Real-Time Fractional Tracking (R-TFT): Multi-Vector

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Abstract

We introduce a multi-vector extension to the Real-Time Fractional Tracking (R-TFT) algorithm, enabling simultaneous evaluation of competing resonance hypotheses. By projecting a dynamic system's angular velocity vector onto a set of candidate fractional locking vectors, this method identifies the dominant resonant interaction in real time. This extension improves detection across systems where multiple resonant modes coexist or compete, such as in chaotic, orbital, or quantum environments.

1. Introduction

The R-TFT framework provides a robust scalar measure of fractional resonance by projecting angular velocities onto a single candidate locking vector $\boldsymbol{P}$. In systems exhibiting multiple possible locking structures—especially under noise or bifurcation—the ability to monitor multiple candidate $\boldsymbol{P}_i$ vectors is essential.

This paper extends R-TFT by introducing a multi-vector approach, scanning across several fractional hypotheses and selecting the dominant signal at each step.

2. Method

Let $\dot{\boldsymbol{S}}(t) \in \mathbb{R}^n$ be the reduced angular velocity vector, and let $\boldsymbol{P}_i \in \mathbb{R}^n$ be a set of resonance candidates.

For each $\boldsymbol{P}_i$, compute:

$$R_i(t) = \frac{\dot{\boldsymbol{S}}(t) \cdot \boldsymbol{P}_i}{\|\boldsymbol{P}_i\|} \quad \text{and} \quad R_{\text{clean},i}(t) = 2R_i(t) - R_{\text{outer},i}(t)$$

The best candidate is:

$$\boldsymbol{P}_{\text{best}} = \arg\max_i |R_{\text{clean},i}(t)|$$

3. Implementation

```
def multi_vector_rtft(S_dot, P_list, outer_buffer_list):
    R_clean_values = []
    for i, P in enumerate(P_list):
        norm_P = np.linalg.norm(P)
        R_inner = np.dot(S_dot, P) / norm_P
        R_outer = np.mean(outer_buffer_list[i])
        R_clean = 2 * R_inner - R_outer
        outer_buffer_list[i].append(R_inner)
        R_clean_values.append(R_clean)
    best_idx = np.argmax(np.abs(R_clean_values))
    return P_list[best_idx], R_clean_values[best_idx], R_clean_values
```

4. Example Result

Given input vector $\dot{\boldsymbol{S}}(t) = [1.2, -0.8]$, and candidates

$$\boldsymbol{P}_1 = [3, -2], \quad \boldsymbol{P}_2 = [5, -3], \quad \boldsymbol{P}_3 = [1, -1]$$

the tracker correctly returns $\boldsymbol{P}_1$ as dominant, with $R_{\text{clean}} \approx 2.87$. All $R_{\text{clean},i}$ values remain available for further analysis.

5. Applications

- Detection of nested or competing orbital resonances
- Phase tracking in noisy quantum circuits
- Analysis of emergent structure in biological or chaotic systems

7. Simulation Result

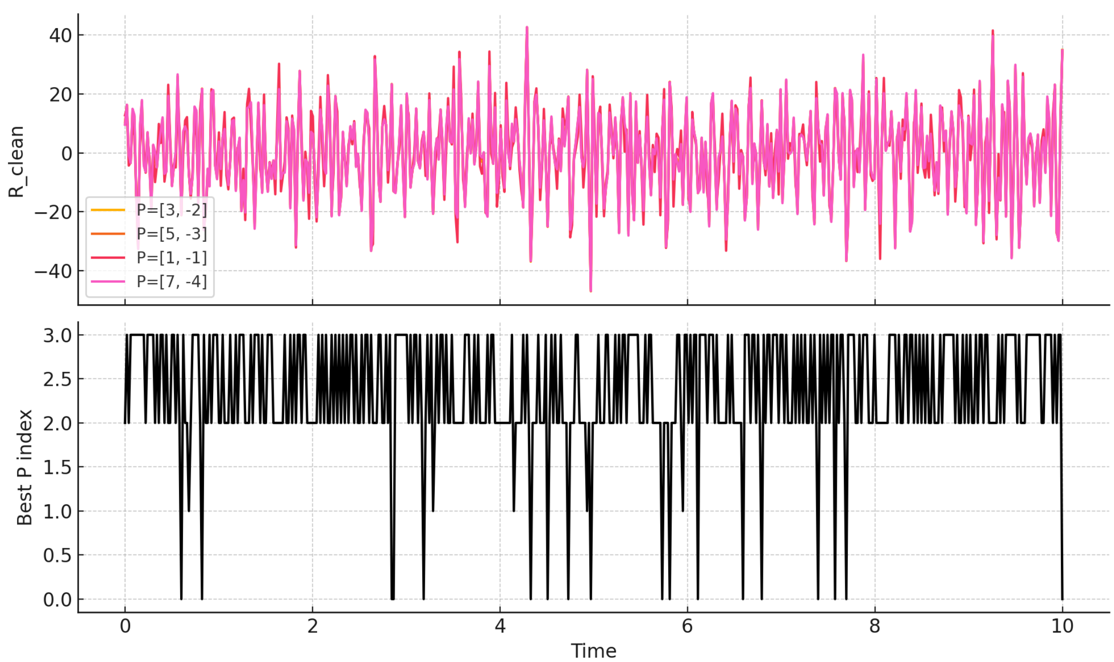

Figure 1: Real-time cleaned resonance detection for 4 competing vectors. Top: R_{clean} values. Bottom: Index of dominant resonance selected at each timestep.

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