

Introduction to the ZX-calculus

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Outline

Introduction

The ZX-calculus

Notation

Equational theory

Applications

Optimisation of quantum circuits

Quantum error correction

Variants and extensions

Conclusions

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Motivation: quantum circuit optimisation

Quantum computational resources are limited, so we need to use them efficiently.

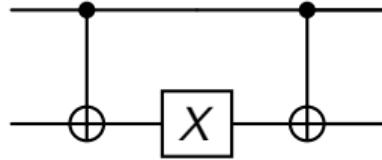
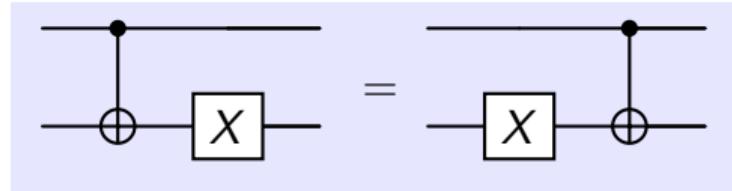
- ▶ Given a quantum circuit, can we find a more efficient circuit that describes the same linear map?

Motivation: quantum circuit optimisation

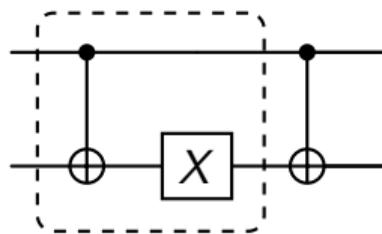
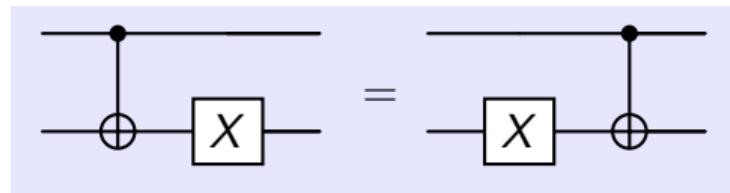
Quantum computational resources are limited, so we need to use them efficiently.

- ▶ Given a quantum circuit, can we find a more efficient circuit that describes the same linear map?
- ▶ If someone gives you a circuit and claims it is a more efficient version of the circuit you want to run, how can you check that the two circuits really describe the same linear map?

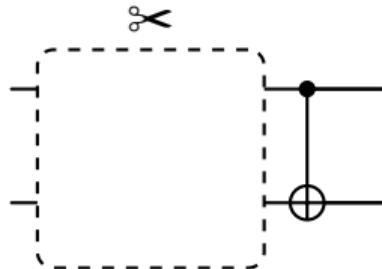
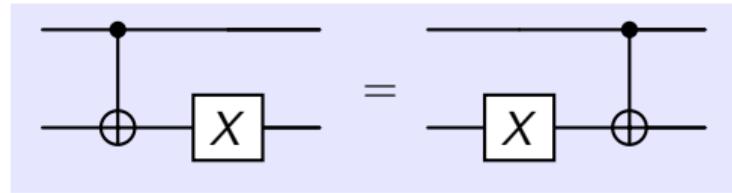
Rewriting quantum circuits



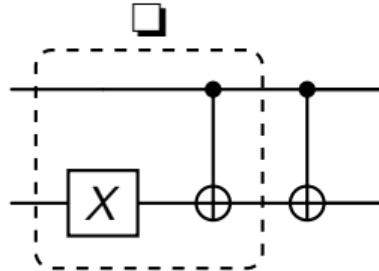
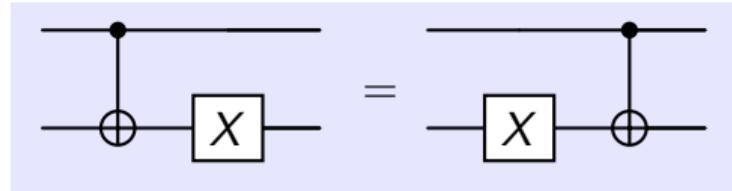
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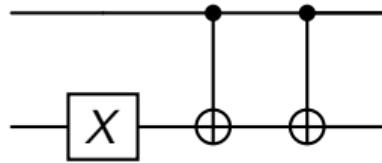
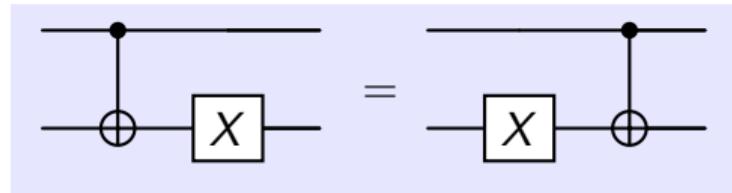
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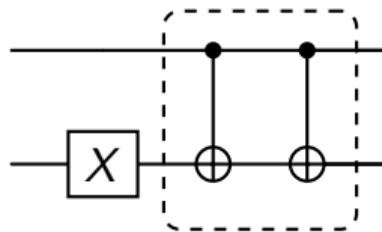
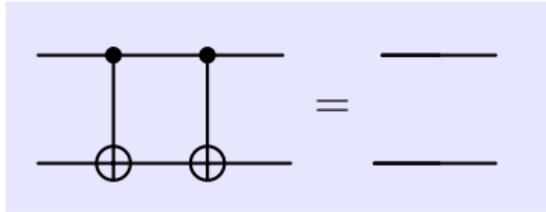
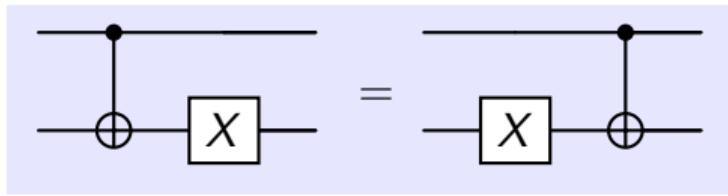
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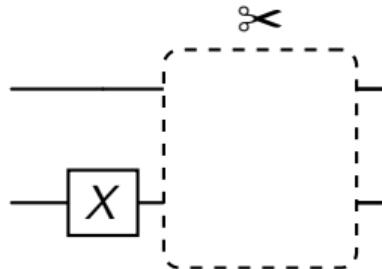
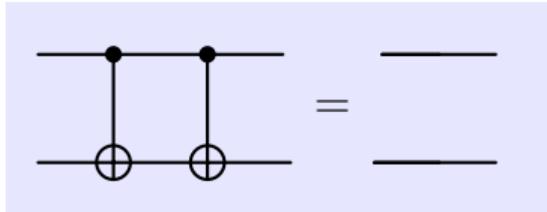
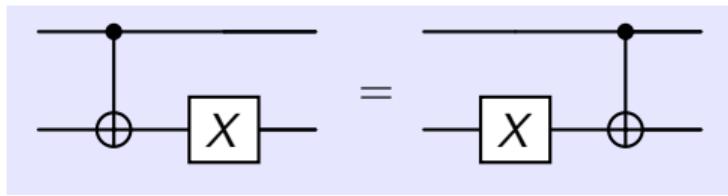
Rewriting quantum circuits



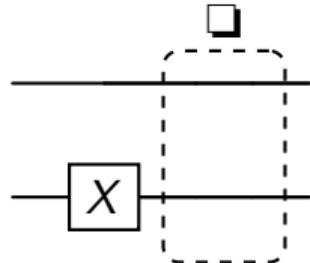
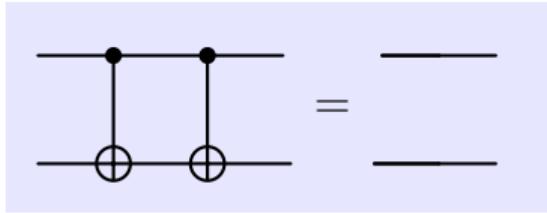
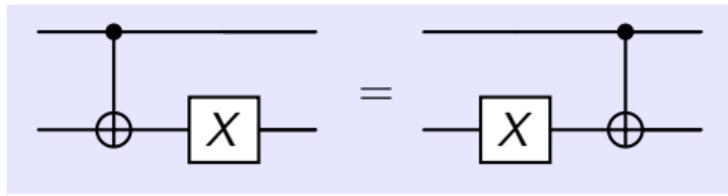
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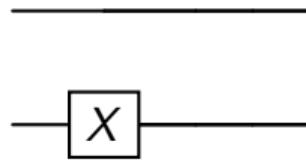
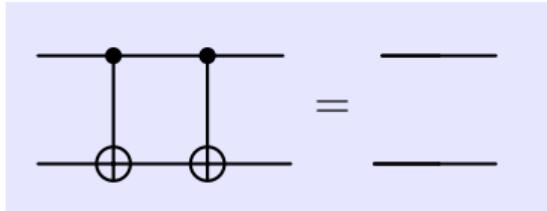
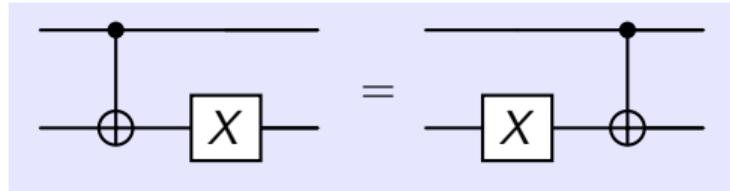
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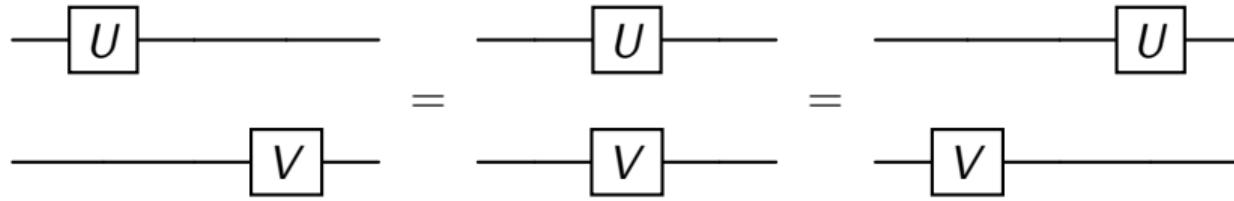
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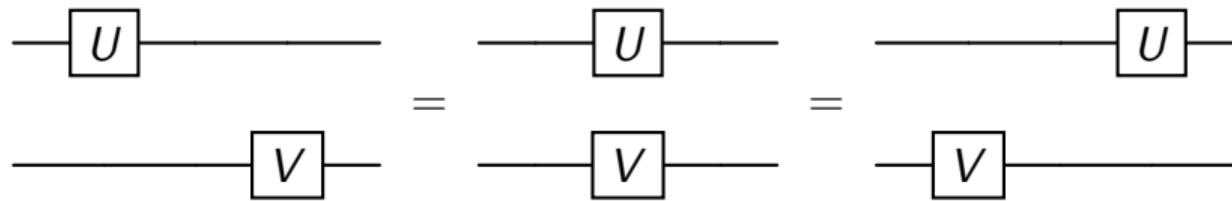
Rewriting quantum circuits



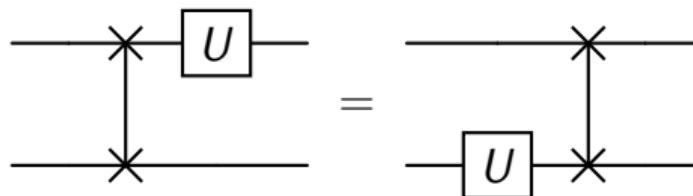
Some circuit equations are particularly intuitive



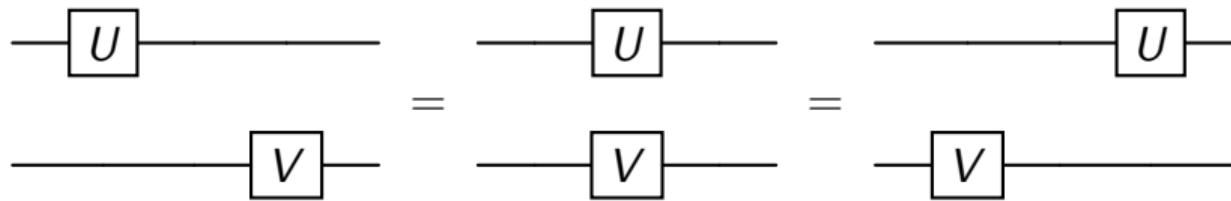
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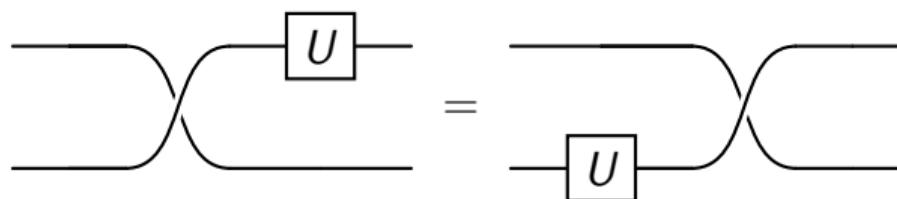
Notation can make a difference, for example the two symbols for the SWAP gate



Some circuit equations are particularly intuitive



Notation can make a difference, for example the two symbols for the SWAP gate



Equality up to scalar factor

$$\begin{array}{c} \text{---} \boxed{X} \text{---} \boxed{S} \text{---} \\ \rightsquigarrow \end{array} \quad \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ i & 0 \end{pmatrix}$$

$$\begin{array}{c} \text{---} \boxed{S^\dagger} \text{---} \boxed{X} \text{---} \\ \rightsquigarrow \end{array} \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix} = \begin{pmatrix} 0 & -i \\ 1 & 0 \end{pmatrix} = -i \begin{pmatrix} 0 & 1 \\ i & 0 \end{pmatrix}$$

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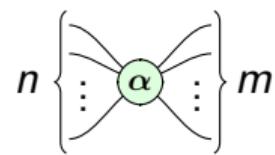
The ZX-calculus components: (mostly) spiders instead of gates

Hadamard gate

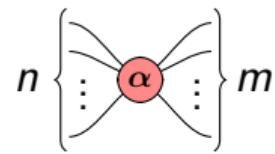
[Coecke & Duncan 2008]



Z-spider



X-spider



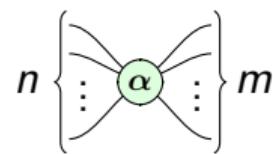
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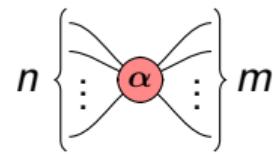
[Coecke & Duncan 2008]

$$\text{---} \square \text{---} \rightsquigarrow |+\rangle\langle 0| + |-\rangle\langle 1| = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Z-spider



X-spider



The ZX-calculus components: (mostly) spiders instead of gates

Hadamard gate

[Coecke & Duncan 2008]

$$\text{---} \square \text{---} \rightsquigarrow |+\rangle\langle 0| + |-\rangle\langle 1| = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Z-spider

$$n \left\{ \begin{array}{c} \text{---} \\ \vdots \\ \text{---} \end{array} \text{---} \right. \alpha \left. \begin{array}{c} \text{---} \\ \vdots \\ \text{---} \end{array} \right\} m \rightsquigarrow |\underbrace{0 \dots 0}_{m}\rangle\langle \underbrace{0 \dots 0}_n| + e^{i\alpha} |\underbrace{1 \dots 1}_m\rangle\langle \underbrace{1 \dots 1}_n| = \begin{pmatrix} 1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & e^{i\alpha} \end{pmatrix}$$

X-spider

$$n \left\{ \begin{array}{c} \text{---} \\ \vdots \\ \text{---} \end{array} \text{---} \right. \alpha \left. \begin{array}{c} \text{---} \\ \vdots \\ \text{---} \end{array} \right\} m$$

The ZX-calculus components: (mostly) spiders instead of gates

Hadamard gate

[Coecke & Duncan 2008]

$$\text{---} \square \text{---} \rightsquigarrow |+\rangle\langle 0| + |-\rangle\langle 1| = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Z-spider

$$n \left\{ \begin{array}{c} \text{---} \\ \vdots \\ \text{---} \end{array} \text{---} \right\} m \rightsquigarrow |\underbrace{0 \dots 0}_m\rangle\langle\underbrace{0 \dots 0}_n| + e^{i\alpha} |\underbrace{1 \dots 1}_m\rangle\langle\underbrace{1 \dots 1}_n| = \begin{pmatrix} 1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & e^{i\alpha} \end{pmatrix}$$

X-spider

$$n \left\{ \begin{array}{c} \text{---} \\ \vdots \\ \text{---} \end{array} \text{---} \right\} m \rightsquigarrow |\underbrace{+ \dots +}_m\rangle\langle\underbrace{+ \dots +}_n| + e^{i\alpha} |\underbrace{- \dots -}_m\rangle\langle\underbrace{- \dots -}_n|$$

A closer look at Z-spiders: unitaries

$$\text{---} \circled{\alpha} \text{---} \quad \leadsto \quad |0\rangle\langle 0| + e^{i\alpha} |1\rangle\langle 1| \quad = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{pmatrix}$$

A closer look at Z-spiders: unitaries

$$\text{---} \circledcirc \alpha \text{---} \rightsquigarrow |0\rangle\langle 0| + e^{i\alpha} |1\rangle\langle 1| = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{pmatrix}$$

$$\text{---} \circledcirc \pi \text{---} \rightsquigarrow |0\rangle\langle 0| - |1\rangle\langle 1| = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = Z$$

A closer look at Z-spiders: states

$$\textcircled{\alpha} \longrightarrow \rightsquigarrow |0\rangle + e^{i\alpha} |1\rangle = \begin{pmatrix} 1 \\ e^{i\alpha} \end{pmatrix}$$

A closer look at Z-spiders: states

$$\textcircled{\alpha} \text{---} \rightsquigarrow |0\rangle + e^{i\alpha} |1\rangle = \begin{pmatrix} 1 \\ e^{i\alpha} \end{pmatrix}$$

$$\textcircled{0} \text{---} \rightsquigarrow |0\rangle + |1\rangle = \sqrt{2} |+\rangle = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

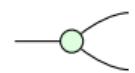
A closer look at Z-spiders: states

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$$\textcircled{0} \text{---} \rightsquigarrow |0\rangle + |1\rangle = \sqrt{2} |+\rangle = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\textcircled{\pi} \text{---} \rightsquigarrow |0\rangle - |1\rangle = \sqrt{2} |-\rangle = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

A closer look at Z-spiders: the copy map



\rightsquigarrow

$$|00\rangle\langle 0| + |11\rangle\langle 1|$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$$

A closer look at X-spiders: unitaries

$$\text{---} \circled{\alpha} \text{---} \quad \leadsto \quad |+\rangle\langle +| + e^{i\alpha} |-\rangle\langle -| = e^{i\alpha/2} \begin{pmatrix} \cos \frac{\alpha}{2} & i \sin \frac{\alpha}{2} \\ i \sin \frac{\alpha}{2} & \cos \frac{\alpha}{2} \end{pmatrix}$$

A closer look at X-spiders: unitaries

$$\text{---} \circled{\alpha} \text{---} \rightsquigarrow |+\rangle\langle +| + e^{i\alpha} |-\rangle\langle -| = e^{i\alpha/2} \begin{pmatrix} \cos \frac{\alpha}{2} & i \sin \frac{\alpha}{2} \\ i \sin \frac{\alpha}{2} & \cos \frac{\alpha}{2} \end{pmatrix}$$

$$\text{---} \circled{\pi} \text{---} \rightsquigarrow |+\rangle\langle +| - |-\rangle\langle -| = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = X$$

A closer look at X-spiders: states

$$\bullet - \quad \rightsquigarrow \quad |+\rangle + |-\rangle = \sqrt{2} |0\rangle \quad = \sqrt{2} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

A closer look at X-spiders: states

$$\bullet - \rightsquigarrow |+\rangle + |-\rangle = \sqrt{2} |0\rangle = \sqrt{2} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\pi - \rightsquigarrow |+\rangle - |-\rangle = \sqrt{2} |1\rangle = \sqrt{2} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

A closer look at X-spiders: measurement outcomes

$$\text{---} \bullet \quad \rightsquigarrow \quad \langle + | + \langle - | = \sqrt{2} \langle 0 | \quad = \sqrt{2} \begin{pmatrix} 1 & 0 \end{pmatrix}$$

$$\text{---} \bullet_\pi \quad \rightsquigarrow \quad \langle + | - \langle - | = \sqrt{2} \langle 1 | \quad = \sqrt{2} \begin{pmatrix} 0 & 1 \end{pmatrix}$$

$$\text{---} \bullet_{a\pi} \quad \rightsquigarrow \quad \langle + | + (-1)^a \langle - | \quad = \sqrt{2} \begin{pmatrix} 1-a & a \end{pmatrix}$$

A closer look at X-spiders: the parity map

$$\text{Diagram: } \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \quad \rightsquigarrow \quad |+\rangle\langle++| + |-\rangle\langle--| \quad = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

Wires in the ZX-calculus

$$\text{——} \rightsquigarrow |0\rangle\langle 0| + |1\rangle\langle 1| = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Wires in the ZX-calculus

$$\begin{array}{ccc} \text{---} & \rightsquigarrow & |0\rangle\langle 0| + |1\rangle\langle 1| \\ & & = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ \text{X} & \rightsquigarrow & |00\rangle\langle 00| + |10\rangle\langle 01| + |01\rangle\langle 10| + |11\rangle\langle 11| \\ & & = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

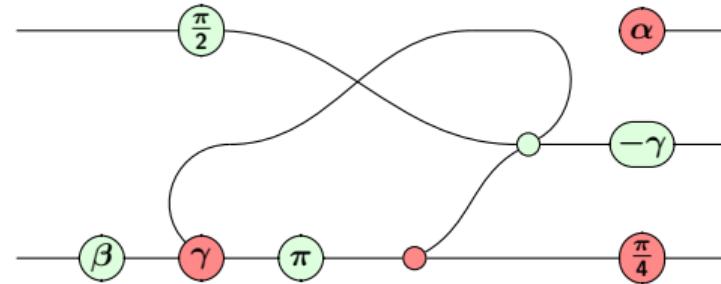
Wires in the ZX-calculus

$$\begin{array}{c} \text{---} \\ \times \\) \end{array} \rightsquigarrow \begin{array}{lcl} |0\rangle\langle 0| + |1\rangle\langle 1| & = & \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ |00\rangle\langle 00| + |10\rangle\langle 01| + |01\rangle\langle 10| + |11\rangle\langle 11| & = & \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \\ \langle 00| + \langle 11| & = & (1 \ 0 \ 0 \ 1) \end{array}$$

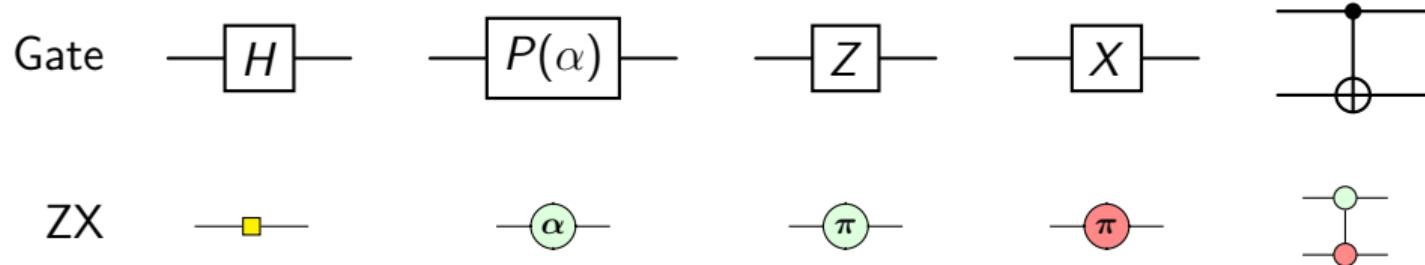
Wires in the ZX-calculus

$$\begin{array}{c} \text{---} \\ \text{X} \\ \text{) } \\ \text{(} \end{array} \quad \rightsquigarrow \quad \begin{array}{l} |0\rangle\langle 0| + |1\rangle\langle 1| \\ |00\rangle\langle 00| + |10\rangle\langle 01| + |01\rangle\langle 10| + |11\rangle\langle 11| \\ \langle 00| + \langle 11| \\ |00\rangle + |11\rangle \end{array} \quad = \begin{array}{c} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \\ (1 \ 0 \ 0 \ 1) \\ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \end{array}$$

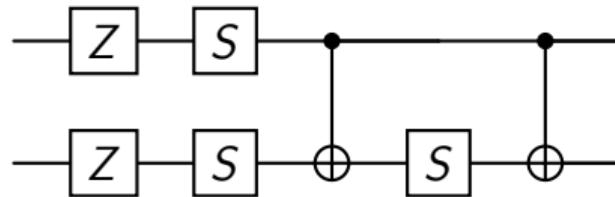
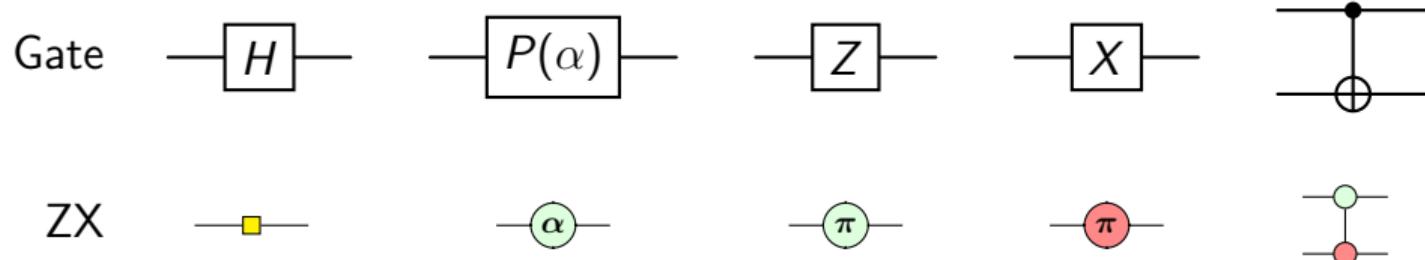
Building ZX-diagrams



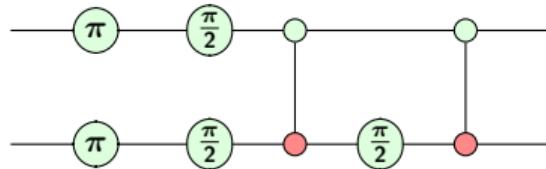
Translating circuits into ZX-diagrams



Translating circuits into ZX-diagrams



\rightsquigarrow



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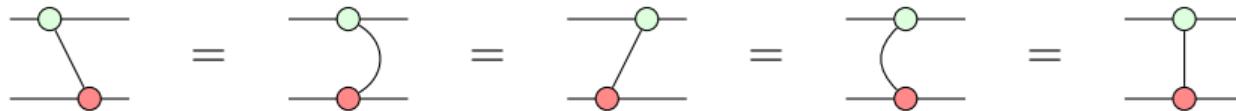
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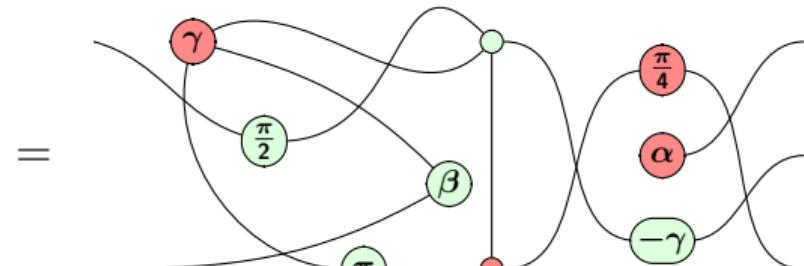
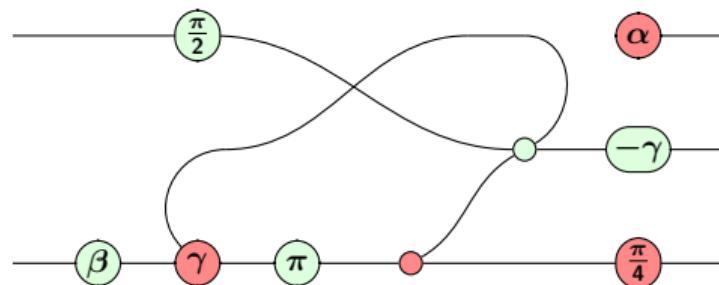
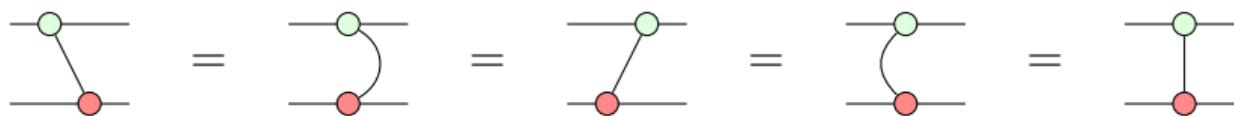
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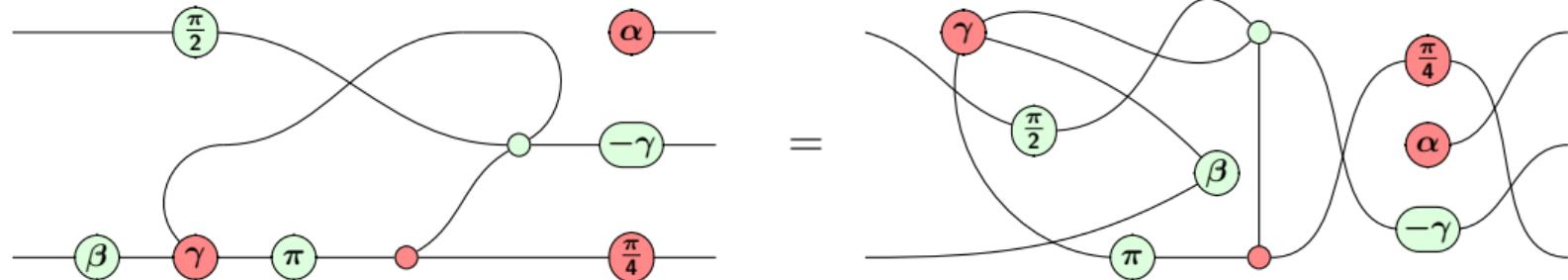
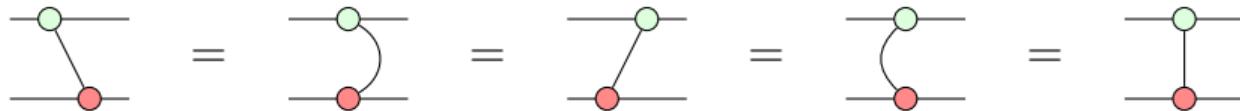
Only connectivity matters



Only connectivity matters



Only connectivity matters



This is made mathematically rigorous using monoidal category theory.

The identity rule

$$\text{---} \circ \text{---} = \text{---} = \text{---} \bullet \text{---}$$

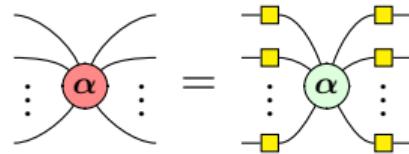
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$$\text{---} \circ \text{---} = \text{---} = \text{---} \bullet \text{---}$$

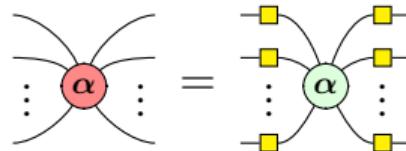
Because only connectivity matters, we also get:

$$\text{---} \circ \text{---} = \text{---} = \text{---} \bullet \text{---} \quad \text{and} \quad \text{---} \bullet \text{---} = \text{---} = \text{---} \circ \text{---}$$

The colour change rule



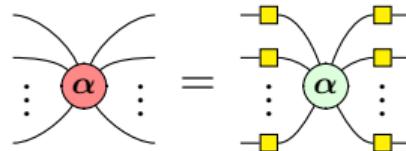
The colour change rule



Two Hadamard gates in a row cancel:



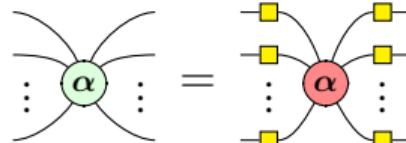
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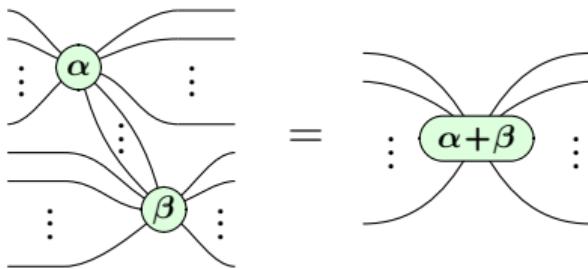
All diagram equations hold with Z- and X-spiders swapped, for example:



The spider rule

$$\begin{array}{c} \text{Diagram showing two green circles labeled } \alpha \text{ and } \beta \text{ with multiple lines connecting them.} \\ = \\ \text{Diagram showing a single green oval labeled } \alpha + \beta \text{ with multiple lines connecting it.} \end{array}$$

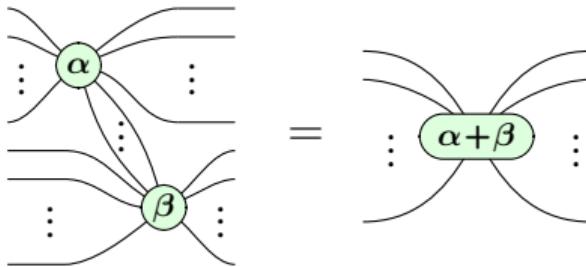
The spider rule



For example:

$$-\langle\alpha\rangle\langle\beta\rangle- = -\langle\alpha+\beta\rangle-$$

The spider rule

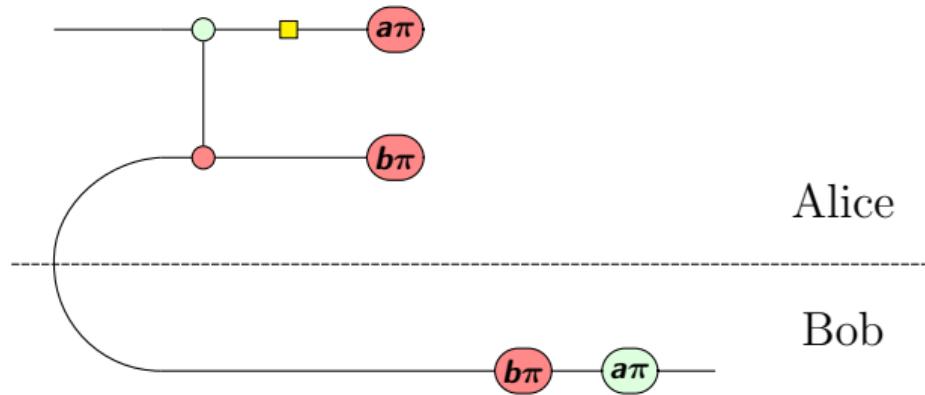


For example:

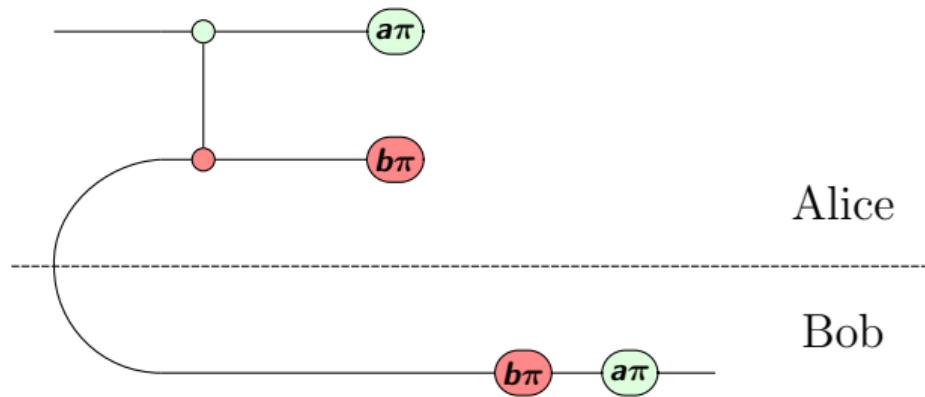
$$-\circlearrowleft \alpha \circlearrowright \beta - = -\circlearrowleft \alpha + \beta \circlearrowright -$$

A diagram showing three configurations of strands. The first configuration has two strands: one red strand with a dot at the bottom and one green strand with a dot at the top, labeled α . They are connected by a vertical strand. The second configuration is identical. The third configuration has the strands swapped: the green strand is at the top with a dot, and the red strand is at the bottom with a dot. All three configurations are separated by equals signs.

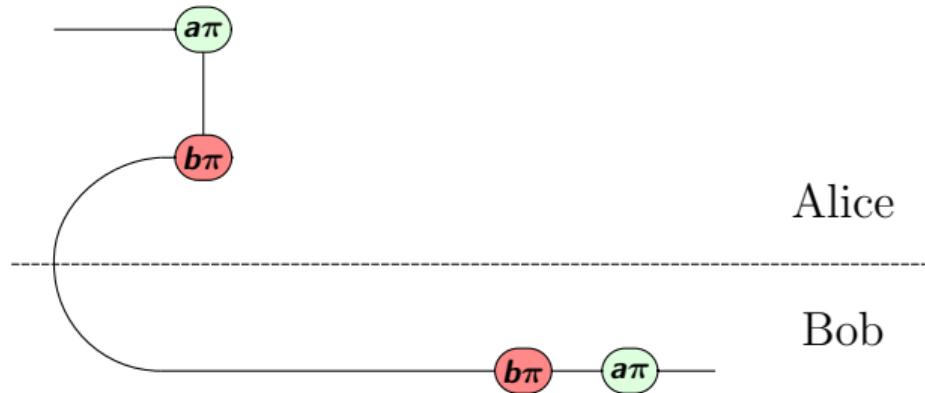
Example: quantum teleportation



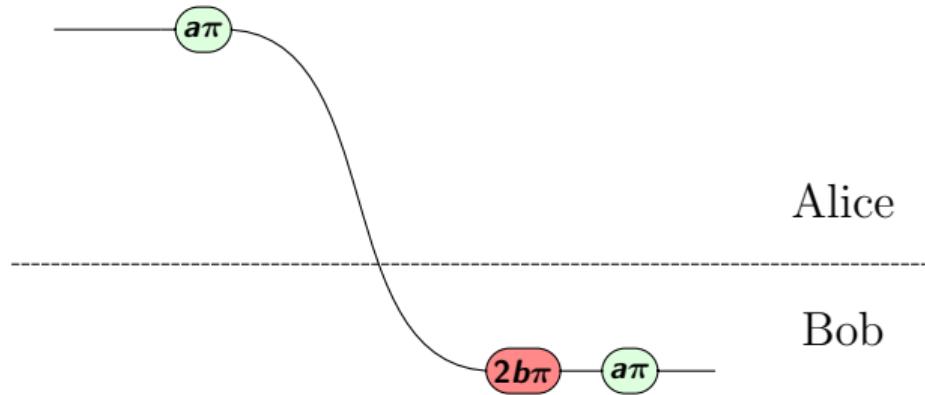
Example: quantum teleportation



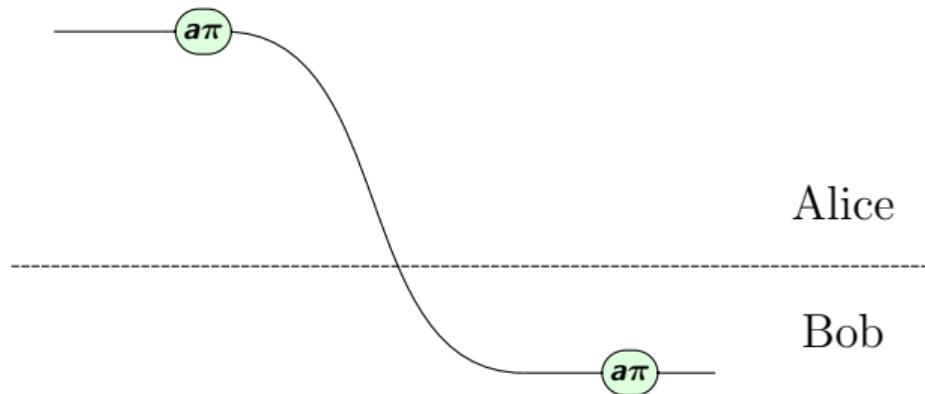
Example: quantum teleportation



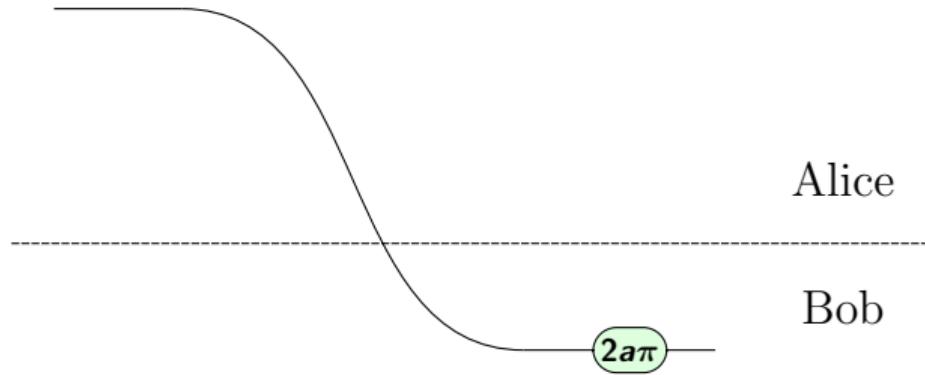
Example: quantum teleportation



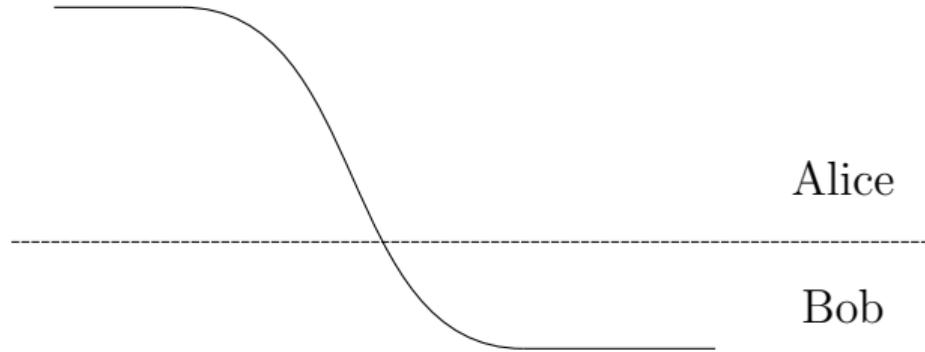
Example: quantum teleportation



Example: quantum teleportation



Example: quantum teleportation



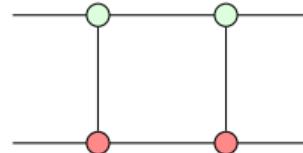
The Hopf rule

$$\text{---} \textcolor{lightgreen}{\circ} \textcolor{red}{\circ} \text{---} = \text{---} \textcolor{lightgreen}{\circ} \textcolor{red}{\circ} \text{---}$$

The Hopf rule

$$\text{---} \circlearrowleft \text{---} = \text{---} \circlearrowleft \text{---}$$

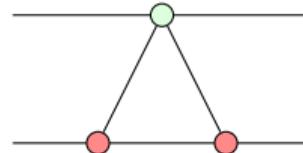
Example: two CNOTs cancel



The Hopf rule

$$\text{---} \circ \text{---} = \text{---} \circ \text{---}$$

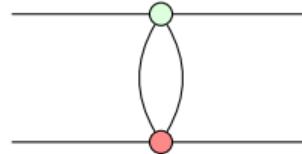
Example: two CNOTs cancel



The Hopf rule

$$\text{---} \circlearrowleft \circlearrowright \text{---} = \text{---} \circlearrowleft \text{---} \circlearrowright \text{---}$$

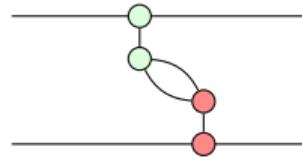
Example: two CNOTs cancel



The Hopf rule

$$\text{---} \textcolor{lightgreen}{\circ} \textcolor{red}{\circ} \text{---} = \text{---} \textcolor{lightgreen}{\circ} \textcolor{red}{\circ} \text{---}$$

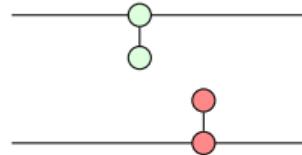
Example: two CNOTs cancel



The Hopf rule

$$\text{---} \circlearrowleft \circlearrowright \text{---} = \text{---} \circlearrowleft \text{---} \circlearrowright \text{---}$$

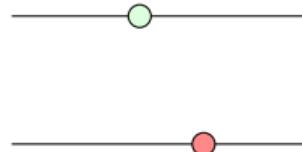
Example: two CNOTs cancel



The Hopf rule

$$\text{---} \circ \text{---} = \text{---} \circ \text{---}$$

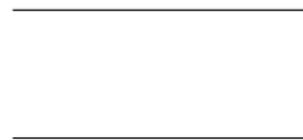
Example: two CNOTs cancel



The Hopf rule

$$\text{---} \circlearrowleft \bullet = \text{---} \circlearrowleft \text{---}$$

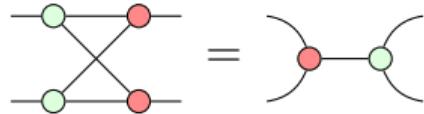
Example: two CNOTs cancel



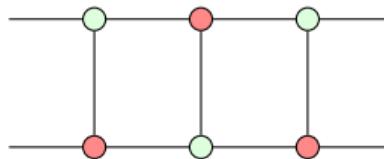
The bialgebra rule

$$\begin{array}{c} \text{Diagram 1: Two horizontal lines with four nodes (green top-left, red top-right, green bottom-left, red bottom-right). The top-left and bottom-right nodes are connected by a diagonal line, and the top-right and bottom-left nodes are connected by another diagonal line.} \\ = \\ \text{Diagram 2: Two separate nodes, one red and one green, connected by a single horizontal line. Each node has a curved line extending from its right side.} \end{array}$$

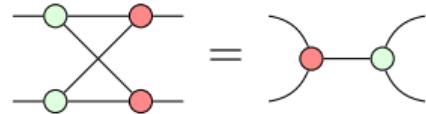
The bialgebra rule



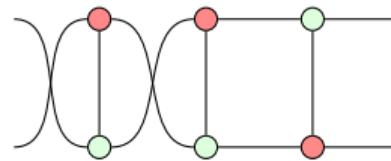
Example: three CNOTs make a SWAP



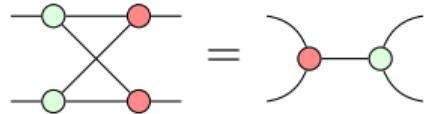
The bialgebra rule



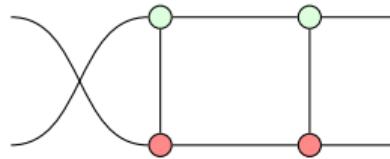
Example: three CNOTs make a SWAP



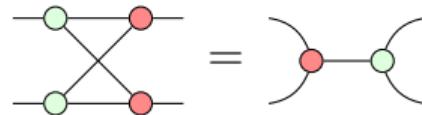
The bialgebra rule



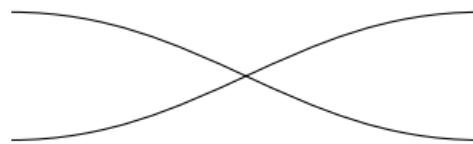
Example: three CNOTs make a SWAP



The bialgebra rule



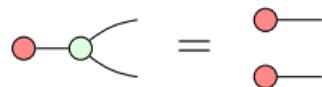
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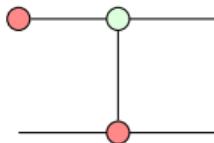
The copy rule

$$\text{Diagram: A red dot connected by a horizontal line to a green dot, which has two curved lines branching downwards.} = \text{Diagram: Two red dots connected by a horizontal line, one above the other.}$$

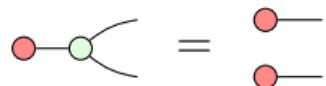
The copy rule



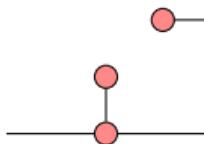
Example: plugging $|0\rangle$ into the control of a CNOT means the gate does nothing



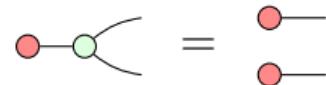
The copy rule



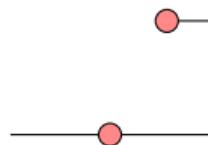
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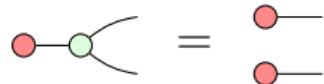
The copy rule



Example: plugging $|0\rangle$ into the control of a CNOT means the gate does nothing



The copy rule



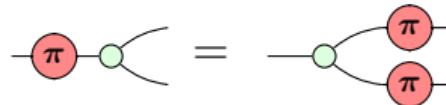
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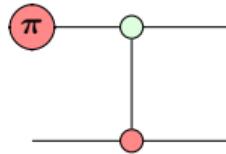
The π -copy rule

$$\text{Diagram illustrating the } \pi\text{-copy rule:}$$
$$\text{Left side: } \text{Red circle } \pi - \text{horizontal line} - \text{Green circle} - \text{Two curved lines branching right.}$$
$$\text{Right side: } \text{Green circle} - \text{horizontal line} - \text{Red circle } \pi \text{ (top)} - \text{Red circle } \pi \text{ (bottom)}$$
$$\text{Curved line connects the top red circle to the bottom red circle.}$$

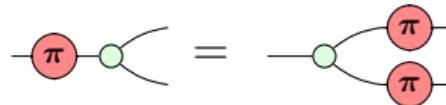
The π -copy rule



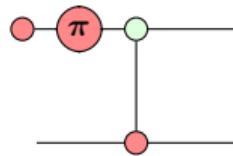
Example: plugging $|1\rangle$ into the control of a CNOT flips the second qubit



The π -copy rule



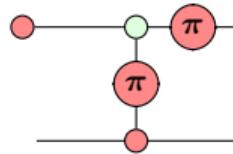
Example: plugging $|1\rangle$ into the control of a CNOT flips the second qubit



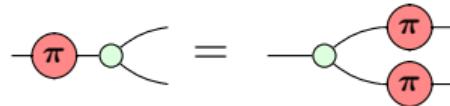
The π -copy rule

$$\begin{array}{c} \text{---} \textcolor{red}{\circlearrowleft} \textcolor{green}{\circlearrowright} \\ \textcolor{red}{\pi} \quad \textcolor{green}{\pi} \end{array} = \begin{array}{c} \text{---} \textcolor{green}{\circlearrowright} \text{---} \textcolor{red}{\circlearrowleft} \\ \textcolor{green}{\pi} \quad \textcolor{red}{\pi} \end{array}$$

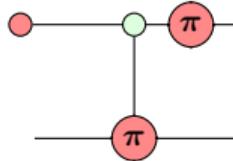
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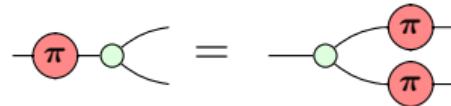
The π -copy rule



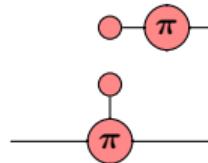
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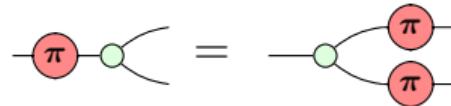
The π -copy rule



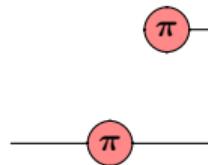
Example: plugging $|1\rangle$ into the control of a CNOT flips the second qubit



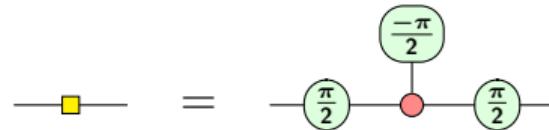
The π -copy rule



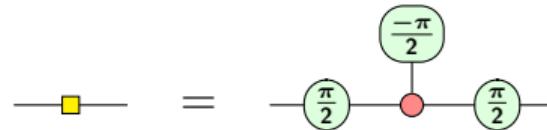
Example: plugging $|1\rangle$ into the control of a CNOT flips the second qubit



Two more rules about single-qubit operations



Two more rules about single-qubit operations



(with a complicated relationship between the phase angles)

The complete set of (scalar-free) ZX-calculus rewrite rules

$$\begin{array}{c} \text{Diagram showing two green nodes } \alpha \text{ and } \beta \text{ with multiple outgoing wires.} \\ \vdots \quad \vdots \\ \alpha \quad \beta \end{array} = \begin{array}{c} \text{Diagram showing a single green node } \alpha + \beta \text{ with all outgoing wires from both } \alpha \text{ and } \beta. \\ \vdots \quad \vdots \end{array}$$

$$\begin{array}{c} \text{Diagram showing a red node } \alpha \text{ with multiple outgoing wires.} \\ \vdots \quad \vdots \\ \alpha \end{array} = \begin{array}{c} \text{Diagram showing a green node } \alpha \text{ with multiple outgoing wires, each passing through a yellow square.} \\ \vdots \quad \vdots \\ \alpha \end{array}$$

$$\begin{array}{c} \text{Diagram showing two red nodes connected by a crossing line.} \\ \text{Diagram showing two red nodes connected by a crossing line.} \end{array} = \begin{array}{c} \text{Diagram showing two red nodes connected by a crossing line.} \\ \text{Diagram showing two red nodes connected by a crossing line.} \end{array}$$

$$\begin{array}{c} \text{Diagram showing a red node connected to a green node.} \\ \text{Diagram showing a red node connected to a green node.} \end{array} = \begin{array}{c} \text{Diagram showing a red node connected to a green node.} \\ \text{Diagram showing a red node connected to a green node.} \end{array}$$

$$\begin{array}{c} \text{Diagram showing a yellow square on a wire.} \\ \text{Diagram showing a yellow square on a wire.} \end{array} = \begin{array}{c} \text{Diagram showing a green node } \frac{\pi}{2} \text{ followed by a red node } \frac{-\pi}{2} \text{ followed by a green node } \frac{\pi}{2}. \\ \text{Diagram showing a green node } \frac{\pi}{2} \text{ followed by a red node } \frac{-\pi}{2} \text{ followed by a green node } \frac{\pi}{2}. \end{array}$$

$$\begin{array}{c} \text{Diagram showing a green node on a wire.} \\ \text{Diagram showing a green node on a wire.} \end{array} = \begin{array}{c} \text{Diagram showing a blank wire.} \\ \text{Diagram showing a blank wire.} \end{array} = \begin{array}{c} \text{Diagram showing a red node on a wire.} \\ \text{Diagram showing a red node on a wire.} \end{array}$$

$$\begin{array}{c} \text{Diagram showing three nodes } \alpha, \beta, \gamma \text{ in sequence.} \\ \text{Diagram showing three nodes } \alpha, \beta, \gamma \text{ in sequence.} \end{array} = \begin{array}{c} \text{Diagram showing three nodes } \alpha', \beta', \gamma' \text{ in sequence.} \\ \text{Diagram showing three nodes } \alpha', \beta', \gamma' \text{ in sequence.} \end{array}$$

Only connectivity matters.

$$\begin{array}{c} \text{Diagram showing a green node connected to a red node.} \\ \text{Diagram showing a green node connected to a red node.} \end{array} = \begin{array}{c} \text{Diagram showing a green node.} \\ \text{Diagram showing a red node.} \end{array}$$

$$\begin{array}{c} \text{Diagram showing a red node connected to a green node.} \\ \text{Diagram showing a red node connected to a green node.} \end{array} = \begin{array}{c} \text{Diagram showing a green node.} \\ \text{Diagram showing a red node.} \end{array}$$

The complete set of (scalar-free) ZX-calculus rewrite rules

$$\begin{array}{c} \text{Diagram showing two nodes } \alpha \text{ and } \beta \text{ with multiple outgoing wires.} \\ \vdots \quad \vdots \\ \alpha \quad \beta \\ \vdots \quad \vdots \end{array} = \begin{array}{c} \text{Diagram showing a single node } \alpha + \beta \text{ with all outgoing wires from } \alpha \text{ and } \beta. \\ \vdots \quad \vdots \end{array}$$

$$\begin{array}{c} \text{Diagram showing a red node } \alpha \text{ with multiple outgoing wires.} \\ \vdots \quad \vdots \\ \alpha \\ \vdots \quad \vdots \end{array} = \begin{array}{c} \text{Diagram showing a green node } \alpha \text{ with four yellow squares (gates) attached to its wires.} \\ \vdots \quad \vdots \\ \alpha \\ \vdots \quad \vdots \end{array}$$

$$\begin{array}{c} \text{Diagram showing two nodes: one green and one red, connected by two crossing wires.} \\ \text{Diagram showing a red node connected to a green node.} \end{array} = \begin{array}{c} \text{Diagram showing a red node connected to a green node.} \\ \text{Diagram showing a red node connected to a green node.} \end{array}$$

$$\begin{array}{c} \text{Diagram showing a red node connected to a green node.} \\ \text{Diagram showing a red node connected to a green node.} \end{array} = \begin{array}{c} \text{Diagram showing a red node connected to a green node.} \\ \text{Diagram showing a red node connected to a green node.} \end{array}$$

$$\begin{array}{c} \text{Diagram showing a green node connected to a red node.} \\ \text{Diagram showing a red node connected to a green node.} \end{array} = \begin{array}{c} \text{Diagram showing a red node connected to a green node.} \\ \text{Diagram showing a red node connected to a green node.} \end{array}$$

$$\begin{array}{c} \text{Diagram showing a sequence of three nodes: } \alpha, \beta, \gamma. \\ \text{Diagram showing a sequence of three nodes: } \alpha', \beta', \gamma'. \end{array} = \begin{array}{c} \text{Diagram showing a sequence of three nodes: } \alpha', \beta', \gamma'. \\ \text{Diagram showing a sequence of three nodes: } \alpha', \beta', \gamma'. \end{array}$$

Only connectivity matters.

The complete set of ZX-calculus rewrite rules

$$\begin{array}{c} \text{Diagram showing two green nodes labeled } \alpha \text{ and } \beta \text{ with multiple outgoing wires.} \\ \vdots \quad \vdots \\ \text{Diagram showing a single green node labeled } \alpha + \beta \text{ with all outgoing wires from the original nodes.} \end{array} =$$

$$\begin{array}{c} \text{Diagram showing a red node labeled } \alpha \text{ with multiple outgoing wires.} \\ \vdots \quad \vdots \\ \text{Diagram showing a green node labeled } \alpha \text{ with four yellow squares (ZX-gates) attached to its wires.} \end{array} =$$

$$\begin{array}{c} \text{Diagram showing two green nodes and two red nodes connected in a loop.} \\ \text{Diagram showing a red node connected to a green node.} \end{array} =$$

$$\begin{array}{c} \text{Diagram showing two red nodes connected in a loop.} \\ \text{Diagram showing a red node connected to a green node.} \end{array} =$$

$$\begin{array}{c} \text{Diagram showing a yellow square (ZX-gate).} \\ \text{Diagram showing a sequence of three nodes: green (\frac{\pi}{2}), red, and green (\frac{\pi}{2}).} \end{array} =$$

$$\begin{array}{c} \text{Diagram showing a green node connected to a wire.} \\ \text{Diagram showing a wire.} \\ \text{Diagram showing a red node connected to a wire.} \end{array} = \quad =$$

$$\begin{array}{c} \text{Diagram showing three nodes: green } \alpha, red } \beta, \text{ and green } \gamma \text{ in sequence.} \\ \text{Diagram showing three nodes: red } \alpha', \text{ green } \beta', \text{ and red } \gamma' \text{ in sequence.} \end{array} =$$

Only connectivity matters.

$$\begin{array}{c} \text{Diagram showing a green node (\frac{\pi}{4}) connected to a red node (-\frac{\pi}{4}).} \\ \text{Diagram showing a red node (-\frac{\pi}{4}) connected to a green node (\frac{\pi}{4}).} \end{array} =$$

Outline

Introduction

The ZX-calculus

Notation

Equational theory

Applications

Optimisation of quantum circuits

Quantum error correction

Variants and extensions

Conclusions

Outline

Introduction

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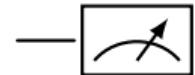
Quantum error correction

Variants and extensions

Conclusions

Single-qubit measurements

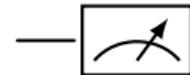
Have already seen computational basis measurements:



$$-\text{---} \xrightarrow{\text{a}\pi} \rightsquigarrow \sqrt{2}(a - 1 - a) = \begin{cases} \sqrt{2}\langle 0| & \text{if } a = 0 \\ \sqrt{2}\langle 1| & \text{if } a = 1 \end{cases}$$

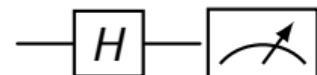
Single-qubit measurements

Have already seen computational basis measurements:



$$\xrightarrow{\text{---} \textcolor{red}{a\pi}} \sqrt{2}(a - 1 - a) = \begin{cases} \sqrt{2}\langle 0| & \text{if } a = 0 \\ \sqrt{2}\langle 1| & \text{if } a = 1 \end{cases}$$

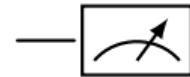
Other measurement shorthands:



$$\xrightarrow{\text{---} \textcolor{teal}{a\pi}} \sqrt{2}(1 - (-1)^a) = \begin{cases} \sqrt{2}\langle +| & \text{if } a = 0 \\ \sqrt{2}\langle -| & \text{if } a = 1 \end{cases}$$

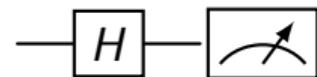
Single-qubit measurements

Have already seen computational basis measurements:

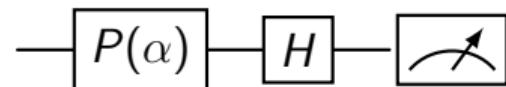


$$-\text{---} \textcolor{red}{\text{a}\pi} \rightsquigarrow \sqrt{2} (a \quad 1 - a) = \begin{cases} \sqrt{2} \langle 0| & \text{if } a = 0 \\ \sqrt{2} \langle 1| & \text{if } a = 1 \end{cases}$$

Other measurement shorthands:



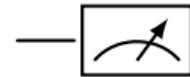
$$-\text{---} \textcolor{teal}{\text{a}\pi} \rightsquigarrow \sqrt{2} (1 \quad (-1)^a) = \begin{cases} \sqrt{2} \langle +| & \text{if } a = 0 \\ \sqrt{2} \langle -| & \text{if } a = 1 \end{cases}$$



$$-\text{---} \textcolor{lightgreen}{\alpha + \text{a}\pi} \rightsquigarrow \sqrt{2} (1 \quad \pm e^{i\alpha}) = \langle 0| \pm e^{i\alpha} \langle 1|$$

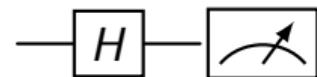
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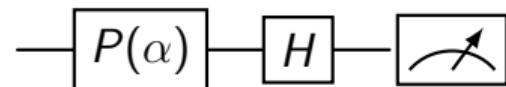


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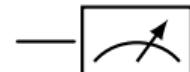


$$-\text{---} \textcolor{green}{\alpha + a\pi} \rightsquigarrow \sqrt{2}(1 \pm e^{i\alpha}) = \langle 0| \pm e^{i\alpha} \langle 1|$$

Similarly, can also consider more complicated ZX-diagrams to represent a single measurement, e.g. $-\textcolor{red}{\frac{\pi}{2}} \textcolor{red}{\text{---}} \textcolor{green}{\alpha + a\pi}$.

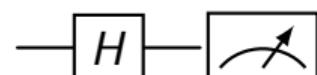
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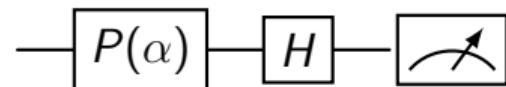


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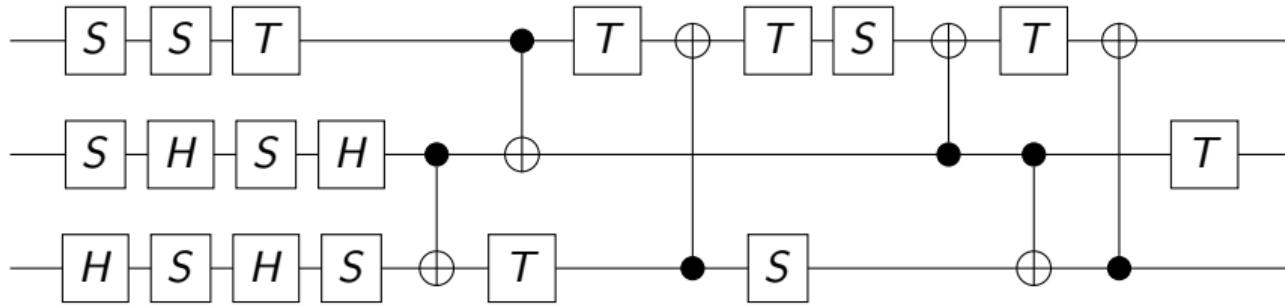


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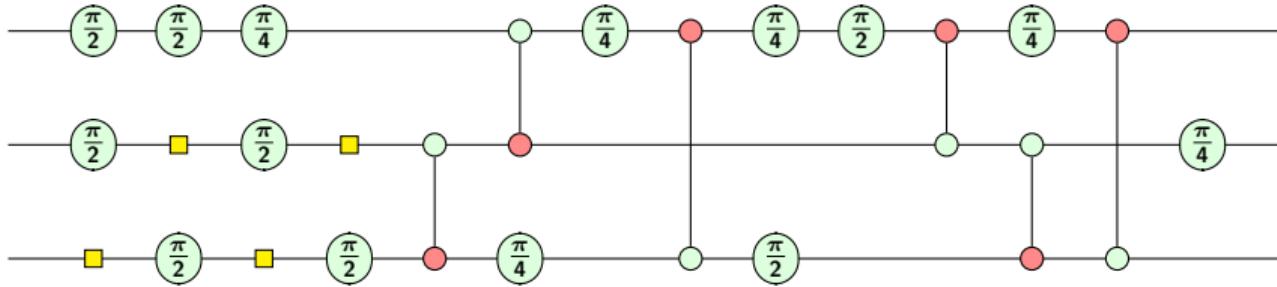
Similarly, can also consider more complicated ZX-diagrams to represent a single measurement, e.g. $-\textcolor{red}{\frac{\pi}{2}} - \textcolor{green}{\alpha + a\pi}$.

Leave out variable to represent post-selected measurements, e.g. $-\textcolor{green}{\alpha}$.

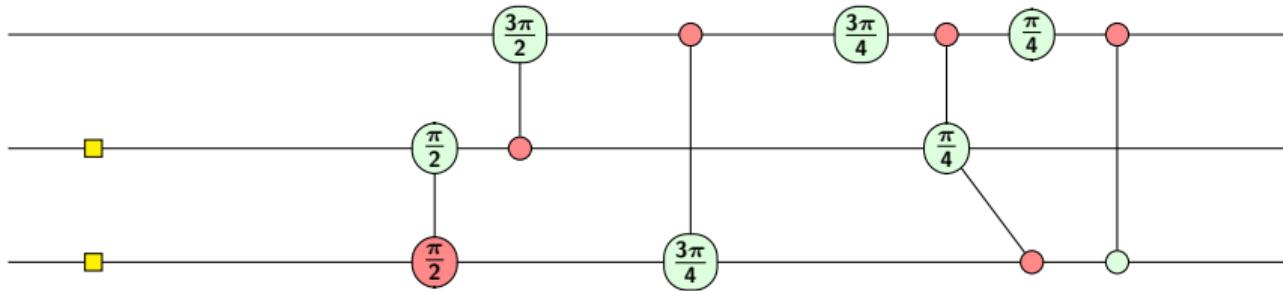
Optimising quantum computations using the ZX-calculus



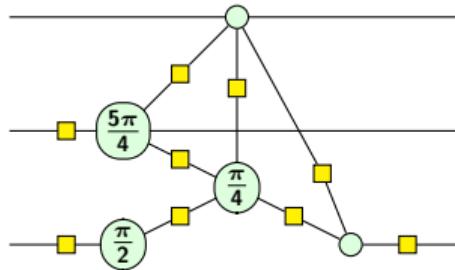
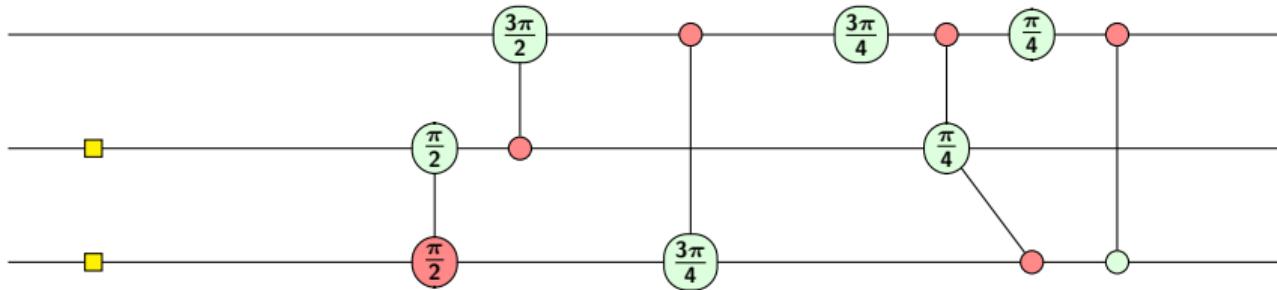
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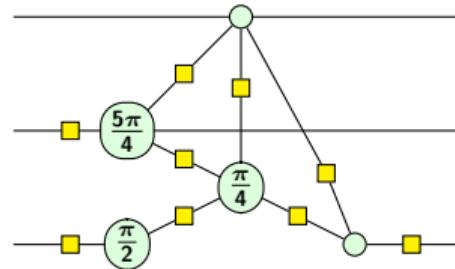
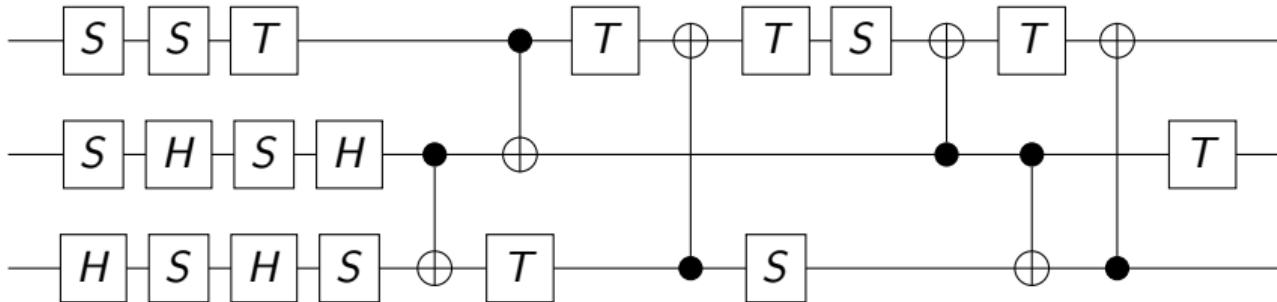
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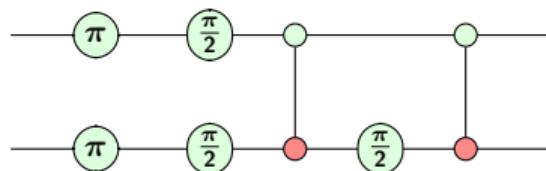
Problem: translating ZX-diagrams to circuits is $\#P$ -hard [de Beaudrap et al. 2022]

Two models of quantum computation

Two models of quantum computation

quantum circuit model

[Deutsch 1989]



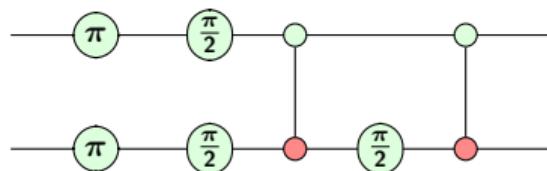
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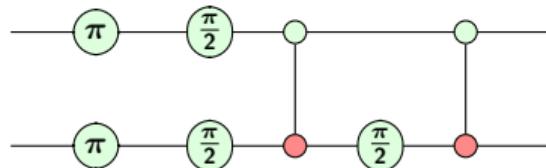


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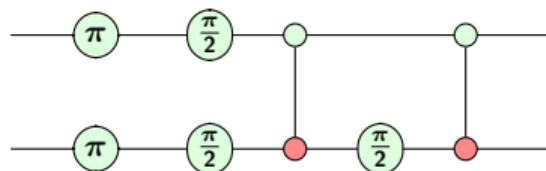


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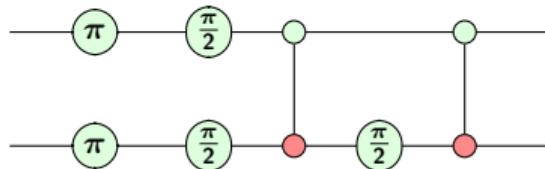


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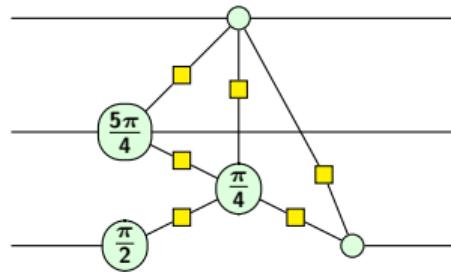
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one-way model

[Raussendorf & Briegel 2001]

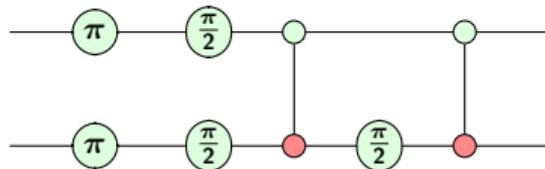


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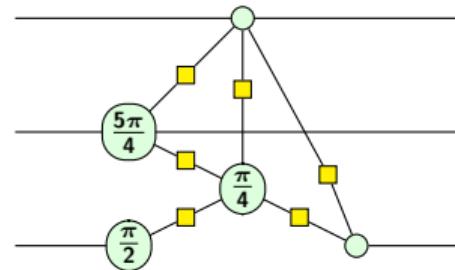
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one-way model

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- ▶ initialise entangled ‘graph state’ (can be made independent of computation)

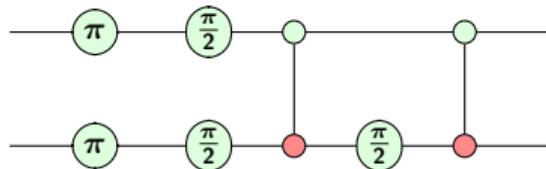


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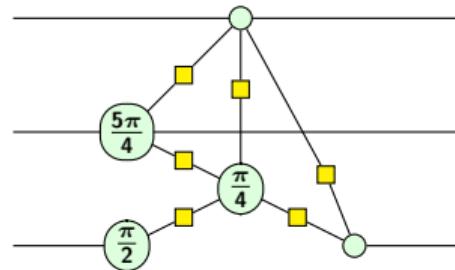
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one-way model

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- ▶ computation driven by successive adaptive single-qubit measurements

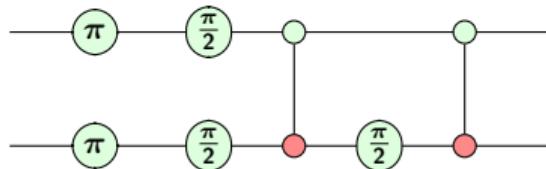


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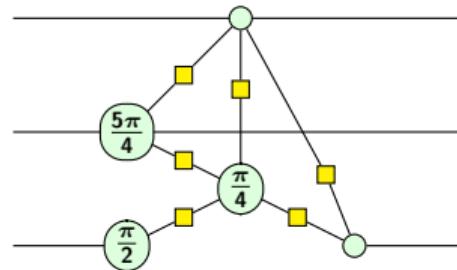
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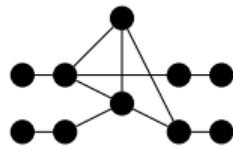
[Raussendorf & Briegel 2001]

- ▶ initialise entangled ‘graph state’ (can be made independent of computation)
- ▶ computation driven by successive adaptive single-qubit measurements
- ▶ if goal is state preparation, may need Pauli corrections at the end



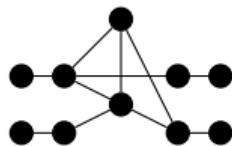
Graph states as ZX-diagrams

Take a simple graph, e.g.

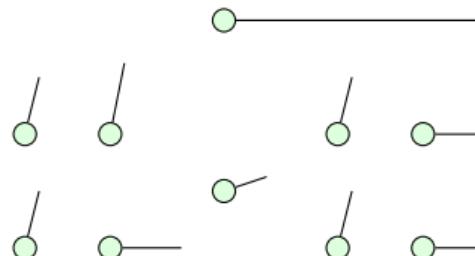


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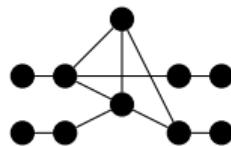


- ▶ For each vertex, prepare a qubit in the state $|+\rangle$.

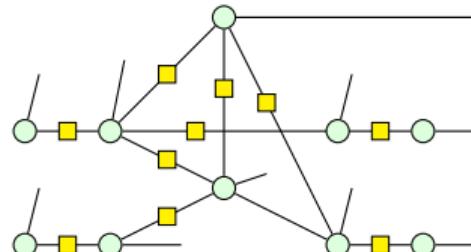


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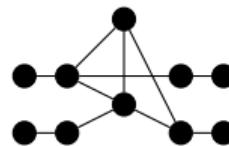


- ▶ For each vertex, prepare a qubit in the state $|+\rangle$.
- ▶ For each edge, apply a controlled-Z gate .

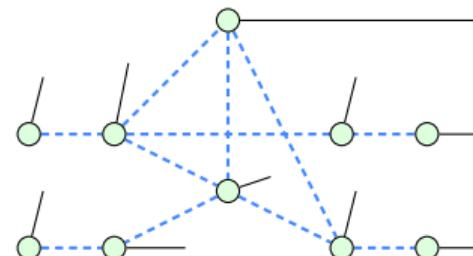
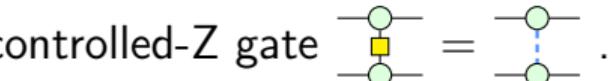


Graph states as ZX-diagrams

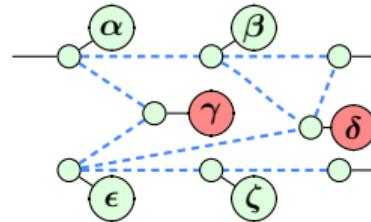
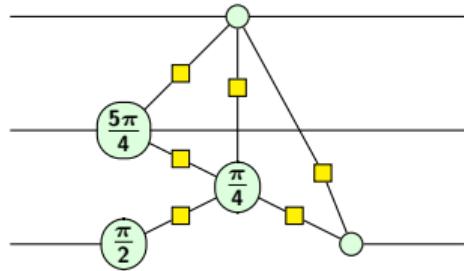
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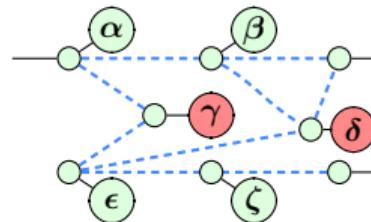
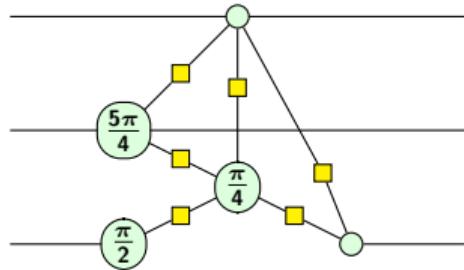
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Representing one-way computations in the ZX-calculus



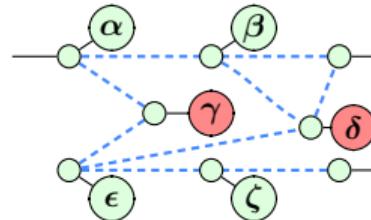
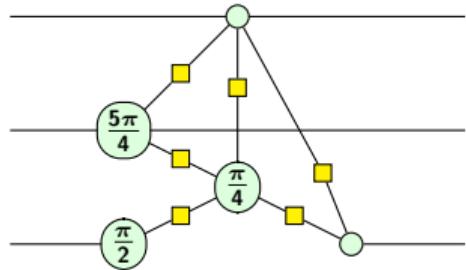
Representing one-way computations in the ZX-calculus



Bring any ZX-calculus diagram into 'MBQC-form' by

- ▶ Colour-changing all red spiders to green.

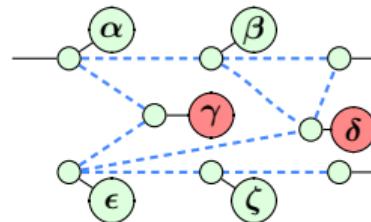
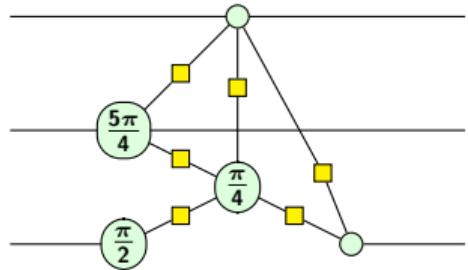
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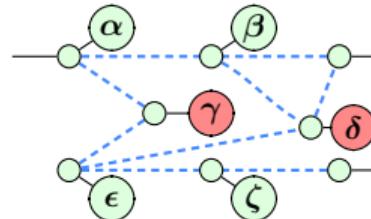
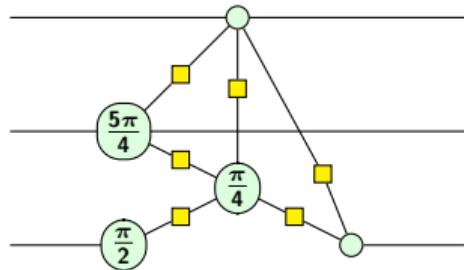
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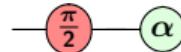
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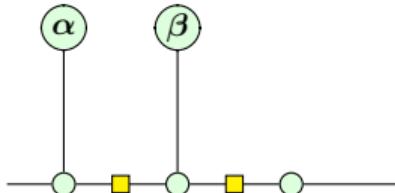
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Can allow different measurement effects:



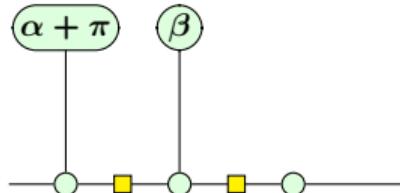
Correcting undesired measurement outcomes

Suppose we want to implement the operation $\alpha - \beta$ in the one-way model:



Correcting undesired measurement outcomes

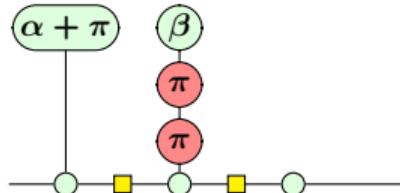
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Suppose the first measurement yields the undesired outcome.

Correcting undesired measurement outcomes

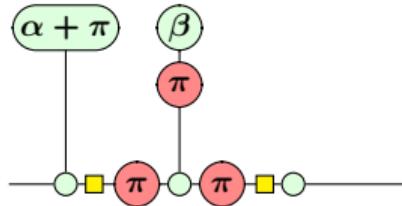
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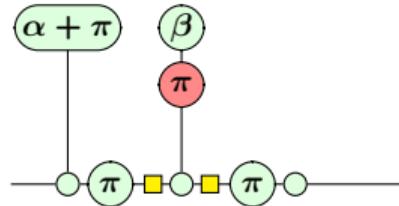
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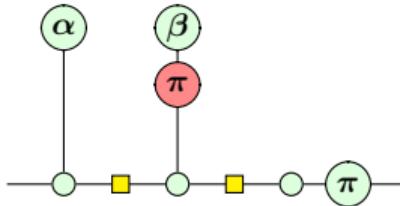
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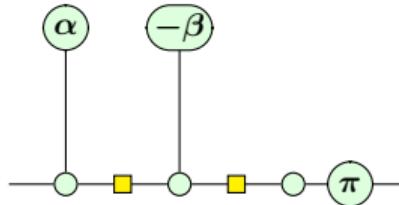
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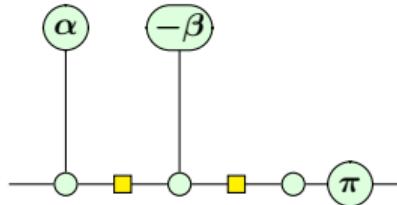
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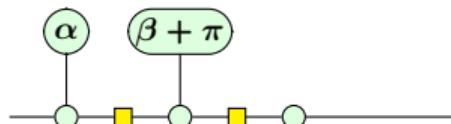
Suppose the first measurement yields the undesired outcome. This can be compensated by flipping the second angle to $-\beta$ and applying Z to the output.

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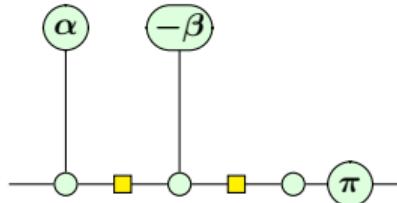
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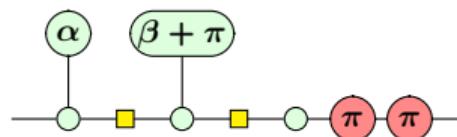
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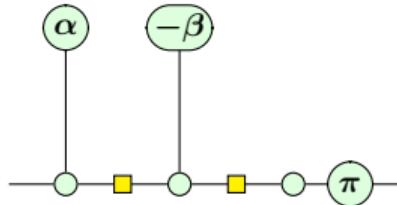
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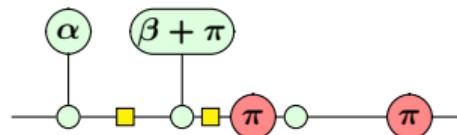
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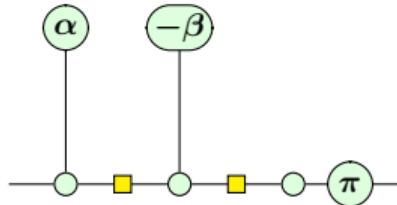
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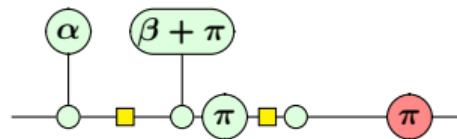
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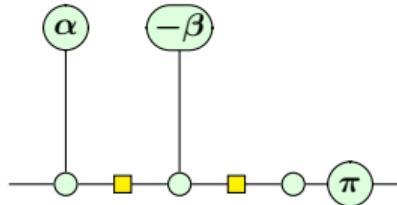
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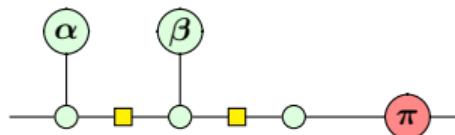
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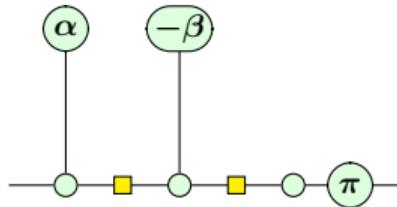
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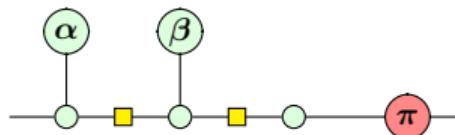
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Similarly, suppose the second measurement yields the undesired outcome. This can be corrected by applying X to the output, without affecting the first measurement. The two correction procedures are compatible with each other.

Determinism in the one-way model allows circuit extraction

If for a one-way computation, there exist:

- ▶ a partial order on the qubits, and

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There exists an efficient algorithm which ‘extracts a circuit’ from any ZX-diagram that corresponds to a one-way computation with flow.

[Duncan et al. 2020; B. et al. 2021; Simmons 2021]

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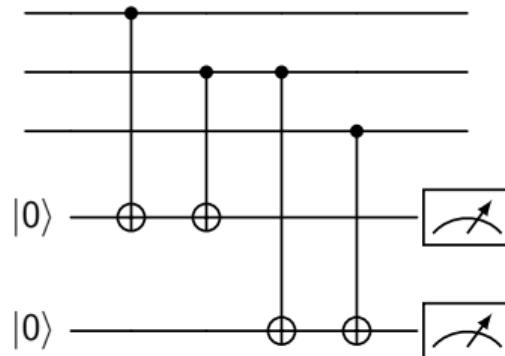
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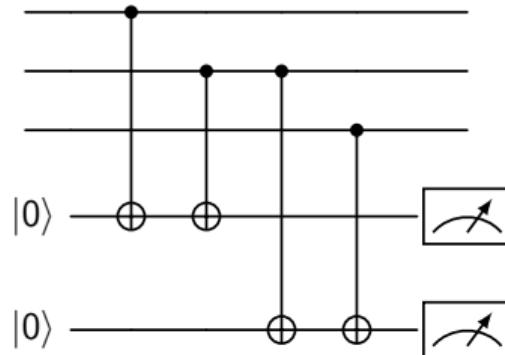
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$$\alpha|000\rangle + \beta|111\rangle \rightsquigarrow 00$$

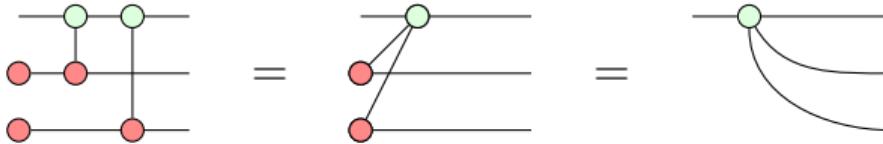
$$\alpha|100\rangle + \beta|011\rangle \rightsquigarrow 10$$

$$\alpha|010\rangle + \beta|101\rangle \rightsquigarrow 11$$

$$\alpha|001\rangle + \beta|110\rangle \rightsquigarrow 01$$

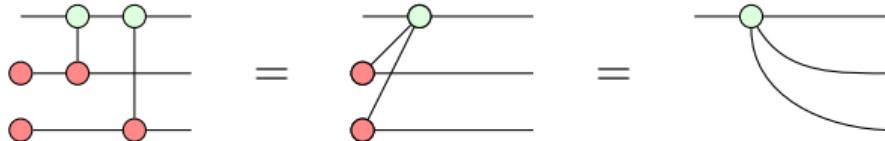
The bit flip code in the ZX-calculus

Encoder:

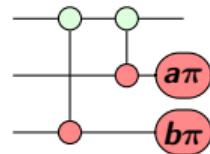


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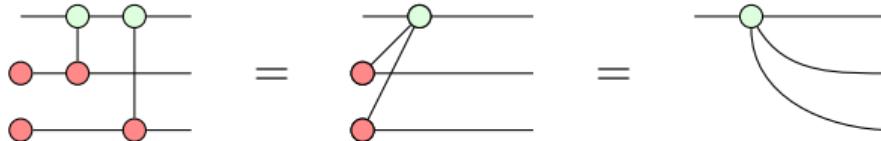


Use the adjoint of this map to decode; this will involve measurements:

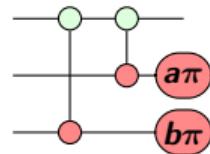


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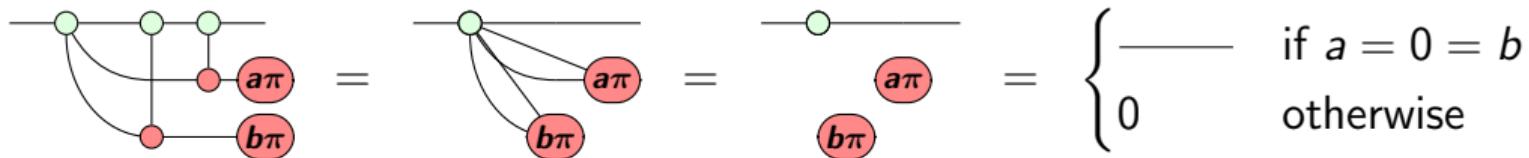
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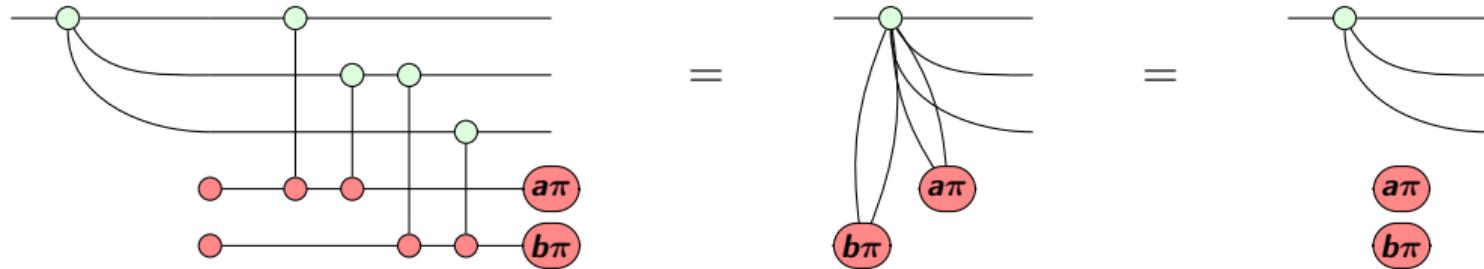


Encoding and decoding when there are no errors yields outcomes $a = 0 = b$:



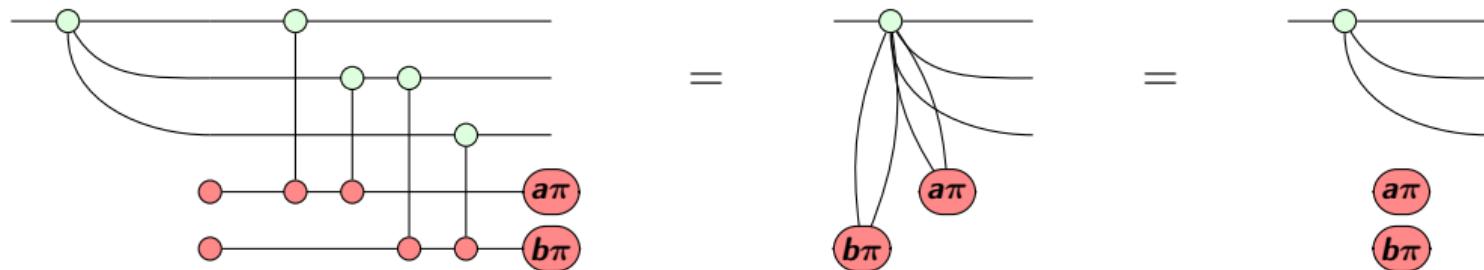
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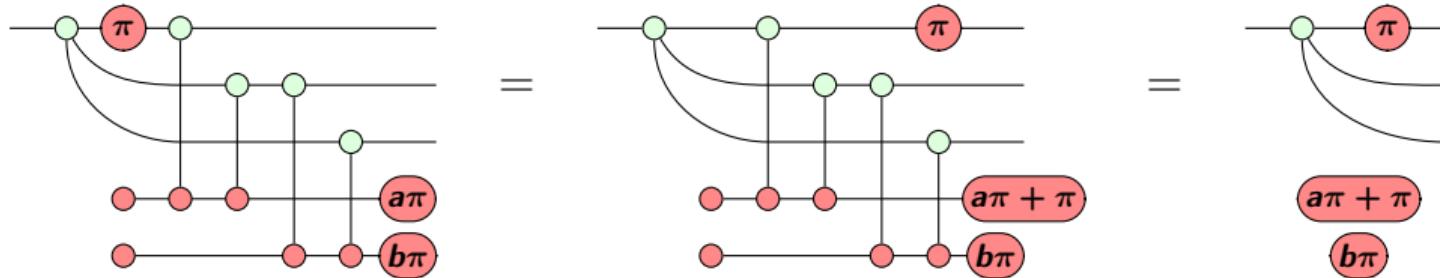


The detectors

No errors:



A bit-flip error on the first qubit changes the first parity measurement:



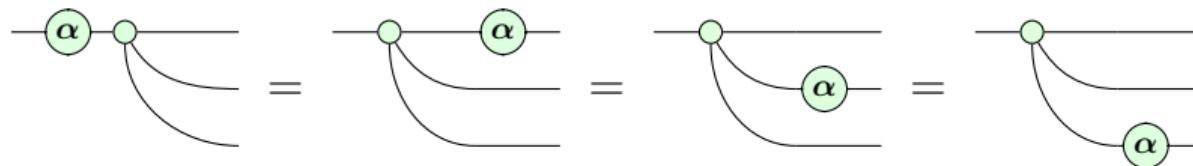
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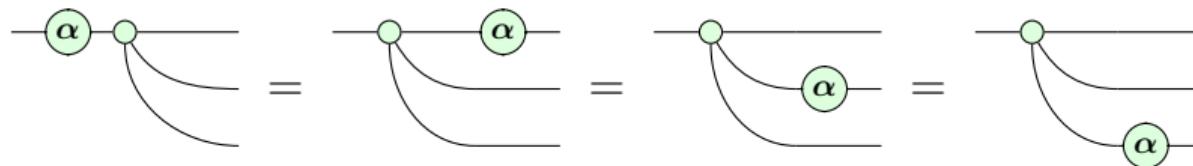
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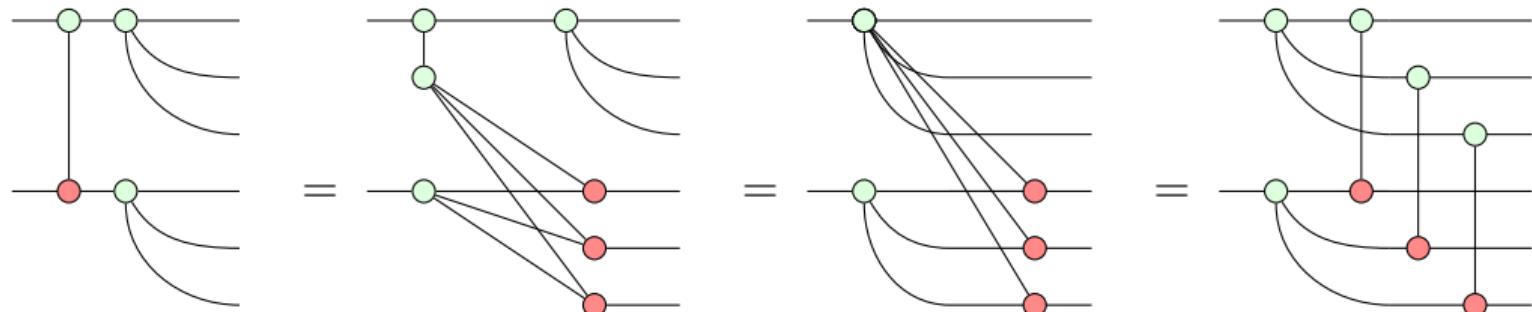
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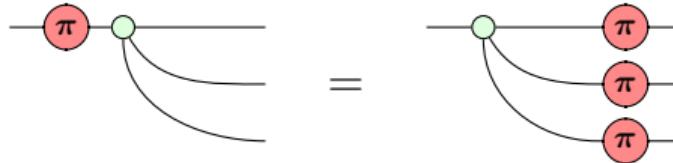


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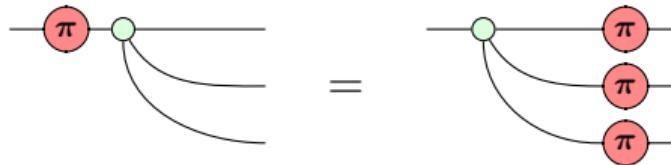
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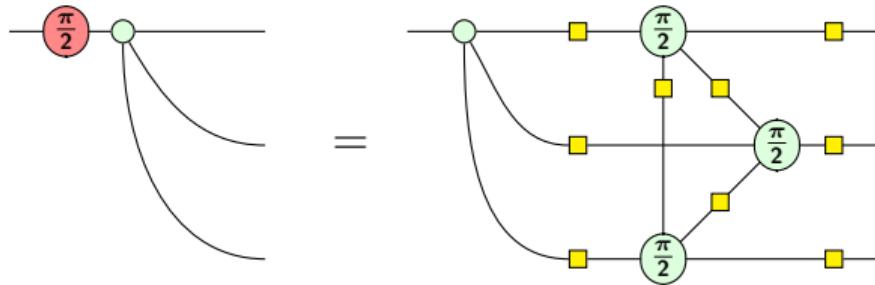


Fault-tolerant computation on encoded data

Logical X gate:



Other X -rotations are not transverse, e.g. for $\frac{\pi}{2}$:



Need to use more complicated techniques to implement these gates fault-tolerantly.

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The ZW-calculus

Z spider, $z \in \mathbb{C}$

[Hadzihasanovic 2015; Hadzihasanovic, Ng, Wang 2018]

$$n \left\{ \begin{array}{c} \text{---} \\ z \\ \text{---} \end{array} \right\} m \quad \rightsquigarrow \quad | \underbrace{0 \dots 0}_m \rangle \langle \underbrace{0 \dots 0}_n | + z | \underbrace{1 \dots 1}_m \rangle \langle \underbrace{1 \dots 1}_n | = \begin{pmatrix} 1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & z \end{pmatrix}$$

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$$\text{---} \otimes \text{---} \rightsquigarrow |00\rangle\langle 00| + |01\rangle\langle 10| + |10\rangle\langle 01| - |11\rangle\langle 11| = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Some properties of the ZW-calculus

The two three-legged spiders of the ZW-calculus represent the two inequivalent types of 3-qubit entangled states:



$$\rightsquigarrow |000\rangle + |111\rangle$$



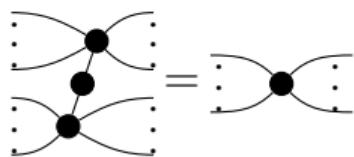
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$$\text{Diagram with open circle} \rightsquigarrow |000\rangle + |111\rangle \quad \text{Diagram with solid black dot} \rightsquigarrow |001\rangle + |010\rangle + |100\rangle$$

W is a pseudo-spider because its ‘spider fusion’ rule is slightly different:

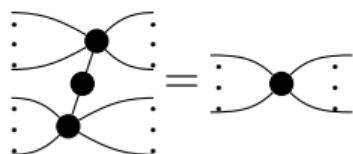


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$$\text{Diagram 5} \neq \text{Diagram 6}$$

The ZW-calculus has a complete equational theory, in fact it was the first universal graphical calculus to be proved complete.

The ZH-calculus

Z spider

[B. & Kissinger 2018]

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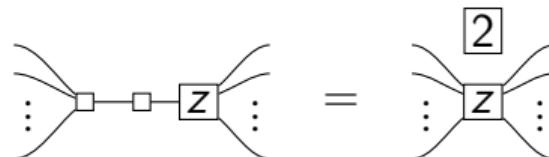
$$\begin{array}{ccc} \vdots & \text{---} \square \text{---} \square \text{---} \boxed{Z} \text{---} \vdots & = & \boxed{2} \\ \text{---} \square \text{---} \square \text{---} \boxed{Z} \text{---} \vdots & & \vdots & \vdots \end{array}$$

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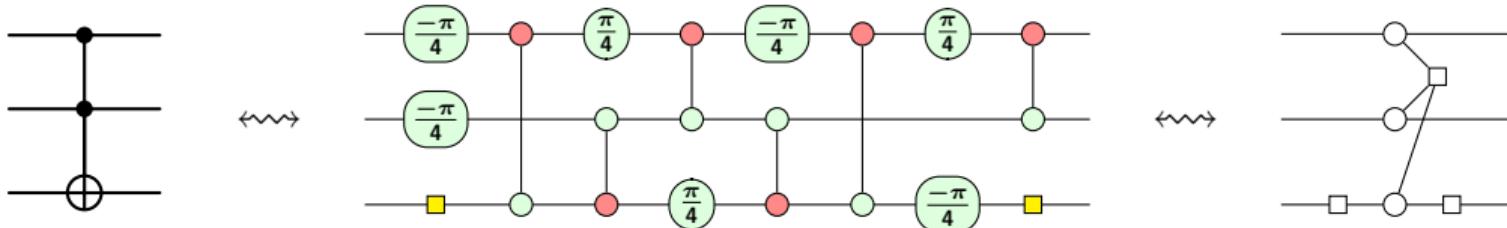
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The ZH-calculus is particularly useful for representing ‘classical non-linearity’, such as in multi-controlled gates, for example the Toffoli gate:



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The above examples become:

$$|+\rangle\langle+| = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\frac{1}{2} |0\rangle\langle 0| + \frac{1}{2} |1\rangle\langle 1| = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Graphical calculi for mixed states and classical information

Change the interpretation of the existing generators by combining each generator with its complex conjugate, e.g. for the ZX-calculus:

$$\text{CPM} \left(\begin{array}{c|c} : & \alpha \\ \hline \alpha & : \end{array} \right) = \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array}$$

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$$\text{CPM} \left(-\parallel \right) = \text{---}$$

Then we can represent superpositions and probabilistic mixtures:

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \rightsquigarrow \text{CPM} \left(\text{---} \right) = \begin{array}{c} \text{---} \\ | \quad | \\ \text{---} \end{array} \quad \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \rightsquigarrow \text{CPM} \left(\text{---} \parallel \right) = \begin{array}{c} \text{---} \\ | \quad | \\ \text{---} \end{array} = \text{---}$$

Equations for the new generator, and application

All existing ZX-calculus equations remain true and it suffices to add four new equations involving $\neg\parallel$

[Carette et al. 2019]

$$\text{---} \bullet \parallel = \bullet \text{---} \textcolor{lightgreen}{\bullet}$$

$$\text{---} \blacksquare \parallel = \text{---} \parallel$$

$$\text{---} (\alpha) \parallel = \text{---} \parallel$$

$$\begin{array}{c} \textcolor{lightgreen}{\bullet} \text{---} \textcolor{red}{\bullet} \\ | \qquad | \\ \textcolor{red}{\bullet} \text{---} \parallel \\ | \qquad | \\ \textcolor{lightgreen}{\bullet} \text{---} \parallel \end{array} = \text{---} \parallel$$

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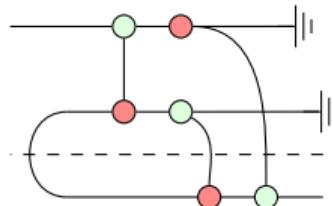
$$\bullet \parallel = \bullet \circ \bullet$$

$$-\square \parallel = __\parallel$$

$$-\langle \alpha \rangle \parallel = __\parallel$$

$$\begin{array}{c} \bullet \\ \bullet \\ -\end{array} \parallel = \begin{array}{c} \bullet \\ \bullet \\ -\end{array} \parallel$$

For example, this allows to represent the classical information flow in quantum teleportation:

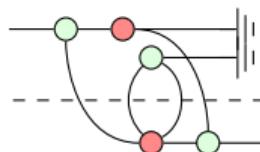


Equations for the new generator, and application

All existing ZX-calculus equations remain true and it suffices to add four new equations involving $\dashv\parallel$ [Carette et al. 2019]

$$\begin{array}{c} \text{---} \bullet \parallel = \text{---} \bullet \text{---} \\ \text{---} \square \parallel = \text{---} \parallel \\ \text{---} \alpha \parallel = \text{---} \parallel \\ \text{---} \text{---} \parallel = \text{---} \parallel \end{array}$$

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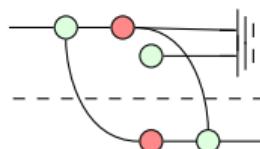


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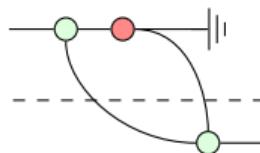


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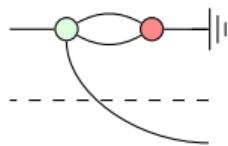
$$\text{red circle} \parallel = \text{red circle} \text{ green circle}$$

$$\text{yellow square} \parallel = \text{empty line} \parallel$$

$$\text{green circle } \alpha \parallel = \text{empty line} \parallel$$

$$\begin{array}{c} \text{green circle} \\ \text{red circle} \\ \text{green circle} \end{array} \parallel = \begin{array}{c} \text{empty line} \parallel \\ \text{empty line} \parallel \end{array}$$

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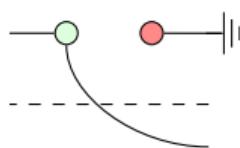


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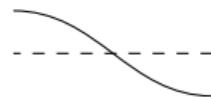


Equations for the new generator, and application

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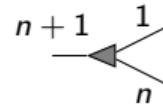
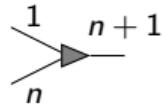
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For example, this allows to represent the classical information flow in quantum teleportation:



Scalable notation

Bundle multiple wires into one and introduce two new components to *gather* and *split* these bundles:

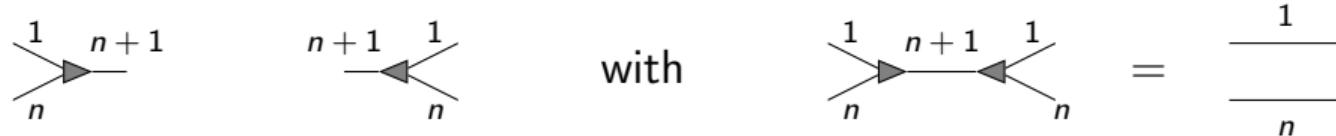


with

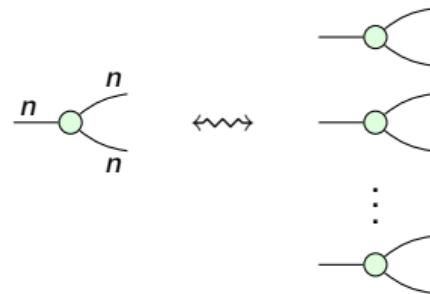
$$\begin{array}{c} 1 \quad n+1 \quad 1 \\ \nearrow \quad \searrow \\ n \quad \quad \quad n \end{array} = \frac{1}{n}$$

Scalable notation

Bundle multiple wires into one and introduce two new components to *gather* and *split* these bundles:

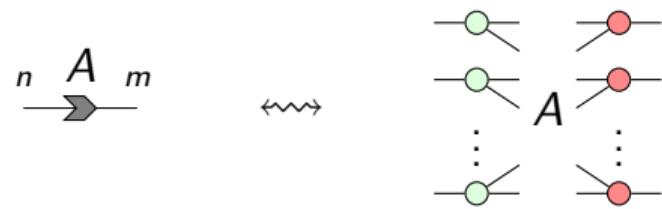


In addition to combining multiple wires, also allow multiple copies of other generators, e.g.

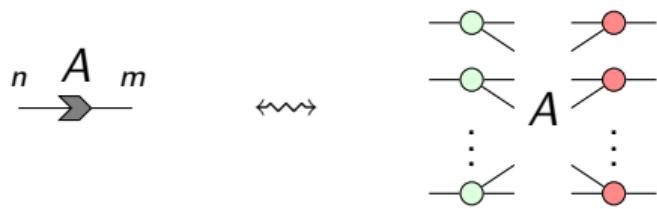


[Carette, Horsman, Perdrix 2019]

Unified notation and reasoning for diagrams with similar structure



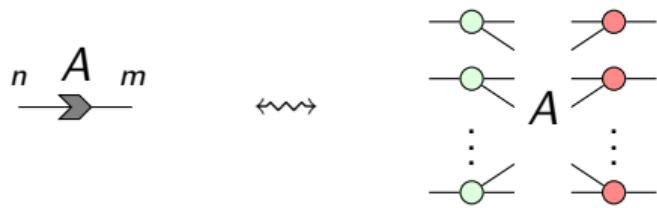
Unified notation and reasoning for diagrams with similar structure



Applying the matrix arrow labelled by a biadjacency matrix A to a computational basis state $|\mathbf{x}\rangle$ for some $\mathbf{x} \in \{0, 1\}^n$:

$$|\mathbf{x}\rangle \xrightarrow{A} |\mathbf{Ax}\rangle$$

Unified notation and reasoning for diagrams with similar structure



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This has applications in quantum error correcting codes and many other areas of quantum computing.

Outline

Introduction

The ZX-calculus

Notation

Equational theory

Applications

Optimisation of quantum circuits

Quantum error correction

Variants and extensions

Conclusions

Summary

Graphical languages for representing and reasoning about quantum computations.

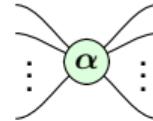
- ▶ Graphical rewriting: cutting and pasting of diagrams



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Graphical languages for representing and reasoning about quantum computations.

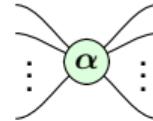
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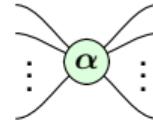
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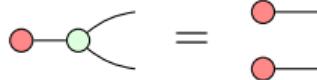
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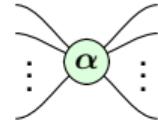


- ▶ A set of (fairly) simple rewrite rules

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Graphical languages for representing and reasoning about quantum computations.

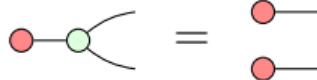
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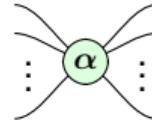


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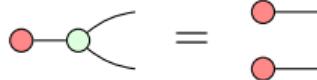
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More information about the ZX-calculus is at <https://zxcalculus.com/>. See also [⟨hal-05322779⟩](https://hal.archives-ouvertes.fr/hal-05322779) (in French).