A Two Level Dependent Session Type Theory

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1 Syntax

$$
\begin{array}{rlcl}
\text{variables} & x, y, z & & \\
\text{channels} & c, d & & \\
\text{sorts} & s, r, t & ::= & \mathrm{U} \mid \mathrm{L} \\
\text{roles} & \rho & ::= & + \mid - \\
\text{terms} & m, n, A, B, C & ::= & x \mid c \mid s \\
 & & \mid & \Pi_t(x:A).B \mid \Pi_t\{x:A\}.B \mid \Sigma_t(x:A).B \mid \Sigma_t\{x:A\}.B \\
 & & \mid & \lambda_t(x:A).m \mid \lambda_t\{x:A\}.m \mid \langle m, n\rangle_t \mid \{m, n\}_t \\
 & & \mid & m\ n \mid \mathrm{R}^{\Sigma}_{[z]A}(m, [x,y]n) \mid \mu(x:A).m \\
 & & \mid & 1 \mid () \mid 2 \mid \mathrm{true} \mid \mathrm{false} \mid \mathrm{R}^{2}_{[z]A}(m, n_1, n_2) \\
 & & \mid & \mathrm{T}\,A \mid \mathrm{return}\ m \mid \mathrm{let}\ x \leftarrow m\ \mathrm{in}\ n \\
 & & \mid & \mathrm{proto} \mid \mathrm{end}\rho \mid \rho\{x:A\}.B \mid \rho(x:A).B \mid \rho\mathrm{Ch}\,A \\
 & & \mid & \mathrm{fork}\,(x:A).m \mid \mathrm{recv}\,m \mid \mathrm{recv}\,\{m\} \mid \mathrm{send}\,m \mid \mathrm{send}\,\{m\} \\
 & & \mid & \mathrm{close}\,m \mid \mathrm{wait}\,m \\
\text{values} & u, v & ::= & x \mid c \mid \lambda_t(x:A).m \mid \lambda_t\{x:A\}.m \mid \langle u, v\rangle_t \mid \{v, m\}_t \\
 & & \mid & () \mid \mathrm{true} \mid \mathrm{false} \mid \mathrm{return}\ v \mid \mathrm{let}\ x \leftarrow v\ \mathrm{in}\ m \\
 & & \mid & \mathrm{fork}\,(x:A).m \mid \mathrm{recv}\,v \mid \mathrm{recv}\,\{v\} \mid \mathrm{send}\,v \mid \mathrm{send}\,\{v\} \\
 & & \mid & \mathrm{send}\,u\ v \mid \mathrm{send}\,\{v\}\ m \mid \mathrm{close}\,v \mid \mathrm{wait}\,v \\
\text{process} & o, p, q & \mid & \langle m\rangle \mid p \mid q \mid \nu cd.p
\end{array}
$$

2 Static Fragment

Sort Order

$$\frac{}{\mathrm{U} \sqsubseteq s} \qquad\qquad \frac{}{\mathrm{L} \sqsubseteq \mathrm{L}}$$

Static Context

$$\frac{}{\epsilon \vdash} \qquad\qquad \frac{\Gamma \vdash \qquad \Gamma \vdash A : s \qquad x \in \mathit{fresh}(\Gamma)}{\Gamma, x : A \vdash}$$

Core Typing

$$\frac{\Gamma \vdash}{\Gamma \vdash s : \mathrm{U}} \qquad \frac{\Gamma, x : A \vdash}{\Gamma, x : A \vdash x : A} \qquad \frac{\Gamma \vdash A : s \qquad \Gamma, x : A \vdash B : r}{\Gamma \vdash \Pi_t(x : A).B : t} \qquad \frac{\Gamma \vdash A : s \qquad \Gamma, x : A \vdash B : r}{\Gamma \vdash \Pi_t\{x : A\}.B : t}$$

$$\frac{\Gamma, x : A \vdash m : B}{\Gamma \vdash \lambda_t(x : A).m : \Pi_t(x : A).B} \qquad \frac{\Gamma, x : A \vdash m : B}{\Gamma \vdash \lambda_t\{x : A\}.m : \Pi_t\{x : A\}.B} \qquad \frac{\Gamma, x : A \vdash m : A}{\Gamma \vdash \mu(x : A).m : A}$$

$$\frac{\Gamma \vdash m : \Pi_t(x : A).B \qquad \Gamma \vdash n : A}{\Gamma \vdash m\ n : B[n/x]} \qquad \frac{\Gamma \vdash m : \Pi_t\{x : A\}.B \qquad \Gamma \vdash n : A}{\Gamma \vdash m\ n : B[n/x]}$$

$$\frac{s \sqsubseteq t \qquad r \sqsubseteq t \qquad \Gamma \vdash A : s \qquad \Gamma, x : A \vdash B : r}{\Gamma \vdash \Sigma_t(x : A).B : t} \qquad \frac{r \sqsubseteq t \qquad \Gamma \vdash A : s \qquad \Gamma, x : A \vdash B : r}{\Gamma \vdash \Sigma_t\{x : A\}.B : t}$$

$$\frac{\Gamma \vdash \Sigma_t(x : A).B \qquad \Gamma \vdash m : A \qquad \Gamma \vdash n : B[m/x]}{\Gamma \vdash \langle m, n\rangle_t : \Sigma_t(x : A).B} \qquad \frac{\Gamma \vdash \Sigma_t\{x : A\}.B \qquad \Gamma \vdash m : A \qquad \Gamma \vdash n : B[m/x]}{\Gamma \vdash \{m, n\}_t : \Sigma_t\{x : A\}.B}$$

$$\frac{\Gamma, z : \Sigma_t(x : A).B \vdash C : s \qquad \Gamma \vdash m : \Sigma_t(x : A).B \qquad \Gamma, x : A, y : B \vdash n : C[\langle x, y\rangle_t/z]}{\Gamma \vdash \mathrm{R}^{\Sigma}_{[z]C}(m, [x,y]n) : C[m/z]}$$

$$\frac{\Gamma, z : \Sigma_t\{x : A\}.B \vdash C : s \qquad \Gamma \vdash m : \Sigma_t\{x : A\}.B \qquad \Gamma, x : A, y : B \vdash n : C[\{x, y\}_t/z]}{\Gamma \vdash \mathrm{R}^{\Sigma}_{[z]C}(m, [x,y]n) : C[m/z]}$$

$$\frac{\Gamma \vdash B : s \qquad \Gamma \vdash m : A \qquad A \simeq B}{\Gamma \vdash m : B}$$

Data Typing

$$\frac{\Gamma \vdash}{\Gamma \vdash 1 : \mathrm{U}} \qquad \frac{\Gamma \vdash}{\Gamma \vdash () : 1} \qquad \frac{\Gamma \vdash}{\Gamma \vdash 2 : \mathrm{U}} \qquad \frac{\Gamma \vdash}{\Gamma \vdash \mathrm{true} : 2} \qquad \frac{\Gamma \vdash}{\Gamma \vdash \mathrm{false} : 2}$$

$$\frac{\Gamma, z : 2 \vdash A : s \qquad \Gamma \vdash m : 2 \qquad \Gamma \vdash n_1 : A[\mathrm{true}/z] \qquad \Gamma \vdash n_2 : A[\mathrm{false}/z]}{\Gamma \vdash \mathrm{R}^2_{[z]A}(m, n_1, n_2) : A[m/z]}$$

Monadic Typing

$$\frac{\Gamma \vdash A : s}{\Gamma \vdash \mathrm{T}\, A : \mathrm{L}} \qquad \frac{\Gamma \vdash m : A}{\Gamma \vdash \mathrm{return}\ m : \mathrm{T}\, A} \qquad \frac{\Gamma \vdash B : s \qquad \Gamma \vdash m : \mathrm{T}\, A \qquad \Gamma, x : A \vdash n : \mathrm{T}\, B}{\Gamma \vdash \mathrm{let}\ x : A \leftarrow m\ \mathrm{in}\ n : \mathrm{T}\, B}$$

Session Typing

$$\frac{\Gamma \vdash}{\Gamma \vdash \mathrm{proto} : \mathrm{U}} \qquad \frac{\Gamma \vdash}{\Gamma \vdash \mathrm{end}\rho : \mathrm{proto}} \qquad \frac{\Gamma, x : A \vdash B : \mathrm{proto}}{\Gamma \vdash \rho\{x : A\}.B : \mathrm{proto}} \qquad \frac{\Gamma, x : A \vdash B : \mathrm{proto}}{\Gamma \vdash \rho(x : A).B : \mathrm{proto}} \qquad \frac{\Gamma \vdash A : \mathrm{proto}}{\Gamma \vdash \rho\mathrm{Ch}\, A : \mathrm{L}}$$

$$\frac{\Gamma \vdash \qquad \epsilon \vdash A : \mathrm{proto}}{\Gamma \vdash c : \rho\mathrm{Ch}\, A} \qquad \frac{\Gamma, x : +\mathrm{Ch}\, A \vdash m : \mathrm{T}\, 1}{\Gamma \vdash \mathrm{fork}\, (x : A).m : \mathrm{T}\, (-\mathrm{Ch}\, A)}$$

$$\frac{\Gamma \vdash m : \rho_1\mathrm{Ch}\, (\rho_2(x : A).B) \qquad \rho_1\, \mathrm{xor}\, \rho_2 = ?}{\Gamma \vdash \mathrm{recv}\, m : \mathrm{T}\, (\Sigma_{\mathrm{L}}(x : A).\rho_1\mathrm{Ch}\, B)} \qquad \frac{\Gamma \vdash m : \rho_1\mathrm{Ch}\, (\rho_2\{x : A\}.B) \qquad \rho_1\, \mathrm{xor}\, \rho_2 = ?}{\Gamma \vdash \mathrm{recv}\, \{m\} : \mathrm{T}\, (\Sigma_{\mathrm{L}}\{x : A\}.\rho_1\mathrm{Ch}\, B)}$$

$$\frac{\Gamma \vdash m : \rho_1\mathrm{Ch}\, (\rho_2(x : A).B) \qquad \rho_1\, \mathrm{xor}\, \rho_2 = !}{\Gamma \vdash \mathrm{send}\, m : \Pi_{\mathrm{L}}(x : A).\mathrm{T}\, (\rho_1\mathrm{Ch}\, B)} \qquad \frac{\Gamma \vdash m : \rho_1\mathrm{Ch}\, (\rho_2\{x : A\}.B) \qquad \rho_1\, \mathrm{xor}\, \rho_2 = !}{\Gamma \vdash \mathrm{send}\, \{m\} : \Pi_{\mathrm{L}}\{x : A\}.\mathrm{T}\, (\rho_1\mathrm{Ch}\, B)}$$

$$\frac{\Gamma \vdash m : \rho_1\mathrm{Ch}\, (\mathrm{end}\rho_2) \qquad \rho_1\, \mathrm{xor}\, \rho_2 = ?}{\Gamma \vdash \mathrm{wait}\, m : \mathrm{T}\, 1} \qquad \frac{\Gamma \vdash m : \rho_1\mathrm{Ch}\, (\mathrm{end}\rho_2) \qquad \rho_1\, \mathrm{xor}\, \rho_2 = !}{\Gamma \vdash \mathrm{close}\, m : \mathrm{T}\, 1}$$

3 Dynamic Fragment

Dynamic Context

$$\frac{}{\epsilon;\epsilon\vdash}\qquad\frac{\Gamma;\Delta\vdash\quad \Gamma\vdash A:s\quad x\in\mathit{fresh}(\Gamma)}{\Gamma,x:A;\Delta,x:_s A\vdash}\qquad\frac{\Gamma;\Delta\vdash\quad \Gamma\vdash A:s\quad x\in\mathit{fresh}(\Gamma)}{\Gamma,x:A;\Delta\vdash}$$

Context Merge

$$\frac{}{\epsilon\uplus\epsilon=\epsilon}\qquad\frac{\Delta_1\uplus\Delta_2=\Delta\quad x\in\mathit{fresh}(\Delta)}{(\Delta_1,x:_{\mathrm{U}}A)\uplus(\Delta_2,x:_{\mathrm{U}}A)=(\Delta,x:_{\mathrm{U}}A)}$$

$$\frac{\Delta_1\uplus\Delta_2=\Delta\quad x\in\mathit{fresh}(\Delta)}{(\Delta_1,x:_{\mathrm{L}}A)\uplus\Delta_2=(\Delta,x:_{\mathrm{L}}A)}\qquad\frac{\Delta_1\uplus\Delta_2=\Delta\quad x\in\mathit{fresh}(\Delta)}{\Delta_1\uplus(\Delta_2,x:_{\mathrm{L}}A)=(\Delta,x:_{\mathrm{L}}A)}$$

Context Constraint

$$\frac{}{\epsilon\triangleright s}\qquad\frac{\Delta\triangleright\mathrm{U}}{\Delta,x:_{\mathrm{U}}A\triangleright\mathrm{U}}\qquad\frac{\Delta\triangleright\mathrm{L}}{\Delta,x:_s A\triangleright\mathrm{L}}$$

Core Typing

$$\frac{\epsilon;\Gamma,x:A;\Delta,x:_s A\vdash\quad \Delta\triangleright\mathrm{U}}{\epsilon;\Gamma,x:A;\Delta,x:_s A\vdash x:A}\qquad\frac{\Theta;\Gamma,x:A;\Delta,x:_s A\vdash m:B\quad \Theta\triangleright t\quad \Delta\triangleright t}{\Theta;\Gamma;\Delta\vdash\lambda_t(x:A).m:\Pi_t(x:A).B}$$

$$\frac{\Theta;\Gamma,x:A;\Delta\vdash m:B\quad \Theta\triangleright t\quad \Delta\triangleright t}{\Theta;\Gamma;\Delta\vdash\lambda_t\{x:A\}.m:\Pi_t\{x:A\}.B}\qquad\frac{\epsilon;\Gamma,x:A;\Delta,x:_{\mathrm{U}}A\vdash m:A\quad \Delta\triangleright\mathrm{U}}{\epsilon;\Gamma;\Delta\vdash\mu(x:A).m:A}$$

$$\frac{\Theta_1;\Gamma;\Delta_1\vdash m:\Pi_t(x:A).B\quad \Theta_2;\Gamma;\Delta_2\vdash n:A}{\Theta_1\uplus\Theta_2;\Gamma;\Delta_1\uplus\Delta_2\vdash m\ n:B[n/x]}\qquad\frac{\Theta;\Gamma;\Delta\vdash m:\Pi_t\{x:A\}.B\quad \Gamma\vdash n:A}{\Theta;\Gamma;\Delta\vdash m\ n:B[n/x]}$$

$$\frac{\Gamma\vdash\Sigma_t(x:A).B:t\quad \Theta_1;\Gamma;\Delta_1\vdash m:A\quad \Theta_2;\Gamma;\Delta_2\vdash n:B[m/x]}{\Theta_1\uplus\Theta_2;\Gamma;\Delta_1\uplus\Delta_2\vdash\langle m,n\rangle_t:\Sigma_t(x:A).B}$$

$$\frac{\Gamma\vdash\Sigma_t\{x:A\}.B:t\quad \Gamma\vdash m:A\quad \Theta;\Gamma;\Delta\vdash n:B[m/x]}{\Theta;\Gamma;\Delta\vdash\{m,n\}_t:\Sigma_t\{x:A\}.B}$$

$$\frac{\begin{array}{c}\Gamma,z:\Sigma_t(x:A).B\vdash C:s\quad \Theta_1;\Gamma;\Delta_1\vdash m:\Sigma_t(x:A).B\\ \Theta_2;\Gamma,x:A,y:B;\Delta_2,x:_{r1}A,y:_{r2}B\vdash n:C[\langle x,y\rangle_t/z]\end{array}}{\Theta_1\uplus\Theta_2;\Gamma;\Delta_1\uplus\Delta_2\vdash\mathrm{R}^{\Sigma}_{[z]C}(m,[x,y]n):C[m/z]}$$

$$\frac{\begin{array}{c}\Gamma,z:\Sigma_t\{x:A\}.B\vdash C:s\quad \Theta_1;\Gamma;\Delta_1\vdash m:\Sigma_t\{x:A\}.B\\ \Theta_2;\Gamma,x:A,y:B;\Delta_2,y:_r B\vdash n:C[\{x,y\}_t/z]\end{array}}{\Theta_1\uplus\Theta_2;\Gamma;\Delta_1\uplus\Delta_2\vdash\mathrm{R}^{\Sigma}_{[z]C}(m,[x,y]n):C[m/z]}$$

$$\frac{\Gamma\vdash B:s\quad \Theta;\Gamma;\Delta\vdash m:A\quad A\simeq B}{\Theta;\Gamma;\Delta\vdash m:B}$$

Data Typing

$$\frac{\Gamma;\Delta\vdash\qquad \Delta\triangleright\mathrm{U}}{\epsilon;\Gamma;\Delta\vdash ():1}\qquad\frac{\Gamma;\Delta\vdash\qquad \Delta\triangleright\mathrm{U}}{\epsilon;\Gamma;\Delta\vdash \mathrm{true}:2}\qquad\frac{\Gamma;\Delta\vdash\qquad \Delta\triangleright\mathrm{U}}{\epsilon;\Gamma;\Delta\vdash \mathrm{false}:2}$$

$$\frac{\Gamma,z:2\vdash A:s\qquad \Theta_1;\Gamma;\Delta_1\vdash m:2\qquad \Theta_2;\Gamma;\Delta_2\vdash n_1:A[\mathrm{true}/z]\qquad \Theta_2;\Gamma;\Delta_2\vdash n_2:A[\mathrm{false}/z]}{\Theta_1\mathbin{\dot\cup}\Theta_2;\Gamma;\Delta_1\mathbin{\dot\cup}\Delta_2\vdash \mathrm{R}^2_{[z]A}(m,n_1,n_2):A[m/z]}$$

Monadic Typing

$$\frac{\Theta;\Gamma;\Delta\vdash m:A}{\Theta;\Gamma;\Delta\vdash \mathrm{return}\ m:\mathrm{T}\,A}\qquad\frac{\Gamma\vdash B:s\qquad \Theta_1;\Gamma;\Delta_1\vdash m:\mathrm{T}\,A\qquad \Theta_2;\Gamma,x:A;\Delta_2,x:_r A\vdash n:\mathrm{T}\,B}{\Theta_1\mathbin{\dot\cup}\Theta_2;\Gamma;\Delta_1\mathbin{\dot\cup}\Delta_2\vdash \mathrm{let}\ x\leftarrow m\ \mathrm{in}\ n:\mathrm{T}\,B}$$

Session Typing

$$\frac{\Gamma;\Delta\vdash\qquad \epsilon\vdash A:\mathrm{proto}\qquad \Delta\triangleright\mathrm{U}}{c:_{\mathrm{L}}\rho\mathrm{Ch}\,A;\Gamma;\Delta\vdash c:\rho\mathrm{Ch}\,A}\qquad\frac{\Theta;\Gamma,x:+\mathrm{Ch}\,A;\Delta,x:_{\mathrm{L}}+\mathrm{Ch}\,A\vdash m:\mathrm{T}\,1}{\Theta;\Gamma;\Delta\vdash \mathrm{fork}\,(x:A).m:\mathrm{T}\,(-\mathrm{Ch}\,A)}$$

$$\frac{\Theta;\Gamma;\Delta\vdash m:\rho_1\mathrm{Ch}\,(\rho_2(x:A).B)\qquad \rho_1\ \mathrm{xor}\ \rho_2={?}}{\Theta;\Gamma;\Delta\vdash \mathrm{recv}\,m:\mathrm{T}\,(\Sigma_{\mathrm{L}}(x:A).\rho_1\mathrm{Ch}\,B)}\qquad\frac{\Theta;\Gamma;\Delta\vdash m:\rho_1\mathrm{Ch}\,(\rho_2\{x:A\}.B)\qquad \rho_1\ \mathrm{xor}\ \rho_2={?}}{\Theta;\Gamma;\Delta\vdash \mathrm{recv}\,\{m\}:\mathrm{T}\,(\Sigma_{\mathrm{L}}\{x:A\}.\rho_1\mathrm{Ch}\,B)}$$

$$\frac{\Theta;\Gamma;\Delta\vdash m:\rho_1\mathrm{Ch}\,(\rho_2(x:A).B)\qquad \rho_1\ \mathrm{xor}\ \rho_2={!}}{\Theta;\Gamma;\Delta\vdash \mathrm{send}\,m:\Pi_{\mathrm{L}}(x:A).\mathrm{T}\,(\rho_1\mathrm{Ch}\,B)}\qquad\frac{\Theta;\Gamma;\Delta\vdash m:\rho_1\mathrm{Ch}\,(\rho_2\{x:A\}.B)\qquad \rho_1\ \mathrm{xor}\ \rho_2={!}}{\Theta;\Gamma;\Delta\vdash \mathrm{send}\,\{m\}:\Pi_{\mathrm{L}}\{x:A\}.\mathrm{T}\,(\rho_1\mathrm{Ch}\,B)}$$

$$\frac{\Theta;\Gamma;\Delta\vdash m:\rho_1\mathrm{Ch}\,(\mathrm{end}\rho_2)\qquad \rho_1\ \mathrm{xor}\ \rho_2={?}}{\Theta;\Gamma;\Delta\vdash \mathrm{wait}\,m:\mathrm{T}\,1}\qquad\frac{\Theta;\Gamma;\Delta\vdash m:\rho_1\mathrm{Ch}\,(\mathrm{end}\rho_2)\qquad \rho_1\ \mathrm{xor}\ \rho_2={!}}{\Theta;\Gamma;\Delta\vdash \mathrm{close}\,m:\mathrm{T}\,1}$$

4 Process Fragment

Process Typing

$$\frac{\Theta; \epsilon; \epsilon \vdash m : \mathrm{T}\,1}{\Theta \vdash \langle m \rangle} \qquad \frac{\Theta_1 \vdash p \qquad \Theta_2 \vdash q}{\Theta_1 \uplus \Theta_2 \vdash p \mid q} \qquad \frac{\Theta, c :_{\mathrm{L}} \rho\mathrm{Ch}\, A, d :_{\mathrm{L}} \neg\rho\mathrm{Ch}\, A \vdash p}{\Theta \vdash \nu cd.p}$$

Structural Congruence

Structural congruence is the least congruence relation with the following properties.

$$\frac{}{p \mid q \equiv q \mid p} \qquad \frac{}{o \mid (p \mid q) \equiv (o \mid p) \mid q} \qquad \frac{}{p \mid \langle \mathrm{return}\ () \rangle \equiv p}$$

$$\frac{}{\nu cd.p \mid q \equiv \nu cd.(p \mid q)} \qquad \frac{}{\nu cd.p \equiv \nu dc.p} \qquad \frac{}{\nu cd.\nu c'd'.p \equiv \nu c'd'.\nu cd.p}$$

Reductions

$$\frac{p \Rrightarrow q}{o \mid p \Rrightarrow o \mid q} \qquad \frac{p \Rrightarrow q}{\nu cd.p \Rrightarrow \nu cd.q} \qquad \frac{p \equiv p' \qquad p' \Rrightarrow q' \qquad q' \equiv q}{p \Rrightarrow q} \qquad \frac{m \rightsquigarrow\!\!\!\!\rightarrow m'}{\langle m \rangle \Rrightarrow \langle m' \rangle}$$

$$\frac{}{\langle \mathrm{let}\ x \leftarrow \mathrm{return}\ v\ \mathrm{in}\ m \rangle \Rrightarrow \langle m[v/x] \rangle} \qquad \frac{}{\langle \mathrm{let}\ x \leftarrow \mathrm{fork}\,(y : A).m\ \mathrm{in}\ n \rangle \Rrightarrow \nu cd.(\langle n[c/x] \rangle \mid \langle m[d/y] \rangle)}$$

$$\frac{}{\begin{array}{c}\nu cd.(\langle \mathrm{let}\ x \leftarrow \mathrm{send}\, c\ v\ \mathrm{in}\ n_1 \rangle \mid \langle \mathrm{let}\ y \leftarrow \mathrm{recv}\, d\ \mathrm{in}\ n_2 \rangle) \Rrightarrow \\ \nu cd.(\langle \mathrm{let}\ x \leftarrow \mathrm{return}\ c\ \mathrm{in}\ n_1 \rangle \mid \langle \mathrm{let}\ y \leftarrow \mathrm{return}\ \langle v, d \rangle_L\ \mathrm{in}\ n_2 \rangle)\end{array}}$$

$$\frac{}{\begin{array}{c}\nu cd.(\langle \mathrm{let}\ x \leftarrow \mathrm{send}\,\{c\}\ m\ \mathrm{in}\ n_1 \rangle \mid \langle \mathrm{let}\ y \leftarrow \mathrm{recv}\,\{d\}\ \mathrm{in}\ n_2 \rangle) \Rrightarrow \\ \nu cd.(\langle \mathrm{let}\ x \leftarrow \mathrm{return}\ c\ \mathrm{in}\ n_1 \rangle \mid \langle \mathrm{let}\ y \leftarrow \mathrm{return}\ \{m, d\}_L\ \mathrm{in}\ n_2 \rangle)\end{array}}$$

$$\frac{}{\begin{array}{c}\nu cd.(\langle \mathrm{let}\ x \leftarrow \mathrm{close}\, c\ \mathrm{in}\ m \rangle \mid \langle \mathrm{let}\ y \leftarrow \mathrm{wait}\, d\ \mathrm{in}\ n \rangle) \Rrightarrow \\ \langle \mathrm{let}\ x \leftarrow \mathrm{return}\ ()\ \mathrm{in}\ m \rangle \mid \langle \mathrm{let}\ x \leftarrow \mathrm{return}\ ()\ \mathrm{in}\ n \rangle\end{array}}$$