26

577

HERNANDEZ-IGLESIAS, C., and F. HERNANDEZ-IGLESIAS. 1981. Causality and the independence phenomenon, the case of the demand for money. J Econometrics 15:247–263.

LJUNG, G. M., and G. E. P. BOX. 1978. On a measure of lack of fit in time series models. Biometrika 65(2):297–303.

PIERCE, D. A. 1977. Relationships—and the lack thereof—between economic time series, with special reference to money and interest rates. J Am Stat Assoc 72(351):11–26.

SIMS, C. A. 1972. Money, income and causality. The Am Econ Rev 62:540–552.

WIENER, A. 1956. The theory of prediction. *In* Modern mathematics for engineers, Series 1 (E. F. Beckenback, ed), p 165–190. McGraw-Hill, New York. 514 p.

Forest Sci., Vol. 31, No. 2, 1985, pp. 414–418

Short-Term Projection Accuracy of Five Asymptotic Height-Age Curves for Loblolly Pine

John A. Brewer, Paul Y. Burns, and Quang V. Cao

ABSTRACT. Height-age data from the Southwide Pine Seed Source Study of loblolly pine plantations were used to evaluate short-term projection accuracy of five nonlinear asymptotic height-age curves: Cilliers-Van Wyk, Chapman-Richards, first-degree logistic, Gompertz, and Lundqvist. Upper asymptotes were selected for each plot by two alternative methods: fixed at a reasonable value, and estimated from the data through age 20. Accuracy was assessed by comparing observed top heights at age 25 with predicted top heights at this age. For short-term projections of height-age curves, the Cilliers-Van Wyk, Chapman-Richards, and Lundqvist models show promise, especially when the upper asymptote is fixed. FOREST SCI. 31:414–418.

ADDITIONAL KEY WORDS. *Pinus taeda,* growth models, height growth.

GROWTH MODELING is a major area of continuing forest research. The ability to predict tree or stand height is essential to forest management. Research effort has been devoted to the construction of tree- or stand-height growth models, many of which are based on nonlinear, asymptotic height-age curves. Since consecutive height measurements indicate a monotonic, nondecreasing trend, a predictive relationship can be established by hypothesizing that tree height is a function of time. To identify and best fit the data pattern of a particular species growing in specific conditions, various height-age curves have been proposed and evaluated. While most of these curves are suitable for trend analysis of the entire life span of trees, the least-squares fit of these models with early tree-height data may result in a variety of curve shapes with an upper asymptote which may be too low or too high; thus the models may produce unreliable predictions at ages beyond the data range. One method to circumvent this problem is to fix the upper asymptote at a constant value while estimating all other parameters in the model.

The predictive capability of a height-age curve has limitations. Pienaar and Turnbull (1973) believed that a rational model deduced from allometric theory is useful for extrapolation, whereas Pienaar and Shiver (1980) cautioned against a projection for more than 3 years beyond the data range. Kilpatrick and Savill (1981) suggested that height-age curves apply only within the age range of data. In general, the weight placed on extrapolations should decrease as the distance beyond the observed data increases.

The authors are respectively, Operations Research Analyst, U.S. Minerals Management Service, P.O. Box 7944, Metairie, LA 70010; Professor and Assistant Professor, School of Forestry, Wildlife, and Fisheries, Louisiana Agricultural Experiment Station, Louisiana State University Agricultural Center, Baton Rouge, LA 70803. The authors thank the Committee on Southern Forest Tree Improvement and the USDA Forest Service for the data used in this study. Manuscript received 12 March 1982.

In order to use height-age curves effectively, knowledge of their extrapolative accuracy is required. The objective of this study was to evaluate the projection capability of five height-age curves based on both fixed and estimated asymptotes.

Models Chosen for Evaluation

Many nonlinear, asymptotic height-age curves have been used in forestry for projecting tree- or stand-height development from current height and/or for classifying stand productivity. Based on a review of the literature, the following equations were evaluated in this study:

Cilliers-Van Wyk:	$H = A\,[1 - \exp(-b(t-c))]$	(1)
Chapman-Richards[1]:	$H = A\,[1 - \exp(-bt)]^c$	(2)
First-degree logistic:	$H = A\,[1 + b\exp(ct)]^{-1}$	(3)
Gompertz:	$H = Ab^{c^t}$	(4)
Lundqvist:	$H = A\exp(-bt^{-c})$	(5)

where

H = top height in meters,
t = stand age in years,
A = upper asymptote ($H \to A$ when $t \to \infty$), and
b and c = parameters to be estimated from the data.

The Cilliers-Van Wyk model was modified by Grut (1977) from an earlier form by Cilliers and Van Wyk (1938). The resulting model is similar to a curve widely known in mathematical biology as the monomolecular growth curve, which is based on the law of diminishing returns. Since the slope of the curve decreases from the beginning, this growth curve lacks an inflection point. The model was tested with favorable results on height data of pinaster pine (*Pinus pinaster* Ait.) in South Africa by Grut (1977).

The Chapman-Richards model was suggested by Richards (1959) as a generalization of the allometric model derived by von Bertalanffy (1957) from studies of the growth of aquatic and terrestrial organisms. Recent applications of this model in constructing site index curves for various species include research done by Bailey and others (1973), Graney and Burkhart (1973), Trousdell and others (1974), Grut (1977), Burkhart and Tennent (1977 a, b), and Pienaar and Shiver (1980), to name a few. The Chapman-Richards model was also employed by Pienaar and Turnbull (1973) for modeling basal area growth and yield of slash pine (*Pinus elliottii* Engelm.) and spruce (*Picea abies* (L.) Karst.), Kilpatrick and Savill (1981) for top height growth of Sitka spruce (*Picea sitchensis* (Bong.) Carr.), Green (1981) for basal area growth of individual loblolly pine (*Pinus taeda* L.) trees, and Murphy (1983) for basal area projection of loblolly pine stands.

The first-degree logistic form, developed by Pearl and Reed (1923) for population growth, was investigated as a height-growth model by Liu (1976) and Grut (1977).

Winsor (1932) presented the Gompertz model, originally used in human mortality studies, as a biological growth model. It was tested by Liu (1976), Grut (1977), and Kilpatrick and Savill (1981) on height-growth data.

The Lundqvist model has been used to define site index curves for grand fir (*Abies grandis* (Dougl.) Lindl.) in the Inland Empire (Stage 1963), and was evaluated by Grut (1977) on pinaster pine height data.

Karish and Borden (1976) found that strong correlations existed among parameter estimates in five growth models, causing biological interpretations of these parameters to be invalid. They suggested reducing the number of parameters to two, either by fixing some of them or by expressing some of them as functions of the other parameters.

---

[1] Normally, the Chapman-Richards curve is presented with an additional parameter to define the origin. This parameter is unnecessary in the present context.

TABLE 1. Fixed and estimated upper asymptotes for each plot.

Plot	Top height at age 20	Top height at age 25	Fixed upper asymptote	Estimated upper asymptote[a]				
				Eq. (1)	Eq. (2)	Eq. (3)	Eq. (4)	Eq. (5)
	meters							
1	18.90	22.71	34	26.24	21.20[b]	19.34	20.31	33.37
2	16.46	18.75	30	23.47	18.38	16.75	17.62	28.25
3	14.63	16.76	27	20.51	16.40	14.82	15.58	25.37
4	14.78	18.23	27	19.78	16.41	15.01	15.71	24.92
5	17.37	18.29	32	23.29	19.44	17.63	18.47	30.66
6	19.81	22.20	36	27.29	22.02	20.18	21.15	33.29
7	17.98	21.79	33	32.36	20.45	18.47	19.66	31.88
8	19.81	21.64	36	28.63	22.09	20.15	21.21	33.47

[a] Estimated upper asymptotes were derived from data through age 20.

[b] Underlined estimated upper asymptotes are lower than corresponding observed top height at age 25.

Data

Height-age data through age 25 from the Southwide Pine Seed Source Study, available through the cooperation of the USDA Forest Service, were used in our study. Wells and Wakeley (1966) have provided general information about this regional research project, which included 15 loblolly pine seed sources planted at 13 locations across the geographic range of this species. For our study, we selected one plot at random from each of the following locations: (1) eastern North Carolina; (2) western South Carolina; (3) southwestern Georgia; (4) west-central Georgia; (5) central Alabama, Coosa County; (6) central Alabama, Talledega County; (7) north-central Mississippi; and (8) southwestern Arkansas. Each plot replication consisted originally of 11 rows of 11 trees each at a spacing of 1.8 m by 1.8 m. The interior 49 trees (plot size 0.0165 ha) were measured at ages 1, 3, 5, 10, 15, 20, and 25 years.

Top height, defined as the average height of the 100 largest-diameter trees per ha (two largest-diameter trees on the 0.0165 ha plot), was computed for each of the eight test plots at each of the seven measurement ages.

Parameter Estimation

In the current study, fixed as well as estimated upper asymptotes were investigated. In the fixed-asymptote analysis, the upper asymptote was assigned a constant value individually for each plot, and the remainder of the model parameters were estimated from the data. In the estimated-asymptote approach, all model parameters, including the upper asymptote, were estimated from the data.

In an attempt to determine a reasonable fixed upper asymptote, a sensitivity analysis was conducted in which the upper asymptote was allowed to vary in 2-m intervals between 16 and 50 m, and data through age 20 were fitted to the models. Results indicated that the "optimum" upper asymptote—that which produced accurate extrapolation at age 25—varied depending on the plot and also on the model used, ranging from 18 to more than 50 m.

Since the sensitivity analysis failed conclusively to produce a reasonable asymptote, we decided to use predicted top heights at age 80 as fixed upper asymptotes. Top heights at age 80 were predicted from observed top height at age 20 by use of site index equations presented by Clutter and Lenhart (1968) and Smalley and Bower (1971); these two numbers were then averaged.

Regression coefficients of the five models (with both fixed and estimated asymptotes)

TABLE 2. Error percentages for predicting top height at age 25 from equations based on data through age 20.

Plot	Eq. (1)	Eq. (2)	Eq. (3)	Eq. (4)	Eq. (5)
Upper asymptotes estimated from the data			*Error percent*[a]		
1	11.7	13.7	15.7	14.3	12.4
2	7.1	9.3	11.5	10.0	8.1
3	8.2	10.3	12.6	11.1	9.2
4	14.5	16.3	18.4	17.0	15.2
5	0.1	2.1	4.6	3.0	0.8
6	6.4	8.6	10.7	9.2	7.3
7	11.9	14.5	16.4	15.0	13.4
8	3.0	5.5	7.8	6.2	4.2
Mean error	7.9	10.0	12.2	10.7	8.8
Mean absolute error	7.9	10.0	12.2	10.7	8.8
Upper asymptotes fixed					
1	4.0	0.3	−14.7	−7.5	9.3
2	−0.5	−5.5	−21.7	−13.6	4.4
3	0.4	−3.6	−20.1	−12.1	6.1
4	5.9	2.9	−12.1	−5.1	11.8
5	−9.7	−12.9	−30.6	−22.4	−2.5
6	−2.2	−6.6	−22.8	−14.9	3.4
7	8.7	0.1	−15.3	−6.5	9.7
8	−4.5	−10.1	−26.8	−18.4	0.3
Mean error	0.3	−4.4	−20.5	−12.6	5.3
Mean absolute error	4.5	5.2	20.5	12.6	5.9

[a] Error percent = 100 $(H - \hat{H})/H$, where H = observed top height at age 25, and $\hat{H}$ = predicted top height at age 25.

were estimated through age 20 for each plot by use of the Gauss-Newton method for nonlinear regression (SAS Institute 1982).

Fixed and estimated upper asymptotes for each plot are presented in Table 1. Estimated upper asymptotes for equations (2), (3), and (4) for the most part were lower than the observed age-25 top heights.

Evaluation of Extrapolation Capability

Table 2 shows error percentages for predicting top height at age 25. The upper portion of the table covers the case of upper asymptotes estimated from the data; the lower portion shows values for models with fixed upper asymptotes.

Where asymptotes were estimated from the data, top heights at age 25 were underestimated by all five equations for all eight plots. The underestimation was especially pronounced for equations (2), (3), and (4).

Where asymptotes were fixed, equations (3) and (4) produced age-25 top heights which were consistently too high, but equations (1), (2), and (5) had relatively low extrapolation errors—below 6 percent for both mean error and mean absolute error.

This study indicates the need to choose a height-age model for a given data set with care. When upper asymptotes were fixed, the Cilliers-Van Wyk, Chapman-Richards, and Lundqvist equations produced reasonably accurate short-term projections (5 years beyond the data range). When asymptotes were estimated from the data, extrapolations made from these equations were less accurate, confirming the need expressed by Knight (1968) for caution in estimating upper asymptotes from the data.

The first-degree logistic and Gompertz models provided relatively high error percentages whether the upper asymptotes were fixed or estimated.

For short-term projections of height-age curves, the Cilliers-Van Wyk, Chapman-Richards, and Lundqvist models show promise, especially when the upper asymptote is fixed at a reasonable value.

Literature Cited

Bailey, R. L., W. F. Mann, and T. E. Campbell. 1973. Slash pine site index in the West Gulf. USDA Forest Serv Res Note SO-169, 4 p.

Burkhart, H. E., and R. B. Tennent. 1977a. Site index equations for radiata pine in New Zealand. N Z J Forest Sci 7:408–416.

Burkhart, H. E., and R. B. Tennent. 1977b. Site index equations for Douglas fir in Kaingaroa Forest. N Z J Forest Sci 7:417–419.

Cilliers, A. C., and J. H. Van Wyk. 1938. Mathematical expression for the growth of trees in their dependence on time and density of stocking. Ann Univ Stellenbosch 16, 36 p.

Clutter, J. L., and J. D. Lenhart. 1968. Site index curves for old-field loblolly pine plantations in the Georgia Piedmont. Ga For Res Council Rep 22—Series 1, 4 p.

Graney, D. L., and H. E. Burkhart. 1973. Polymorphic site index curves for shortleaf pine in the Ouachita Mountains. USDA Forest Serv Res Pap SO-85, 12 p.

Green, E. J. 1981. Models of stand basal area distributions, individual tree basal area growth, and height-diameter relationships for loblolly pine. Ph D thesis, Va Polytech Inst and State Univ, Blacksburg, Va. 127 p. (Diss Abstr Int 42:3892-B.)

Grut, M. 1977. Equations for calculating height increment and site index of pinaster pine. S Afr For J 102:43–50.

Karish, J. F., and F. Y. Borden. 1976. Parameter correlation effects in non-linear mathematical models for biological growth. Research Briefs, Sch For Resources, Coll of Agric, Agr Exp Stn, Penn State Univ, p 11–15.

Kilpatrick, D. J., and P. S. Savill. 1981. Top height growth curves for Sitka spruce in northern Ireland. Forestry 54(1):31–39.

Knight, W. 1968. Asymptotic growth: an example of nonsense disguised as mathematics. J Fish Res Bd Can 25:1303–1307.

Liu, C. J. 1976. An examination of some subsystems for a model of tree volume growth. Ph D thesis, La State Univ, Baton Rouge. 221 p. (Diss Abstr 37:2593-B.)

Murphy, P. A. 1983. A nonlinear timber yield equation system for loblolly pine. Forest Sci 29:582–591.

Pearl, R., and L. J. Reed. 1923. On the mathematical theory of population growth. Metron 3: 6–19.

Pienaar, L. V., and B. D. Shiver. 1980. Dominant height growth and site index curves for loblolly pine plantations in the Carolina flatwoods. South J Appl For 4:54–59.

Pienaar, L. V., and K. J. Turnbull. 1973. The Chapman-Richards generalization of von Bertalanffy's growth model for basal area growth and yield in even-aged stands. Forest Sci 19:2–22.

Richards, F. J. 1959. A flexible growth function for empirical use. J Exp Bot 10:290–300.

SAS Institute. 1982. SAS user's guide: statistics. SAS Institute, Cary, N C. 584 p.

Smalley, G. W., and D. R. Bower. 1971. Site index curves for loblolly and shortleaf pine plantations on abandoned fields in Tennessee, Alabama, and Georgia highlands. USDA Forest Serv Res Note SO-126, 6 p.

Stage, A. R. 1963. A mathematical approach to polymorphic site index curves for grand fir. Forest Sci 9:167–180.

Trousdell, K. B., D. E. Beck, and F. T. Lloyd. 1974. Site index for loblolly pine in the Atlantic Coastal Plain of the Carolinas and Virginia. USDA Forest Serv Res Pap SE-115, 11 p.

von Bertalanffy, L. 1957. Quantitative laws in metabolism and growth. Quart Rev Biol 32:217–231.

Wells, O. O., and P. C. Wakeley. 1966. Geographic variation in survival, growth, and fusiform rust infection of planted loblolly pine. Forest Sci Monogr 11, 40 p.

Winsor, C. P. 1932. The Gompertz curve as a growth curve. Proc Natl Acad Sci 18:1–8.