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Class: CS3010.01 - Numerical Methods
Date: 10/21/2020
Assignment: Programming Project 3

**a. $2x^3 - 11.7x^2 + 17.7x - 5$**

I choose the following starting points for the methods to find $x_1 = 0.36509$

- Bisection: $a_n = 0, b_n = 1$
- False-Position: $a_n = 0, b_n = 1$
- Newton Raphson: $x_n = 0$
- Secant: $x_{n-1} = 0, x_n = 1$
- Modified Secant: $x_n = 0.5$

Comment: With the same starting points (0, 1), **Newton Raphson Method** converges the fastest with 3 iterations (0 inclusive) and **Bisection Method** converges the slowest with 8 iterations (0 inclusive). Besides, the **Modified Secant Method** converges at 2 iterations (0 inclusive) with a starting point of $x_n = 0.5$.

I choose the following starting points for the methods to find $x_2 = 1.92174$

- Bisection: $a_n = 1.5, b_n = 2$
- False-Position: $a_n = 1.5, b_n = 2$
- Newton Raphson: $x_n = 1.5$
- Secant: $x_{n-1} = 1.5, x_n = 2$
- Modified Secant: $x_n = 1.5$

Comment: With the same starting points (1.5, 2), **False Position Method** and **Secant Method** converge the fastest with 1 iterations (0 inclusive), and **Bisection Method** converges the slowest with 4 iterations (0 inclusive).

I choose the following starting points for the methods to find $x_3 = 3.56316$

- Bisection: $a_n = 3.5, b_n = 4$
- False-Position: $a_n = 3.5, b_n = 4$
- Newton Raphson: $x_n = 3.5$
- Secant: $x_{n-1} = 3.5, x_n = 4$
- Modified Secant: $x_n = 3.5$

Comment: With the same starting points (3.5, 4), **False Position Method**, **Newton Raphson Method, Secant Method, Modified Secant Method** converge the fastest with 1 iterations (0 inclusive), and **Bisection Method** converges the slowest with 3 iterations (0 inclusive).

**b. f(x) = x + 10 - x cosh(50/x)**

I choose the following starting points for the methods to find $x_1$ = 0.36509

- Bisection: $a_n$ = 123, $b_n$ = 127
- False-Position: $a_n$ = 123, $b_n$ = 127
- Newton Raphson: $x_n$ = 123
- Secant: $x_{n-1}$ = 123, $x_n$ = 127
- Modified Secant: $x_n$ = 123

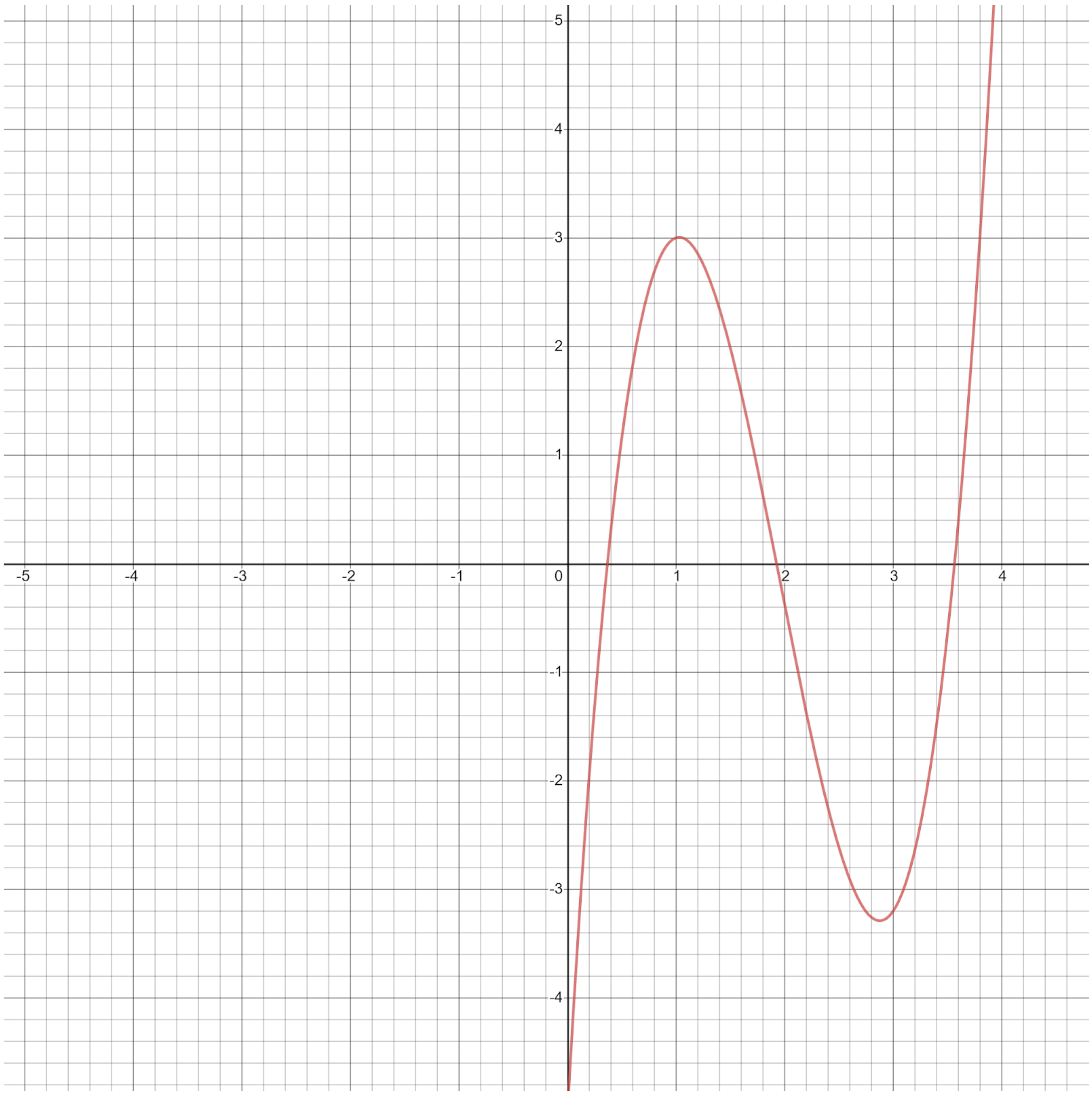
Comment: With the same starting points (123, 127), all 5 methods converge at 1 iteration (0 inclusive).

**In the program, I use data type *double* to store values to calculate each method. The data type *double* allows more accuracy as it can hold 8 bytes and store about 15 digits after the decimal point. However, I round the numbers to four decimal places for a cleaner output.**

**a. $f(x) = 2x^3 - 11.7x^2 + 17.7x - 5$**

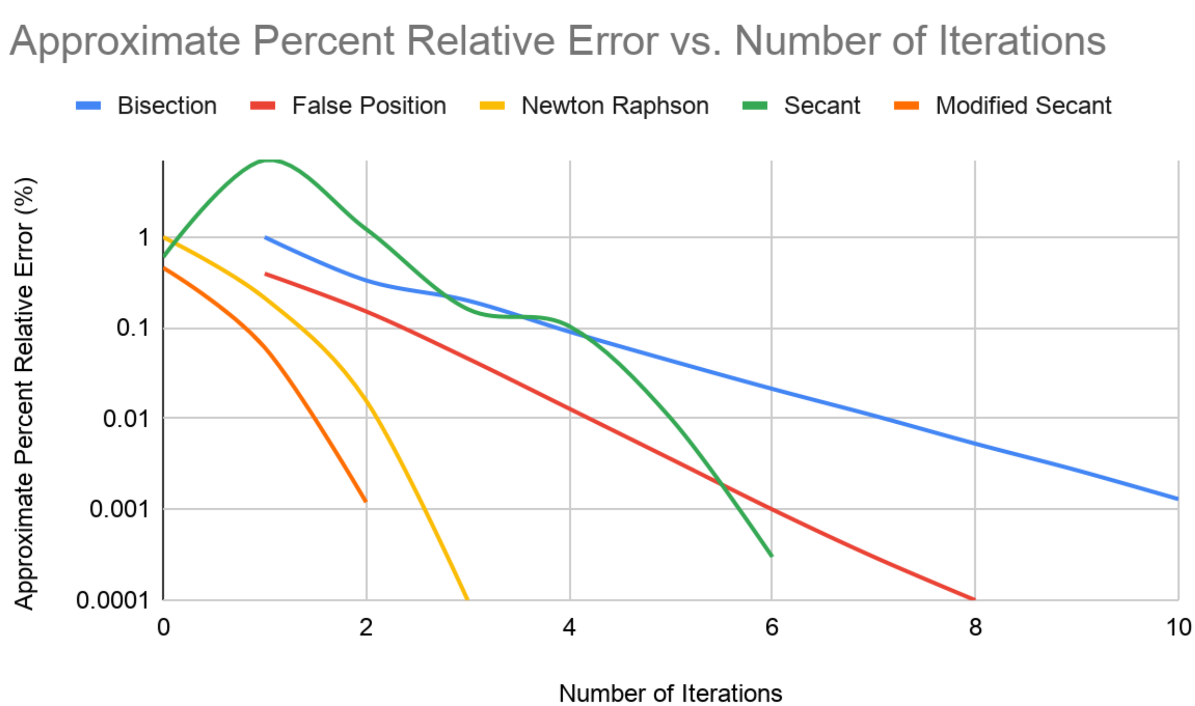
The roots are:

- $x_1 \approx 0.36509...$
- $x_2 \approx 1.92174...$
- $x_3 \approx 3.56316...$



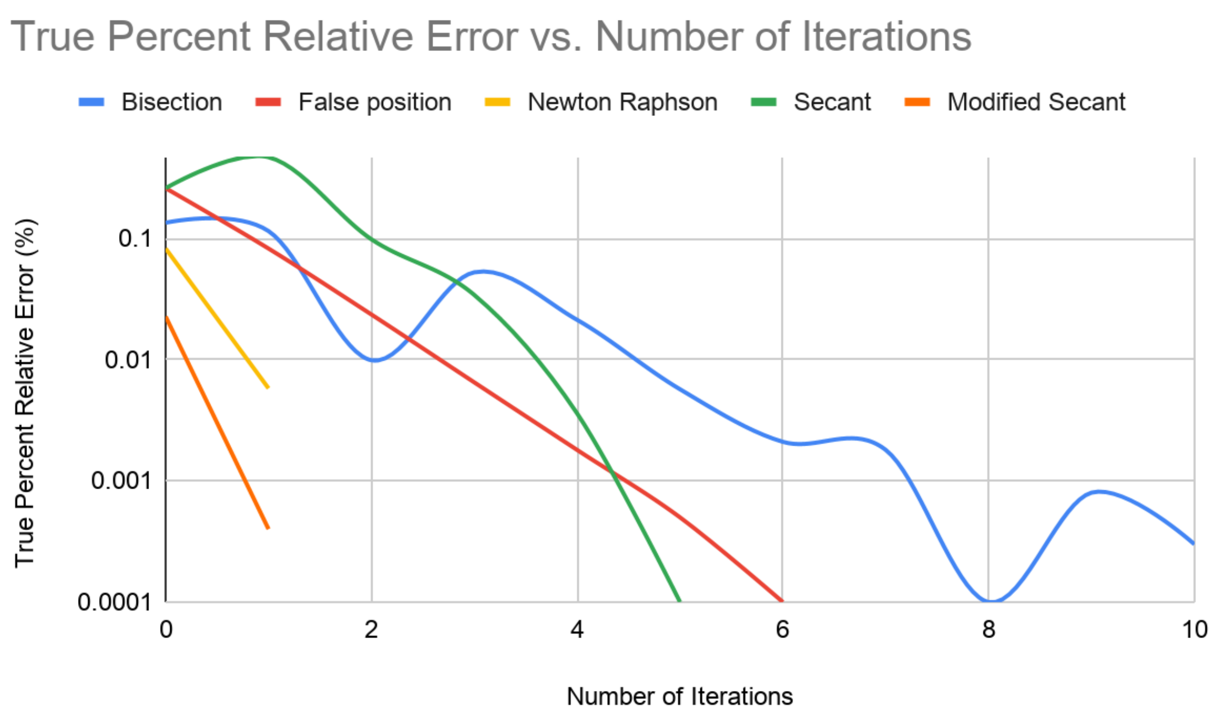
**Finding roots $x_1$ = 0.36509**

| Approximate Percent Relative Error vs. Number of Iterations | | | | | |
|---|---|---|---|---|---|
| n | Bisection | False position | Newton Raphson | Secant | Modified Secant |
| 0 | | | 1 | 0.6 | 0.4602 |
| 1 | 1 | 0.3961 | 0.2138 | 7.0417 | 0.061 |
| 2 | 0.3333 | 0.1517 | 0.0158 | 1.2231 | 0.0012 |
| 3 | 0.2 | 0.046 | 0.0001 | 0.1611 | 0 |
| 4 | 0.0909 | 0.0129 | 0 | 0.1044 | 0 |
| 5 | 0.0435 | 0.0036 | 0 | 0.0101 | 0 |
| 6 | 0.0213 | 0.001 | 0 | 0.0003 | 0 |
| 7 | 0.0108 | 0.0003 | 0 | 0 | 0 |
| 8 | 0.0053 | 0.0001 | 0 | 0 | 0 |
| 9 | 0.0027 | 0 | 0 | 0 | 0 |
| 10 | 0.0013 | 0 | 0 | | 0 |




Comment: The graph of Secant Method has a peak at 7.0417 before it goes down, compared to the rest of the methods goes down gradually, with Modified Secant Method being the fastest and Bisection Method being the slowest.

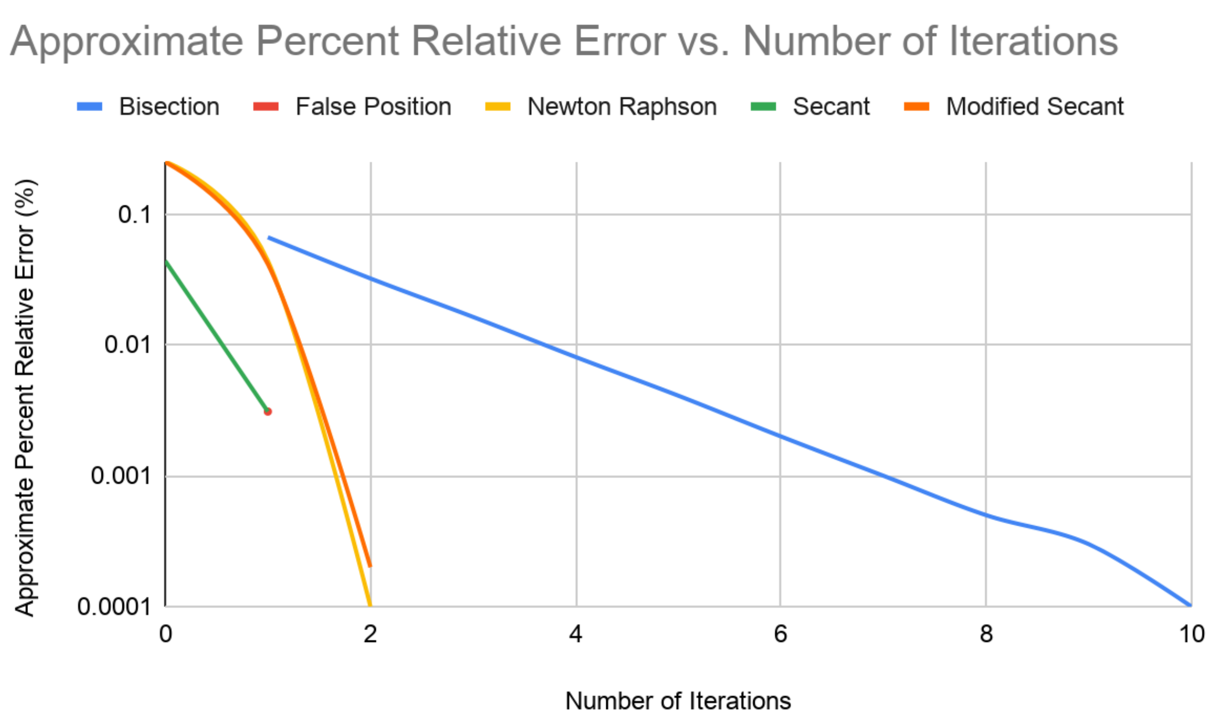
| True Percent Relative Error vs. Number of Iterations | | | | | |
|---|---|---|---|---|---|
| n | Bisection | False position | Newton Raphson | Secant | Modified Secant |
| 0 | 0.1349 | 0.2599 | 0.0826 | 0.2599 | 0.0227 |
| 1 | 0.1151 | 0.0826 | 0.0058 | 0.4685 | 0.0004 |
| 2 | 0.0099 | 0.0236 | 0 | 0.0985 | 0 |
| 3 | 0.0526 | 0.0065 | 0 | 0.0342 | 0 |
| 4 | 0.0213 | 0.0018 | 0 | 0.0036 | 0 |
| 5 | 0.0057 | 0.0005 | 0 | 0.0001 | 0 |
| 6 | 0.0021 | 0.0001 | 0 | 0 | 0 |
| 7 | 0.0018 | 0 | 0 | 0 | 0 |
| 8 | 0.0001 | 0 | 0 | 0 | 0 |
| 9 | 0.0008 | 0 | 0 | 0 | 0 |
| 10 | 0.0003 | 0 | 0 | | 0 |




<u>Comment</u>: The graph of the Bisection Method is unstable with multiple peaks and troughs compared to the rest of the methods, which go down in a rather stable manner. Modified Secant Method and Newton Raphson converge the fastest and Bisection Method converges the slowest.

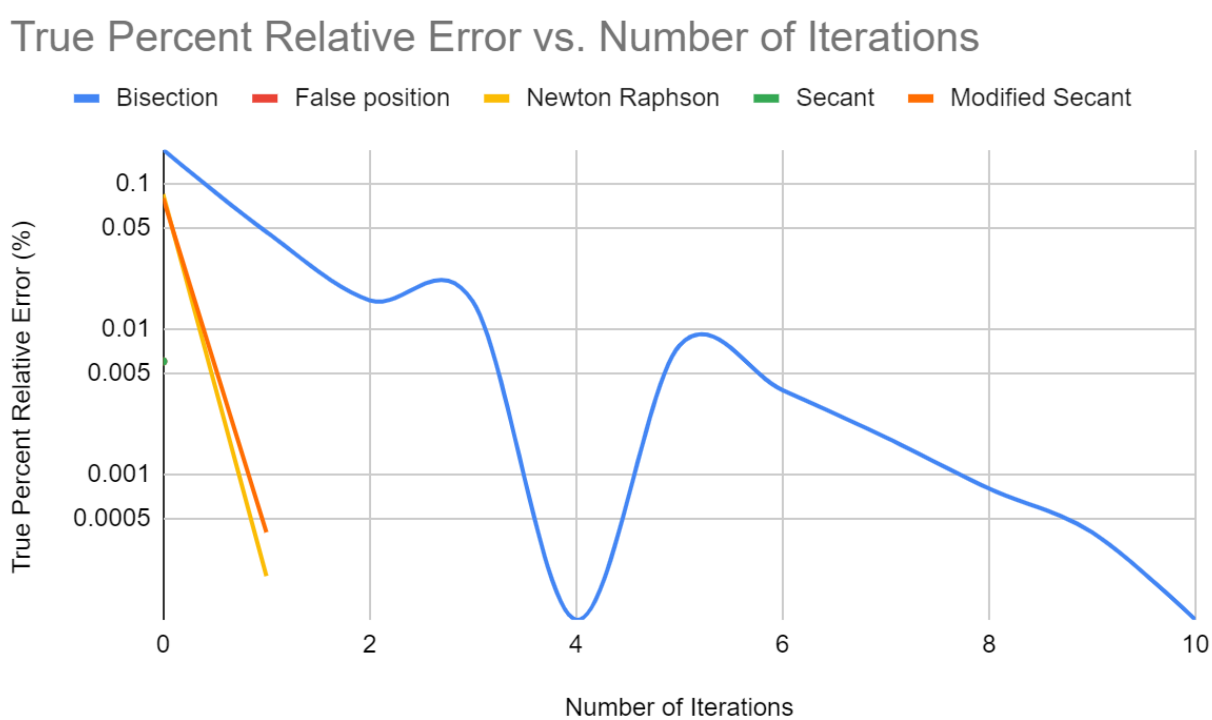
**Finding roots $x_2$ = 1.92174**

| Approximate Percent Relative Error vs. Number of Iterations | | | | | |
|---|---|---|---|---|---|
| n | Bisection | False position | Newton Raphson | Secant | Modified Secant |
| 0 | | | 0.2524 | 0.044 | 0.2505 |
| 1 | 0.0667 | 0.0031 | 0.0442 | 0.0031 | 0.0416 |
| 2 | 0.0323 | 0 | 0.0001 | 0 | 0.0002 |
| 3 | 0.0164 | 0 | 0 | 0 | 0 |
| 4 | 0.0081 | 0 | 0 | 0 | 0 |
| 5 | 0.0041 | 0 | 0 | 0 | 0 |
| 6 | 0.002 | 0 | 0 | 0 | 0 |
| 7 | 0.001 | 0 | 0 | 0 | 0 |
| 8 | 0.0005 | 0 | 0 | | 0 |
| 9 | 0.0003 | 0 | 0 | | 0 |
| 10 | 0.0001 | 0 | 0 | | 0 |




Comment: The graph of the Modified Secant Method and Newton Raphson method quite overlap each other and converge simultaneously. The graph of False Position method is a point since it only needs one iteration to converge (fastest), followed by Secant Method. The slowest graph to converge is the Bisection Method.

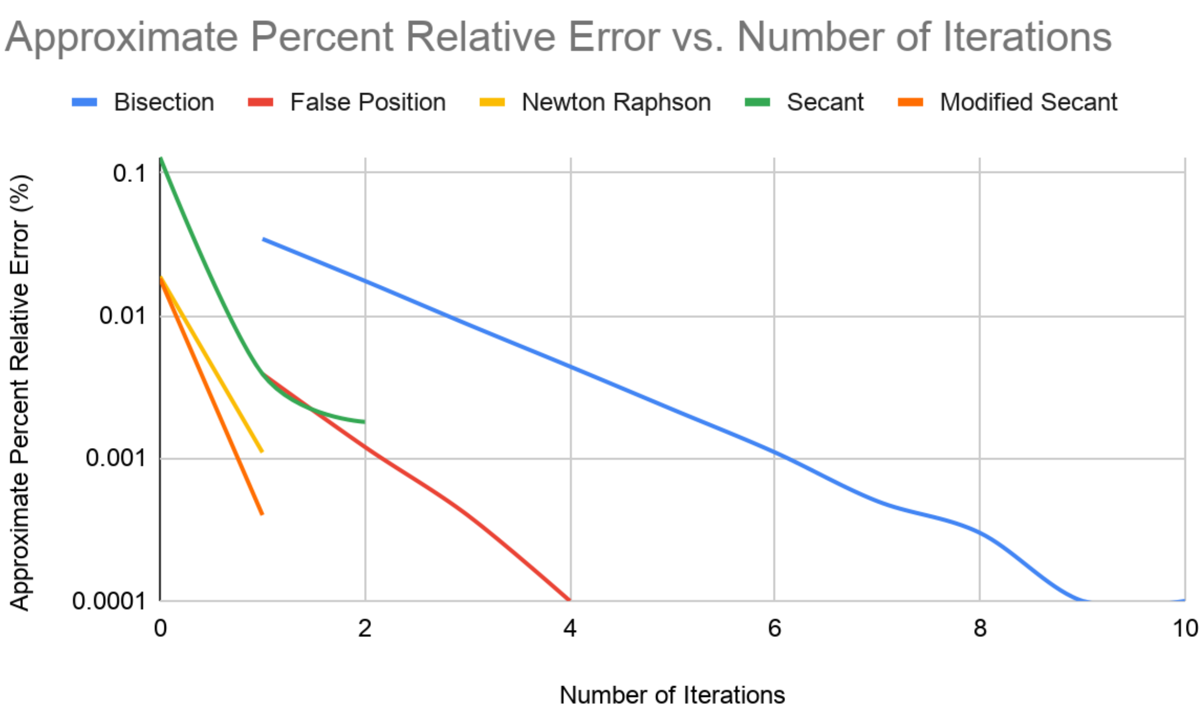
| True Percent Relative Error vs. Number of Iterations | | | | | |
|---|---|---|---|---|---|
| n | Bisection | False position | Newton Raphson | Secant | Modified Secant |
| 0 | 0.1717 | 0.006 | 0.0847 | 0.006 | 0.0795 |
| 1 | 0.0467 | 0 | 0.0002 | 0 | 0.0004 |
| 2 | 0.0158 | 0 | 0 | 0 | 0 |
| 3 | 0.0155 | 0 | 0 | 0 | 0 |
| 4 | 0.0001 | 0 | 0 | 0 | 0 |
| 5 | 0.0077 | 0 | 0 | 0 | 0 |
| 6 | 0.0038 | 0 | 0 | 0 | 0 |
| 7 | 0.0018 | 0 | 0 | 0 | 0 |
| 8 | 0.0008 | 0 | 0 | | 0 |
| 9 | 0.0004 | 0 | 0 | | 0 |
| 10 | 0.0001 | 0 | 0 | | 0 |




Comment: The graph of the Bisection Method has a trough at 4 iteration, which error of 0.0001. However, it goes back up and gradually converges at 10 iterations. The graph of False Position Method and Secant Method are a dot because as they converge at 1 iteration. Newton Raphson Method and Modified Secant method converge at 2 iterations with a linear graph of negative slope.

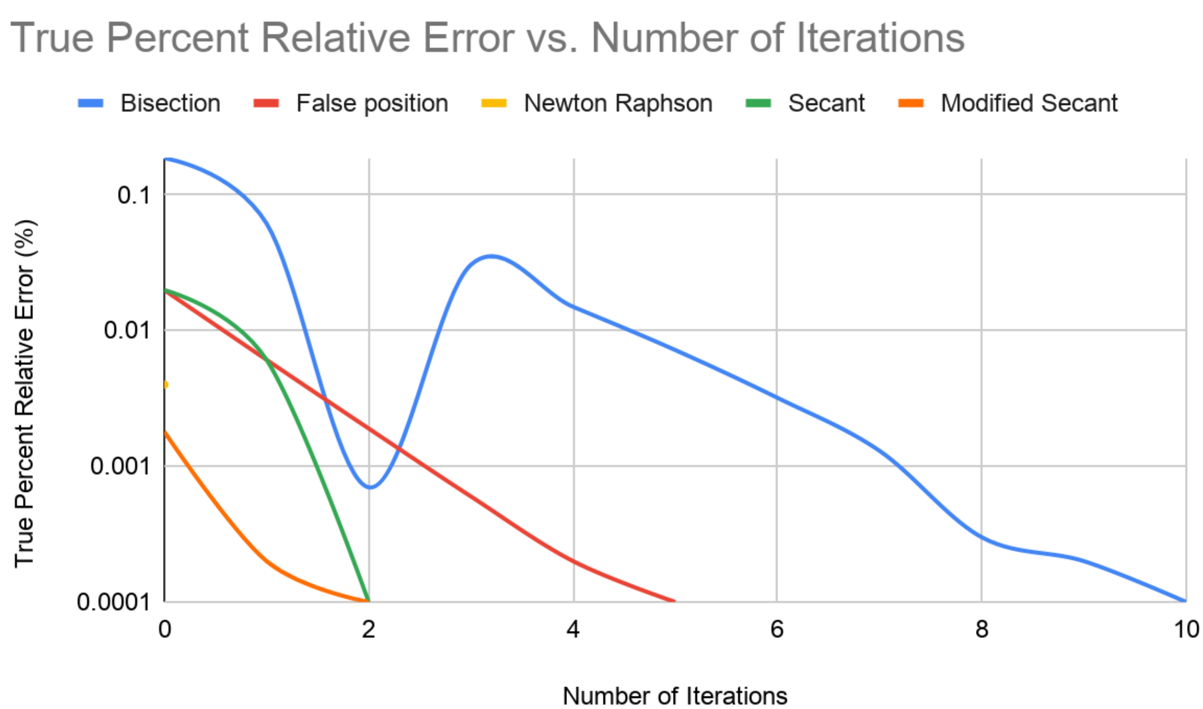
**Finding roots $x_3$ = 3.56316**

| Approximate Percent Relative Error vs. Number of Iterations | | | | | |
|---|---|---|---|---|---|
| n | Bisection | False position | Newton Raphson | Secant | Modified Secant |
| 0 | | | 0.0188 | 0.1289 | 0.0182 |
| 1 | 0.0345 | 0.0039 | 0.0011 | 0.0039 | 0.0004 |
| 2 | 0.0175 | 0.0012 | 0 | 0.0018 | 0 |
| 3 | 0.0087 | 0.0004 | 0 | 0 | 0 |
| 4 | 0.0044 | 0.0001 | 0 | 0 | 0 |
| 5 | 0.0022 | 0 | 0 | 0 | 0 |
| 6 | 0.0011 | 0 | 0 | 0 | 0 |
| 7 | 0.0005 | 0 | 0 | | 0 |
| 8 | 0.0003 | 0 | 0 | | 0 |
| 9 | 0.0001 | 0 | 0 | | 0 |
| 10 | 0.0001 | 0 | 0 | | 0 |




<u>Comment</u>: All the graphs go down gradually, with Modified Secant Method and Newton Raphson Method being the fastest at 1 iteration, and Bisection Method being the slowest at 9 iterations.

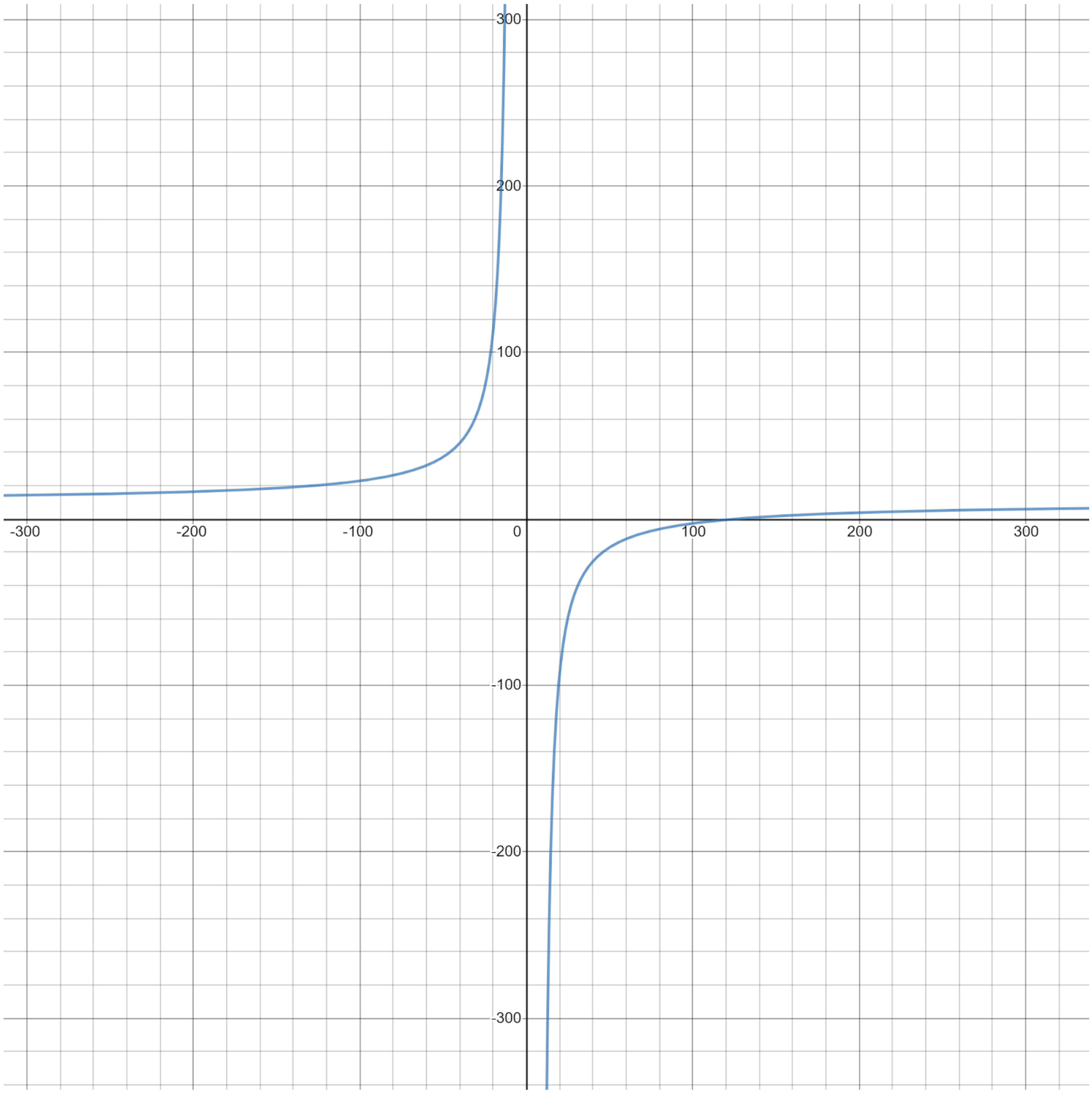
| True Percent Relative Error vs. Number of Iterations | | | | | |
|---|---|---|---|---|---|
| n | Bisection | False position | Newton Raphson | Secant | Modified Secant |
| 0 | 0.1868 | 0.0199 | 0.004 | 0.0199 | 0.0018 |
| 1 | 0.0618 | 0.0061 | 0 | 0.0061 | 0.0002 |
| 2 | 0.0007 | 0.0019 | 0 | 0.0001 | 0.0001 |
| 3 | 0.0306 | 0.0006 | 0 | 0 | 0 |
| 4 | 0.015 | 0.0002 | 0 | 0 | 0 |
| 5 | 0.0072 | 0.0001 | 0 | 0 | 0 |
| 6 | 0.0032 | 0 | 0 | 0 | 0 |
| 7 | 0.0013 | 0 | 0 | | 0 |
| 8 | 0.0003 | 0 | 0 | | 0 |
| 9 | 0.0002 | 0 | 0 | | 0 |
| 10 | 0.0001 | 0 | 0 | | 0 |




<u>Comment</u>: The graph of the Bisection Method has a trough at 2 iterations and then goes up again before it gradually converges at 10 iterations. The graph of the Newton Raphson Method is a dot as it converges at 1 iteration.

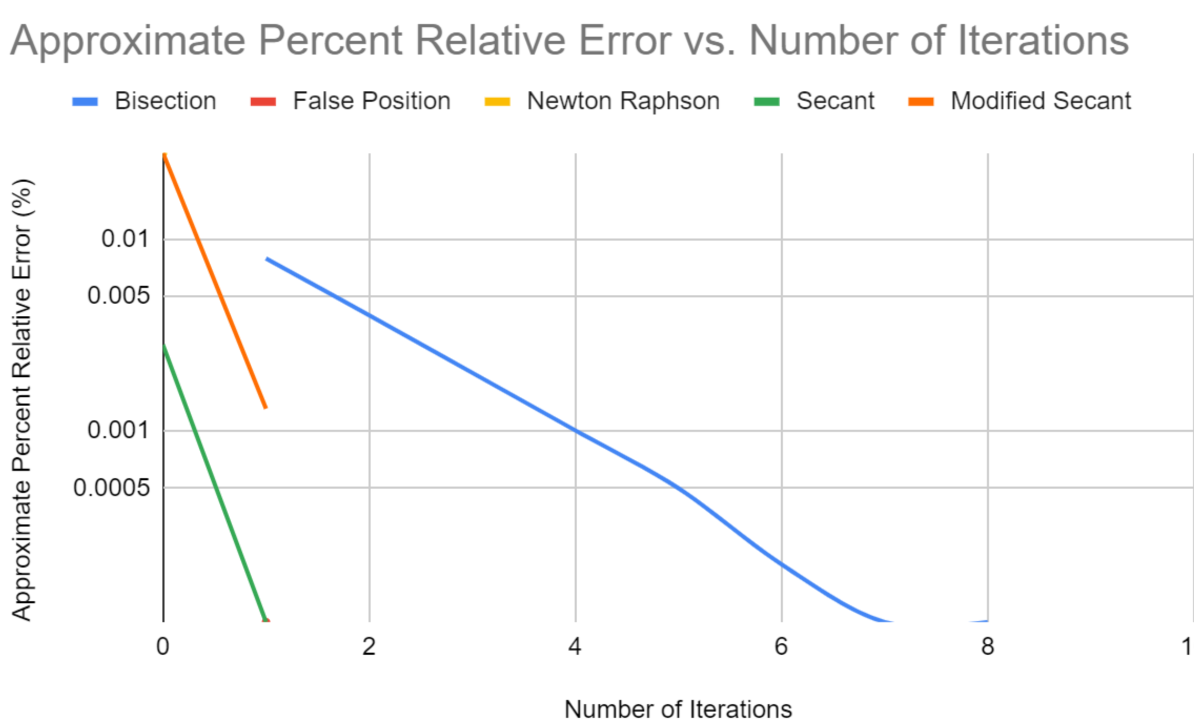
## b. f(x) = x + 10 - x cosh(50/x)

The root is: **x ≈ 126.632**



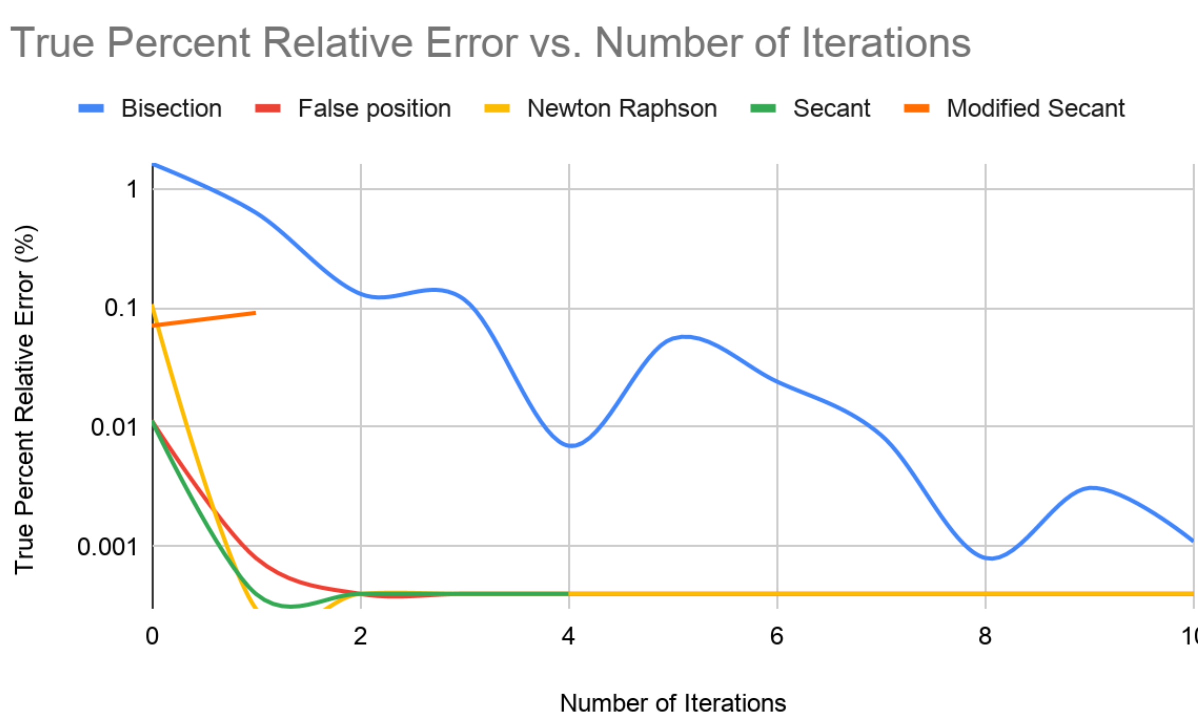
**Finding root x = 126.632**

| Approximate Percent Relative Error vs. Number of Iterations | | | | | |
|---|---|---|---|---|---|
| n | Bisection | False position | Newton Raphson | Secant | Modified Secant |
| 0 | | | 0.0279 | 0.0028 | 0.0281 |
| 1 | 0.0079 | 0.0001 | 0 | 0.0001 | 0.0013 |
| 2 | 0.004 | 0 | 0 | 0 | |
| 3 | 0.002 | 0 | 0 | 0 | |
| 4 | 0.001 | 0 | 0 | 0 | |
| 5 | 0.0005 | 0 | 0 | | |
| 6 | 0.0002 | 0 | 0 | | |
| 7 | 0.0001 | 0 | 0 | | |
| 8 | 0.0001 | 0 | 0 | | |
| 9 | 0 | 0 | 0 | | |
| 10 | 0 | 0 | 0 | | |




Comment: The graph of the Modified Secant Method, Secant Method, and Bisection Method goes down in a rather linear manner. The graph of False Position is a dot as it converges at 1 iteration. The graph of the Bisection Method still takes the longest to converge at 8 iterations.

| True Percent Relative Error vs. Number of Iterations | | | | | |
|---|---|---|---|---|---|
| n | Bisection | False position | Newton Raphson | Secant | Modified Secant |
| 0 | 1.632 | 0.0114 | 0.1078 | 0.0114 | 0.0711 |
| 1 | 0.632 | 0.0008 | 0.0003 | 0.0004 | 0.0912 |
| 2 | 0.132 | 0.0004 | 0.0004 | 0.0004 | |
| 3 | 0.118 | 0.0004 | 0.0004 | 0.0004 | |
| 4 | 0.007 | 0.0004 | 0.0004 | 0.0004 | |
| 5 | 0.0555 | 0.0004 | 0.0004 | | |
| 6 | 0.0242 | 0.0004 | 0.0004 | | |
| 7 | 0.0086 | 0.0004 | 0.0004 | | |
| 8 | 0.0008 | 0.0004 | 0.0004 | | |
| 9 | 0.0031 | 0.0004 | 0.0004 | | |
| 10 | 0.0011 | 0.0004 | 0.0004 | | |




<u>Comment</u>: The graph of the Modified Secant Method diverges, whereas the rest of the methods finally converge. After many iterations, the graph of False Position Method, Newton Raphson Method and Secant Method cannot go below error of 0.0004. The graph of the Bisection Method still takes the longest to converge.