

Figure 1: plot of chunk unnamed-chunk-2

1. **Problem**

Consider the polynomial function below.

$$f(x) = -6x^3 + 7x^2 - 4x + 3$$

Multiple choice: describe the end behavior.

- (a) Down-down: when $x \rightarrow -\infty$ then $f(x) \rightarrow -\infty$ and when $x \rightarrow \infty$ then $f(x) \rightarrow -\infty$.
- (b) Up-down: when $x \rightarrow -\infty$ then $f(x) \rightarrow \infty$ and when $x \rightarrow \infty$ then $f(x) \rightarrow -\infty$.
- (c) Down-up: when $x \rightarrow -\infty$ then $f(x) \rightarrow -\infty$ and when $x \rightarrow \infty$ then $f(x) \rightarrow \infty$.
- (d) Up-up: when $x \rightarrow -\infty$ then $f(x) \rightarrow \infty$ and when $x \rightarrow \infty$ then $f(x) \rightarrow \infty$.

Solution

The degree of the polynomial is 3, which is odd. The highest-degree term's coefficient is negative.

Thus the end behavior is up-down: when $x \rightarrow -\infty$ then $f(x) \rightarrow \infty$ and when $x \rightarrow \infty$ then $f(x) \rightarrow -\infty$.

2. **Problem**

Consider the polynomial function below.

$$f(x) = -4x^7 - 5x^6 + 6x^5 - 7x^3$$

How many roots, including complex roots and counting multiplicity, does the polynomial have?

Or, in other words, based on the degree of the polynomial, what is the most number of possible real roots?

Solution

The Fundamental Theorem of Algebra tells us that the number of complex roots, when counting for multiplicity, is equal to the degree of the polynomial.

The degree of the polynomial is just the largest exponent.

Here are the (numerically solved, approximate) roots:

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## Error in py_call_impl(callable, dots$args, dots$keywords): NoConvergence: convergence to
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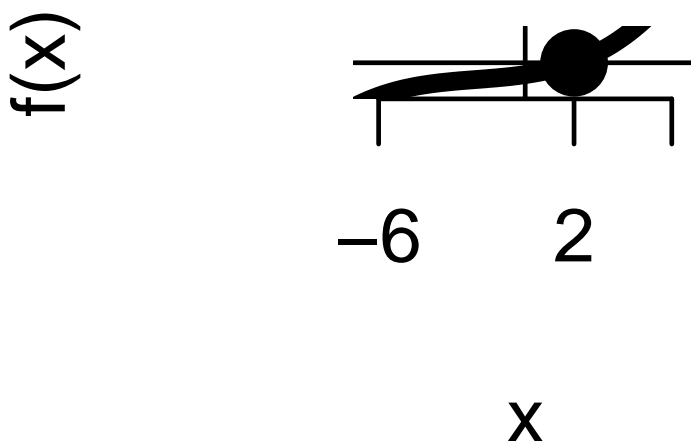
3. Problem

Draw the polynomial function $f(x)$ shown in factored and expanded forms. Be sure to indicate the roots, where the function is positive or negative, and end behavior.

$$f(x) = 2(x - 2)(x + 4 - 5i)(x + 4 + 5i)$$

$$f(x) = 2x^3 + 12x^2 + 50x - 164$$

Solution



4. Problem

Factor the polynomial function $f(x)$ to determine the roots.

$$f(x) = -3x^5 + 3x^4 - 3x^3 - 63x^2 + 570x + 1224$$

Solution

Factor.

$$f(x) = -3(x-4)(x+2)(x+3)(x-1-4i)(x-1+4i)$$

The roots:

$$x|_{f(x)=0} = [-3, -2, 4, 1-4i, 1+4i]$$