Written Re-Exam of the Course
Time Series Econometrics 2ST111 2019

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This is a five hours open-book exam. Please read carefully and answer all questions. The answers shall be clearly written, concise and relevant, and all steps shall be well explained. The total score is 100 points.

You can bring the textbook, the printed materials offered by the teachers (slides, notes), and any paper or printed articles that are relevant to the course. You are not allowed to use any calculator, computer, smart-phone or any devices with internet or bluetooth connection. You can bring paper dictionary, but the electronic dictionary is not allowed. You can use pen, pencil, eraser and ruler. You are not allowed to share any books, notes, papers, tools or devices with others during the exam.

Part 1: Autoregressive Moving Average Model (52 points)

Consider the following AR(2) model:

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + c + \epsilon_t \tag{1}$$

where c is a constant, ϵ_t is a white noise process with zero mean and variance σ^2. And the error is assumed to be weakly exogenous to the lagged Y_t, i.e. $\mathrm{E}(Y_{t-k}\varepsilon_t) = 0$, for any integer $k > 0$.

1. (4 points) State the (weak or covariance) stationarity condition for the AR(2) model.
2. (4 points) Suppose that the sequence Y_t is weakly stationary. Derive explicitly the unconditional expectation $\mu = \mathrm{E}(Y_t)$ in terms of c, ϕ_1, ϕ_2 and σ^2.

Suppose now that $\phi_1 = 0.3$, $\phi_2 = 0.1$, $c = 1$, and $\sigma^2 = 1$.

3. (4 points) Verify that the sequence Y_t is weakly stationary.

Now consider the ARMA(1,1) model

$$Y_t = \phi Y_{t-1} + \varepsilon_t + \theta \varepsilon_{t-1} \tag{2}$$

where ϵ_t is a white noise process with zero mean and variance σ^2. And the error is assumed to be weakly exogenous to the lagged Y_t, i.e. $\mathrm{E}(Y_{t-k}\varepsilon_t) = 0$, for any integer $k > 0$.

4. (4 points) Again, state the (weak or covariance) stationarity condition.
5. (6 points) Compute $\mathrm{E}(Y_t\varepsilon_t)$ in terms of c, ϕ, θ and σ^2. Does it depend on time t?
6. (6 points) Compute $\mathrm{E}(Y_t\varepsilon_{t-1})$ in terms of c, ϕ, θ and σ^2. Does it depend on time t?
7. (6 points) Compute $\mathrm{E}(Y_t\varepsilon_{t-k})$ for $k > 1$ in terms of c, ϕ, θ and σ^2. Does it depend on time t?
8. (18 points) Suppose that $\phi + \theta = 1$, and $\sigma^2 = 1$. Calculate (i) the unconditional expectation $\mu = \mathrm{E}(Y_t)$, (ii) the variance $\gamma_0 = \mathrm{Var}(Y_t)$, the covariances $\gamma_1 = \mathrm{Cov}(Y_t, Y_{t-1})$, $\gamma_2 = \mathrm{Cov}(Y_t, Y_{t-2})$, (iii) the two autocorrelations ρ_1 and ρ_2, in terms of ϕ and θ.

   (Hint for the variance and autocovariances, use the Yule-Walker approach)

Part 2: Dickey-Fuller Test, Unit Roots and Cointegration (48 points)

Consider the regression model

$$\Delta Y_t = \delta + \rho Y_{t-1} + \gamma_1 \Delta Y_{t-1} + \gamma_2 \Delta Y_{t-2} + \varepsilon_t \tag{3}$$

where the initial values, Y_{-2}, Y_{-1} and Y_0 are given and the error term is assumed to be independently and identically distributed with zero mean and constant variance.

1. (8 points) Show how the equation in (3) is related to an autoregressive model for Y_t.
2. (8 points) Now, let Y_t be a monthly time series for the percentage yield of a 10 years US government bond covering the period $t = 1955:1 - 2002:12$. The results of an OLS estimation of equation (3) is reported in Table 1, while Table 2 contains the corresponding critical values. Use the information in the tables to construct a test for the hypothesis that Y_t has a unit root. Explain the test and the outcome.
3. (8 points) Some macroeconomic theories predict that the real interest rate should remain constant in equilibrium. In a stochastic environment where shocks repeatedly hit the economy, a testable implication is that the real interest rate should be stationary. Let Z_t be the year-on-year inflation rate, and define the realized real interest rate as $X_t = Y_t - Z_t$, where Y_t is the nominal interest rate measured using the bond mentioned in the previous question. Given your conclusion regarding the order of integration of Y_t, what does the theory suggest on the order of integration of the inflation rate, Z_t?

Table 1: Modelling ΔY_t by OLS for 1955:4 – 2002:12

	Coefficient	Std.Error	t-value
Constant	0.049972	0.03080	1.62
ΔY_{t-1}	0.394837	0.04074	9.69
ΔY_{t-1}	-0.221742	0.04083	-5.43
Y_{t-1}	-0.007030	0.00420	-1.67
$\hat{\sigma}$	0.265579	RSS	40.13
R^2	0.153091	F(3, 569)	34.28
No. of observations	573		

Table 2: 1% and 5% critical values for augmented Dickey-Fuller tests

	No Constant No Trend		Constant No Trend		Constant Trend	
Sample Size	1%	5%	1%	5%	1%	5%
$T = 25$	-2.66	-1.95	-3.75	-3.00	-4.38	-3.60
$T = 50$	-2.62	-1.95	-3.58	-2.93	-4.15	-3.50
$T = 100$	-2.60	-1.95	-3.51	-2.89	-4.04	-3.45
$T = 250$	-2.58	-1.95	-3.46	-2.88	-3.99	-3.43
$T = 500$	-2.58	-1.95	-3.44	-2.87	-3.98	-3.42
$T = \infty$	-2.58	-1.95	-3.43	-2.86	-3.96	-3.41

4. (8 points) Table 3 reports the results of OLS applied to the regression model

$$\Delta X_t = \tilde{\delta} + \tilde{\rho} X_{t-1} + \tilde{\gamma}_1 \Delta X_{t-1} + \tilde{\varepsilon}_t. \tag{4}$$

Use the information in the tables to infer if the US data is in accordance with the theoretical prediction.

5. (16 points) Instead of the real interest rate, $X_t = Y_t - Z_t$, with a unit coefficient to inflation, we could consider a different linear combination

$$X_t^* = Y_t - \beta Z_t. \tag{5}$$

Explain how the parameter β can be estimated and outline a procedure for testing if $\beta = (1 - \beta_1)'$ is a cointegrating vector.

Table 3: Modelling ΔX_t by OLS for 1956:3 – 2002:12

	Coefficient	Std.Error	t-value
Constant	0.045980	0.02490	1.85
ΔX_{t-1}	0.252356	0.04090	6.16
X_{t-1}	-0.017406	0.00686	-2.54
$\hat{\sigma}$	0.389264	RSS	84.70
R^2	0.070025	F(2, 559)	21.05
No. of observations	562		