VIETNAM NATIONAL UNIVERSITY, HO CHI MINH CITY
UNIVERSITY OF TECHNOLOGY
FACULTY OF COMPUTER SCIENCE AND ENGINEERING

MATHEMATICAL MODELING (CO2011)

Assignment

Dynamical systems in forecasting Greenhouse Micro-climate

Advisor: Nguyen Tien Thinh
Students: Ngo Tu Duy - 1752133 (CC01)
Truong Van Quang Dat - 1652142 (CC03)
Quach Dang Giang - 1952044 (CC02)
Hoang Tran Viet Long - 1652350 (CC01)
Nguyen Dinh Tuan - 1552411 (CC02)

HO CHI MINH CITY, DECEMBER 2020

Contents

1 Member list & Workload **2**

2 Exercise 1 **2**
2.1 Problem 1a 2
2.2 Problem 1b 3
2.3 Problem 1c 3
2.4 Problem 1d 4
2.5 Problem 1e 6

3 Exercise 2 **6**
3.1 Problem 2a 6
3.2 Problem 2b 9

4 Exercise 3 **10**

5 Exercise 4 **11**
5.1 Exercise 4a 11
5.2 Exercise 4b 12
5.3 Conclusion 12

6 Exercise 5 **13**
6.1 The model for the vapor pressure 13
6.2 Programs calculate the net CO_2 flux 15

1 Member list & Workload

No.	Fullname	Student ID	Problems	Percentage of work
1	Ngo Tu Duy	1752133	50% 4 50% 5	20 %
2	Truong Van Quang Dat	1652142	10% 2 50% 5	20%
3	Quach Dang Giang (Leader)	1952044	70% 1 30% 2 40% 3	20 %
4	Hoang Tran Viet Long	1652350	20% 1 50% 2 60% 3	20 %
5	Nguyen Dinh Tuan	1552411	10% 1 10% 2 50% 4	20 %

2 Exercise 1

2.1 Problem 1a

Dynamical systems are mathematical objects used to model physical phenomena whose state (or instantaneous description) changes over time. These models are used in financial and economic forecasting, environmental modeling, medical diagnosis, industrial equipment diagnosis, and a host of other applications. To study dynamical systems mathematically, we represent them in terms of differential equations.

The state of dynamical system at an instant of time is described by a point in an n-dimensional space called the **state space** (the dimension n depends on how complicated the systems is) . As time passes the point moves, sweeping out a curve in the state space, called a **trajectory** of the system (mathematically, this curve is a solution of the governing differential equations)

A system can be classified according to different criteria, such as

System Classification	
Either	***Or***
Deterministic	Stochastic
Discrete	Continous
Linear	Nonlinear
Autonomous	Nonautomonous

A deterministic system has an entirely predictable behavior. The system is fully understood, and it is possible to predict exactly what will happen. A stochastic model, on the other hand, possess inherent randomness that make it impossible to predict its behavior.

A discrete system changes the state variables only at a countable number of points in time. Dynamical systems with discrete time, like the ideal coin toss, have their states evaluated only after certain discrete intervals. In the case of the coin toss, the smooth tumbling and bouncing of the coin is ignored, and its state is only viewed when it has come to equilibrium. These points in time are the ones at which the event occurs/change in state. A continuous system, however, change in a continuous way, and not abruptly from one state to another, ex: A pendulum swinging back and forth,...

In a linear system, function describing the system behavior must satisfied 2 basic properties:

$$Addictivity : f(x + y) = f(x) + f(y)$$
$$Homogeneity : f(\alpha x) = \alpha f(x)$$

A nonliner system's function simply doesn't satisfied the above requirements.

An autonomous system is a system of ordinary differential equations, which do not depend on the independent variable. If the independent variable is time, we call it time-invariant system.

A nonautonomous system is a system that doesn't satisfied the above requirements.

In this assignment, we use the dynamical system especially the first order differential equation to predict the changes of CO2 inside a greenhouse model

2.2 Problem 1b

The first order differential equations in the dynamical system with the initial value y_0 and t_0 can be re-written as:

$$\begin{cases} \frac{dy}{dx} & = f(x,y) \\ y\left(x_0\right) & = y_0 \end{cases}$$

Suppose we have a linear differential equation

$$y + p(x)y = g(x)$$

in which p and g are continous functions on the open interval I = (α, β). Then for each $t \in I$ there exists a unique solution $y = \phi(t)$ to the differential equation $\frac{dy}{dx} + p(x)y = g(x)$ that also satisfies the initial value condition $y(x_0) = y_0$

2.3 Problem 1c

A differential equation is an equation which contains one or more terms and the derivatives of one variable (i.e., dependent variable) with respect to the other variable (i.e., independent variable). The order of the equation is equal to the highest order of the derivative in the equation, therefore a first-order differential equation only have first-order derivative in their equation

$$\frac{dy}{dx} = f(x)$$

We often have to find a particular solution which specifies the value of the unknown function at a given point in the domain. This is called an initial-value problem. For example, solve the diferential equation:

$$\dot{y} = 2(25 - y)$$

Knowing that

$$y(0) = 40$$

We can see that if y(t) = 25 then it will satisfy the first equation but y(0) = 25 $\neq$ 40. So y(t) = 8 is not a solution. So as long as y is not 8, we can rewrite the equation as

$$\frac{dy}{dt}\frac{1}{25-y} = 2$$
$$\Leftrightarrow \frac{1}{25-y}dy = 2dt$$
$$\Leftrightarrow \int \frac{1}{25-y}dy = \int 2dt$$
$$\Leftrightarrow ln|25-y| * (-1) = 2t + C_0$$
$$\Leftrightarrow ln|25-y| = -2t - C_0 = -2t + C$$
$$\Leftrightarrow |25-y| = e^{-2t+C}$$
$$\Leftrightarrow y - 25 = \pm e^C e^{-2t}$$
$$\Leftrightarrow y = 25 \pm e^C e^{-2t} = 25 + Ae^{-2t}$$

A is some non-zero constant. Since we want y(0) = 40, we substitute and solve for A:

$$40 = 25 + Ae^0$$

$$A = 15$$

Thus $y = 25 + 15e^{-2t}$ is the solution of the differential equation.

2.4 Problem 1d

- Explicit Euler

In mathematics and computational science, the Euler method (also called forward Euler method) is a first-order numerical procedure for solving ordinary differential equations (ODEs) with a given initial value. It is the most basic explicit method for numerical integration of ordinary differential equations and is the simplest Runge–Kutta method.

Explicit methods calculate the state of the system at a later time from the state of the system at the current time without the need to solve algebraic equations. For the method, we begin by choosing a step size or Δt. The size of Δt determines the accuracy of the approximate solutions as well as the number of computations. Graphically this method produces a series of line segments, which thereby approximates the solution curve.

Given $\frac{dy}{dt} = f(t, y)$ with $y(t_0) = y_0$ we approximate the path of the solution by:

1. Step size: First, we choose the step size h which is the size of the increments along the t-axis that we will use in approximation. Smaller increments tend to give more accurate answers, but then there are more steps to compute.
2. Compute slope: Compute the slope $\frac{dy}{dt} = f(t_0, y_0)$
3. Get next point: The next point is $t_1 = t_0 + h$ and $y_1 = y_0 + f(t_0, y_0)h$
4. Repeat: : Repeat the last two steps with (t_1, y_1)

- Explicit Runge–Kutta of order 4

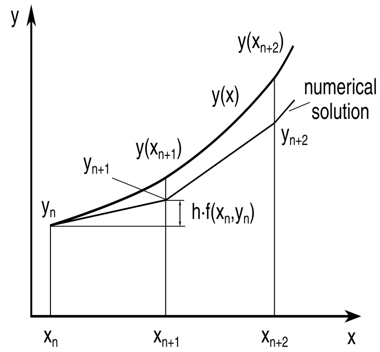

Figure 1: Explicit Euler metho

In numerical analysis, the Runge–Kutta methods are a family of implicit and explicit iterative methods, which include the well-known routine called the Euler Method, used in temporal discretization for the approximate solutions of ordinary differential equations. The most widely known member of the Runge–Kutta family is generally referred to as "RK4", the "classic Runge–Kutta method" or simply as "the Runge–Kutta method".

Supoose we have a problem: $\frac{dy}{dt} = f(t, y)$ with $y(t_0) = y_0$ With y is an unknown function (scalar or vector) of time t, which we would like to approximate. We also know the rate of change $\frac{dy}{dt}$ is a function of y and t; and the initial value. First, we choose the step sizehwhich is the size of the increments along thet-axis that we will use in approximation. We define

$$y_{n+1} = y_n + \frac{1}{6}h(k_1 + 2k_2 + 3k_3 + 4k_4$$

$$t_{n+1} = t_n + h$$

for n = 0,1,2,3,..... with

$$k_1 = f(t_n, y_n)$$

$$k_2 = f(t_n + \frac{h}{2}, y_n + h\frac{k_1}{2})$$

$$k_3 = f(t_n + \frac{h}{2}, y_n + h\frac{k_2}{2})$$

$$k_4 = f(t_n + h, y_n + hk_3)$$

Here, by using the RK4 approximation, we get the next value y_{n+1} using present value y_n and the weighted average of four increments, where each increment is the product of the size of the interval, h and an estimated slope specified by function f on the right-hand side of the differential equation.

k_1 is the slope at the beginning of the interval, using y (Euler's method).

k_2 is the slope at the midpoint of the interval, using y and k_1.

k_3 is also the slope at the midpoint, but now using y and k_2.

k_4 is the slope at the end of the interval, using y and k_3

In averaging the four slopes, greater weight is given to the slopes at the midpoint.

2.5 Problem 1e

Given that,

$$\dot{y} = 2(25 - y)$$

and

$$y(0) = 40$$

Find y(0.1)
We know the solution is $y = 25 + 15e^{-2t}$. We choose h = 0.1
Euler method:

$$\begin{aligned} y(0.1) &= y_0 + hf(y_0, t_0) \\ &= 40 + 0.1(2(25 - 40)) \\ &= 37 \end{aligned}$$

RK4 method:

$$k_1 = 2(25 - 40) = -30$$

$$k_2 = 2(25 - 38.5) = -27$$

$$k_3 = 2(25 - 38.65) = -27.3$$

$$k_4 = 2(25 - 37.2) = -24.54$$

$y(0.1) = y(0) + \frac{1}{6}h(k_1 + 2k_2 + 3k_3 + 4k_4) = 36.599$
True y(0.1) $= 25 + 15e^{-2*0.1} \approx 37.28$

3 Exercise 2

3.1 Problem 2a

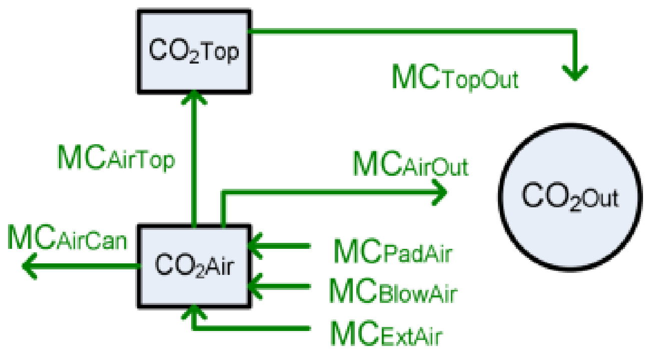

Figure 2: The CO_2 flow inside and outside a greenhouse

The two main equation to calculate the CO_2 concentration in the lower and upper compartments is

$$\begin{aligned} cap_{CO_2Air}\dot{CO}_{2Air} = &MC_{BlowAir} + MC_{ExtAir} + MC_{PadAir} \\ &-MC_{AirCan} - MC_{AirTop} - MC_{AirOut} \end{aligned} \tag{1}$$

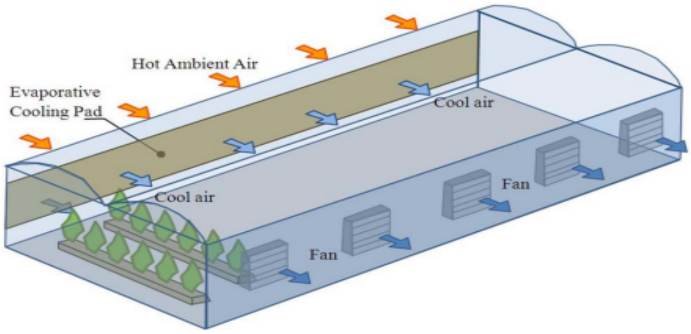

Figure 3: The movement of CO_2 through the pad system and fan system.

$$\text{cap}_{\text{CO}_2\text{Top}}\, \dot{CO_2}\text{Top} = MC_{\text{AirTop}} - MC_{\text{TopOut}} \tag{2}$$

First, the equation to calculate MC_{BlowAir} is:

$$MC_{\text{BlowAir}} = \frac{\eta_{\text{Heat } CO_2} U_{\text{Blow}}\, P_{\text{Blow}}}{A_{Flr}} \tag{3}$$

Similarly, the amount of CO_2 that is pumped into the greenhouse by the third party that supplies CO_2 is given by

$$MC_{\text{ExtAir}} = \frac{U_{ExtCO_2} * \phi_{ExtCO_2}}{A_{Flr}} \tag{4}$$

The following formula is used to calculate MC_{PadAir} :

$$MC_{\text{PadAir}} = f_{\text{Pad}}\left(CO_{2Out} - CO_{2\text{ Air}}\right) = \frac{U_{Pad}\phi_{\text{Pad}}}{A_{Flr}}\left(CO_{2Out} - CO_{2\text{ Air}}\right) \tag{5}$$

The net flux of CO_2 from the lower compartment to the upper compartment of the greenhouse is more complicated and it depends on the difference in temperature and air density between the two compartments.

$$MC_{\text{AirTop}} = f_{ThScr}\left(CO_2\text{Air} - CO_{2\text{Top}}\right) \tag{6}$$

with f_{ThScr} being:

$$f_{ThScr} = U_{ThScr} K_{ThScr} \left|T_{Air} - T_{Top}\right|^{\frac{2}{3}} + \left(1 - U_{ThScr}\right)\left[\frac{g\left(1 - U_{ThScr}\right)}{2\rho_{Air}^{Mean}}\left|\rho_{Air} - \rho_{Top}\right|\right]^{\frac{1}{2}} \tag{7}$$

Similarly, for the net $C0_2$ flux from the inside to the outside of the greenhouse, let consider the following formula:

$$MC_{\text{AirOut}} = \left(f_{\text{Ventside}} + f_{\text{VentForced}}\right)\left(CO_{2\text{Air}} - CO_{2Out}\right) \tag{8}$$

To generalize the model for many different types of greenhouses, the following general formula $f_{VentRoofSide}$ (ms^{-1}) is used to set the formula for $f_{VentSize}$

$$f_{\text{VentRoofSide}} = \frac{C_d}{A_{\text{Flr}}}\left[\frac{U_{\text{Roof}}^2\, U_{\text{Side}}^2\, A_{\text{Roof}}^2\, A_{\text{Side}}^2}{U_{\text{Roof}}^2 A_{\text{Roof}}^2 + U_{\text{Side}}^2\, A_{\text{Side}}^2} \cdot \frac{2gh_{\text{SideRoof}}\,(T_{\text{Air}} - T_{\text{Out}})}{T_{\text{Air}}^{\text{Mean}}} + \left(\frac{U_{\text{Roof}}\, A_{\text{Roof}} + U_{\text{Side}}\, A_{\text{Side}}}{2}\right)^2 C_w v_{\text{Wind}}^2\right]^{\frac{1}{2}} \tag{9}$$

In addition, this topic also explores insect screens on ventilation openings and ventilators and the leakage coefficient of the greenhouse. In the presence of an insect screen, the movement speed of the air currents through the ventilation areas will be reduced by a factor

$$\eta_{\text{Ins}Scr} = \zeta_{\text{Ins}Scr}\,(2 - \zeta_{\text{Ins}Scr}) \tag{10}$$

where η_{InsScr} is the porosity (dimensionless), which is the ratio of the area of the holes in the screen to the total area of the screen. Given the leakage coefficient $c_{leakage}$, which depends on the greenhouse type and is dimensionless, the air-exchange rate is added an amount of approximately 50% of the leakage rate

$$f_{\text{leakage}} = \begin{cases} 0.25 \cdot c_{\text{leakage}}\,, & v_{\text{Wind}} < 0.25 \\ v_{\text{Wind}} \cdot c_{\text{leakage}}\,, & v_{\text{Wind}} \geq 0.25 \end{cases} \tag{11}$$

The $f_{VentSide}$ is given by the following

$$f_{\text{Ventside}} = \begin{cases} \eta_{\text{Ins } Scr} f''_{Ventside} + 0.5 f_{leakage}, \eta side \geq \eta_{Side_Thr} \\ \eta_{\text{Ins } Scr}\,[U_{\text{ThScr}}\, f''_{\text{Ventside}} \\ + (1 - U_{\text{ThScr}}\,)\, f_{\text{Vent Roof Side}}\, \eta_{\text{Side}}\,] + 0.5 f_{\text{leakage}}, \eta side < \eta_{Side_Thr} \end{cases} \tag{12}$$

The flux $f_{VentForced}$ by the fan system inside the greenhouse is calculated as follows

$$f_{VentForced} = \frac{\eta_{\text{InsScr}} U_{VentForced} \phi_{VentForced}}{A_{Flr}} \tag{13}$$

Similarly to MC_{AirOut}, the net CO2 flux from the greenhouse to outside the greenhouse through the roof openings is calculated by using the formula

$$MC_{TopOut} = f_{VentRoof}\,(CO_{2Top} - CO_{2Out}) \tag{14}$$

where $f_{VentRoof}$ is the flux rate through the roof openings and is given by

$$f_{\text{VentRoof}} = \begin{cases} \eta_{\text{Ins}Scr} f''_{\text{VentRoof}} + 0.5 f_{\text{leakage}}, \eta roof \geq \eta_{Roof_Thr} \\ \eta_{\text{Ins}Scr}\,[U_{\text{ThScr}} f''_{\text{VentRoof}} \\ + (1 - U_{\text{ThScr}})\, f_{VentRoofSide} \eta_{\text{Side}}\,] + 0.5 f_{\text{leakage}}, \eta_{Roof} < \eta_{Side_Thr} \end{cases} \tag{15}$$

with

$$f''_{VentRoof} = \frac{C_d U_{Roof} A_{Roof}}{2A_{Flr}}\left[\frac{gh_{Vent}\,(T_{Air} - T_{Out})}{2T_Air}^{\text{Mean}} + C_w v_{Wind}^2\right]^{\frac{1}{2}} \tag{16}$$

Finally, we need to describe the amount of CO2 that is absorbed into the leaves due to photosynthesis.

$$MC_{\text{AirCan}} = M_{CH_2O} h_{C_{Buf}} (P - R) \tag{17}$$

with

$$h_{C_{Buf}} = \begin{cases} 0, & C_{Buf} > C_{Buf}^{Max} \\ 1, & C_{Buf} \leq C_{Buf}^{Max} \end{cases} \tag{18}$$

. Usually, the respiration rate R is negligible compared to the photosynthetic rate P and can be omitted or calculated as about 1% of the photosynthetic rate. To simplify the assignment, $h_{C_{Buf}}$ will always have a value of 1, meaning that C_{Buf} will have no effect on the CO2 fluctuation.

We will use Equation (9.10) and relevant ones in reference [Van11] to solve Equation (18) in this exercise with assumption that PAR_{Can} is a constant

3.2 Problem 2b

We use data for coefficients given by "A methodology for model-based greenhouse design" by Vanthoor, , C. Stanghellini, E.J. van Henten and P.H.B. de Visser as well as "A model-based greenhouse design method" by Bram Vanthoor. The data for $CO2_{Air}$ and $CO2_{Top}$ came from https://github.com/CEAOD/Data

```
#define A_Flr 78000
#define A_Roof 14040
#define A_Side 0
#define M_CH2O 30000000
#define cap_CO2Top 0.4
#define cap_CO2Air 4.7
#define Muy_ExtCO2 720000
#define f_pad 0
#define K_ThScr 0.00025
#define P_InsScr 1
#define T_MeanAir 293
#define Cd 0.65
#define Cw 0.09
#define g 9.8
#define h_Vent 0.97
#define Muy_VentForced 0
#define Muy_Pad 0
#define h_CBuf 1
#define M_CH2O 0.03
#define P_Blow 0
#define v_Wind 2.9
#define p_Mean_Air 2.99
#define c_leakage 0.001
#define n_Side_Thr 0.9
#define n_Side 0.1
```

Figure 4: Declaring constants

We have a function named doublerand used to randomize the control value in the range [0,1]

The function read_record is used to read the .csv input file

After that, we simply write the code using all the formulas mentioned in problem a of exercise 2

```
double double_rand(double min, double max)
{
    double scale = rand() / (double)RAND_MAX; /* [0, 1.0] */
    return min + scale * (max - min);      /* [min, max] */
}
```

Figure 5: Randomize function

```
if (v_Wind < 0.25)f_leakage = 0.25*c_leakage;
else f_leakage = v_Wind * c_leakage;

n_InsScr = P_InsScr * (2 - P_InsScr);

if (n_Side > n_Side_Thr)f_VentSide = n_InsScr * f_VentRoofSide_A0 + 0.5*f_leakage;
else f_VentSide = n_InsScr * (U_ThScr*f_VentRoofSide_A0 + (1 - U_ThScr)*f_VentRoofSide*n_Side) + 0.5*f_leakage;

f_VentForced = n_InsScr * U_VentForced*Muy_VentForced / A_Flr;
f_ThScr = U_ThScr * K_ThScr*pow((T_Air - T_Top), 0.66) + (1 - U_ThScr)*pow(((g*(1 - U_ThScr) / (2 * p_Mean_Air))*abs(p_Air - p_Top)),
f_VentRoofSide = (Cd / A_Flr)*pow((((U_Roof*U_Roof*U_Side*U_Side*A_Roof*A_Roof*A_Side*A_Side) / (pow(U_Roof, 2)*pow(A_Roof, 2) + pow(
f_VentRoofSide_A0 = (Cd / A_Flr)*pow((((U_Roof*U_Roof*U_Side*U_Side * 0 * 0 * A_Side*A_Side) / (pow(U_Roof, 2)*pow(0, 2) + pow(U_Side
f_VentRoof = (Cd*U_Roof*A_Roof / (2 * A_Flr))*pow((g*h_Vent*(T_Air - T_Out) / (2 * T_MeanAir)) + Cw * pow(v_Wind, 2), 0.5);

//Ex2
MC_AirOut = (f_VentSide + f_VentForced)*(CO2_Air - CO2_Out);
MC_PadAir = (U_Pad*Muy_Pad / A_Flr)*(CO2_Out - CO2_Air);
MC_AirTop = f_ThScr * (CO2_Air - CO2_Top);
MC_ExtAir = U_ExtCO2 * Muy_ExtCO2 / A_Flr;
MC_BlowAir = n_heatCO2*U_Blow*P_Blow/A_Flr;
MC_TopOut = (n_InsScr * f_VentRoof +0.5 *f_leakage) * (CO2_Top - CO2_Out);
MC_AirCan = M_CH2O * h_CBuf*100;
CO2_Air_Rate = (MC_BlowAir + MC_ExtAir + MC_PadAir - MC_AirCan - MC_AirTop - MC_AirOut) / cap_CO2Air;
CO2_Top_Rate = (MC_AirTop - MC_TopOut) / cap_CO2Top;
```

Figure 6: Main code

4 Exercise 3

Ex3 give us that the temperature and density of air difference are constant so it means that $C\dot{O}2_{Air}$ and $C\dot{O}2_{Top}$ depend on data of CO2 concentration. However, when we did some test with data, results were almost approximately same as the first time we have done.

Follow the constant variables as the exercise 2.b above, dx and constant temperature and density of air difference as below:

p_Air = 3; p_Top = 2.987; T_Air = 295; T_Top = 291; T_Out = 288.9;

Note: There is NO Heater, NO Fan and NO sidewalls CO2 emission so that $CO2_{Heater->Air}$ and $CO2_{Pad->Air}$ are 0 but there are still leakages on sidewalls so that $CO2_{Air->Out}$ still has a non-zero value.

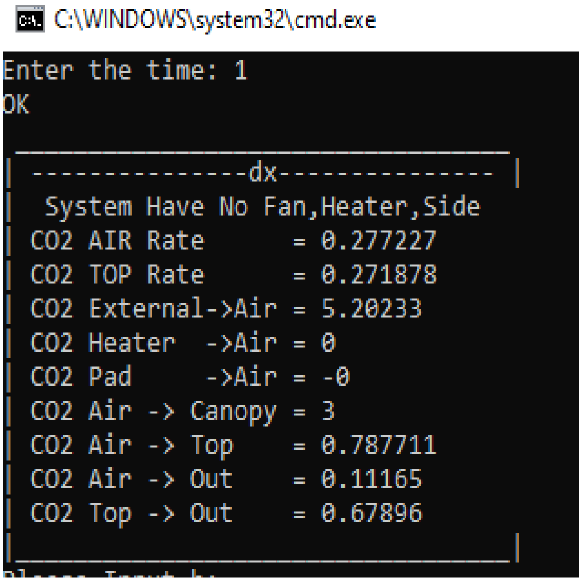

Figure 7: Output

5 Exercise 4

5.1 Exercise 4a

Follow Euler Mehtod, we have:
$CO_{2Air(thEx4a)} = CO_{2Air} + CO_{2AirRate} * h$
$CO_{2Top(thEx4a)} = CO_{2Top} + CO_{2TopRate} * h$ Follow order 4 Runge-Kutta method, we have:
$k_1 = f(tn, xn);$
$k_2 = f(tn + 0.5 * h, xn + 0.5 * h * k_2);$
$k_3 = f(tn + 0.5 * h, xn + 0.5 * h * k_2);$
$k_4 = f(tn + h, xn + 0.5 * h * k_3);$
$xn + 1 = xn + (\frac{1}{6}) * h * (k_1 + 2k_2 + 2k_3 + k_4);$
However, as we tested the derivatives of $CO_{2AirTop}$ over the derivatives of time are constant so that: $k_1 = k_2 = k_3 = k_4 = CO_{2AirTopRate}$. We have:
$CO_{2AirTop(t+h)} = CO_{2AirTop} + (\frac{1}{6}) * h * (6 * CO_{2AirTopRate})$
Where,
$CO_{2Air(thEx4a)}$ is CO_{2Air} at $t + h$;
$CO_{2Top(thEx4a)}$ is CO_{2Top} at $t + h$;
$CO_{2AirTop(t+h)}$ is CO_{AirTop} at $t + h$;
CO_{2Air} is CO_{2Air} concentration at t;
CO_{2Top} is CO_{2Top} concentration at t;
$CO_{2AirTop}$ is $CO_{2AirTop}$ concentration at t;
$CO_{2AirRate}$ is rate change of CO_{2Air} concentration as derivative of CO_{2Air} over dt;
$CO_{2TopRate}$ is rate change of CO_{2Top} concentration as derivative of CO_{2Top} over dt;
$CO_{2AirTopRate}$ is rate change of $CO_{2AirTop}$ concentration as derivative of $CO_{2AirTop}$ over dt;

5.2 Exercise 4b

Using Euler method in exercise 1, (figure 8)

```
//Euler
CO2_Air_t_h_Ex4a = CO2_Air + CO2_Air_Rate * h;
CO2_Top_t_h_Ex4a = CO2_Top + CO2_Top_Rate * h;
```

Figure 8: Euler Method

We already have the CO2_Air_Rate and CO2_Top_Rate in exercise 2, and h is given in this exercise, thus we already have everything we need to execute the method.

Below is the result of the programm

```
Enter the time: 1
OK
 _______________________________________
| ---------------dx---------------      |
|  System Have No Fan,Heater,Side
| CO2 AIR Rate      = 0.277227
| CO2 TOP Rate      = 0.271878
| CO2 External->Air = 5.20233
| CO2 Heater   ->Air = 0
| CO2 Pad      ->Air = -0
| CO2 Air -> Canopy = 3
| CO2 Air -> Top    = 0.787711
| CO2 Air -> Out    = 0.11165
| CO2 Top -> Out    = 0.67896
|_______________________________________|
Please Input h: 2
CO2_Air at t+h = 601.474
CO2_Top at t+h = 581.464
CO2_Air at t + 5 = 602.306
CO2_Top at t + 5 = 582.279
CO2_Air at t + 10 = 603.692
CO2_Top at t + 10 = 583.639
Press any key to continue . . .
```

Figure 9: Programm output

5.3 Conclusion

By comparing the output with the measured data below, we can see that: With $t = 1 \longrightarrow t + 5 = 6, t + 10 = 11$

- CO2_Air at 6 min is 602.306, CO2_Top at 6 min = 582.279

  Compare to actual data CO_{2Air} and CO_{2Top} being 602.66 and 582.66 at 6 minutes respectively

- CO2_Air at 6 min is 603.692, CO2_Top at 6 min = 583.639

  Compare to actual data CO_{2Air} and CO_{2Top} being 603.8 and 583.8 at 6 minutes respectively

It is clear that we have a very small margin of error (around 0.02%). Thus, the programm have a very good accuracy.

```
1,600.92,523.92,580.92
2,601.2,524.2,581.2
3,601.51,524.51,581.51
4,601.94,524.94,581.94
5,602.21,525.21,582.21
6,602.66,525.66,582.66
7,602.93,525.93,582.93
8,603.16,526.16,583.16
9,603.41,526.41,583.41
10,603.67,526.67,583.67
11,603.8,526.8,583.8
12,604.11,527.11,584.11
```

Figure 10: Measured data

6 Exercise 5

6.1 The model for the vapor pressure

In this greenhouse climate model, the greenhouse functions heating, insulation, shading, cooling, CO_2 enrichment, humidification and de-humidification are fulfilled by one or more techniques such as a direct air heater, a boiler, an industrial heat source, a geothermal source and passive buffer.

For the improvement of a model-based plan strategy, these strategies are viewed as adequately nonexclusive for a wide scope of areas everywhere on the world. Explicit nearby answers for energy creation, energy change or atmosphere adjustment, for example, co-generation of heat and electricity, artificial photosynthetic lighting, an active heat buffer, a heat pump and a solar heat collector, lie outside the scope of this study.

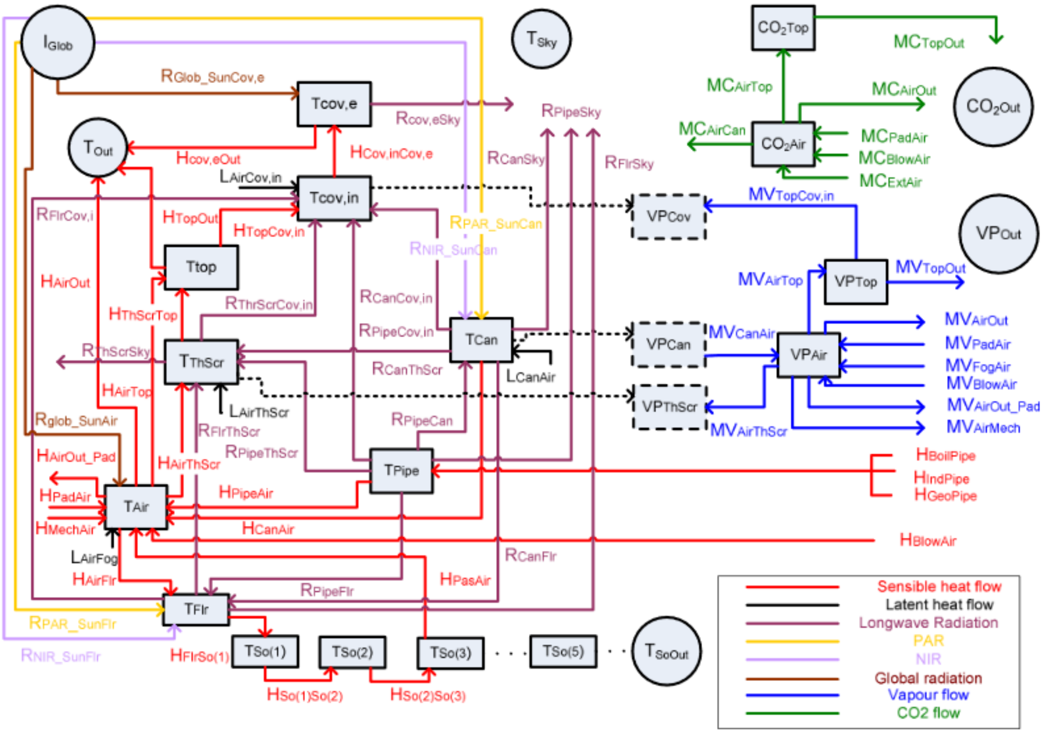

Figure 11: *Overview of the state variables (blocks), semi-state variables (dotted blocks), external climate inputs (circles) and fluxes (arrows) of the greenhouse model. Coloured arrows represent the various energy and mass fluxes (legend at the bottom right).*

Thank to De Zwart (1996), this model was based on the greenhouse climate modelling study of him. This model was added some extra elements and parts because of the current purpose.The following model elements were implemented: the design elements presented below in Figure 3. A lumped cover description to combine the impact of different cover layers on indoor climate; the internal and external cover temperature are state variables of the model to describe the impact of cover insulation on indoor climate; a description of the far infrared radiation (FIR) transmission through the cover, which is needed for films that partially transmit FIR; a description of both roof and side ventilation; a description of the impact of insect screens on ventilation rate; and a description of the near infrared radiation (NIR) absorption of both canopy and floor, which depend on the optical properties of the cover and floor.Since optimization of the greenhouse structure properties, i.e. greenhouse dimensions, roof slope and vent orientation and location, exceeded the purpose of our design method, the model was simplified by not distinguishing between diffuse and direct solar radiation and by assuming that the greenhouse cover transmission coefficient was independent on solar angle.

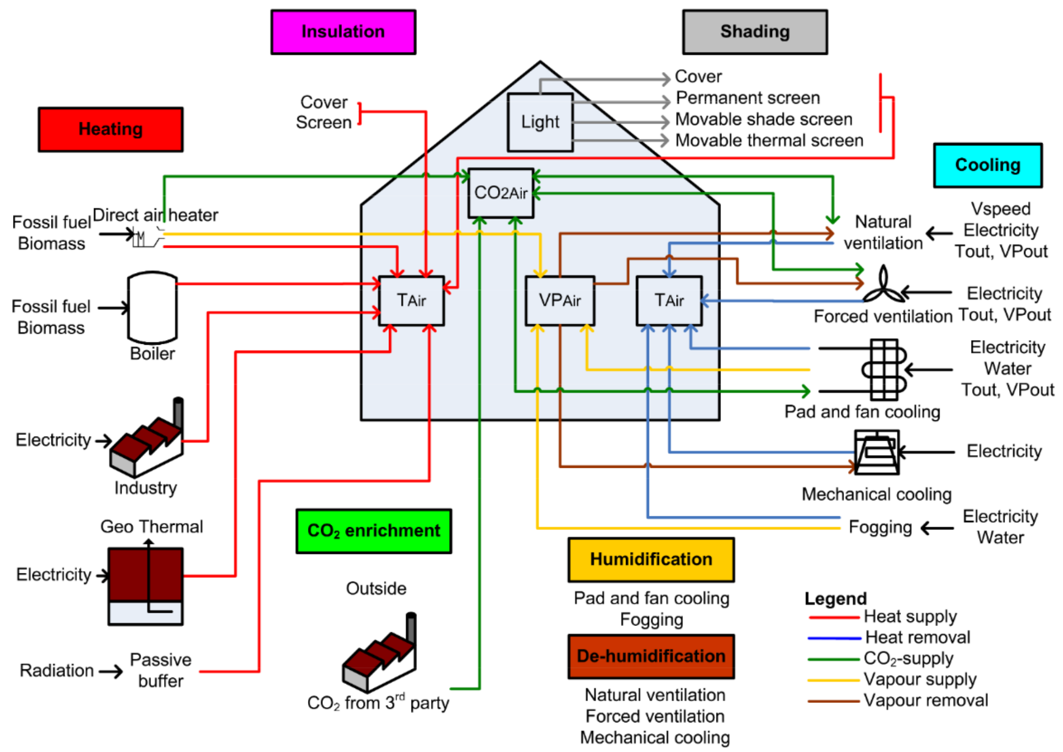

Figure 12: *Selected functions (coloured boxes), and design elements (text blocks and pictures below the accompanying functions), needed for the greenhouse design method to manage the greenhouse climate (transparent boxes inside the greenhouse). The coloured arrows represent the various energy and mass fluxes (legend at the bottom right).*

An overview of the states, the energy and mass fluxes of the greenhouse model is presented in Figure 2 above. This model is based on the following assumptions:

- Greenhouse air is considered to be a "perfectly stirred tank" which means that no spatial differences in temperature, vapor pressure and the CO_2 concentration occur. Therefore all the model fluxes are described per square metre of greenhouse floor.
- Describing the effect of the thermal screen on the indoor climate, the greenhouse air is divided into compartments: one below and one above the thermal screen.

The conditions of the model are totally portrayed by differential conditions. The time subordinates of the states to time are introduced by a dab over the state image. All images are characterized in the Nomenclature.

6.2 Programs calculate the net CO_2 flux

The two main equation to calculate the vapor pressure concentration in the lower and upper compartments is

$$\begin{aligned} cap_{VP_{Air}} \dot{VP}_{Air} = MV_{CanAir} + MV_{PadAir} + MV_{FogAir} + MV_{BlowAir} - MV_{AirThScr} \\ - MV_{AirTop} - MV_{AirOut} - MV_{Airout_{Pad}} - MV_{AirMech} \end{aligned} \tag{19}$$

$$cap_{VP_{Top}}\dot{VP}_{Top} = MV_{AirTop} - MV_{TopCov,in} - MV_{TopOut} \quad (20)$$

However, in our model only has:

$$MV_{CanAir} = -VEC_{CanAir}(VP_{Can} - VP_{Air}) \quad (21)$$

$$VEC_{CanAir} = \frac{2\rho_{Air}c_{p.Air}LAI}{\Delta H\gamma(r_b+r_s)} \quad (22)$$

Since the canopy temperature was not measured, it was assumed to be equal to the greenhouse air temperature.

Density of air at sea level:	$\rho_{Air} = 1.20$	kgm^{-3}
Specific heat capacity of the air:	$c_{p.Air} = 1 * 10^{-3}$	$JK^{-1}kg^{-1}$
Latent heat of evaporation:	$\Delta H = 2.45 * 10^6$	$Jkg^{-1}water$
Stefan BoltzMann constant:	$\sigma = 5.670 * 10{-}8$	$Wm^{-2}K^{-4}$
Psychrometric constant:	$\gamma = 65.8$	PaK^{-1}
Boundary layer resistance of the canopy for vapor transport:	$r_b = 275$	sm^{-1}
Stomatal resistance of the canopy:	$r_s = r_{s.min}.rf(R_{Can}).$ $rf(CO_{2Air_{ppm}}.rf(VP_{Can} - VP_{Air})$	sm^{-1}
The minimum canopy resistance for transpiration :	$r_{s.min} = 82.0$	sm^{-1}

$$rf(R_{Can}) = \frac{R_{Can}+c_{evap1}}{R_{Can}+c_{evap2}}$$
$$rf(CO_{2Air} = 1 + c_{evap3}(\eta_{mg_{ppm}CO_{2Air}-200})^2$$
$$rf(VP_{Can} - VPAir = 1 + c_{evap4}(VP_{Can} - VP_{Air})^2$$

Coefficient of the stomatal resistance model to account for radiation effect	$c_{evap1} = 4.30$	Wm^{-2}
Coefficient of the stomatal resistance model to account for radiation effect	$c_{evap2} = 0.54$	Wm^{-2}
Coefficient of the stomatal resistance model to account CO_2 effect	$c^{day}_{evap3} = 6.1 * 10{-}7$ $c^{night}_{evap3} = 1.1{*}10{-}11$	ppm^{-2}
Coefficient of the stomatal resistance model to account for vapor pressure difference	$c^{day}_{evap4} = 4.3 * 10{-}6$ $c^{night}_{evap4} = 5.2 * 10{-}6$	Pa^{-2}

The general form of a vapour flux accompanying an air flux is described by:
$MV_{12} = \frac{M_{Water}}{R}f_{12}(\frac{VP_1}{T_1+273.15} - \frac{VP_2}{T2+273.15} \quad kgm^{-2}s^{-1}$

where MV_{12} is the vapor flux from location 1 to location 2, $f_{12}(m^3m^{-2}s^{-1})$ is the air flux from location 1 to location 2, $T_1(^oC)$ is the tempertue at location 1 and $T_2(^oC)$ is the temperature at location 2. The vapor fluxes MV_{AirTop}, and MV_{TopOut} are described above analogously. Where by their accompanying air fluxes are f_{ThScr} (the flux through the thermal screen), $f_{VentRoof}$ (flux due to roof ventilation) respectively. We have VP_{Top} and VP_{Air} as input, while $VP_{Out} = 1.0$
$f_{ThScr} = U_{ThScr}K_{ThScr}|T_{Air} - T_{Out}|^{0.66} + \frac{1-U_{ThScr}}{\rho^{Mean}_{Air}}(0.5\rho^{Mean}_{Air}(1 - U_{ThScr})g|\rho_{Air} - \rho_{Out}|)^{0.5}$
$K_{ThScr} = 0.00025 \quad \rho^{Mean}_{Air} = 2.99 \quad m^3m^{-2}s^{-1}$

The density of the air is elevation dependent and by assuming a mean air temperature of 20^oC the density of the air is calculated by:

$\rho_{Air} = \rho_{Air0} exp(\frac{gM_{Air}h_{Elevation}}{293.15R}) \qquad kgm^{-3}$

Density of air at sea level $\rho_{Air0} = 1.20kgm^{-3}$ and $h_{Elevation} = 1470(m)$

Molar mass of air: $M_{Air} = 28.96kgkmol^{-1}$

$R(Jkmol^{-1}K^{-1})$ is the molar gas constant $g = 9.81$

$$f_{\text{VentRoof}} = \begin{cases} \eta_{\text{Ins}Scr} f''_{\text{VentRoof}} + 0.5 f_{\text{leakage}} & \eta roof \geq \eta_{Roof_Thr} \\ \eta_{\text{Ins}Scr} \left[U_{\text{ThScr}} f''_{\text{VentRoof}} \right. & \\ \left. + (1 - U_{\text{ThScr}}) f''_{VentRoofSide} \eta_{\text{Side}} \right] + 0.5 f_{\text{leakage}} & if \eta_{Roof} < \eta_{Side_Thr} \\ & m^3 m^{-2} s^{-1} \end{cases} \tag{23}$$

The natural ventilation rate due to roof ventilation is described by Boulard and Baille (1995):

$f''_{VentRoof} = \frac{U_{Roof}A_{Roof}C_d}{2A_{Flr}} \sqrt{\frac{gh_{Vent}}{2} \frac{T_{Air} - T_{Out}}{T + 273.15} + C_w v^2_{Wind}} \qquad m^3m^{-2}s^{-1}$

$\eta_{Roof_Thr} = 0.9$ is the ration between the roof vent area and total ventilation are above no chimney effect and was assumed. Our model $\eta_{Roof} = 1$ since we only have top roof vent, and no insect screen to reduce ventilation rate: $f_{VentRoof} = f''_{VentRoof} + 0.5f_{leakage}$

Furthermore the ventilation rate of the greenhouse is influenced by the greenhouse leakage rate which depends on wind speed and is described by:

$$f_{\text{leakage}} = \begin{cases} 0.25 * c_{leakage}, & v_w ind < 0.25 \qquad m^3m^{-2}s^{-1} \\ c_{leakage * v_{wind}}, & v_{wind} \geq 0.25 \end{cases} \tag{24}$$

Because our $v_{wind} = 0.29$ so we $2nd$ formula: $c_{leakage} * v_{wind}$, where $cleakage = 1 * 10^{-4}$

To calculate A_{Roof}, we multiply the specific roof ventilation area $A_{Roof}/A_{Flr} = 0.18$ to the surface of the greenhouse floor $A_{Flr} = 7,8 * 10^4 m^2$, which result is $14040(m^2)$.

$C_d = C_d^{Gh}(1 - \eta_{ShScrC_d} U_{ShSc})$

$C_w = C_w^{Gh}(1 - \eta_{ShScrC_w} U_{ShSc})$

Where $C_d^{Gh} = 0.65$ and$C_w^{Gh} = 0.9$ for Texas model, since we do not have moving shading screen so the parameter that determines the effect of the movable shading screen on the discharge coefficient $\eta_{ShScrC_d} = 0$ and $ShScrC_w = 0$. Other needed variables is: $h_{vent} = 0.97$, $T_{Out} = 23.9C$, $VP_{out} = 1.3$, $T_{Air} = 6.8$, $VP_{Air} = 12.8$.

References

[1] Bram HE Vanthoor. *A model-based greenhouse design method.* 2011

[2] Vanthoor, C. Stanghellini, E.J. van Henten and P.H.B. de Visser. *A methodology for model-based greenhouse design*

[3] David Katzin, Simon van Mourik, Frank Kempkes, Eldert J. van Henten *GreenLight - An open source model for greenhouses with supplemental lighting: Evaluation of heat requirements under LED and HPS lamps*