The Calculus of Linear Constructions — Technical Report

Qiancheng Fu

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1 Introduction

This extended report is meant to accompany our paper of the same title. Here, we describe the meta-theory of CILC and their proofs in detail. All the results presented here have been formalized and proven correct in the Coq Proof Assistant.

2 Syntax of CLC (`clc_ast.v`)

$$
\begin{array}{lcll}
i & := & 0 \mid 1 \mid 2 \;... & \text{universe levels} \\
s, t & ::= & U \mid L & \text{sorts} \\
m, n, A, B, M & ::= & U_i \mid L_i \mid x & \text{expressions} \\
 & \mid & (x :_s A) \to B & \\
 & \mid & (x :_s A) \multimap B & \\
 & \mid & \lambda x :_s A.n & \\
 & \mid & m\ n &
\end{array}
$$

3 Reduction and Equality of CLC (`clc_ast.v`)

$$\frac{m_1 \rightsquigarrow^* n \qquad m_2 \rightsquigarrow^* n}{m_1 \equiv m_2 : A}\textsc{Join} \qquad \frac{}{(\lambda x :_s A.m)\ n \rightsquigarrow m[n/x]}\textsc{Step-}\beta \qquad \frac{A \rightsquigarrow A'}{\lambda x :_s A.m \rightsquigarrow \lambda x :_s A'.m}\textsc{Step-}\lambda\textsc{L}$$

$$\frac{m \rightsquigarrow m'}{\lambda x :_s A.m \rightsquigarrow \lambda x :_s A.m'}\textsc{Step-}\lambda\textsc{R} \qquad \frac{A \rightsquigarrow_p A'}{(x :_s A) \to B \rightsquigarrow (x :_s A') \to B}\textsc{Step-L}\to$$

$$\frac{B \rightsquigarrow_p B'}{(x :_s A) \to B \rightsquigarrow (x :_s A) \to B'}\textsc{Step-R}\to \qquad \frac{A \rightsquigarrow_p A'}{(x :_s A) \multimap B \rightsquigarrow (x :_s A') \multimap B}\textsc{Step-L}\multimap$$

$$\frac{B \rightsquigarrow_p B'}{(x :_s A) \multimap B \rightsquigarrow (x :_s A) \multimap B'}\textsc{Step-R}\multimap \qquad \frac{m \rightsquigarrow m'}{m\ n \rightsquigarrow m'\ n}\textsc{Step-AppL} \qquad \frac{n \rightsquigarrow n'}{m\ n \rightsquigarrow m\ n'}\textsc{Step-AppR}$$

4 Confluence of CLC (`clc_confluence.v`)

4.1 Parallel Reduction

To prove the confluence property of CLC, we employ the standard technique utilizing parallel reductions.

$$\frac{}{x \rightsquigarrow_p x}\textsc{Pstep-Var} \qquad \frac{}{s_i \rightsquigarrow_p s_i}\textsc{Pstep-Sort} \qquad \frac{m \rightsquigarrow_p m'}{}$$