Time Series Econometrics: Home work assignment 1

Yukai Yang
Department of Statistics
Uppsala University

Abstract

Please write your report in LaTeX. The report should be clearly written such that it is easy to understand what is done and why. Please attach any computer code in an appendix.

1 Problem 1

Consider the second-order difference equation ($p = 2$)

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + w_t.$$

Using direct multiplication, show that

1. the effect on y_{t+3} of a one-unit increase in w_t is
$$\phi_1^3 + 2\phi_1\phi_2 \tag{1}$$

2. the effect on y_{t+4} of a one-unit increase in w_t is
$$\phi_1^4 + 3\phi_1^2\phi_2 + \phi_2^2 \tag{2}$$

2 Problem 2

Consider the same difference equation

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + w_t,$$

where $\phi_1 = 3/4$ and $\phi_2 = -1/8$.

1. Using the eigenvalues of the matrix $\mathbf{F}$, show that

$$\frac{\partial y_{t+j}}{\partial w_t} = \left(\frac{1}{2}\right)^{j-1} - \left(\frac{1}{4}\right)^{j}. \tag{3}$$

2. For $j = 3$ and $j = 4$, verify that (1) and (2) produce the same results as (3).

3. Is the system stable? Motivate your answer.

3 Problem 3

Let $\{y_t\}_{t=-\infty}^{\infty}$ be given by

$$\mathbf{z}_s = \begin{pmatrix} y_{2s-1} \\ y_{2s} \end{pmatrix}, \quad s = 0, \pm 1, \pm 2, \ldots$$

where $\mathbf{z}_s$ is iid $N(\mathbf{0}, \mathbf{\Sigma})$, with

$$\mathbf{\Sigma} = \begin{pmatrix} 1 & \gamma \\ \gamma & 1 \end{pmatrix}.$$

Using standard results for the multivariate normal distribution,

1. verify that $y_t \sim N(0,1)$ for all $t = 0, \pm 1, \pm 2, \ldots$

2. show that if $\gamma \neq 0$, then $\{y_t\}_{t=-\infty}^{\infty}$ is neither strictly stationary nor covariance stationary

Hint: In (ii), compare the distribution of $\mathbf{z}_1$ with that of $(y_2, y_3)'$.

4 Problem 4

Let $\{\epsilon_t\}_{t=-\infty}^{\infty}$ be a white noise process and $\theta \neq 0$. Consider the two $MA(1)$ processes $\{y_t\}_{t=-\infty}^{\infty}$ and $\{\tilde{y}_t\}_{t=-\infty}^{\infty}$ given by

$$y_t = \mu + \epsilon_t + \theta\epsilon_{t-1}$$

and

$$\tilde{y}_t = \mu + \tilde{\epsilon}_t + \tilde{\theta}\tilde{\epsilon}_{t-1}$$

respectively, where $\tilde{\epsilon}_t = \theta\epsilon_t$ and $\tilde{\theta} = 1/\theta$.

Verify that $E(y_t) = E(\tilde{y}_t) = \mu$ and

$$E(y_t - \mu)(y_{t-j} - \mu) = E(\tilde{y}_t - \mu)(\tilde{y}_{t-j} - \mu),$$

for $j = 0, 1, 2, \ldots$.[1]

5 Problem 5

Consider the simple $AR(1)$ process

$$(1 - \phi L)y_t = \epsilon_t$$

where $\{\epsilon_t\}_{t=-\infty}^{\infty}$ is a white noise process.

1. Show, by recursive substitution, that

$$y_{t+s} = \phi^s y_t + \sum_{i=0}^{s-1} \phi^i \epsilon_{t+s-i}.$$

2. Use the above formula to compute the conditional expectation $E(y_{t+s}|I_t)$, where I_t is the information set available at time t.

[1] That is, verify that for any invertible $MA(1)$ representation, there is a noninvertible $MA(1)$ representation with the same first and second moments as the invertible representation.