Written Re-Exam of the Course Time Series Econometrics 2ST111 Fall 2018

Yukai Yang
Department of Statistics, Uppsala University
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UPPSALA UNIVERSITET

This is a four hours open-book exam. Please read carefully and answer all questions. The answers shall be clearly written, concise and relevant, and all steps shall be well explained. The total score is 100 points.

You can bring the textbook, the printed materials offered by the teachers (slides, notes), and any paper or printed articles that are relevant to the course. You are not allowed to use any calculator, computer, smart-phone or any devices with internet or bluetooth connection. You can bring paper dictionary, but the electronic dictionary is not allowed. You can use pen, pencil, eraser and ruler. You are not allowed to share any books, notes, papers, tools or devices with others during the exam.

Good luck!

Questions

1. Consider an AR(p) process X_t and a MA(q) process Y_t where the order p and q are known. Suppose that X_t and Y_τ are mutually independent for any t and τ. Show that the sum $X_t + Y_t$ is an ARMA(p^*, q^*) process. What are the orders p^* and q^* then?

2. Suppose that the process $\{y_t\}_{t=-\infty}^{\infty}$ is generated by

$$y_t = \mu + 2\varepsilon_t\varepsilon_{t-1} + \varepsilon_t \tag{1}$$

where μ is a constant and $\{\varepsilon_t\}_{t=-\infty}^{\infty}$ is an independent white noise process

(a) Show that y_t is covariance stationary. Is y_t still covariance stationary if ε_t is white noise process but not independent?

(b) Consider the process generated by

$$z_t = y_t - \mu \tag{2}$$

for $t = 0, \pm 1, \pm 2, ...$ Is z_t a martingale difference sequence (MDS)? Motivate your answer.

3. Consider the univariate model

$$y_t = x_t'\beta + u_t \tag{3}$$

for $t = 1, 2, ...,$ where y_t is the dependent variable, the vector x_t are the explanatory variables, the vector β are the coefficients, and u_t is the error. Inside the vector x_t, besides the constant ones, there can be either (1) strictly exogenous stochastic variables or (2) lagged y_t. And the error u_t is heteroskedastic, and can be either (i) white noise or (ii) autocorrelated.

In Lecture 5, we have introduced the two cases

Case 5:

(a) x_t stochastic

(b) $u|X \sim N(0, \sigma^2 V)$

(c) V is a known positive definite matrix

where u is the vector $(u_1, u_2, ...)'$ and X is the matrix $(x_1', x_2', ...)'$.

Case 6:

a) x_t stochastic, including perhaps lags of y

b) $x_t u_t$ is an MDS

c) $E(u_t^2 x_t x_t') = \Omega_t$ (positive definite) and

(i) $\sum_{t=1}^{T} \frac{\Omega_t}{T} \to \Omega$

(ii) $\sum_{t=1}^{T} \frac{u_t^2 x_t x_t'}{T} \xrightarrow{p} \Omega$

d)-e) x_t and u_t well-behaved such that certain asymptotic results apply

There can be four possible cases (combinations) which are (1)+(i), (1)+(ii), (2)+(i) and (2)+(ii). Try to match the four possible cases with Case 5 and Case 6 in Lecture 5. Explain why.

hint: answer the questions like "which of the two cases 5 or 6 can be applied on (1)+(i)" and etc.

4. How do your understand the sentence: "a lag polynomial is stable"? What is the relation between "stable" and "absolutely summable" in lag polynomials?

5. For model evaluation or diagnostics, what tests will you do after estimating the model? State and briefly explain them.

hint: fine if you cannot remember the names of the tests, but you should know what they are testing for and against.

6. Find the unconditional mean, variance and the autocorrelations of an ARMA(1,1) process by solving the Yule-Walker equations.

7. Consider the following univariate random walk model

$$y_t = y_{t-1} + u_t \tag{4}$$

with the error term following a stationary AR(1) process

$$u_t = \phi u_{t-1} + \varepsilon_t \tag{5}$$

where $|\phi| < 1$ and ε_t is an *i.i.d.* sequence with zero mean and variance σ^2. Show how you obtain the Beveridge-Nelson decomposition of y_t, and interpret it (show explicitly the trend, stochastic trend, cycle, and initial conditions).

8. Suppose that the true data generating process is as follows

$$Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + ... + A_p Y_{t-p} + c + \varepsilon_t. \tag{6}$$

where Y_t is a vector of dimension n, matrices A_i, $i = 1, ..., p$, are coefficients, c is a vector of intercepts, and ε_t is the error vector. There is/are unit root(s) in the dynamic model, and Y_t is I(1). Note that the dimension n can take value $1, 2, ...$, and when $n = 1$, the model is univariate.

(a) Since Y_t is I(1), there is only one unit root in the system. True or false? If false, can it be true and when?

(b) Sometimes one may take the first order difference of the data and then run the regression

$$\Delta Y_t = B_1 \Delta Y_{t-1} + B_2 \Delta Y_{t-2} + ... + B_p \Delta Y_{t-p} + c + \eta_t. \tag{7}$$

in order to get rid of the non-stationarity. Is this a good scheme? Explain why.