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Predictions of individual-tree and whole-stand attributes for loblolly pine plantations

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Abstract

In this study, a new approach was developed for constraining an individual-tree model such that it provides reasonable tree- and stand-level predictions of survival and growth. Data from 100 plots randomly selected from the Southwide Seed Source Study of loblolly pine (*Pinus taeda* L.) were used to fit the equations. Another 100 plots, randomly selected from the remaining plots, were used for validation. Results showed that the new approach produced results that were comparable to those from the same tree model constrained with number of trees and basal area in each diameter-class. Both of these two constrained tree models were slightly better than the unconstrained tree model in predicting tree and stand attributes. All of the above methods, however, were outperformed by the disaggregative model, in which outputs from the individual-tree model were adjusted with stand growth predictions from a whole-stand model. The disaggregative approach provided the best predictions of tree- and stand-level survival and growth. It bridges the gaps between individual-tree models and whole-stand models and should be seriously considered as an alternative to the constrained modeling approach.

Keywords: Constrained models; Disaggregative models; Tree diameter; Survival probability; *Pinus taeda*

1. Introduction

Models that simulate survival and growth of individual trees in a forest stand are called individual-tree models. Such models provide the most detailed information at tree level (Belcher et al., 1982; Burkhart et al., 1987; Zhang et al., 1997; Cao et al., 2002), as compared to size-class distribution models at diameter-class level (Clutter and Bennett, 1965; Lenhart and Clutter, 1971; Smalley and Bailey, 1974; Matney and Sullivan, 1982; Hafley et al., 1982) or whole-stand models at stand level (Murphy and Farrar, 1982; Lloyd and Harms, 1986; Pienaar and Harrison, 1989; Ochi and Cao, 2003). The higher resolution of the individual-tree models is also typically accompanied by problems of error accumulation when computing stand summaries. The resulting stand predictions frequently suffer from lack of accuracy and precision, prompting some forest managers to opt for whole-stand models if stand-level outputs are of primary concern.

There are two approaches used by researchers to overcome this drawback. The first approach involves adjusting diameter predictions from individual-tree models such that the resulting stand basal area equals that predicted by a whole-stand model (Harrison and Daniels, 1988; Dhote, 1994; Moore et al., 1994; McTague and Stansfield, 1994, 1995; Ritchie and Hann, 1997a; Qin and Cao, 2006). Ritchie and Hann (1997b) called this the disaggregative approach, in which information obtained from an individual-tree model is used to disaggregate stand growth (which is predicted by a whole-stand model) among trees in the tree list.

In the second approach, Zhang et al. (1997) used the multi-response parameter estimation developed by Bates and Watts (1987, 1988) to constrain a tree survival and diameter growth model to optimize for both tree and diameter-class levels. They found that the constrained model produced similar individual-tree diameter projections to the unconstrained model, but better predictions for basal area of each diameter-class and for total stand basal area. The constrained model did not improve survival predictions because of the occurrence of low mortality in their data.

The objectives of this study were to: (1) develop a third approach, in which an individual-tree model was constrained by

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optimizing for both tree and stand levels and (2) evaluate all three approaches against the unconstrained tree model in their abilities to predict both individual-tree and whole-stand attributes.

2. Data

Data used in this study were from loblolly pine (*Pinus taeda* L.) plantations in the Southwide Seed Source Study, which included 15 seed sources planted at 13 locations across 10 southern states (Wells and Wakeley, 1966). Each plot of 0.0164 ha consisted of 49 trees planted at a 1.8 m × 1.8 m spacing. Diameters of these trees were measured at ages 10, 15, 20 and 25 years.

The fitting data set, consisting of 100 plots, was randomly selected from the original data. Furthermore, only one 5-year growth period was randomly chosen from each plot. This data set was used for fitting the models. The validation data sets were formed by randomly selecting one-hundred 5-year growing intervals, one from each plot, from the remaining plots. Table 1 shows summary statistics for the fit and validation data sets.

3. Methods

3.1. Unconstrained individual-tree model

Preliminary analysis revealed that the following individual-tree model was most suitable to predict tree survival and diameter growth from the fit data set.

$$\hat{p}_{ij} = [1 + \exp(\alpha_1 + \alpha_2 N_{1,i} + \alpha_3 B_{1,i} + \alpha_4 d_{1,ij})]^{-1} \quad (1a)$$

$$\hat{d}_{2,ij} = d_{1,ij} + \beta_1 \left(\frac{A_2}{A_1}\right)^{\beta_2} H_{1,i}^{\beta_3} B_{1,i}^{\beta_4} d_{1,ij}^{\beta_5} \quad (1b)$$

where $\hat{p}_{ij}$ is the predicted probability that tree j in plot i is alive at age A_2, given that it was alive at age A_1; $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m_i$, n the number of plots in the fit data set, m_i the number of trees in plot i, $N_{1,i}$ the number of trees per ha in plot i at age A_1, $B_{1,i}$ the stand basal area (m^2/ha) of plot i at age A_1, $H_{1,i}$ the average dominant and codominant height (m) in plot i at age A_1, $d_{1,ij}$ the diameters (cm) of tree j in plot i at age A_1, $\hat{d}_{2,ij}$ the predicted diameters (cm) of tree j in plot i at age A_2 and α's and β's are the regression parameters.

3.2. Approach 1: linking the individual-tree and whole-stand models using disaggregation

In this approach, outputs from the individual-tree model were adjusted such that their sums matched outputs from the whole-stand model. The following whole-stand model was used to predict number of trees per ha ($\hat{N}_{2,i}$) and basal area per ha ($\hat{B}_{2,i}$) at age A_2 (standard error in parentheses is put immediately after each parameter

Table 1
Summary statistics for stand- and tree-level attributes, by data type and growth period

Attribute	Growth period (years)		
	Age (10–15 years)	Age (15–20 years)	Age (20–25 years)
Fitting data set			
Number of growth periods	34	33	33
Beginning (age A_1)			
Number of trees/ha	1649 (946)[a]	1413 (823)	1146 (546)
Basal area (m^2/ha)	17.64 (9.21)	27.30 (11.08)	30.23 (10.33)
Number of trees/plot	27 (16)	23 (13)	19 (9)
Tree diameter (cm)	11.2 (3.3)	15.2 (3.9)	17.8 (4.5)
End (age A_2)			
Number of trees/ha	1425 (905)	1011 (468)	987 (568)
Basal area (m^2/ha)	25.60 (12.36)	28.70 (9.99)	31.80 (13.62)
Number of trees/plot	23 (15)	17 (8)	16 (9)
Tree diameter (cm)	14.5 (4.2)	18.4 (4.9)	19.6 (5.1)
Validation data set			
Number of growth periods	37	34	29
Beginning (age A_1)			
Number of trees/ha	1852 (821)	1308 (634)	1025 (490)
Basal area (m^2/ha)	19.70 (8.70)	27.36 (10.62)	29.31 (11.63)
Number of trees/plot	30 (13)	21 (10)	17 (8)
Tree diameter (cm)	11.2 (3.1)	15.8 (4.1)	18.5 (4.6)
End (age A_2)			
Number of trees/ha	1623 (763)	1016 (463)	928 (468)
Basal area (m^2/ha)	29.42 (10.48)	31.08 (11.99)	33.63 (13.84)
Number of trees/plot	27 (13)	17 (8)	15 (8)
Tree diameter (cm)	14.7 (4.0)	19.2 (4.7)	20.8 (5.5)

[a] For each pair, the first number is the mean and the second (in parentheses) is the standard deviation.

estimate).

$$\hat{N}_{2,i} = \frac{N_{1,i}}{\left\{\begin{array}{c} 1+\exp(16.3197_{(3.1577)} - 42.4204_{(8.6584)}\mathrm{RS}_{1,i} \\ -0.7466_{(0.1405)}H_{1,i} - 0.0269_{(0.0060)}N_{1,i}/A_1 \\ +50.2622_{(16.9155)}/A_1) \end{array}\right\}}$$
$$R^2 = 0.88;\ \ \mathrm{RMSE} = 245\ \mathrm{trees/ha} \tag{2a}$$

$$\hat{B}_{2,i} = B_{1,i}\{1+\exp(-3.3259_{(0.7531)} - 0.7800_{(0.1688)}B_{1,i}/A_1 + 41.0393_{(7.4586)}/A_1)\}$$
$$R^2 = 0.65;\ \ \mathrm{RMSE} = 7.35\ \mathrm{m^2/ha} \tag{2b}$$

where $\mathrm{RS}_{1,i} = (10{,}000/N_{1,i})^{0.5}/H_{1,i}$ is the relative spacing of plot i at age A_1.

Tree survival probability was adjusted with a simple power function.

$$\tilde{p}_{ij} = \hat{p}_{ij}^{r_p} \tag{3a}$$

where $\hat{p}_{ij}$ is the unadjusted survival probability for tree j in plot i, computed from Eq. (1a), $\tilde{p}_{ij}$ the adjusted survival probability for tree j in plot i, r_p the adjusting coefficient for number of trees, to be iteratively solved such that $\sum_{j=1}^{m_i} \tilde{p}_{ij} = s\hat{N}_{2,i}$, $\hat{N}_{2,i}$ the number of trees per ha in plot i at age A_2, predicted from Eq. (2a) and s is the plot size in ha.

Tree diameter growth was adjusted by use of the proportional growth method (Qin and Cao, 2006).

$$\tilde{d}_{2,ij}^2 = d_{1,ij}^2 + r_d(\hat{d}_{2,ij}^2 - d_{1,ij}^2) \tag{3b}$$

where $r_d = \left\{(s\hat{B}_{2,i}/c) - \sum_j(\tilde{p}_{ij}d_{1,ij}^2)\right\}/\sum_{j=1}^{m_i}(\tilde{p}_{ij}[\hat{d}_{2,ij}^2 - d_{1,ij}^2])$ is the adjusting coefficient for diameter such that $c\sum_{j=1}^{m_i}\tilde{p}_{ij}\tilde{d}_{2,ij}^2 = s\hat{B}_{2,i}$, $\hat{d}_{2,ij}$ the unadjusted diameter at age A_2 for tree j in plot i, predicted from Eq. (1b), $\tilde{d}_{2,ij}$ the adjusted diameter at age A_2 for tree j in plot i, $\hat{B}_{2,i}$ the stand basal area (m^2/ha) of plot i at age A_2, predicted from Eq. (2b) and $c = \pi/40{,}000$ is a constant to convert diameters in cm to basal area in m^2.

3.3. Approach 2: constraining the individual-tree model with diameter-class attributes

This approach was introduced by Zhang et al. (1997), who used the multi-response parameter estimation developed by Bates and Watts (1987, 1988) to constrain an individual-tree model to optimize for both tree and diameter-class levels. Tree survival and diameter growth were predicted from the following systems of equations.

$$\begin{cases} \hat{p}_{ij} = [1+\exp(\alpha_1 + \alpha_2 N_{1,i} + \alpha_3 B_{1,i} + \alpha_4 d_{1,ij})]^{-1} \\ \hat{n}_{2,ik} = \sum_{j=1}^{n_{1,ik}} \hat{p}_{ij} \end{cases} \tag{4a}$$

$$\begin{cases} \hat{d}_{2,ij} = d_{1,ij} + \beta_1\left(\frac{A_2}{A_1}\right)^{\beta_2} H_{1,i}^{\beta_3} B_{1,i}^{\beta_4} d_{1,ij}^{\beta_5} \\ \hat{b}_{2,ik} = c\sum_{j=1}^{n_{1,ik}} \hat{p}_{ij}\hat{d}_{2,ij}^2 \end{cases} \tag{4b}$$

where $n_{1,ik}$ and $b_{1,ik}$ are the number of trees and basal area of the kth diameter-class in plot i at age A_1 and $\hat{n}_{2,ik}$ and $\hat{b}_{2,ik}$ are the predicted number of trees and basal area of the kth diameter-class in plot i at age A_2.

Parameters from the system of Eq. (4a) were estimated by use of a program written in SAS/IML (SAS Institute Inc., 2004), and parameters from system (4b) were subsequently estimated from another program, also written in SAS/IML.

3.4. Approach 3: constraining the individual-tree model with stand attributes

Modified from approach 2, this new approach substituted stand attributes for diameter-class attributes, so that the resulting constrained tree model was optimized for both tree and stand levels. Tree survival and diameter growth were predicted from the following systems of equations.

$$\begin{cases} \hat{p}_{ij} = [1+\exp(\alpha_1 + \alpha_2 N_{1,i} + \alpha_3 B_{1,i} + \alpha_4 d_{1,ij})]^{-1} \\ \hat{N}_{2,i} = \sum_{j=1}^{m_i} \hat{p}_{ij}/s \end{cases} \tag{5a}$$

$$\begin{cases} \hat{d}_{2,ij} = d_{1,ij} + \beta_1\left(\frac{A_2}{A_1}\right)^{\beta_2} H_{1,i}^{\beta_3} B_{1,i}^{\beta_4} d_{1,ij}^{\beta_5} \\ \hat{B}_{2,i} = \left(\frac{c}{s}\right)\sum_{j=1}^{m_i} \hat{p}_{ij}\hat{d}_{2,ij}^2 \end{cases} \tag{5b}$$

4. Evaluations

The validation data set was used to evaluate the different methods, based on their abilities to predict tree survival and diameter. Evaluation statistics for predicting tree survival probability include MD which is the mean difference, where differences are deviations between observed and predicted survival probabilities, MAD the mean absolute difference and $-2\ln(L) = -2\left\{\sum_i\sum_j p_{ij}\ln(\hat{p}_{ij}) + \sum_i\sum_j(1-p_{ij})\ln(1-\hat{p}_{ij})\right\}$. For approach 1, $\hat{p}_{ij}$ in the above formula for $-2\ln(L)$ is replaced with $\tilde{p}_{ij}$.

The criteria for evaluating the methods in terms of tree diameter were mean difference, mean absolute difference, and fit index (FI). The latter expresses the relative amount of variation explained by the model (similar to R^2).

In addition to tree-level evaluation, the methods were evaluated based on their predictions of stand attributes such as number of trees and basal area per hectare. Again, the statistics MD, MAD and FI were used in the evaluation.

Table 2
Parameter estimates (and standard errors) of the unconstrained and constrained individual-tree models

Parameter	Unconstrained tree model	Diameter-class constrained tree model	Stand-level constrained tree model
Tree survival equation[a]			
α_1	1.3586 (0.3942)	3.2641 (0.2278)	2.5992 (0.2698)
α_2	−0.0010 (0.0001)	−0.0017 (0.0001)	−0.0014 (0.0001)
α_3	0.1042 (0.0098)	0.1836 (0.0056)	0.1860 (0.0079)
α_4	−0.2902 (0.0245)	−0.5474 (0.0188)	−0.5430 (0.0272)
Tree diameter growth equation[b]			
β_1	0.7168 (0.1697)	0.3775 (0.0717)	0.8199 (0.1520)
β_2	2.0192 (0.2664)	3.0735 (0.2138)	1.9270 (0.2071)
β_3	−1.0111 (0.0797)	−0.8910 (0.0646)	−0.9266 (0.0648)
β_4	−0.3166 (0.0181)	−0.2903 (0.0151)	−0.2832 (0.0154)
β_5	1.5117 (0.0535)	1.47996 (0.0421)	1.3806 (0.0449)

Variables are defined in text.
[a] $\hat{p}_{ij} = [1 + \exp(\alpha_1 + \alpha_2 N_{1,i} + \alpha_3 B_{1,i} + \alpha_4 d_{1,ij})]^{-1}$.
[b] $\hat{d}_{2,ij} = d_{1,ij} + \beta_1 \left(\frac{A_2}{A_1}\right)^{\beta_2} H_{1,i}^{\beta_3} B_{1,i}^{\beta_4} d_{1,ij}^{\beta_5}$.

5. Results and discussion

Table 2 presents the parameter estimates and their standard errors for the unconstrained and constrained individual-tree models obtained from the three methods. All parameters were significant at the 5% level.

Statistics in Table 3 show consistent results for both tree- and stand-level evaluations, based on 5-year growth intervals from the validation data set. The unconstrained tree model ranked last in all criteria, except −2 ln(L) for tree survival probability. The two methods used to constrain the tree model (approaches 2 and 3) produced very similar statistics in all categories. On the other hand, the disaggregative method (approach 1) outperformed the rest of the models in all evaluation statistics, except MD for tree diameter.

Long-term (10- and 15-year) growth predictions were also performed by conducting two and three consecutive 5-year projections for each plot in the validation data set. Even though the performance of the long-term projections deteriorated over time as expected, the ranking among the four methods remained unchanged.

5.1. Two methods for constraining the tree model

The two constrained tree models produced similar evaluation statistics in terms of tree survival probability

Table 3
Tree-level and stand-level evaluation statistics for four methods

Statistic[a]	Unconstrained tree model	Diameter-class constrained tree model	Stand-level constrained tree model	Disaggregative tree model
Tree level				
Tree survival probability				
MD	0.035	0.013	0.014	***−0.002***[b]
MAD	0.236	***0.199***	***0.199***	***0.199***
−2 ln(L)	1661	1684	1681	***1459***
Tree diameter				
MD	0.175	0.162	***0.022***	−0.124
MAD	0.966	0.953	0.954	***0.824***
FI	0.940	0.941	0.941	***0.954***
Stand level				
Stand density (trees/ha)				
MD	51	20	21	***−2***
MAD	186	173	172	***133***
FI	0.867	0.874	0.874	***0.903***
Stand basal area (m^2/ha)				
MD	2.104	0.819	0.305	***0.155***
MAD	4.356	3.799	3.572	***2.230***
FI	0.774	0.812	0.820	***0.913***

[a] MD, mean difference; MAD, mean absolute difference, $-2\ln(L) = -2\{\sum p_j \ln(p_j) + \sum(1-p_j)\ln(1-p_j)\}$, where p_j is predicted survival probability of tree j and fit index = FI $= 1 - (\sum(y_j - \hat{y}_j)^2 / \sum(y_j - \bar{y})^2)$, where y_j and $\hat{y}_j$ are the observed and predicted values of tree diameter, stand density or stand basal area and $\bar{y}$ is the mean of y.
[b] For each evaluation statistic, the bold italic number denotes the best among all models.

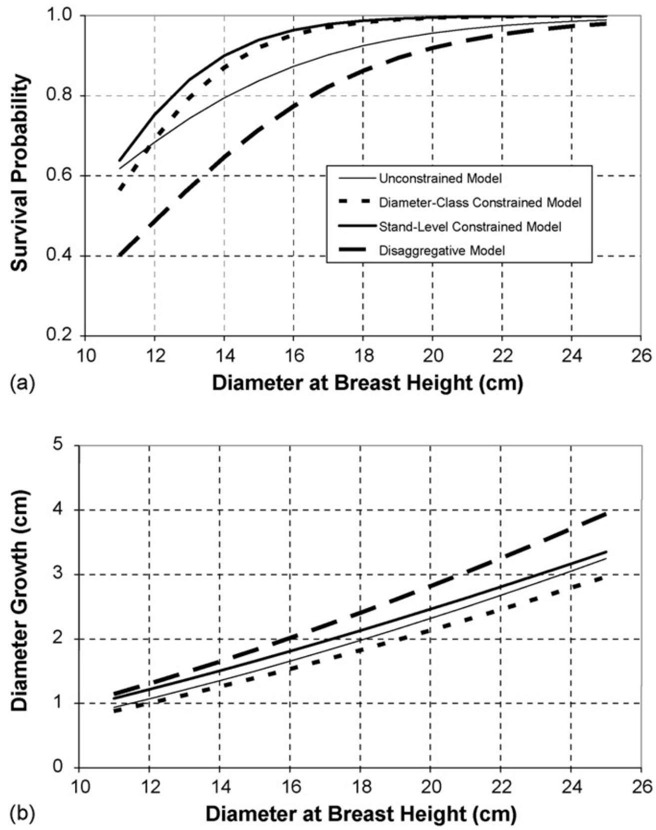

Fig. 1. Graphs of predicted tree survival probability (a) and diameter growth (b) vs. tree diameter from four methods for plot 732322 (A_1 = 20 years, A_2 = 25 years, H_1 = 16.5 m, B_1 = 21.5 m^2/ha and N_1 = 854 trees/ha).

and stand density predictions. This can be explained by examining a graph of predicted survival probability versus tree diameter for an example plot (Fig. 1a). Curves for these two constrained models are close to each other, indicating that their tree survival predictions were also similar.

The curve of diameter growth versus tree diameter appeared higher for the stand-level constrained model as compared to the diameter-class constrained model (Fig. 1b); this might contribute to a smaller tree diameter MD value for the stand-level constrained model. Overall, the two constrained models produced tree diameter predictions that were also comparable. The stand-level constrained model had an edge in predicting stand basal area, but the difference in evaluation statistics between the two constrained models was slight.

Results from this data set confirmed findings by Zhang et al. (1997) that similar individual-tree diameter projections were obtained for the constrained and unconstrained models, but the constrained models produced better predictions for stand basal area. Results for tree survival probability were mixed, but the constrained models did improve stand density projections. Zhang et al. (1997) did not find improvement by their constrained model in survival predictions, probably because of the occurrence of low mortality in their data.

5.2. Disaggregation method

The disaggregation method ranked first in all but one of the evaluation statistics. Compared to the unconstrained tree model, the disaggregation approach reduced the $-2\ln(L)$ value from 1661 to 1459 for tree survival probability, while increasing fit index values from 0.940 to 0.954 for tree diameter, from 0.867 to 0.903 for stand density and from 0.774 to 0.913 for stand basal area. Similar improvements were reported by Qin and Cao (2006) for short (4–7 years), medium (10–12 years) and long (15–17) projections. Qin and Cao (2006) showed that outputs from a tree model would be greatly improved when adjusted with observed stand attributes. As a result, they cautioned that the performance of the disaggregative tree model greatly depends on the quality of the stand attributes predicted from a whole-stand model.

The disaggregation approach also outscored the two constrained models in predicting both tree and stand attributes. Even with the addition of diameter-class or stand-level constraints, the two constrained models could not match the performance of the disaggregative model in providing for tree-level predictions and especially stand-level predictions.

6. Conclusions

A new approach was developed in this study for constraining an individual-tree model such that it provides reasonable tree- and stand-level predictions of survival and growth. This approach produced results that were comparable to those from the same tree model constrained with number of trees and basal area in each diameter-class. Both of these two constrained tree models did slightly better than the unconstrained tree model in predicting tree and stand attributes. Furthermore, users would not have to take extra steps when applying the constrained models.

The extra steps required in disaggregating stand growth (predicting from a whole-stand model) by use of information from the individual-tree model paid dividends in the case of this data set. The disaggregative approach provided the best predictions of tree- and stand-level survival and growth. It bridges the gaps between individual-tree models and whole-stand models and should be seriously considered as an alternative to the constrained modeling approach.

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References

Bates, D.M., Watts, D.G., 1987. A generalized Gauss–Newton procedure for multi-response parameter estimation. SIAM J. Sci. Stat. Comput. 8, 4–55.

Bates, D.M., Watts, D.G., 1988. Nonlinear Regression Analysis and its Application. Wiley, New York, 365 pp.

Belcher, D.M., Holdway, M.R., Brand, G.J., 1982. A description of STEMS—the stand and tree evaluation and modeling system. In: USDA For. Serv. Gen. Tech. Rep. NC-79, 18 pp.

Burkhart, H.E., Farrar, K.D., Amateis, R.L., Daniels, R.F., 1987. Simulation of individual-tree growth and stand development in loblolly pine plantations on cutover, site-prepared areas, vol. FWS-1-87. School of Forest and Wildlife Resources, VA. Polytech. Institute and State University, 47 pp.

Cao, Q.V., Li, S., McDill, M.E., 2002. Developing a system of annual tree growth equations for the loblolly pine—shortleaf pine type in Louisiana. Can. J. For. Res. 32, 2051–2059.

Clutter, J.L., Bennett, F.A., 1965. Diameter distribution in old-field slash pine plantations. In: Ga. For. Res. Counc. Rep., vol. 13, 9 pp.

Dhote, J.F., 1994. Hypotheses about competition for light and water in even-aged common beech (*Fagus silvatica* L.). For. Ecol. Manage. 69, 219–232.

Hafley, W.L., Smith, W.D., Buford, M.A., 1982. A new yield prediction model for unthinned loblolly pine plantations. In: N.C. State Univ. Tech. Rep., vol. 1, South. For. Res. Center, School of For. Res., 65 pp.

Harrison, W.C., Daniels, R.F., 1988. A new biomathematical model for growth and yield of loblolly pine plantations. In: Ek, A.R., Shifley, S.R., Burk, T.E. (Eds.), Forest Growth Modeling and Prediction, vol. 1. USDA For. Serv. Gen. Tech. Rep. NC-120. pp. 293–304.

Lenhart, J.D., Clutter, J.L., 1971. Cubic-foot yield tables for old-field loblolly pine plantations in the Georgia Piedmont. In: Ga. For. Res. Counc. Rep. No. 22. Ser. 3, 12 pp.

Lloyd, F.T., Harms, W.R., 1986. An individual stand growth model for mean plant size based on the rule of self-thinning. Ann. Bot. 57, 681–688.

Matney, T.G., Sullivan, A.D., 1982. Compatible stand and stock tables for thinned loblolly pine stands. For. Sci. 28, 161–171.

McTague, J.P., Stansfield, W.F., 1994. Stand and tree dynamics of uneven-aged ponderosa pine. For. Sci. 40, 289–302.

McTague, J.P., Stansfield, W.F., 1995. Stand, species, and tree dynamics of an uneven-aged, mixed conifer forest type. Can. J. For. Res. 25, 803–812.

Moore, J.A., Zhang, L., Newberry, J.D., 1994. Effects of intermediate silvicultural treatments on the distribution of within-stand growth. Can. J. For. Res. 24, 398–404.

Murphy, P.A., Farrar Jr., R.M., 1982. Interim models for based area and volume projection of uneven-aged loblolly-shortleaf pine stands. South. J. Appl. For. 6, 115–119.

Ochi, N., Cao, Q.V., 2003. A comparison of compatible and annual growth models. For. Sci. 49, 285–290.

Pienaar, L.V., Harrison, W.M., 1989. Simultaneous growth and yield prediction equations for *Pinus elliottii* plantations in Zululand. South Afr. For. J. 149, 48–53.

Qin, J., Cao, Q.V., 2006. Using disaggregation to link individual-tree and whole-stand growth models. Can. J. For. Res. 36, 953–960.

Ritchie, M.W., Hann, D.W., 1997a. Evaluation of individual-tree and disaggregative prediction methods for Douglas-fir stands in western Oregon. Can. J. For. Res. 27, 207–216.

Ritchie, M.W., Hann, D.W., 1997b. Implications of disaggregation in forest growth and yield modeling. For. Sci. 43, 223–233.

SAS Institute Inc., 2004. SAS/IML 9.1 User's Guide, vols. 1 and 2. SAS Institute Inc., Cary, NC, 1040 pp.

Smalley, G.W., Bailey, R.L., 1974. Yield tables and stand structure for loblolly pine plantations in Tennessee, Alabama, and Georgia highlands. In: USDA For. Serv. Res. Pap. SO-96, 81 pp.

Wells, O.O., Wakeley, P.C., 1966. Geographic variation in survival, growth, and fusiform rust infection of planted loblolly pine. For. Sci. Monogr. 11, 40.

Zhang, S., Amateis, R.L., Burkhart, H.E., 1997. Constraining individual tree diameter increment and survival models for loblolly pine plantations. For. Sci. 43, 414–423.