Part 1

AK-0003-MDY

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + c + \varepsilon_t \quad (1)$$

4/4

1. The AR(2) process is stationary, if its corresponding difference equation is stable.
The AR(2) model can be written as
$$(1 - \phi_1 L - \phi_2 L^2) Y_t = c + \varepsilon_t \quad (2)$$
The difference equation (1) is stable provided that the roots of (3) lie outside the unit circle.
$$(1 - \phi_1 z - \phi_2 z^2) = 0 \quad (3)$$

2. 4/4
$$E(Y_t) = \phi_1 E(Y_{t-1}) + \phi_2 E(Y_{t-2}) + c + E(\varepsilon_t)$$
which gives us
$$E(Y_t) = \mu = \phi_1 \mu + \phi_2 \mu + c + 0 \implies$$
$$\implies c = \mu - \phi_1 \mu - \phi_2 \mu = \mu(1 - \phi_1 - \phi_2) \implies$$
$$\implies \mu = \frac{c}{(1 - \phi_1 - \phi_2)}$$

4/4

3. $(1 - 0{,}3z - 0{,}1z^2 = (1 - 0{,}5z)(1 + 0{,}2z)$

It is weakly stationary since the eigenvalues 0,5 and 0,2 are both inside the unit circle.

Part 1

AK-0003-MOY

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4/4

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$\Rightarrow \mu = \frac{c}{(1 - \phi_1 - \phi_2)}$

3. 4/4 $(1 - 0{,}3 z - 0{,}1 z^2 = (1 - 0{,}5 z)(1 + 0{,}2 z)$

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Part 1

AK-0003-MDY

4/4

4. The stationarity of an ARMA(1,1) process depends entirely on the Autoregressive parameter ϕ_1. The roots needs to lie outside the unit circle

5. 6/6

$$Y_t = \phi Y_{t-1} + \varepsilon_t + \theta \varepsilon_{t-1}$$

$$E(Y_t \varepsilon_t) = \phi E(Y_{t-1}\varepsilon_t) + E(\varepsilon_t \varepsilon_t) + \theta E(\varepsilon_{t-1}\varepsilon_t) =$$

$$= 0 + \sigma^2 + 0 =$$

$$= \sigma^2$$

No, it does not depend on time t

0/6

6. $E(Y_t \varepsilon_{t-1}) = \phi E(\overset{=\sigma^2}{Y_{t-1}\varepsilon_{t-1}}) + E(\varepsilon_t \varepsilon_{t-1}) + \theta E(\varepsilon_{t-1}^2) =$

$$\overset{?}{=} \phi E(\underline{Y_{t-2}\varepsilon_{t-1} + \varepsilon_{t-1}^2 + \theta \varepsilon_{t-2}\varepsilon_{t-1}}) + E(\varepsilon_t \varepsilon_{t-1}) + \theta(\varepsilon_{t-1}^2) =$$

$$= \phi \sigma^2 + \sigma^2 = \sigma^2(\phi + 1)$$

Yes, it depends on time t

7. $E(Y_t \varepsilon_{t-k}) = \phi E(Y_{t-k-1}\varepsilon_{t-k} + \varepsilon_{t-k}^2 + \theta \varepsilon_{t-k-1}\varepsilon_{t-k}) +$

$$+ E(\varepsilon_t \varepsilon_{t-k}) + \theta E(\varepsilon_{t-1}\varepsilon_{t-k}) = \cancel{\sigma^2} \quad 0$$

No, it does not depend on time t.

2/6

Part 1

AX-0003-MDY

8. 4/18

Unconditional mean:

$$E(y_t) = c^{?} + \phi E(Y_{t-1}) + \theta E(\varepsilon_{t-1}) + E(\varepsilon_t) \Rightarrow$$

$$\Rightarrow (1-\phi) E(Y_t) = c \Rightarrow E(Y_t) = \frac{c}{1-\phi} \quad =0$$

variance:

$$\gamma_0 = Var(Y_t) = E(Y_t - \mu)^2 = \int_{-\infty}^{\infty} (y_t - \mu_t)^2 f_{Y_t}(y_t)\, dy_t$$

$$\gamma_0 = E(Y_t - \mu)^2 = E(\varepsilon_t^2) = \sigma^2 = 1$$

covariances

$$\gamma_1 = \phi_1 \gamma_0 = \phi_1$$

$$\gamma_2 = \phi_1 \gamma_0 + \gamma_1 = 2\phi_1$$

autocorrelations

$$\rho_1 = \frac{\gamma_1}{\gamma_0} = \frac{\phi_1}{1} = \phi_1$$

$$\rho_2 = \frac{\gamma_2}{\gamma_0} = \frac{2\phi_1}{1} = 2\phi_1$$

0/8

Part 2

AK-0003-MDY

2. We want to test if Y_t has a unit root.
4/8
Our null hypothesis will then be

$H_0 : \rho = 1$ $\rho = 0$

and the alternative

$H_1 : |\rho| < 1$

Assumptions
$\alpha = 0$
$\delta = 0$

Our t-value for Y_{t-1} is -1,67 which is bigger than the results for the no constant, no trend test on the 1% and 5% significance level. And therefore we cannot reject the null that there exists a unit root on any significance level.

mostly?

2. 4

6/8

3. From 2 we the that the order of integration for Y_t is I(1). The theory the suggests that Z_t should also be I(1)

What about the cointegration vector?

Part 2

AK-0003-MDY

4/8

4. Well if we take alook at the t-value for X_{t-1} we can see that it is -2,54 which is actually less than the -1,95 for the no constant no trend ADF test on the 5% significance level which means that we can reject the null that there is a unit root. And it is therefore in accordance with the theoretical prediction

why?

2.4

6/16