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Self-Thinning Trajectories of Chinese Fir Plantations in Southern China

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Chinese fir (*Cunninghamia lanceolata* [Lamb.] Hook.) is the most important conifer species for timber production with a wide geographic distribution in southern China. Self-thinning is a dynamic equilibrium between forest growth and mortality in fully stocked stands. In this study, the segmented model was used to analyze the trajectory of stand density and quadratic mean diameter of Chinese fir plantations. Results showed that a linear-quadratic-linear model could simulate the self-thinning trajectories of Chinese fir plantations well. For a given province, the self-thinning slope was invariant with changes in planting densities. The self-thinning trajectories in Jiangxi and Fujian provinces were similar and were different from those in Guangxi and Sichuan provinces. The slope of the self-thinning line of Chinese fir plantations in Sichuan province was much steeper than that of the other three provinces (−4.2169 versus −2.4453). These slopes were steeper than the value of −1.605 as proposed by Reineke for many species.

Keywords: Chinese fir plantations, joint point, self-thinning line, segmented model

The self-thinning rule describes the mortality related to competition among trees within even-aged stands (Hynynen 1993). The rule has been a topic of research and discussion for more than 80 years (Reineke 1933, Yoda et al. 1963, Comeau et al. 2010, Burkhart 2013). In forestry, the self-thinning concept has been applied to develop relative density indices (Curtis 1970, Drew and Flewelling 1979), construct stand density management diagrams (Stankova and Shibuya 2007, Newton 2012), and predict stand growth (Cao 1994, Pretzsch and Biber 2005).

The segmented regression technique (Gallant and Fuller 1973) has been applied in forestry to model tree taper (Max and Burkhart 1976, Cao et al. 1980), height-age relationship (Borders et al. 1984), and self-thinning trajectories (Cao and Dean 2008, VanderSchaaf and Burkhart 2008). The relationship between the log of quadratic mean diameter, $\ln(Q)$, and the log of stand density, $\ln(N)$, in self-thinned, pure and even-aged forests, or plantations was never considered linear throughout the entire range of tree density because it includes competition (Westoby 1984). Cao and Dean (2008) reported that a linear-quadratic-linear model can be used to describe the classic $\ln(N)$-$\ln(Q)$ relationship for even-aged trees of monocultures. The first linear segment is a horizontal line that denotes a constant density due to little or no mortality when the stand is young. As competition sets in, trees begin to die while the remaining trees keep increasing their diameters. The resulting trajectory is a concave curve, which can be modeled by a quadratic function. Next, the stand reached the maximum size-density stage, where the relationship between $\ln(N)$ and $\ln(Q)$ is linear (del Rio et al. 2001), as observed by Reineke (1933).

Chinese fir (*Cunninghamia lanceolata* [Lamb.] Hook.), a fast growing evergreen coniferous tree, is one of the most important tree species for timber production in southern China. As a native species, Chinese fir has been widely planted for more than 1,000 years (Zhang et al. 2013). Understanding the self-thinning behavior of Chinese fir is essential to better grasp its intraspecific mortality patterns, which can lead to its more efficient management. Although studies have been conducted to describe the self-thinning boundary line of Chinese fir stands (Wu and Hong 2000, Sun et al. 2011), most of the stands were in one region.

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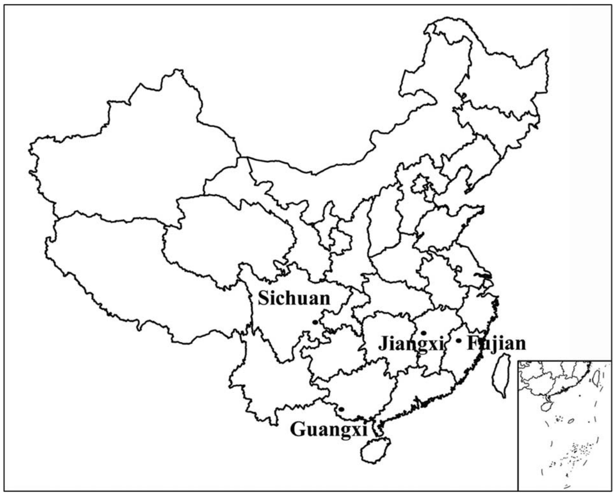

Figure 1. Locations of the Chinese fir sites in South China.

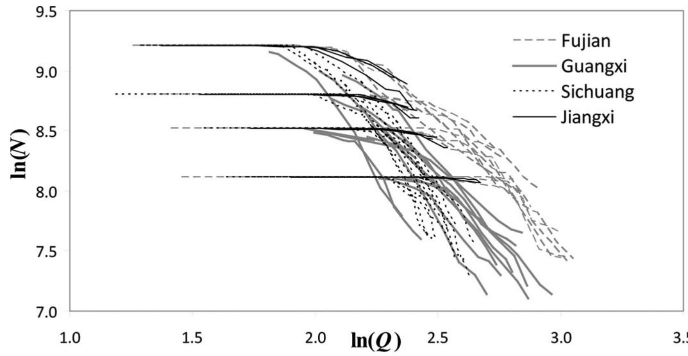

Figure 2. Relationship between log of stand density and quadratic mean dbh (4 sites × 3 repetitions × 4 densities = 48 lines).

The objective of this study was to apply segmented regression techniques to model the reciprocal relationship between stand density and quadratic mean diameter of Chinese fir plantations located in southern China. Specifically, the objectives were as follows: (1) to determine whether the slope of the self-thinning line varies with different planting densities for a given province, (2) to determine whether the self-thinning slopes vary with provinces, (3) to determine whether the self-thinning slopes could be combined for some provinces, and (4) to determine whether the self-thinning slope of Chinese fir is close to the value of −1.605 as reported by Reineke (1933).

Materials and Methods

Data

Data for this study were from Chinese fir (*Cunninghamia lanceolata* [Lamb.[Hook.) stands established in 1981 in four provinces (sites) in southern China: Fujian, Jiangxi, Guangxi, and Sichuan (Figure 1). The study sites located in Shaowu city, Fujian province, and Fenyi County, Jiangxi province, both have a middle-subtropical climate. In Guangxi province, the site located in Pingxiang city has a southern-subtropical climate. However, in Sichuan province, the site located in Naxi County has a northern-subtropical climate. Table 1 describes the weather conditions and soil type at each site. In each site, the soil type is laterite, and the plots were planted in a randomized block design with repeated measurements with the following tree spacings: 2 m × 1.5 m (3,333 trees/ha), 2 m × 1 m (5,000 trees/ha), 1 m × 1.5 m (6,667 trees/ha), and 1 m × 1 m (10,000 trees/ha). Each spacing level was replicated three times. Each plot comprised an area of 20 m × 30 m and a buffer zone consisting of two lines of similarly treated trees surrounding each plot. Diameters of all trees in the plots were measured after the tree heights reached 1.3 m. More than 50 trees in each plot were tagged and measured for total height. Each plot was remeasured every 1–3 years.

Measurements were performed in each winter at first and every 2 or 3 years afterward (Table 1). In the winter of 1998, there was a snow storm in Jiangxi Province, resulting in numerous dead trees. We therefore excluded the data after 1999 from Jiangxi Province. The trajectories of stand density in terms of number of trees per hectare (N) and quadratic mean diameter (Q) for these measurements are shown in Figure 2, and the summary statistics for Q and N by province are described in Table 2.

Methods

In the early stage, stand density does not affect mortality because intraspecific competition has not yet started. The trajectory is close to a horizontal line. As the trees continue to increase in size, they begin to compete for limited resources such as light, water, and nutrients. Density-related mortality in this second stage results in a concave trajectory (del Rio et al. 2001), which can be modeled by a quadratic function. In the third stage, the stand trajectory is a self-thinning line (Reineke 1933), describing a maximum size-density relationship that involves maximum allowable stand density for a given quadratic mean diameter.

A linear-quadratic-quadratic segmented model, used by Cao and Dean (2008) to describe the self-thinning trajectories, has the following form:

$$y = y_0 + b(x - a_1)^2 I_1 + d(x - a_2)^2 I_2 + \varepsilon \tag{1}$$

where $y = \ln(N)$; $x = \ln(Q)$; $y_0 = \ln(N_0)$; N_0 = planting density; $I_j = 1$ if $x > a_j$, and 0 otherwise, $j = 1, 2$; and b, d, a_1, and a_2 are regression parameters.

The linear-quadratic-linear model used in this study requires the last segment to be linear. Therefore, parameter d should satisfy the constraint, $d = -b$, resulting in the following segmented model:

Table 1. Weather conditions, soil type, and measurement age by location.

City/county	Province	Coordinates	Average temperature (° C)	Average precipitation (mm)	Soil type	Elevation (m)	Measurement age
Shaowu	Fujan	27°05′ N, 117°43′ E	17.7	1,768	Laterite	300	6, 7, 8, 9, 10, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31
Fenyi	Jiangxi	27°30′ N, 114°33′ E	17.2	1,656	Laterite, yellow	250	6, 7, 8, 9, 10, 12, 14, 16, 18, 20
Pingxiang	Guangxi	28°12′ N, 105°10′ E	20.5	1,300	Laterite	500	11, 12, 13, 14, 15, 16, 18, 20, 22, 24, 26, 28, 30
Naxi	Sichuan	28°10′ N, 105°10′ E	17.5	1,129	Laterite	440	7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 20, 23, 25, 27, 29, 31, 34

Table 2. **Summary statistics for quadratic mean diameter and number of trees per hectare by province.**

		Q				*N*			
Province	*n*	Mean	Min	Max	SD	Mean	Min	Max	SD
		................(cm)................							
Fujian	192	11.26	3.50	21.09	4.35	5,120	1,616	10,000	2,262.45
Jiangxi	124	9.20	3.95	14.48	2.47	5,977	3,183	10,000	2,237.00
Guangxi	132	11.82	6.15	19.37	2.90	3,627	1,216	9,500	1,815.02
Sichuan	192	9.70	14.26	3.27	2.42	4,646	1,433	10,000	2,212.24

n = total number of measurements for all plots. Min, minimum; max, maximum.

$$y = y_0 + b(x - a_1)^2 I_1 - b(x - a_2)^2 I_2 + \varepsilon \qquad (2)$$

The last segment, where $I_1 = I_2 = 1$, can be put in the following form:

$$y = y_0 + b(a_1^2 - a_2^2)^2 + 2b(a_2 - a_1)x + \varepsilon$$

or

$$y = y_0 + b(a_1^2 - a_2^2)^2 + cx + \varepsilon$$

where $c = 2b(a_2 - a_1)$, or $a_2 = a_1 + (c/2b)$.

Equation (2) can be rewritten as:

$$y = y_0 + b(x - a_1)^2 I_1 - b\left(x - a_1 - \frac{c}{2b}\right)^2 I_2 + \varepsilon \qquad (3)$$

where

$$I_1 = \begin{cases} 1, \text{ if } x > a_1 \\ 0, \text{ otherwise} \end{cases}, \quad I_2 = \begin{cases} 1, \text{ if } x > a_1 + \dfrac{c}{2b} \\ 0, \text{ otherwise} \end{cases}$$

Equation 3 has three parameters to be estimated: b, c, and a_1. Note that a_2 from Equation 2 is replaced with c, which is the slope of the line in the last segment.

Results and Discussion

In this study, a linear-quadratic-linear segmented model was used to model the self-thinning trajectories of Chinese fir plantations. Equation 3 was fitted with remeasured data in each plot. Data from Jiangxi province were not used to fit Equation 3 because data were unavailable except in the early stages.

Effects of Provinces and Planting Densities One Self-Thinning Line

Based on an analysis of variance, values of the slope (c) of the self-thinning line were significantly different ($P = 0.0025$) in the four provinces (blocks) but were not significantly different ($P = 0.2570$) for the four planting densities (treatments). For a given province, the self-thinning slope did not vary with planting densities. The result that the slope of the self-thinning line was invariant to changes in planting densities was consistent with findings from previous studies (Puettmann et al. 1993, Tang et al. 1995, Turnblom and Burk. 2000, VanderSchaaf and Burkhart 2007).

A General Model for All Planting Densities

The first joint point, a_1, of Equation 3 was inversely related to y_0 (Figure 3). Lower values of a_1 occurred at higher y_0, indicating that mortality started sooner at the higher planting density because of competition for limited resources occurring at earlier ages as planting density increases (VanderSchaaf and Burkhart 2008). The first joint point corresponds to a relative density characteristic of full-site occupancy, at 30% of maximum density (Solomon and Zhang 2002) or 35% of maximum density (Long 1985). In this study, the relative densities corresponding to the joint point a_1 averaged 30% for stands with planting density $N_0 = 3{,}333$ trees/ha, 26% for $N_0 = 5{,}000$ trees/ha, 24% for $N_0 = 6{,}667$ trees/ha, and 20% for $N_0 = 10{,}000$ trees/ha.

Parameter a_1 can be expressed as a linear function of y_0 as follows:

$$a_1 = a_{11} + a_{12} y_0 \qquad (4)$$

Equation 3 can be reformulated to express current stand density in terms of past stand density:

$$y_2 = y_1 + b\left\{ (x_2 - a_{11} - a_{12}y_0)^2 I_{12} - \left(x_2 - a_{11} - a_{12}y_0 - \frac{c}{2b}\right)^2 I_{22} - (x_1 - a_{11} - a_{12}y_0)^2 I_{11} + \left(x_1 - a_{11} - a_{12}y_0 - \frac{c}{2b}\right)^2 I_{21} \right\} \qquad (5)$$

where x_i and y_i are values of x and y at time i, respectively,

$$I_{1i} = \begin{cases} 1, \text{ if } x_i > a_{11} + a_{12}y_0 \\ 0, \text{ otherwise} \end{cases}, \text{ and}$$

$$I_{2i} = \begin{cases} 1, \text{ if } x_i > a_{11} + a_{12}y_0 + \dfrac{c}{2b}, \ i = 1,2 \\ 0, \text{ otherwise} \end{cases}$$

Parameters a_{11}, a_{12}, and c were considered as fixed-effects parameters (common to all plots), whereas parameter b was assumed to contain an additional random component (specific to individual plots). Measurements from consecutive growth periods from each province were used to estimate the parameters of Equation 5 by use of the SAS procedure NLMIXED (SAS Institute, Inc. 2011).

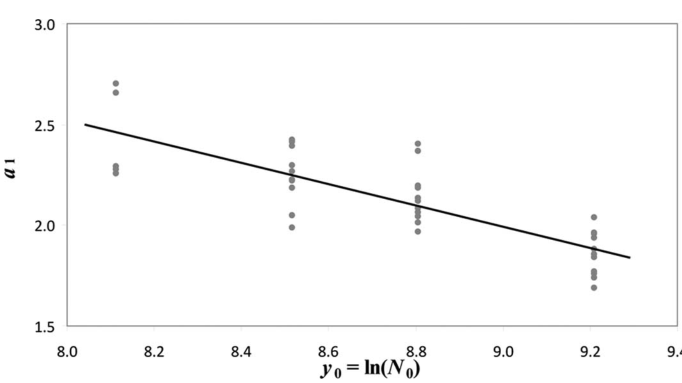

Figure 3. Relationship between joint point a_1 and the natural log of planting density (12 observations at each density).

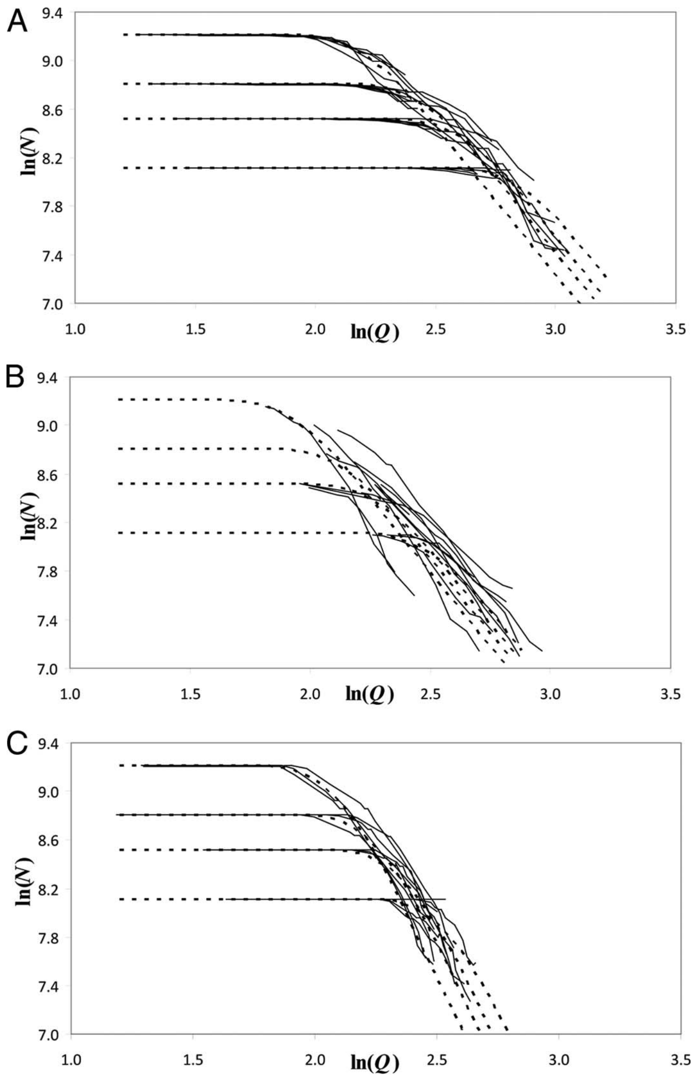

Figure 4. Observed (solid lines) and predicted (dotted lines) size-density trajectories for the provinces (a, Jiangxi and Fujian; b, Guangxi; c, Sichuan).

Combining Data from Jiangxi and Fujian Provinces

It is apparent from Figure 2 that the trajectories from Jiangxi and Fujian provinces are similar. A likelihood ratio test confirmed that data from these two provinces can be combined ($P = 0.2206$) to fit Equation 5. The plots in these two provinces are approximately 300 km apart and belong to the same climate region; thus, the self-thinning trajectories of Chinese fir in the two provinces have a similar trend. Jiangxi and Fujian also have the highest number of Chinese fir plantations in China (Wu 1984).

On the other hand, separate equations must be used for the Guangxi and Sichuan provinces because their parameters were significantly different ($P = 0.0001$). We found that the segmented model (Equation 5) adequately described the self-thinning trajectories of Chinese fir plantations in southern China (Figure 4).

Combining Slopes

The slopes (c) of the self-thinning lines seem to be similar for Jiangxi, Fujian, and Guangxi provinces. This was confirmed by a likelihood ratio test ($P = 0.0833$) that involved the full model (separate sets of parameters) and the reduced model, in which Equation 5 was fitted with a common slope, c, and separate remaining parameters for Jiangxi/Fujian and Guangxi provinces (Table 3).

Table 3. Parameter estimates of Equation 5 for two groups of provinces.

Province	n	b	a_{11}	a_{12}	c	s^2	Var(b)
Sichuan	192	−4.1901	5.5619	−0.4048	−4.2169	0.0014	2.3518
Guangxi, Jiangxi	448	−2.552	6.4307	−0.516	−2.4453	0.0015	0.4323
and Fujian		−2.6121	7.4132	−0.5924			

s^2 is the error variance of Equation 5, and Var(b) is the variance of random parameter b. All parameters were significantly different from zero ($P < 0.0001$).

The slope of the self-thinning line of Chinese fir plantations in Sichuan province was much steeper than that of the remaining three provinces (−4.2169 versus −2.4453). The most suitable annual precipitation for Chinese fir ranges from 1,300 to 2,000 mm (Wu 1984); however, the annual precipitation in Naxi County, Sichuan province, is lower than 1,200 mm. Furthermore, the highest temperature in summer in Naxi County reaches 41° C and stays there for several days, which is not conducive for Chinese fir growth. In addition, Sichuan is the marginal distribution area for Chinese fir, and the site quality is lower than those in the other three provinces (Wu 1984), which causes additional mortality and makes the slope of the self-thinning line of Chinese fir steeper than those for the other three provinces.

Comparison with Previous Studies

Reineke (1933) found that the slope of the self-thinning line was −1.605 for 12 of 14 species. The slope was slightly steeper for slash pine (*Pinus elliottii*) and appreciably steeper for shortleaf pine (*Pinus echinata*) (approximately −1.76 and −1.94, respectively, as computed from Figure 7) (Reineke (1933)). In this study, the slopes of Chinese fir, −2.4453 and −4.2169, were significantly steeper than −1.605 as proposed by Reineke (1933). The slope of the self-thinning line varied among species (Pretzsch and Biber 2005). In mixed species stands of western hemlock (*Tsuga heterophylla*) and Sitka spruce (*Picea sitchensis*), Poage et al. (2007) showed that the slopes for individual stands ranged from −2.192 to −1.310. Weiskittel et al. (2009) reported that the slopes were −1.5473 for Douglas-fir (*Pseudotsuga menziesii*), −1.2208 for red alder (*Alnus rubra*), and −1.7336 for western hemlock. The slopes for oaks (*Quercus* spp.), common beech (*Fagus sylvatica*), and Norway spruce (*Picea abies*) were −1.605, −1.76, −1.86, and −1.85, respectively (Rivoire and Le Moguedec 2012). Charru et al. (2012) also reported that the slope was species dependent, ranging from −1.941 to −1.615 for 11 temperate species in France.

In addition, the debate on whether the slope of the self-thinning line is invariant can also be explained by the variety of statistical methods that have been used to examine the species self-thinning line (Weiskittel et al. 2009). Zhang et al. (2005) used different methods to estimate the self-thinning slopes of eastern white pine (*Pinus strobus*) and found that the slopes varied for different methods. VanderSchaaf and Burkhart (2007) reported that the slope of the mixed-effects model for loblolly pine (*Pinus taeda*) varied widely, ranging from −2.5 to −1.2.

Departure from the Self-Thinning Line

Many researchers agree that a departure from a self-thinning line would occur (del Rio et al. 2001, Yang and Titus 2002, Monserud et al. 2005). As trees die, the residual trees cannot continue to fully occupy canopy gaps, and the thinning trajectory turns away from the self-thinning line (VanderSchaaf and Burkhart 2008). Some authors have depicted the divergence from the self-thinning line as linear (Christensen and Peet 1981, Lonsdale 1990, VanderSchaaf and Burkhart 2008), whereas others expressed it as a curve (Zeide 1985, Cao et al. 2000). The stage of departure from the self-thinning line is most likely related to the length of time since the occurrence of self-thinning (Weller 1991, VanderSchaaf and Burkhart 2008). For our data, departure did not occur because the stands in this study were not old enough.

Conclusions

In this study, the linear-quadratic-linear segmented model (Equation 5) was used to describe the self-thinning trajectories of Chinese fir plantations in southern China. Remeasured data from Chinese fir stands provided evidence to support the use of this model and justify the existence of the self-thinning line. For a given province, the slope of the self-thinning line was invariant with changes among the four planting densities: 3,333, 5,000, 6,667, and 10,000 trees/ha). The self-thinning slopes differ among provinces. The combined self-thinning slope was −2.4453 for Jiangxi/Fujian and Guangxi provinces, which was different from the slope for Sichuan province (−4.2169). These slope values for Chinese fir plantations were much steeper than the −1.605 value reported by Reineke (1933). We also found that divergence from the self-thinning line did not occur because the stands were not old enough. Overall, the segmented model (Equation 5) adequately described the self-thinning trajectories of Chinese fir plantations in southern China.

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