

## 0.1 Syntax

$Ar$	$::= s_k \mid (x :_U M) \rightarrow Ar$
$Pos$	$::= X \vec{m} \mid (x :_s M) \rightarrow Pos \mid (x :_s M) \multimap Pos$
$Co_U$	$::= X \vec{m} \mid (- :_U P) \rightarrow Co'_U \mid (x :_U M) \rightarrow Co'_U$
$Co'_U$	$::= X \vec{m} \mid (- :_U P) \multimap Co'_U \mid (x :_U M) \multimap Co'_U$
$Co_L$	$::= X \vec{m} \mid (- :_s P) \rightarrow Co'_L \mid (x :_U M) \rightarrow Co'_L \mid (- :_L M) \rightarrow Co'_L$
$Co'_L$	$::= X \vec{m} \mid (- :_s P) \multimap Co'_L \mid (x :_U M) \multimap Co'_L \mid (- :_L M) \multimap Co'_L$

## 0.2 Introduction Rules

$$\frac{(\forall i = 1 \dots n) \quad \text{arity}(A, s_k) \quad \text{constructor}_s(C_i, X) \quad \frac{|\Gamma| \quad \Gamma \vdash A : U_{k+1} \quad \Gamma, X :_U A \vdash C_i : U_k}{\Gamma \vdash \text{Ind}_s(X : A)\{C_1 \dots C_n\} : A}}{\frac{|\Gamma| \quad \Gamma \vdash \text{Ind}_s(X : A)\{C_1 \dots C_n\} : A \quad 1 \leq i \leq n}{\Gamma \vdash \text{Constr}(i, I_s) : C_i[I_s/X]}}$$

## 0.3 Non-Dependent Elimination

$$\begin{aligned} ((x :_{s'} M) \rightarrow P)\{s\} &= U \\ ((x :_{s'} M) \multimap P)\{s\} &= L \\ (X \vec{m})\{s\} &= s \end{aligned}$$
  

$$\begin{aligned} ((- :_U P) \rightarrow C)\{X, Q, s\} &= (- :_U P) \rightarrow (- :_{s'} P[Q/X]) \multimap C\{X, Q, s\} && (\text{where } s' = P\{s\}) \\ ((- :_U P) \multimap C)\{X, Q, s\} &= (- :_U P) \multimap (- :_{s'} P[Q/X]) \multimap C\{X, Q, s\} && (\text{where } s' = P\{s\}) \\ ((- :_L P) \rightarrow C)\{X, Q, s\} &= (- :_{s'} P[Q/X]) \rightarrow C\{X, Q, s\} && (\text{where } s' = P\{s\}) \\ ((- :_L P) \multimap C)\{X, Q, s\} &= (- :_{s'} P[Q/X]) \multimap C\{X, Q, s\} && (\text{where } s' = P\{s\}) \\ ((x :_s M) \rightarrow C)\{X, Q, s\} &= (x :_s M) \rightarrow C\{X, Q, s\} \\ ((x :_s M) \multimap C)\{X, Q, s\} &= (x :_s M) \multimap C\{X, Q, s\} \\ (X \vec{m})\{X, Q, s\} &= Q \vec{m} \end{aligned}$$

$$\begin{aligned} ((- :_s P) \rightarrow C)\{X, Q\} &= (- :_s P) \multimap C\{X, Q\} \\ ((- :_s P) \multimap C)\{X, Q\} &= (- :_s P) \multimap C\{X, Q\} \\ ((x :_s M) \rightarrow C)\{X, Q\} &= (x :_s M) \multimap C\{X, Q\} \\ ((x :_s M) \multimap C)\{X, Q\} &= (x :_s M) \multimap C\{X, Q\} \\ (X \vec{m})\{X, Q\} &= Q \vec{m} \end{aligned}$$

$$\frac{(\forall i = 1 \dots n) \quad \frac{|\Gamma_2| \quad \Gamma_1 \ddagger \Gamma_2 \ddagger \Gamma}{\Gamma_2 \vdash Q : (\vec{x} :_U \vec{A}) \rightarrow s' \quad \Gamma_2 \vdash f_i : C_i\{I_s, Q, s'\}}}{\Gamma \vdash \text{Rec}(c, Q)\{f_1 \dots f_n\} : (Q \vec{a})}$$

$$\frac{(\forall i = 1 \dots n) \quad \frac{\Gamma_1 \ddagger \Gamma_2 \ddagger \Gamma}{\Gamma_2 \vdash Q : (\vec{x} :_U \vec{A}) \rightarrow s' \quad \Gamma_2 \vdash f_i : C_i\{I_s, Q\}}}{\Gamma \vdash \text{Elim}(c, Q)\{f_1 \dots f_n\} : (Q \vec{a})}$$

## 0.4 Dependent Elimination

$$\begin{aligned} ((x :_s M) \rightarrow P)\{Q, p\} &= (x :_s M) \rightarrow P\{Q, (p \ x)\} \\ ((x :_s M) \multimap P)\{Q, p\} &= (x :_s M) \multimap P\{Q, (p \ x)\} \\ (X \vec{m})\{Q, p\} &= Q \vec{m} \ p \end{aligned}$$

$$\begin{aligned}
((\_ :_U P) \rightarrow C)\{X, Q, s, c\} &= (p :_U P) \rightarrow (\_ :_{s'} P\{Q, p\}) \multimap C\{X, Q, s, (c\ p)\} && (\text{where } s' = P\{s\}) \\
((\_ :_U P) \multimap C)\{X, Q, s, c\} &= (p :_U P) \multimap (\_ :_{s'} P\{Q, p\}) \multimap C\{X, Q, s, (c\ p)\} && (\text{where } s' = P\{s\}) \\
((x :_U M) \rightarrow C)\{X, Q, s, c\} &= (x :_U M) \rightarrow C\{X, Q, s, (c\ x)\} \\
((x :_U M) \multimap C)\{X, Q, s, c\} &= (x :_U M) \multimap C\{X, Q, s, (c\ x)\} \\
(X \overrightarrow{m})\{X, Q, s, c\} &= (Q \overrightarrow{m}\ c)
\end{aligned}$$

$$\begin{aligned}
((\_ :_U P) \rightarrow C)\{X, Q, c\} &= (p :_U P) \multimap C\{X, Q, (c\ p)\} \\
((\_ :_U P) \multimap C)\{X, Q, c\} &= (p :_U P) \multimap C\{X, Q, (c\ p)\} \\
((x :_U P) \rightarrow C)\{X, Q, c\} &= (x :_U M) \multimap C\{X, Q, (c\ x)\} \\
((x :_U P) \multimap C)\{X, Q, c\} &= (x :_U M) \multimap C\{X, Q, (c\ x)\} \\
(X \overrightarrow{m})\{X, Q, c\} &= (Q \overrightarrow{m}\ c)
\end{aligned}$$

$$\frac{\Gamma_1 \vdash c : (I_U \overrightarrow{a}) \quad \Gamma_2 \vdash Q : (\overrightarrow{x} :_U \overrightarrow{A}) \rightarrow (\_ :_U I_U \overrightarrow{x}) \rightarrow s' \quad \Gamma_2 \vdash f_i : C_i\{I_U, Q, s', Constr(i, I_U)\}}{(\forall i = 1 \dots n) \quad |\Gamma_2| \quad \Gamma_1 \ddagger \Gamma_2 \ddagger \Gamma \quad \Gamma \vdash Rec(c, Q)\{f_1 | \dots | f_n\} : (Q \overrightarrow{a}\ c)}$$

$$\frac{\Gamma_1 \vdash c : (I_U \overrightarrow{a}) \quad \overline{\Gamma}_2 \vdash Q : (\overrightarrow{x} :_U \overrightarrow{A}) \rightarrow (\_ :_U I_U \overrightarrow{x}) \rightarrow s' \quad \Gamma_2 \vdash f_i : C_i\{I_U, Q, Constr(i, I_U)\}}{\Gamma \vdash Elim(c, Q)\{f_1 | \dots | f_n\} : (Q \overrightarrow{a}\ c)}$$

## 0.5 Conversion

$$\begin{aligned}
((\_ :_U P) \rightarrow C)[X, F, f] &= \lambda p. C[X, F, (f\ p\ \lambda \overrightarrow{x}. (F \overrightarrow{m}\ (p\ \overrightarrow{x})))]) \\
((\_ :_U P) \multimap C)[X, F, f] &= \lambda p. C[X, F, (f\ p\ \lambda \overrightarrow{x}. (F \overrightarrow{m}\ (p\ \overrightarrow{x})))]) \\
((\_ :_L P) \rightarrow C)[X, F, f] &= \lambda p. C[X, F, (f\ \lambda \overrightarrow{x}. (F \overrightarrow{m}\ (p\ \overrightarrow{x})))]) \\
((\_ :_L P) \multimap C)[X, F, f] &= \lambda p. C[X, F, (f\ \lambda \overrightarrow{x}. (F \overrightarrow{m}\ (p\ \overrightarrow{x})))]) \\
((x :_s M) \rightarrow C)[X, F, f] &= \lambda x. C[X, F, (f\ x)] \\
((x :_s M) \multimap C)[X, F, f] &= \lambda x. C[X, F, (f\ x)] \\
(X \overrightarrow{m})[X, F, f] &= f
\end{aligned}$$

$$\begin{aligned}
((\_ :_s P) \rightarrow C)[X, f] &= \lambda p. C[X, (f\ p)] \\
((\_ :_s P) \multimap C)[X, f] &= \lambda p. C[X, (f\ p)] \\
((x :_s M) \rightarrow C)[X, f] &= \lambda x. C[X, (f\ x)] \\
((x :_s M) \multimap C)[X, f] &= \lambda x. C[X, (f\ x)] \\
(X \overrightarrow{m})[X, f] &= f
\end{aligned}$$

$$Fun\_Rec(I_s, Q, \overrightarrow{f}) = \lambda \overrightarrow{x}. \lambda c. Rec(c, Q)\{\overrightarrow{f}\}$$

$$\begin{aligned}
Rec((Constr(i, I_s) \overrightarrow{m}), Q)\{\overrightarrow{f}\} &\rightarrow_\iota (C_i[I_s, Fun\_Rec(I_s, Q, \overrightarrow{f}), f_i] \overrightarrow{m}) \\
Elim((Constr(i, I_s) \overrightarrow{m}), Q)\{\overrightarrow{f}\} &\rightarrow_\iota (C_i[I_s, f_i] \overrightarrow{m})
\end{aligned}$$