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Evaluation of Methods for Calibrating a Tree Taper Equation

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With recent advances in laser technology, it is now more and more affordable to measure upper-stem diameters accurately. In this paper, diameter measurement at a point halfway between the tree tip and breast height was used to calibrate a taper equation to improve predictions of diameters along the tree bole. Different calibration methods were investigated, including the use of fixed- and mixed-effects models, as well as quantile regression models. Felled-tree data collected in a loblolly pine (*Pinus taeda* L.) plantation showed that all calibration methods yielded improved results as compared to those from the uncalibrated taper model. Furthermore, the mixed-model approach and the quantile regression method based on five quantiles performed slightly better than the other calibration methods, based on two evaluation statistics. The methods presented in this paper should be applicable to data sets of loblolly pine or of other conifer species as well.

Keywords: *Pinus taeda*, loblolly pine, quantile regression, mixed model, segmented regression

Taper equations are developed to describe tree profiles and are therefore useful for computing tree merchantable volume for any utilization standard. When a taper equation is constrained such that its curve passes through a measured upper-stem diameter, the result is improved prediction of tree taper anywhere on the bole. Upper-stem diameter measurements can now be obtained by use of better and more affordable laser dendrometers, which are able to produce a mean bias of only 0.13 cm for measuring diameters (Parker and Matney 1999, Williams et al. 1999, Kalliovirta et al. 2004). The Max and Burkhart (1976) taper equation was conditioned by Czaplewski and McClure (1986) to pass through dbh and an upper-stem diameter (at 5.27 m). The result was a 10–25% reduction in root mean squared error as compared to the unconditioned model. Cao (2009) calibrated the original Max and Burkhart (1976) taper equation for dbh and an upper-stem diameter and obtained an increase in R^2 value from 0.953 to 0.975.

Juha Lappi started to solve the problem of local calibration in forestry by using mixed-effects models on tree taper (Lappi 1986) and continued to model dominant height (Lappi and Bailey 1988), height and volume (Lappi 1991), and height-diameter relationship (Lappi 1997). Recently, mixed-effects models have been employed to predict tree taper because they take into account the correlation among multiple diameter measurements on an individual stem (Garber and Maguire 2003, Leites and Robinson 2004, Trincado and Burkhart 2006, Lejeune et al. 2009, Yang et al. 2009). In these

models, fixed-effects parameters are common to all trees in the sample, whereas random effects are specific to each individual tree. A mixed-effects taper model can be calibrated for a particular tree by predicting the random effects from one or more upper-stem diameter measurements (Cao and Wang 2011).

Quantile regression (Koenker and Bassett 1978) has been widely used to describe the response variable given a set of explanatory variables in the last few years. Whereas the mean regression estimator addresses only the conditional mean or the central effects of the covariates, the quantile regression estimator can quantify the entire conditional distribution of the response variable and give an overall assessment of the covariate effects at different quantiles of the response (Koenker 2004). There have been recent attempts at fitting quantile regression to longitudinal data (Geraci and Bottai 2007, Karlsson 2008). In forestry, quantile regression has been applied to model self-thinning boundary lines (Zhang et al. 2005, Ducey and Knapp 2010), spread rates of insects (Evans and Gregoire 2007) and diseases (Evans and Finkral 2010), diameter percentiles (Mehtätalo 2008), prediction errors of tree- and stand-growth simulators (Mäkinen et al. 2008), impacts of prescribed burning (Boer et al. 2009), and assessment errors in forest inventory data (Mäkinen et al. 2010). A set of quantile regression equations can be used to model tree taper. When an upper-stem diameter is encompassed by two taper equations based on different quantiles, a calibrated taper equation to pass through that point can be derived from the two quantile regressions.

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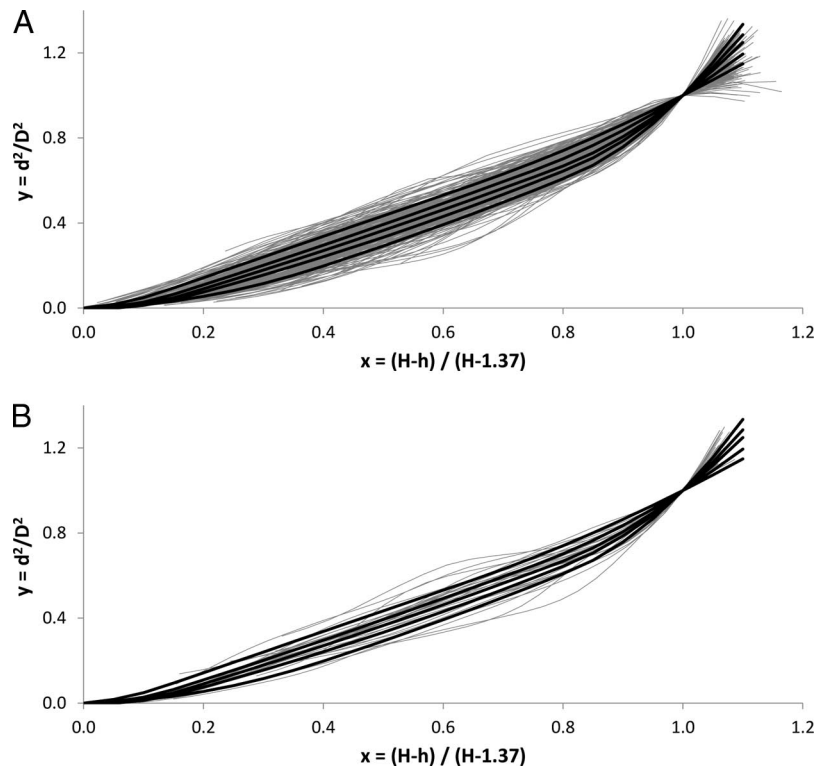


Figure 1. Graphs of observed tree diameters (gray) from (A) the validation data and (B) the independent data. The taper curves (black) are generated from the fit data by quantile regressions based on five quantiles (0.1, 0.3, 0.5, 0.7, and 0.9).

The objective of this study was to evaluate different methods, based on fixed-effects, mixed-effects, and quantile regression models, for calibrating a taper equation when an upper-stem diameter measurement is available.

Data

Data used in this study were from a loblolly pine (*Pinus taeda* L.) plantation at the Hill Farm Research Station, Homer, Louisiana. Site index (base age 25 years) for the study area ranged from 18.6 to 23.2 m. Diameter measurements were taken from trees felled in a thinning at age 21. The data consisted of 272 trees, with dbh ranging from 12.1 to 48.9 cm and total height from 9.1 to 23.4 m. There was a total of 6,385 outside-bark diameter measurements, taken at 64-cm intervals starting from the stump to the tree tip.

The data were randomly divided into a fit data set and a validation data set, each containing 136 trees. Parameters of the taper equations were estimated from the fit data. The predictions from the validation data were used to compute evaluation statistics for the different methods.

Further evaluation was performed on an independent data set, collected from a 29-year-old loblolly pine plantation, also at the Hill Farm Research Station, Homer, Louisiana. A total of 807 diameter measurements were taken every 64-cm intervals from 29 felled trees, which ranged from 16.8 to 33.8 cm in dbh, and 16.1 to 24.0 m in total height. Figure 1 shows diameter measurements along the tree bole for the validation and independent data sets.

Methods

Cao et al. (1980) evaluated six taper equations and concluded that the Max and Burkhart (1976) taper equation was superior in predicting tree taper. This segmented taper equation consists of

three different quadratic functions joined at two join points; therefore, it is flexible enough to describe complicated tree profiles. In this paper, the Max and Burkhart (1976) taper equation was constrained to pass through dbh. The modified form of this taper equation is as follows

$$\hat{y}(x_{ij}) = b_1^* x_{ij} + b_2 x_{ij}^2 + b_3 I_1(x_{ij} - a_1)^2 + b_4 I_2(x_{ij} - a_2)^2 \quad (1)$$

where

$\hat{y}(x_{ij})$ = predicted value of y for a given value of x_{ij}

$y_{ij} = y(x_{ij}) = d_{ij}^2/D_i^2$

D_i = dbh in cm of tree i , ($i = 1, 2, \dots, N$)

N = number of trees in the sample

d_{ij} = the j th upper-stem diameter outside bark in cm measured at height h_{ij} on

tree i ($j = 1, 2, \dots, n_i$)

n_i = number of diameter measurements for tree i

$x_{ij} = (H_i - h_{ij})/(H_i - 1.37)$ = relative height from the tree tip of the j th measurement on tree i

H_i = total height of tree i

a_1 and a_2 = join points to be estimated from the data

$I_k = 1$ if $x_{ij} > a_k$, and 0 otherwise, $k = 1, 2$

b_m 's = regression coefficients, $m = 2, 3, 4$

Note that b_1^* is not a parameter but is computed from the following constraint

$$b_1^* = 1 - b_2 - b_3 I_1'(1 - a_1)^2 - b_4 I_2'(1 - a_2)^2 \quad (2)$$

where $I_k' = 1$ if $a_k < 1$, and 0 otherwise, $k = 1, 2$. This constraint ensures that $\hat{y}(1) = 1$, or

$d_{ij} = D_i$, when $h_{ij} = 1.37$ m.

Table 1. Estimates of parameters of the fixed- and mixed-effects regression models and quantile regressions at five quantiles (τ).

Type	b_2	b_3	b_4	a_1	a_2	σ^2	σ_{u3}^2	σ_{u4}^2	σ_{u3u4}
Fixed-effects model	1.2816	-1.1326	2.3618	0.2716	0.7816	0.00223			
Mixed-effects model	1.5638	-1.5223	1.7832	0.2595	0.7155	0.00030	0.2156	2.9994	-0.6922
Quantile regression									
$\tau = 0.1$	1.1024	-0.8330	4.3611	0.3882	0.8117				
$\tau = 0.3$	2.6240	-2.3291	3.2865	0.1687	0.8093				
$\tau = 0.5$	2.8523	-2.6957	2.6064	0.1681	0.7899				
$\tau = 0.7$	2.9053	-2.7813	1.4020	0.1552	0.7542				
$\tau = 0.9$	2.7687	-2.7768	0.6482	0.1367	0.6414				

Fixed-Effects Model

Cao (2009) proposed a calibration procedure such that the taper curve passed through dbh and an upper-stem diameter. He obtained best results when the diameter for calibration (d_{i0}) was measured at $x_{i0} = 0.5$. This relative height of 0.5 makes sense because it is the midpoint between the tree tip and breast height, where the relative squared diameters (y_{ij}) are constrained to be 0 and 1, respectively.

The calibrated taper equation (Cao and Wang 2011) is obtained by modifying the least squares parameter, b_2 , as follows

$$y^*(x_{ij}) = b_1^*x_{ij} + b_2^*x_{ij}^2 + b_3I_1(x_{ij} - a_1)^2 + b_4I_2(x_{ij} - a_2)^2 \quad (3)$$

where

$$b_2^* = 2 - 4y_{i0} - b_3[2(1 - a_1)^2 - J_1(1 - 2a_1)^2] - b_4[2(1 - a_2)^2 - J_2(1 - 2a_2)^2],$$

$$b_1^* = 1 - b_2^* - b_3(1 - a_1)^2 - b_4I_2'(1 - a_2)^2,$$

$$y_{i0} = y(x_{i0}) = d_{i0}^2/D_i^2,$$

and $J_k = 1$ if $x_{i0} > a_k$, and 0 otherwise, $k = 1, 2$.

Mixed-Effects Model

In a mixed-effects model for Equation 1, all parameters are expressed as fixed-effects parameters (common to all trees), and some parameters also have additional random components (specific to individual trees). Various combinations of the parameters (b_2 , b_3 , b_4 , a_1 , and a_2) are candidates for random parameters. Equation 1 can be expressed in matrix form as follows

$$\mathbf{y}_i = \mathbf{f}(\mathbf{b}, \mathbf{u}_i, \mathbf{x}_i) + \boldsymbol{\varepsilon}_i \quad (4)$$

where $\mathbf{y}_i = [y_{i1}, y_{i2}, \dots, y_{im}]^T$, $\mathbf{x}_i = [x_{i1}, x_{i2}, \dots, x_{im}]^T$, $\boldsymbol{\varepsilon}_i = [\varepsilon_{i1}, \varepsilon_{i2}, \dots, \varepsilon_{im}]^T$ and \mathbf{b} and \mathbf{u}_i are column vectors of fixed- and random-effects parameters, respectively. The $\boldsymbol{\varepsilon}_i$ and \mathbf{u}_i are assumed to follow the multivariate normal distributions with mean 0 and variance \mathbf{R} and \mathbf{D} , respectively. Here, \mathbf{R} is a diagonal matrix because the $\boldsymbol{\varepsilon}_i$'s are assumed to be independent, having variance σ^2 . Matrix \mathbf{D} is not diagonal because the \mathbf{u}_i 's are assumed to be correlated to one another. SAS procedure NLMIXED (SAS Institute, Inc. 2011) were used to obtain parameters of Equation 4.

The random parameters \mathbf{u}_i for tree i can be computed by use of the first-order Taylor series expansion (Meng and Huang 2009)

$$\hat{\mathbf{u}}_i^{k+1} = \hat{\mathbf{D}}\mathbf{Z}_i^T(\mathbf{Z}_i\hat{\mathbf{D}}\mathbf{Z}_i^T + \hat{\mathbf{R}})^{-1}[\mathbf{y}_i - \mathbf{f}(\hat{\mathbf{b}}, \hat{\mathbf{u}}_i^k, \mathbf{x}_i) + \mathbf{Z}_i\hat{\mathbf{u}}_i^k], \quad (5)$$

where

$\hat{\mathbf{u}}_i^k$ is estimate of the random parameters for tree i at the k th iteration.

$\hat{\mathbf{D}}$ is estimate of \mathbf{D} , the variance-covariance matrix for \mathbf{u}_i .

Table 2. Evaluation statistics^a computed for the validation and independent data for the uncalibrated and four calibrated models: fixed-effects, mixed-effects, and quantile regression taper models.

Model	MD	RMSE
Validation data		
Uncalibrated model	-0.065	1.158
Calibrated: Fixed-effects model	-0.085	0.834
Mixed-effects model	-0.034 ^b	0.786
5 quantile regressions ^c	-0.111	0.777
3 quantile regressions ^d	-0.089	0.810
Independent data		
Uncalibrated model	-0.218	1.336
Calibrated: Fixed-effects model	0.089	0.941
Mixed-effects model	0.105	0.904
5 quantile regressions	0.033	0.937
3 quantile regressions	0.048	0.955

^a MD = mean difference, $RMSE = [\sum_{i=1}^N \sum_{j=1}^{n_i} (d_{ij} - \hat{d}_{ij})^2] / \sum_{i=1}^N n_i$, where d_{ij} and \hat{d}_{ij} are observed and predicted upper-stem diameter, respectively.

^b Underlined number for each evaluation statistic denotes the best statistic for that data set.

^c Quantile regressions based on five quantiles (0.1, 0.3, 0.5, 0.7, and 0.9).

^d Quantile regressions based on three quantiles (0.1, 0.5, and 0.9).

$$\mathbf{Z}_i = \left. \frac{\partial \mathbf{f}(\mathbf{b}, \mathbf{u}_i, \mathbf{x}_i)}{\partial \mathbf{u}_i} \right|_{\hat{\mathbf{b}}, \hat{\mathbf{u}}_i},$$

$\hat{\mathbf{R}}$ is estimate of \mathbf{R} , the variance-covariance matrix for $\boldsymbol{\varepsilon}_i$

y_i is the $m \times 1$ vector of observed diameters

m is number of measurements used in localizing the diameter growth model ($m = 1$ in this case)

An iterative procedure was used to estimate \mathbf{u}_i , whose starting value was set at zero ($\hat{\mathbf{u}}_i^0 = \mathbf{0}$). Equation 5 was used to repeatedly update the value for $\hat{\mathbf{u}}_i$ until the absolute difference between two successive iterations was smaller than a predetermined tolerance limit. The result was an approximation of the empirical best linear unbiased predictor (EBLUP) for random effects.

Quantile Regression

The same form in Equation 1 was used to predict the τ th taper quantile

$$\hat{y}_\tau(x_{ij}) = b_{1\tau}^*x_{ij} + b_{2\tau}^*x_{ij}^2 + b_{3\tau}I_1(x_{ij} - a_{1\tau})^2 + b_{4\tau}I_2(x_{ij} - a_{2\tau})^2 \quad (6)$$

where $\hat{y}_\tau(x_{ij})$ is predicted value of the τ th quantile of y given x_{ij} , and $b_{1\tau}^*$ is computed in a manner similar to Equation 2.

In contrast to the mean regression technique, which employs the least-squares procedure, parameters from the quantile regression are obtained by minimizing

$$S = \sum_{y(x_{ij}) \in A} \tau[y(x_{ij}) - \hat{y}_\tau(x_{ij})] + \sum_{y(x_{ij}) \notin A} (1 - \tau)[y(x_{ij}) - \hat{y}_\tau(x_{ij})] \quad (7)$$

where A is the set of $y(x_{ij}) \geq \hat{y}_\tau(x_{ij})$ when $x_{ij} \leq 1$, and $y(x_{ij}) < \hat{y}_\tau(x_{ij})$ when $x_{ij} > 1$. The curve for a lower quantile is situated lower than the one for a higher quantile when $x_{ij} \leq 1$, and the opposite is true for $x_{ij} > 1$ because the two curves cross at the point $x_{ij} = 1$.

A set of q quantile regressions was developed for each of the two groups, by use of SAS procedure NLP (SAS Institute, Inc. 2010). For each upper-stem diameter measurement in the validation data, the goal was to identify the two closest quantile regression curves.

If the upper-stem diameter measurement was encompassed by the m th and $(m+1)$ st quantile regressions, i.e., $\hat{y}_m(x_{i0}) \leq y_{i0} \leq \hat{y}_{m+1}(x_{i0})$, a modified taper curve that passed through this point was generated by interpolation

$$\tilde{y}_m(x_{ij}) = \alpha \hat{y}_m(x_{ij}) + (1 - \alpha) \hat{y}_{m+1}(x_{ij}), \quad (8)$$

where $\alpha = \frac{\hat{y}_{m+1}(x_{i0}) - y_{i0}}{\hat{y}_{m+1}(x_{i0}) - \hat{y}_m(x_{i0})}$ is the interpolation ratio.

If the upper-stem diameter was above the highest (q th) quantile regression curve, Equation 8 was still appropriate, with \hat{y}_m defined as \hat{y}_{q-1} and \hat{y}_{m+1} as \hat{y}_q . The method became extrapolation in nature.

Table 3. Similarities between site index curves and taper curves.

Action	Site index curves	Taper curves
Given	Stand height at some age	Tree diameter at some height
Select	Two closest curves	Two closest curves
Predict	Stand height at a given age	Tree diameter at a given height
Use	Interpolation method	Interpolation method

Similarly, if the upper-stem diameter was below the lowest (first) quantile regression curve, \hat{y}_m and \hat{y}_{m+1} in Equation 8 were defined as \hat{y}_1 and \hat{y}_2 , respectively. Furthermore, to avoid negative predictions in this case, the calibrated prediction was the maximum of $\tilde{y}(x_{ij})$ and $y_{i0} \hat{y}_1(x_{ij}) / \hat{y}_1(x_{i0})$; the latter was obtained by interpolating between $\hat{y}_1(x_{ij})$ and $y = 0$.

Evaluation

Parameters of the taper equations estimated from the fit data were used to predict for the validation and independent data sets. The performance of all models was evaluated by use of two evaluation statistics: mean difference (MD) between observed and predicted diameters and root mean squared error (RMSE), defined by $RMSE = [\sum_{i=1}^N \sum_{j=1}^{n_i} (d_{ij} - \hat{d}_{ij})^2] / \sum_{i=1}^N n_i$, where \hat{d}_{ij} is predicted upper-stem diameter at the j th measurement on the i th tree.

Results and Discussion

Among various combinations of the parameters, the combination of random parameters in b_3 and b_4 resulted in lowest values of Akaike's information criterion and Bayesian information criterion. The final mixed-effects model was

$$\hat{y}(x_{ij}) = b_1^* x_{ij} + b_2 x_{ij}^2 + (b_3 + u_3) I_1(x_{ij} - a_1)^2 + (b_4 + u_4) I_2(x_{ij} - a_2)^2 \quad (9)$$

where u_3 and u_4 are random effects.

Estimates of parameters of the fixed- and mixed-effects regression models and five quantile regressions were obtained from the fit

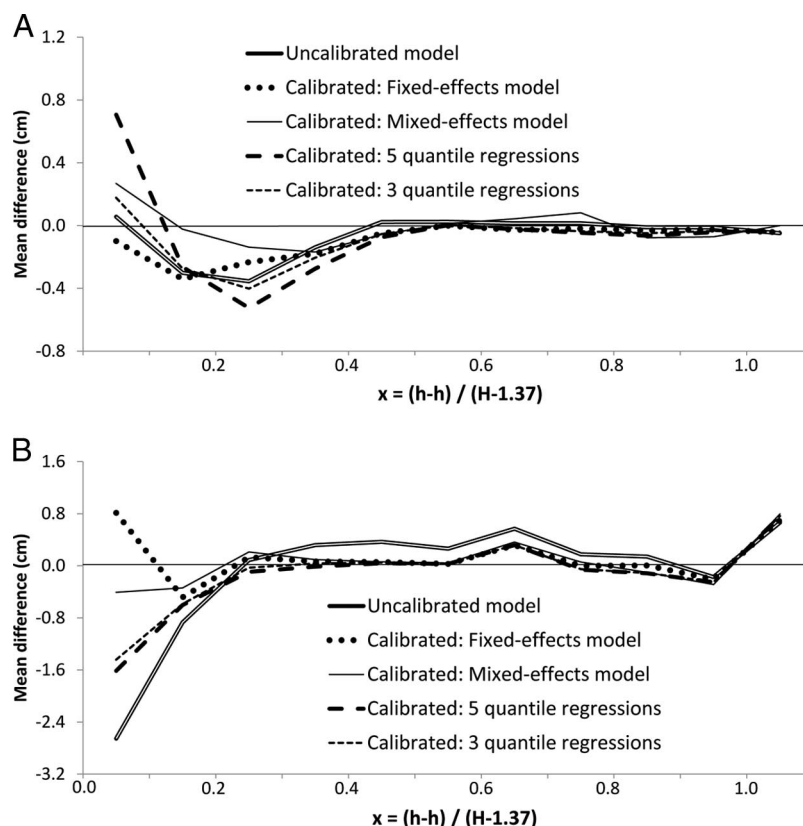


Figure 2. MD computed along the tree bole for various methods from (A) the validation data and (B) the independent data.

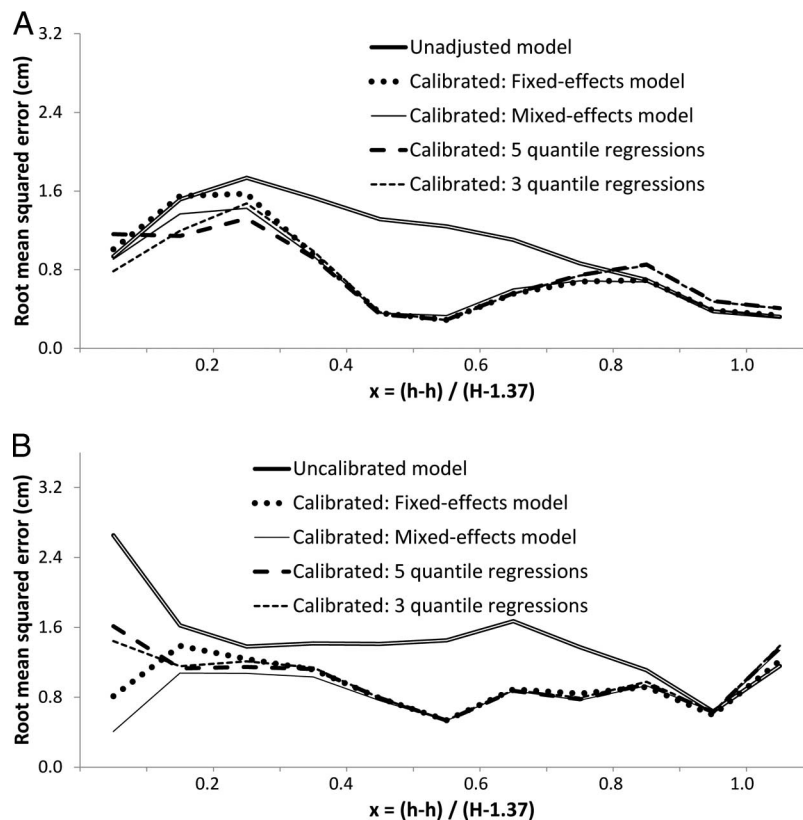


Figure 3. RMSE computed along the tree bole for various methods from (A) the validation data and (B) the independent data.

data (Table 1). The set of five quantile regressions, with τ varying from 0.1 to 0.9, is shown with observed diameter measurements from the validation data (Figure 1A) and independent data (Figure 1B).

Table 2 presents evaluation statistics based on the validation and independent data for the uncalibrated taper model and the calibrated model based on fixed and mixed effects. The same statistics are shown for systems of quantile regressions based on five quantiles (0.1, 0.3, 0.5, 0.7, and 0.9) and three quantiles (0.1, 0.5, and 0.9).

Uncalibrated Model versus Calibrated Model

The taper equation calibrated for d_{i0} performed better than did the uncalibrated model. For the validation data, the calibrated model based on mixed effects produced a 48% reduction in MD and 32% reduction in RMSE, as compared to the uncalibrated equation. The reduction was similar for the independent data: 52% in MD and 32% in RMSE. These results were similar to findings obtained by Cao (2009) and Cao and Wang (2011).

Note that the uncalibrated model was itself constrained to pass through dbh. Cao (2009) showed that the taper model performed much worse without this constraint.

For the validation data, the MD values along the tree bole appeared similar for both calibrated and uncalibrated models (Figure 2A). Differences between observed and predicted taper were more pronounced at the top half of the stem than for the bottom half. This might be due to variations of tree taper within the crown. For the independent data (Figure 2B), the uncalibrated model, on average, produced negative MD values (underprediction) for $x < 0.3$ and positive MD values (overprediction) otherwise. The constraint

for d_{i0} clearly helped lower the RMSE values (Figure 3A and B) of the calibrated models as compared to the uncalibrated model for both validation and independent data sets.

Quantile Regression Models

Table 2 shows that the five-quantile system was better than the three-quantile system in terms of RMSE (0.777 versus 0.810 for the validation data and 0.937 versus 0.955 for the independent data). Results based on MD values were mixed: the five-quantile system was slightly more biased (-0.111 versus -0.089) for the validation data but less biased (0.033 versus 0.048) for the independent data.

The use of quantile regression to predict tree taper in this study resembles the use of site index curves to predict stand height (average height of the dominants and codominants). Table 3 summarizes the similarities between the two approaches.

The goal in both approaches is to either find a curve that passes through the given observation or interpolates from the two closest curves to that observation.

Mixed-Effects Models versus Quantile Regression Models

The mixed model produced the best MD value for the validation data and the best RMSE value for the independent data. Conversely, the five-quantile system yielded the best RMSE value for the validation data and the best MD value for the independent data. Figures 2 and 3 reveal that the performances of these two models were similar along the tree bole.

Calibration Methods

The calibration based on fixed-effects models changed only the value for parameter b_2 , while keeping the remaining parameters

intact. On the other hand, the mixed model approach modified values for both b_3 and b_4 , resulting in a more flexible calibrated curve. This calibration procedure for the mixed model resulted in a separate taper model for each tree. Note that all parameters of the mixed model could have random components, but the result would be overfitting for these taper data. If the data contained trees sampled in multiple plots (or stands), a hierarchical model with grouping of plots and trees within plots was possible.

The quantile regression approach should be the most flexible because all parameters of the regression based on various quantiles are different (Table 1), producing taper curves of different shapes (Figure 1). However, the quantile regression approach treated each individual observation as independent of one another, ignoring the fact that multiple measurements were made from each tree. The quantile regression method relied on the assumption that observations from the same tree keep their relative positions along the tree bole, i.e., all diameter measurements from one tree should have similar quantiles. This assumption was violated when crossover occurred for some trees. Furthermore, the constraint that the quantile regression curves cross at breast height results in an assumption that trees having larger diameters above breast height should have smaller diameters below breast height and vice versa. The performance of quantile regression in modeling tree taper depends largely on how well these two assumptions held.

Karlsson (2008) introduced a weighted quantile regression method in which observations in each subject (or tree in this study) were weighted by the amount of variation of $y(x_{ij}) - \hat{y}(x_{ij})$. Different weights suggested by Karlsson (2008) were tried in this study, but convergence was not attained for some quantiles. The weighted quantile regression method was not successfully implemented here. The reason might be because the taper function was too complicated, containing three submodels and a constraint at breast height.

Summary and Conclusions

Taper data from felled loblolly pine trees showed that upper-stem diameter measurements improved the accuracy and precision of diameter predictions along the tree bole. The mixed-effects model and the quantile regression system based on five quantiles were the overall best methods, based on the two evaluation statistics. However, the values of these statistics were similar for the four calibration methods that any method could do a good job of producing better diameter predictions when a diameter measurement at the middle of the bole was available.

This application of quantile regression to modeling tree taper appeared successful. The various shapes of curves from different quantiles allowed the flexibility of taper curves to accommodate a diameter observed at a certain height. A system involving five quantile regressions seemed to perform slightly better than one with only three quantile regressions.

Although the diameter measured at the midpoint between the tree tip and breast height was used, based on Cao's (2009) recommendation, the method can be applied to diameters measured at anywhere on the tree bole. The method presented in this paper should also be applicable to data sets of loblolly pine or of other conifer species as well.

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Appendix: SAS Program for Quantile Regression

```
data one;
input dbh th dob h;
y = (dob/dbh)**2;
z = (th - h)/(th - 1.37);
proc nlp data = one tech = nmsimp;
min f; * f is the objective function to be minimized;
decvar b2 = 1.1, b3 = -0.8, b4 = 4.2, a1 = 0.4, a2 = 0.8;
tau = 0.1;
b1 = 1 - b2 - (1>a1)*b3*(1 - a1)**2 - (1>a2)*b4*(1 -
a2)**2;
yhat = b1*x + b2*x**2 + (x>a1)*b3*(x - a1)**2 +
(x>a2)*b4*(x - a2)**2;
if y ge yhat
then f = tau*(y - yhat);
else f = (1 - tau)*(yhat - y);
title '10% Quantile Regression for Tree Taper'.
```