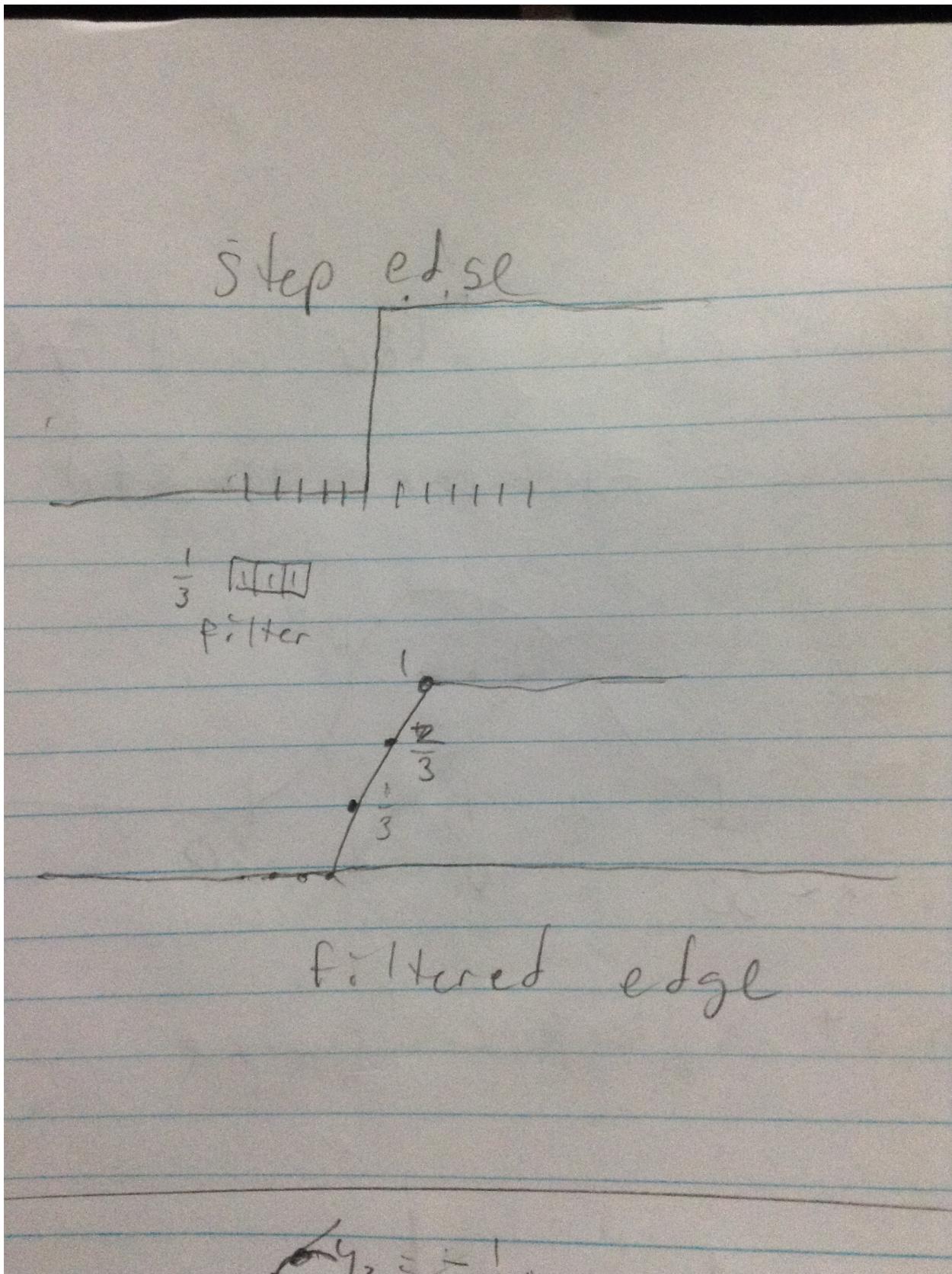


ECG 782 HW 4

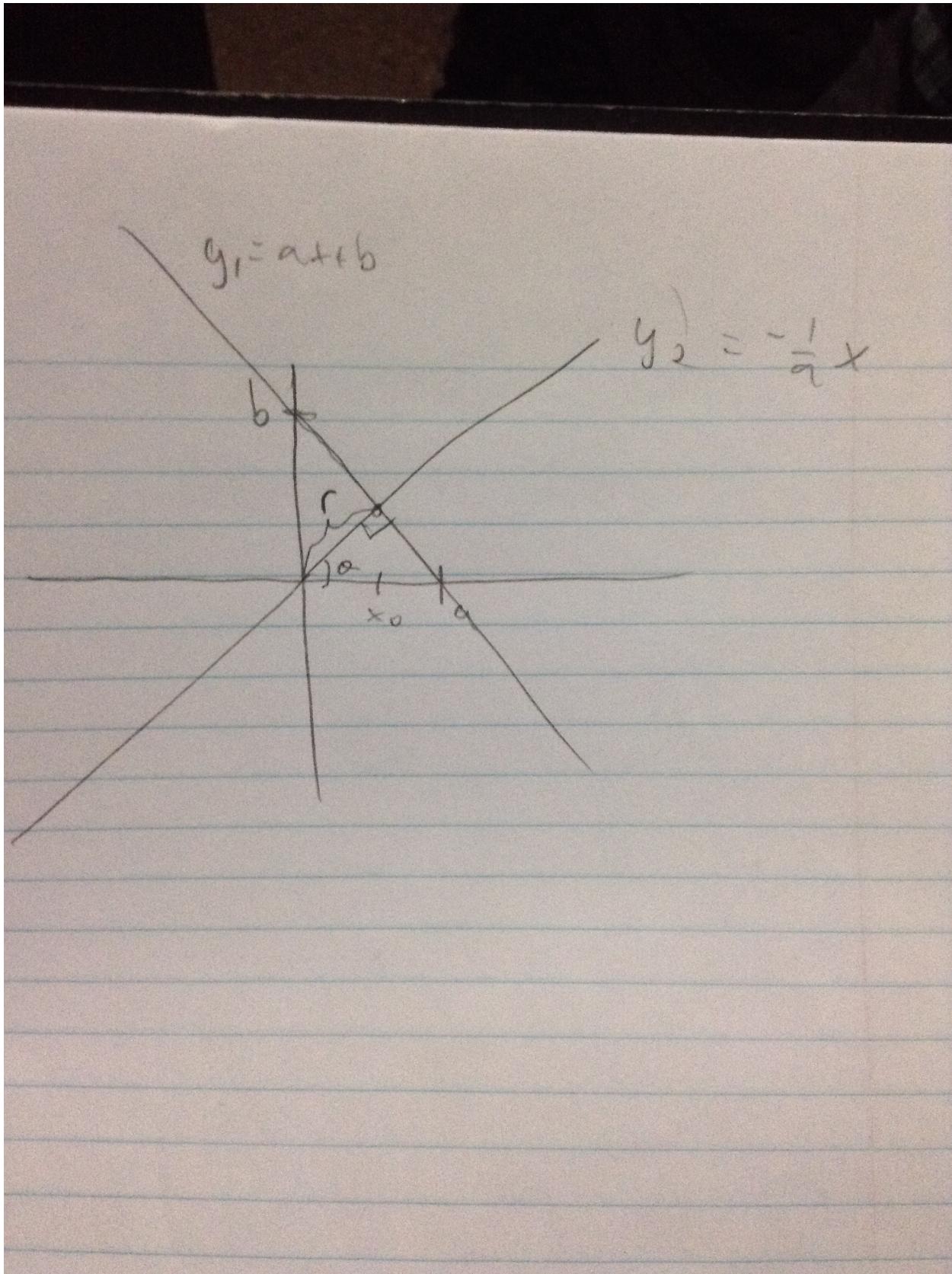
Carlo Lopez-Tello

2015-10-15

1 Problem 10.6



2 Problem 10.22



The slope of $y_2 = -\frac{1}{a}x$. This implies that $\theta = \text{atan}(-\frac{1}{a})$. We can find x_0 by solving the following equation.

$$y_1 = y_2$$

$$ax + b = -\frac{1}{a}x$$

$$b = -\frac{1}{a}x - ax$$

$$b = \left(\frac{1}{a} + a\right) - x$$

$$-b = \left(\frac{1}{a} + a\right)x$$

$$-\frac{b}{\left(\frac{1}{a} + a\right)} = x$$

$$x_0 = x$$

$$x_0 = -\frac{b}{\left(\frac{1}{a} + a\right)}$$

We can find r by solving the following equation.

$$r \cos(\theta) = x_0$$

$$r = \frac{x_0}{\cos(\theta)}$$

$$r = \frac{-\frac{b}{\left(\frac{1}{a} + a\right)}}{\cos(\text{atan}(-\frac{1}{a}))}$$

If $y = -3x + 2$ then $a = -3$ and $b = 2$

$$\theta = \text{atan}(-\frac{1}{a}) = \text{atan}(-\frac{1}{-3}) = 18.4^\circ$$

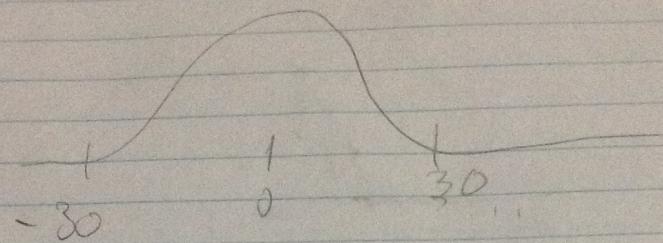
$$x_0 = -\frac{b}{\left(\frac{1}{a} + a\right)} = -\frac{2}{\left(\frac{1}{-3} + -3\right)} = .6$$

$$r = \frac{x_0}{\cos(\theta)} = \frac{.6}{\cos(18.4^\circ)} = \frac{.6}{.95} = .63$$

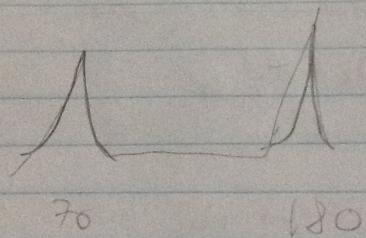
3 Problem 10.36

mean intensity = 180 and 70

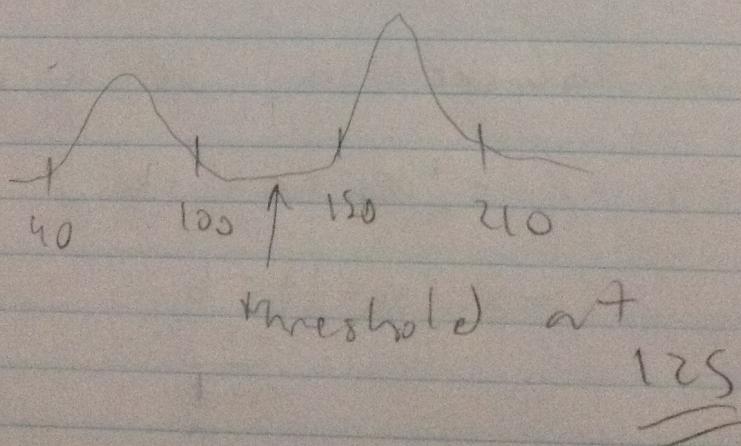
noise = 0 mean 10° std



hist before noise



hist after noise



threshold at

125

4 Problem 4

The error for a given T threshold value can be obtained from the following:

$$E(T) = P_1 \int_T^\infty p_1(z) + P_2 \int_{-\infty}^T p_2(z)$$

$$\frac{d}{dt} E(T) = -P_1 p_1(T) + P_2 p_2(T)$$

$$0 = -P_1 p_1(T) + P_2 p_2(T)$$

$$P_1 p_1(T) = P_2 p_2(T)$$

Since $P_2 = P_1$

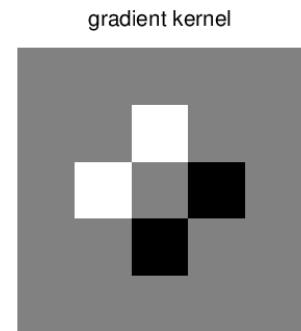
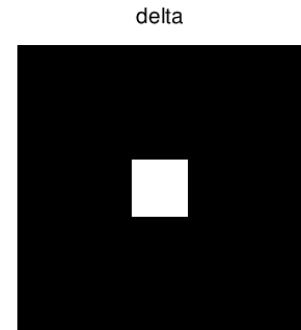
$$p_1(T) = p_2(T)$$

$$1 - .5T = -.5 + .5T$$

$$1.5 = T$$

5 Problem 5

5.1 Gradient Kernel



The kernel used is [.5 0 -.5] in each direction.

5.2 Implement a simple version of the Canny edge detector

```

function [E,M,A] = canny(I,sig,tau)

smoothedImage = imsmooth(I,"Gaussian",sig);

horizontalFilter = [-1 -1 -1;0 0 0;1 1 1];
horizontalEdges = imfilter(smoothedImage,horizontalFilter);

verticalFilter = horizontalFilter';
verticalEdges = imfilter(smoothedImage,verticalFilter);

edgeGradients = sqrt(verticalEdges.^2 + horizontalEdges.^2);
edgeGradients = normalizeImage(edgeGradients);
M = edgeGradients;

edgeDirections = abs(atan2(verticalEdges,horizontalEdges));
edgeDirections = roundAngleValues(edgeDirections);
A = edgeDirections;
edgeGradients = nonMaxSuppression(edgeGradients, edgeDirections);
E = (edgeGradients > tau);

end

function [roundedImage] = roundAngleValues(image)
image(isnan(image)) = 0;
image(image > 0 & image < 22.5*pi/180) = 0;
image(image > 157.5*pi/180 & image < 180*pi/180) = 0;
image(image > 22.5*pi/180 & image < 67.5*pi/180) = 45;
image(image > 67.5*pi/180 & image < 112.5*pi/180) = 90;
image(image > 112.5*pi/180 & image < 157.5*pi/180) = 135;
roundedImage = image;
end

function [suppressedImage] = nonMaxSuppression(gradImage, angleImage)
for i=2:rows(gradImage)-1
    for j=2:columns(gradImage)-1
        switch(angleImage(i,j))
            case 0
                gradImage(i,j) = nonMaxPixel(gradImage(i,j),gradImage(i,j-1),gradImage(i,j+1));
            case 45
                gradImage(i,j) = nonMaxPixel(gradImage(i,j),gradImage(i-1,j+1),gradImage(i+1,j-1));
            case 90
                gradImage(i,j) = nonMaxPixel(gradImage(i,j),gradImage(i+1,j),gradImage(i-1,j));
            case 135
                gradImage(i,j) = nonMaxPixel(gradImage(i,j),gradImage(i-1,j-1),gradImage(i+1,j+1));
            otherwise

```

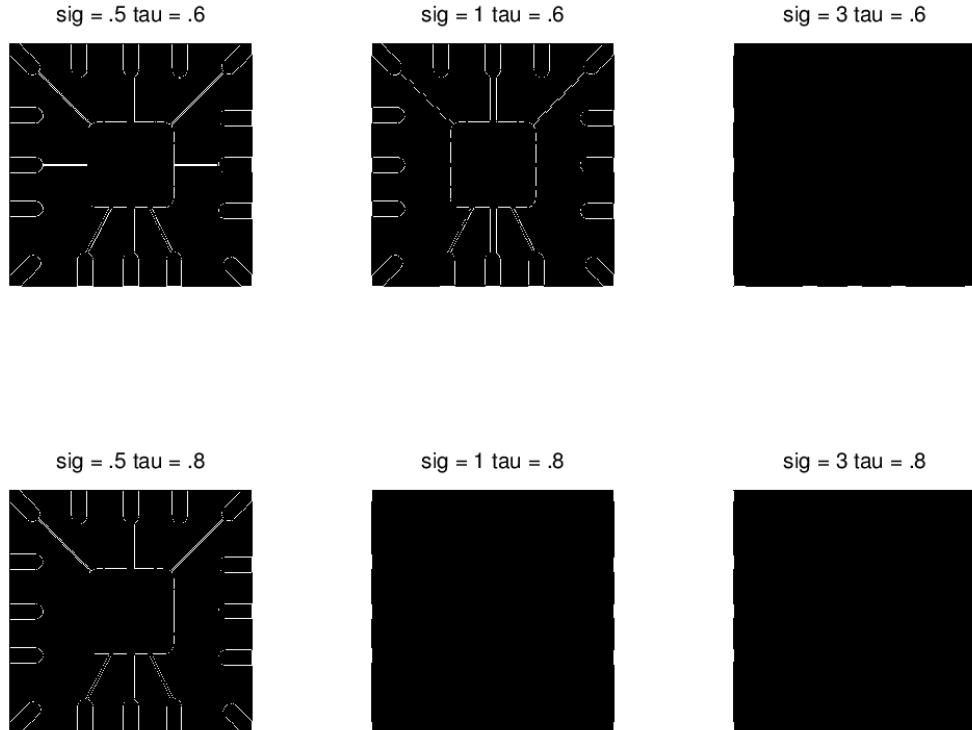
```

        gradImage(i,j) = nonMaxPixel(gradImage(i,j),gradImage(i,j-1),
                                      gradImage(i,j+1));
    end
end
suppressedImage = gradImage;
end

function [pixelValue] = nonMaxPixel(currentPixel,adjPixel1,adjPixel2)
    if(currentPixel < adjPixel1 || currentPixel < adjPixel2)
        pixelValue = 0;
    else
        pixelValue = currentPixel;
    end
end

```

5.3 Apply the canny edge detector to wirebondmask.tif



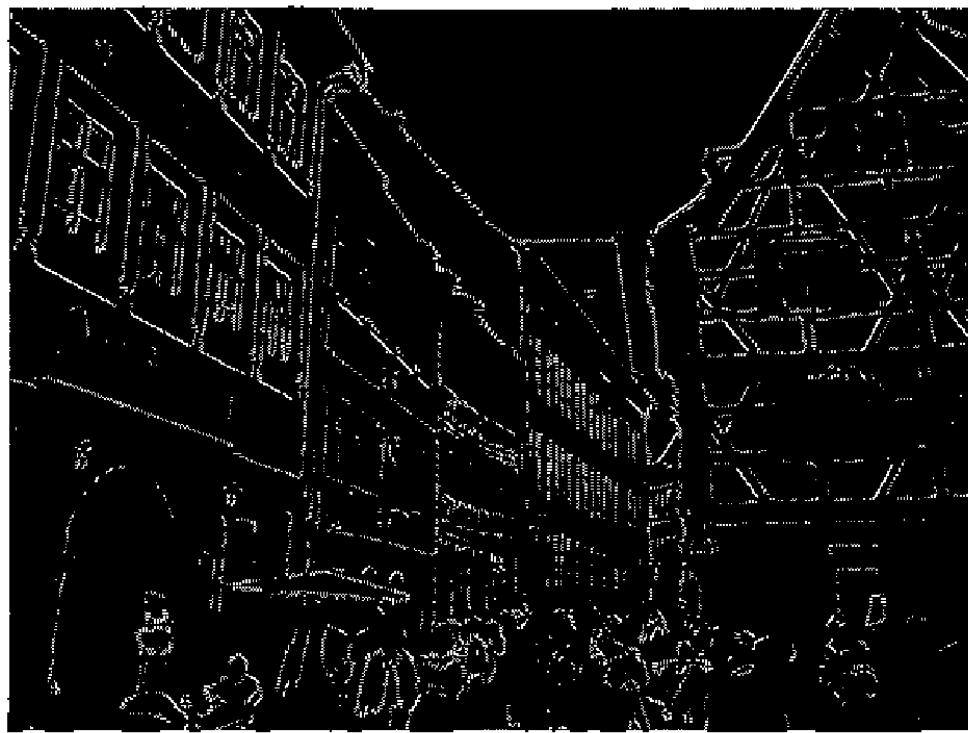
Applying bigger filters to smooth the image before edge detection appears to decrease the intensity of the edges. This can be seen by the smaller amount of edges that are visible when using the same threshold value. This problem appears to be caused by border artifacts in the images.

5.4 Apply the canny edge detector to city.jpg

edges using edge function with default parameters



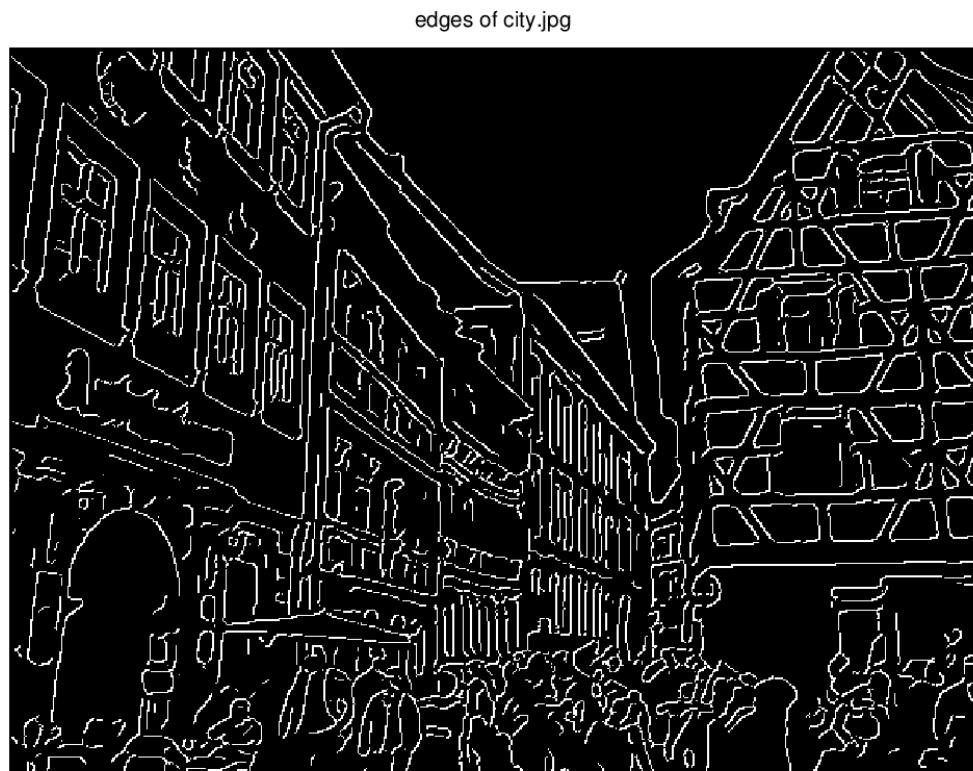
edges using custom canny function with sig = 1 and tau = .15



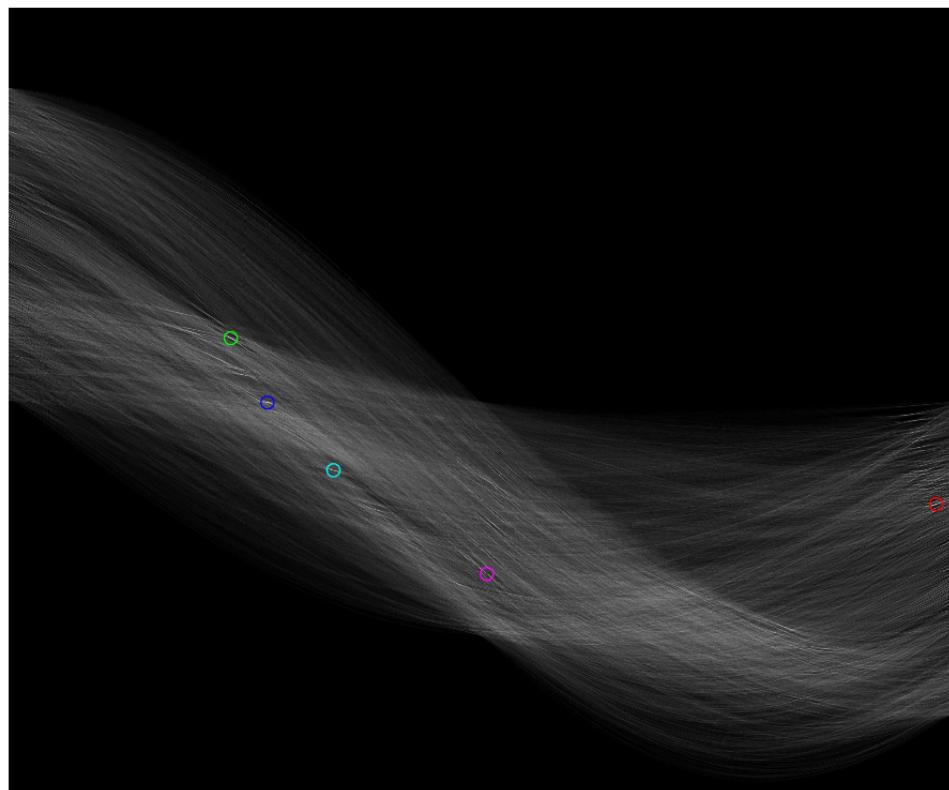
The results of the matlab function are much better than the custom made one. The difference in performance can be attributed to the lack of hysteresis thresholding and border artifacts. The border artifacts mess up the thresholding, since they have a much higher intensity, while the lack of hysteresis thresholding makes the edges look choppy.

6 Problem 6

6.1 Apply the line hough transform to city.jpg



hough transform of city.jpg

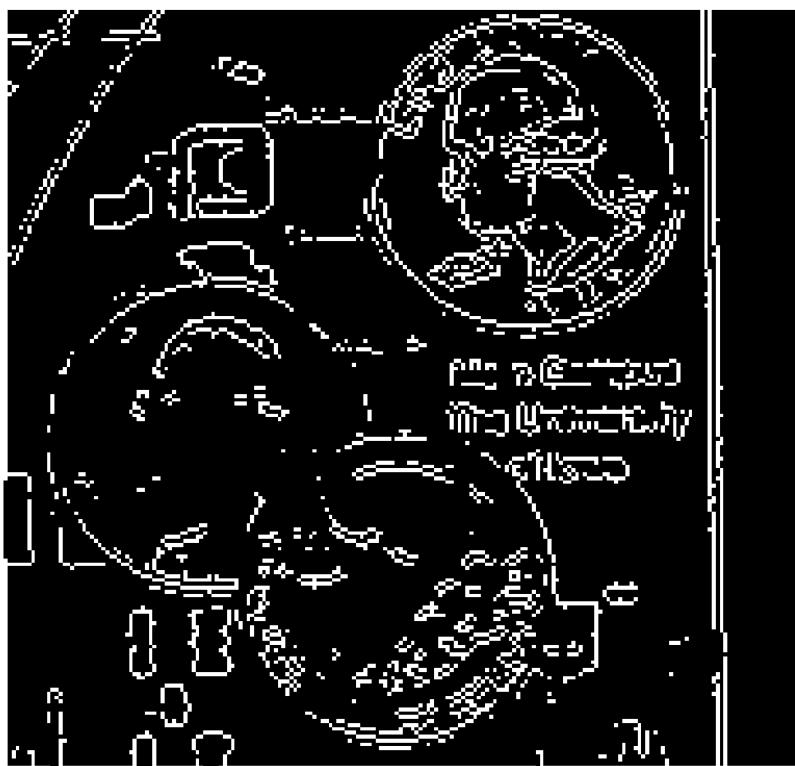


city.jpg + lines from hough transform

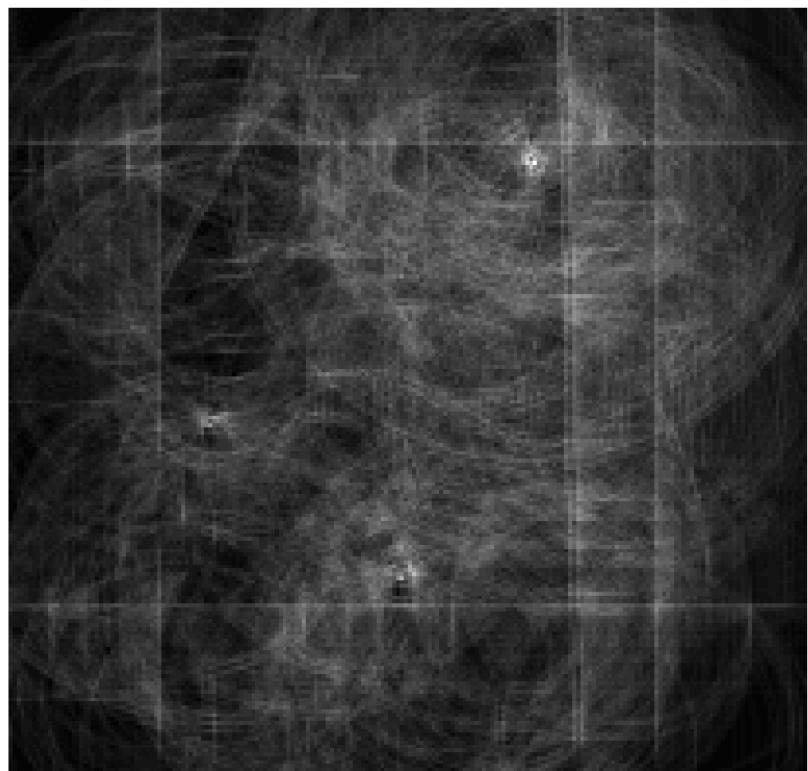


6.2 Apply the circle hough transform to quarters.bmp

edges of quarters.bmp



hough transform of quarters.bmp



circles from hough transform + quarters.bmp



7 Code

Part 5.A

```
x = [0 0 0 0 0;
      0 0 0 0 0;
      0 0 1 0 0;
      0 0 0 0 0;
      0 0 0 0 0;]
[dx,dy] = gradient(x)
figure(1)
subplot(2,1,1)
imshow(x)
title('delta');
subplot(2,1,2)
imshow(dx+dy+.5);
title('gradient_kernel');
print(figure(1), 'partA.png');
```

Part 5.B

```

function [E,M,A] = canny(I,sig,tau)

smoothedImage = imsmooth(I,"Gaussian",sig);

horizontalFilter = [-1 -1 -1;0 0 0;1 1 1];
horizontalEdges = imfilter(smoothedImage,horizontalFilter);

verticalFilter = horizontalFilter';
verticalEdges = imfilter(smoothedImage,verticalFilter);

edgeGradients = sqrt(verticalEdges.^2 + horizontalEdges.^2);
edgeGradients = normalizeImage(edgeGradients);
M = edgeGradients;

edgeDirections = abs(atan2(verticalEdges,horizontalEdges));
edgeDirections = roundAngleValues(edgeDirections);
A = edgeDirections;
edgeGradients = nonMaxSuppression(edgeGradients, edgeDirections);
E = (edgeGradients > tau);

end

function [roundedImage] = roundAngleValues(image)
image(isnan(image)) = 0;
image(image > 0 & image < 22.5*pi/180) = 0;
image(image > 157.5*pi/180 & image < 180*pi/180) = 0;
image(image > 22.5*pi/180 & image < 67.5*pi/180) = 45;
image(image > 67.5*pi/180 & image < 112.5*pi/180) = 90;
image(image > 112.5*pi/180 & image < 157.5*pi/180) = 135;
roundedImage = image;
end

function [suppressedImage] = nonMaxSuppression(gradImage, angleImage)
for i=2:rows(gradImage)-1
    for j=2:columns(gradImage)-1
        switch(angleImage(i,j))
            case 0
                gradImage(i,j) = nonMaxPixel(gradImage(i,j),gradImage(i,j-1),gradImage(i,j+1));
            case 45
                gradImage(i,j) = nonMaxPixel(gradImage(i,j),gradImage(i-1,j+1),gradImage(i+1,j-1));
            case 90
                gradImage(i,j) = nonMaxPixel(gradImage(i,j),gradImage(i+1,j),gradImage(i-1,j));
            case 135
                gradImage(i,j) = nonMaxPixel(gradImage(i,j),gradImage(i-1,j-1),gradImage(i+1,j+1));
            otherwise

```

```

        gradImage(i,j) = nonMaxPixel(gradImage(i,j),gradImage(i,j-1),
                                      gradImage(i,j+1));
    end
end
suppressedImage = gradImage;
end

function [pixelValue] = nonMaxPixel(currentPixel,adjPixel1,adjPixel2)
    if(currentPixel < adjPixel1 || currentPixel < adjPixel2)
        pixelValue = 0;
    else
        pixelValue = currentPixel;
    end
end

```

Part 5.C

```
image = imread('wirebond-mask.tif');
image = double(image)/255;
sig = [.5, 1, 3];
t = [.6, .8];
k = 1;
figure(1);

for i=1:length(t)
    for j=1:length(sig)
        edges = canny(image, sig(j), t(i));
        subplot(2,3,k);
        imshow(edges);
        k = k+1;
    end
end
```

Part 5.D

```
image = imread('city.jpg');
image = rgb2gray(image);
edges = edge(image, 'canny');
figure(1);
title('edges - using edge function with default parameters');
imshow(edges);
image = double(image);
[edges, gradMag, gradAngle] = canny(image, 1, .15);
figure(2);
title('edges - using custom canny function with sig = .5 and tau = .1');
imshow(edges);
```

Utility Code

```
function [ normalizedImage ] = normalizeImage( image )
    MI = min(min(image));
    MA = max(max(image));
    normalizedImage = (image-MI) ./ (MA-MI);
end
```

Part 5.A

```

image = imread('city.jpg');
image = rgb2gray(image);

edges = edge(image, 'Canny');
theta = pi*(-90:.1:90)/180;
[houghTransform,R] = hough_line(edges, theta);
figure(1);
houghTransform = normalizeImage(houghTransform);
title('hough_transform_of_city.jpg');
imshow(houghTransform);
[dIx, thetaIx] = find(houghTransform == max(max(houghTransform)));

figure(2)
imshow(image)
title('city.jpg + lines from hough_transform');
hold on

figure(1)
title('hough_transform_of_city.jpg');
k = 1;

hold on
colors = ['r' 'g' 'b' 'm' 'c'];
for i=1:5
    [dIx, thetaIx] = find(houghTransform == max(max(houghTransform)));
    x = -rows(image)*2:1:rows(image)*2;
    d = R(dIx(1));
    th = theta(thetaIx(1));
    figure(1)
    plot(thetaIx(1), dIx(1), [colors(i) 'o']);
    y = (x*cos(th) - d)/sin(th);
    figure(2)
    plot(x, -y, colors(i));
    for j= dIx(1)-10:dIx(1)+10
        for k = thetaIx(1)-10:thetaIx(1)+10
            if j > 0 & k > 0
                houghTransform(j, k) = 0;
            end
        end
    end
end
hold off
figure(3)
title('edges_of_city.jpg');
imshow(edges);

print(figure(1), 'partA1.png');
print(figure(2), 'partA2.png');
print(figure(3), 'partA3.png');

```

Part 5.B

```

function [ accumulator ] = houghCircle(image, radius)
    accScale = 1;
    [X,Y] = meshgrid(1:1/ accScale : columns(image) ,1:1/ accScale : rows(image))
    ;
    X = X-ceil(columns(image)/2);
    Y = Y-ceil(rows(image)/2);
    accumulator = zeros(rows(X) ,columns(X));
    for i=1:rows(image)
        for j=1:columns(image);
            if (image(i,j) == 1);
                x0 = j-ceil(columns(image)/2);
                y0 = i-ceil(columns(image)/2);
                Z = (X-x0).^2 + (Y-y0).^2 > radius.^2 -radius
                    & (X-x0).^2 + (Y-y0).^2 < radius.^2 +
                    radius;
                accumulator = accumulator + Z;
            end
        end
    end
    accumulator = normalizeImage( accumulator );
    %
    figure(2)
    imshow( accumulator );

    circles = zeros(rows(image) ,columns(image));
    temp = accumulator;
    [X,Y] = meshgrid(1:1:columns(image) ,1:1:rows(image));

    for i=1:2
        [y,x] = find(temp == max(max(temp)));
        for j=1:length(x)
            x0 = round(x(j,1)/accScale);
            y0 = round(y(j,1)/accScale);
            Z = ((X-x0).^2 + (Y-y0).^2 == radius.^2);
            circles = circles + Z;
            temp(temp == max(max(temp))) = 0;
        end
    end
    figure(3)
    imshow( circles );
    %
end

```

```

%[X,Y] = meshgrid (-100:1:100,-100:1:100);
%Z = (X-20).^2 + (Y-20).^2 > 990 & (X-20).^2 + (Y-20).^2 < 1100;
%imshow(Z);
radius = 38;
image = imread('quarters.bmp');
edgeImage = edge(image, 'Sobel');
figure(1)
title('edges_of_quarters.bmp');
imshow(edgeImage);

accumulator = houghCircle(edgeImage, radius);

figure(2)
title('hough_transform_of_quarters.bmp');
imshow(accumulator)

circles = zeros(rows(image), columns(image));
[X,Y] = meshgrid(1:1:columns(image), 1:1:rows(image));
X = X-ceil(columns(image)/2);
Y = Y-ceil(rows(image)/2);
k = 1;

for i=1:3
    [colIx, rIx] = find(accumulator == max(max(accumulator)));
    x0 = rIx(1)-ceil(columns(image)/2);
    y0 = colIx(1)-ceil(rows(image)/2);
    Z = (X-x0).^2 + (Y-y0).^2 > radius.^2 - radius & (X-x0).^2 + (Y-y0).^2
        < radius.^2 + radius;
    circles = circles + Z;
    for j= colIx(1)-10:colIx(1)+10
        for k = rIx(1)-10:rIx(1)+10
            if j > 0 & k > 0
                accumulator(j,k) = 0;
            end
        end
    end
end
figure(3);
title('circles_from_hough_transform+_quarters.bmp');
imshow(circles*255+image);

print(figure(1), 'partB1.png');
print(figure(2), 'partB2.png');
print(figure(3), 'partB3.png');
%
```