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Distributed optimal control law design for a class of higher order linear multi-agent systems and its application to Euler-Lagrangian systems

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Abstract: The paper is concerned with the distributed optimization problem for a class of higher order linear multi-agent systems. The target is to cooperatively optimize a performance function formed by a sum of convex local cost functions. To deal with the higher order dynamic, the integrator backstepping idea is introduced to break the integrator chains. Moreover, the adaptive gain technique is proposed such that the control law design can be performed in a fully distributed way, i.e., without utilizing the information of Laplacian matrix. Furthermore, the proposed scheme is applied to the control law design for the consensus of networked uncertain Euler-Lagrangian systems under optimization constraint. The effectiveness of the proposed methods is validated through some numerical simulations of a group of networked uncertain robot manipulators.

Key Words: Consensus, Distributed optimization, Higher order system, Adaptive gain, Integrator backstepping, Euler-Lagrangian system.

1 Introduction

The consensus or coordination problem of a group of dynamical systems or the so-called multi-agent system has received a lot of attention since the pioneering work [1] and [2]. This is due to the fact in the modern industrial society the multiple agents are required to carry out task in a cooperative way. For example, unmanned air vehicles (UAVs), rendezvous and flocking, sensor networks and formation of robots. With the progress of the research on the multi-agent systems, the researchers begin to focus on the property of the consensus value, for example it is required to minimize the sum of individual convex functions. This can be seen as the distributed optimization problem (DOP) and has many applications in modern society such as wireless resource allocation [3] and machine learning [4].

The distributed optimization or computation problem was firstly studied in [5] and has received renewed interest in the recent years. In the seminal work [6], the gradient based algorithm is proposed and the states of the single-integrator multi-agent system reach a consensus value that is the solution of a convex optimization problem. Build on this framework, many algorithms are proposed to solve the distributed optimization problem, just name a few [7]-[12] and the references therein. From the viewpoint of control of the multi-agent system, the distributed optimization problem can be reformulated as the controller design problem such that the multi-agent system can reach consensus and the target cost function is minimized on the consensus value. Note that all the multi-agent models in [7]-[12] are single-integrators. In [14] and [15] the DOP for the double-integrator multi-agent systems are considered. However, the real systems may have higher order dynamics such as robot systems. The key point for designing the control law for the higher order system is to break the control design problem into a sequence of controller design problem for lower order or even scalar subsystems. When it comes to the DOP of higher order linear multi-agent systems the control design becomes more challenging due to the gradient term. By introducing the integrator backstepping idea and the novel gradient estimation term, the higher order linear multi-agent system can reach consensus under optimization constraint.

The Euler-Lagrangian (EL) system is an important nonlinear model, which can represent many practical systems such as mechanical dynamics and power systems [22]. It is no surprise that the multi-agent system with agents in the form of Lagrangian systems has received a lot of attention [23, 24] and the references therein. The consensus problem of EL systems under optimization constraint is considered in [16]. However, the controller gain should be chosen based on the information of Laplacian matrix. Therefore, the control law design in [16] is not in a strictly distributed sense. In this paper the adaptive gain technique is introduced such that control gain is a time-varying and non-decreasing function. Therefore, the control law design is independent of the Laplacian matrix, i.e., in a fully distributed way.

In [16], the graph is required to be undirected. The extension from undirected graph to the more general directed graph received attention in the recent years. To our best knowledge the first result on the directed graph of distributed optimization problem of continuous system is given in [9] where the graph is allowed to be weight balanced. A recent result [19] that deals with the single-integrator multi-agent system considers the directed graph. An optimal consensus problem for higher-order multi-agent systems is considered in [25] under directed graph. The main contributions of the paper are summarized in the follows:

- Firstly, the novel distributed control laws are proposed such that the higher order linear multi-agent system can reach consensus under optimization constraint. The integrator backstepping idea is introduced to break the integrator chain into two interconnected subsystems. With the help of novel control law, the origin of the two subsystems is globally asymptotically stable and hence

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the distributed optimization problem is solved.

- Secondly, the adaptive gain technique is introduced such that the controller gain is time-varying and non-decreasing. Therefore, the control law design can be performed in a fully distributed way, i.e., without using the maximum eigenvalue of the Laplacian matrix that is utilized in [16].
- Finally, the graph only requires to be weight balanced in the current paper. For the consensus problem of EL systems under optimization constraint such as in [16], the graph is assumed to be undirected.

The organization of the paper is as follows. Section II gives the formulation of the DOP with related preliminaries. Section III presents the distributed control law design for a class of higher order integrator chain system. Then, in Section IV the proposed control scheme is applied to the consensus problem of networked uncertain EL systems under optimization constraint. The simulation results on a group of uncertain robot manipulators are given in Section V. Finally, Section VI gives the concluding remarks.

2 Preliminaries and Problem formulation

2.1 Notations and definitions

Throughout the paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional real vector space and real matrix space of size $n\times m$, respectively, $\|\cdot\|$ is the Euclidean norm of a vector. The symbol $\otimes$ denotes the Kronecker product. The property $(A\otimes B)(C\otimes D)=(AC)\otimes(BD)$ holds for the matrices A,B,C,D with proper dimensions. A^T and A^{-1} mean the transpose and the inverse (if invertible) of the matrix A, respectively. I_n represents a n-dimensional identity matrix. $\mathbf{1}_n$ means a n-dimensional column vector with each entry being 1, i.e., $\mathbf{1}_n=[1\ 1\ ,\cdots,1]^T$. diag$[x_1,x_2,\cdots,x_n]$ means the $n\times n$ diagonal matrix with its diagonal elements as $x_1,x_2,\cdots,x_n$. For a differentiable function $f(\cdot):\mathbb{R}^n\to\mathbb{R}$, $\nabla f(\cdot)$ means the gradient of $f(\cdot)$.

2.2 Graph theory

The communication topology of the multi-agent system with N agents is described by a triplet $\mathcal{G}=\{\mathcal{V},\mathcal{E},\mathcal{A}\}$, where $\mathcal{V}=\{1,2,\cdots,N\}$ is the set of agents or nodes, $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ is the edge set, and $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{N\times N}$ is a weighted adjacency matrix. If the edge $(i,j)\in\mathcal{E}$, agent i can obtain information from agent j and hence $a_{ij}>0$. Otherwise, agent i cannot obtain information from agent j and $a_{ij}=0$. A directed path from node i_1 to node i_n is a sequence of ordered edges in the form of $(i_1,i_2),(i_2,i_3),\cdots,(i_{n-1},i_n)$ where $i_1,i_2,\cdots,i_n\in\mathcal{V}$ and $(i_1,i_2),(i_2,i_3),\cdots,(i_{n-1},i_n)\in\mathcal{E}$. A directed graph is called strongly connected if there exists a directed path connecting every pair of nodes in $\mathcal{V}$. The Laplacian matrix $L=[l_{ij}]\in\mathbb{R}^{N\times N}$ of a directed graph is defined as $l_{ii}=\sum_{j=1}^N a_{ij}$ and $l_{ij}=-a_{ij}, i\neq j$. A directed graph is called weight balanced if $\sum_j a_{ji}=0$ or $\mathbf{1}_N^T L=0$. Introduce the following lemmas.

Lemma 2.1 *[21] When the graph $\mathcal{G}$ is strongly connected and the matrix $L\in\mathbb{R}^{N\times N}$ is the corresponding Laplacian matrix, L has a simple zero eigenvalue corresponding to the right eigenvector $\mathbf{1}_N$ and all nonzero eigenvalues have positive real parts.*

Lemma 2.2 *[21] Let $r=[r_1,r_2,\cdots,r_n]^T$, $r_i>0, i=1,\cdots,n$, be the left eigenvector of the Laplacian matrix L corresponding to the zero eigenvalue and $R=$ diag$[r_1,r_2,\cdots,r_n]$. Then, $\min_{\iota^T x=0, x\neq 0}\frac{x^T\check{L}x}{x^Tx}>\frac{\lambda_2(\check{L})}{N}$ where $\check{L}=RL+L^TR$, ι is any vector with positive entries.*

Throughout the paper, the following assumption is made on the multi-agent system communication graph topology.

Assumption 2.1 *The communication graph $\mathcal{G}$ is strongly connected and weight balanced.*

2.3 Problem formulation

The paper considers the linear multiagent system with each agent taking the following integrator chain form:

$$\begin{aligned}\dot{x}_{1i}&=x_{2i}\\ \dot{x}_{2i}&=x_{3i}\\ &\vdots\\ \dot{x}_{\rho i}&=u_i,\quad i=1,2,\cdots,n,\end{aligned}\tag{1}$$

where $x_{ji},u_i\in\mathbb{R}^m$ are the states and input, respectively.

The control target is to design the distributed control laws $u_i, i=1,\cdots,n$, for the multi-agent system (1) such that the states reach consensus, i.e., $\lim_{t\to\infty}(x_{1i}-x^\star)=0, \forall i\in\{1,2,\cdots,N\}$, where $x^\star$ is a constant vector which is the so-called group decision value. Furthermore, the decision value $x^\star$ should also satisfy the following optimal condition

$$x^\star=\arg\min\sum_{i=1}^N f_i(x),\tag{2}$$

where $f_i(x):\mathbb{R}^m\to\mathbb{R}$ is the convex cost function. It is assumed that $f_i(\cdot)$ is only accessible by the i-th agent. Moveover, $\nabla f_i(\cdot)$ has linear form with the homegeneous positive slope. Therefore, $x^\star$ is the unique global minimizer of the optimization problem.

3 Distributed control law design for (1)

In this section we will propose the distributed controller design for the multiagent system (1). Throughout the analysis of the section, it is assumed the $x_{ji}\in\mathbb{R}$ for the brevity of the presentation. However, the current results can be trivially extended to the case $x_{ji}\in\mathbb{R}^m, m>1$ by including Kronecker product. Introduce the following distributed controller

$$
\begin{aligned}
\dot{\zeta}_i =&(\hat{k}_i + l_i)\sum_{j=1}^{N} a_{ij}[\delta_1(x_{1i} - x_{1j}) + \cdots + \delta_{\rho-1}(x_{\rho-1,i} \\
&- x_{\rho-1,j}) + (x_{\rho i} - x_{\rho j})] \\
u_i =&-(\hat{k}_i + l_i)\sum_{j=1}^{N} a_{ij}[\delta_1(x_{1i} - x_{1j}) + \cdots + \delta_{\rho-1}(x_{\rho-1,i} \\
&- x_{\rho-1,j}) + (x_{\rho i} - x_{\rho j})] - \nabla f_i(x_{1i}) - \sum_{j=1}^{N} a_{ij}(\zeta_i - \zeta_j) \\
\dot{\hat{k}}_i =&[\sum_{j=1}^{N} a_{ij}\delta_1(x_{1i} - x_{1j}) + \cdots + \sum_{j=1}^{N} a_{ij}\delta_{\rho-1}(x_{\rho-1,i} \\
&- x_{\rho-1,j}) + \sum_{j=1}^{N} a_{ij}(x_{\rho i} - x_{\rho j})][\sum_{j=1}^{N} a_{ij}\delta_1(x_{1i} - x_{1j}) \\
&+ \cdots + \sum_{j=1}^{N} a_{ij}\delta_{\rho-1}(x_{\rho-1,i} - x_{\rho-1,j}) \\
&+ \sum_{j=1}^{N} a_{ij}(x_{\rho i} - x_{\rho j})]
\end{aligned} \tag{3}
$$

where $l_i = \dot{\hat{k}}_i$, $\delta_1, \delta_2, \cdots, \delta_{\rho-1}$ are positive constants that are chosen such that the polynomial $s^{\rho-1} + \delta_{\rho-1}s^{\rho-2} + \cdots + \delta_2 s + \delta_1$ is Hurwitz stable. Therefore, the closed-loop system consisting of (1) and (3) can be written as

$$
\begin{aligned}
\dot{x}_1 =&x_2 \\
\dot{x}_2 =&x_3 \\
&\vdots \\
\dot{x}_\rho =&-(\hat{k} + l)L(\delta_1 x_1 + \delta_2 x_2 + \cdots + \delta_{\rho-1}x_{\rho-1} + x_\rho) \\
&- \nabla f(x_1) - L\zeta \\
\dot{\zeta} =&(\hat{k} + l)L(\delta_1 x_1 + \delta_2 x_2 + \cdots + \delta_{\rho-1}x_{\rho-1} + x_\rho) \\
\dot{\hat{k}}_i =&[\sum_{j=1}^{N} a_{ij}\delta_1(x_{1i} - x_{1j}) + \cdots + \sum_{j=1}^{N} a_{ij}\delta_{\rho-1}(x_{\rho-1,i} - x_{\rho-1,j}) \\
&+ \sum_{j=1}^{N} a_{ij}(x_{\rho i} - x_{\rho j})][\sum_{j=1}^{N} a_{ij}\delta_1(x_{1i} - x_{1j}) + \cdots \\
&+ \sum_{j=1}^{N} a_{ij}\delta_{\rho-1}(x_{\rho-1,i} - x_{\rho-1,j}) + \sum_{j=1}^{N} a_{ij}(x_{\rho i} - x_{\rho j})]
\end{aligned} \tag{4}
$$

where $x_j = [x_{j1}, x_{j2}, \cdots, x_{jn}], j = 1, \cdots, \rho$, $\hat{k} = \mathrm{diag}[\hat{k}_1, \hat{k}_2, \cdots, \hat{k}_n]$ and $l = \mathrm{diag}[l_1, l_2, \cdots, l_n]$.

Lemma 3.1 *If the graph is strongly connected and $\hat{k}_i(0) > 0$ for $i = 1, \cdots, n$, the equilibrium point of the closed-loop system (4) is on the set $\{x_1 = \mathbf{1}_n \otimes x^\dagger, x_2 = \cdots = x_\rho = 0, \zeta = \zeta^\dagger, \hat{k} = k^\dagger\}$, where $x^\dagger$ is a constant scalar, $\zeta^\dagger, k^\dagger \in \mathbb{R}^n$ are constant vectors.*

Proof: Suppose the equilibrium point of the closed-loop system (4) is $\{\bar{x}_1, \cdots, \bar{x}_\rho, \bar{\zeta}, \bar{k}\}$. It is inferred from (4) that $0 = \bar{x}_2 = \cdots = \bar{x}_\rho$ and $0 = \delta_1(\hat{k} + l)Lx_1$. Therefore, $x_1 = \mathbf{1}_n \otimes x^\dagger$ for some $x^\dagger \in \mathbb{R}$ due to Lemma 2.1 and the fact that $\delta_1(\hat{k} + l)$ is a positive definite matrix. □

Define $\tilde{x}_1 = x_1 - \mathbf{1}_N \otimes x^\star$, $\tilde{x}_2 = x_2$ and $\tilde{\zeta} = \zeta - [\zeta_1^\star, \cdots, \zeta_N^\star]^T$ where $\zeta_i^\star$ is defined as $\sum_{j=1}^{N} a_{ij}(\zeta_i^\star - \zeta_j^\star) = -\nabla f_i(x^\star)$. Thus, (4) can be transformed to the following system

$$
\begin{aligned}
\dot{\tilde{x}}_1 =&\tilde{x}_2 \\
\dot{\tilde{x}}_2 =&\tilde{x}_2 \\
&\vdots \\
\dot{\tilde{x}}_\rho =&-(\hat{k} + l)L(\delta_1\tilde{x}_1 + \delta_2\tilde{x}_2 + \cdots + \delta_{\rho-1}\tilde{x}_{\rho-1} + \tilde{x}_\rho) \\
&- g(x_1, x^\star) - L\tilde{\zeta} \\
\dot{\tilde{\zeta}} =&(\hat{k} + l)L(\delta_1\tilde{x}_1 + \delta_2\tilde{x}_2 + \cdots + \delta_{\rho-1}\tilde{x}_{\rho-1} + \tilde{x}_\rho) \\
\dot{\hat{k}}_i =&[\sum_{j=1}^{N} a_{ij}\delta_1(\tilde{x}_{1i} - \tilde{x}_{1j}) + \cdots + \sum_{j=1}^{N} a_{ij}\delta_{\rho-1}(\tilde{x}_{\rho-1,i} - \tilde{x}_{\rho-1,j}) \\
&+ \sum_{j=1}^{N} a_{ij}(\tilde{x}_{\rho i} - \tilde{x}_{\rho j})][\sum_{j=1}^{N} a_{ij}\delta_1(\tilde{x}_{1i} - \tilde{x}_{1j}) \\
&+ \cdots + \sum_{j=1}^{N} a_{ij}\delta_{\rho-1}(\tilde{x}_{\rho-1,i} - \tilde{x}_{\rho-1,j}) + \sum_{j=1}^{N} a_{ij}(\tilde{x}_{\rho i} - \tilde{x}_{\rho j})]
\end{aligned} \tag{5}
$$

where $g(x_1, x^\star) = \nabla f(x_1) - \nabla f(x^\star)$.

Next a proposition is given to show that the origin is an asymptotically stable equilibrium point of (5).

Proposition 3.1 *Under Assumption 2.1 and $\hat{k}_i(0) > 0, i = 1, \cdots, n$, the equilibrium point $\{\tilde{x}_1 = \cdots = \tilde{x}_\rho = 0, \tilde{\zeta} = \bar{\zeta}, \hat{k} = \bar{k}\}$ where $\bar{\zeta}, \bar{k} \in \mathbb{R}^n$ are constant diagonal matrices, is an asymptotically stable equilibrium point of (5).*

Proof: The proof is omitted due to space limit. □

The results in Lemma 2.1 and Proposition 3.1 are summarized in the following theorem.

Theorem 3.1 *Under Assumption 2.1, the consensus problem of the multi-agent system (1) under the optimization constraint (2) can be solved by the distributed control law (3) provided $\hat{k}_i(0) > 0, i = 1, \cdots, n$.*

It is worth to note that when the relative degree $\rho = 2$, the system (1) is in the following double-integrator form:

$$
\begin{aligned}
\dot{x}_{1i} =&x_{2i} \\
\dot{x}_{2i} =&u_i, \quad i = 1, 2, \cdots, n,
\end{aligned} \tag{6}
$$

Therefore, the control law (3) can be designed in the following form:

$$
\begin{aligned}
\dot{\zeta}_i =& (\hat{k}_i + l_i) \sum_{j=1}^{N} a_{ij}[(x_{1i} - x_{1j}) + (x_{2i} - x_{2i})] \\
u_i =& -(\hat{k}_i + l_i) \sum_{j=1}^{N} a_{ij}[(x_{1i} - x_{1j}) + (x_{2i} - x_{2i})] \\
& - \nabla f_i(x_i) - \sum_{j=1}^{N} a_{ij}(\zeta_i - \zeta_j) \\
\dot{\hat{k}}_i =& [\sum_{j=1}^{N} a_{ij}(x_{1i} - x_{1j}) + \sum_{j=1}^{N} a_{ij}(x_{2i} - x_{2j})][\sum_{j=1}^{N} a_{ij}(x_{1i} \\
& - x_{1j}) + \sum_{j=1}^{N} a_{ij}(x_{2i} - x_{2j})]
\end{aligned} \tag{7}
$$

where $l_i = \dot{\hat{k}}_i$. Thus, the following corollary is achieved.

Corollary 3.1 *Under Assumption* 2.1*, the consensus problem of the multi-agent system* (6) *under the optimization constraint* (2) *can be solved by the distributed control law* (7) *provided* $\hat{k}_i(0) > 0, i = 1, \cdots, n$.

4 Application to networked Euler-Lagrangian systems

This section considers the distributed control law design for a team of networked uncertain Euler-Lagrangian (EL) systems subject to optimization constraint as in (2). Each EL system can be modeled as the following form [22]:

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) = \tau_i, i = 1, \cdots, n \tag{8}$$

where $q_i \in \mathbb{R}^m$ is the generalized configuration coordinates, $M_i(q_i) \in \mathbb{R}^{m\times m}$ is the positive definite inertia matrix, $C_i(q_i, \dot{q}_i)\dot{q}_i$ are the Coriolis and centrifugal forces, $G_i(q_i)$ are the forces due to the potential field, $\tau_i \in \mathbb{R}^m$ is the vector of external forces acting on the system. Due to the results in [22], the EL system (8) satisfies the following property:

P1. $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew symmetric.

P2. The matrices $M_i(q_i)$, $C_i(q_i, \dot{q}_i)$ and the vector $G_i(q_i)$ depend linearly on a column vector θ_i of unknown constant parameters.

P2 implies that:

$$M_i(q_i)x + C(q_i, \dot{q}_i)y + G_i(q_i) = \Lambda_i(q_i, \dot{q}_i, x, y)\theta_i \tag{9}$$

where $\Lambda_i(q_i, \dot{q}_i, x, y)$ is a matrix of known functions. Before presenting the distributed control law, the following auxiliary variables are defined as in [22]:

$$\dot{\hat{q}}_i = x_{2i} - (q_i - x_{1i}), i = 1, \cdots, n, \tag{10}$$

where x_{1i} and x_{2i} are the states of auxiliary double-integrator systems (6). Therefore, it is easily achieved by (9) that

$$M_i(q_i)\ddot{\hat{q}}_i + C(q_i, \dot{q}_i)\dot{\hat{q}}_i + G_i(q_i) = \Lambda_i(q_i, \dot{q}_i, \ddot{\hat{q}}_i, \dot{\hat{q}}_i)\theta_i \tag{11}$$

Now we are ready to present the following distributed control law:

$$\tau_i = -\epsilon_i[(\dot{q}_i - x_{2i}) + (q_i - x_{1i})] + \Lambda_i(q_i, \dot{q}_i, \ddot{\hat{q}}_i, \dot{\hat{q}}_i)\hat{\theta}_i, \tag{12a}$$

$$\dot{\hat{\theta}}_i = -\Lambda_i^T(q_i, \dot{q}_i, \ddot{\hat{q}}_i, \dot{\hat{q}}_i)[(\dot{q}_i - x_{2i}) + (q_i - x_{1i})] \tag{12b}$$

$$\dot{x}_{1i} = x_{2i} \tag{12c}$$

$$\dot{x}_{2i} = -(\hat{k}_i + l_i)e_i - \nabla f_i(x_i) - \sum_{j=1}^{N} a_{ij}(\zeta_i - \zeta_j) \tag{12d}$$

$$\dot{\zeta}_i = (\hat{k}_i + l_i)e_i \tag{12e}$$

$$\dot{\hat{k}}_i = e_i^2 \tag{12f}$$

where $\epsilon_i > 0$ is the design parameter and $e_i = \sum_{j=1}^{N} a_{ij}[(x_{1i} - x_{1j}) + (x_{2i} - x_{2i})]$, $\hat{\theta}_i$ is the estimation of θ_i and $l_i = \dot{\hat{k}}_i$.

Remark 4.1 (12) *mainly consists of two parts.* $(12a)$ *and* $(12b)$ *are the tracking controller such that* q_i *and* $\dot{q}_i$ *track* x_{1i} *and* x_{2i}*, respectively.* $(12c) - (12f)$ *is the auxiliary double-integrator system that* $\lim_{t\to\infty} x_{1i} = x^\star$ *based on the results in Corollary 3.1.*

Theorem 4.1 *Under Assumption* 2.1*, the consensus problem of the networked Euler-Lagrangian system* (8) *under the optimization constraint* (2) *can be solved by the distributed control law* (12) *provided* $\hat{k}_i(0) > 0, i = 1, \cdots, n$.

Proof: Putting the tracking control law $(12a)$ and $(12b)$ in (8) and we have that

$$
\begin{aligned}
M(q)\dot{s} =& \Lambda(q, \dot{q}, \ddot{\hat{q}}, \dot{\hat{q}})\tilde{\theta} - \epsilon s - C(q, \dot{q})s \\
\dot{\tilde{\theta}} =& -\Lambda^T(q, \dot{q}, \ddot{\hat{q}}, \dot{\hat{q}})s
\end{aligned} \tag{13}
$$

where $s = [s_1^T \quad s_2^T \quad \cdots \quad s_n^T]^T$ and $s_i = (\dot{q}_i - x_{2i}) + (q_i - x_{1i})$, $M(q) = \text{diag}[M_1(q_1), M_2(q_2), \cdots, M_n(q_n)]$, $C(q, \dot{q}) = \text{diag}[C_1(q_1, \dot{q}_1), C_2(q_2, \dot{q}_2), \cdots, C_n(q_n, \dot{q}_n)]$, $\Lambda(q, \dot{q}, \ddot{\hat{q}}, \dot{\hat{q}}) = \text{diag}[\Lambda_1, \Lambda_2, \cdots, \Lambda_n]$, $\tilde{\theta} = [\tilde{\theta}_1^T \quad \tilde{\theta}_2^T \quad \cdots \quad \tilde{\theta}_n^T]^T$ and $\tilde{\theta}_i = \hat{\theta}_i - \theta_i$, $\epsilon = \text{diag}[\epsilon, \epsilon, \cdots, \epsilon] \otimes I_2$. Define Lyapunov function as

$$V_e = \frac{1}{2}s^T M(q)s + \frac{1}{2}\tilde{\theta}^T\tilde{\theta}$$

whose derivative along (13) satisfies $\dot{V}_e \leq -s^T\epsilon s$. By invoking the Barbalat's lemma, s converges to the origin or $(\dot{q}_i - x_{2i}) + (q_i - x_{1i}) \to 0$. Therefore, $\lim_{t\to\infty} q_i = x_{1i}$. This finished the tracking proof. The optimization result or $\lim_{t\to\infty} x_{1i} = x^\star$ can be easily concluded from Corollary 3.1. It is concluded that $\lim_{t\to\infty} q_i = x^\star$ that means the consensus problem of the networked EL systems is solved under the optimization constraint. □

Remark 4.2 *Compared to the controller* (21) *in [16],* (12) *has the following remarkable advantages: 1) In* (21) *of [16], the control gain* k *should be chosen based on the* $\lambda_2(L)$ *and the gradient of the cost function* θ*. While these informations are not needed in* (12) *due to the adaptive gain technique used. 2) In* (21) *of [16], the sum of the initial conditions of the gradient estimation system should be equal to zero, i.e.,*

$\sum_{i=1}^{N} v_i = 0$. (12) *does not have this requirement. 3) The graph in [16] is required to be undirected while the current paper only requires the graph to be weight balanced.*

5 Simulation

In this section we apply the proposed method to the multi-agent system consisting of five two-link robot manipulators . Therefore, the system (8) with $m = 2$ is written in the following detailed form [22]:

$$\begin{bmatrix} M_{i,11}(q_i) & M_{i,12}(q_i) \\ M_{i,21}(q_i) & M_{i,22}(q_i) \end{bmatrix} \begin{bmatrix} \ddot{q}_{i1} \\ \ddot{q}_{i2} \end{bmatrix} + \begin{bmatrix} C_{i,11}(q_i,\dot{q}_i) & C_{i,12}(q_i,\dot{q}_i) \\ C_{i,21}(q_i,\dot{q}_i) & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_{i1} \\ \dot{q}_{i2} \end{bmatrix} + \begin{bmatrix} G_{i1}(q_i) \\ G_{i2}(q_i) \end{bmatrix} = \begin{bmatrix} \tau_{i1} \\ \tau_{i2} \end{bmatrix}, \tag{14}$$

where $M_{i,11}(q_i) = M_{1i}\ell_{i,c1}^2 + M_{2i}\ell_{1i}^2 + M_{2i}\ell_{c2,i}^2 + 2M_{2i}\ell_{1i}\ell_{c2,i}\cos q_{i2} + \frac{1}{12}M_{1i}\ell_{i1}^2 + \frac{1}{12}M_{2i}\ell_{i2}^2$, $M_{i,12}(q_i) = M_{2i}\ell_{c2,i}^2 + M_{2i}\ell_{1i}\ell_{c2,i}\cos q_{i2} + \frac{1}{12}M_{2i}\ell_{i2}^2$, $M_{i,22}(\cdot) = M_{i2}\ell_{c2,i}^2 + \frac{1}{12}M_{2i}\ell_{2i}$, $C_{i,11}(\cdot) = -M_{2i}\ell_{1i}\ell_{c2,i}\sin q_{i2}\dot{q}_{i2}$, $C_{i,12}(\cdot) = -M_{2i}\ell_{1i}\ell_{c2,i}\sin q_{i2}(\dot{q}_{i1} + \dot{q}_{i2})$, $C_{i,21}(\cdot) = M_{2i}\ell_{1i}\ell_{c2,i}\sin(q_{i2})\dot{q}_{i1}$, $G_{i,1}(\cdot) = (M_{1i}\ell_{c1,i} + M_{2i}\ell_{1i})g\cos q_{i1} + M_2\ell_{c2,i}g\cos(q_{i1} + q_{i2})$, $G_{i,2}(\cdot) = M_{2i}\ell_{c2,i}g\cos(q_{i1} + q_{i2})$. The definitions for the parameters in (14) are given in Fig. 1 where the diagram for a single two-link robot is given. Note that g is the gravitational acceleration; for $j = 1, 2$, q_{ij} denotes the joint angle, M_{ji} denotes the mass of link j of i-th robot; ℓ_{ji} denotes the length of link j of i-th robot; $\ell_{cj,i}$ denotes the distance from the previous joint to the center of mass of link j of i-th robot.

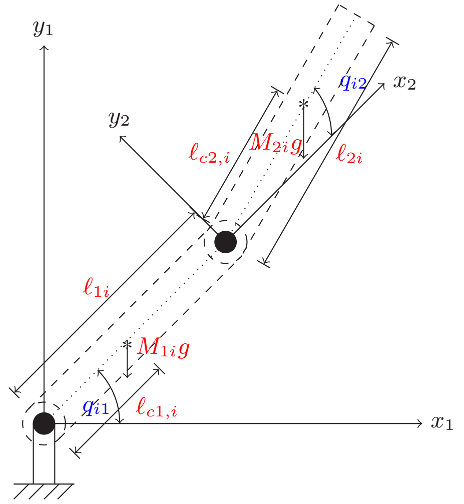

Fig. 1: A two-link planar robot.

The target is to design the distributed control law (12) such that $\lim_{t\to\infty} q_i = q^\star, 1 \le i \le 5$ and moreover $q^\star$ satisfies

$$q^\star = \arg\min \sum_{i=1}^{5} \|q^\star - q_i(0)\|^2 \tag{15}$$

where $q_i(0) \in \mathbb{R}^2$ is the initial condition of q_i. The current distributed optimization problem is corresponding to a real problem: five robots are moving in a 2-dimensional plane and they are controlled to reach a consensus point such that the sum of the distances from the initial points to the consensus point is minimized. The network topology is given in Fig. 2. In the simulations, the initial conditions of q_{i1}

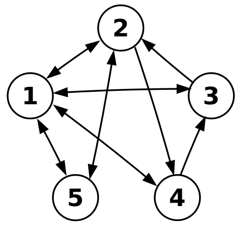

Fig. 2: The strongly connected and balanced graph for the five manipulators

and q_{i2} of the five robots (14) are $[1 \quad -2]$, $[-8 \quad -5]$, $[-6 \quad -1]$, $[7 \quad 0]$, and $[-3 \quad 1]$, respectively. Setting $M_{1i} = 10\text{kg}, g = 0\text{N/kg}, \ell_{1i} = 1\text{m}, \ell_{c1,i} = 0.5\text{m}$ and $\ell_{2i} = 1\text{m}$ for $i = 1, \cdots, 5$. As for the unknown payloads M_{2i}, we allow $5\text{kg} \le M_{2i} \le 6\text{kg}$ and $0.5\text{m} \le \ell_{c2,i} \le 0.6\text{m}$. It is seen from Fig. 3 that $[q_{i1}, q_{i2}]$ converges to $[-1.5, -0.8]$ that is the global minimizer of the cost function (15). Furthermore, it is seen from Fig. 4 that $\hat{k}_i, i = 1, \cdots, 5$ converges to constant, respectively.

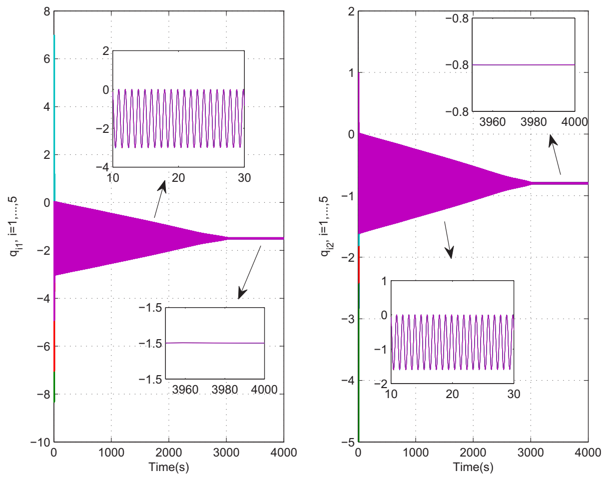

Fig. 3: The trajectories of q_{i1} and q_{i2} for $i = 1, \cdots, 5$.

6 Conclusion

In the paper the distributed optimization problem of a class of higher order linear multi-agent systems is considered. The adaptive gain technique is introduced such that the design of the control law can be performed in a fully distributed way, i.e., without using the information of Laplacian matrix. Furthermore, the requirement that the graph should be undirected is relaxed to a weight balanced one. The future work will focus on the nonlinear multi-agent systems and time-varying cost functions.

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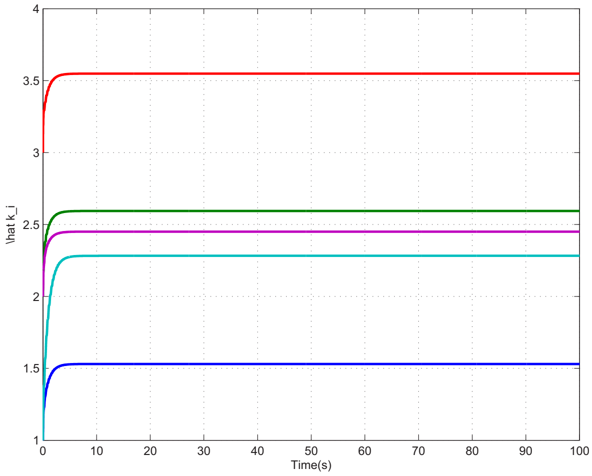

Fig. 4: The trajectories of $\hat{k}_i$ for $i = 1, \cdots, 5$.

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