Assignment 3 Jack (Quan Cheng) Xie
Professor Yukai Yang 4 October 2022
Time Series Econometrics

Problem 1.

Q1.1.

Let us define

$$\mathbf{y}_t = \begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix}, \tag{1}$$

$$\mathbf{\Phi}_1 = \begin{pmatrix} \phi_{1,1}^{(1)} & \phi_{1,2}^{(1)} \\ \phi_{2,1}^{(1)} & \phi_{2,2}^{(1)} \end{pmatrix} = \begin{pmatrix} 0.750 & 0.5 \\ 0 & 0.700 \end{pmatrix}, \tag{2}$$

$$\mathbf{\Phi}_2 = \begin{pmatrix} \phi_{1,1}^{(2)} & \phi_{1,2}^{(2)} \\ \phi_{2,1}^{(2)} & \phi_{2,2}^{(2)} \end{pmatrix} = \begin{pmatrix} -0.125 & 0 \\ 0 & -0.100 \end{pmatrix}, \tag{3}$$

$$\varepsilon_t = \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix} \overset{iid}{\sim} N(\mathbf{0}, \mathbf{\Omega}), \quad \mathbf{\Omega} \in \mathbb{R}^{2\times 2}. \tag{4}$$

Then we can rewrite the VAR(2) system as

$$\begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix} = \begin{pmatrix} 0.750 & 0.5 \\ 0 & 0.700 \end{pmatrix} \begin{pmatrix} y_{1,t-1} \\ y_{2,t-1} \end{pmatrix} + \begin{pmatrix} -0.125 & 0 \\ 0 & -0.100 \end{pmatrix} \begin{pmatrix} y_{1,t-2} \\ y_{2,t-2} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix}, \tag{5}$$

or equivalently,

$$\mathbf{y}_t = \mathbf{\Phi}_1 \mathbf{y}_{t-1} + \mathbf{\Phi}_2 \mathbf{y}_{t-2} + \varepsilon_t. \tag{6}$$

Now define

$$\xi_t = \begin{pmatrix} \mathbf{y}_t \\ \mathbf{y}_{t-1} \end{pmatrix} = \begin{pmatrix} y_{1,t} \\ y_{2,t} \\ y_{1,t-1} \\ y_{2,t-1} \end{pmatrix}, \tag{7}$$

$$\mathbf{F} = \begin{pmatrix} \mathbf{\Phi}_1 & \mathbf{\Phi}_2 \\ \mathbf{I} & \mathbf{0} \end{pmatrix} = \begin{pmatrix} 0.75 & 0.5 & -0.125 & 0 \\ 0 & 0.7 & 0 & -0.1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \tag{8}$$

$$\mathbf{v}_t = \begin{pmatrix} \varepsilon_t \\ \mathbf{0} \end{pmatrix} = \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ 0 \\ 0 \end{pmatrix}, \tag{9}$$

where by i.i.d. of $\varepsilon_t \sim N(\mathbf{0}, \mathbf{\Omega})$, we have

$$E[\mathbf{v_t}\mathbf{v}_\tau'] = \begin{cases} \mathbf{Q}, & t = \tau \\ \mathbf{0}, & t \neq \tau, \end{cases} \quad \mathbf{Q} = \begin{pmatrix} \mathbf{\Omega} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}. \tag{10}$$

Then we can rewrite the VAR(2) process in (6) as a VAR(1) process,

$$\begin{pmatrix} \mathbf{y}_t \\ \mathbf{y}_{t-1} \end{pmatrix} = \begin{pmatrix} \mathbf{\Phi}_1 & \mathbf{\Phi}_2 \\ \mathbf{I} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{y}_{t-1} \\ \mathbf{y}_{t-2} \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ \mathbf{0} \end{pmatrix} \tag{11}$$

$$= \begin{pmatrix} y_{1,t} \\ y_{2,t} \\ y_{1,t-1} \\ y_{2,t-1} \end{pmatrix} = \begin{pmatrix} 0.75 & 0.5 & -0.125 & 0 \\ 0 & 0.7 & 0 & -0.1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} y_{1,t-1} \\ y_{2,t-1} \\ y_{1,t-2} \\ y_{2,t-2} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ 0 \\ 0 \end{pmatrix} \tag{12}$$

or equivalently,

$$\xi_t = \mathbf{F}\xi_{t-1} + \mathbf{v}_t. \tag{13}$$

Q1.2. As defined in Hamilton, a vector process is covariance-stationary if $E[\mathbf{y}_t]$ and $E[\mathbf{y}_t\mathbf{y}_{t-j}]$ for all $j \in \mathbb{N}$ are independent of t.

According to Hamilton's comment to proposition 10.1, $\mathbf{y}_t$ is covariance-stationary if all eigenvalues $\lambda \in \mathbb{C}$ of $\mathbf{F}$ are in the unit circle.

The eigenvalues of $\mathbf{F}$ are calculated with R in the appendix code. We have that

$$\lambda \in \{0.5,\ 0.25,\ 0.2\}\,. \tag{14}$$

Since $|\lambda| < 1$ for each eigenvalue λ, the VAR process is stable and covariance-stationary.

Problem 2.

Q2.1.

At first glance, a lag length of 4 does not seem to be optimal. Table 1 show different evaluation criteria, where the row index is lag periods. A lag length of 4 does not perform best in any of the criteria. A lag length of 3 performs better in all criteria than lag 4 except FPE, where they are the same.

Table 1: VARselect Criteria

	AIC(n)	HQ(n)	SC(n)	FPE(n)
1	-2.68	-2.58	-2.44	0.07
2	-3.11	-2.94	-2.69	0.04
3	-3.25	-3.01	-2.65	0.04
4	-3.22	-2.90	-2.45	0.04
5	-3.20	-2.82	-2.26	0.04
6	-3.32	-2.86	-2.20	0.04
7	-3.28	-2.75	-1.98	0.04
8	-3.30	-2.70	-1.82	0.04
9	-3.36	-2.69	-1.70	0.04
10	-3.38	-2.64	-1.55	0.03

Q2.2. The results of Tables 1B.i to 1B.iii are replicated in Tables 2 to 4 below, where column h is the forecast horizon.

Table 2: Variance Decomposition of Inflation (Percentage Points)

h	Forecast SE	Inflation	Unemployment	Interest.Rate
1	0.98	100	0	0
4	1.43	88	10	1
8	1.85	83	16	1
12	2.07	83	15	2

Table 3: Variance Decomposition of Unemployment (Percentage Points)

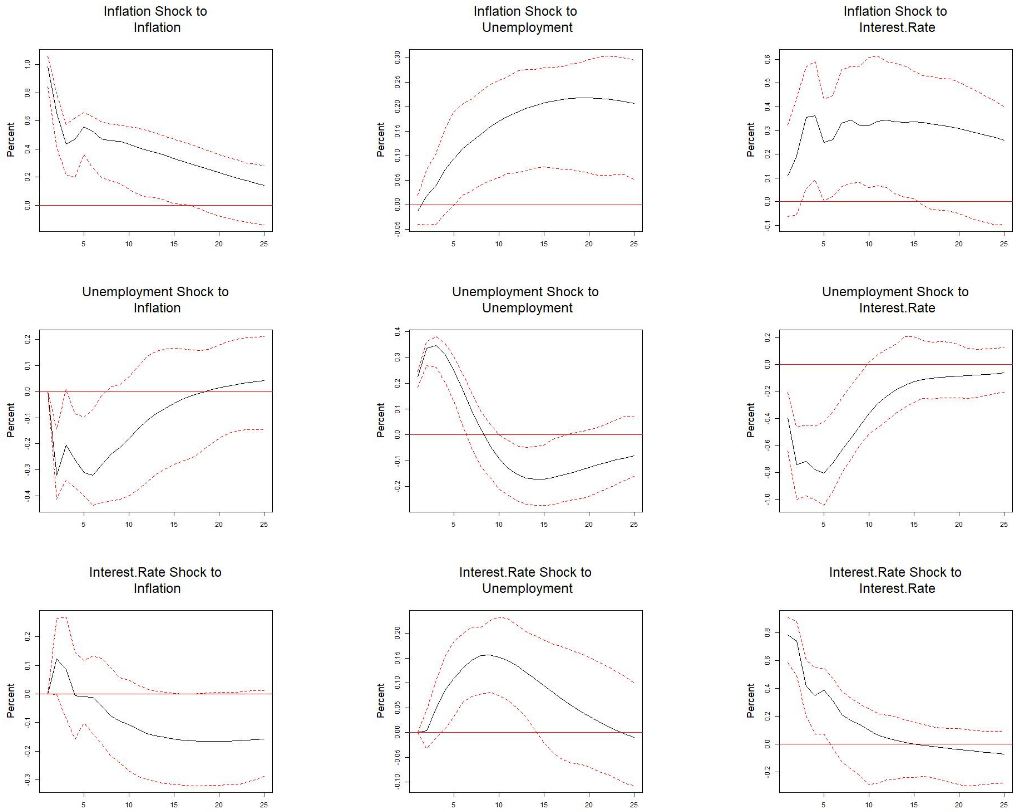
h	Forecast SE	Inflation	Unemployment	Interest.Rate
1	0.23	0	100	0
4	0.63	2	96	2
8	0.80	10	76	13
12	0.95	21	60	19

Table 4: Variance Decomposition of Interest.Rate (Percentage Points)

h	Forecast SE	Inflation	Unemployment	Interest.Rate
1	0.88	2	20	79
4	1.90	8	51	40
8	2.48	11	60	29
12	2.67	15	59	26

Q2.3.

Figure 1 is recreated below, where the x-axes are lag periods after the 1 percentage point increase shock.



A Appendix: R Code

```
library(vars)
library(xtable)

# Question 1. ####

# setup lag coefficients
Phi1 <- matrix(c(0.75, 0, 0.5, 0.7), 2, 2)
Phi2 <- matrix(c(-0.125, 0, 0, -0.1), 2, 2)

print(Phi1)
print(Phi2)

# VAR(1) matrix
F <- rbind(cbind(Phi1, Phi2), cbind(diag(2), matrix(0, 2, 2)))

# check VAR(1) coefficient matrix
F

# check eigenvalues
eigen(F)


# Question 2.1. ####

data <- read.delim("sw2001.txt", header=TRUE)
data <- ts(data[,-1], start = 1960, frequency = 1)

colnames(data) <- c("Inflation", "Unemployment", "Interest Rate")

# plot data
plot(data)

# select
varselect <- VARselect(data)
varselect

tab <- xtable(t(varselect$criteria),
       type="latex",
       caption=paste('VARselect Criteria'),
       label = "tab:VARselect",
       align = "|c|cccc|"
       )

print(tab,
      file=paste("hw3/HW3-VARselect", ".tex", sep=""),
      caption.placement="top"
)

# AIC and BIC
ABIC <- matrix(NA, ncol = 2, nrow = 10)
colnames(ABIC) <- c('AIC', 'BIC')

for (i in 1 : nrow(ABIC)){

  var <- VAR(data, p=i)
  ABIC[i, 1] <- AIC(var)
```

```
  ABIC[i, 2] <- BIC(var)

}

tab <- xtable(ABIC,
              type="latex",
              caption=paste('AIC and BIC'),
              label = "tab:ABIC",
              align = "|c|cc|"
              )

print(tab,
      file=paste("hw3/HW3-ABIC", ".tex", sep=""),
      caption.placement="top",
)

# Question 2.2. ####

# generate VAR
var <- VAR(data, p=4)

# forecast horizons
horizons <- c(1, 4, 8, 12)

# forecast errors variance decomposition
vd <- fevd(var, n.ahead=12)

# forecast covariance matrix
# https://github.com/cran/vars/blob/master/R/fevd.varest.R
msey <- vars:::.fecov(var, n.ahead=12)

# get mean squared errors
mse <- matrix(NA, nrow = length(horizons), ncol = length(vd))
colnames(mse) <- names(vd)

for (i in 1:nrow(mse)){
  mse[i,] <- diag(msey[,,horizons][,,i])
}

# standard errors
se <- sqrt(mse)

# variance decompositon tables
for (j in 1:length(vd)){

  # index specific periods
  vd_pct <- vd[[j]][horizons,]

  # express in percentage points
  vd_pct <- vd_pct * 100

  # add forecast horizons and MSE
  res <- cbind(horizons, se[, j], vd_pct)
  colnames(res)[1:2] <-  c("h", "Forecast SE")

  # show results
  name <- names(vd)[j]
  print(name)
```

```
  print(res)

  # make latex table
  tab <- xtable(res,
                type="latex",
                digits=c(0,0,2,0,0,0),
                caption=paste('Variance Decomposition of', name,
                                   '(Percentage Points)'
                              ),
                label=paste('tab:VD-', j, sep=""),
                align = "c|c|c|ccc|"
                )

  print(tab,
        file=paste("hw3/HW3-table", j, ".tex", sep=""),
        include.rownames = FALSE,
        caption.placement="top"
        )
}


# Question 2.3. ####

# names of variables
varlist <- names(vd)

i <- 1

for (imp in varlist){

  for (resp in varlist){

    # file save name
    jpegname <- paste("hw3/HW3-", i, ".jpg", sep='')
    i = i + 1

    # save plot
    jpeg(jpegname, width = 650, height = 500)

    # generate IRF
    irf_res <- irf(var, impulse = imp, response = resp, n.ahead = 24)

    # title
    title <- paste(imp, 'Shock to\n', resp)

    # plot
    plot(irf_res,
         main=title, sub='',
         ylab='Percent', xlab='Lag',
         cex=2, cex.lab=1.5)

    dev.off()
  }
}
```

Time Series Econometrics:
Home work assignment 3

Yukai Yang
Department of Statistics
Uppsala University

Abstract

Please write your report in LaTeX. The report should be clearly written such that it is easy to understand what is done and why. Please attach any computer code in an appendix.

1 Problem 1

Let

$$\begin{aligned} y_{1,t} &= 0.750 y_{1,t-1} - 0.125 y_{1,t-2} + 0.5 y_{2,t-1} + \varepsilon_{1,t} \\ y_{2,t} &= 0.700 y_{2,t-1} - 0.100 y_{2,t-2} + \varepsilon_{2,t} \end{aligned}$$

where all $\begin{pmatrix} \varepsilon_{1,t} & \varepsilon_{2,t} \end{pmatrix}$ are i.i.d. $N(\mathbf{0}, \mathbf{\Omega})$ for some 2×2 matrix $\mathbf{\Omega}$.

1. Write the system on the form (10.1.11) in Hamilton.
2. Is the system covariance-stationary?

2 Problem 2

Download the Stock and Watson (2001) data from Studentportalen.

1. Stock and Watson chose a lag length of 4. Does this seem appropriate?

2. Replicate their variance decomposition analysis (Table 1B.i-1B.iii).[1]
3. Replicate their impulse response analysis (Figure 1).

[1]Note: You may present the table as a figure instead.