A Two Level Dependent Session Type Theory

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1 Syntax

variables	x, y, z		
channels	c, d		
sorts	s, r, t	$::=$	$\mathrm{U} \mid \mathrm{L}$
roles	ρ	$::=$	$+ \mid -$
terms	m, n, A, B, C	$::=$	$x \mid c \mid s$
		$\mid$	$\Pi_t(x : A).B \mid \Pi_t\{x : A\}.B \mid \Sigma_t(x : A).B \mid \Sigma_t\{x : A\}.B$
		$\mid$	$\lambda_t(x : A).m \mid \lambda_t\{x : A\}.m \mid \langle m, n\rangle_t \mid \{m, n\}_t$
		$\mid$	$m\ n \mid m\ \{n\} \mid \mathrm{R}^{\Sigma}_{[z]A}(m, [x, y]n) \mid \mu(x : A).m$
		$\mid$	$1 \mid () \mid 2 \mid \mathrm{true} \mid \mathrm{false} \mid \mathrm{R}^{2}_{[z]A}(m, n_1, n_2)$
		$\mid$	$\mathrm{T}\ A \mid \mathrm{return}\ m \mid \mathrm{let}\ x \Leftarrow m\ \mathrm{in}\ n$
		$\mid$	$\mathrm{proto} \mid \mathrm{end}\rho \mid \rho(x : A).B \mid \rho\{x : A\}.B \mid \rho\mathrm{Ch}\ A$
		$\mid$	$\mathrm{fork}\ (x : A).m \mid \mathrm{recv}\ m \mid \underline{\mathrm{recv}}\ m \mid \mathrm{send}\ m \mid \underline{\mathrm{send}}\ m$
		$\mid$	$\mathrm{close}\ m \mid \mathrm{wait}\ m$
values	u, v	$::=$	$x \mid c \mid \lambda_t(x : A).m \mid \lambda_t\{x : A\}.m \mid \langle u, v\rangle_t \mid \{v, m\}_t$
		$\mid$	$() \mid \mathrm{true} \mid \mathrm{false} \mid \mathrm{return}\ v \mid \mathrm{let}\ x \Leftarrow v\ \mathrm{in}\ m$
		$\mid$	$\mathrm{fork}\ (x : A).m \mid \mathrm{recv}\ v \mid \underline{\mathrm{recv}}\ v \mid \mathrm{send}\ v \mid \underline{\mathrm{send}}\ v$
		$\mid$	$\mathrm{send}\ v\ u \mid \underline{\mathrm{send}}\ v\ \{m\} \mid \mathrm{close}\ v \mid \mathrm{wait}\ v$
process	o, p, q	$\mid$	$\langle m\rangle \mid p \mid q \mid \nu cd.p$

2 Static Fragment

Sort Order

$$\frac{}{\mathrm{U} \sqsubseteq s} \qquad \frac{}{\mathrm{L} \sqsubseteq \mathrm{L}}$$

Static Context

$$\frac{}{\epsilon \vdash} \qquad \frac{\Gamma \vdash \quad \Gamma \vdash A : s \quad x \in \mathit{fresh}(\Gamma)}{\Gamma, x : A \vdash}$$

Core Typing

$$\frac{\Gamma \vdash}{\Gamma \vdash s : \mathrm{U}} \qquad \frac{\Gamma, x : A \vdash}{\Gamma, x : A \vdash x : A} \qquad \frac{\Gamma \vdash A : s \quad \Gamma, x : A \vdash B : r}{\Gamma \vdash \Pi_t(x : A).B : t} \qquad \frac{\Gamma \vdash A : s \quad \Gamma, x : A \vdash B : r}{\Gamma \vdash \Pi_t\{x : A\}.B : t}$$

$$\frac{\Gamma, x : A \vdash m : B}{\Gamma \vdash \lambda_t(x : A).m : \Pi_t(x : A).B} \qquad \frac{\Gamma, x : A \vdash m : B}{\Gamma \vdash \lambda_t\{x : A\}.m : \Pi_t\{x : A\}.B} \qquad \frac{\Gamma, x : A \vdash m : A}{\Gamma \vdash \mu(x : A).m : A}$$

$$\frac{\Gamma \vdash m : \Pi_t(x : A).B \quad \Gamma \vdash n : A}{\Gamma \vdash m\ n : B[n/x]} \qquad \frac{\Gamma \vdash m : \Pi_t\{x : A\}.B \quad \Gamma \vdash n : A}{\Gamma \vdash m\ \{n\} : B[n/x]}$$

$$\frac{s \sqsubseteq t \quad r \sqsubseteq t \quad \Gamma \vdash A : s \quad \Gamma, x : A \vdash B : r}{\Gamma \vdash \Sigma_t(x : A).B : t} \qquad \frac{r \sqsubseteq t \quad \Gamma \vdash A : s \quad \Gamma, x : A \vdash B : r}{\Gamma \vdash \Sigma_t\{x : A\}.B : t}$$

$$\frac{\Gamma \vdash \Sigma_t(x : A).B : t \quad \Gamma \vdash m : A \quad \Gamma \vdash n : B[m/x]}{\Gamma \vdash \langle m, n\rangle_t : \Sigma_t(x : A).B} \qquad \frac{\Gamma \vdash \Sigma_t\{x : A\}.B : t \quad \Gamma \vdash m : A \quad \Gamma \vdash n : B[m/x]}{\Gamma \vdash \{m, n\}_t : \Sigma_t\{x : A\}.B}$$

$$\frac{\Gamma, z : \Sigma_t(x : A).B \vdash C : s \quad \Gamma \vdash m : \Sigma_t(x : A).B \quad \Gamma, x : A, y : B \vdash n : C[\langle x, y\rangle_t/z]}{\Gamma \vdash \mathrm{R}^{\Sigma}_{[z]C}(m, [x, y]n) : C[m/z]}$$

$$\frac{\Gamma, z : \Sigma_t\{x : A\}.B \vdash C : s \quad \Gamma \vdash m : \Sigma_t\{x : A\}.B \quad \Gamma, x : A, y : B \vdash n : C[\{x, y\}_t/z]}{\Gamma \vdash \mathrm{R}^{\Sigma}_{[z]C}(m, [x, y]n) : C[m/z]}$$

$$\frac{\Gamma \vdash B : s \quad \Gamma \vdash m : A \quad A \simeq B}{\Gamma \vdash m : B}$$

Data Typing

$$\frac{\Gamma \vdash}{\Gamma \vdash 1 : \mathrm{U}} \qquad \frac{\Gamma \vdash}{\Gamma \vdash () : 1} \qquad \frac{\Gamma \vdash}{\Gamma \vdash 2 : \mathrm{U}} \qquad \frac{\Gamma \vdash}{\Gamma \vdash \mathrm{true} : 2} \qquad \frac{\Gamma \vdash}{\Gamma \vdash \mathrm{false} : 2}$$

$$\frac{\Gamma, z : 2 \vdash A : s \qquad \Gamma \vdash m : 2 \qquad \Gamma \vdash n_1 : A[\mathrm{true}/z] \qquad \Gamma \vdash n_2 : A[\mathrm{false}/z]}{\Gamma \vdash \mathrm{R}^2_{[z]A}(m, n_1, n_2) : A[m/z]}$$

Monadic Typing

$$\frac{\Gamma \vdash A : s}{\Gamma \vdash \mathrm{T}\, A : \mathrm{L}} \qquad \frac{\Gamma \vdash m : A}{\Gamma \vdash \mathrm{return}\ m : \mathrm{T}\, A} \qquad \frac{\Gamma \vdash B : s \qquad \Gamma \vdash m : \mathrm{T}\, A \qquad \Gamma, x : A \vdash n : \mathrm{T}\, B}{\Gamma \vdash \mathrm{let}\ x \Leftarrow m\ \mathrm{in}\ n : \mathrm{T}\, B}$$

Session Typing

$$\frac{\Gamma \vdash}{\Gamma \vdash \mathrm{proto} : \mathrm{U}} \qquad \frac{\Gamma \vdash}{\Gamma \vdash \mathrm{end}\rho : \mathrm{proto}} \qquad \frac{\Gamma, x : A \vdash B : \mathrm{proto}}{\Gamma \vdash \rho\{x : A\}.B : \mathrm{proto}} \qquad \frac{\Gamma, x : A \vdash B : \mathrm{proto}}{\Gamma \vdash \rho(x : A).B : \mathrm{proto}} \qquad \frac{\Gamma \vdash A : \mathrm{proto}}{\Gamma \vdash \rho \mathrm{Ch}\, A : \mathrm{L}}$$

$$\frac{\Gamma \vdash \qquad \epsilon \vdash A : \mathrm{proto}}{\Gamma \vdash c : \rho \mathrm{Ch}\, A} \qquad \frac{\Gamma, x : +\mathrm{Ch}\, A \vdash m : \mathrm{T}\, 1}{\Gamma \vdash \mathrm{fork}\, (x : A).m : \mathrm{T}\, (-\mathrm{Ch}\, A)}$$

$$\frac{\Gamma \vdash m : \rho_1 \mathrm{Ch}\, (\rho_2(x : A).B) \qquad \rho_1 \,\mathrm{xor}\, \rho_2 = ?}{\Gamma \vdash \mathrm{recv}\, m : \mathrm{T}\, (\Sigma_{\mathrm{L}}(x : A).\rho_1 \mathrm{Ch}\, B)} \qquad \frac{\Gamma \vdash m : \rho_1 \mathrm{Ch}\, (\rho_2\{x : A\}.B) \qquad \rho_1 \,\mathrm{xor}\, \rho_2 = ?}{\Gamma \vdash \underline{\mathrm{recv}}\, m : \mathrm{T}\, (\Sigma_{\mathrm{L}}\{x : A\}.\rho_1 \mathrm{Ch}\, B)}$$

$$\frac{\Gamma \vdash m : \rho_1 \mathrm{Ch}\, (\rho_2(x : A).B) \qquad \rho_1 \,\mathrm{xor}\, \rho_2 = !}{\Gamma \vdash \mathrm{send}\, m : \Pi_{\mathrm{L}}(x : A).\mathrm{T}\, (\rho_1 \mathrm{Ch}\, B)} \qquad \frac{\Gamma \vdash m : \rho_1 \mathrm{Ch}\, (\rho_2\{x : A\}.B) \qquad \rho_1 \,\mathrm{xor}\, \rho_2 = !}{\Gamma \vdash \underline{\mathrm{send}}\, m : \Pi_{\mathrm{L}}\{x : A\}.\mathrm{T}\, (\rho_1 \mathrm{Ch}\, B)}$$

$$\frac{\Gamma \vdash m : \rho_1 \mathrm{Ch}\, (\mathrm{end}\rho_2) \qquad \rho_1 \,\mathrm{xor}\, \rho_2 = ?}{\Gamma \vdash \mathrm{wait}\, m : \mathrm{T}\, 1} \qquad \frac{\Gamma \vdash m : \rho_1 \mathrm{Ch}\, (\mathrm{end}\rho_2) \qquad \rho_1 \,\mathrm{xor}\, \rho_2 = !}{\Gamma \vdash \mathrm{close}\, m : \mathrm{T}\, 1}$$

3 Dynamic Fragment

Dynamic Context

$$\frac{}{\epsilon; \epsilon \vdash} \qquad \frac{\Gamma; \Delta \vdash \qquad \Gamma \vdash A : s \qquad x \in \mathit{fresh}(\Gamma)}{\Gamma, x : A; \Delta, x :_s A \vdash} \qquad \frac{\Gamma; \Delta \vdash \qquad \Gamma \vdash A : s \qquad x \in \mathit{fresh}(\Gamma)}{\Gamma, x : A; \Delta \vdash}$$

Context Merge

$$\frac{}{\epsilon \mathbin{\dot\cup} \epsilon = \epsilon} \qquad \frac{\Delta_1 \mathbin{\dot\cup} \Delta_2 = \Delta \qquad x \in \mathit{fresh}(\Delta)}{(\Delta_1, x :_{\mathrm{U}} A) \mathbin{\dot\cup} (\Delta_2, x :_{\mathrm{U}} A) = (\Delta, x :_{\mathrm{U}} A)}$$

$$\frac{\Delta_1 \mathbin{\dot\cup} \Delta_2 = \Delta \qquad x \in \mathit{fresh}(\Delta)}{(\Delta_1, x :_{\mathrm{L}} A) \mathbin{\dot\cup} \Delta_2 = (\Delta, x :_{\mathrm{L}} A)} \qquad \frac{\Delta_1 \mathbin{\dot\cup} \Delta_2 = \Delta \qquad x \in \mathit{fresh}(\Delta)}{\Delta_1 \mathbin{\dot\cup} (\Delta_2, x :_{\mathrm{L}} A) = (\Delta, x :_{\mathrm{L}} A)}$$

Context Constraint

$$\frac{}{\epsilon \triangleright s} \qquad \frac{\Delta \triangleright \mathrm{U}}{\Delta, x :_{\mathrm{U}} A \triangleright \mathrm{U}} \qquad \frac{\Delta \triangleright \mathrm{L}}{\Delta, x :_s A \triangleright \mathrm{L}}$$

Core Typing

$$\frac{\epsilon; \Gamma, x : A; \Delta, x :_s A \vdash \qquad \Delta \triangleright \mathrm{U}}{\epsilon; \Gamma, x : A; \Delta, x :_s A \vdash x : A} \qquad \frac{\Theta; \Gamma, x : A; \Delta, x :_s A \vdash m : B \qquad \Theta \triangleright t \qquad \Delta \triangleright t}{\Theta; \Gamma; \Delta \vdash \lambda_t(x : A).m : \Pi_t(x : A).B}$$

$$\frac{\Theta; \Gamma, x : A; \Delta \vdash m : B \qquad \Theta \triangleright t \qquad \Delta \triangleright t}{\Theta; \Gamma; \Delta \vdash \lambda_t\{x : A\}.m : \Pi_t\{x : A\}.B} \qquad \frac{\epsilon; \Gamma, x : A; \Delta, x :_{\mathrm{U}} A \vdash m : A \qquad \Delta \triangleright \mathrm{U}}{\epsilon; \Gamma; \Delta \vdash \mu(x : A).m : A}$$

$$\frac{\Theta_1; \Gamma; \Delta_1 \vdash m : \Pi_t(x : A).B \qquad \Theta_2; \Gamma; \Delta_2 \vdash n : A}{\Theta_1 \mathbin{\dot\cup} \Theta_2; \Gamma; \Delta_1 \mathbin{\dot\cup} \Delta_2 \vdash m\ n : B[n/x]} \qquad \frac{\Theta; \Gamma; \Delta \vdash m : \Pi_t\{x : A\}.B \qquad \Gamma \vdash n : A}{\Theta; \Gamma; \Delta \vdash m\ \{n\} : B[n/x]}$$

$$\frac{\Gamma \vdash \Sigma_t(x : A).B : t \qquad \Theta_1; \Gamma; \Delta_1 \vdash m : A \qquad \Theta_2; \Gamma; \Delta_2 \vdash n : B[m/x]}{\Theta_1 \mathbin{\dot\cup} \Theta_2; \Gamma; \Delta_1 \mathbin{\dot\cup} \Delta_2 \vdash \langle m, n \rangle_t : \Sigma_t(x : A).B}$$

$$\frac{\Gamma \vdash \Sigma_t\{x : A\}.B : t \qquad \Gamma \vdash m : A \qquad \Theta; \Gamma; \Delta \vdash n : B[m/x]}{\Theta; \Gamma; \Delta \vdash \{m, n\}_t : \Sigma_t\{x : A\}.B}$$

$$\frac{\begin{array}{c}\Gamma, z : \Sigma_t(x : A).B \vdash C : s \qquad \Theta_1; \Gamma; \Delta_1 \vdash m : \Sigma_t(x : A).B \\ \Theta_2; \Gamma, x : A, y : B; \Delta_2, x :_{r1} A, y :_{r2} B \vdash n : C[\langle x, y \rangle_t / z]\end{array}}{\Theta_1 \mathbin{\dot\cup} \Theta_2; \Gamma; \Delta_1 \mathbin{\dot\cup} \Delta_2 \vdash \mathrm{R}^{\Sigma}_{[z]C}(m, [x, y]n) : C[m/z]}$$

$$\frac{\begin{array}{c}\Gamma, z : \Sigma_t\{x : A\}.B \vdash C : s \qquad \Theta_1; \Gamma; \Delta_1 \vdash m : \Sigma_t\{x : A\}.B \\ \Theta_2; \Gamma, x : A, y : B; \Delta_2, y :_r B \vdash n : C[\{x, y\}_t / z]\end{array}}{\Theta_1 \mathbin{\dot\cup} \Theta_2; \Gamma; \Delta_1 \mathbin{\dot\cup} \Delta_2 \vdash \mathrm{R}^{\Sigma}_{[z]C}(m, [x, y]n) : C[m/z]}$$

$$\frac{\Gamma \vdash B : s \qquad \Theta; \Gamma; \Delta \vdash m : A \qquad A \simeq B}{\Theta; \Gamma; \Delta \vdash m : B}$$

Data Typing

$$\frac{\Gamma; \Delta \vdash \qquad \Delta \triangleright \mathrm{U}}{\epsilon; \Gamma; \Delta \vdash () : 1} \qquad \frac{\Gamma; \Delta \vdash \qquad \Delta \triangleright \mathrm{U}}{\epsilon; \Gamma; \Delta \vdash \mathrm{true} : 2} \qquad \frac{\Gamma; \Delta \vdash \qquad \Delta \triangleright \mathrm{U}}{\epsilon; \Gamma; \Delta \vdash \mathrm{false} : 2}$$

$$\frac{\Gamma, z : 2 \vdash A : s \qquad \Theta_1; \Gamma; \Delta_1 \vdash m : 2 \qquad \Theta_2; \Gamma; \Delta_2 \vdash n_1 : A[\mathrm{true}/z] \qquad \Theta_2; \Gamma; \Delta_2 \vdash n_2 : A[\mathrm{false}/z]}{\Theta_1 \dot\cup \Theta_2; \Gamma; \Delta_1 \dot\cup \Delta_2 \vdash \mathrm{R}^2_{[z]A}(m, n_1, n_2) : A[m/z]}$$

Monadic Typing

$$\frac{\Theta; \Gamma; \Delta \vdash m : A}{\Theta; \Gamma; \Delta \vdash \mathrm{return}\ m : \mathrm{T}\,A} \qquad \frac{\Gamma \vdash B : s \qquad \Theta_1; \Gamma; \Delta_1 \vdash m : \mathrm{T}\,A \qquad \Theta_2; \Gamma, x : A; \Delta_2, x :_r A \vdash n : \mathrm{T}\,B}{\Theta_1 \dot\cup \Theta_2; \Gamma; \Delta_1 \dot\cup \Delta_2 \vdash \mathrm{let}\ x \Leftarrow m\ \mathrm{in}\ n : \mathrm{T}\,B}$$

Session Typing

$$\frac{\Gamma; \Delta \vdash \qquad \epsilon \vdash A : \mathrm{proto} \qquad \Delta \triangleright \mathrm{U}}{c :_{\mathrm{L}} \rho\mathrm{Ch}\,A; \Gamma; \Delta \vdash c : \rho\mathrm{Ch}\,A} \qquad \frac{\Theta; \Gamma, x : +\mathrm{Ch}\,A; \Delta, x :_{\mathrm{L}} +\mathrm{Ch}\,A \vdash m : \mathrm{T}\,1}{\Theta; \Gamma; \Delta \vdash \mathrm{fork}\,(x : A).m : \mathrm{T}\,(-\mathrm{Ch}\,A)}$$

$$\frac{\Theta; \Gamma; \Delta \vdash m : \rho_1\mathrm{Ch}\,(\rho_2(x : A).B) \qquad \rho_1\ \mathrm{xor}\ \rho_2 = ?}{\Theta; \Gamma; \Delta \vdash \mathrm{recv}\,m : \mathrm{T}\,(\Sigma_{\mathrm{L}}(x : A).\rho_1\mathrm{Ch}\,B)} \qquad \frac{\Theta; \Gamma; \Delta \vdash m : \rho_1\mathrm{Ch}\,(\rho_2\{x : A\}.B) \qquad \rho_1\ \mathrm{xor}\ \rho_2 = ?}{\Theta; \Gamma; \Delta \vdash \underline{\mathrm{recv}}\,m : \mathrm{T}\,(\Sigma_{\mathrm{L}}\{x : A\}.\rho_1\mathrm{Ch}\,B)}$$

$$\frac{\Theta; \Gamma; \Delta \vdash m : \rho_1\mathrm{Ch}\,(\rho_2(x : A).B) \qquad \rho_1\ \mathrm{xor}\ \rho_2 = !}{\Theta; \Gamma; \Delta \vdash \mathrm{send}\,m : \Pi_{\mathrm{L}}(x : A).\mathrm{T}\,(\rho_1\mathrm{Ch}\,B)} \qquad \frac{\Theta; \Gamma; \Delta \vdash m : \rho_1\mathrm{Ch}\,(\rho_2\{x : A\}.B) \qquad \rho_1\ \mathrm{xor}\ \rho_2 = !}{\Theta; \Gamma; \Delta \vdash \underline{\mathrm{send}}\,m : \Pi_{\mathrm{L}}\{x : A\}.\mathrm{T}\,(\rho_1\mathrm{Ch}\,B)}$$

$$\frac{\Theta; \Gamma; \Delta \vdash m : \rho_1\mathrm{Ch}\,(\mathrm{end}\rho_2) \qquad \rho_1\ \mathrm{xor}\ \rho_2 = ?}{\Theta; \Gamma; \Delta \vdash \mathrm{wait}\,m : \mathrm{T}\,1} \qquad \frac{\Theta; \Gamma; \Delta \vdash m : \rho_1\mathrm{Ch}\,(\mathrm{end}\rho_2) \qquad \rho_1\ \mathrm{xor}\ \rho_2 = !}{\Theta; \Gamma; \Delta \vdash \mathrm{close}\,m : \mathrm{T}\,1}$$

4 Process Fragment

Process Typing

$$\frac{\Theta; \epsilon; \epsilon \vdash m : \mathrm{T}\,1}{\Theta \vdash \langle m \rangle} \qquad \frac{\Theta_1 \vdash p \qquad \Theta_2 \vdash q}{\Theta_1 \uplus \Theta_2 \vdash p \mid q} \qquad \frac{\Theta, c :_{\mathrm{L}} \rho\mathrm{Ch}\, A, d :_{\mathrm{L}} \neg\rho\mathrm{Ch}\, A \vdash p}{\Theta \vdash \nu cd.p}$$

Structural Congruence

Structural congruence is the least congruence relation with the following properties.

$$p \mid q \equiv q \mid p \qquad o \mid (p \mid q) \equiv (o \mid p) \mid q \qquad p \mid \langle \mathrm{return}\ () \rangle \equiv p$$

$$\nu cd.p \mid q \equiv \nu cd.(p \mid q) \qquad \nu cd.p \equiv \nu dc.p \qquad \nu cd.\nu c'd'.p \equiv \nu c'd'.\nu cd.p$$

Reductions

$$\frac{p \Rrightarrow q}{o \mid p \Rrightarrow o \mid q} \qquad \frac{p \Rrightarrow q}{\nu cd.p \Rrightarrow \nu cd.q} \qquad \frac{p \equiv p' \qquad p' \Rrightarrow q' \qquad q' \equiv q}{p \Rrightarrow q} \qquad \frac{m \rightsquigarrow\!\!\!\!\rightarrow m'}{\langle m \rangle \Rrightarrow \langle m' \rangle}$$

$$\frac{}{\langle \mathrm{let}\ x \Leftarrow \mathrm{return}\ v\ \mathrm{in}\ m \rangle \Rrightarrow \langle m[v/x] \rangle} \qquad \frac{}{\langle \mathrm{let}\ x \Leftarrow \mathrm{fork}\,(y : A).m\ \mathrm{in}\ n \rangle \Rrightarrow \nu cd.(\langle n[c/x] \rangle \mid \langle m[d/y] \rangle)}$$

$$\frac{}{\begin{array}{c}\nu cd.(\langle \mathrm{let}\ x \Leftarrow \mathrm{send}\, c\ v\ \mathrm{in}\ n_1 \rangle \mid \langle \mathrm{let}\ y \Leftarrow \mathrm{recv}\, d\ \mathrm{in}\ n_2 \rangle) \Rrightarrow \\ \nu cd.(\langle \mathrm{let}\ x \Leftarrow \mathrm{return}\ c\ \mathrm{in}\ n_1 \rangle \mid \langle \mathrm{let}\ y \Leftarrow \mathrm{return}\ \langle v, d \rangle_L\ \mathrm{in}\ n_2 \rangle)\end{array}}$$

$$\frac{}{\begin{array}{c}\nu cd.(\langle \mathrm{let}\ x \Leftarrow \underline{\mathrm{send}}\, c\ \{m\}\ \mathrm{in}\ n_1 \rangle \mid \langle \mathrm{let}\ y \Leftarrow \underline{\mathrm{recv}}\, d\ \mathrm{in}\ n_2 \rangle) \Rrightarrow \\ \nu cd.(\langle \mathrm{let}\ x \Leftarrow \mathrm{return}\ c\ \mathrm{in}\ n_1 \rangle \mid \langle \mathrm{let}\ y \Leftarrow \mathrm{return}\ \{m, d\}_L\ \mathrm{in}\ n_2 \rangle)\end{array}}$$

$$\frac{}{\begin{array}{c}\nu cd.(\langle \mathrm{let}\ x \Leftarrow \mathrm{close}\, c\ \mathrm{in}\ m \rangle \mid \langle \mathrm{let}\ y \Leftarrow \mathrm{wait}\, d\ \mathrm{in}\ n \rangle) \Rrightarrow \\ \langle \mathrm{let}\ x \Leftarrow \mathrm{return}\ ()\ \mathrm{in}\ m \rangle \mid \langle \mathrm{let}\ x \Leftarrow \mathrm{return}\ ()\ \mathrm{in}\ n \rangle\end{array}}$$