1 LOGICAL TYPING

1.1 Global Context

$$\frac{}{\epsilon \vdash \mathit{global}}\textsc{global-empty} \qquad \frac{\Gamma \vdash \mathit{global} \qquad \Gamma \vdash m : A}{\Gamma, x := m : A \vdash \mathit{global}}\textsc{global-def}$$

$$(\forall i)\,\frac{\begin{array}{l}\Gamma \vdash \mathit{global} \qquad \Gamma, \Theta \vdash s : \mathrm{U} \\ \Gamma, \mathrm{ind}\ \{d : \Theta.s\}\ [], \Theta_i \vdash \mathrm{ind}\ \{d, \overline{m_{ij}}^{j}\} : s \qquad \mathit{positive}(d, \Theta_i) \qquad d \notin \mathit{FV}(\overline{m_{ij}}^{j})\end{array}}{\Gamma, \mathrm{ind}\ \{d : \Theta.s\}\ [\overline{c_i : \Theta_i.\mathrm{ind}\ \{d, \overline{m_{ij}}^{j}\}}^{i}\ ] \vdash \mathit{global}}\textsc{global-ind}_0$$

$$(\forall i)\,\frac{\begin{array}{l}\Gamma \vdash \mathit{global} \qquad \Gamma, \Theta \vdash s : \mathrm{U} \\ \Gamma, \mathrm{ind}\ (d : \Theta.s)\ [], \Theta_i \vdash \mathrm{ind}\ (d, \overline{m_{ij}}^{j}) : s \qquad \mathit{positive}(d, \Theta_i) \qquad d \notin \mathit{FV}(\overline{m_{ij}}^{j})\end{array}}{\Gamma, \mathrm{ind}\ (d : \Theta.s)\ [\overline{c_i : \Theta_i.\mathrm{ind}\ (d, \overline{m_{ij}}^{j})}^{i}\ ] \vdash \mathit{global}}\textsc{global-ind}_1$$

1.2 Local Context

$$\frac{\Gamma \vdash \mathit{global}}{\Gamma \vdash \mathit{local}}\textsc{local-empty} \qquad \frac{\Gamma \vdash \mathit{local} \qquad \Gamma \vdash A : s}{\Gamma, x : A \vdash \mathit{local}}\textsc{local-var}$$

1.3 Typing

$$\frac{\Gamma \vdash \mathit{local}}{\Gamma \vdash s : \mathrm{U}}\textsc{logic-sort} \qquad \frac{\Gamma \vdash \mathit{local} \qquad x := m : A \in \Gamma}{\Gamma \vdash x : A}\textsc{logic-def} \qquad \frac{\Gamma \vdash \mathit{local} \qquad x : A \in \Gamma}{\Gamma \vdash x : A}\textsc{logic-var}$$

$$\frac{\Gamma \vdash A : s \qquad \Gamma, x : A \vdash B : r}{\Gamma \vdash \Pi_t\{x : A\}.B : t}\textsc{logic-}\Pi_0 \qquad \frac{\Gamma \vdash A : s \qquad \Gamma, x : A \vdash B : r}{\Gamma \vdash \Pi_t(x : A).B : t}\textsc{logic-}\Pi_1$$

$$\frac{\Gamma, x : A \vdash m : B}{\Gamma \vdash \lambda_t\{x : A\}.B : \Pi_t\{x : A\}.B}\textsc{logic-}\lambda_0 \qquad \frac{\Gamma, x : A \vdash m : B}{\Gamma \vdash \lambda_t(x : A).B : \Pi_t(x : A).B}\textsc{logic-}\lambda_1$$

$$\frac{\Gamma \vdash m : \Pi_t\{x : A\}.B \qquad \Gamma \vdash n : A}{\Gamma \vdash m\ \{n\} : B[n/x]}\textsc{logic-app}_0 \qquad \frac{\Gamma \vdash m : \Pi_t(x : A).B \qquad \Gamma \vdash n : A}{\Gamma \vdash m\ n : B[n/x]}\textsc{logic-app}_1$$

$$\frac{\mathrm{ind}\ \{d : \Theta.s\}\ [\overline{c_i : \Theta_i.\mathrm{ind}\ \{d, \overline{m_{ij}}^j\}}^i\,] \in \Gamma \qquad \Gamma \vdash \overline{n_k}^k : \Theta}{\Gamma \vdash \mathrm{ind}\ \{d, \overline{n_k}^k\} : s}\textsc{logic-ind}_0$$

$$\frac{\mathrm{ind}\ (d : \Theta.s)\ [\overline{c_i : \Theta_i.\mathrm{ind}\ (d, \overline{m_{ij}}^j)}^i\,] \in \Gamma \qquad \Gamma \vdash \overline{n_k}^k : \Theta}{\Gamma \vdash \mathrm{ind}\ (d, \overline{n_k}^k) : s}\textsc{logic-ind}_1$$

$$\frac{\mathrm{ind}\ \{d : \Theta.s\}\ [\overline{c_i : \Theta_i.\mathrm{ind}\ \{d, \overline{m_{ij}}^j\}}^i\,] \in \Gamma \qquad \Gamma \vdash \overline{n_k}^k : \Theta_i}{\Gamma \vdash \mathrm{constr}\ \{c_i, \overline{n_k}^k\} : \mathrm{ind}\ \{d, \overline{m_{ij}}^j\}[\overline{n_k}^k/\Theta_i]}\textsc{local-constr}_0$$

$$\frac{\mathrm{ind}\ (d : \Theta.s)\ [\overline{c_i : \Theta_i.\mathrm{ind}\ (d, \overline{m_{ij}}^j)}^i\,] \in \Gamma \qquad \Gamma \vdash \overline{n_k}^k : \Theta_i}{\Gamma \vdash \mathrm{constr}\ (c_i, \overline{n_k}^k) : \mathrm{ind}\ (d, \overline{m_{ij}}^j)[\overline{n_k}^k/\Theta_i]}\textsc{local-constr}_1$$

$$\frac{\begin{array}{l}\mathrm{ind}\ \{d : \Theta.s\}\ [\overline{c_i : \Theta_i.\mathrm{ind}\ \{d, \overline{m_{ij}}^j\}}^i\,] \in \Gamma \\ \Gamma \vdash m : \mathrm{ind}\ \{d, \overline{n_k}^k\} \qquad \Gamma, \Theta, x : \mathrm{ind}\ \{d, \Theta\} \vdash A : s \qquad \Gamma, \Theta_i \vdash n_i : A[\overline{m_{ij}}^j/\Theta, \mathrm{ind}\ \{d, \overline{m_{ij}}^j\}/x]\end{array}}{\Gamma \vdash \mathrm{case}\ \{m, \Theta(x : \mathrm{ind}\ \{d, \Theta\}).A\}\ [\overline{\Theta_i.n_i}^i\,] : A[\overline{n_k}^k/\Theta, m/x]}$$

$$\frac{\begin{array}{l}\mathrm{ind}\ (d : \Theta.s)\ [\overline{c_i : \Theta_i.\mathrm{ind}\ \{d, \overline{m_{ij}}^j\}}^i\,] \in \Gamma \\ \Gamma \vdash m : \mathrm{ind}\ (d, \overline{n_k}^k) \qquad \Gamma, \Theta, x : \mathrm{ind}\ (d, \Theta) \vdash A : s \qquad \Gamma, \Theta_i \vdash n_i : A[\overline{m_{ij}}^j/\Theta, \mathrm{ind}\ \{d, \overline{m_{ij}}^j\}/x]\end{array}}{\Gamma \vdash \mathrm{case}\ (m, \Theta(x : \mathrm{ind}\ (d, \Theta)).A)\ [\overline{\Theta_i.n_i}^i\,] : A[\overline{n_k}^k/\Theta, m/x]}$$