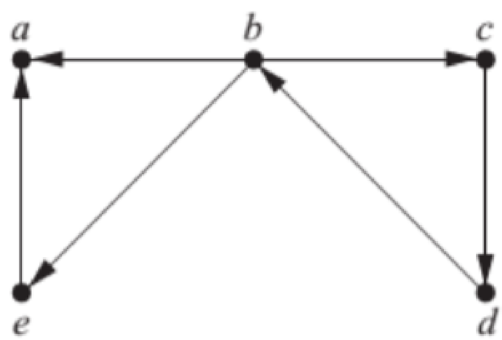
GROUP 1984 —— MEMBER LIST			
No.	**Name**	**ID**	**Role**
1	Quach Dang Giang	1952044	Leader, 25% (of the current work)
2	Huynh Phuoc Thien	1952463	Member, 35%
3	Tran Nguyen Anh Khoa	1911419	Member, 40%

The number of mandatory exercise: 23/23

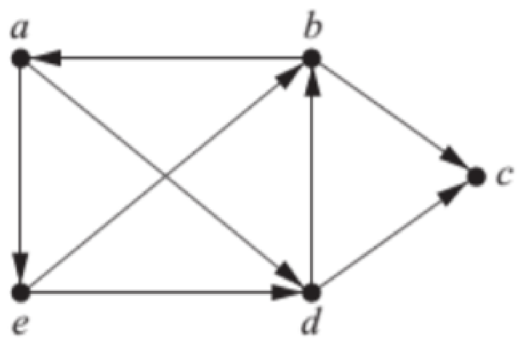
Question 1

a)



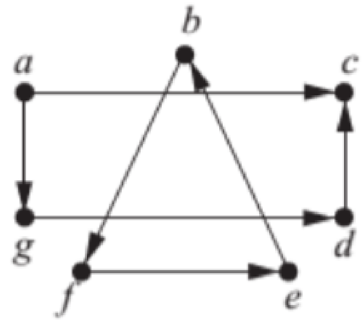
The graph is weakly connected because there is no path from a to any other vertices.

b)



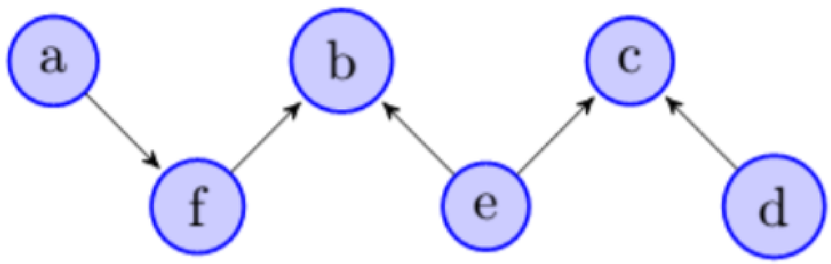
The graph is weakly connected because there is no path from a to any other vertices.

c)



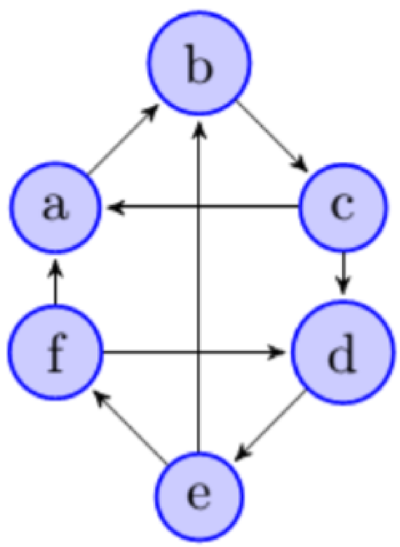
The graph is neither strongly connected nor weakly connected because there is no path from a to b in the underlying undirected graph.

d)



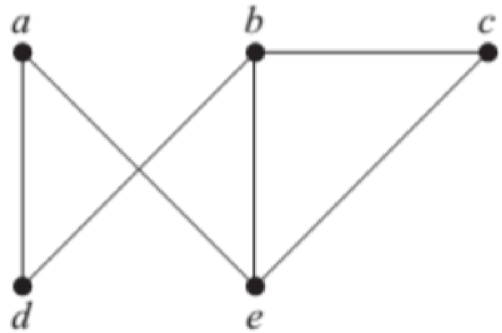
The graph is weakly connected because there is no path from b to any other vertices.

e)



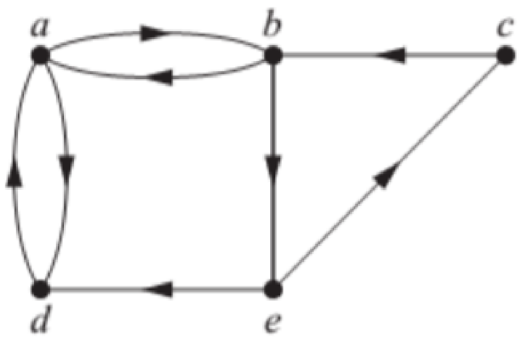
The graph is strongly connected.

Question 2



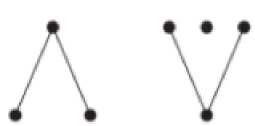
a, e, b, c, b form a path of length 4, this path is not simple because edge bc is passed 2 times
c, b, d, a, e, c form path of length 5, this path is simple and circuit.

Question 3

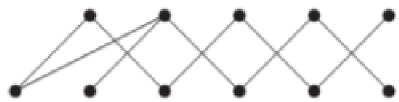


a, b, e, c, b form a simple path of length 4
a, d, a, d, a form a circuit path of length 4

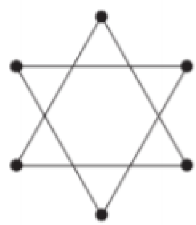
Question 4

a) 

The graph is not a connected graph

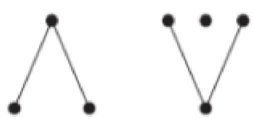
b) 

The graph is a connected graph

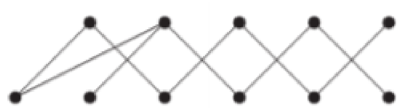
c) 

The graph is not a connected graph

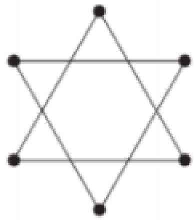
Question 5

a) 

Graph a) has 3 connected components

b) 

Graph b) has 1 connected component

c) 

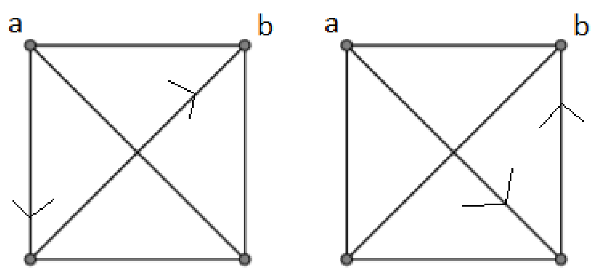
Graph b) has 2 connected components

Question 6

Let the two different vertices be a and b.

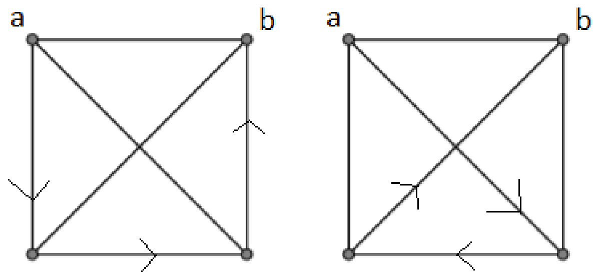
a)

There are two path of length 2

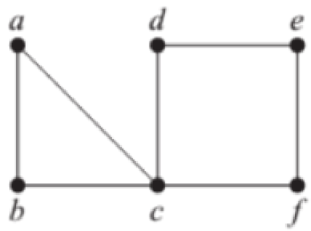


b)

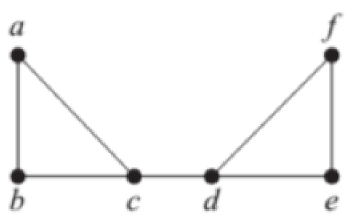
There are two path of length 3



Question 7

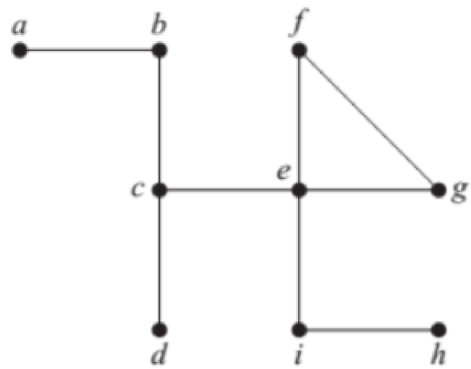
a) 

Cut vertex: c

b) 

Cut vertex: c, d

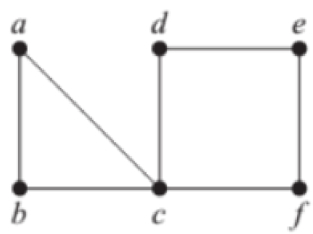
c)



Cut vertex: b, c, e, i

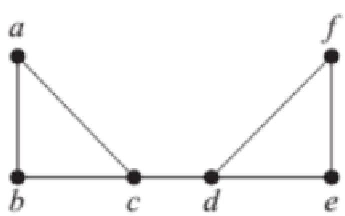
Question 8

a)

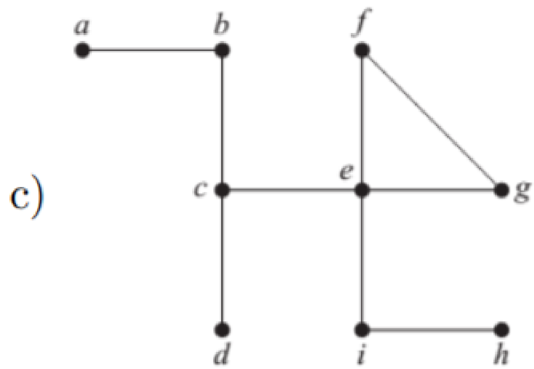


This graph has no cut edge

b)



Cut edge: cd

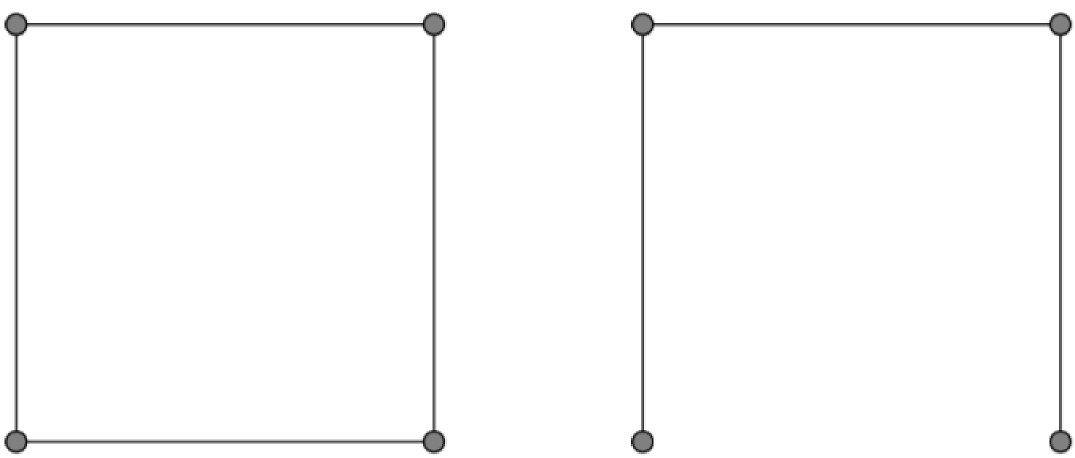


Cut edge: ab, bc, cd, ce, ei, ih

Question 9

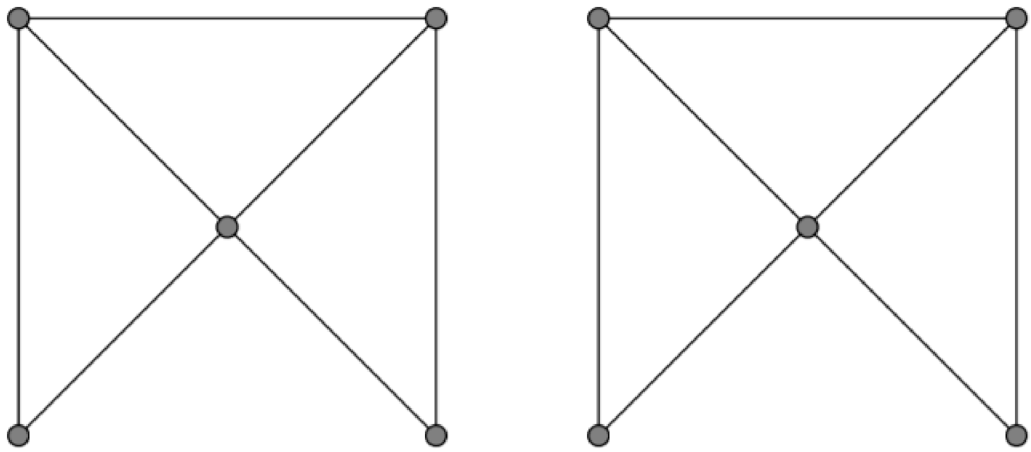
a)

C_n has no cut edge because if we remove any edge, the graph still remain a connected component Example: C_4



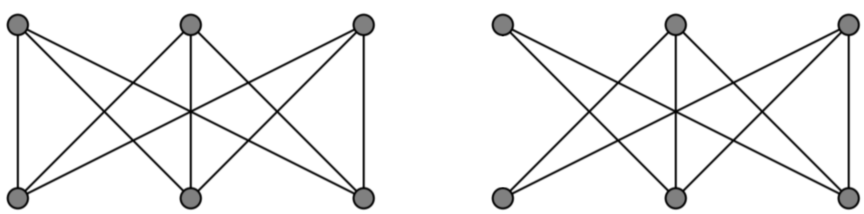
b)

K_n has no cut edge because if we remove any edge, the graph still remain a connected component
Example: K_4



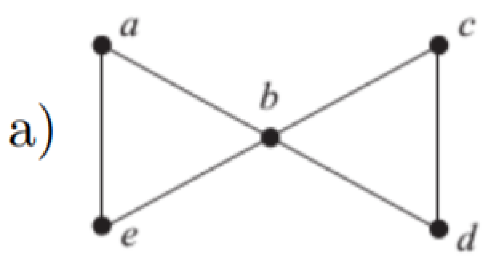
c)

K_n has no cut edge because if we remove any edge, the graph still remain a connected component
Example: K_4



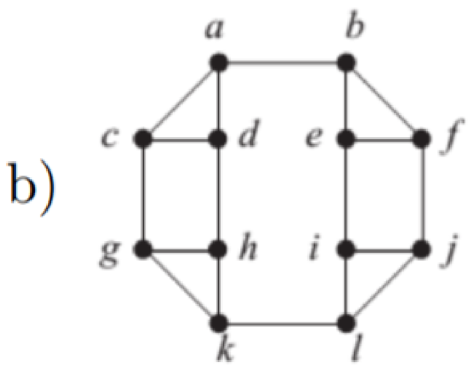
Question 10

a)



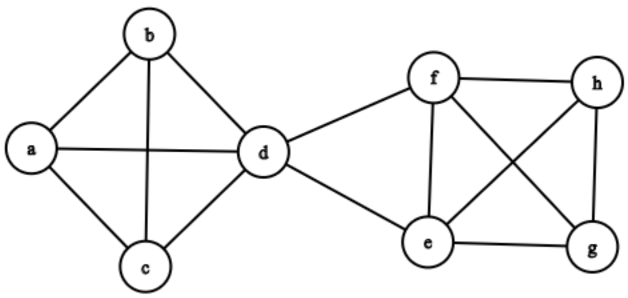
$\kappa(G) = 1, \lambda(G) = 2$

b)



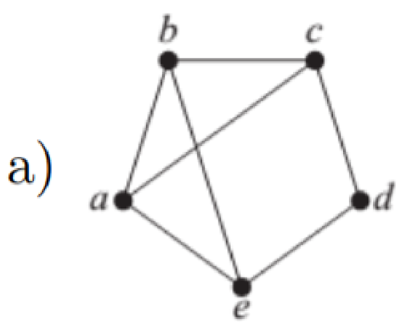
$\kappa(G) = 2, \lambda(G) = 2$

Question 11



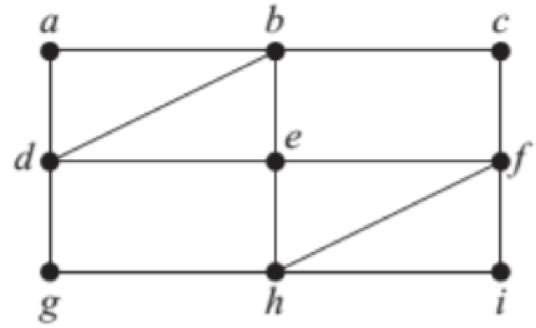
Question 12

An undirected graph has an Euler circuit if all vertex's degree are odd and has an Euler path if every vertex have even degree or exactly two vertex have odd degree.



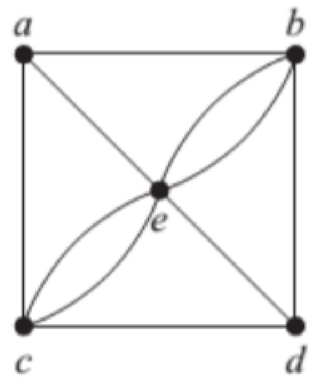
Graph a) has 4 odd degree vertices
⇒ Graph a) has no Euler circuit and has no Euler path

b)



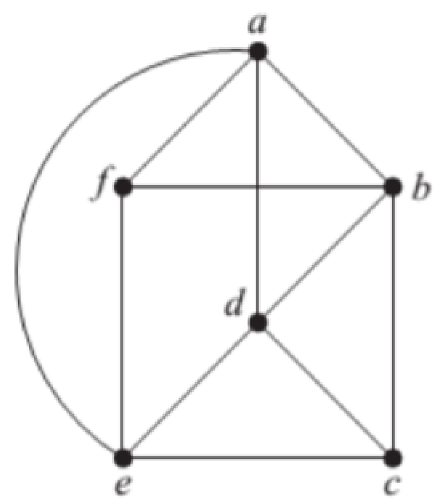
All vertex in graph b) are even degree
Graph b) has Euler circuit
Path sequence: {a,b},{b,d},{d,e},{e,f},{f,h},{h,e},{e,b},{b,c},{b,c},{c,f},{f,i},{i,h},{h,g},{g,d},{d,a}

c)



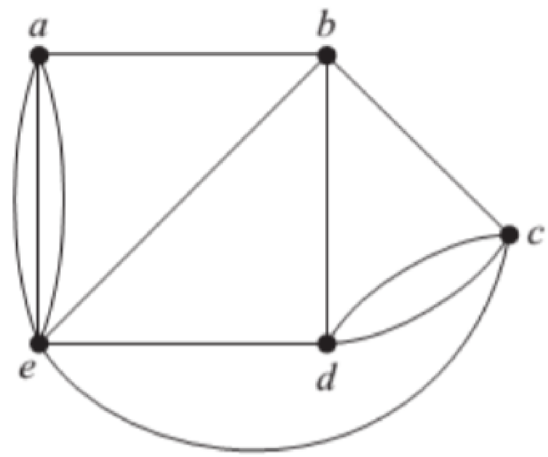
Graph c) has 2 odd degree vertices
$\Rightarrow$ Graph c) has no Euler circuit and has an Euler path
Path sequence: {a,b},{b,e},{e,a},{a,c},{c,e},{e,b},{b,d},{d,e},{e,c},{c,d}

d)



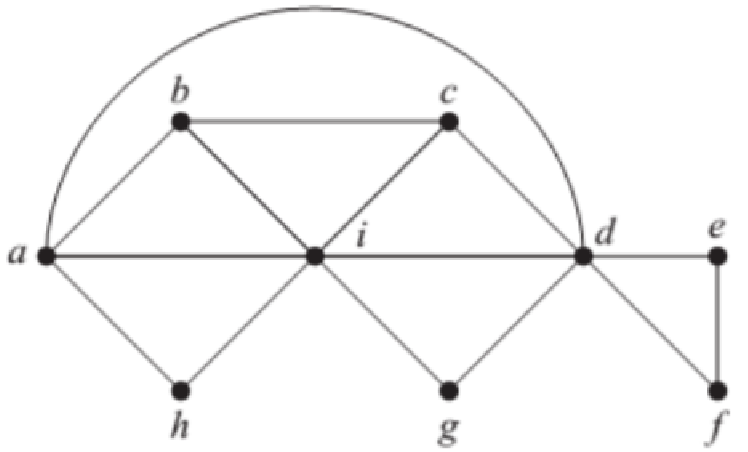
Graph d) has 2 odd degree vertices
⇒ Graph d) has no Euler circuit and has an Euler path
Path sequence: {f,a},{a,e},{e,d},{d,b},{b,a},{a,d},{d,c},{c,b},{b,f},{f,e},{e,c}

e)



All vertex in graph b) are even degree
Graph e) has Euler circuit
Path sequence: {a,b},{b,e},{e,d},{d,b},{b,c},{c,d},{d,c},{c,e},{e,a},{a,e},{e,a}

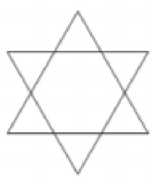
f)



Graph c) has 2 odd degree vertices
$\Rightarrow$ Graph c) has no Euler circuit and has an Euler path
Path sequence: {b,a},{a,i},{i,h},{h,a},{a,d},{d,e},{e,f},{f,d},{d,g},{g,i},{i,d},{d,c},{c,i},{i,b},{b,c}

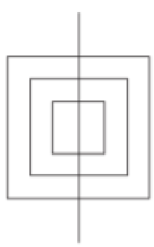
Question 13

a)



The graph has no odd degree vertex so it contains an Euler circuit. $\Rightarrow$ The picture shown can be drawn with a pencil in a continuous motion without lifting the pencil or retracing part of the picture.

b)



The graph has two odd degree vertices so it contains an Euler path. $\Rightarrow$ The picture shown can be drawn with a pencil in a continuous motion without lifting the pencil or retracing part of the picture.

Question 14

To form an Euler path, the degree of all vertices must be even degree.

a)

The degree of each vertex of K_n has Euler path then $n-1$. So if K_n has Euler circuit then $n-1$ must be an even number $\Rightarrow n$ must be an odd number and $n \geqslant 1$.

b)

For $n \geqslant 3$ (the requirement to form a cycle graph) the degree of every vertices of C_n is 2. So if C_n has Euler circuit then $n \geqslant 3$.

c)

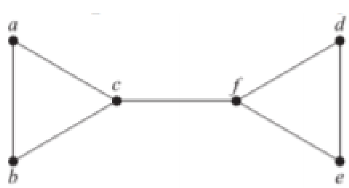
For $n \geqslant 3$ (the requirement to form a wheel graph) the degree of most vertices of C_n is 3, then it contain odd degree vertex. So W_n has no Euler circuit no matter the value of n.

d)

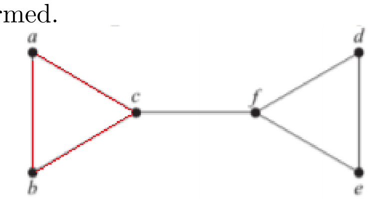
The degree of every vertices of Q_n is n. So if K_n has Euler circuit then n must be an even number and $n \geqslant 1$.

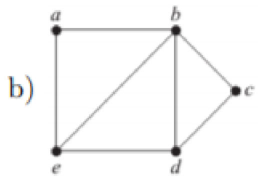
Question 15

a)

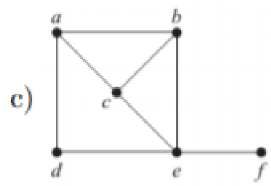


Graph a) has no Hamilton circuit because in the process of finding Hamilton circuit, a new subcircuit is formed.

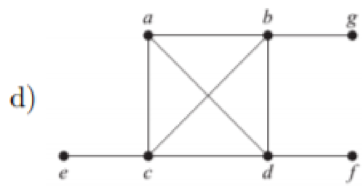




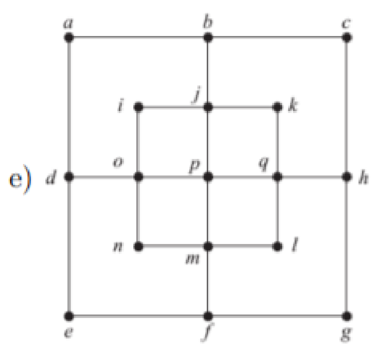
Graph b) has Hamilton circuit.
Path sequence: {a,b},{b,e},{e,d},{d,c},{c,a}



Graph c) has no Hamilton circuit because $deg(f) = 1$.

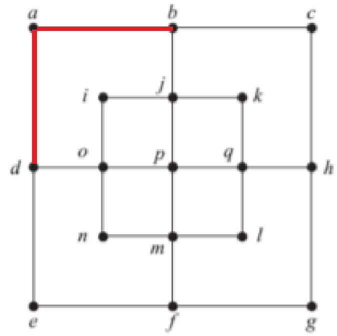


Graph d) has no Hamilton circuit because $deg(e) = deg(f) = deg(g) = 1$.

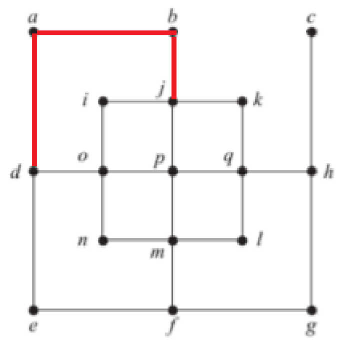


First, we start from vertex a.

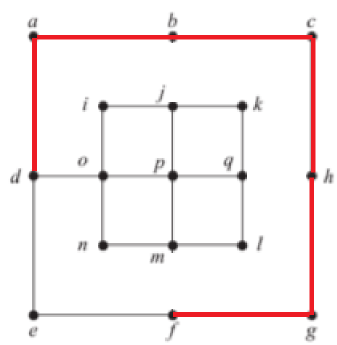
e)



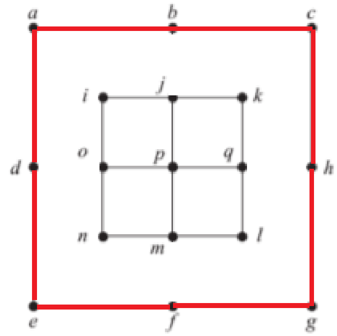
From now, if we take the path {b,j}:



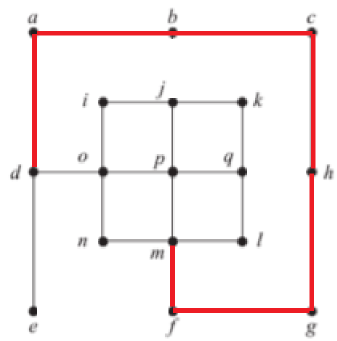
Then $deg(c) = 1$, violating with the rule
So we have to take the path {b,c}, {c,h}, {h,g}, {g,f}



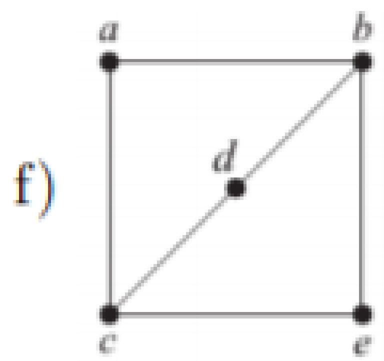
From now, if we take the path {f,e} then {e,d}:



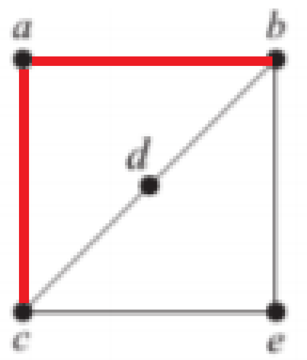
A new subcircuit will be formed, violated with the rule so that we can not take this path.
We take {f,m} instead.



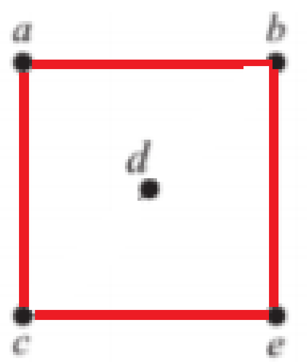
Then $deg(c) = 1$, violating with the rule.
Conclusion: The graph has no Hamilton circuit.

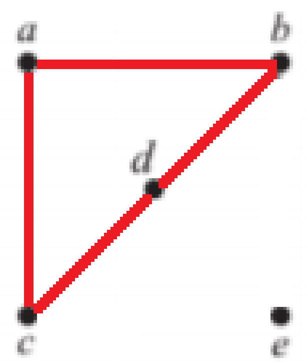


First, we start from vertex a.



Now, we have two different path, {b,d} or {b,e}.





Both way give us the same result that the paths form a new subcircuit, which violate with the rule.
Conclusion: The graph has no Hamilton circuit.

Question 16

a)

Exist Hamilton circuit in $K_n$for all $n \geqslant 3$

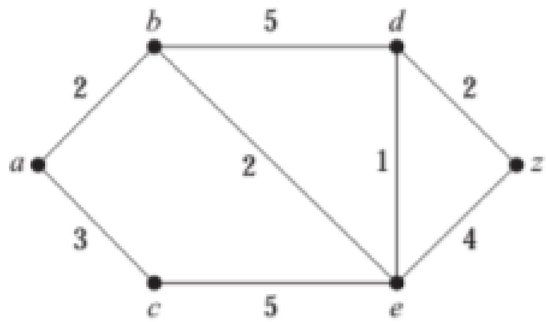
b)

Exist Hamilton circuit in $C_n$for all $n \geqslant 3$

c)

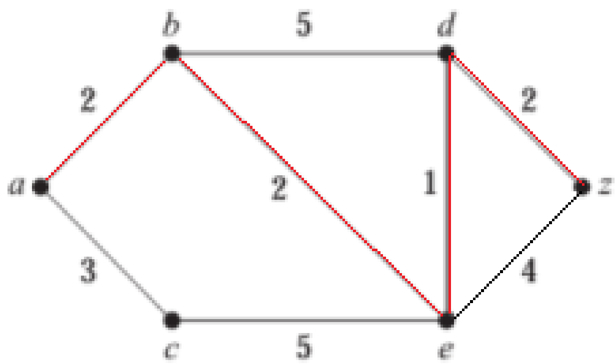
Exist Hamilton circuit in $W_n$for all $n \geqslant 3$

Question 17

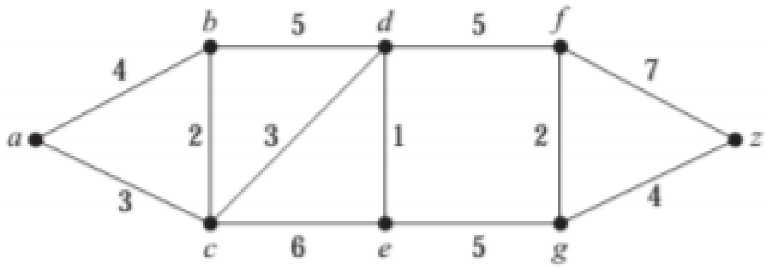
a) 

	a	b	c	d	e	z
θ	0	∞	∞	∞	∞	∞
a	0	2	3	∞	∞	∞
b	0	2	3	7	4	∞
c	0	2	3	7	4	∞
d	0	2	3	7	4	9
e	0	2	3	5	4	8
d	0	2	3	5	4	7

Path sequence: {a,b}, {b,e}, {e,d}, {d,z}

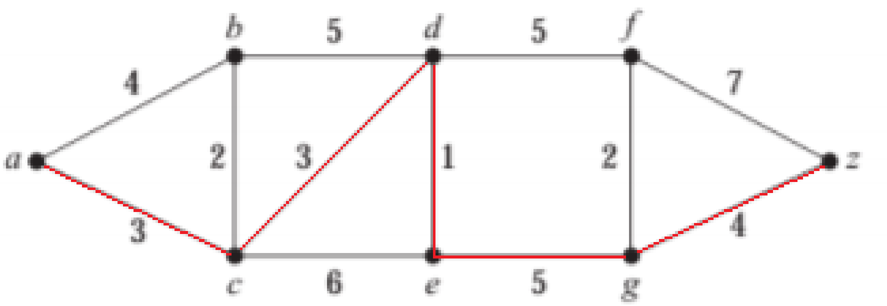


b)

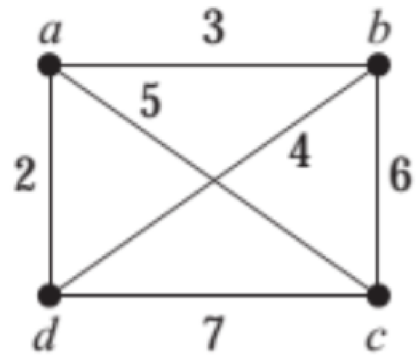


	a	b	c	d	e	f	g	z
θ	0	∞	∞	∞	∞	∞	∞	∞
a	0	4	3	∞	∞	∞	∞	∞
b	0	4	3	9	∞	∞	∞	∞
c	0	4	3	6	9	∞	∞	∞
d	0	4	3	6	7	11	∞	∞
e	0	4	3	6	7	11	12	∞
f	0	4	3	6	7	11	12	18
g	0	4	3	6	7	11	12	16

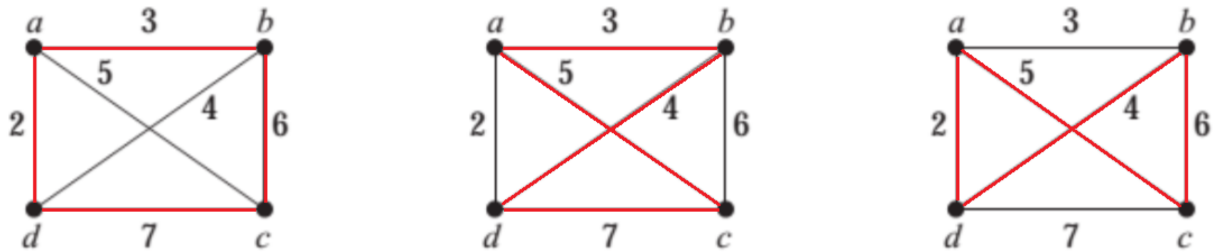
Path sequence: {a,c},{c,d},{d,e},{e,g},{g,z}



Question 18

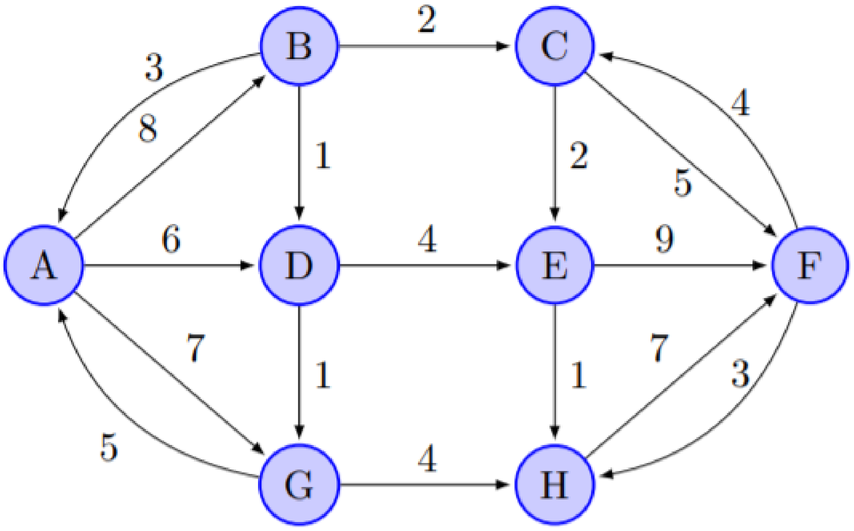


There are three possible combination of edge to form a Hamilton path.



Because the weight is the same no matter starting point we chose, let's start with a.
The path {a,b},{b,c},{c,d},{d,a} have the weight of 18
The path {a,b},{b,d},{d,c},{c,a} have the weight of 19
The path {a,d},{d,b},{b,c},{c,a} have the weight of 17
So the path {a,d},{d,b},{b,c},{c,a} have the minimum weight.

Question 19



Using Floyd Warshal algorithm, we have these matrix. The final result is $L^{(8)}$

$$L^{(0)} = \begin{bmatrix} 0_0 & 8_0 & \infty_0 & 6_0 & \infty_0 & \infty_0 & 7_0 & \infty_0 \\ 3_0 & 0_0 & 2_0 & 1_0 & \infty_0 & \infty_0 & \infty_0 & \infty_0 \\ \infty_0 & \infty_0 & 0_0 & \infty_0 & 2_0 & 5_0 & \infty_0 & \infty_0 \\ \infty_0 & \infty_0 & \infty_0 & 0_0 & 4_0 & \infty_0 & 1_0 & \infty_0 \\ \infty_0 & \infty_0 & \infty_0 & \infty_0 & 0_0 & 9_0 & \infty_0 & 1_0 \\ \infty_0 & \infty_0 & 4_0 & \infty_0 & \infty_0 & 0_0 & \infty_0 & 3_0 \\ 5_0 & \infty_0 & \infty_0 & \infty_0 & \infty_0 & \infty_0 & 0_0 & 4_0 \\ \infty_0 & \infty_0 & \infty_0 & \infty_0 & \infty_0 & 7_0 & \infty_0 & 0_0 \end{bmatrix} \quad L^{(1)} = \begin{bmatrix} 0_0 & 8_0 & \infty_0 & 6_0 & \infty_0 & \infty_0 & 7_0 & \infty_0 \\ 3_0 & 0_0 & 2_0 & 1_0 & \infty_0 & \infty_0 & 10_1 & \infty_0 \\ \infty_0 & \infty_0 & 0_0 & \infty_0 & 2_0 & 5_0 & \infty_0 & \infty_0 \\ \infty_0 & \infty_0 & \infty_0 & 0_0 & 4_0 & \infty_0 & 1_0 & \infty_0 \\ \infty_0 & \infty_0 & \infty_0 & \infty_0 & 0_0 & 9_0 & \infty_0 & 1_0 \\ \infty_0 & \infty_0 & 4_0 & \infty_0 & \infty_0 & 0_0 & \infty_0 & 3_0 \\ 5_0 & 13_1 & \infty_0 & 11_1 & \infty_0 & \infty_0 & 0_0 & 4_0 \\ \infty_0 & \infty_0 & \infty_0 & \infty_0 & \infty_0 & 7_0 & \infty_0 & 0_0 \end{bmatrix}$$

$$L^{(2)} = \begin{bmatrix} 0_0 & 8_0 & 10_2 & 6_0 & \infty_0 & \infty_0 & 7_0 & \infty_0 \\ 3_0 & 0_0 & 2_0 & 1_0 & \infty_0 & \infty_0 & 10_1 & \infty_0 \\ \infty_0 & \infty_0 & 0_0 & \infty_0 & 2_0 & 5_0 & \infty_0 & \infty_0 \\ \infty_0 & \infty_0 & \infty_0 & 0_0 & 4_0 & \infty_0 & 1_0 & \infty_0 \\ \infty_0 & \infty_0 & \infty_0 & \infty_0 & 0_0 & 9_0 & \infty_0 & 1_0 \\ \infty_0 & \infty_0 & 4_0 & \infty_0 & \infty_0 & 0_0 & \infty_0 & 3_0 \\ 5_0 & 13_1 & 15_2 & 11_1 & \infty_0 & \infty_0 & 0_0 & 4_0 \\ \infty_0 & \infty_0 & \infty_0 & \infty_0 & \infty_0 & 7_0 & \infty_0 & 0_0 \end{bmatrix} \quad L^{(3)} = \begin{bmatrix} 0_0 & 8_0 & 10_2 & 6_0 & 12_3 & 15_3 & 7_0 & \infty_0 \\ 3_0 & 0_0 & 2_0 & 1_0 & 4_3 & 7_3 & 10_1 & \infty_0 \\ \infty_0 & \infty_0 & 0_0 & \infty_0 & 2_0 & 5_0 & \infty_0 & \infty_0 \\ \infty_0 & \infty_0 & \infty_0 & 0_0 & 4_0 & \infty_0 & 1_0 & \infty_0 \\ \infty_0 & \infty_0 & \infty_0 & \infty_0 & 0_0 & 9_0 & \infty_0 & 1_0 \\ \infty_0 & \infty_0 & 4_0 & \infty_0 & 6_3 & 0_0 & \infty_0 & 3_0 \\ 5_0 & 13_1 & 15_2 & 11_1 & 17_3 & 20_3 & 0_0 & 4_0 \\ \infty_0 & \infty_0 & \infty_0 & \infty_0 & \infty_0 & 7_0 & \infty_0 & 0_0 \end{bmatrix}$$

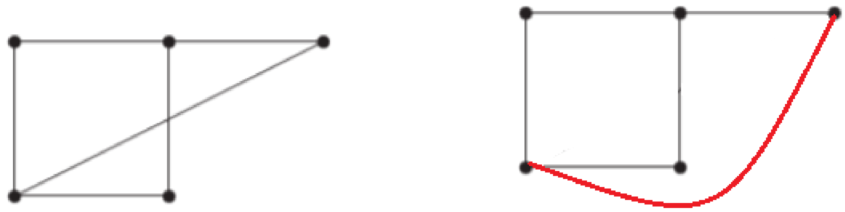
$$L^{(4)} = \begin{bmatrix} 0_0 & 8_0 & 10_2 & 6_0 & 10_4 & 15_3 & 7_0 & \infty_0 \\ 3_0 & 0_0 & 2_0 & 1_0 & 4_3 & 7_3 & 2_4 & \infty_0 \\ \infty_0 & \infty_0 & 0_0 & \infty_0 & 2_0 & 5_0 & \infty_0 & \infty_0 \\ \infty_0 & \infty_0 & \infty_0 & 0_0 & 4_0 & \infty_0 & 1_0 & \infty_0 \\ \infty_0 & \infty_0 & \infty_0 & \infty_0 & 0_0 & 9_0 & \infty_0 & 1_0 \\ \infty_0 & \infty_0 & 4_0 & \infty_0 & 6_3 & 0_0 & \infty_0 & 3_0 \\ 5_0 & 13_1 & 15_2 & 11_1 & 15_4 & 20_3 & 0_0 & 4_0 \\ \infty_0 & \infty_0 & \infty_0 & \infty_0 & \infty_0 & 7_0 & \infty_0 & 0_0 \end{bmatrix} \quad L^{(5)} = \begin{bmatrix} 0_0 & 8_0 & 10_2 & 6_0 & 10_4 & 15_3 & 7_0 & 11_5 \\ 3_0 & 0_0 & 2_0 & 1_0 & 4_3 & 7_3 & 2_4 & 5_5 \\ \infty_0 & \infty_0 & 0_0 & \infty_0 & 2_0 & 5_0 & \infty_0 & 3_5 \\ \infty_0 & \infty_0 & \infty_0 & 0_0 & 4_0 & 13_5 & 1_0 & 5_5 \\ \infty_0 & \infty_0 & \infty_0 & \infty_0 & 0_0 & 9_0 & \infty_0 & 1_0 \\ \infty_0 & \infty_0 & 4_0 & \infty_0 & 6_3 & 0_0 & \infty_0 & 3_0 \\ 5_0 & 13_1 & 15_2 & 11_1 & 15_4 & 20_3 & 0_0 & 4_0 \\ \infty_0 & \infty_0 & \infty_0 & \infty_0 & \infty_0 & 7_0 & \infty_0 & 0_0 \end{bmatrix}$$

$$L^{(6)} = \begin{bmatrix} 0_0 & 8_0 & 10_2 & 6_0 & 10_4 & 15_3 & 7_0 & 11_5 \\ 3_0 & 0_0 & 2_0 & 1_0 & 4_3 & 7_3 & 2_4 & 5_5 \\ \infty_0 & \infty_0 & 0_0 & \infty_0 & 2_0 & 5_0 & \infty_0 & 3_5 \\ \infty_0 & \infty_0 & 17_6 & 0_0 & 4_0 & 13_5 & 1_0 & 5_5 \\ \infty_0 & \infty_0 & 13_6 & \infty_0 & 0_0 & 9_0 & \infty_0 & 1_0 \\ \infty_0 & \infty_0 & 4_0 & \infty_0 & 6_3 & 0_0 & \infty_0 & 3_0 \\ 5_0 & 13_1 & 15_2 & 11_1 & 15_4 & 20_3 & 0_0 & 4_0 \\ \infty_0 & \infty_0 & 11_6 & \infty_0 & 13_6 & 7_0 & \infty_0 & 0_0 \end{bmatrix} \quad L^{(7)} = \begin{bmatrix} 0_0 & 8_0 & 10_2 & 6_0 & 10_4 & 15_3 & 7_0 & 11_5 \\ 3_0 & 0_0 & 2_0 & 1_0 & 4_3 & 7_3 & 2_4 & 5_5 \\ \infty_0 & \infty_0 & 0_0 & \infty_0 & 2_0 & 5_0 & \infty_0 & 3_5 \\ 6_7 & 14_7 & 16_7 & 0_0 & 4_0 & 13_5 & 1_0 & 5_5 \\ \infty_0 & \infty_0 & 13_6 & \infty_0 & 0_0 & 9_0 & \infty_0 & 1_0 \\ \infty_0 & \infty_0 & 4_0 & \infty_0 & 6_3 & 0_0 & \infty_0 & 3_0 \\ 5_0 & 13_1 & 15_2 & 11_1 & 15_4 & 20_3 & 0_0 & 4_0 \\ \infty_0 & \infty_0 & 11_6 & \infty_0 & 13_6 & 7_0 & \infty_0 & 0_0 \end{bmatrix}$$

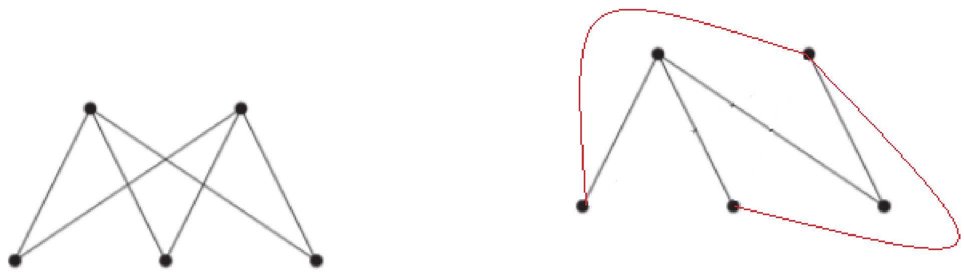
$$L^{(8)} = \begin{bmatrix} 0_0 & 8_0 & 10_2 & 6_0 & 10_4 & 15_3 & 7_0 & 11_5 \\ 3_0 & 0_0 & 2_0 & 1_0 & 4_3 & 7_3 & 2_4 & 5_5 \\ \infty_0 & \infty_0 & 0_0 & \infty_0 & 2_0 & 5_0 & \infty_0 & 3_5 \\ 6_7 & 14_7 & 16_7 & 0_0 & 4_0 & 12_8 & 1_0 & 5_5 \\ \infty_0 & \infty_0 & 12_8 & \infty_0 & 0_0 & 8_8 & \infty_0 & 1_0 \\ \infty_0 & \infty_0 & 4_0 & \infty_0 & 6_3 & 0_0 & \infty_0 & 3_0 \\ 5_0 & 13_1 & 15_2 & 11_1 & 15_4 & 11_8 & 0_0 & 4_0 \\ \infty_0 & \infty_0 & 11_6 & \infty_0 & 13_6 & 7_0 & \infty_0 & 0_0 \end{bmatrix}$$

Question 20

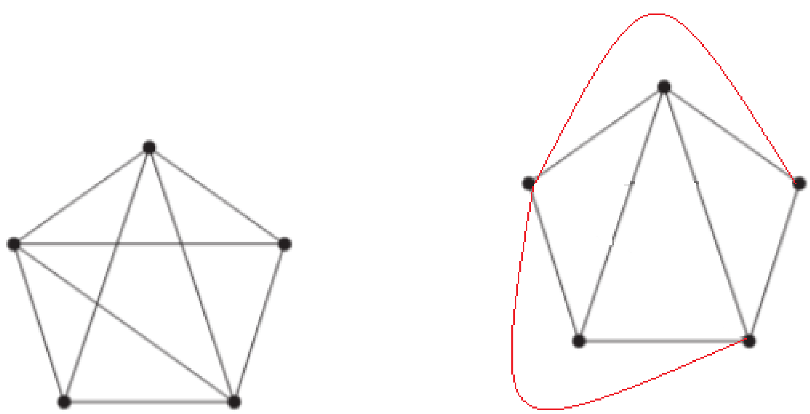
a)



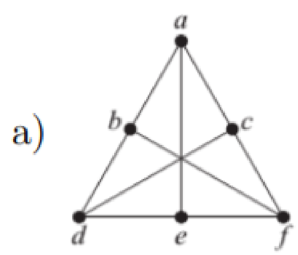
b)



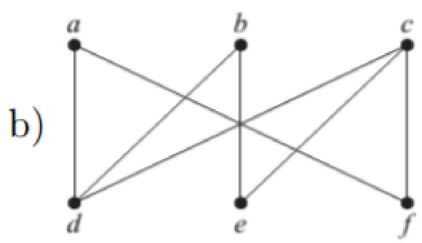
c)



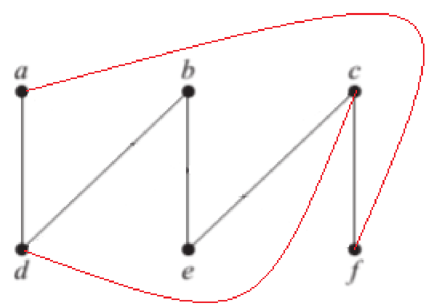
Question 21



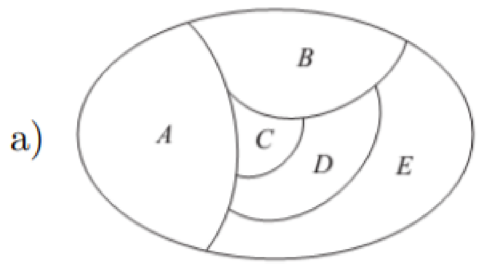
Graph a) is $K_{3,3}$. Base on Kuratowski's theorem, the graph is nonplanar.



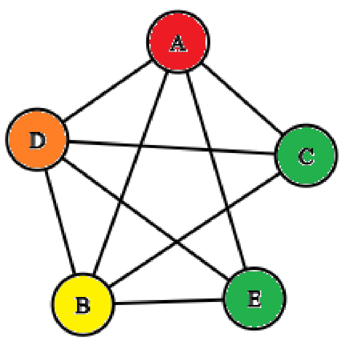
Graph b) is planar, so it can be draw again in order to have no pair of cross edges.



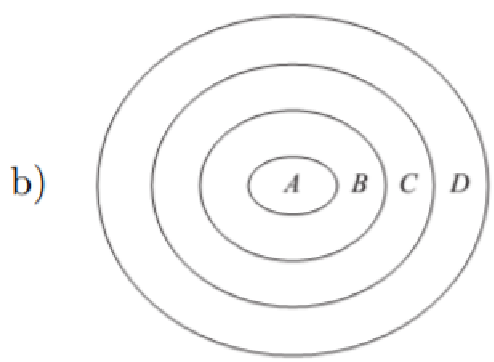
Question 22



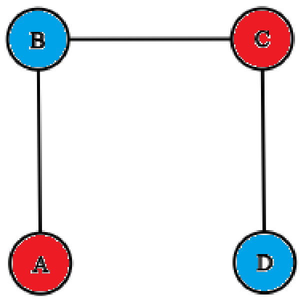
The dual graph of map a):



At least 4 colors needed.



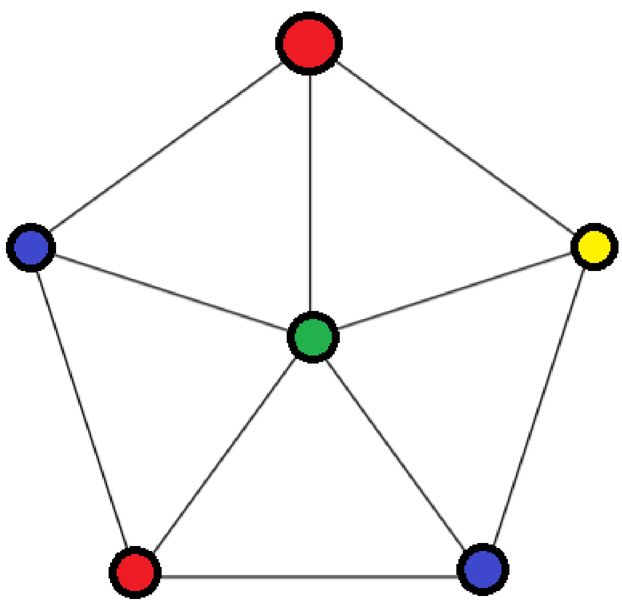
The dual graph of map b):



At least 2 colors needed.

Question 23

If n is odd number, the chromatic number is 4 since the vertices of cycle can be given three color and the center vertex is given fourth color.
Example: W_5



If n is odd number, the chromatic number is 4 since the vertices of cycle can be given three color and the center vertex is given fourth color.
Example: W_4