−2.4637
c_1	−0.0045	−0.0045	−0.0044	−0.0040	−0.0044	−0.0041
c_2	16.0584	17.6789	16.7167	15.3133	17.5353	16.4020
c_3	−0.0885	−0.0853	−0.0855	−0.0856	−0.0885	−0.0865
c_4	−0.0003	−0.0002	−0.0002	−0.0003	−0.0002	−0.0002

$$R_i = k - \frac{(k-1)(S_i - S_{min})}{S_{max} - S_{min}} \text{ for maximization objective.} \tag{15}$$

where R_i = the relative rank of method i (i = 1, 2, …, k), k = number of methods evaluated, S_i = the evaluation statistic produced by method i, S_{min} = the minimum value of S_i, and S_{max} = the maximum value of S_i. This ranking system results in a real number for R_i and is preferable to the traditional integer ranks. For either minimization of maximization objective, the best method receives a rank of 1 whereas the worst method a rank of k.

4. Results

Table 3 shows parameter estimates, by group and estimation method. Evaluation statistics are shown for predicting attributes at the stand level for both estimation methods (Table 4). After a relative rank was computed separately for each statistic of each method, an overall rank was calculated based on the sum of all ranks for each method. The Direct approach, in which outputs were predicted directly from the stand-level models (Eqs. (2)–(3)), were also evaluated against the Summation approach (stand outputs were obtained by aggregating outputs from tree-level models, Eqs. (5)–(6)). Based on the overall ranks, the Direct approach performed better than the Summation approach, with a rank of 1.00 and 2.22 for the Simultaneous and Sequential estimation methods, respectively (Table 4).

Table 5 presents evaluation statistics for predicting tree diameter and survival probability. For both estimation methods, the Disaggregation approach provided better diameter prediction than the Unadjusted approach. The reverse was true for tree survival prediction. The combined rankings for both tree diameter and survival for the Disaggregation approach were 1.00 (Sequential Estimation) and 1.96 (Simultaneous Estimation), ahead of the rankings of the Unadjusted approach at 3.66 and 4.00 for the Sequential and Simultaneous Estimation, respectively. The ROC curves for the Sequential and Simultaneous Estimation methods are presented in Fig. 1 for the Disaggregation approach. Fig. 2 shows the residual plot of tree diameters for the Disaggregated Sequential Estimation method.

A final rank was computed for each method, based on the sum of the ranks for all evaluation statistics at both stand and tree levels (Table 6). Both estimation methods performed equally well after disaggregation, with ranks of 1.00 for the Sequential Estimation and 1.10 for the Simultaneous Estimation methods. Without disaggregation, the two estimations methods were ranked at the bottom, 4.00 for the Sequential Estimation and 3.40 for the Simultaneous Estimation.

5. Discussion

5.1. Annual growth prediction

A compatible stand-level model is step-invariant because it is obtained by integrating a growth function, whereas the annual growth model is also step-invariant owing to the process of summing over the growth period (Ochi and Cao, 2003). Because no integration is needed, the annual growth model does not have to be built on integrable functions and therefore can take on flexible functional forms. Ochi and Cao (2003) found that the annual growth approach performed better than

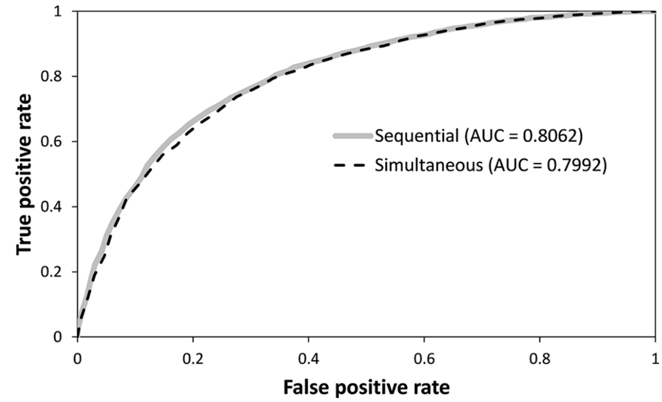

Fig. 1. Receiver operating characteristic (ROC) curves for the Sequential and Simultaneous Estimation methods, both disaggregated from the stand models.

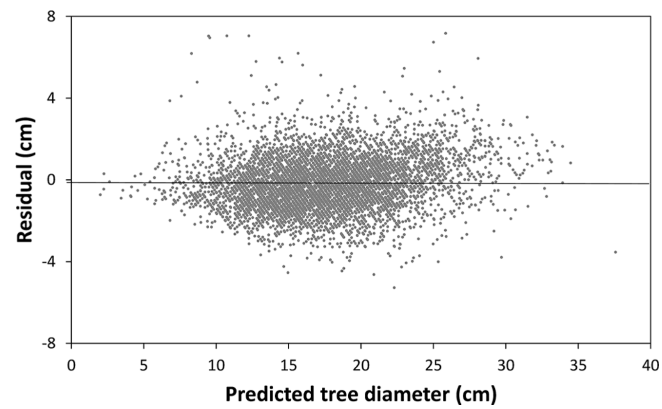

Fig. 2. Residual plot of tree diameters for the Disaggregated Sequential Estimation method. The graph for the Disaggregated Simultaneous Estimation method is similar to this one and is not shown.

Table 6
Final ranks based on both stand and tree levels, by estimation and disaggregation methods.

Estimation method	Disaggregated?	Final rank
Sequential	No	4.00
	Yes	1.00
Simultaneous	No	3.40
	Yes	1.10

Table 5
Evaluation statistics for tree-level prediction, by estimation method.

Variable	Evaluation statistics	Sequential Estimation		Simultaneous Estimation	
		Unadjusted	Disaggregated	Unadjusted	Disaggregated
Tree diameter	MD	−0.7423	−0.1550	−0.7335	−0.2961
	MAD	1.2432	1.0532	1.2373	1.0752
	FI	0.9065	0.9290	0.9075	0.9268
Tree survival probability	MD	0.0010	0.0115	−0.0020	−0.0426
	MAD	0.2506	0.2558	0.2498	0.2392
	AUC	0.8132	0.8062	0.8081	0.7992
Sum of the ranks		17.07	11.25	17.81	13.36
Overall Rank		***3.66***	***1.00***	***4.00***	***1.96***

the compatible growth and yield approach. The annual growth approach has been successfully applied to stand-level models (Ochi and Cao, 2003; Qin and Cao, 2006; Zhang et al., 2010; Dean et al., 2013) and tree-level models (McDill and Amateis, 1993; Cao, 1994, 2000; Cao et al., 2002; Nord-Larsen, 2006; Qin and Cao, 2006; Cao and Strub, 2008; Crecente-Campoa et al., 2010; Zhang et al., 2010; Coble et al., 2012). This approach appears to be a logical choice for modeling growth data with intervals of different lengths, such as those used in the current study.

5.2. Simultaneous estimation

5.2.1. Stand models

Burkhart and Sprinz (1984) simultaneously estimated coefficients of two equations for predicting stand basal area and volume by use of a squared error loss function (Reed and Green, 1984). Van Deusen (1988) showed that this method is equivalent to the technique of SUR (Seemingly Unrelated Regression). The SUR approach has been used to estimate coefficients in systems of equations to predict stand-level attributes (Ochi and Cao, 2003; Qin and Cao, 2006; Zhang et al., 2010).

Because stand basal area could follow different trajectories: increasing, increasing and leveling off, or increasing and then decreasing, the approach employed in this study was to use an indirect approach rather than modeling basal area directly. Stand basal area was computed as a product of number of trees per ha (modeled as a non-increasing function) and quadratic mean diameter (modeled as a non-decreasing function). Parameters of the stand-level system (Eqs. (7a)–(7c)) were simultaneously obtained by means of SUR, which optimized the prediction of N_{2i}, Q_{2i}, and B_{2i}.

5.2.2. Tree models

Because the objective is to maximize the likelihood function of a tree survival model and to minimize the sum of squared errors of a tree diameter growth model, the techniques of squared error loss function (Reed and Green, 1984) and SUR (Van Deusen, 1988) do not work if one desires to simultaneously estimate parameters of this system of tree models. In order to optimize for both tree survival and diameter prediction, this study adopted a method that maximized the combined likelihood function (Cao and Strub, 2008; Coble et al., 2012). The combined likelihood for each tree was either (1) the probability of that tree being dead if it is dead, or (2) the product of the probability of that tree surviving and the probability of its diameter equaling the predicted diameter if the tree is alive.

5.2.3. Both stand and tree models

Cao (2017a) developed another type of combined likelihood method to optimize for both tree-and stand-level survival. The method was modified in this study to include stand basal area and tree diameter. The result was a combined log-likelihood which was the sum of the log-likelihood values for tree and stand levels. These log-likelihood values were scaled by dividing by their respective optimum values, obtained when the models were fitted separately. The goal was to find a set of parameters for Eqs. (5) and (6) to produce predictions that performed equally well at both tree and stand levels.

5.3. Disaggregation

Summation of outputs from tree models often suffers from lack of accuracy and precision because of error accumulation. In this study, stand-level prediction obtained directly from the stand models was better than the aggregate of outputs from the tree models (Table 4), confirming previous findings (Cao, 2006, 2014; Qin and Cao, 2006; Zhang et al., 2011; Hevia et al., 2015).

It therefore makes sense to constrain outputs from a tree model so that their sum matches outputs from a stand-level model. The disaggregated tree models, in addition to giving rise to numerical consistency, has been reported to improve tree-level predictions over the unadjusted models in previous studies (Cao, 2006, 2014; Qin and Cao, 2006; Zhang et al., 2011; Hevia et al., 2015), even though exceptions did exist (Cao, 2017a). By evaluating adjustment methods from observed and predicted stand survival, Cao (2010) concluded that the success of disaggregation depends largely on the quality of stand prediction. Cao (2017a) found through simulation that the disaggregation method should produce better tree survival predictions if the fit index for the stand survival model exceeded 0.80. Disaggregation yielded mixed results in this study. It improved tree diameter predictions, maybe because they were disaggregated from the quadratic mean diameter model with a fit index of 0.92. However, disaggregation was worse in predicting tree survival, maybe because the stand survival model had a fit index of 0.80. Apparently, a better fit index for the stand survival model is necessary for the disaggregation method to perform well in predicting tree survival.

Even with the mixed results produced by the Disaggregation method for tree-level predictions, the final rankings (Table 6) based on both tree- and stand-level predictions revealed that, overall, the Disaggregation method performed better than the Unadjusted method for both types of parameter estimation.

5.4. A unified system

In this study, the proposed unified system has two components. The first component, as developed by Cao (2017b), deals with survival at both tree and stand levels. The tree survival model is the stand survival model with an added term, $b_5\left(\widehat{d}_{ijt} - Ds_{it}\right)$, where $Ds_{it} = b_6\widehat{Q}_{it}^{\,b_7}$ is the tree diameter at which tree survival probability equals stand survival rate. Note that when $\widehat{d}_{ijt} = Ds_{it}$, this term approaches zero, and the tree survival model defaults to the stand survival model (equation (2)). The second and also new component treats tree diameter and quadratic mean diameter growth in a similar manner. The tree diameter model is the quadratic mean diameter model with an extra term, which would disappear when $\widehat{d}_{ijt} = \widehat{Q}_{it}$, defaulting the tree diameter model to the stand-level diameter model (Eq. (3)).

Daniels and Burkhart (1988) introduced the concept of an integrated system, which relies on a common mathematical structure for modeling tree and stand growth. This system is integrated because it can be

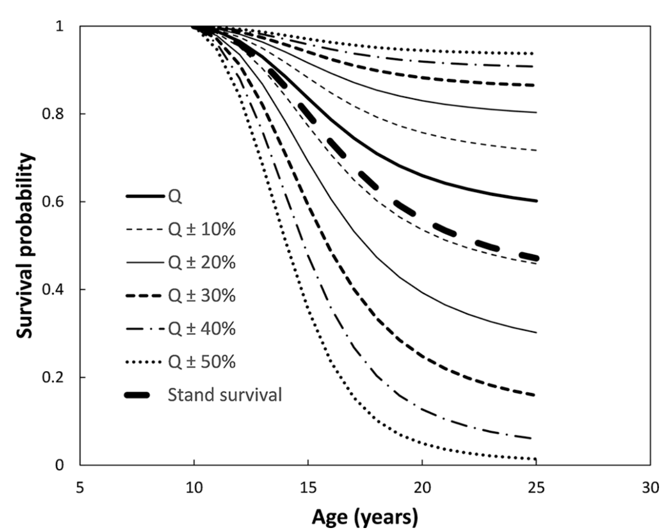

Fig. 3. Survival probabilities over time of trees having $(Q \pm x)$ as starting diameters at age 10, where × varies from 0% to 50% of quadratic mean diameter (Q). For each pair, the top curve denotes $(Q + x)$ and the bottom curve $(Q - x)$. Also shown in thick dashed line is stand-level survival.

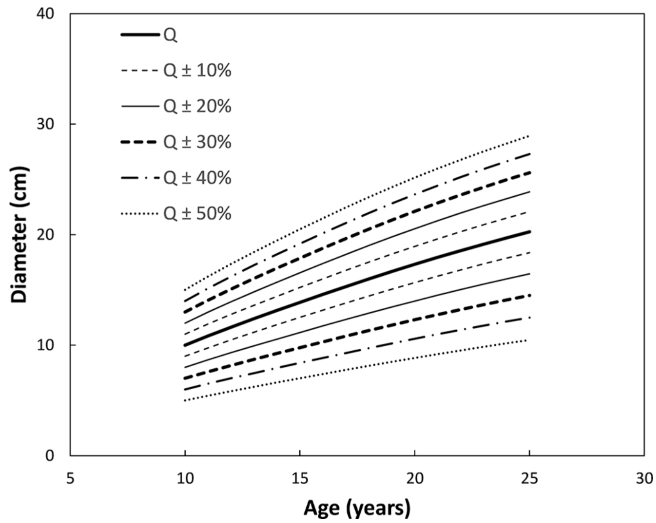

Fig. 4. Diameter growth of trees having ($Q \pm x$) as starting diameters at age 10, where × varies from 0% to 50% of quadratic mean diameter (Q). For each pair, the top curve denotes ($Q + x$) and the bottom curve ($Q - x$).

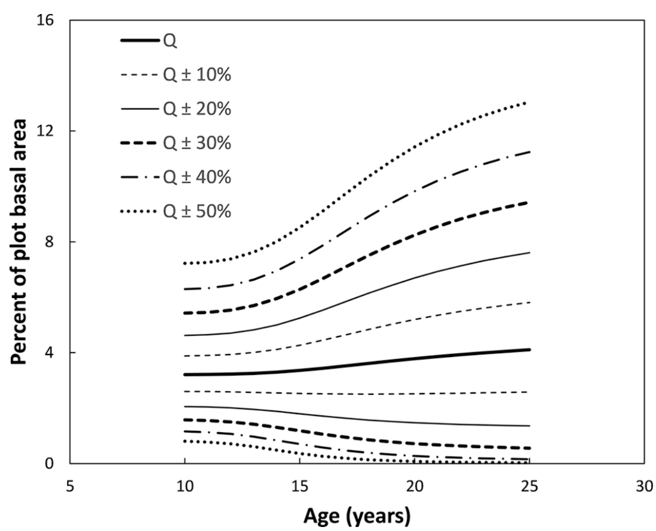

Fig. 5. Change through time of tree basal area as a percent of plot basal area.

applied at any level of resolution. The system developed in the current study is an extension of Cao's (2017b) system, and can predict growth at both tree and stand levels. In addition to being an "integrated" system because its equations at different resolution levels are structurally similar, it is also a "unified" system because each of the two tree models collapses to its corresponding stand model at an appropriate tree diameter. In addition, disaggregation not only ensures numerical consistency, but also delivered an overall better performance than the unadjusted models.

Fig. 3 displays patterns over time of survival probabilities for trees having various diameters at age 10. The stand attributes at age 10 are $H = 9$ m, $N = 1900$ trees/ha, and $Q = 10$ cm. The graph shows that the trajectories of tree and stand survival closely match when tree diameter is equal to Ds, or 90% of Q in this example. A similar graph is constructed for diameter growth (Fig. 4). Fig. 5 presents how surviving tree basal area, as a percent of plot basal area, varies through the years. The pattern shows that the contribution of tree basal area to plot basal area increases over the years for large trees, decreases for smaller trees, and remains the same when the tree diameter equals 90% of Q. This is the same tree with survival probability matching the stand survival rate (Fig. 3).

6. Summary and conclusions

The unified system developed in this study comprises two components, one to predict survival and the other to predict diameter, both at tree and stand levels. The tree survival model defaults to the stand survival model when tree diameter reaches a certain value. Similarly, the tree diameter growth model defaults to the quadratic mean diameter model when tree diameter is equal to quadratic mean diameter. The annual growth approach was used to accommodate growth intervals of different lengths. Parameters of the unified system can be estimated together (Simultaneous Estimation) or separately in two phases (Sequential Estimation). Also investigated was the Disaggregation method in which tree attributes (survival probabilities and diameters) were adjusted to match outputs from the stand-level models (number of trees and basal area per unit area).

Results indicated that, overall, the Disaggregation approach clearly gave a better performance than did the Unadjusted approach. On the other hand, the Sequential Estimation narrowly surpassed the Simultaneous Estimation method (overall rank of 1.00 versus 1.10). Parameters of the unified system should be estimated by use of the Sequential Estimation method because it is a simpler method which involves only a subset of the parameters at each estimation phase.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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