Homework 1

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Exercise 1

(a)

The marginal distribution $p(x)$:

X	1	2	3	4	5
$p(x)$	0.16	0.17	0.11	0.22	0.34

The marginal distribution $p(y)$:

Y	1	2	3
$p(y)$	0.26	0.47	0.47

(b)

Conditional distribution $p(x|Y = y_1)$:

X	1	2	3	4	5
$p(x\|Y = y_1)$	$\frac{0.01}{0.26} = \frac{1}{26}$	$\frac{0.02}{0.26} = \frac{1}{13}$	$\frac{0.03}{0.26} = \frac{3}{26}$	$\frac{0.1}{0.26} = \frac{5}{13}$	$\frac{0.1}{0.26} = \frac{5}{13}$

Conditional distribution $p(x|Y = y_3)$:

X	1	2	3	4	5
$p(x\|Y = y_3)$	$\frac{0.1}{0.27} = \frac{10}{27}$	$\frac{0.05}{0.27} = \frac{5}{27}$	$\frac{0.03}{0.27} = \frac{1}{9}$	$\frac{0.05}{0.27} = \frac{5}{27}$	$\frac{0.04}{0.27} = \frac{4}{27}$

Exercise 2: Show that: $E_X[X] = E_Y[E_X[x|y]]$

We consider the left-hand side:

$$\begin{aligned}
E_Y[E_X[x|y]] &= \Sigma_y E(X|Y = y)P(Y = y) \\
&= \Sigma_y \Sigma_x x P(X = x|Y = y)P(Y = y) \\
&= \Sigma_y \Sigma_x x P(Y = y|X = x)P(X = x) \\
&= \Sigma_x x P(X = x)\Sigma_y P(Y = y|X = x) \\
&(note : \Sigma_y P(Y = y|X = x) = 1) \\
&= \Sigma_x x P(X = x) \\
&= E_X[X] = \text{right-hand side}
\end{aligned}$$

Exercise 3

$P(X) = 0.207$

$P(Y) = 0.5$

$P(X|Y) = 0.365$

(a) Dùng cả X và Y:
$P(X,Y) = P(X|Y).P(Y) = 0.365 \times 0.5 = 0.1825$

(b) Dùng Y và biết rằng người đó không dùng X:
$P(Y|\bar{X}) = \frac{P(\bar{X}|Y).P(Y)}{P(\bar{X})} = \frac{(1 - P(X|Y)).P(Y)}{1 - P(X)} = \frac{(1 - 0.365) \times 0.5}{1 - 0.207} = 0.4004$

Exercise 4
$E[X] = \mu$
$Var(X) = \Sigma(X - \mu)^2.p_i$
$= E[(X - \mu)^2]$
$= E[X^2 - 2\mu X + \mu^2]$
$= E[X^2] - 2\mu E[X] + \mu^2$
$= E[X^2] - 2E[X]E[X] + (E[X])^2$
$= E[X^2] - 2(E[X])^2 + (E[X])^2$
$= E[X^2] - (E[X])^2$
So, $Var(X) = E[X^2] - (E[X])^2$

Exercise 5
Set C for car and G for goat. We can see that there are 3 cases to order 1 car and 2 goats.

	Door 1	Door 2	Door 3
Case 1	C	G	G
Case 2	G	C	G
Case 3	G	G	C

Suppose that we choose Door 1. $P(A)$ be the probability of choosing correctly the car. In case 1, if we change the choice we will get the goat and if we keep the first choice we will get the car. In case 2 and 3 if we change the door, we can get the car and vice versa.
If not change the Door $P(A) = \frac{1}{3}$
If change the Door $P(A) = \frac{2}{3}$
Therefore, we should change choice in second change in order to have higher probability of getting a car.