A Growth and Yield Model for Improved Eastern Cottonwood Plantations in the Lower Mississippi Delta[1]

Quang V. Cao, *School of Forestry, Wildlife, and Fisheries, Louisiana Agricultural Experiment Station, Louisiana State University Agricultural Center, Baton Rouge, LA 70803, and* **Kenneth M. Durand**, *MacMillan Bloedel Inc., P.O. Box 667, Marion, AL 36756.*

*ABSTRACT. A compatible growth and yield model was developed based on remeasurement data collected from 183 plots on unthinned improved eastern cottonwood (*Populus deltoides *Bartr.) plantations in the lower Mississippi Delta. The Sullivan and Clutter (1972) equation form was selected for predicting cubic-foot volume yield and projecting volume from site index and initial age and basal area. Yield equations explained 97% and 94%, respectively, of the variations in total outside bark and merchantable inside bark volumes. Mean annual increment of merchantable volume culminated between 8 and 15 years, depending on site index and initial basal area.*

South. J. Appl. For. 15(4):213–216.

Eastern cottonwood (*Populus deltoides* Bartr.) is the fastest growing tree species in the South (Krinard and Johnson 1984). Interest in growing cottonwood in plantations began about 1960 and increased for several years thereafter. One reason for the interest in producing cottonwood is its fast growth; another is the steady progress made in tree improvement programs (Johnson and Kerr 1976). An estimated 40,000 to 50,000 acres of eastern cottonwood were planted in the lower Mississippi Delta from 1961 through the mid-1970's (Burkhardt and Krinard 1976).

A growth and yield model for cottonwood grown in the Mississippi River Valley was introduced by Alexander (1976). However, this model did not possess the desirable characteristics of compatibility between growth and yield as described by Clutter (1963); i.e., yields could not be obtained by summation of the predicted growth through the appropriate growth periods. The objective of this study was to develop a compatible growth and yield model to predict whole stand volume for improved eastern cottonwood plantations in the lower Mississippi River Delta.

DATA

Data for this study consisted of diameter and total height measurements taken over an 11-year period from 1975 through 1985 in improved eastern cottonwood plantations. Along the Mississippi River between St. Francisville, LA, and Vicksburg, MS, 183 plots were located by superimposing a square grid system across 20,000 ac of the cottonwood plantations and selecting plot centers at random. No thinning occurred on any of the plots. These permanent plots were established using point sampling technique (Beers and Miller 1964) with a basal area factor of 10. The plots were measured up to 7 times on 1- or 2-year intervals over the 11-year period. Each measurement was considered an observation.

Stand volume per acre was computed using individual tree volume equations published by Krinard (1988) for eastern cottonwood grown in Mississippi River alluvium:

$$TVOB = 0.06 + 0.002221\,D^2H \qquad (1)$$

and

$$MVIB = -0.86 + 0.001904\,D^2H \qquad (2)$$

where

$TVOB$ = total tree volume outside bark in cubic feet from a 1-ft stump to the tree tip
$MVIB$ = merchantable tree volume inside bark in cubic feet from a 1-ft stump to a 3-in. top
D = diameter at breast height (dbh) in inches
H = total height in feet

Average height of dominant and codominant trees was calculated as the mean height of the tallest 50% of the surviving trees at each plot. For plots that were 10 years or older, site index (or dominant and codominant height at base age 10 years) was computed by linear interpolation. Younger plots required the prediction of site index from the oldest measurement at each plot, using Cao and Durand's (1991) site index equation. Table 1 shows the distribution of sample observations by site index, age, and basal area per acre, and also the summary statistics for merchantable volume per acre by site index and age.

DEVELOPMENT OF A GROWTH AND YIELD MODEL

Clutter (1963) defined compatible growth and yield models as those in which yield was obtained through mathematical integration of growth. Noncompatible growth and yield models, on the other hand, produce different values for yield and cumulative growth. His system of equations was further refined by Sullivan and Clutter

[1] Approved for publication by the Director of the Louisiana Agricultural Experiment Station as manuscript number 90-22-4592. Appreciation is extended to the late Mr. Don Alexander of Cavenham Corp. for supplying data for this study, and to Mr. Pat Weber of James River Corp. for his advice.

Table 1. Distribution of sample observations and summary statistics for merchantable volume (inside bark to a 3-in.-top) per acre, by site index (base age 10 years), stand age, and basal area per acre.

Site index (ft)	Stand age (yr)	Stand Basal Area (ft^2/ac) 10	20	30	40	50	60	70	80	90	100	110	120 +	Total	Stand merchantable volume Min.	Ave.	Max.
		No. of observations													ft^3/ac		
40	8				1									1	312	312	312
50	3		1											1	138	138	138
	5		1	1										2	62	85	108
	6		2	1	1									4	62	182	347
	7	1	2											3	56	120	238
	8	1			1	2								4	167	429	697
	9		1		1									2	131	226	321
	10	1					1							2	188	261	334
	11				1									1	556	556	556
	12					1		1						2	946	1151	1356
	13					1				1				2	681	1288	1895
	14					1		1						2	1026	1048	1070
	16					1								1	1310	1310	1310
	17									1				1	2064	2064	2064
	18								1					1	2287	2287	2287
60	3	1												1	40	40	40
	4	2			1									3	51	181	361
	5		2											2	113	166	219
	6		2					1						3	128	375	677
	7	1	3	1	1	1	1							8	76	386	862
	8		1		1		2							4	248	652	924
	9			1	1	3	1							6	540	798	1058
	10	1				1				1				3	215	928	1720
	11		1		1	1								3	346	760	1172
	12							1						1	1693	1693	1693
	13						1							1	1014	1014	1014
	14						1		1					2	612	1233	1853
70	3	2	2	3	1									8	2	140	305
	4		2	1										3	123	252	471
	5	2	1	2	6	1	1							13	67	337	918
	6			2	1	4	1							8	283	644	975
	7	1	2	1	5		4		1		1			15	179	749	1341
	8	2	1		1	3	2	1						10	224	756	1327
	9					2	2		2	1				7	792	1219	1874
	10		1	3		1	2	2						9	463	976	1385
	11		1		2	1		1						5	584	1080	1772
	12		2					1		1				4	549	1372	2255
	13									1				1	1688	1688	1688
	14		2	1		1		1	1		2			8	480	1506	2339
	15			1		1	1							3	797	1333	1889
80	2			1										1	114	114	114
	3		4	2		1								7	48	201	704
	4	2	4	3	2									11	32	271	582
	5		1	3	2	3	1							10	310	586	1128
	6		1	2	2	3	1							9	326	685	1130
	7				2	1	3	1	1					8	696	1026	1475
	8		3		2	2	1	1	2		1			12	420	1086	2039
	9		1	1	2	1							1	6	473	1213	2447
	10			1	1		1	1						4	865	1233	1800
	11				1	3	1							5	1262	1332	1410
	12								2					2	1784	1883	1982
	13						1				1			2	1326	1810	2295
	15				1									1	1229	1229	1229
90	2			1										1	142	142	142
	3	1	2	2	7	1								13	106	291	608
	4		5	4	5	2		2	1					19	88	592	1775
	5	1	1		1	1	3	1	1		1			10	108	953	1675
	6	2	1	3		2	1	3	3		1			16	190	1070	2257
	7	1	2	1	1		1	1	1	4				12	220	1270	2048
	8			3	2	1	2		1	1	3			13	756	1505	2773
	9		1	1	1	1		1						5	542	1092	1559
	10				1	1								2	1082	1235	1387
	11			1			1	1	1	1				5	861	1975	2513
	12						1							1	1822	1822	1822

Table 1. (Continued)

Site index (ft)	Stand age (yr)	Stand Basal Area (ft²/ac) 10	20	30	40	50	60	70	80	90	100	110	120+	Total	Stand merchantable volume Min.	Ave.	Max.
		No. of observations													ft³/ac		
	13											1		1	2802	2802	2802
	14								1		1			2	2242	2298	2353
	15								1					1	2667	2667	2667
100	2		1											1	34	34	34
	3		1	4	4	1								10	104	393	673
	4				3	4	5	1		1				14	578	980	1820
	5	1	1		1	1		1						5	142	710	1398
	6	2				1	1	1	2		4			11	172	1625	2424
	7			1			2		1					4	560	1369	1865
	8	1			1			1	2				2	7	256	1998	4201
	9				2		1							3	1078	1355	1676
	13					1								1	1610	1610	1610
	14							1						1	2345	2345	2345
110	3					1								1	752	752	752
	4								1					1	1538	1538	1538
	All	26	59	52	70	58	47	27	27	13	15	1	3	398	2	897	4201

(1972) to yield both analytically and numerically compatible growth and yield predictions.

Sullivan and Clutter's (1972) equation form was used in this study to simultaneously predict growth and yield of cottonwood plantations in the lower Mississippi Delta in terms of total volume and merchantable volume per acre:

$$\ln TV_1 = a_0 + a_1 S + a_2/A_1 + a_3 \ln B_1 \quad (3)$$

$$\ln MV_1 = b_0 + b_1 S + b_2/A_1 + b_3 \ln B_1 \quad (4)$$

$$\ln B_2 = A_1/A_2 \ln B_1 + c_1 (1 - A_1/A_2) + c_2 S (1 - A_1/A_2) \quad (5)$$

$$\ln TV_2 = \ln TV_1 + a_2 (1/A_2 - 1/A_1) + a_3 (\ln B_2 - \ln B_1) \quad (6)$$

$$\ln MV_2 = \ln MV_1 + b_2 (1/A_2 - 1/A_1) + b_3 (\ln B_2 - \ln B_1) \quad (7)$$

where

$\ln x$ = natural logarithm of x

TV_i = total outside-bark volume in ft³/ac at time i

MV_i = merchantable inside-bark volume in ft³/ac at time i

B_i = stand basal area in ft²/ac at time i

A_i = stand age in years at time i

S = site index in feet (base age 10 years)

a_i, b_i, and c_i = parameters to be estimated from the data

The parameter estimation method introduced by Borders (1989) was employed to obtain the coefficients of the above system (Table 2). The yield equations [(3) and (4)] explained 97% and 94% of the variations, respectively. Since each plot was remeasured several times, the growth equations [(5), (6), and (7)] were applied to all possible growth pairs in the data set to determine their ability to project stand attributes (Table 3). As expected, accuracy and precision of growth projections decreased as projection length increased.

Table 2. Parameter estimates of the cottonwood growth and yield growth and their approximate standard errors.

Parameter	Estimate	Approximate standard error
a_0	2.64098	0.05063
a_1	0.00868	0.00045
a_2	−3.27063	0.08713
a_3	1.09103	0.01149
b_0	2.12838	0.09997
b_1	0.01411	0.00084
b_2	−5.04889	0.18972
b_3	1.08576	0.02329
c_1	4.44967	0.20933
c_2	0.00722	0.00251

Table 3. Evaluation statistics for stand projection equations (5, 6, and 7).[a]

Projection length (yr)	No. of observations	Basal area projection difference[b] Mean	Std. dev.	Total volume projection difference[b] Mean	Std. dev.	Merchantable volume projection difference[b] Mean	Std. dev.
		(%)					
1	125	1.57	8.35	4.40	12.93	13.50	21.94
2	258	−3.39	18.39	−1.13	23.14	8.80	29.98
3	49	−4.46	24.72	1.26	28.49	15.99	36.91
4	108	−3.11	23.60	0.41	29.82	14.79	41.04
5	49	−23.71	41.64	−27.54	64.50	−6.03	72.90
6	31	−10.21	28.26	−9.72	47.37	1.44	61.47
7	31	−26.48	45.46	−26.44	63.04	4.76	59.49
8	4	−60.35	73.23	−82.11	104.17	−67.00	100.00
9	6	−11.90	36.98	−13.30	52.02	20.43	66.72
11	1	34.96	—	70.86	—	91.52	—
13	1	23.30	—	−32.74	—	−35.55	—

[a] Projection equations:
$\ln B_2 = A_1/A_2 \ln B_1 + 4.44967 (1 - A_1/A_2) + 0.00722\, S (1 - A_1/A_2)$
$\ln TV_2 = \ln TV_1 - 3.27063 (1/A_2 - 1/A_1) + 1.09103 (\ln B_2 - \ln B_1)$
$\ln MV_2 = \ln MV_1 - 5.04889 (1/A_2 - 1/A_1) + 1.08576 (\ln B_2 - \ln B_1)$
[b] Projection difference = actual − projected.

Table 4. Age at maximum mean annual increment (MAI) for merchantable volume, by site index and stand density at age 3.

Site index (ft)	Stand basal area (ft^2/ac) at age 3: 10	20	30	40	50
	(age at max. MAI)				
50	13.2	11.0	9.6	8.7	8.0
60	13.5	11.2	9.9	8.9	8.2
70	13.7	11.4	10.1	9.2	8.4
80	13.9	11.7	10.3	9.4	8.7
90	14.2	11.9	10.6	9.6	8.9
100	14.4	12.1	10.8	9.9	9.2
110	14.6	12.4	11.1	10.1	9.4

ROTATION AGE DETERMINATION

Mean annual increment (MAI) can be used as one criterion to determine rotation age. Selecting the rotation age at the age of maximum MAI optimizes the average annual volume production (Clutter et al. 1983). Table 4 shows that MAI of merchantable volume (ib to a 3-in. top) reaches a maximum when the stand is between 8 and 15 years old, depending on site index and initial stand density (basal area/ac at age 3). Culmination of MAI is reached at a slightly higher age when site index increases, but occurs at a lower age when initial stand density increases.

CONCLUSIONS

The system of equations presented in this paper produces compatible growth and yield predictions for improved eastern cottonwood grown on short rotations in the lower Mississippi Delta. These equations should provide reliable short-term growth projections in terms of basal area, total volume, and merchantable volume per acre. Mean annual increment of pulpwood volume culminated at a young stand age between 8 and 15 years, based on the growth equations. Results from this analysis explain the short rotations for pulpwood that have been predominantly applied to cottonwood plantations in the South. □

Literature Cited

ALEXANDER, D.S. 1976. Growth and yield of cottonwood plantations—prediction and realization. P. 405–413 *in* Proc. Symp. on Eastern Cottonwood and Related Species, Thielges, B.A., and S.B. Land, Jr. (eds.). La. State Univ., Baton Rouge.

BEERS, T.W., AND C.I. MILLER. 1964. Point sampling: Research results, theory, and applications. Purdue Univ. Agric. Exp. Stn. Res. Bull. 786. 56 p.

BORDERS, B.E. 1989. Systems of equations in forest stand modeling. For. Sci. 35:548–556.

BURKHARDT, E.C., AND R.M. KRINARD. 1976. Summary of the 1976 cottonwood plantation survey. P. 428–431 *in* Proc. Symp. on Eastern Cottonwood and Related Species, Thielges, B.A., and S.B. Land, Jr., (eds.). La. State Univ., Baton Rouge.

CAO, Q.V., AND K.M. DURAND. 1991. Site index curves for eastern cottonwood plantations in the lower Mississippi Delta. South. J. Appl. For. 15(1):28–30.

CLUTTER, J.L. 1963. Compatible growth and yield models of loblolly pine. For. Sci. 9:354–371.

CLUTTER, J.L., ET AL. 1983. Timber management: A quantitative approach. Wiley, New York. 333 p.

JOHNSON, R.L., AND E. KERR. 1976. The cottonwood success story. South. Lumberman 233(2896):78–79.

KRINARD, R.M. 1988. Volume equations for plantation cottonwood trees (*Populus deltoides*). USDA For. Serv. South. For. Exp. Stn. Res. Note SO-347. 4 p.

KRINARD, R.M., AND R.L. JOHNSON. 1984. Cottonwood plantation growth through 20 years. USDA For. Serv. South. For. Exp. Stn. Res. Paper SO-212. 11 p.

SULLIVAN, A.D., AND J.L. CLUTTER. 1972. A simultaneous growth and yield model for loblolly pine. For. Sci. 18:76–86.