Instruction: *Type your answers to the following questions provided by LaTeX and submit a zipped file (included .pdf and .tex files) to E-learning (Sakai) by group (only 4-5 members in each group). Only team leader will submit it. One page per problem. Please use the solution template which is provided; summarize the work of each member in percentage (%).*

GROUP 1984 —— MEMBER LIST			
No.	**Name**	**ID**	**Role**
1	Quach Dang Giang	1952044	Leader, 30% (of the current work)
2	Huynh Phuoc Thien	1952463	Member, 30%
3	Tran Nguyen Anh Khoa	1911419	Member, 40%

The number of mandatory exercise: 5/5

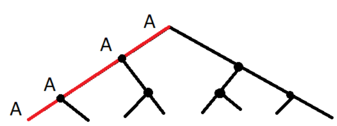
Problem 1

1.1

We have: Each person may have competed 0,1,2,3 or 4 matches. However, it is impossible that 1 person have competed 4 matches AND 1 person have competed 0 match at the same time. Thus, at any moment of the tournament there are always two players having identical number of games played

1.2

Imagine the tournament is a binary tree in which the teams are the leaves, the matches are the internal vertices. With n leaves, we can create n-1 internal vertices. So, let n = 8 then we have 8 teams and 7 matches happened. Let team A always won. 2 teams create an internal vertices, so if team A always won, team A will go through 3 matches which are similar to the vertices (According to the graph)



Thus, there exists a team that has played at least three matches if n - 1 competition games were happening.

1.3

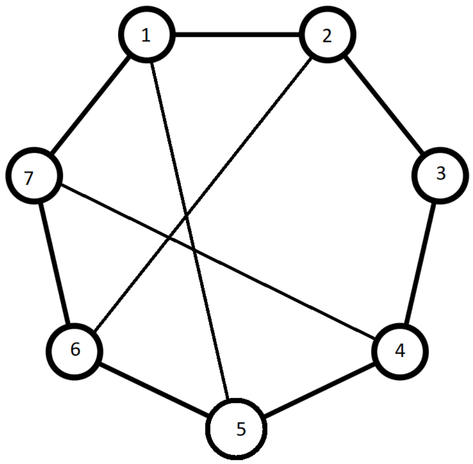
Define a graph where each vertex corresponds to a participant and where two vertices are adjacent iff the two participants they represent know each other. Take a vertex a of maximum degree. We claim that this vertex is adjacent to every other vertex. Suppose, on the contrary, that there is a vertex b which is not adjacent to a. Then every pair of vertices in the neighbourhood N(a) of a are adjacent. Furthermore, at least one of the vertices in N(a) is adjacent to b. The degree of this vertex is larger than that of a, a contradiction.

1.4

Consider 6 people represented by dots in a circle. Let every dot be connected to every other dot by a line and let the lines be the color red if the two people know each other and blue if they do not know each other. Consider any of the 6 dots, say dot x. We can see that x is connected to 5 other dots by a line and by the pigeonhole principle 3 of these lines are the same color, say red (the proof is analogous if we choose blue instead of red) . Now examine the 3 dots connected to x. If any of those 3 people are connected by a red line, then we have found a red triangle which represents 3 people who know each other. If the 3 people are not connected by any red lines, then all 3 of them are connected by blue lines forming a blue triangle which represents 3 people not knowing each other. Thus in a group of 6 people there will always be 3 people who know each other or 3 people who do not know each other.

1.5

Consider 7 people represented by dots in a circle. Let each dot connected with 3 others by a line and each line represented the letter being sent. Each dot only have 3 line connected to, so when we have connected all the lines to the dots, we have 1 dot that just has 2 lines connected to. So, there is one person who did not write back to his sender. As we can see from the graph, the third dot just have 2 lines connected to it.



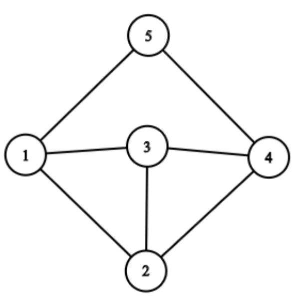
Problem 2

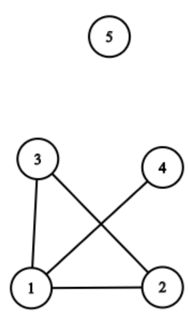
2.1

	K_n	C_n	W_n	$K_{m,n}$	Q_n
Edges	$\frac{n(n-1)}{2}$	n	$2n$	$m.n$	$n.2^{n-1}$
Vertices	n	n	$n+1$	$m+n$	2^n

2.2

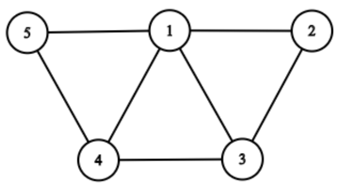
There exist simple graphs including vertices that their degree are respectively 3,3,3,3,2 and 3,2,2,1,0





2.3

There are 7 edges in a graph which has vertices of degree 4,3,3,2,2



2.4

The number of edges of $\overline{G}$ is the number of edges of the complete graph based on the number of G vertices minus the number of G edges

$$\Rightarrow \overline{e} = \frac{v(v-1)}{2} - e$$

2.5

Apply equation from 2.4 that we have proved

$$\overline{e} = \frac{v(v-1)}{2} - e \Rightarrow 13 = \frac{v(v-1)}{2} - 15 \Rightarrow v = 8$$

$\Rightarrow$ G has 8 vertices

Problem 3

3.1

Matrix 1:

	a	b	c	d
a	0	1	1	1
b	1	0	0	1
c	1	0	0	1
d	1	1	1	0

	e_1	e_2	e_3	e_4	e_5
a	1	0	0	1	1
b	1	1	0	0	0
c	0	0	1	1	0
d	0	1	1	0	1

Matrix 2:

	a	b	c	d
a	0	0	1	1
b	0	0	1	2
c	1	1	0	1
d	0	2	1	0

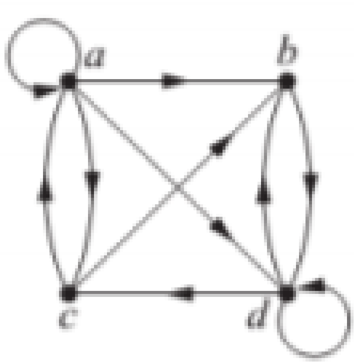
	e_1	e_2	e_3	e_4	e_5
a	1	0	0	0	0
b	0	0	1	1	1
c	1	1	0	0	1
d	0	1	1	1	0

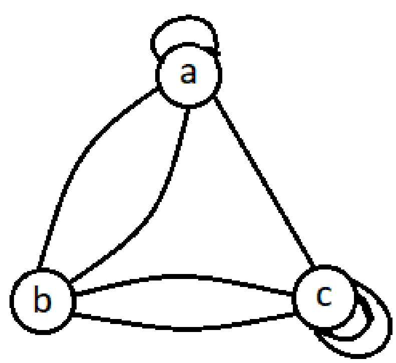
Matrix 3:

	a	b	c	d
a	1	1	1	1
b	0	0	0	1
c	1	1	0	0
d	0	1	1	1

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
a	-1	0	0	0	1	-1	0	0	-1	0
b	1	1	-1	0	0	0	0	0	0	-1
c	0	0	0	1	-1	1	0	0	0	1
d	0	-1	1	-1	0	0	0	0	1	0

3.2





3.3

Using Handshaking Theorem a v vertices and e edges graph, since the degree of any vertex of the graph is greater of equal than m, we have:

$$2e = \sum_{vertex \in V} deg(vertex) = deg(vertex_1) + deg(vertex_2) + ... + deg(vertex_v) \geqslant m.v$$

$$\Rightarrow \frac{2e}{v} \geqslant m$$

Proving similarly, we also have:

$$\frac{2e}{v} \leqslant M$$

Then

$$m \leqslant \frac{2e}{v} \leqslant M$$

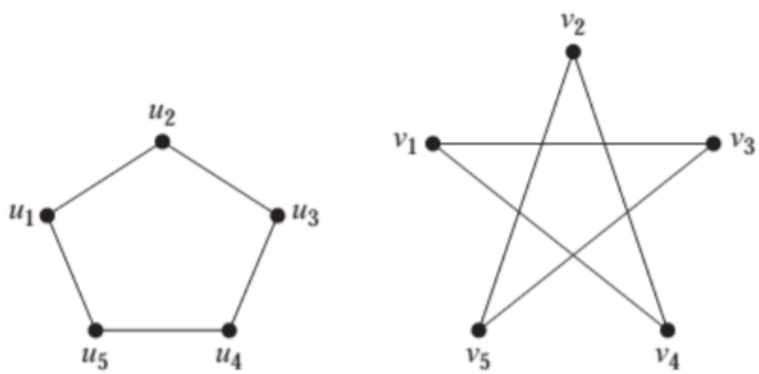
Problem 4

a)

The two graph are isomorphic. Isomorphism function $f : U \to V$ with

$$f(u_1) = v_1, f(u_2) = v_2, f(u_3) = v_4, f(u_4) = v_5, f(u_5) = v_3$$

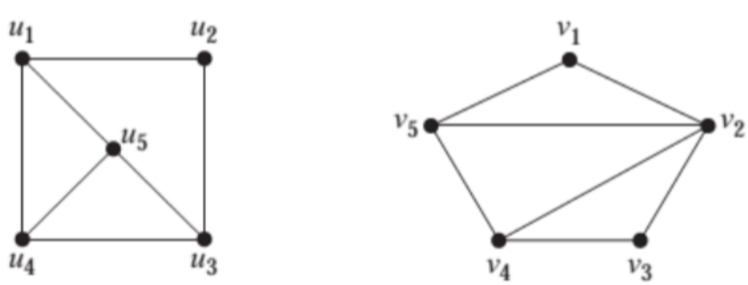
b)



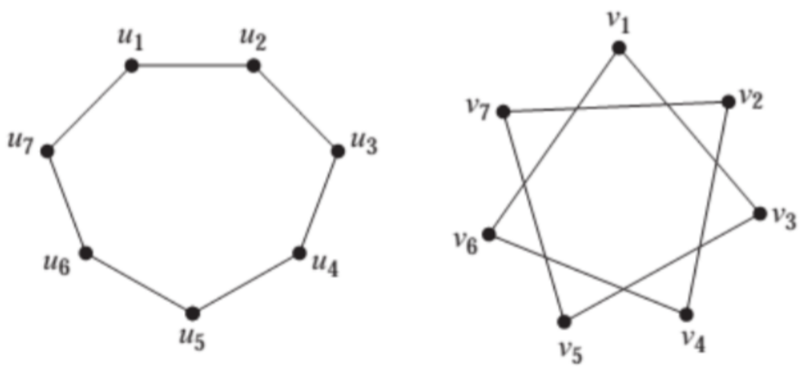
The two graph are isomorphic. Isomorphism function $f : U \to V$ with

$$f(u_1) = v_1, f(u_2) = v_3, f(u_3) = v_5, f(u_4) = v_2, f(u_5) = v_4$$

c)



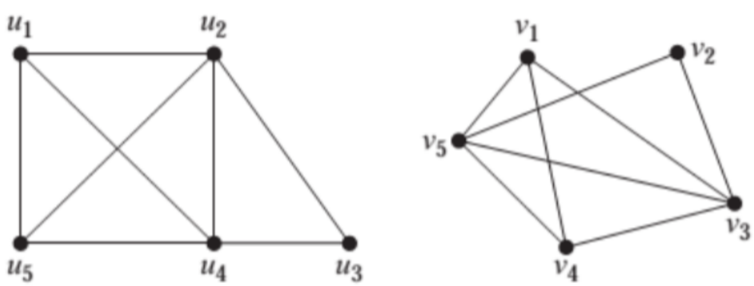
The two graph are not isomorphic because the degree of v_2 is 4 but no vertex in the first graph has degree of 4 d)



The two graph are isomorphic. Isomorphism function $f : U \to V$ with

$$f(u_1) = v_1, f(u_2) = v_3, f(u_3) = v_5, f(u_4) = v_7, f(u_5) = v_2, f(u_6) = v_4, f(u_7) = v_6$$

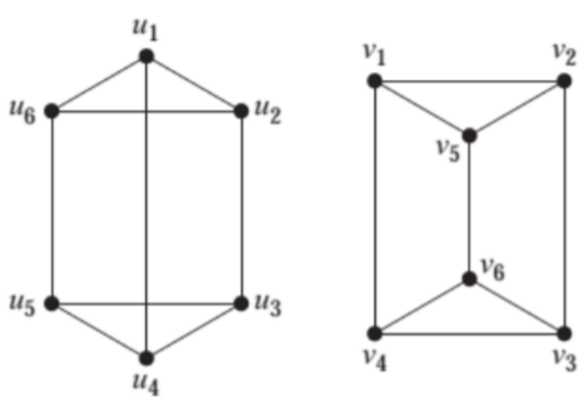
e)



The two graph are isomorphic. Isomorphism function $f : U \to V$ with

$$f(u_1) = v_1, f(u_2) = v_3, f(u_3) = v_2, f(u_4) = v_5, f(u_5) = v_4$$

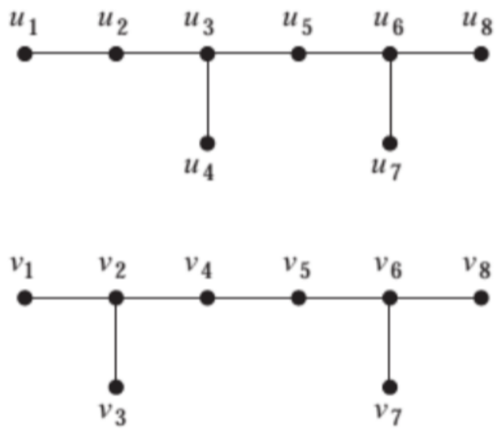
f)



The two graph are isomorphic. Isomorphism function $f : U \to V$ with

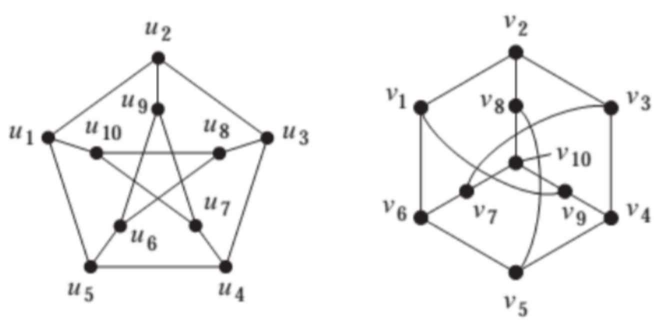
$$f(u_1) = v_5, f(u_2) = v_2, f(u_3) = v_3, f(u_4) = v_6, f(u_5) = v_4, f(u_6) = v_1$$

g)



The two graph are not isomorphic since their traversal path are different to each other

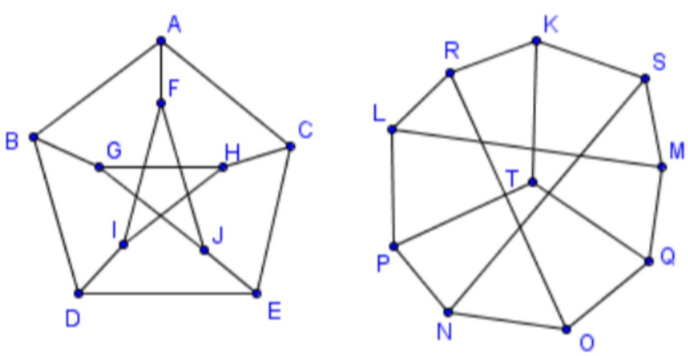
h)



The two graph are isomorphic. Isomorphism function $f : U \to V$ with

$f(u_1) = v_1, f(u_2) = v_2, f(u_3) = v_3, f(u_4) = v_7, f(u_5) = v_6, f(u_6) = v_5, f(u_7) = v_{10}, f(u_8) = v_4, f(u_9) = v_2, f(u_{10}) = v_9$

i)

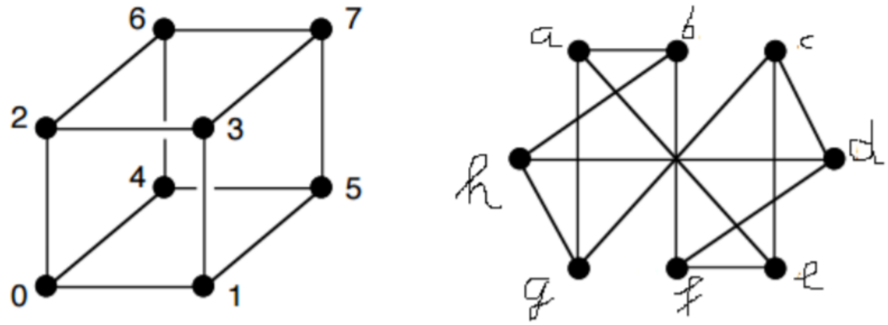


The two graph are isomorphic. Isomorphism function $f : U \to V$ with

$$f(A) = K, f(B) = R, f(C) = S, f(D) = L, f(E) = M, f(F) = T, f(I) = P, f(J) = Q,$$

$$f(H) = N, f(G) = O$$

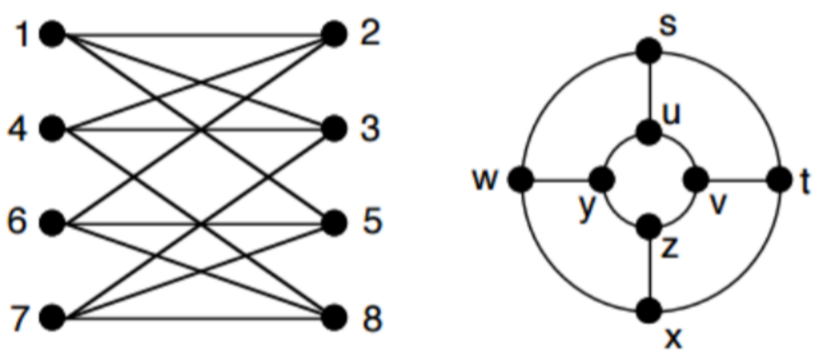
j)



The two graph are isomorphic. Isomorphism function $f : U \to V$ with

$$f(a) = 0, f(b) = 1, f(c) = 6, f(d) = 7, f(e) = 4, f(f) = 5, f(g) = 2, f(h) = 3$$

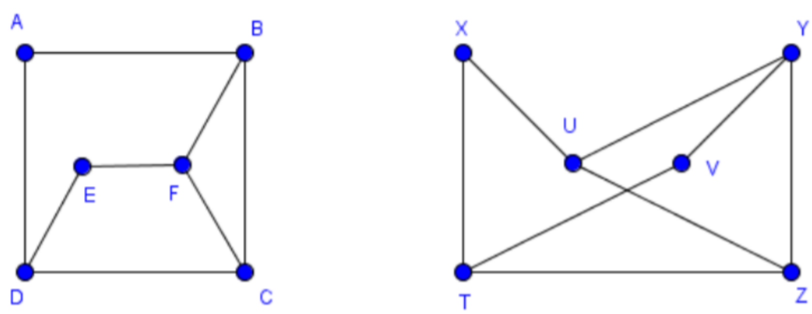
k)



The two graph are isomorphic. Isomorphism function $f : U \to V$ with

$$f(1) = s, f(2) = w, f(3) = t, f(4) = x, f(5) = u, f(6) = y, f(7) = v, f(8) = z$$

l)



The two graph are isomorphic. Isomorphism function $f : U \to V$ with

$$f(A) = V, f(B) = Y, f(C) = Z, f(D) = T, f(E) = X, f(F) = U$$