A Comparison of Compatible and Annual Growth Models

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ABSTRACT. Compatible growth and yield models are desirable because they provide the same growth estimates regardless of length of growth periods. However, the compatibility constraints restrict the number of possible models. This restriction might be overcome by using models that predict annual stand growth based on periodic measurements. The advantages of this approach are (1) the flexibility allowed in building annual growth models without constraints, and (2) the step-invariance property maintained by these models. An annual growth model was developed in this study that predicts yield based on information from the previous year. For 162 plots from the Southwide Seed Source Study of loblolly pine (*Pinus taeda* L.), the annual growth model provided better predictions of stand survival, basal area, and volume than two compatible growth models. For. Sci. 49(2):285–290.

Key Words: Compatible growth and yield models, step-invariant models, loblolly pine, *Pinus taeda.*

Growth and yield models were defined by Clutter (1963) as compatible when the algebraic form of the yield model can be derived by mathematical integration of the growth model. Sullivan and Clutter (1972) further developed a system of equations that were numerically compatible. In their model, growth projected directly from age A_1 to age A_3 is identical to the growth predicted in two steps, from A_1 to A_2 and then from A_2 to A_3. This property is termed "step-invariance." The compatibility between growth and yield is highly desirable because it ensures consistent results when forest managers consider various alternatives in managing their stands. The drawback is that compatibility places constraints on the system of equations that may reduce the accuracy of the model.

This limitation might be overcome by using models that predict annual stand growth. They are not necessarily conceptually compatible, but they may provide estimates that are step-invariant. Furthermore, these annual growth models do not suffer from restrictions caused by compatibility constraints and thus are more flexible than their compatible counterparts. The objectives of this study were to develop and evaluate such an annual growth model.

Data

Data were from the Southwide Seed Source Study, conducted on 15 loblolly pine (*Pinus taeda* L.) stands planted at 13 locations across 10 southern states (Wells and Wakeley 1966, Wells 1983). This covered the greater part of the natural range of loblolly pine. A total of 162 plots from four seed sources (northeastern Mississippi, southeastern Louisiana, eastern Texas, and southwestern Arkansas) was used in the current study. Survival, diameters, and heights of the 49 measurement trees on each plot of size 0.0164 ha were recorded every 5 to 7 yr from age 10 to age 25 or 27 (Table 1). Stand height (average height of the dominant and codominant trees) was computed for each plot by taking the average height of the tallest 50% of the trees in the plot.

The data were randomly divided into a fit data set and a validation data set. The fit data set (65% of the data) was used to obtain parameter estimates for the growth and yield systems. The validation data set (35%) that represents the general population was used to evaluate the performance of the models. Table 2 presents summary statistics for the two data sets.

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Table 1. Distribution of 162 plots used in this study, by measurement age.

Measurement age	Number of plots
10, 15, 20, 25	114
10, 15, 20, 27	12
10, 15, 22, 27	12
10, 16, 20, 27	24
Total	162

Procedures

Height-Age Relationship

The following Bailey and Clutter's (1974) height-age equation was selected for predicting height growth of dominants and codominant trees:

$$H_{t+q} = \exp\ \{\lambda_1 + [\ln\ (H_t) - \lambda_1]\ (A_{t+q} / A_t)^{\lambda_2}\} + \varepsilon \quad (1)$$

where H_t and H_{t+q} = stand heights at ages A_t and A_{t+q}, respectively, A_t = age in years at the beginning of the growth period, q = length of growth period in years, λ_i's = regression coefficients, and ε = random error.

Simultaneous estimation of parameters was not possible when the height growth equation was included with other equations in the growth and yield systems because the iterative process did not converge. Therefore the parameters of the height-age equation were estimated using the ordinary least squares (OLS) method, separately from the other equations of the growth and yield systems.

Growth Models

Two published compatible models and one new annual growth model were evaluated in this study. Nonoverlapping growth intervals (from 4 to 7 yr) were used to fit the models. In addition to the above stand height component, each model contains equations to project number of surviving trees, basal area, and volume on a per hectare basis. Since cross-equation correlations existed among error components of these models, a method suggested by Borders (1989) was employed to simultaneously estimate parameters of the above models. The fitting procedure was carried out using option SUR (seemingly unrelated regression) of SAS procedure MODEL (SAS Institute Inc. 1993).

Pienaar and Harrison's (1989) Model.—The following compatible model was developed for slash pine (*Pinus elliottii* Engelm.) planted in South Africa:

$$N_{t+q} = \{ N_t^{\alpha_1} + \alpha_2[(A_{t+q} / 100)^{\alpha_3} - (A_t / 100)^{\alpha_3}]\} + \varepsilon \quad (2a.1)$$

$$\begin{aligned} B_{t+q} = \exp\ \{&\ln\ (B_t) + \beta_1(1 / A_{t+q} - 1 / A_t) \\ &+ \beta_2[\ln\ (\hat{N}_{t+q}) - \ln\ (N_t)] \\ &+ \beta_3[\ln\ (\hat{H}_{t+q}) - \ln\ (H_t)] \\ &+ \beta_4[\ln\ (\hat{N}_{t+q}) / A_{t+q} - \ln\ (N_t) / A_t] \\ &+ \beta_5[\ln\ (\hat{H}_{t+q}) / A_{t+q} - \ln\ (H_t) / A_t]\ \} + \varepsilon \end{aligned} \quad (2b)$$

$$\begin{aligned} V_{t+q} = \exp\ \{&\ \ln\ (V_t) + \gamma_1[\ln\ (\hat{H}_{t+q}) - \ln\ (H_t)] \\ &+ \gamma_2[\ln\ (\hat{N}_{t+q}) - \ln\ (N_t)] \\ &+ \gamma_3[\ln\ (\hat{B}_{t+q}) - \ln\ (B_t)]\ \} + \varepsilon \end{aligned} \quad (2b)$$

where N_j, B_j, and V_j = number of trees, basal area (m^2), and volume (m^3) per hectare at age A_j; $\hat{Y}$ = predicted value of Y; α_i's, β_i's, and γ_i's = regression coefficients; and ε = random error.

Table 2. Summary statistics for stand-level attributes, by data type.

Attribute *	*n*	Min	Mean	Max	SD
		........(Fit data set)........			
N	315	122	1191	2807	584
H	315	8.0	16.7	24.9	3.3
B	315	4.23	30.91	53.95	9.53
V	315	25.98	241.72	498.07	94.04
S	315	11.1	19.4	23.8	2.4
ΔN	315	–296	–63	0	67
ΔH	315	0.1	0.7	1.3	0.2
ΔB	315	–4.41	0.77	3.93	1.28
ΔV	315	–31.52	12.61	38.32	9.92
		........(Validation data set)........			
N	170	122	1247	2807	575
H	170	9.3	16.3	24.0	3.4
B	170	7.73	30.47	50.53	8.06
V	170	46.70	233.95	548.24	86.49
S	170	14.4	19.3	25.8	2.6
ΔN	170	–253	–65	0	68
ΔH	170	0.2	0.7	1.2	0.2
ΔB	170	–3.07	0.84	2.65	1.16
ΔV	170	–25.21	12.89	31.81	8.76

* Notation:
N = number of surviving trees per hectare,
H = stand height (average height of dominant and codominant trees) in m,
B = stand basal area in m^2/ha,
V = stand volume in m^3/ha,
S = site index (base age 25 yr) in m, and
ΔY = annual growth of *Y*, where *Y* is *N*, *H*, *B*, or *V*.

Equation (2a.1) is a survival function developed by Clutter and Jones (1980). Its parameter α_2 was not significant for the data set used in this study (P = 0.35). Therefore this equation needed to be replaced by another survival equation. The following equation form, derived from the Weibull probability density function, was selected because it performed better than other survival equations in a preliminary evaluation:

$$N_{t+q} = N_t \, (A_{t+q} / A_t)^{\alpha_2 - 1} \exp\{ \alpha_1 \, (A_{t+q}^{\alpha_2} - A_t^{\alpha_2}) \} + \varepsilon \qquad (2a.2)$$

Sullivan and Clutter's (1972) Model.—This compatible model, developed for thinned loblolly pine natural stands in the southern United States, includes the following equations:

$$N_{t+q} = N_t \, (A_{t+q} / A_t)^{\alpha_2 - 1} \exp\{ \alpha_1 \, (A_{t+q}^{\alpha_2} - A_t^{\alpha_2}) \} + \varepsilon \qquad (3a)$$

$$B_{t+q} = \exp\{(A_t / A_{t+q}) \ln(B_t) + (1 - A_t / A_{t+q}) \, (\beta_1 + \beta_2 S)\} + \varepsilon \qquad (3b)$$

$$V_{t+q} = \exp\{\gamma_1 + \gamma_2 S + \gamma_3 / A_{t+q} + \gamma_4 \ln(\hat{B}_{t+q})\} + \varepsilon \qquad (3c)$$

where S = site index in m at base age 25 yr.

The original Sullivan-Clutter model includes only equations to predict per hectare basal area and volume (3b and 3c). Equation (3a) was added to this system to predict stand survival; this equation was identical in form to equation (2a.2) above.

Annual Growth Model

Annual growth projection is a common approach in modeling the growth of individual trees. McDill and Amateis (1993) and Cao (2000) discussed methods of estimating the parameters of annual tree growth equations from periodic measurements. Annual projection has also been employed to model the growth of an entire stand (Cao 1994, Cao et al. 2000). An advantage of annual projection is that model outputs maintain the property of step invariance. Annual changes in stand density and volume were described in a recursive manner as follows:

Year (t + 1)

$$\hat{N}_{t+1} = \exp\{\alpha_1 + \ln(N_t)\,(\alpha_2 + \alpha_3 A_t / A_{t+1}) + \ln(H_t)\,(\alpha_4 + \alpha_5 A_t / A_{t+1})\} \qquad (4.a.1)$$

$$\hat{B}_{t+1} = \exp\{(A_t / A_{t+1}) \ln(B_t) + (1 - A_t / A_{t+1})\,[\beta_1 + \beta_2 \ln(Ht) + \beta_3 \ln(N_t)]\} \qquad (4.b.1)$$

$$\hat{V}_{t+1} = \exp\{(A_t / A_{t+1}) \ln(V_t) + (1 - A_t / A_{t+1})\,[\gamma_1 + \gamma_2 \ln(H_t) + \gamma_3 \ln(N_t) + \gamma_4 \ln(B_t)]\} \qquad (4.c.1)$$

Year (t + 2)

$$\hat{N}_{t+2} = \exp\{\alpha_1 + \ln(\hat{N}_{t+1})(\alpha_2 + \alpha_3 A_{t+1} / A_{t+2}) + \ln(\hat{H}_{t+1})\,(\alpha_4 + \alpha_5 A_{t+1} / A_{t+2})\} \qquad (4.a.2)$$

$$\hat{B}_{t+2} = \exp\{(A_{t+1} / A_{t+2}) \ln(\hat{B}_{t+1}) + (1 - A_{t+1} / A_{t+2})\,[\beta_1 + \beta_2 \ln(\hat{H}_{t+1}) + \beta_3 \ln(\hat{N}_{t+1})]\} \qquad (4.b.2)$$

$$\hat{V}_{t+2} = \exp\{(A_{t+1} / A_{t+2}) \ln(\hat{V}_{t+1}) + (1 - A_{t+1} / A_{t+2})\,[\gamma_1 + \gamma_2 \ln(\hat{H}_{t+1}) + \gamma_3 \ln(\hat{N}_{t+1}) + \gamma_4 \ln(\hat{B}_{t+1})]\} \qquad (4.c.2)$$

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Year (t + q)

$$N_{t+q} = \exp\{\alpha_1 + \ln(\hat{N}_{t+q-1})\,(\alpha_2 + \alpha_3 A_{t+q-1} / A_{t+q}) + \ln(\hat{H}_{t+q-1})(\alpha_4 + \alpha_5 A_{t+q-1} / A_{t+q})\} + \varepsilon \qquad (4.a.q)$$

$$B_{t+q} = \exp\{(A_{t+q-1} / A_{t+q}) \ln(\hat{B}_{t+q-1}) + (1 - A_{t+q-1} / A_{t+q})\,[\beta_1 + \beta_2 \ln(\hat{H}_{t+q-1})(\beta_3 \ln(\hat{N}_{t+q-1})]\} + \varepsilon \qquad (4.b.q)$$

$$V_{t+q} = \exp\{(A_{t+q-1} / A_{t+q}) \ln(\hat{V}_{t+q-1}) + (1 - A_{t+q-1} / A_{t+q})\,[\gamma_1 + \gamma_2 \ln(\hat{H}_{t+q-1}) + \gamma_3 \ln(\hat{N}_{t+q-1}) + \gamma_4 \ln(\hat{B}_{t+q-1})]\} + \varepsilon \qquad (4.c.q)$$

Model Evaluation

The models were evaluated based on statistics computed from the validation data set. Each plot was measured at four different times: t_1 (age 10), t_2 (age 15 or 16), t_3 (age 20 or 22), and t_4 (age 25 or 27). This allowed us to evaluate the models at three projection lengths. Short projection length (from 4 to 7 yr) included intervals from t_1 to t_2, t_2 to t_3, and t_3 to t_4. Medium projection length (from 10 to 12 yr) covered intervals from t_1 to t_3 and from t_2 to t_4, and long projection length (from 15 to 17 yr) spanned from t_1 to t_4.

Evaluation statistics were computed for each projection length. The following criteria were used.

Mean difference:

$$MD = \Sigma(Y_{i,\,t+q} - \hat{Y}_{i,\,t+q}) / n$$

Mean absolute difference:

$$MAD = \Sigma\,|Y_{i,\,t+q} - \hat{Y}_{i,\,t+q}| / n$$

Fit index:

$$FI = 1 - \Sigma(Y_{i,\,t+q} - \hat{Y}_{i,\,t+q})^2 \,/\, \Sigma(Y_{i,\,t+q} - \overline{Y}_{t+q})^2$$

where $Y_{i,\,t+q}$ and $\hat{Y}_{i,\,t+q}$ = observed and predicted values, respectively, of the stand attribute (N, B, or V) of plot i at time $(t + q)$, $\overline{Y}_{t+q}$ = mean value of the stand attribute at time $(t + q)$, n = number of plot observations, and the sum includes values of i from 1 to n.

Results and Discussion

The estimates (and standard errors) for the parameters of the height-age equation (1) were λ_1 = 3.8937 (0.0892) and λ_2 = 0.6434 (0.0467). Table 3 shows the parameter estimates and their standard errors of the two compatible models and the annual growth model. Parameters β_5 of Pienaar and Harrison's (1989) model and β_2 of Sullivan and Clutter's (1972) model were not significant ($P > 0.48$); the two models were refitted without these parameters.

The evaluation statistics computed from the three models were presented in Table 4 for different stand attributes and for three projection lengths. The annual growth model was ranked first based on all 27 statistics. It consistently performed better than the two compatible models for all projection lengths. In the case of short projection periods, the annual growth model improved the fit index from 0.7417 and 0.7419 to 0.8273 for stand survival, from 0.5096 and 0.5387 to 0.6055 for stand basal area, and from 0.7048 and 0.7135 to 0.7483 for stand volume. The annual growth model also reduced the mean absolute differences and the mean differences in predicting all stand attributes. The above trends continued for long-term projections (10 yr and above). The performance of all three models deteriorated as projection intervals increased, but the annual growth model still maintained its superior rankings compared to the two compatible models. For all projection lengths, the mean differences computed from predictions by the annual growth model were less than 3%. These small bias values were surprising considering the long projection periods.

Evaluation statistics from the two compatible models were similar, with Pienaar and Harrison's (1989) model ranked last for 22 out of 27 statistics. Both models produced some negative fit indices for the longest projection periods (15 to 17 yr). Negative fit indices indicate that using the means to predict growth fared better than using the predictions from the models in these cases.

One advantage that the annual growth model held over the compatible models is that it included both current measures of stand density (tree number and basal area) as independent variables to predict future stand density. Compatible models did not have that luxury. Due to restrictions of compatible growth and yield models, one of the two measures of stand density in these models could not be predicted from the other.

It should be noted that compatible growth models are special cases of annual growth models because compatible models can be rewritten in the form of annual growth. In building compatible growth models, it is necessary to restrict oneself to a smaller pool of possible models even possibly at the expense of model predicting ability. Since the class of annual growth models is a superset of the class of compatible models, annual growth models allow modelers more flexibility in selection of independent variables and equation forms.

A desirable property of compatible growth and yield models is that they are step-invariant, meaning identical predictions are obtained if the stand is projected in one step (directly from age A_t to age A_{t+q2}) or in two steps (from A_t to A_{t+q1} and then from A_{t+q1} to A_{t+q2}):

$$\begin{aligned} Y_{t+q2} &= Y_t + \int_{A_t}^{A_{t+q2}} g(x)\,dx \\ &= Y_t + \int_{A_t}^{A_{t+q1}} g(x)\,dx + \int_{A_t}^{A_{t+q2}} g(x)\,dx, \end{aligned} \tag{5}$$

where Y_k is stand attribute (survival, basal area, or volume) at time A_k, and $g(x)$ is instantaneous stand growth at time x.

Annual growth models also produce step-invariant results as follows.

$$Y_{t+q2} = Y_t + \sum_{i=A_{t+1}}^{A_{t+q2}} g_i = Y_t + \sum_{i=A_{t+1}}^{A_{t+q1}} g_i + \sum_{i=A_{t+q1+1}}^{A_{t+q2}} g_i, \tag{6}$$

Table 3. Parameter estimates and their standard errors (SE) of three models.

		Model type					
		Pienaar and Harrison (1989)		Sullivan and Clutter (1972)		Annual growth	
Attribute	Parameter	Estimate	SE	Estimate	SE	Estimate	SE
Stand survival (trees/ha)	α_1	–0.0254	0.0043	–0.0304	0.0049	0.3151	0.0829
	α_2	1.3068	0.0620	1.2254	0.0608	1.9611	0.1330
	α_3					–1.0624	0.1406
	α_4					–2.8228	0.3681
	α_5					2.9605	0.3970
Stand basal area (m^2/ha)	β_1	–64.3974	6.4448	3.6804	0.0708	9.1393	1.4782
	β_2	0.4115	0.0523	*	*	–0.9683	0.2500
	β_3	1.6345	0.2448			–0.3834	0.1369
	β_4	8.9383	0.8097				
	β_5	*	*				
Stand volume (m^3/ha)	γ_1	–0.3064	0.0359	1.0703	0.1646	14.1724	1.3672
	γ_2	0.3375	0.0617	0.0541	0.0035	–1.2649	0.2847
	γ_3	1.3192	0.0553	–7.6711	0.9263	–0.9614	0.1491
	γ_4			1.0935	0.0377	0.7393	0.1948

* This parameter was not significant ($P > 0.48$) and was omitted from the model.

Table 4. Evaluation statistics resulting from three models, by projection length and stand attribute.

		Model type		
Attribute	Evaluation statistic*	Pienaar and Harrison (1989)	Sullivan and Clutter (1972)	Annual growth
Fit data set				
Stand survival	MD	***60.6***[†]	42.5	6.3[†]
	MAD	***236.5***	231.8	192.2
	FI	0.7396	***0.7391***	0.8049
Stand basal area	MD	***1.09***	1.08	0.05
	MAD	4.83	***4.86***	4.46
	FI	0.6012	***0.5806***	0.6329
Stand volume	MD	***7.26***	5.04	0.74
	MAD	***37.90***	37.34	35.52
	FI	***0.7283***	0.7298	0.7437
Validation data set				
	.... [Short projection length—4 to 7 yr (n = 170)]			
Stand survival	MD	***71.4***	52.9	13.0
	MAD	***244.5***	241.0	196.1
	FI	***0.7417***	0.7419	0.8273
Stand basal area	MD	***1.51***	1.33	0.17
	MAD	***4.69***	4.50	3.89
	FI	***0.5096***	0.5387	0.6055
Stand volume	MD	***11.23***	5.44	4.04
	MAD	36.26	***36.36***	31.63
	FI	***0.7048***	0.7125	0.7483
	.... [Medium projection length—10 to 12 yr (n = 113)]			
Stand survival	MD	***66.5***	28.9	–9.1
	MAD	223.4	***225.0***	209.3
	FI	***0.4713***	0.4846	0.5453
Stand basal area	MD	***1.90***	1.28	–0.66
	MAD	5.77	***5.82***	5.26
	FI	***0.2599***	0.2748	0.3464
Stand volume	MD	***14.34***	14.16	0.30
	MAD	***50.65***	50.36	48.07
	FI	***0.4224***	0.4699	0.4834
	.... [Long projection length—15 to 17 yr (n = 56)]			
Stand survival	MD	***187.4***	139.4	27.1
	MAD	***275.5***	262.8	238.5
	FI	***0.1211***	0.2158	0.3756
Stand basal area	MD	***4.46***	2.97	0.20
	MAD	***7.30***	6.98	5.73
	FI	***–0.1473***	–0.0652	0.1905
Stand volume	MD	29.78	***38.89***	7.32
	MAD	60.52	***69.76***	55.37
	FI	0.1593	***–0.0099***	0.2503

* MD = mean difference, MAD = mean absolute difference, and FI = fit index. Formulas for these statistics are given in the text.
[†] For each evaluation statistic, an underlined number indicates the best among three models, whereas a bold italic number denotes the worst.

where g_i is growth between year $(i - 1)$ and year i. Equation (6) is similar to equation (5), with the integrals in (5) being replaced by the sums in (6).

The step-invariance property was further examined by estimating and evaluating a growth and yield system similar to (4.a.1–4.c.1), but with A_{t+1} replaced by A_{t+q}. The new model did not involve annual projection and therefore was not step-invariant. Table 5 shows that the results were similar evaluation statistics for short projection periods but much poorer predictions for long-term projection (10 yr and longer).

Table 5. Evaluation statistics resulting from a model that is not step-invariant, by projection length and stand attribute.

			Validation data set		
Attribute	Evaluation statistic*	Fit data set	Short projection length (4–7 yr)	Medium projection length (10–12 yr)	Long projection length (15–17 yr)
Stand survival	MD	5.0	7.3	-239.9	-515.4
	MAD	200.7	206.9	364.0	594.3
	FI	0.7606	0.7901	-0.5145	-3.1094
Stand basal area	MD	0.13	0.24	-1.88	-4.37
	MAD	4.46	3.88	5.70	7.39
	FI	0.6349	0.6080	0.2551	-0.0775
Stand volume	MD	1.20	4.02	0.68	15.14
	MAD	35.59	31.78	49.24	57.27
	FI	0.7423	0.7445	0.4625	0.2326

* MD = mean difference, MAD = mean absolute difference, and FI = fit index. Formulas for these statistics are given in the text.

This suggests that a noncompatible growth and yield system might be improved if it is rewritten and its parameters estimated in the form of annual projection.

Conceptually, the annual growth model is not compatible with the yield model because it is not derivative of the yield model. However, the annual growth model as formulated in Equations (4.a.1–4.c.q) does provide annual predictions that are step invariant. Results from this study show that the annual growth model provided better predictions of stand survival, basal area, and volume than the two compatible growth models, and that the annual growth model can offer flexibility and acceptable performance without sacrificing the step-invariance property.

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