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Leader-following bipartite consensus of multiple uncertain Euler-Lagrange systems over signed switching digraphs

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ABSTRACT

In this paper, we investigate the leader-following bipartite consensus problem of multiple uncertain Euler-Lagrange (EL) systems over signed switching networks. We first extend the distributed observer for a linear leader system over signed communication networks from the static case to the jointly connected switching case. Based on such a distributed observer and the certainty equivalence principle, we further show that, under the assumptions that the signed switching network is structurally balanced and jointly connected, the leader-following bipartite consensus problem of multiple uncertain EL systems over signed switching networks is solvable by a class of distributed adaptive control law. Our result will then be applied to the leader-following bipartite consensus problem of a group of two-link robot manipulators.

1. Introduction

Over the past few years, the distributed control of multi-agent systems over signed graphs has received considerable attention. In practice, a signed digraph can represent, say, allied/adversary relationships such as two parties in politics since cooperation and competition often coexist, trade war between countries or opinion dynamics in trust-mistrust social networks [22]. A fundamental problem in the distributed control of multi-agent systems over signed graphs is the so-called bipartite consensus, which is to design a distributed control law to achieve modulus consensus for each agent with antagonistic interactions [1].

Early work on the bipartite consensus focused on multiple single-integrator systems. For example, in [1], Altafini showed that the bipartite consensus for multiple single-integrator systems can be achieved if and only if the signed graph is structurally balanced and connected. The result in [1] was then extended to the general linear multi-agent systems [6,23,24,26]. Like the consensus problem over unsigned graphs, the bipartite consensus can also be classified into leaderless case [1] and leader-following case [11,13,20,21]. The bipartite tracking problem was studied in [11] with a scalar leader system, and in [6,18,21] with a high-order leader system. In [9], the robust bipartite output regulation for a class of general linear systems was studied, which aims to achieve asymptotic modulus tracking in the presence of a class of disturbances. Almost all the publications mentioned above assumed that the signed graphs were static. Recently, the distributed control of multi-agent systems over signed switching graphs were considered in some papers. For example, the output bipartite consensus of heterogeneous linear multi-agent systems was investigated in [19] by a distributed dynamic output feedback control law. The bipartite synchronization of a class of nonlinear systems satisfying the one-sided Lipschitz condition over signed switching networks was studied in [25]. Other relevant work on bipartite consensus over signed switching topologies can be found in [7].

EL systems are an important class of nonlinear systems that can model robots, spacecraft systems, etc [8,10]. The cooperative control problems of multiple EL systems over unsigned communication networks have been thoroughly studied in quite a few papers [3,4,14]. In particular, Ref. [3] solved the leader-following consensus problem of multiple EL systems over jointly connected switching networks via a distributed observer approach. More recently, the study of the cooperative control problems of multiple EL systems has been extended to the signed communication networks. For example, Ref. [5] considered the swarming behaviors of multiple EL systems with uncertain parameters over signed static communication networks.

In this paper, we will further study the leader-following bipartite consensus problem of multiple EL systems over signed switching communication networks. Compared with [5], our paper offers at least three new features. First, our digraph is a signed switching

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graph satisfying jointly connected condition. Under the jointly connected condition, the signed digraph can be disconnected at any time instant. Thus, the approach in [5] does not apply to our problem. Second, the leader's signal in [5] is an exactly known time function. In contrast, like in [3], we use an autonomous linear system called leader system to generate a large class of signals such as step functions with arbitrarily large unknown amplitudes, and sinusoidal functions with arbitrarily large unknown amplitudes and arbitrarily unknown initial phases as well as their linear combinations. Third, unlike in [5] where the pinning control strategy together with a second-order auxiliary system over cooperation-competition networks was used, we employ the distributed observer approach, which, together with the certainty equivalence principle, leads to a distributed adaptive control law to solve our problem. It is noted that the distributed observer for a linear leader system was first proposed for unsigned static communication networks in [15] and then was extended to the unsigned switching communication networks in [17]. The distributed observer for a linear leader system over unsigned static communication networks in [15] was extended to the signed static communication networks in [2]. The main technical challenge in this paper is summarized as follows. First, we need to establish the distributed observer over signed jointly connected switching networks, which can be disconnected at every time instant. Second, since the communication graph of multiple Euler-Lagrange systems is jointly connected, the closed-loop system is discontinuous. Thus, the classical Barbalat's lemma does not apply to our problem. We have to further employ the generalized Barbalat's lemma to furnish the stability analysis.

The rest of the paper is organized as follows. Section 2 briefly summarizes some preliminaries and formulates the problem. In Section 3, we establish the distributed observer for a linear leader system over signed switching digraphs. We synthesize a class of distributed adaptive control law to solve the problem in Section 4. As an application of our main result, we solve the bipartite consensus problem for a group of two-link manipulators in Section 5. We close this paper in Section 6 with some remarks.

Notation. The symbol $\otimes$ denotes Kronecker product. $||x||$ denotes the Euclidean norm of a vector $x \in \mathbb{R}^n$ and $||A||$ is the induced norm of a matrix $A \in \mathbb{R}^{m\times n}$ by the Euclidean norm. The symbol $\mathbf{1}_N$ represents an N-dimensional column vector with all elements being 1. For column vectors a_i, $i = 1, 2, \dots, s$, $\mathrm{col}(a_1, \dots, a_s) = [a_1^T, \dots, a_s^T]^T$.

2. Preliminaries and problem formulation

As in [3], we consider the multiple uncertain EL systems as follows:

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) = \tau_i, i = 1, 2, \dots, N \tag{1}$$

where $q_i \in \mathbb{R}^n$ is the vector of generalized coordinates, $\tau_i \in \mathbb{R}^n$ denotes the generalized force vector, $M_i(q_i) \in \mathbb{R}^{n\times n}$ is the inertia matrix which is symmetric, $C_i(q_i, \dot{q}_i)\dot{q}_i \in \mathbb{R}^n$ represents the vector of Coriolis and centripetal forces, and $G_i(q_i) \in \mathbb{R}^n$ is the gravity term. We assume the EL systems satisfy the following three properties:

Property 1. *There exist positive constants k_m, k_M, k_c and k_g such that $k_m I_N \le M_i(q_i) \le k_M I_N$, $||C_i(q_i, \dot{q}_i)|| \le k_c||\dot{q}_i||$, $||G_i(q_i)|| \le k_g$.*

Property 2. *The matrix $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew symmetric.*

Property 3. The left hand side of (1) can be *linearly parameterized in the sense that, for all $x, y \in \mathbb{R}^n$,*

$$M_i(q_i)x + C_i(q_i, \dot{q}_i)y + G_i(q_i) = Y_i(q_i, \dot{q}_i, x, y)\Theta_i \tag{2}$$

where $Y_i(q_i, \dot{q}_i, x, y)$ called regression matrices are known and the constant matrices Θ_i are unknown.

Let $q_0 \in \mathbb{R}^n$ denote the reference signal, which is assumed to be generated by the following linear exosystem:

$$\begin{aligned} \dot{v} &= Sv \\ q_0 &= Fv \end{aligned} \tag{3}$$

where $v \in \mathbb{R}^m$, and $S \in \mathbb{R}^{m\times m}$ and $F \in \mathbb{R}^{n\times m}$ are two known constant matrices.

As in [3], the multiple EL systems (1) together with the exosystem (3) is considered as a multi-agent system of $N+1$ agents with the exosystem as the leader and all subsystems of (1) as followers. The communication network of this multi-agent system is represented by a signed switching digraph $\mathcal{G}^s_{\sigma(t)} = \{\mathcal{V}, \mathcal{E}_{\sigma(t)}, \mathcal{A}^s_{\sigma(t)}\}$[1] where $\mathcal{V} = \{0, 1, \dots, N\}$ with the node 0 associated with the exosystem and the other N nodes associated with the N subsystems of system (1), and $(j, i) \in \mathcal{E}_{\sigma(t)}$, $i \neq j$, $i, j = 0, 1, \dots, N$, if and only if the control τ_i of the subsystem i, $i = 1, \dots, N$, can access the state q_j, $j = 0, 1, \dots, N$[2] The elements of the matrix $\mathcal{A}^s_{\sigma(t)} \in \mathbb{R}^{(N+1)\times(N+1)}$ are denoted by $a_{ij}(t)$, $i, j = 0, 1, \dots, N$.

Three assumptions are listed as follows:

Assumption 2.1. All eigenvalues of S have non-positive real parts and those eigenvalues with zero real parts are semi-simple.

Assumption 2.2. The signed switching digraph $\mathcal{G}^s_{\sigma(t)}$ is structurally balanced.

Assumption 2.3. There exists an infinite subsequence $\{i_k\}$ of $\{i : i = 0, 1, \dots, \}$ satisfying $t_{i_{k+1}} - t_{i_k} < \nu$ for some positive ν such that the union graph $\cup_{j=i_k}^{i_{k+1}-1} \mathcal{G}^s_{\sigma(t_j)}$ contains a spanning tree with the node 0 as the root.

Remark 2.1. Under Assumption 2.1, the reference signal is the combination of exponentially decayed time functions or sinusoidal functions of any amplitude determined by the initial condition. Thus, the reference signal is bounded. Assumption 2.3 is called jointly connected condition [12]. This assumption is the mildest assumption on a switching graph since, under this assumption, the communication graph can be disconnected at any time instant. Assumption 2.2 is standard in the literature of bipartite consensus problem [1]. Under Assumption 2.2, without loss of generality, there exists some integer $0 \le k < N$ such that $\mathcal{V} = \mathcal{V}_1 \cup \mathcal{V}_2$ where $\mathcal{V}_1 = \{0, 1, \dots, k\}$ and $\mathcal{V}_2 = \{k+1, k+2, \dots, N\}$, and $a_{ij} \le 0$, $\forall i \in \mathcal{V}_p$, $j \in \mathcal{V}_q$ where p, $q \in \{1, 2\}$, $p \neq q$; otherwise, $a_{ij} \ge 0$. For $i = 0, 1, \dots, N$, let

$$\phi_i = \begin{cases} 1, & i \in \mathcal{V}_1 \\ -1, & i \in \mathcal{V}_2 \end{cases} \tag{4}$$

$\bar{\Phi} = \mathrm{diag}\,(\phi_0, \phi_1, \dots, \phi_N)$. Then, by Lemma 1 of [1], for all $t \ge 0$, $\bar{\Phi}\mathcal{A}^s_{\sigma(t)}\bar{\Phi}$ has all nonnegative entries.

We consider the following class of control laws:

$$\begin{aligned} \tau_i &= f_i(q_i, \dot{q}_i, \eta_i, \hat{\Theta}_i, \eta_j - \eta_i, j \in \mathcal{N}_i(t)) \\ \dot{\eta}_i &= g_i(\eta_i, \eta_j - \eta_i, j \in \mathcal{N}_i(t)) \\ \dot{\hat{\Theta}}_i &= h_i(q_i, \dot{q}_i, \eta_i, \eta_j - \eta_i, j \in \mathcal{N}_i(t)), \quad i = 1, \dots, N \end{aligned} \tag{5}$$

where $\eta_0 = v$, for $i = 1, \dots, N$, η_i represents the state of the distributed observer to be introduced later, $\hat{\Theta}_i$ is the estimation of the unknown parameter matrices Θ_i, and f_i, g_i, and h_i are some functions to be designed. The control law (5) consists of N compensators. Since the ith compensator can use the information of the jth subsystem if and only if j is a neighbor of i, such a control law is called a distributed feedback control law.

We now formulate our problem as follows:

[1] See Appendix for terminologies of signed digraphs.

[2] Since the exosystem does not have a control input, $(j, 0) \notin \mathcal{E}$, $\forall j$.

Problem 2.1. [Leader-following Bipartite Consensus of Multiple EL Systems] Given (1), (3) and a signed switching digraph $\mathcal{G}^s_{\sigma(t)}$, design a distributed control law in the form of (5) such that, for any initial condition $q_i(0), \dot{q}_i(0), \eta_i(0)$ and $v(0)$, $q_i(t), \dot{q}_i(t), \eta_i(t)$ exist and are bounded for all $t \geq 0$ and the solutions to (1) satisfy

$$\lim_{t\to\infty}(q_i(t) - q_0(t)) = 0,\ \lim_{t\to\infty}(\dot{q}_i(t) - \dot{q}_0(t)) = 0,\ \ i \in \mathcal{V}_1$$
$$\lim_{t\to\infty}(q_i(t) + q_0(t)) = 0,\ \lim_{t\to\infty}(\dot{q}_i(t) + \dot{q}_0(t)) = 0,\ \ i \in \mathcal{V}_2$$

Remark 2.2. In the special case where the digraph is unsigned, that is, $\mathcal{V}_1 = \{0, 1, \ldots, N\}$ and $\mathcal{V}_2$ is an empty set, the above problem reduces to the leader-following consensus problem of multiple EL systems over unsigned switching graph as studied in [3,4]. It is also noted that Ref. [5] investigated the swarming behaviors of multiple EL systems over signed static digraphs.

3. The distributed observer over signed switching digraphs

In this section, we first present a stability result for a linear switching system and then establish the distributed observer for the leader system (3) over signed switching digraphs. For this purpose, let $\mathcal{L}^s_{\sigma(t)}$ be the Laplacian of the digraph $\mathcal{G}^s_{\sigma(t)}$, and $\mathcal{H}^s_{\sigma(t)}$ be the matrix consisting of the last N rows and the last N columns of $\mathcal{L}^s_{\sigma(t)}$. We have the following result:

Lemma 3.1. *Under Assumptions 2.2 and 2.3, the following system*

$$\dot{x} = -\mathcal{H}^s_{\sigma(t)}x \tag{6}$$

is exponentially stable.

Proof. Let $\mathcal{A}_{\sigma(t)} = \bar{\Phi}\mathcal{A}^s_{\sigma(t)}\bar{\Phi}$. By Remark 2.1, the digraph $\mathcal{G}_{\sigma(t)} = (\mathcal{V}, \mathcal{E}_{\sigma(t)}, \mathcal{A}_{\sigma(t)})$ is an unsigned switching graph with the same node set and the same edge set as the signed switching graph $\mathcal{G}^s_{\sigma(t)} = (\mathcal{V}, \mathcal{E}_{\sigma(t)}, \mathcal{A}^s_{\sigma(t)})$. Thus, the digraph $\mathcal{G}_{\sigma(t)}$ also satisfies Assumption 2.3. Let $\mathcal{L}_{\sigma(t)}$ be the Laplacian of the digraph $\mathcal{G}_{\sigma(t)}$ and let $\mathcal{H}_{\sigma(t)}$ be the matrix consisting of the last N rows and the last N columns of $\mathcal{L}_{\sigma(t)}$. Since the digraph $\mathcal{G}_{\sigma(t)}$ is unsigned and satisfies Assumption 2.3, by Corollary 4 in [17], the following system

$$\dot{x} = -\mathcal{H}_{\sigma(t)}x \tag{7}$$

is exponentially stable.

Let $\Phi = \text{diag}\ (\phi_1, \phi_2, \ldots, \phi_N)$. It can be verified that $\mathcal{H}_{\sigma(t)} = \Phi\mathcal{H}^s_{\sigma(t)}\Phi = \Phi^{-1}\mathcal{H}^s_{\sigma(t)}\Phi$. Thus, the exponential stability of (7) implies that of (6). □

Next, we recall the distributed observer for the leader system (3) over unsigned switching digraphs proposed in [17] as follows:

$$\dot{\eta}_i = S\eta_i + \mu\sum_{j=0}^{N}a_{ij}(t)(\eta_j - \eta_i) \tag{8}$$

where μ is some positive constant and $\eta_0 = v$. It was shown in [17] that, under Assumptions 2.1 and 2.3, for any initial condition $\eta_i(0)$, $i = 0, 1, \ldots, N$, the solution to (8) exists and is such that

$$\lim_{t\to\infty}(\eta_i(t) - v(t)) = 0 \tag{9}$$

That is why (8) is called a distributed observer for the leader system (3). The distributed observer (8) was recently extended to the signed static graph in [2] as follows:

$$\dot{\eta}_i = S\eta_i + \mu\sum_{j=1}^{N}(a_{ij}\eta_j - |a_{ij}|\eta_i) + \mu a_{i0}(\phi_i\eta_0 - \eta_i) \tag{10}$$

where a_{i0} is assumed to be nonnegative.

Here we further propose the distributed observer for the leader system (3) over signed switching digraphs as follows:

$$\dot{\eta}_i = S\eta_i + \mu\sum_{j=0}^{N}\Big(a_{ij}(t)\eta_j - |a_{ij}(t)|\eta_i\Big) \tag{11}$$

Then we have the following result:

Lemma 3.2. *Under Assumptions 2.1 to 2.3, for any initial condition $\eta_i(0)$, $i = 0, 1, \ldots, N$, the solution of (11) exists and is such that*

$$\begin{aligned}&\lim_{t\to\infty}(\eta_i(t) - v(t)) = 0,\ \ i \in \mathcal{V}_1\\ &\lim_{t\to\infty}(\eta_i(t) + v(t)) = 0,\ \ i \in \mathcal{V}_2\end{aligned} \tag{12}$$

Proof. For $i = 0, 1, \ldots, N$, let

$$w_i = \phi_i v \tag{13}$$

Then

$$\dot{w}_i = \phi_i\dot{v} = \phi_i S v = S w_i \tag{14}$$

Let

$$\xi_i(t) \quad = \eta_i(t) - w_i(t),\ \ i = 0, 1, \ldots, N \tag{15}$$

Then

$$\begin{aligned}&\dot{\xi}_i(t) = \dot{\eta}_i(t) - \dot{w}_i(t)\\ &= S(\eta_i(t) - w_i(t)) + \mu\Big(\textstyle\sum_{j=0}^{N}a_{ij}(t)\eta_j(t) - |a_{ij}(t)|\eta_i(t)\Big)\\ &= S\xi_i(t) + \mu\Big(\textstyle\sum_{j=0}^{N}(a_{ij}(t)(\xi_j(t) + w_j(t)) - |a_{ij}(t)|(\xi_i(t) + w_i(t))\Big)\\ &= S\xi_i(t) + \mu\Big(\textstyle\sum_{j=0}^{N}(a_{ij}(t)\xi_j(t) - |a_{ij}(t)|\xi_i(t) + a_{ij}(t)w_j(t) - |a_{ij}(t)|w_i(t))\Big)\end{aligned} \tag{16}$$

Note that

$$a_{ij}(t)w_j(t) - |a_{ij}(t)|w_i(t) = a_{ij}(t)\phi_j v(t) - |a_{ij}(t)|\phi_i v(t) \tag{17}$$

Under Assumption 2.2, if $i, j \in \mathcal{V}_1$ or $i, j \in \mathcal{V}_2$, then $|a_{ij}(t)| = a_{ij}(t)$ and $\phi_j = \phi_i$. Thus,

$$a_{ij}(t)w_j(t) - |a_{ij}(t)|w_i(t) \quad = a_{ij}(t)(\phi_j - \phi_i)v(t) = 0$$

Also, under Assumption 2.2, if $i \in \mathcal{V}_1, j \in \mathcal{V}_2$ or $i \in \mathcal{V}_2, j \in \mathcal{V}_1$, then $|a_{ij}(t)| = -a_{ij}(t)$, $\phi_i = -\phi_j$. Thus,

$$\begin{aligned}a_{ij}(t)w_j(t) - |a_{ij}(t)|w_i(t) &= a_{ij}(t)\phi_j v(t) - |a_{ij}(t)|\phi_i v(t)\\ &= a_{ij}(t)\phi_j v(t) + a_{ij}(t)(-\phi_j)v(t)\\ &= 0\end{aligned}$$

Therefore,

$$\dot{\xi}_i(t) = S\xi_i(t) + \mu\Big(\sum_{j=0}^{N}a_{ij}(t)\xi_j(t) - |a_{ij}(t)|\xi_i(t)\Big) \tag{18}$$

Let $\xi = \text{col}(\xi_1, \ldots, \xi_N)$ and note $\xi_0 = 0$. Then it can be verified that ξ is governed by the following equation:

$$\dot{\xi} = (I_N \otimes S - \mu\mathcal{H}^s_{\sigma(t)} \otimes I_m)\xi \tag{19}$$

Since, under Assumptions 2.2 and 2.3, the linear system (6) is exponentially stable, and the matrix S satisfies Assumption 2.1, by Lemma 2 in [17], the origin of the system (19) is exponentially stable for any $\mu > 0$. Hence, we conclude that $\lim_{t\to\infty}\xi(t) = 0$, which implies (12). □

Remark 3.1. Lemma 3.2 has generalized the distributed observer (10) proposed in [2] in two ways. First, in [2], it is assumed that $a_{i0} \geq 0$ while in (11), we allow a_{i0} to be any real number. Second, if the digraph $\mathcal{G}^s_{\sigma(t)}$ is static, then our result reduces to Proposition 1 of [2]. It is noted that, if the digraph $\mathcal{G}^s_{\sigma(t)}$ is static, then system (19) reduces to the following time-invariant system:

$$\dot{\xi} = (I_N \otimes S - \mu\mathcal{H}^s \otimes I_m)\xi \tag{20}$$

The stability of the Eq. (20) can be easily determined by the locations of the eigenvalues of the matrix $\mathcal{H}^s$. In contrast, here we need to first establish the exponential stability of (6) to conclude the exponential stability of the system (19).

Remark 3.2. Let

$$\bar{\eta}_i = \sum_{j=0}^{N}(a_{ij}(t)\eta_j - |a_{ij}(t)|\eta_i) \tag{21}$$

We claim, under Assumptions 2.1 to 2.3, $\lim_{t\to\infty}\bar{\eta}_i(t)=0$ exponentially. In fact, by Lemma 3.2, $\xi(t)$ and $\dot{\xi}(t)$ converge to zero exponentially. Thus, from (11), for any $\mu>0$,

$$\begin{aligned}\lim_{t\to\infty}\mu\bar{\eta}_i(t) &= \lim_{t\to\infty}(\dot{\eta}_i-S\eta_i)\\ &= \lim_{t\to\infty}\left((\dot{\xi}_i+\dot{w}_i)-S(\xi_i+w_i)\right)\\ &= \lim_{t\to\infty}(Sw_i-Sw_i)=0\end{aligned} \tag{22}$$

exponentially.

4. Leader-following bipartite consensus of multiple EL systems

In this section, we design a class of distributed adaptive control law based on the distributed observer to solve the leader-following bipartite consensus of multiple EL systems over signed switching digraphs. As in [3], we first define the following quantities.

Let

$$\dot{q}_{ri}=FS\eta_i-\alpha(q_i-F\eta_i) \tag{23}$$

where α is a positive constant and

$$\ddot{q}_{ri}=FS\dot{\eta}_i-\alpha(\dot{q}_i-F\dot{\eta}_i) \tag{24}$$

According to Property 3, there exists a regression matrix

$$Y_i=Y_i(q_i,\dot{q}_i,\dot{q}_{ri},\ddot{q}_{ri}) \tag{25}$$

such that

$$Y_i\Theta_i=M_i(q_i)\ddot{q}_{ri}+C_i(q_i,\dot{q}_i)\dot{q}_{ri}+G_i(q_i) \tag{26}$$

Let

$$s_i=\dot{q}_i-\dot{q}_{ri}=\dot{q}_i-FS\eta_i+\alpha(q_i-F\eta_i) \tag{27}$$

Consider the following distributed control law:

$$\begin{aligned}\dot{\eta}_i &= S\eta_i+\mu\textstyle\sum_{j=0}^{N}\left(a_{ij}(t)\eta_j-|a_{ij}(t)|\eta_i\right)\\ s_i &= \dot{q}_i-FS\eta_i+\alpha(q_i-F\eta_i)\\ \dot{\hat{\Theta}}_i &= -\Lambda_iY_i^Ts_i\\ \tau_i &= -K_is_i+Y_i\hat{\Theta}_i\end{aligned} \tag{28}$$

where Λ_i and K_i are positive definite constant matrices. Now, it is ready to give our main result:

Theorem 4.1. *Under Assumptions 2.1–2.3, the distributed control law (28) solves Problem 2.1.*

Proof. The proof is somehow similar to that of [3]. For the presentation to be self-contained, we still provide a detailed proof here. Substituting (28) into (1) yields

$$M_i(q_i)\ddot{q}_i+C_i(q_i,\dot{q}_i)\dot{q}_i+G_i(q_i)=-K_is_i+Y_i\hat{\Theta}_i \tag{29}$$

or equivalently,

$$\begin{aligned}&M_i(q_i)\ddot{q}_i+C_i(q_i,\dot{q}_i)\dot{q}_i+G_i(q_i)-Y_i\Theta_i\\ &=-K_is_i+Y_i\hat{\Theta}_i-Y_i\Theta_i\\ &=-K_is_i+Y_i\tilde{\Theta}_i\end{aligned} \tag{30}$$

where $\tilde{\Theta}_i=\hat{\Theta}_i-\Theta_i$. Using Property 3 in (30) gives

$$\begin{aligned}&M_i(q_i)\ddot{q}_i+C_i(q_i,\dot{q}_i)\dot{q}_i+G_i(q_i)-M_i(q_i)\ddot{q}_{ri}\\ &-C_i(q_i,\dot{q}_i)\dot{q}_{ri}-G_i(q_i)=-K_is_i+Y_i\tilde{\Theta}_i\end{aligned} \tag{31}$$

which can be rewritten as follows:

$$M_i(q_i)\dot{s}_i+C_i(q_i,\dot{q}_i)s_i+K_is_i=Y_i\tilde{\Theta}_i \tag{32}$$

Let

$$\begin{aligned}q&=\mathrm{col}(q_1,\ldots,q_N),\ \dot{q}=\mathrm{col}(\dot{q}_1,\ldots,\dot{q}_N)\\ s&=\mathrm{col}(s_1,\ldots,s_N),\ \tilde{\Theta}=\mathrm{col}(\tilde{\Theta}_1,\ldots,\tilde{\Theta}_N)\end{aligned} \tag{33}$$

and

$$\begin{aligned}M(q)&=\mathrm{diag}(M_1(q_1),\ldots,M_N(q_N))\\ C(q,\dot{q})&=\mathrm{diag}(C_1(q_1,\dot{q}_1),\ldots,C_N(q_N,\dot{q}_N))\\ K&=\mathrm{diag}(K_1,\ldots,K_N)\\ Y&=\mathrm{diag}(Y_1,\ldots,Y_N)\\ \Lambda&=\mathrm{diag}(\Lambda_1,\ldots,\Lambda_N)\end{aligned} \tag{34}$$

Then we have

$$\begin{aligned}Y\tilde{\Theta}&=M(q)\dot{s}+C(q,\dot{q})s+Ks\\ \dot{\tilde{\Theta}}&=-\Lambda Y^Ts\end{aligned} \tag{35}$$

Let

$$V=\tfrac{1}{2}(s^TM(q)s+\tilde{\Theta}^T\Lambda^{-1}\tilde{\Theta}) \tag{36}$$

Then, using Property 2, the derivative of V along the trajectory of the closed-loop system is given by

$$\begin{aligned}\dot{V}&=s^TM(q)\dot{s}+\tfrac{1}{2}s^T\dot{M}(q)s+\tilde{\Theta}^T\Lambda^{-1}\dot{\tilde{\Theta}}\\ &=s^T(-C(q,\dot{q})s-Ks+Y\tilde{\Theta})+\tfrac{1}{2}s^T\dot{M}(q)s+\tilde{\Theta}^T\Lambda^{-1}\dot{\tilde{\Theta}}\\ &=-s^TKs+s^TY\tilde{\Theta}-\tilde{\Theta}^T\Lambda^{-1}\Lambda Y^Ts\\ &=-s^TKs\le 0\end{aligned} \tag{37}$$

Since V is positive definite and $\dot{V}$ is negative semi-definite, s and $\tilde{\Theta}$ are both bounded. To conclude that $\lim_{t\to\infty}s(t)=0$, differentiating (37) gives

$$\ddot{V}=-2s^TK\dot{s} \tag{38}$$

From (27), s is continuous, and

$$\dot{s}_i=\ddot{q}_i-FS\dot{\eta}_i+\alpha(\dot{q}_i-F\dot{\eta}_i) \tag{39}$$

However, since the digraph is switching, $\dot{\eta}_i$ and hence $\dot{s}_i$ are not continuous at switching time instants, we cannot use Barbalat's lemma to conclude $\lim_{t\to\infty}s(t)=0$. Thus, like in [3], we need to appeal to the generalized Barbalat's lemma as can be found in Corollary 1 of [16]. To this end, note that by (11) and (21), (27) can be rewritten as follows:

$$s_i-\mu F\bar{\eta}_i=\dot{q}_i-F\dot{\eta}_i+\alpha(q_i-F\eta_i) \tag{40}$$

(40) is a stable first-order linear differential equation with $q_i-F\eta_i$ as the state and $s_i-\mu F\bar{\eta}_i$ as the input. Since s_i is bounded and $\bar{\eta}_i$ tends to the origin by Remark 3.1, the input $s_i-\mu F\bar{\eta}_i$ is bounded. Thus, we conclude that both $q_i-F\eta_i$ and $\dot{q}_i-F\dot{\eta}_i$ are bounded. Under Assumption 2.1, v and $\dot{v}$ are bounded. Since $\lim_{t\to\infty}(\eta_i(t)-\phi_iv(t))=0$ and $\lim_{t\to\infty}(\dot{\eta}_i(t)-\phi_i\dot{v}(t))=0$, $\eta_i(t)$ and $\dot{\eta}_i(t)$ are bounded. Thus q_i and $\dot{q}_i$ are bounded. Since $\dot{q}_{ri}=FS\eta_i-\alpha(q_i-F\eta_i)$ and $\ddot{q}_{ri}=FS\dot{\eta}_i-\alpha(\dot{q}_i-F\dot{\eta}_i)$, we conclude that $\dot{q}_{ri}$ and $\ddot{q}_{ri}$ are bounded. By (26), Y_i is also bounded.

Since $M(q)$ satisfies Property 1, $M(q)^{-1}$ is bounded for all q. By the first equation of (35), we conclude that $\dot{s}$ is also bounded. Hence, $\ddot{V}$ is a bounded over $t\in[0,\infty)$, that is, there exists a positive real number γ such that

$$\sup_{t_i\le t\le t_{i+1},i=0,1,2,\ldots}|\ddot{V}(t)|\le\gamma \tag{41}$$

Thus, by Corollary 1 of [16], we conclude that $\lim_{t\to\infty}s(t)=0$.

Since both s_i and $\bar{\eta}_i$ tend to 0 as t tends to infinity, from (40), we conclude that both $q_i-F\eta_i$ and $\dot{q}_i-F\dot{\eta}_i$ also tend to 0 as t tends to infinity. Hence,

$$\begin{aligned}\lim_{t\to\infty}(q_i-\phi_iq_0)&=\lim_{t\to\infty}(q_i-F\eta_i)+F(\eta_i-\phi_iv)=0\\ \lim_{t\to\infty}(\dot{q}_i-\phi_i\dot{q}_0)&=\lim_{t\to\infty}(\dot{q}_i-F\dot{\eta}_i)+F(\dot{\eta}_i-\phi_i\dot{v})=0\end{aligned}$$

□

5. An example

In this section, we apply our approach to design a distributed control law of the form (28) to solve the leader-following bipartite consensus problem of four two-link manipulators over a signed switching digraph. The leader-following consensus problem for the

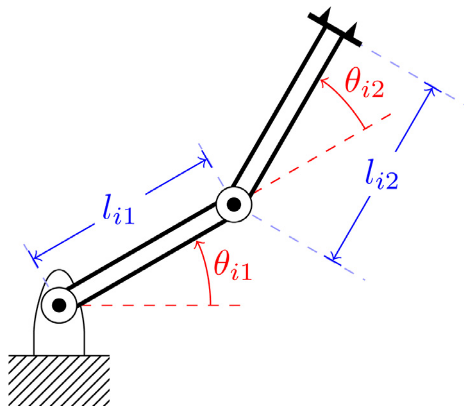

Fig. 1. Two-link manipulator.

Table 1
Numerical values of various parameters.

Notation	Quantity	Value
J_{i1}	Moments of inertia of the first link ($kg \cdot m^2$)	$J_{11} = J_{41} = 0.21$ $J_{21} = 0.19$, $J_{31} = 0.23$
J_{i2}	Moments of inertia of the second link ($kg \cdot m^2$)	$J_{12} = 0.42$, $J_{22} = 0.40$ $J_{32} = 0.39$, $J_{42} = 0.41$
m_{i1}	Masses of the first link (kg)	$m_{11} = 1.02$, $m_{21} = 1.01$ $m_{31} = 0.96$, $m_{41} = 1.04$
m_{i2}	Masses of the second link (kg)	$m_{12} = 1.12$, $m_{22} = 1.07$ $m_{32} = 1.15$, $m_{42} = 1.09$
l_{i1}	Lengths of the first link (m)	1
l_{i2}	Lengths of the second link (m)	1
g	Gravity constant ($m \cdot s^2$)	9.8

same group of two-link manipulators over an unsigned switching digraph was studied in [3].

The two-link manipulator is illustrated in Fig. 1, and the motion equation of each manipulator is taken from page 69 of [8] and is given as follows:

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) = \tau_i, i = 1, 2, \dots, 4 \tag{42}$$

where $q_i = \mathrm{col}(\theta_{i1}, \theta_{i2})$,

$$M_i(q_i) = \begin{bmatrix} a_{i1} + a_{i2} + 2a_{i3}\cos(\theta_{i2}) & a_{i2} + a_{i3}\cos(\theta_{i2}) \\ a_{i2} + a_{i3}\cos(\theta_{i2}) & a_{i2} \end{bmatrix}$$

$$C_i(q_i, \dot{q}_i) = \begin{bmatrix} -a_{i3}\sin(\theta_{i2})\dot{\theta}_{i2} & -a_{i3}\sin(\theta_{i2})(\dot{\theta}_{i1} + \dot{\theta}_{i2}) \\ a_{i3}\sin(\theta_{i2})\dot{\theta}_{i1} & 0 \end{bmatrix}$$

$$G_i(q_i) = \begin{bmatrix} a_{i4}g\cos(\theta_{i1}) + a_{i5}g\cos(\theta_{i1} + \theta_{i2}) \\ a_{i5}g\cos(\theta_{i1} + \theta_{i2}) \end{bmatrix}$$

and $a_{i1} = J_{i1} + m_{i2}l_{i1}^2$, $a_{i2} = J_{i2} + 0.25m_{i2}l_{i2}^2$, $a_{i3} = 0.5m_{i2}l_{i1}l_{i2}$, $a_{i4} = (0.5m_{i1} + m_{i2})l_{i1}$, $a_{i5} = 0.5m_{i2}l_{i2}$.

The numerical values of various parameters are listed in Table 1.

Let $\Theta_i = \mathrm{col}(a_{i1}, a_{i2}, a_{i3}, a_{i4}, a_{i5})$. With $x = \mathrm{col}(x_1, x_2)$ and $y = \mathrm{col}(y_1, y_2)$, it can be verified that

$$M_i(q_i)x + C_i(q_i, \dot{q}_i)y + G_i(q_i) = Y_i(q_i, \dot{q}_i, x, y)\Theta_i \tag{43}$$

where $Y_i = \begin{bmatrix} Y_{i11} & Y_{i12} & Y_{i13} & Y_{i14} & Y_{i15} \\ Y_{i21} & Y_{i22} & Y_{i23} & Y_{i24} & Y_{i25} \end{bmatrix}$ and

$Y_{i11} = x_1, \quad Y_{i12} = x_1 + x_2$
$Y_{i13} = 2\cos(\theta_{i2})x_1 + \cos(\theta_{i2})x_2 - \sin(\theta_{i2})\dot{\theta}_{i2}y_1 - \sin(\theta_{i2})(\dot{\theta}_{i1} + \dot{\theta}_{i2})y_2$
$Y_{i14} = g\cos(\theta_{i1}), \quad Y_{i15} = g\cos(\theta_{i1} + \theta_{i2})$
$Y_{i21} = 0, \quad Y_{i22} = x_1 + x_2, \quad Y_{i23} = \theta_{i2}x_1 + \sin(\theta_{i2})\dot{\theta}_{i1}y_1$
$Y_{i24} = 0, \quad Y_{i25} = g\cos(\theta_{i1} + \theta_{i2})$

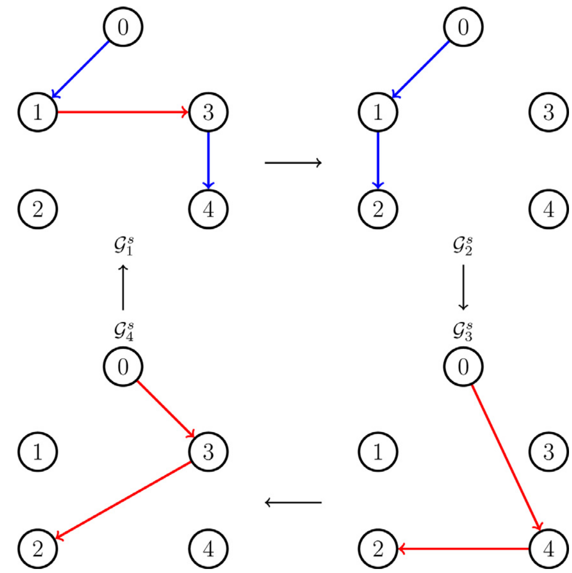

Fig. 2. Structurally balanced signed switching digraph $\mathcal{G}_{\sigma(t)}^s$.

The reference signal is given by

$$q_0(t) = \begin{bmatrix} \frac{\pi}{6}\cos(\pi t) + \frac{\pi}{2} \\ \frac{2\pi}{3}\cos(\pi t) \end{bmatrix} \tag{44}$$

which can be generated by the exosystem of the form (3) with

$$S = \begin{bmatrix} 0 & \pi & 0 \\ -\pi & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad F = \begin{bmatrix} 0 & \frac{\pi}{6} & \frac{\pi}{2} \\ 0 & \frac{2\pi}{3} & 0 \end{bmatrix} \tag{45}$$

It can be verified that Assumption 2.1 is satisfied.

The signed switching digraph is shown in Fig. 2 with the switching signal being defined as follows:

$$\sigma(t) = \begin{cases} 1, & sT \le t < (s + 0.25)T \\ 2, & (s + 0.25)T \le t < (s + 0.5)T \\ 3, & (s + 0.5)T \le t < (s + 0.75)T \\ 4, & (s + 0.75)T \le t < (s + 1)T \end{cases} \tag{46}$$

where $T = 0.5sec$ and $s = 0, 1, 2, \dots$. For convenience, all the non-zero elements of the adjacent matrix are assumed to be either 1 (blue lines) or -1 (red lines).

It can be verified that Assumptions 2.2 and 2.3 are satisfied. The matrices associated with the four digraphs $\mathcal{G}_i^s$ for $i = 1, 2, 3, 4$ are given as follows:

$$\mathcal{H}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}, \quad \mathcal{H}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$
$$\mathcal{H}_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathcal{H}_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Thus, a distributed control law of the form (28) can be designed with $\mu = 10, \alpha = 10, K_i = 20I_2, \Lambda_i = 10I_5$.

The performance of the control law is simulated with randomly chosen initial conditions. Figs. 3 and 4 show the time responses of the tracking errors and the generalized coordinates of each EL system. It can be seen that the objective of the leader-following bipartite consensus is achieved satisfactorily.

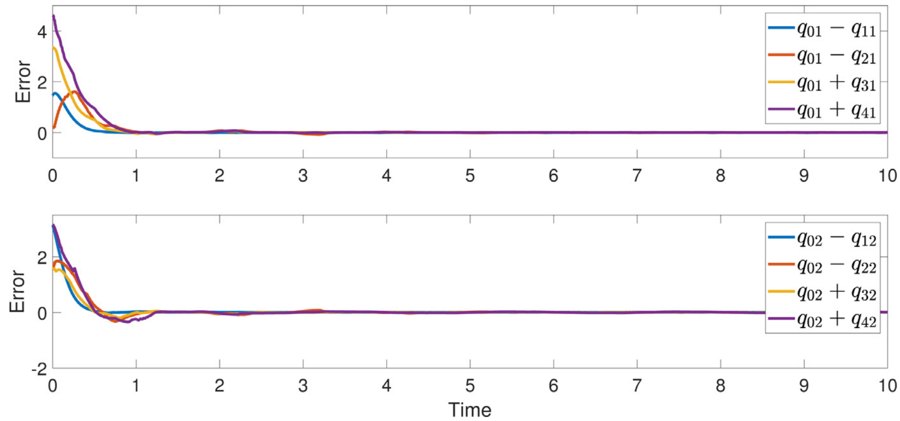

Fig. 3. Tracking error.

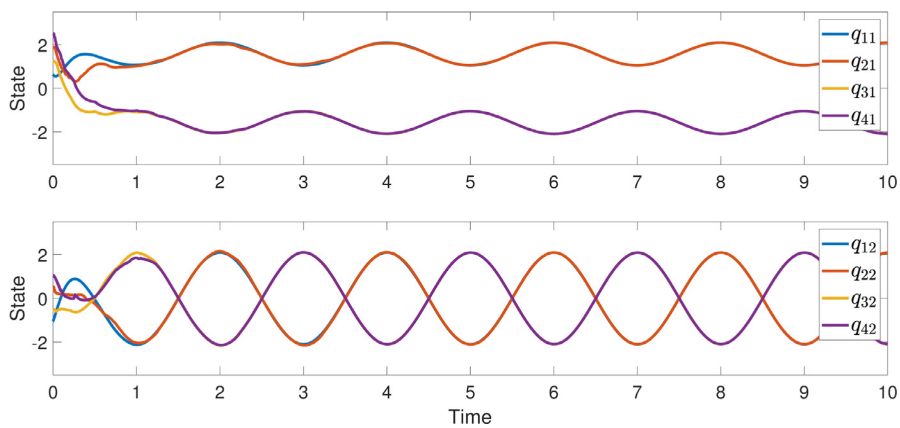

Fig. 4. Generalized coordinate.

6. Conclusion

In this paper, we have investigated the leader-following bipartite consensus problem of multiple uncertain EL systems over signed switching networks. We have shown that, under some standard assumptions, the leader-following bipartite consensus problem of multiple uncertain EL systems over signed switching networks is solvable by a class of distributed adaptive control law. Our result has been applied to the leader-following bipartite consensus problem of a group of two-link robot manipulators. The key technique used in this paper is the distributed observer for a linear leader system over signed switching communication networks. Like the unsigned case, the distributed observer approach is a quite general approach and can be applied to solve some other bipartite control problems such as the bipartite attitude synchronization problem of multiple rigid spacecraft systems and the bipartite output regulation problem of linear multi-agent systems over signed switching graphs. These problems will be considered in our future research.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

Dong Liang: Conceptualization, Methodology, Software, Formal analysis, Investigation, Writing - original draft. **Jie Huang:** Validation, Writing - review & editing, Supervision, Funding acquisition.

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Appendix

A signed digraph is represented by $\mathcal{G}^s = (\mathcal{V}, \mathcal{E}, \mathcal{A}^s)$ where $\mathcal{V} = \{0, 1, 2, \ldots, N\}$ is the node set with the node 0 as the leader and node i, $i = 1, 2, \ldots, N$, as followers, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set, and $\mathcal{A}^s = [a_{ij}] \in \mathbb{R}^{(N+1)\times(N+1)}$ is the adjacency matrix of the digraph $\mathcal{G}^s$. For $i, j = 0, 1, \ldots, N$, $j \neq i$, the notation (j, i) denotes an edge of $\mathcal{E}$ from node j to node i, and the elements a_{ij} of the matrix $\mathcal{A}^s$ are such that $a_{ii} = 0$, and, for $i \neq j$, $a_{ij} \neq 0$ if and only if $(j, i) \in \mathcal{E}$. Let $\mathcal{N}_i = \{j \mid (j, i) \in \mathcal{E}\}$, which is called the neighbor set of the node i. The edge is said to be undirected if $(i, j) \in \mathcal{E}$ implies and is implied by $(j, i) \in \mathcal{E}$. If there exists a set of edges $(i_1, i_2), (i_2, i_3), \ldots, (i_{k-1}, i_k)$ in a directed graph (digraph), then this set of edges is called a directed path from node i_1 to node i_k. In such a case, node i_k is said to be reachable from node i_1. A digraph is said to contain a spanning tree if there exists a node i that can reach all other nodes. In such a case, the node i is said to be the root of the spanning tree. A pair of edges $(i, j), (j, i) \in \mathcal{E}$ sharing the same nodes is called a digon. Throughout this paper, we assume that $a_{ij}a_{ji} \geq 0$, which means that none of any digons can admit opposite signs. The signed digraph is structurally balanced if and only if there exists a bipartition of $\mathcal{V}$ into two nonempty subsets $\mathcal{V}_1$ and $\mathcal{V}_2$ with $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$ and $\mathcal{V}_1 \cap \mathcal{V}_2 = \varnothing$, such that, $a_{ij} \leq 0$, $\forall v_i \in \mathcal{V}_p$, $v_j \in \mathcal{V}_q$ where $p, q \in \{1, 2\}$, $p \neq q$; otherwise, $a_{ij} \geq 0$. A time function $\sigma(t) : [0, +\infty) \to \mathcal{P} = \{1, 2, \ldots, n_0\}$ with some positive integer n_0 is said to be a piecewise constant switching signal if there exists a sequence $\{t_j : j = 0, 1, 2, \ldots\}$ satisfying $t_0 = 0, t_{j+1} - t_j \geq \tau$ for some positive constant τ such that, for all $t \in [t_j, t_{j+1})$, $\sigma(t) = p$ for some $p \in \mathcal{P}$. $\mathcal{P}$ is called the switching index set, t_j is called the switching instant, and τ is called a dwell time. Given a piecewise constant switching signal $\sigma(t)$ and a set of n_0 static signed digraphs $\mathscr{G}_l^s = (\mathscr{V}, \mathscr{E}_l, \mathscr{A}_l^s)$, $l = 1, \ldots, n_0$. The signed time-varying digraph $\mathcal{G}_{\sigma(t)}^s = (\mathcal{V}, \mathcal{E}_{\sigma(t)}, \mathcal{A}_{\sigma(t)}^s)$ is called a signed switching digraph. The signed switching digraph is structurally balanced if and only if there exists a bipartition of $\mathcal{V}$ into two nonempty subsets $\mathcal{V}_1$ and $\mathcal{V}_2$ with $\mathcal{V}_1\cup\mathcal{V}_2=\mathcal{V}$ and $\mathcal{V}_1\cap\mathcal{V}_2=\varnothing$, such that, for $l=1,\cdots,n_0$, the elements of $\mathcal{A}^{s}_l$ denoted by a_{ij}^l satisfy $a_{ij}^l \leq 0$, $\forall v_i \in \mathcal{V}_p$, $v_j \in \mathcal{V}_q$ where $p,q \in \{1,2\}$, $p \neq q$; otherwise, $a_{ij}^l \geq 0$. The Laplacian matrix $\mathcal{L}_{\sigma(t)}^s$ of a signed switching digraph $\mathcal{G}_{\sigma(t)}^s$ is defined by $\mathcal{L}_{\sigma(t)}^s = \mathcal{C}_{\sigma(t)}^s - \mathcal{A}_{\sigma(t)}^s$ where the degree matrix $\mathcal{C}_{\sigma(t)}^s$ is diagonal and $[\mathcal{C}_{\sigma(t)}^s]_i = \sum_{j \in \mathcal{N}_i(t)} |a_{ij}(t)|$.

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