ANNUAL TREE GROWTH PREDICTIONS BASED ON PERIODIC MEASUREMENTS

Quang V. Cao
School of Renewable Natural Resources
Louisiana State University Agricultural Center
Baton Rouge, LA 70803
qcao@lsu.edu

ABSTRACT

A method was developed to estimate parameters of a system of annual tree diameter growth, height growth, and survival equations, based on periodic measurements. Data from 100 plots randomly selected from the Southwide Seed Source Study of loblolly pine (*Pinus taeda* L.) were used to fit the equations. Another 100 plots, also randomly selected from the same study, were used to evaluate this method against the constant rate approach that assumes constant tree survival and growth rates during the growing period. Results showed that the variable rate method out-performed the constant rate method in predicting both tree-level and stand-level attributes, especially for long projection periods. The variable rate method was superior because it accounted for the ever-changing annual rates of tree survival and growth.

INTRODUCTION

Three main components of a distance-independent individual tree model include tree diameter and height growth equations and a tree survival function. Individual tree growth equations have been used in predicting annual or periodic increment in diameter (Belcher et al. 1982, Amateis et al. 1989, Zhang et al. 1997, Lessard et al. 2001, Mabvurira and Miina 2002) and height (Ritchie and Hann 1986, Courbaud et al. 1993, Lynch and Murphy 1995, Golser and Hasenauer 1997). The probability that a tree survives a period has been modeled using logistic functions (Hamilton 1974, Hamilton and Edwards 1976, Monserud 1976, Buchman 1979, 1983, Zhang et al. 1997, Monserud and Sterba 1999), although some exceptions exist (Amateis et al. 1989, Guan and Gertner 1991a, 1991b).

Since trees are not measured every year but often at some interval, it is not a straightforward procedure to predict annual tree growth and survival. Interpolation methods were developed by McDill and Amateis (1993) for modeling one variable (e.g. tree height) and later generalized by Cao et al. (in press) for many variables (e.g. tree diameter, height, and crown ratio). These interpolation methods were shown to perform better than the averaging method that assumes a constant growth for the entire period. Predicting annual tree survival from periodic measurements is particularly taxing. The survival probability is often assumed to remain constant during the growing period (Hamilton and Edwards 1976, Monserud 1976). Cao (2000) developed an iterative method to predict annual diameter growth and survival for individual trees. This method was shown to be superior to the averaging approach because it accounted for variable rates of survival and diameter growth.

The objective of this study is to extend Cao's (2000) method to estimate parameters of an individual tree model that consists of annual tree survival, diameter growth, and height growth equations, based on data measured at some interval.

DATA

Data were from the Southwide Seed Source Study, which involved 15 loblolly pine (*Pinus taeda* L.) seed sources planted at 13 locations across 10 southern states (Wells and Wakeley 1966). Seedlings were planted at a 1.8 m x 1.8 m spacing. Each plot of size 0.0164 ha consisted of 49 trees measured at ages t_1 (10 years), t_2 (15 or 16), t_3 (20 or 22), and t_4 (25 or 27).

A subset (100 plots) of the original data was randomly selected as the fit data set, to be used for fitting the models. Furthermore, only one growing period was randomly chosen from each plot. The fit data set therefore contained mostly 5-year growth periods (84 plots), with some intervals of 4 years (6 plots), 6 years (4 plots), and 7 years (6 plots).

Another 100 plots were randomly selected from the remaining original data to form the validation data set. All six possible growing periods (t_1–t_2, t_1–t_3, t_1–t_4, t_2–t_3, t_2–t_4, and t_3–t_4) from these plots were used to evaluate the models. Table 1 shows summary statistics for stand- and tree-level attributes for the fit and validation data sets.

Table 1. Means (and standard deviations) of stand- and tree-level attributes, by data type and age

Age	Number of sample plots	Dominant height (m)	Number of trees/ha	Basal area (m^2/ha)	Number of sample trees	Tree diameter (cm)	Tree total height (m)
Fit Data Set							
10	33	8.6 (1.7)	2084 (490)	20.95 (7.25)	1127	10.8 (3.4)	7.7 (2.0)
15	57	12.9 (1.6)	1869 (560)	31.46 (7.36)	1746	14.1 (3.9)	11.6 (2.1)
16	10	16.0 (0.8)	1165 (202)	29.50 (2.81)	191	17.4 (4.2)	14.6 (2.0)
20	59	17.1 (1.9)	1175 (383)	31.93 (8.28)	1135	18.1 (4.4)	15.6 (2.5)
22	8	14.6 (1.1)	808 (221)	16.64 (6.01)	106	15.7 (4.1)	13.3 (2.0)
25	21	20.1 (2.5)	1008 (450)	33.64 (12.17)	347	20.0 (4.8)	18.2 (3.1)
27	12	18.0 (2.6)	747 (233)	25.67 (7.89)	147	20.1 (5.6)	16.2 (3.5)
Validation Data Set							
10	100	9.3 (1.6)	2203 (428)	24.05 (6.48)	3611	11.3 (3.3)	8.3 (1.9)
15	77	12.8 (1.8)	1978 (543)	32.05 (7.38)	2496	13.8 (3.8)	11.6 (2.3)
16	23	15.9 (1.0)	1228 (158)	28.92 (3.37)	463	16.9 (3.7)	14.8 (1.9)
20	93	17.0 (2.3)	1256 (350)	32.23 (7.53)	1915	17.6 (4.2)	15.5 (2.8)
22	7	14.7 (2.5)	941 (168)	18.45 (6.54)	108	15.2 (4.3)	13.5 (2.7)
25	65	19.1 (2.7)	1177 (382)	36.11 (10.02)	1254	19.2 (4.5)	17.4 (3.2)
27	35	20.1 (2.8)	723 (183)	28.14 (8.05)	414	21.5 (5.6)	18.5 (3.6)

METHODS

The following individual tree model comprised of equations predicting annual survival and diameter and height growth was selected after preliminary analyses.

$$d_{i,\,t+1} - d_i = (\alpha_1 + \alpha_2 D_{q,\,t}) \,/\, \{1 + \exp\,(\alpha_3 A_t + \alpha_4 H_t + \alpha_5 N_t + \alpha_6 B_t + \alpha_7 d_{i,\,t} + \alpha_8 h_{i,\,t})\} + \varepsilon_{i,\,t} \qquad (1.a)$$

$$h_{i,\,t+1} - h_{i,\,t} = (\beta_1 + \beta_2 H_t) \,/\, \{1 + \exp\,(\beta_3 + \beta_4 A_t + \beta_5 H_t + \beta_6 B_t + \beta_7 d_{i,\,t} + \beta_8 h_{i,\,t})\} + \varepsilon_{i,\,t} \qquad (1.b)$$

$$p_{i,\,t+1} = [1 + \exp\,(\gamma_1 + \gamma_2 H_t + \gamma_3 B_t + \gamma_4 d_{i,\,t} + \gamma_5 h_{i,\,t})\,]^{-1} \qquad (1.c)$$

where

$d_{i,\,t}$ and $h_{i,\,t}$ = diameter in cm and total height in m, respectively, of tree i at age A_t,
$p_{i,\,t+1}$ = probability that tree i survived the period from age A_t to A_{t+1},
H_t = dominant height (average height of the dominants and codominant heights) in m at age A_t,
N_t = number of trees per ha at age A_t,
B_t = stand basal area in m^2/ha at age A_t,
$D_{q,\,t}$ = quadratic mean diameter in cm at age A_t, and
$\varepsilon_{i,\,t}$ = error term.

Two methods for estimating parameters of the above tree model will be discussed as follows.

Constant Rate Method

In this method, the growth rates of diameter and height of each tree were assumed to be constant during the growth period from age A_t to A_{t+q}, where q is the length of the period. Similarly, the survival probability was also considered constant during this period. Equations (1.a – 1.c) are rewritten as follows.

$$(d_{i,\,t+q} - d_i) / q = (\alpha_1 + \alpha_2 D_{q,\,t}) / \{1 + \exp(\alpha_3 A_t + \alpha_4 H_t + \alpha_5 N_t + \alpha_6 B_t + \alpha_7 d_{i,\,t} + \alpha_8 h_{i,\,t})\} + \varepsilon_{i,\,t} \qquad (2.a)$$

$$(h_{i,\,t+q} - h_{i,\,t}) / q = (\beta_1 + \beta_2 H_t) / \{1 + \exp(\beta_3 + \beta_4 A_t + \beta_5 H_t + \beta_6 B_t + \beta_7 d_{i,\,t} + \beta_8 h_{i,\,t})\} + \varepsilon_{i,\,t} \qquad (2.b)$$

$$p_i = [1 + \exp(\gamma_1 + \gamma_2 H_t + \gamma_3 B_t + \gamma_4 d_{i,\,t} + \gamma_5 h_{i,\,t})]^{-q} \qquad (2.c)$$

where p_i is the probability that tree i survived the period from age A_t to A_{t+q},

A method suggested by Borders (1989) was used to simultaneously estimate parameters of the diameter and height growth equations; this fitting procedure involved the use of option SUR (seemingly unrelated regression) of SAS procedure MODEL (SAS Institute Inc. 1993). Maximum likelihood estimation of parameters of the survival equation was obtained using weighted nonlinear regression (Walker and Duncan 1967).

Variable Rate Method

This method allowed the survival and growth rates to vary from year to year as functions of constantly changing stand variables and tree variables. Annual changes in diameter, height, and survival probability were modeled in a recursive manner as follows.

Year (t+1)

$$\hat{d}_{i,t+1} = d_i + (\alpha_1 + \alpha_2 D_{q,\,t}) / \{1 + \exp(\alpha_3 A_t + \alpha_4 H_t + \alpha_5 N_t + \alpha_6 B_t + \alpha_7 d_{i,\,t} + \alpha_8 h_{i,\,t})\} \qquad (3.a.1)$$

$$\hat{h}_{i,t+1} = h_{i,\,t} + (\beta_1 + \beta_2 H_t) / \{1 + \exp(\beta_3 + \beta_4 A_t + \beta_5 H_t + \beta_6 B_t + \beta_7 d_{i,\,t} + \beta_8 h_{i,\,t})\} \qquad (3.b.1)$$

$$p_{i,\,t+1} = [1 + \exp(\gamma_1 + \gamma_2 H_t + \gamma_3 B_t + \gamma_4 d_{i,\,t} + \gamma_5 h_{i,\,t})]^{-1} \qquad (3.c.1)$$

Year (t+2)

$$\hat{d}_{i,t+2} = \hat{d}_{i,t+1} + (\alpha_1 + \alpha_2 \hat{D}_{q,t+1}) / \{1 + \exp(\alpha_3 A_{t+1} + \alpha_4 \hat{H}_{t+1} + \alpha_5 \hat{N}_{t+1} + \alpha_6 \hat{B}_{t+1} + \alpha_7 \hat{d}_{i,t+1} + \alpha_8 \hat{h}_{i,t+1})\} \qquad (3.a.2)$$

$$\hat{h}_{i,t+2} = \hat{h}_{i,t+1} + (\beta_1 + \beta_2 \hat{H}_{t+1}) / \{1 + \exp(\beta_3 + \beta_4 A_{t+1} + \beta_5 \hat{H}_{t+1} + \beta_6 \hat{B}_{t+1} + \beta_7 \hat{d}_{i,t+1} + \beta_8 \hat{h}_{i,t+1})\} \qquad (3.b.2)$$

$$p_{i,\,t+2} = [1 + \exp(\gamma_1 + \gamma_2 \hat{H}_{t+1} + \gamma_3 \hat{B}_{t+1} + \gamma_4 \hat{d}_{i,t+1} + \gamma_5 \hat{h}_{i,t+1})]^{-1} \qquad (3.c.2)$$

.
.
.

Year (t+q)

$$d_{i,\,t+q} = \hat{d}_{i,t+q-1} + (\alpha_1 + \alpha_2 \hat{D}_{q,t+q-1}) / \{1 + \exp(\alpha_3 A_{t+q-1} + \alpha_4 \hat{H}_{t+q-1} + \alpha_5 \hat{N}_{t+q-1} + \alpha_6 \hat{B}_{t+q-1} + \alpha_7 \hat{d}_{i,t+q-1} + \alpha_8 \hat{h}_{i,t+q-1})\} + \varepsilon_i \qquad (3.a.q)$$

$$h_{i,\,t+q} = \hat{h}_{i,t+q-1} + (\beta_1 + \beta_2 \hat{H}_{t+q-1}) / \{1 + \exp(\beta_3 + \beta_4 A_{t+q-1} + \beta_5 \hat{H}_{t+q-1} + \beta_6 \hat{B}_{t+q-1} + \beta_7 \hat{d}_{i,t+q-1} + \beta_8 \hat{h}_{i,t+q-1})\} + \varepsilon_i \qquad (3.b.q)$$

$$p_{i,\,t+q} = [1 + \exp(\gamma_1 + \gamma_2 \hat{H}_{t+q-1} + \gamma_3 \hat{B}_{t+q-1} + \gamma_4 \hat{d}_{i,t+q-1} + \gamma_5 \hat{h}_{i,t+q-1})]^{-1} \qquad (3.c.q)$$

where the stand-level variables were predicted from the following equations:

$$\hat{H}_{t+s+1} = \exp\{\lambda_1 + [\ln(\hat{H}_{t+s}) - \lambda_1](A_{t+s}/A_{t+s+1})\} \qquad (4.a)$$

$$\ln(\hat{N}_{t+s+1}) = \ln(\hat{N}_{t+s})[\delta_1 + \delta_2 (A_{t+s}/A_{t+s+1})] + \ln(\hat{H}_{t+s})[\delta_3 + \delta_4 (A_{t+s}/A_{t+s+1})] \qquad (4.b)$$

$$\ln(\hat{B}_{t+s+1}) = (A_{t+s}/A_{t+s+1}) \ln(\hat{B}_{t+s}) + \{\tau_1 + \tau_2 [\ln(\hat{H}_{t+s+1}) - \ln(\hat{H}_{t+s})]\} / A_{t+s} \tag{4.c}$$

and the probability that tree i survived the period from age A_t to A_{t+q} is given by

$$p_i = \prod_{s=1}^{q} p_{i,t+s} \tag{5}$$

EVALUATION

The two methods for estimating parameters of the individual tree model were evaluated based on statistics computed from the validation data set. All possible growth intervals were employed in the validation process, allowing the methods to be evaluated at three projection lengths: short (from 4 to 7 years), medium (10 to 12 years), and long (15 to 17 years). The two methods were evaluated based how well they predicted both tree-level attributes (tree diameter, height, and survival probability) and stand-level attributes (number of trees, basal area, and volume per ha). For each projection length, the following evaluation statistics were calculated.

Mean Difference: $MD = \Sigma(y_i - \hat{y}_i)/n$

Mean Absolute Difference: $MAD = \Sigma|y_i - \hat{y}_i|/n$

Fit Index: $FI = 1 - \Sigma(y_i - \hat{y}_i)^2 / \Sigma(y_i - \bar{y}_i)^2$

Log Likelihood: $-2\ln(L) = -2\{\Sigma p_i \ln(p_i) + \Sigma(1-p_i)\ln(1-p_i)\}$

where

y_i = observed value at age A_{t+q} of tree attribute (diameter, height, or survival probability of tree i) or stand attribute (number of trees, basal area, or volume per ha of plot i),

$\hat{y}_i$ and $\bar{y}$ = predicted value and average, respectively, of y_i, and

n = number of observations.

RESULTS AND DISCUSSION

Table 2 shows the parameter estimates for the stand-level model (equations 4.a – 4.c). The parameter estimates for the individual tree model from the constant rate and variable rate methods are presented in Table 3.

Table 2. Estimates and standard errors of parameters in stand-level equations

Attribute	Parameter	Estimate	Std. Error
Dominant height	λ_1	3.9124	0.1885
	λ_2	0.6504	0.0968
Stand survival	δ_1	2.0841	0.1597
	δ_2	-1.1580	0.1722
	δ_3	-2.9659	0.4489
	δ_4	3.1407	0.4817
Basal area	τ_1	3.0693	0.1033
	τ_2	7.8213	1.0135

Evaluation statistics from the two methods computed from the validation data set are listed in Table 4. The variable rate method provided better predictions of tree-level attributes (diameter, height, and survival probability) than did the constant rate method, in terms of lower absolute values of mean difference (MD), lower mean absolute difference (MAD), higher fit index (FI), and lower value of -2 ln (L). As expected, the accuracy and precision of the predictions from both methods suffered as projection length increased. The constant rate method especially did not perform well for long projection periods (15 to 17 years). It tended to underestimate tree diameter and height, as evidenced by close values of MD and MAD for this projection length (2.21 and 2.82, respectively, for tree diameter, and 1.40 and 2.17 for tree height). The mean difference values from the variable rate method increased as projection length increased from approximately 5 years to 15 years

Table 3. Estimates and standard errors of parameters in tree-level equations

Attribute	Parameter	Contant Rate Method		Variable Rate Method	
		Estimate	Std. Error	Estimate	Std. Error
Diameter growth	α_1	2.8121	0.2702	3.3215	0.3259
	α_2	-0.0894	0.0114	-0.1014	0.0122
	α_3	0.0577	0.0122	0.0688	0.0100
	α_4	0.2436	0.0279	0.1787	0.0252
	α_5	0.0003	0.0001	0.0002	0.0001
	α_6	0.0278	0.0061	0.0262	0.0054
	α_7	-0.1307	0.0160	-0.1039	0.0140
	α_8	-0.1967	0.0273	-0.1600	0.0261
Height growth	β_1	1.1655	0.0703	1.2073	0.0881
	β_2	-0.0250	0.0053	-0.0254	0.0063
	β_3	-1.7399	0.3256	-2.2665	0.4341
	β_4	0.1683	0.0238	0.1771	0.0256
	β_5	-0.1033	0.0463	-0.1346	0.0533
	β_6	0.0261	0.0057	0.0202	0.0065
	β_7	-0.2695	0.0343	-0.2822	0.0398
	β_8	0.2538	0.0494	0.3231	0.0649
Tree survival	γ_1	-4.1367	0.2015	-4.7460	0.2483
	γ_2	0.4686	0.0373	0.4940	0.0388
	γ_3	0.0679	0.0067	0.0637	0.0073
	γ_4	-0.1148	0.0225	-0.0714	0.0250
	γ_5	-0.4704	0.0506	-0.5028	0.0578

(-0.01 to 0.67 cm for diameter, -0.03 to 0.23 m for height, and -0.003 to 0.027 for survival probability). It is remarkable that bias resulting from the variable rate method remained reasonably low even for projection periods ranging from 15 to 17 years.

The two methods were also evaluated for their capability to predict stand-level attributes such as number of trees, basal area, and volume per ha (Table 4). Again, the variable rate method consistently produced lower absolute value of MD, lower MAD, and higher FI than did the constant rate approach. Furthermore, results revealed that the ability of this individual tree model to predict stand attributes deteriorated as projection interval increased, even though the variable rate approach always maintained its superiority over the constant rate method. For projection periods ranging from 15 to 17 years, the fit indices (analogous to R^2) from both methods were negative, indicating that using the means to project stand growth fared better than using the predictions from the equations.

The variable rate method as presented in this paper is similar to the iterative method introduced by Cao (2000), except that the stand variables in the variable rate method were predicted from a whole stand model, rather than computed each year by summing up trees in the plots. Consequently, the diameter and height growth equations could be fitted first because they need no information from the survival equation. Parameters from these tree growth equations were then required for fitting the survival equation. This small modification thus eliminated the need for an iterative procedure.

Because stand-level variables and tree-level variables change every year, a method such as the variable rate method that allows diameter and height growth and tree survival probability to vary annually should perform well. This new method is superior to the constant rate approach and can be easily programmed using a statistical package such as SAS. Even though a loblolly pine data set was used in this study, the variable rate approach should be applicable to other species as well.

Table 4. Statistics obtained from evaluation of two methods for estimating parameters of the tree-level model

Attribute	Evaluation Statistic [1/]	Short Projection Length (4 to 7 years)		Medium Projection Length (10 to 12 years)		Long Projection Length (15 to 17 years)	
		Constant Rate	Variable Rate	Constant Rate	Variable Rate	Constant Rate	Variable Rate
Tree diameter	MD	0.37	***-0.01***	0.98	***0.11***	2.21	***0.67***
	MAD	0.97	***0.92***	1.70	***1.50***	2.82	***2.22***
	FI	0.9024	***0.9087***	0.7667	***0.8119***	0.4725	***0.6518***
Tree height	MD	0.35	***-0.03***	0.79	***0.06***	1.40	***0.23***
	MAD	0.99	***0.93***	1.57	***1.38***	2.17	***1.75***
	FI	0.8678	***0.8753***	0.6139	***0.6725***	0.3236	***0.4964***
Tree survival	MD	0.048	***-0.003***	0.071	***-0.028***	0.156	***0.027***
	MAD	0.284	***0.270***	0.368	0.368	0.367	0.367
	-2 ln(L)	7649	***7491***	7874	***7590***	5231	***4248***
Stand density (number of trees/ha)	MD	87	***-1***	141	***-55***	343	***60***
	MAD	235	***228***	287	***267***	398	***291***
	FI	0.7017	***0.7252***	0.1043	***0.2264***	-0.7771	***0.1145***
Stand basal area (m^2/ha)	MD	2.67	***0.27***	5.58	***-0.40***	12.6	***2.63***
	MAD	5.41	***4.96***	8.43	***7.02***	14.08	***9.34***
	FI	0.3374	***0.4301***	0.3117	***0.1202***	-1.9115	***-0.3814***
Stand volume (m^3/ha)	MD	16.47	***1.50***	38.31	***-0.58***	86.91	***18.86***
	MAD	32.21	***29.70***	54.81	***46.25***	97.71	***64.60***
	FI	0.6020	***0.6556***	0.0183	***0.3420***	-1.3672	***-0.1879***

[1/] For each evaluation statistic and each projection length, a bold italic number denotes the better of the two methods. MD = mean difference between observed and predicted values; MAD = mean absolute difference; FI - fit index (analogous to R^2); and L = likelihood function.

REFERENCES

Amateis, R.L., H.E. Burkhart, and T.A. Walsh. 1989. Diameter increment and survival equations for loblolly pine trees growing in thinned and unthinned plantations on cutover, site-prepared lands. South. J. Appl. For. 13:170-174.

Belcher, D.M., M.R. Holdaway, and G.J. Brand. 1982. A description of STEMS – the Stand and Tree Evaluation and Modeling System. USDA For. Serv. Gen. Tech. Rep. NC-79, 18 p.

Borders, B. E. 1989. Systems of equations in forest stand modeling. For. Sci. 35:548-556.

Buchman, R.G. 1979. Mortality functions. Pages 47-55 in A generalized forest growth projection system applied to the Lake States region. USDA For. Serv. Gen. Tech. Rep. NC-49.

Buchman, R.G. 1983. Survival predictions for major Lake States tree species. USDA For. Serv. Res. Pap. NC-233, 7 p.

Burkhart, H.E., K.D. Farrar, R.L. Amateis, and R.F. Daniels. 1987. Simulation of individual tree growth and stand development in loblolly pine plantations on cutover, site-prepared areas. Sch. For. Wildl. Res., Va. Polytech. Inst. & State Univ., Pub. No. FWS-l-87, 47 p.

Cao, Q. V. 2000. Prediction of annual diameter growth and survival for individual trees from periodic measurements. For. Sci. 46:127-131.

Cao Q. V., S. Li, and M. E. McDill. In press. Developing a system of annual tree growth equations for the loblolly pine-shortleaf pine type in Louisiana. Can. J. For. Res. xx:xx-xx.

Courbaud B., F. Houllier, and C. Rupe. 1993. A model for height growth of trees in uneven-aged spruce mountain forest. Annal. Sci. For. 50:337-351.

Golser M., and H. Hasenauer. 1997. Predicting juvenile tree height growth in uneven-aged mixed species stands in Austria. For. Ecol. Manage. 97:133-146.

Guan, B.T., and G. Gertner. 1991a. Using a parallel distributed processing system to model individual tree mortality. Forest Sci. 37:871-885.

Guan, B.T., and G. Gertner. 1991b. Modeling red pine tree survival with an artificial neural network. Forest Sci. 37:1429-1440.

Hamilton, Jr., D.A. 1974. Event probabilities estimated by regression. USDA For. Serv. Res. Pap. INT-152, 18 p.

Hamilton, Jr., D. A., and B.M. Edwards. 1976. Modeling the probability of individual tree mortality. USDA For. Serv. Res. Pap. INT-185, 22 p.

Lessard V. C., R. E. McRoberts, and M. R. Holdaway. 2001. Diameter growth models using Minnesota forest inventory and analysis data. For. Sci. 47:301-310.

Lynch T.B., and P. A. Murphy. 1995. A compatible height prediction and projection system for individual trees in natural, even-aged shortleaf pine stands. For. Sci. 41:194-209.

Mabvurira D., and J. Miina. 2002. Individual-tree growth and mortality models for *Eucalyptus grandis* (Hill) Maiden plantations in Zimbabwe. For. Ecol. Manage. 161:231-245.

McDill, M.E., and R.L. Amateis. 1993. Fitting discrete-time dynamic models having any time interval. Forest Sci. 39:499-519.

Monserud, R.A. 1976. Simulation of forest tree mortality. Forest Sci. 22:438-444.

Monserud R. A., and H. Sterba. 1999. Modeling individual tree mortality for Austrian forest species. For. Ecol. Manage. 113:109-123.

Ritchie M. W., and D. W. Hann. 1986. Development of a tree height growth-model for Douglas-fir. For. Ecol. Manage. 15:135-145.

SAS Institute, Inc. 1993. SAS/ETS User's Guide, Version 6, Second Ed. SAS Institute Inc., Cary, NC. 1,022 p.

Walker, S.H., and D.B. Duncan. 1967. Estimation of the probability of an event as a function of several independent variables. Biometrika 54:167-179.

Wells, O.O., and P.C. Wakeley. 1966. Geographic variation in survival, growth, and fusiform-rust infection of planted loblolly pine. Forest Sci. Monogr. 11. 40 p.

Zhang, S., R.L. Amateis, and H.E. Burkhart. 1997. Constraining individual tree diameter increment and survival models for loblolly pine plantations. Forest Sci. 43:414-423.