# A Two Level Dependent Session Type Theory

Qiancheng Fu[1] and Hongwei Xi[1]

[1]Boston University

February 13, 2023

## 1 Syntax

| | | | |
|---|---|---|---|
| variables | $x, y, z$ | | |
| channels | $c, d$ | | |
| sorts | $s, r, t$ | ::= | $\mathrm{U} \mid \mathrm{L}$ |
| roles | $\rho$ | ::= | $+ \mid -$ |
| terms | $m, n, A, B, C$ | ::= | $x \mid c \mid s$ |
| | | $\mid$ | $\Pi_t(x : A).B \mid \Pi_t\{x : A\}.B \mid \Sigma_t(x : A).B \mid \Sigma_t\{x : A\}.B$ |
| | | $\mid$ | $\lambda_t(x : A).m \mid \lambda_t\{x : A\}.m \mid \langle m, n \rangle_t \mid \{m, n\}_t$ |
| | | $\mid$ | $m\ n \mid \mathrm{R}^{\Sigma}_{[z]A}(m, [x, y]n) \mid \mu(x : A).m$ |
| | | $\mid$ | $1 \mid () \mid 2 \mid \mathrm{true} \mid \mathrm{false} \mid \mathrm{R}^{2}_{[z]A}(m, n_1, n_2)$ |
| | | $\mid$ | $\mathrm{T}\ A \mid \mathrm{return}\ m \mid \mathrm{let}\ x \leftarrow m\ \mathrm{in}\ n$ |
| | | $\mid$ | $\mathrm{proto} \mid \mathrm{end}\rho \mid \rho\{x : A\}.B \mid \rho(x : A).B \mid \rho\mathrm{Ch}\ A$ |
| | | $\mid$ | $\mathrm{fork}\ (x : A).m \mid \mathrm{recv}\ m \mid \mathrm{recv}\ \{m\} \mid \mathrm{send}\ m \mid \mathrm{send}\ \{m\}$ |
| | | $\mid$ | $\mathrm{close}\ m \mid \mathrm{wait}\ m$ |
| values | $u, v$ | ::= | $x \mid c \mid \lambda_t(x : A).m \mid \lambda_t\{x : A\}.m \mid \langle u, v \rangle_t \mid \{v, m\}_t$ |
| | | $\mid$ | $() \mid \mathrm{true} \mid \mathrm{false} \mid \mathrm{return}\ v \mid \mathrm{let}\ x \leftarrow v\ \mathrm{in}\ m$ |
| | | $\mid$ | $\mathrm{fork}\ (x : A).m \mid \mathrm{recv}\ v \mid \mathrm{recv}\ \{v\} \mid \mathrm{send}\ v \mid \mathrm{send}\ \{v\}$ |
| | | $\mid$ | $\mathrm{send}\ u\ v \mid \mathrm{send}\ \{v\}\ m \mid \mathrm{close}\ v \mid \mathrm{wait}\ v$ |
| process | $o, p, q$ | $\mid$ | $\langle m \rangle \mid p \mid q \mid \nu cd.p$ |

## 2 Static Fragment

**Sort Order**

$$\overline{\mathrm{U} \sqsubseteq s} \qquad\qquad \overline{\mathrm{L} \sqsubseteq \mathrm{L}}$$

**Static Context**

$$\overline{\epsilon \vdash} \qquad\qquad \frac{\Gamma \vdash \quad \Gamma \vdash A : s \quad x \in \mathit{fresh}(\Gamma)}{\Gamma, x : A \vdash}$$

**Core Typing**

$$\frac{\Gamma \vdash}{\Gamma \vdash s : \mathrm{U}} \qquad \frac{\Gamma, x : A \vdash}{\Gamma, x : A \vdash x : A} \qquad \frac{\Gamma \vdash A : s \quad \Gamma, x : A \vdash B : r}{\Gamma \vdash \Pi_t(x : A).B : t} \qquad \frac{\Gamma \vdash A : s \quad \Gamma, x : A \vdash B : r}{\Gamma \vdash \Pi_t\{x : A\}.B : t}$$

$$\frac{\Gamma, x : A \vdash m : B}{\Gamma \vdash \lambda_t(x : A).m : \Pi_t(x : A).B} \qquad \frac{\Gamma, x : A \vdash m : B}{\Gamma \vdash \lambda_t\{x : A\}.m : \Pi_t\{x : A\}.B} \qquad \frac{\Gamma, x : A \vdash m : A}{\Gamma \vdash \mu(x : A).m : A}$$

$$\frac{\Gamma \vdash m : \Pi_t(x : A).B \quad \Gamma \vdash n : A}{\Gamma \vdash m\ n : B[n/x]} \qquad \frac{\Gamma \vdash m : \Pi_t\{x : A\}.B \quad \Gamma \vdash n : A}{\Gamma \vdash m\ n : B[n/x]}$$

$$\frac{s \sqsubseteq t \quad r \sqsubseteq t \quad \Gamma \vdash A : s \quad \Gamma, x : A \vdash B : r}{\Gamma \vdash \Sigma_t(x : A).B : t} \qquad \frac{r \sqsubseteq t \quad \Gamma \vdash A : s \quad \Gamma, x : A \vdash B : r}{\Gamma \vdash \Sigma_t\{x : A\}.B : t}$$

$$\frac{\Gamma \vdash \Sigma_t(x : A).B \quad \Gamma \vdash m : A \quad \Gamma \vdash n : B[m/x]}{\Gamma \vdash \langle m, n \rangle_t : \Sigma_t(x : A).B} \qquad \frac{\Gamma \vdash \Sigma_t\{x : A\}.B \quad \Gamma \vdash m : A \quad \Gamma \vdash n : B[m/x]}{\Gamma \vdash \{m, n\}_t : \Sigma_t\{x : A\}.B}$$

$$\frac{\Gamma, z : \Sigma_t(x : A).B \vdash C : s \quad \Gamma \vdash m : \Sigma_t(x : A).B \quad \Gamma, x : A, y : B \vdash n : C[\langle x, y \rangle_t/z]}{\Gamma \vdash \mathrm{R}^{\Sigma}_{[z]C}(m, [x, y]n) : C[m/z]}$$

$$\frac{\Gamma, z : \Sigma_t\{x : A\}.B \vdash C : s \quad \Gamma \vdash m : \Sigma_t\{x : A\}.B \quad \Gamma, x : A, y : B \vdash n : C[\{x, y\}_t/z]}{\Gamma \vdash \mathrm{R}^{\Sigma}_{[z]C}(m, [x, y]n) : C[m/z]}$$

$$\frac{\Gamma \vdash B : s \quad \Gamma \vdash m : A \quad A \simeq B}{\Gamma \vdash m : B}$$

## Data Typing

$$\frac{\Gamma \vdash}{\Gamma \vdash 1 : \mathrm{U}} \qquad \frac{\Gamma \vdash}{\Gamma \vdash () : 1} \qquad \frac{\Gamma \vdash}{\Gamma \vdash 2 : \mathrm{U}} \qquad \frac{\Gamma \vdash}{\Gamma \vdash \mathrm{true} : 2} \qquad \frac{\Gamma \vdash}{\Gamma \vdash \mathrm{false} : 2}$$

$$\frac{\Gamma, z : 2 \vdash A : s \qquad \Gamma \vdash m : 2 \qquad \Gamma \vdash n_1 : A[\mathrm{true}/z] \qquad \Gamma \vdash n_2 : A[\mathrm{false}/z]}{\Gamma \vdash \mathrm{R}^2_{[z]A}(m, n_1, n_2) : A[m/z]}$$

## Monadic Typing

$$\frac{\Gamma \vdash A : s}{\Gamma \vdash \mathrm{T}\,A : \mathrm{L}} \qquad \frac{\Gamma \vdash m : A}{\Gamma \vdash \mathrm{return}\,m : \mathrm{T}\,A} \qquad \frac{\Gamma \vdash B : s \qquad \Gamma \vdash m : \mathrm{T}\,A \qquad \Gamma, x : A \vdash n : \mathrm{T}\,B}{\Gamma \vdash \mathrm{let}\,x : A \leftarrow m\,\mathrm{in}\,n : \mathrm{T}\,B}$$

## Session Typing

$$\frac{\Gamma \vdash}{\Gamma \vdash \mathrm{proto} : \mathrm{U}} \qquad \frac{\Gamma \vdash}{\Gamma \vdash \mathrm{end}\rho : \mathrm{proto}} \qquad \frac{\Gamma, x : A \vdash B : \mathrm{proto}}{\Gamma \vdash \rho\{x : A\}.B : \mathrm{proto}} \qquad \frac{\Gamma, x : A \vdash B : \mathrm{proto}}{\Gamma \vdash \rho(x : A).B : \mathrm{proto}} \qquad \frac{\Gamma \vdash A : \mathrm{proto}}{\Gamma \vdash \rho\mathrm{Ch}\,A : \mathrm{L}}$$

$$\frac{\Gamma \vdash \qquad \epsilon \vdash A : \mathrm{proto}}{\Gamma \vdash c : \rho\mathrm{Ch}\,A} \qquad\qquad \frac{\Gamma, x : +\mathrm{Ch}\,A \vdash m : \mathrm{T}\,1}{\Gamma \vdash \mathrm{fork}\,(x : A).m : \mathrm{T}\,(-\mathrm{Ch}\,A)}$$

$$\frac{\Gamma \vdash m : \rho_1\mathrm{Ch}\,(\rho_2(x : A).B) \qquad \rho_1\,\mathrm{xor}\,\rho_2 =?}{\Gamma \vdash \mathrm{recv}\,m : \mathrm{T}\,(\Sigma_\mathrm{L}(x : A).\rho_1\mathrm{Ch}\,B)} \qquad \frac{\Gamma \vdash m : \rho_1\mathrm{Ch}\,(\rho_2\{x : A\}.B) \qquad \rho_1\,\mathrm{xor}\,\rho_2 =?}{\Gamma \vdash \mathrm{recv}\,\{m\} : \mathrm{T}\,(\Sigma_\mathrm{L}\{x : A\}.\rho_1\mathrm{Ch}\,B)}$$

$$\frac{\Gamma \vdash m : \rho_1\mathrm{Ch}\,(\rho_2(x : A).B) \qquad \rho_1\,\mathrm{xor}\,\rho_2 =!}{\Gamma \vdash \mathrm{send}\,m : \Pi_\mathrm{L}(x : A).\mathrm{T}\,(\rho_1\mathrm{Ch}\,B)} \qquad \frac{\Gamma \vdash m : \rho_1\mathrm{Ch}\,(\rho_2\{x : A\}.B) \qquad \rho_1\,\mathrm{xor}\,\rho_2 =!}{\Gamma \vdash \mathrm{send}\,\{m\} : \Pi_\mathrm{L}\{x : A\}.\mathrm{T}\,(\rho_1\mathrm{Ch}\,B)}$$

$$\frac{\Gamma \vdash m : \rho_1\mathrm{Ch}\,(\mathrm{end}\rho_2) \qquad \rho_1\,\mathrm{xor}\,\rho_2 =?}{\Gamma \vdash \mathrm{wait}\,m : \mathrm{T}\,1} \qquad \frac{\Gamma \vdash m : \rho_1\mathrm{Ch}\,(\mathrm{end}\rho_2) \qquad \rho_1\,\mathrm{xor}\,\rho_2 =!}{\Gamma \vdash \mathrm{close}\,m : \mathrm{T}\,1}$$

# 3 Dynamic Fragment

**Dynamic Context**

$$\frac{}{\epsilon;\epsilon \vdash} \qquad \frac{\Gamma;\Delta \vdash \quad \Gamma \vdash A : s \quad x \in \mathit{fresh}(\Gamma)}{\Gamma, x : A; \Delta, x :_s A \vdash} \qquad \frac{\Gamma;\Delta \vdash \quad \Gamma \vdash A : s \quad x \in \mathit{fresh}(\Gamma)}{\Gamma, x : A; \Delta \vdash}$$

**Context Merge**

$$\frac{}{\epsilon \uplus \epsilon = \epsilon} \qquad \frac{\Delta_1 \uplus \Delta_2 = \Delta \quad x \in \mathit{fresh}(\Delta)}{(\Delta_1, x :_\mathrm{U} A) \uplus (\Delta_2, x :_\mathrm{U} A) = (\Delta, x :_\mathrm{U} A)}$$

$$\frac{\Delta_1 \uplus \Delta_2 = \Delta \quad x \in \mathit{fresh}(\Delta)}{(\Delta_1, x :_\mathrm{L} A) \uplus \Delta_2 = (\Delta, x :_\mathrm{L} A)} \qquad \frac{\Delta_1 \uplus \Delta_2 = \Delta \quad x \in \mathit{fresh}(\Delta)}{\Delta_1 \uplus (\Delta_2, x :_\mathrm{L} A) = (\Delta, x :_\mathrm{L} A)}$$

**Context Constraint**

$$\frac{}{\epsilon \rhd s} \qquad \frac{\Delta \rhd \mathrm{U}}{\Delta, x :_\mathrm{U} A \rhd \mathrm{U}} \qquad \frac{\Delta \rhd \mathrm{L}}{\Delta, x :_s A \rhd \mathrm{L}}$$

**Core Typing**

$$\frac{\epsilon;\Gamma, x : A; \Delta, x :_s A \vdash \quad \Delta \rhd \mathrm{U}}{\epsilon;\Gamma, x : A; \Delta, x :_s A \vdash x : A} \qquad \frac{\Theta;\Gamma, x : A; \Delta, x :_s A \vdash m : B \quad \Theta \rhd t \quad \Delta \rhd t}{\Theta;\Gamma;\Delta \vdash \lambda_t(x : A).m : \Pi_t(x : A).B}$$

$$\frac{\Theta;\Gamma, x : A; \Delta \vdash m : B \quad \Theta \rhd t \quad \Delta \rhd t}{\Theta;\Gamma;\Delta \vdash \lambda_t\{x : A\}.m : \Pi_t\{x : A\}.B} \qquad \frac{\epsilon;\Gamma, x : A; \Delta, x :_\mathrm{U} A \vdash m : A \quad \Delta \rhd \mathrm{U}}{\epsilon;\Gamma;\Delta \vdash \mu(x : A).m : A}$$

$$\frac{\Theta_1;\Gamma;\Delta_1 \vdash m : \Pi_t(x : A).B \quad \Theta_2;\Gamma;\Delta_2 \vdash n : A}{\Theta_1 \uplus \Theta_2;\Gamma;\Delta_1 \uplus \Delta_2 \vdash m\ n : B[n/x]} \qquad \frac{\Theta;\Gamma;\Delta \vdash m : \Pi_t\{x : A\}.B \quad \Gamma \vdash n : A}{\Theta;\Gamma;\Delta \vdash m\ n : B[n/x]}$$

$$\frac{\Gamma \vdash \Sigma_t(x : A).B : t \quad \Theta_1;\Gamma;\Delta_1 \vdash m : A \quad \Theta_2;\Gamma;\Delta_2 \vdash n : B[m/x]}{\Theta_1 \uplus \Theta_2;\Gamma;\Delta_1 \uplus \Delta_2 \vdash \langle m, n \rangle_t : \Sigma_t(x : A).B}$$

$$\frac{\Gamma \vdash \Sigma_t\{x : A\}.B : t \quad \Gamma \vdash m : A \quad \Theta;\Gamma;\Delta \vdash n : B[m/x]}{\Theta;\Gamma;\Delta \vdash \{m, n\}_t : \Sigma_t\{x : A\}.B}$$

$$\frac{\begin{array}{c}\Gamma, z : \Sigma_t(x : A).B \vdash C : s \quad \Theta_1;\Gamma;\Delta_1 \vdash m : \Sigma_t(x : A).B \\ \Theta_2;\Gamma, x : A, y : B; \Delta_2, x :_{r1} A, y :_{r2} B \vdash n : C[\langle x, y \rangle_t/z]\end{array}}{\Theta_1 \uplus \Theta_2;\Gamma;\Delta_1 \uplus \Delta_2 \vdash \mathrm{R}^{\Sigma}_{[z]C}(m, [x, y]n) : C[m/z]}$$

$$\frac{\begin{array}{c}\Gamma, z : \Sigma_t\{x : A\}.B \vdash C : s \quad \Theta_1;\Gamma;\Delta_1 \vdash m : \Sigma_t\{x : A\}.B \\ \Theta_2;\Gamma, x : A, y : B; \Delta_2, y :_r B \vdash n : C[\{x, y\}_t/z]\end{array}}{\Theta_1 \uplus \Theta_2;\Gamma;\Delta_1 \uplus \Delta_2 \vdash \mathrm{R}^{\Sigma}_{[z]C}(m, [x, y]n) : C[m/z]}$$

$$\frac{\Gamma \vdash B : s \quad \Theta;\Gamma;\Delta \vdash m : A \quad A \simeq B}{\Theta;\Gamma;\Delta \vdash m : B}$$

## Data Typing

$$\frac{\Gamma;\Delta \vdash \qquad \Delta \triangleright \mathrm{U}}{\epsilon;\Gamma;\Delta \vdash () : 1} \qquad\qquad \frac{\Gamma;\Delta \vdash \qquad \Delta \triangleright \mathrm{U}}{\epsilon;\Gamma;\Delta \vdash \mathrm{true} : 2} \qquad\qquad \frac{\Gamma;\Delta \vdash \qquad \Delta \triangleright \mathrm{U}}{\epsilon;\Gamma;\Delta \vdash \mathrm{false} : 2}$$

$$\frac{\Gamma, z : 2 \vdash A : s \qquad \Theta_1;\Gamma;\Delta_1 \vdash m : 2 \qquad \Theta_2;\Gamma;\Delta_2 \vdash n_1 : A[\mathrm{true}/z] \qquad \Theta_2;\Gamma;\Delta_2 \vdash n_2 : A[\mathrm{false}/z]}{\Theta_1 \uplus \Theta_2;\Gamma;\Delta_1 \uplus \Delta_2 \vdash \mathrm{R}^2_{[z]A}(m, n_1, n_2) : A[m/z]}$$

## Monadic Typing

$$\frac{\Theta;\Gamma;\Delta \vdash m : A}{\Theta;\Gamma;\Delta \vdash \mathrm{return}\, m : \mathrm{T}\, A} \qquad\qquad \frac{\Gamma \vdash B : s \qquad \Theta_1;\Gamma;\Delta_1 \vdash m : \mathrm{T}\, A \qquad \Theta_2;\Gamma, x : A;\Delta_2, x :_r A \vdash n : \mathrm{T}\, B}{\Theta_1 \uplus \Theta_2;\Gamma;\Delta_1 \uplus \Delta_2 \vdash \mathrm{let}\, x \leftarrow m\, \mathrm{in}\, n : \mathrm{T}\, B}$$

## Session Typing

$$\frac{\Gamma;\Delta \vdash \qquad \epsilon \vdash A : \mathrm{proto} \qquad \Delta \triangleright \mathrm{U}}{c :_{\mathrm{L}} \rho\mathrm{Ch}\, A;\Gamma;\Delta \vdash c : \rho\mathrm{Ch}\, A} \qquad\qquad \frac{\Theta;\Gamma, x : +\mathrm{Ch}\, A;\Delta, x :_{\mathrm{L}} +\mathrm{Ch}\, A \vdash m : \mathrm{T}\, 1}{\Theta;\Gamma;\Delta \vdash \mathrm{fork}\, (x : A).m : \mathrm{T}\, (-\mathrm{Ch}\, A)}$$

$$\frac{\Theta;\Gamma;\Delta \vdash m : \rho_1\mathrm{Ch}\, (\rho_2(x : A).B) \qquad \rho_1\, \mathrm{xor}\, \rho_2 = ?}{\Theta;\Gamma;\Delta \vdash \mathrm{recv}\, m : \mathrm{T}\, (\Sigma_{\mathrm{L}}(x : A).\rho_1\mathrm{Ch}\, B)} \qquad\qquad \frac{\Theta;\Gamma;\Delta \vdash m : \rho_1\mathrm{Ch}\, (\rho_2\{x : A\}.B) \qquad \rho_1\, \mathrm{xor}\, \rho_2 = ?}{\Theta;\Gamma;\Delta \vdash \mathrm{recv}\, \{m\} : \mathrm{T}\, (\Sigma_{\mathrm{L}}\{x : A\}.\rho_1\mathrm{Ch}\, B)}$$

$$\frac{\Theta;\Gamma;\Delta \vdash m : \rho_1\mathrm{Ch}\, (\rho_2(x : A).B) \qquad \rho_1\, \mathrm{xor}\, \rho_2 = !}{\Theta;\Gamma;\Delta \vdash \mathrm{send}\, m : \Pi_{\mathrm{L}}(x : A).\mathrm{T}\, (\rho_1\mathrm{Ch}\, B)} \qquad\qquad \frac{\Theta;\Gamma;\Delta \vdash m : \rho_1\mathrm{Ch}\, (\rho_2\{x : A\}.B) \qquad \rho_1\, \mathrm{xor}\, \rho_2 = !}{\Theta;\Gamma;\Delta \vdash \mathrm{send}\, \{m\} : \Pi_{\mathrm{L}}\{x : A\}.\mathrm{T}\, (\rho_1\mathrm{Ch}\, B)}$$

$$\frac{\Theta;\Gamma;\Delta \vdash m : \rho_1\mathrm{Ch}\, (\mathrm{end}\rho_2) \qquad \rho_1\, \mathrm{xor}\, \rho_2 = ?}{\Theta;\Gamma;\Delta \vdash \mathrm{wait}\, m : \mathrm{T}\, 1} \qquad\qquad \frac{\Theta;\Gamma;\Delta \vdash m : \rho_1\mathrm{Ch}\, (\mathrm{end}\rho_2) \qquad \rho_1\, \mathrm{xor}\, \rho_2 = !}{\Theta;\Gamma;\Delta \vdash \mathrm{close}\, m : \mathrm{T}\, 1}$$

# 4   Process Fragment

## Process Typing

$$\frac{\Theta; \epsilon; \epsilon \vdash m : \mathrm{T}\, 1}{\Theta \vdash \langle m \rangle} \qquad \frac{\Theta_1 \vdash p \qquad \Theta_2 \vdash q}{\Theta_1 \uplus \Theta_2 \vdash p \mid q} \qquad \frac{\Theta, c :_{\mathrm{L}} \rho \mathrm{Ch}\, A, d :_{\mathrm{L}} \neg \rho \mathrm{Ch}\, A \vdash p}{\Theta \vdash \nu cd.p}$$

## Structural Congruence

Structural congruence is the least congruence relation with the following properties.

$$p \mid q \equiv q \mid p \qquad\qquad o \mid (p \mid q) \equiv (o \mid p) \mid q \qquad\qquad p \mid \langle \mathrm{return}\ () \rangle \equiv p$$

$$\nu cd.p \mid q \equiv \nu cd.(p \mid q) \qquad\qquad \nu cd.p \equiv \nu dc.p \qquad\qquad \nu cd.\nu c'd'.p \equiv \nu c'd'.\nu cd.p$$

## Reductions

$$\frac{p \Rightarrow q}{o \mid p \Rightarrow o \mid q} \qquad \frac{p \Rightarrow q}{\nu cd.p \Rightarrow \nu cd.q} \qquad \frac{p \equiv p' \qquad p' \Rightarrow q' \qquad q' \equiv q}{p \Rightarrow q} \qquad \frac{m \rightsquigarrow m'}{\langle m \rangle \Rightarrow \langle m' \rangle}$$

$$\frac{}{\langle \mathrm{let}\ x \leftarrow \mathrm{return}\ v\ \mathrm{in}\ m \rangle \Rightarrow \langle m[v/x] \rangle} \qquad \frac{}{\langle \mathrm{let}\ x \leftarrow \mathrm{fork}\ (y : A).m\ \mathrm{in}\ n \rangle \Rightarrow \nu cd.(\langle n[c/x] \rangle \mid \langle m[d/y] \rangle)}$$

$$\frac{}{\begin{array}{c} \nu cd.(\langle \mathrm{let}\ x \leftarrow \mathrm{send}\ c\ v\ \mathrm{in}\ n_1 \rangle \mid \langle \mathrm{let}\ y \leftarrow \mathrm{recv}\ d\ \mathrm{in}\ n_2 \rangle) \Rightarrow \\ \nu cd.(\langle \mathrm{let}\ x \leftarrow \mathrm{return}\ c\ \mathrm{in}\ n_1 \rangle \mid \langle \mathrm{let}\ y \leftarrow \mathrm{return}\ \langle v, d \rangle_L\ \mathrm{in}\ n_2 \rangle) \end{array}}$$

$$\frac{}{\begin{array}{c} \nu cd.(\langle \mathrm{let}\ x \leftarrow \mathrm{send}\ \{c\}\ m\ \mathrm{in}\ n_1 \rangle \mid \langle \mathrm{let}\ y \leftarrow \mathrm{recv}\ \{d\}\ \mathrm{in}\ n_2 \rangle) \Rightarrow \\ \nu cd.(\langle \mathrm{let}\ x \leftarrow \mathrm{return}\ c\ \mathrm{in}\ n_1 \rangle \mid \langle \mathrm{let}\ y \leftarrow \mathrm{return}\ \{m, d\}_L\ \mathrm{in}\ n_2 \rangle) \end{array}}$$

$$\frac{}{\begin{array}{c} \nu cd.(\langle \mathrm{let}\ x \leftarrow \mathrm{close}\ c\ \mathrm{in}\ m \rangle \mid \langle \mathrm{let}\ y \leftarrow \mathrm{wait}\ d\ \mathrm{in}\ n \rangle) \Rightarrow \\ \langle \mathrm{let}\ x \leftarrow \mathrm{return}\ ()\ \mathrm{in}\ m \rangle \mid \langle \mathrm{let}\ x \leftarrow \mathrm{return}\ ()\ \mathrm{in}\ n \rangle \end{array}}$$