

## 0.1 Syntax

$Ar$	$::= s_k \mid (x :_U M) \rightarrow Ar$
$Pos$	$::= X \vec{m} \mid (x :_s M) \rightarrow Pos \mid (x :_s M) \multimap Pos$
$Co_U$	$::= X \vec{m} \mid (\_ :_U P) \rightarrow Co'_U \mid (x :_U M) \rightarrow Co'_U$
$Co'_U$	$::= X \vec{m} \mid (\_ :_U P) \multimap Co'_U \mid (x :_U M) \multimap Co'_U$
$Co_L$	$::= X \vec{m} \mid (\_ :_s P) \rightarrow Co'_L \mid (x :_U M) \rightarrow Co'_L \mid (\_ :_L M) \rightarrow Co'_L$
$Co'_L$	$::= X \vec{m} \mid (\_ :_s P) \multimap Co'_L \mid (x :_U M) \multimap Co'_L \mid (\_ :_L M) \multimap Co'_L$

## 0.2 Introduction Rules

$\frac{(\forall i = 1 \dots n) \quad \text{arity}(A, s_k) \quad \text{constructor}_s(C_i, X) \quad \begin{array}{c}  \Gamma  \quad \Gamma \vdash A : U_{k+1} \quad \Gamma, X :_U A \vdash C_i : U_k \end{array}}{\Gamma \vdash \text{Ind}_s(X : A)\{C_1   \dots   C_n\} : A}$	$\frac{ \Gamma  \quad \Gamma \vdash \text{Ind}_s(X : A)\{C_1   \dots   C_n\} : A \quad 1 \leq i \leq n}{\Gamma \vdash \text{Constr}(i, I) : C_i[I/X]}$
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## 0.3 Non-Dependent Elimination

$((x :_s M) \rightarrow P)\{s\}$	$= U$
$((x :_s M) \multimap P)\{s\}$	$= L$
$(X \vec{m})\{s\}$	$= s$

$((\_ :_U P) \rightarrow C)\{X, Q, s\}$	$= (\_ :_U P) \rightarrow (\_ :_{s'} P[Q/X]) \multimap C\{X, Q, s\}$	$(\text{where } s' = P\{s\})$
$((\_ :_U P) \multimap C)\{X, Q, s\}$	$= (\_ :_U P) \multimap (\_ :_{s'} P[Q/X]) \multimap C\{X, Q, s\}$	$(\text{where } s' = P\{s\})$
$((\_ :_L P) \rightarrow C)\{X, Q, s\}$	$= (\_ :_{s'} P[Q/X]) \rightarrow C\{X, Q, s\}$	$(\text{where } s' = P\{s\})$
$((\_ :_L P) \multimap C)\{X, Q, s\}$	$= (\_ :_{s'} P[Q/X]) \multimap C\{X, Q, s\}$	$(\text{where } s' = P\{s\})$
$((x :_s M) \rightarrow C)\{X, Q, s\}$	$= (x :_s M) \rightarrow C\{X, Q, s\}$	
$((x :_s M) \multimap C)\{X, Q, s\}$	$= (x :_s M) \multimap C\{X, Q, s\}$	
$(X \vec{m})\{X, Q, s\}$	$= Q \vec{m}$	

$((\_ :_s P) \rightarrow C)\{X, Q\}$	$= (\_ :_s P) \multimap C\{X, Q\}$
$((\_ :_s P) \multimap C)\{X, Q\}$	$= (\_ :_s P) \multimap C\{X, Q\}$
$((x :_s M) \rightarrow C)\{X, Q\}$	$= (x :_s M) \multimap C\{X, Q\}$
$((x :_s M) \multimap C)\{X, Q\}$	$= (x :_s M) \multimap C\{X, Q\}$
$(X \vec{m})\{X, Q\}$	$= Q \vec{m}$

$$\frac{\Gamma_1 \vdash c : (I \vec{a}) \quad \begin{array}{c} (\forall i = 1 \dots n) \quad |\Gamma_2| \quad \Gamma_1 \ddagger \Gamma_2 \ddagger \Gamma \\ \Gamma_2 \vdash Q : (\vec{x} :_U \vec{A}) \rightarrow s' \quad \Gamma_2 \vdash f_i : C_i\{I, Q, s'\} \end{array}}{\Gamma \vdash \text{Rec}(c, Q)\{f_1 | \dots | f_n\} : (Q \vec{a})}$$

$$\frac{\Gamma_1 \vdash c : (I \vec{a}) \quad \begin{array}{c} (\forall i = 1 \dots n) \quad \Gamma_1 \ddagger \Gamma_2 \ddagger \Gamma \\ \Gamma_2 \vdash Q : (\vec{x} :_U \vec{A}) \rightarrow s' \quad \Gamma_2 \vdash f_i : C_i\{I, Q\} \end{array}}{\Gamma \vdash \text{Elim}(c, Q)\{f_1 | \dots | f_n\} : (Q \vec{a})}$$