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Bipartite tracking consensus for multi-agent systems with Lipschitz-Type nonlinear dynamics[☆]

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HIGHLIGHTS

- The bipartite tracking consensus for multi-agent systems with Lipschitz-Type nonlinear dynamics is investigated in this paper.
- To achieve bipartite tracking consensus, existence conditions for the adaptive neighbor-based algorithm is developed.
- Nonzero and zero control inputs of leader in the protocols are taken into account, respectively.

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ABSTRACT

This study concentrates on the bipartite tracking consensus for Lipschitz-Type multi-agent systems (MASs) in the presence of a single leader. Distributed neighbor-based bipartite tracking consensus algorithms with leader's control inputs are developed for both the nonzero and zero cases, under which the bipartite tracking consensus is reached. In contrast to the existing works on this topic, the Lipschitz nonlinear agent dynamics are taken into consideration. Additionally, the control inputs of the leader can be nonzero. Numerical simulations are performed to examine the potentials of results.

1. Introduction

The recent decades have witnessed tremendous attention in the study of control systems including networked systems [1,2] and MASs [3,4] due to the broad applications in real-world systems. In the context of MASs, the agents gather together like a team to perform task rather than single one, which not only reduce costs, but also obtain high flexibility and easy maintenance. This interesting phenomenon is called collective behaviors, e.g. synchronization [5,6], consensus [7,8], swarming [9,10] and flocking [11,12]. Among them, consensus means that each agent agrees upon a common quantity, which is related to the initial states of agents. Tracking consensus is a special case of consensus in which only part of agents has access to the leader's information, and the objective is that each agent follows the leader asymptotically. Hong et al. [13] focused on the consensus problem of a body of agents with signal leader, whose velocity cannot be measured. Zhao et al. [14] addressed the distributed finite-time tracking control problem for second-order MASs. Ge et al. [15] presented a novel event-based control approach to implement the tracking consensus. However, it is observed that aforementioned tracking consensus results are based on collaborative interactions between agents.

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Nowadays, in practice, some agents collaborate, while others compete like in markets or social networks. When both collaborative and antagonistic relations coexist within a group of agents, the agents exhibit a bipartite consensus phenomenon, meaning that the edges can be positive or negative. Such a bipartite consensus framework is generally modeled by signed graph with positive and negative edges, which is distinguished from normal consensus. The aim of bipartite consensus is to guarantee the agents reach agreement regarding identical magnitude but opposite sign. Qin et al. [16] considered the bipartite consensus problem for linear MASs with input saturation under directed interaction topology. Zhao et al. [17] was concerned with the finite-time bipartite consensus problems for second-order MASs with antagonistic interactions. Hu et al. [18] handled the bipartite tracking consensus problem for MASs under coopetition networks, where the agents interact not only collaboratively but also competitively.

To date, consensus problems with general linear dynamics have been widely studied in previous literatures. Wen et al. [19] tackled the problem of bipartite consensus for linear MASs model with single leader. Zhang et al. [20] proposed state feedback and output feedback control laws to solve bipartite consensus problems of general linear MASs over directed signed graphs. Additionally, it is easily observed that the consensus protocol design for nonlinear systems is more complicated than for the linear systems counterpart, owing to its restriction to local stability, network connection and forth on. Li et al. [21] designed distributed adaptive protocols that enable the agents to achieve an agreement for MASs with nonlinear case. Jameel et al. [22] studied the consensus problem for MASs having nonlinear dynamics by output feedback.

Inspired by above considerations, in this study, we make an endeavor to solve the problem of bipartite tracking consensus for MASs having Lipschitz-Type nonlinear dynamics. Contrary to the related literature, the main highlights of this study are twofold. (i) It extends the existing bipartite tracking consensus results to more practical scenario with nonlinear dynamics constraints. Moreover, the control inputs of the leader can be nonzero and unknown to each follower. (ii) By using the general Riccati equation and Lyapunov method, the novel non-smooth protocols based on neighboring agents can be implemented without using any global information.

The rest of this study is summarized as follows. In Section 2, graph theory is briefly reviewed, and the model used is described. In Section 3, we investigate the bipartite tracking consensus problem for nonlinear MASs under antagonistic interactions. Numerical simulations and conclusions are stated in Sections 4 and 5, respectively.

Notations: Throughout this study, I_N denotes the identity matrix of dimension N. $A \otimes B$ represents the Kronecker product of matrices A and B. Let $\lambda_k(\cdot)$ be the k-th smallest eigenvalue. *sign* () is the standard sign function.

2. Preliminaries

2.1. Graph theory

The Leader–follower networks are modeled by a single leader and N followers, where all agents share identical state space $\mathbb{R}$. Weighted graph $\mathcal{G}$ is a triple $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ with vertex set $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$, edge set $\mathcal{E} \subseteq V \times V$ and adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ with $a_{ij} \neq 0$ *if* $\left(v_j, v_i\right) \in \mathcal{E}$. In addition, self-loops are not allowed, i.e., $a_{ii} = 0, \forall i = 1, \ldots, N$. The neighbor set of node is denoted by $N_i = \left\{j : a_{ij} \neq 0\right\}$. We say $\mathcal{G}$ has a path means that there exists a sequence of nodes $i_1, \ldots, i_m$ such that $(i_l, i_{l+1}) \in E, \forall l = 1, \ldots, m-1$. Moreover, $\mathcal{G}$ is connected when any two vertices can be connected via paths.

The Laplacian matrix $L = \left[l_{ij}\right]_{N\times N}$ is given in the form of

$$l_{ij} = \begin{cases} \sum_{k=1}^{N} |a_{ik}|, j = i. \\ -a_{ij}, j \neq i. \end{cases}$$

Lemma 1 ([23]). *Consider signed graph $\mathcal{G}$, the following facts are equivalent:*

(1) $\mathcal{G}$ is structurally balanced;

(2) $\mathcal{G}$ provides two subsets $\mathcal{V}_1$ and $\mathcal{V}_2$, where $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$, $\mathcal{V}_1 \cap \mathcal{V}_2 = 0$, such that $a_{ij} \geq 0$ if $\forall v_i, v_j \in \mathcal{V}_q$ $(q \in \{1, 2\})$, $a_{ij} \leq 0$ if $\forall v_i \in \mathcal{V}_q, \mathcal{V}_j \in \mathcal{V}_r, q \neq r$ $(q, r \in \{1, 2\})$;

(3) $\exists \mathcal{D} = \{D = diag\{d_1, d_2, \ldots, d_N\}, d_i \in \{-1, +1\}\}$ such that $\bar{\mathcal{A}} = D\mathcal{A}D$ has all nonnegative entries and DLD has all positive diagonal entries. Besides, D exists two partitions, such that $\mathcal{V}_1 = \{i|d_i > 0\}$ and $\mathcal{V}_2 = \{i|d_i < 0\}$.

Based on (3) of Lemma 1, set $R = diag(a_{10}, a_{20}, \ldots, a_{N0})$, where $a_{i0} > 0$ if the leader state has access to follower i and $a_{i0} = 0$ otherwise. Define $\bar{L} = DLD + R$, $L_R = L + R$.

2.2. Problem formulation

It is supposed that there is MASs consisting of two layers, i.e., the leader's layer contains 1 agent and the followers' layer contains N agents. The dynamic of the leader–follower agents is formulated as

Followers:

$$\dot{r}_i(t) = Ar_i(t) + Bu_i(t) + f(r_i, t).$$

Leader:

$$\dot{r}_0(t) = Ar_0(t) + Bu_0(t) + f(r_0, t). \tag{1}$$

where $r_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^m$ are the state and control input of agent i. $r_0 \in \mathbb{R}^n$, $u_0 \in \mathbb{R}^m$ are the state and control input of the leader. $f : \mathbb{R} \times \mathbb{R} \times \mathbb{R}^+ \to \mathbb{R}$ is a vector-valued nonlinear function, continuous and differentiable in t. A, B are constant matrices with compatible dimension.

Assumption 1. Assume that subgraph defined on N followers is connected and structurally balanced, meanwhile, each follower has a path to the leader in the graph.

Assumption 2. The pair (A, B) is stabilizable.

Assumption 3. There exists a nonnegative constant k, such that

$$\|f(x, t) - f(y, t)\| \le k\|x - y\|, \forall x, y \in \mathbb{R}^n, t \ge 0. \tag{2}$$

Remark 1. This assumption is called the Lipschitz condition, which is a smoother condition than the consistent continuous.

Lemma 2 ([24]). *Under Assumption 2, there exists a positive definite matrix $P \in \mathbb{R}^{N\times N}$, which make following Riccati equation hold:*

$$A^TP + PA - \alpha PBB^TP + \gamma P = 0, \tag{3}$$

where $\alpha > 0$, and $\gamma > 0$.

Lemma 3 ([25,26]). *For the Kronecker product of two matrices with appropriate dimensions, there hold*

(1) $(A + B) \otimes C = A \otimes C + B \otimes C$.
(2) $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$.

Definition 1. : The bipartite tracking consensus is said to be guaranteed if there holds either

$$\lim_{t\to\infty} \|r_i(t) - r_0(t)\| = 0,$$

or

$$\lim_{t\to\infty} \|r_i(t) + r_0(t)\| = 0. \tag{4}$$

Based upon Lemma 1, equalities given in (4) can be equivalently expressed by

$$\lim_{t\to\infty} \|r_i(t) - d_ir_0(t)\| = 0, i = 1, 2, \ldots, N.$$

3. Main results

In this section, the main theoretical results are derived.

3.1. Bipartite tracking consensus with nonzero control input of the leader

In this subsection, we are at the position to present the bipartite tracking consensus protocol

$$u_i(t) = -K\left[\sum_{j=1}^{N} |a_{ij}| \left(r_i(t) - \operatorname{sign}(a_{ij})\, r_j(t)\right) + a_{i0}\left(r_i(t) - d_i r_0(t)\right)\right] + d_iu_0(t), \tag{5}$$

where $K \in \mathbb{R}^{m\times n}$ is a feedback matrix which will be designed.

Theorem 1. *Presume that Assumptions 1–3 hold. The bipartite tracking consensus for system (1) is achieved by utilizing protocol (5) if $\alpha \le 2\lambda_2(L_R)$, $\gamma \ge 2k$ and $K = B^TP$, where $P > 0$ is the solution of the Riccati equation:*

$$A^TP + PA - \alpha PBB^TP + \gamma P = 0. \tag{6}$$

Proof. Let $\delta_i(t) = r_i(t) - d_ir_0(t)$, $\hat{u}_i(t) = u_i(t) - d_iu_0(t)$. Combining (1) and (5), write the evolution dynamics of $\delta_i(t)$ as

$$\dot{\delta}_i(t) = A\delta_i(t) + B\hat{u}_i(t) + f(r_i, t) - f(d_ir_0, t). \tag{7}$$

In terms of (5), one also obtains

$$\hat{u}_i(t) = -B^T P \left[\sum_{j=1}^{N} \left|a_{ij}\right| \left(r_i(t) - sign\left(a_{ij}\right) r_j(t)\right) + a_{i0}\left(r_i(t) - d_i r_0(t)\right) \right].$$

Owing to $sign\left(a_{ij}\right) d_i = d_j$, one derives

$$\begin{aligned} \hat{u}(t) = & -B^T P \Bigg[\sum_{j=1}^{N} \left|a_{ij}\right| \left[\left(r_i(t) - d_i r_0(t)\right) - sign\left(a_{ij}\right)\left(r_j(t) - d_i r_0(t)\right) \right] \\ & + a_{i0}\left(r_i(t) - d_i r_0(t)\right) \Bigg]. \end{aligned} \tag{8}$$

Substituting (8) into (7), one derives

$$\begin{aligned} \dot{\delta}_i(t) = & A\delta_i(t) - BB^T P \left[\sum_{j=1}^{N} \left|a_{ij}\right| \left(\delta_i(t) - sign\left(a_{ij}\right)\delta_j(t)\right) + a_{i0}\delta_i(t) \right] \\ & + f(r_i, t) - f(d_i r_0, t). \end{aligned} \tag{9}$$

From Assumption 3, one obtains

$$\| f(r_i, t) - f(d_i r_0, t) \| \le k \|\delta_i(t)\|.$$

Then, it is yield from (9) that

$$\begin{aligned} \dot{\delta}_i(t) \le & A\delta_i(t) - BB^T P \left[\sum_{j=1}^{N} \left|a_{ij}\right| \left(\delta_i(t) - sign\left(a_{ij}\right)\delta_j(t)\right) + a_{i0}\delta_i(t) \right] \\ & + kI_N \delta_i(t). \end{aligned} \tag{10}$$

Denote

$$\delta = \left[\delta_1^T, \delta_2^T, \ldots, \delta_N^T\right]^T.$$

The matrix form of (10) is written as

$$\dot{\delta} \le \left(I_N \otimes A - L_R \otimes BB^T P + k\left(I_N \otimes I_N\right)\right)\delta. \tag{11}$$

Introduce the Lyapunov function

$$V = \delta^T \left(I_N \otimes P\right)\delta.$$

Combined with (11), differentiating V produces

$$\begin{aligned} \dot{V} & \le 2\delta^T \left(I_N \otimes PA - L_R \otimes PBB^T P + k\left(I_N \otimes P\right)\right)\delta \\ & = \delta^T \left(I_N \otimes \left(PA + A^T P\right) - 2L_R \otimes PBB^T P + 2k\left(I_N \otimes P\right)\right)\delta \\ & \le \delta^T \left(I_N \otimes \left(PA + A^T P\right) - 2\lambda_2\left(L_R\right) I_N \otimes PBB^T P + 2k\left(I_N \otimes P\right)\right)\delta. \end{aligned}$$

By choosing $\alpha \le 2\lambda_2\left(L_R\right)$, one concludes

$$\dot{V} \le \delta^T \left(I_N \otimes \left(PA + A^T P\right) - \alpha I_N \otimes PBB^T P + 2k\left(I_N \otimes P\right)\right)\delta.$$

With the help of Lemma 3, one gets that

$$\dot{V} \le \delta^T \left[I_N \otimes \left(A^T P + PA - \alpha PBB^T P\right) + 2k\left(I_N \otimes P\right)\right]\delta.$$

Together with Lemma 2, one deduces

$$\dot{V} \le \delta^T \left(2k\left(I_N \otimes P\right) - \gamma\left(I_N \otimes P\right)\right)\delta.$$

Further,

$$\begin{aligned} \dot{V} & \le (2k - \gamma)\,\delta^T \left(I_N \otimes P\right)\delta \\ & = (2k - \gamma)\, V. \end{aligned}$$

Since $\gamma \ge 2k$, one may not hard to obtain $\dot{V} \le 0$. It can be thus derived that $\delta_i(t) \to 0$ as $t \to \infty$. Equivalently, it follows that $r_i(t) \to d_i r_0(t)$ as $t \to \infty$. This completes the proof.

Remark 2. As shown in [27–30], a necessary and sufficient condition for the existence of $P > 0$ to Eq. (3) is that (A, B) is stabilizable. Therefore, one of sufficient condition for the existence of protocol (5) satisfying Theorem 1 is that Assumption 2 holds.

3.2. Bipartite tracking consensus with zero control input of the leader

In this subsection, we will show a special case where the control input of the leader agent is zero, namely, the leader moves in a constant speed. The dynamics of the leader–follower MAS (1) will reduce to the following form.

Followers:

$$\dot{r}_i(t) = Ar_i(t) + Bu_i(t) + f(r_i, t).$$

Leader:

$$\dot{r}_0(t) = Ar_0(t) + f(r_0, t). \tag{12}$$

Similarly, to handle the bipartite tracking consensus, the neighbor-based law for follower agents is constructed as

$$\begin{aligned} u_i(t) = & -K\left[\sum_{j=1}^{N}\left|a_{ij}\right|\left(r_i(t) - \operatorname{sgn}\left(a_{ij}\right)r_j(t)\right) + a_{i0}\left(r_i(t) - d_i r_0(t)\right)\right] \\ & -\operatorname{sgn}\left(B^TP\sum_{j=1}^{N}\left|a_{ij}\right|\left(r_i(t) - \operatorname{sgn}\left(a_{ij}\right)r_j(t)\right) + a_{i0}\left(r_i(t) - d_i r_0(t)\right)\right). \end{aligned} \tag{13}$$

Theorem 2. *Presume that Assumptions 1–3 hold. Presume that Assumptions 1–3 hold. The bipartite tracking consensus with dynamics (12) is solved under protocol (13), if $\alpha \le 2\lambda_2(\bar{L})$, $\gamma \ge 2k$ and $K = B^TP$, where $P > 0$ is the solution of the Riccati equation:*

$$A^TP + PA - \alpha PBB^TP + \gamma P = 0.$$

Proof. Substituting (13) into (12), the closed-loop dynamic can be derived as

$$\begin{aligned} \dot{r}_i(t) = & Ar_i(t) - BB^TP\left[\sum_{j=1}^{N}\left|a_{ij}\right|\left(r_i(t) - \operatorname{sgn}\left(a_{ij}\right)r_j(t)\right) + a_{i0}\left(r_i(t) - d_i r_0(t)\right)\right] \\ & -B\operatorname{sgn}\left(B^TP\left[\sum_{j=1}^{N}\left|a_{ij}\right|\left(r_i(t) - \operatorname{sgn}\left(a_{ij}\right)r_j(t)\right) + a_{i0}\left(r_i(t) - d_i r_0(t)\right)\right]\right) + f(r_i, t). \end{aligned} \tag{14}$$

Eq. (14) can be expressible in compact form as

$$\dot{r} = \left(I_N \otimes A - L_R \otimes BB^TP\right)r - \left(I_N \otimes B\right)\operatorname{sgn}\left(\left(L_R \otimes B^TP\right)r\right) + F(r, t).$$

Denote $\bar{r} = (D \otimes I_n)\, r$, one further has

$$\dot{\bar{r}} = \left(I_N \otimes A - DL_R \otimes BB^TP\right)r - \left(I_N \otimes B\right)\left(D \otimes I_n\right)\operatorname{sgn}\left(\left(L \otimes B^TP\right)r\right) + F(\bar{r}, t). \tag{15}$$

Due to $DD = I_N$ and $D\operatorname{sgn}(s) = \operatorname{sgn}(Ds)$, it is obtained from (15) that

$$\begin{aligned} \dot{\bar{r}} &= \left(I_N \otimes A - DL_RD \otimes BB^TP\right)\bar{r} - \left(I_N \otimes B\right)\operatorname{sgn}\left(\left(DL_RD \otimes B^TP\right)\bar{r}\right) + F(\bar{r}, t) \\ &= \left(I_N \otimes A - \bar{L} \otimes BB^TP\right)\bar{r} - \left(I_N \otimes B\right)\operatorname{sgn}\left(\left(\bar{L} \otimes B^TP\right)\bar{r}\right) + F(\bar{r}, t), \end{aligned}$$

which means

$$\begin{aligned} \dot{\bar{r}}_i(t) = & A\bar{r}_i(t) + BB^TP\sum_{j=0}^{N}\bar{a}_{ij}\left(\bar{r}_j(t) - \bar{r}_i(t)\right) \\ & + B\operatorname{sgn}\left(B^TP\sum_{j=0}^{N}\bar{a}_{ij}\left(\bar{r}_j(t) - \bar{r}_i(t)\right)\right) + f(\bar{r}_i, t(t)), \end{aligned}$$

where $\bar{r}_i(t) = d_i r_i(t)$ and $\bar{a}_{ij} = d_i a_{ij} d_j$.

Let $e_i(t) = \bar{r}_i(t) - r_0(t)$, differentiating $e_i(t)$ produces

$$\begin{aligned}\dot{e}_i(t) &= A\bar{r}_i(t) + BB^TP\sum_{j=0}^{N}\bar{a}_{ij}\left(\bar{r}_j(t) - \bar{r}_i(t)\right) + B\mathrm{sgn}\left(B^TP\sum_{j=0}^{N}\bar{a}_{ij}\left(\bar{r}_j(t) - \bar{r}_i(t)\right)\right)\\ &\quad + f(\bar{r}_i, t) - Ar_0(t) - f(r_0, t)\\ &= Ae_i(t) + BB^TP\sum_{j=0}^{N}\bar{a}_{ij}\left(e_j(t) - e_i(t)\right) + B\mathrm{sgn}\left(B^TP\sum_{j=0}^{N}\bar{a}_{ij}\left(e_j(t) - e_i(t)\right)\right)\\ &\quad + [f(\bar{r}, t) - f(r_0, t)].\end{aligned}$$

According to Assumption 3, one yields

$$\begin{aligned}\dot{e}_i(t) &\le Ae_i(t) + BB^TP\sum_{j=0}^{N}\bar{a}_{ij}\left(e_j(t) - e_i(t)\right) + k\|e_i(t)\|\\ &\quad + B\mathrm{sgn}\left(B^TP\sum_{j=0}^{N}\bar{a}_{ij}\left(e_j(t) - e_i(t)\right)\right).\end{aligned} \tag{16}$$

The condensed expression of (16) is described by

$$\dot{e} \le \left(I_N \otimes A - \bar{L} \otimes BB^TP\right)e - \left(\bar{L} \otimes B\right)\mathrm{sgn}\left(\left(\bar{L} \otimes B^TP\right)e\right) + k\|e\|.$$

Denote $\zeta = \left(\bar{L} \otimes I_n\right)e$, one has

$$\dot{\zeta} \le \left(I_N \otimes A - \bar{L} \otimes BB^TP\right)\zeta - \left(\bar{L} \otimes B\right)\mathrm{sgn}(Y) + k\zeta,$$

where $Y = \left(I_N \otimes B^TP\right)\zeta$.
Select the Lyapunov function candidate as

$$V = e^T(I_N \otimes P)e.$$

Its time derivative is expressed as

$$\dot{V} \le 2\zeta^T\left(I_N \otimes PA - \bar{L} \otimes PBB^TP + kI_N \otimes P\right)\zeta - 2Y^T\left(\bar{L} \otimes I_s\right)\mathrm{sgn}(Y). \tag{17}$$

Additionally,

$$\begin{aligned}&Y\left(\bar{L} \otimes I_s\right)\mathrm{sgn}(Y)\\ &= Y(DLD \otimes I_s)\,\mathrm{sgn}(Y) + Y(H \otimes I_s)\,\mathrm{sgn}(Y)\\ &= \sum_{i=1}^{N}\sum_{j=1}^{N}\bar{a}_{ij}\left(\|Y_i(t)\|_1 - Y_i^T\mathrm{sgn}(Y_i(t))\right) + \sum_{i=1}^{N}a_{i0}\|Y_i(t)\|_1.\end{aligned}$$

Employing the facts $\|Y_i\|_1 \ge Y_i^T\mathrm{sgn}\left(Y_j\right)$, one may get

$$Y\left(\bar{L} \otimes I_s\right)\mathrm{sgn}(Y) \ge \sum_{i=1}^{N}a_{i0}\|Y_i\|_1. \tag{18}$$

Incorporating (18) into (17), one has

$$\dot{V} \le \zeta^T\left(I_N \otimes \left(PA + A^TP\right) - 2\bar{L} \otimes PBB^TP + 2k(I_N \otimes P)\right)\zeta.$$

By choosing $\alpha \le 2\lambda_2\left(\bar{L}\right)$, one obtains

$$\dot{V} \le \zeta^T\left[I_N \otimes \left(A^TP + PA - \alpha PBB^TP\right) + 2k(I_N \otimes P)\right]\zeta.$$

According to Lemma 2, one can deduce

$$\begin{aligned}\dot{V} &\le (2k - \gamma)\zeta^T(I_N \otimes P)\zeta\\ &= (2k - \gamma)V.\end{aligned}$$

Which implies that $\|\zeta(t)\|$ converges to 0 asymptotically since $\gamma \ge 2k$. It implies that $\|r_i(t) - d_ir_0(t)\|$ approaches 0. That is, the bipartite tracking consensus is obtained. This completes the proof. ■

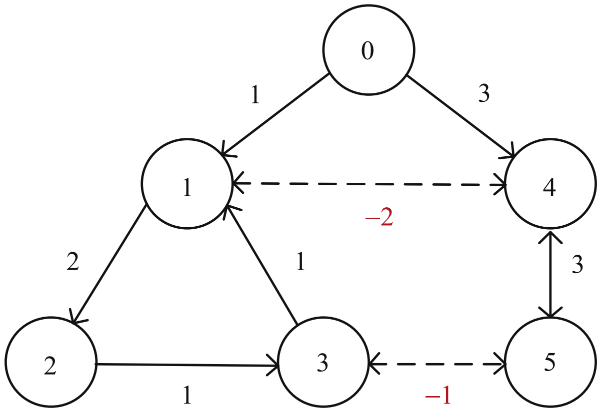

Fig. 1. The interaction topology.

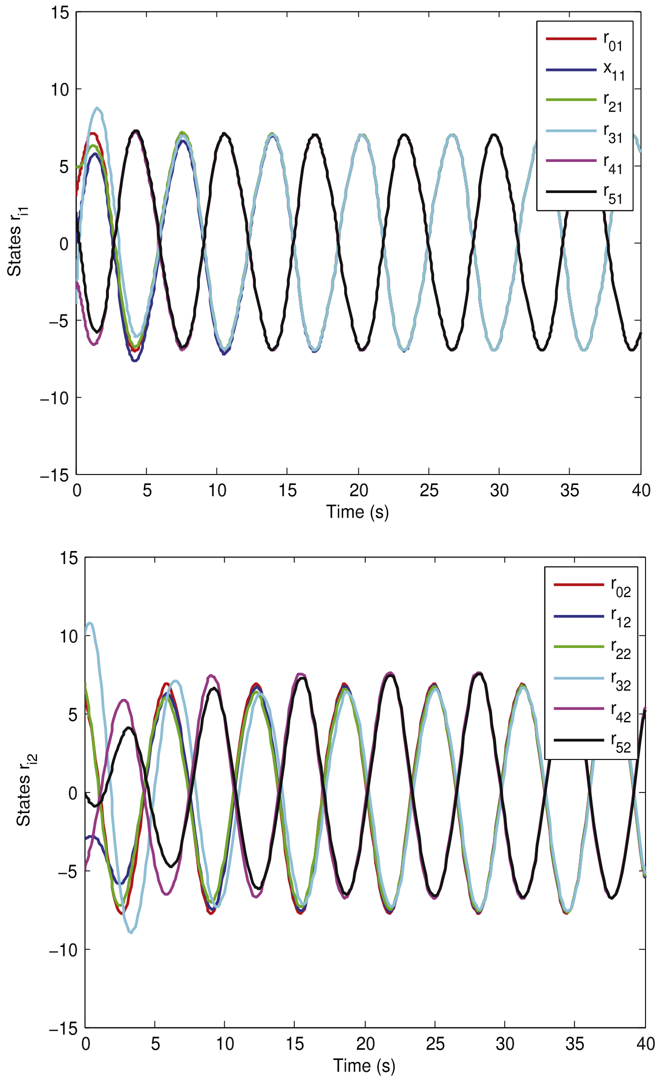

Fig. 2. Trajectories of $r_{i1}(t)$ and $r_{i2}(t)$ $(i = 0, 1, \ldots, 5)$, for case 1.

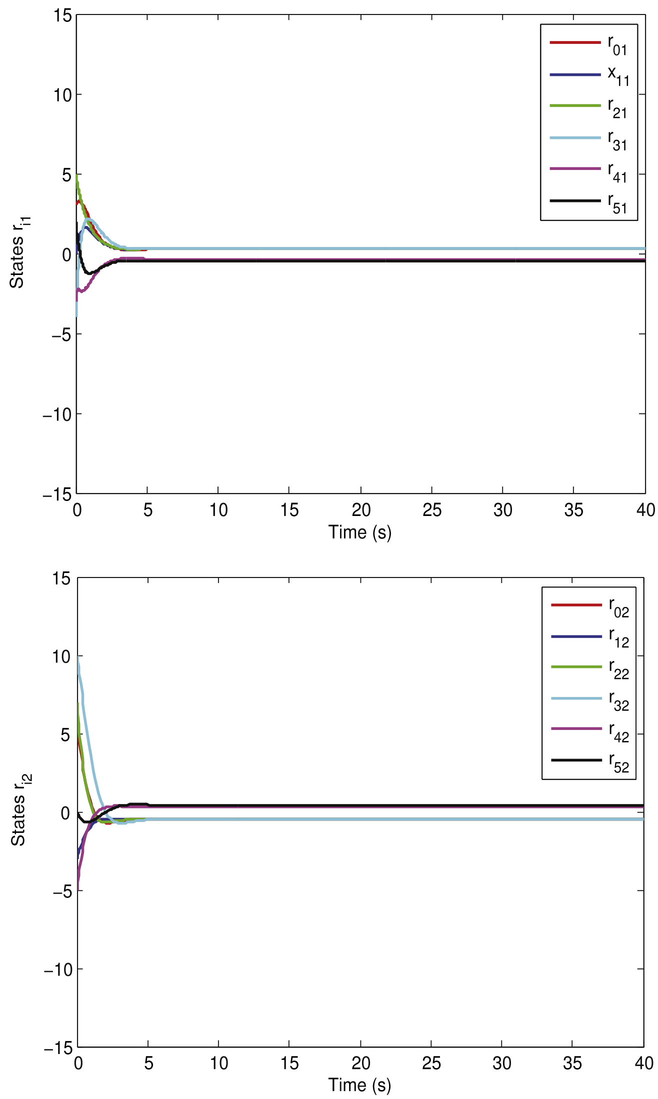

Fig. 3. Trajectories of $r_{i1}(t)$ and $r_{i2}(t)$ $(i = 0, 1, \ldots, 5)$, for case 2.

4. Numerical simulations

In simulations, assuming that a leader and five followers are involved in the collaborative and antagonistic networks, where the nodes indexed from 1 to 5 are the follower agents, and the node indexed by 0 is the leader agent. In Fig. 1, the solid lines represent collaborative relationships while dashed lines indicate antagonistic relationships. Moreover, the interaction topology involving the leader has spanning tree, and is structurally balanced. The followers are split into two groups: $\mathcal{V}_1 = \{1, 2, 3\}$, $\mathcal{V}_2 = \{4, 5\}$.

Next, there are two kinds of nonlinear functions [31,32] are presented to show the organized methods could achieve control requirements.

Case 1: $f(r_i) = [0, -0.34\sin(r_{i1})]^T$.

Case 2: $f(r_i) = [-0.34\cos(r_{i1}) - r_{i1}, -0.34\cos(r_{i2}) - r_{i2}]^T$.

Clearly, it should be mentioned that the nonlinear function $f(r_i)$ in each agent dynamics satisfy the Lipschitz condition (2) with a Lipschitz constant $k = 0.34$. In the two cases, we choose the system parameters as

$$r_i(t) = \begin{bmatrix} r_{i1}(t) \\ r_{i2}(t) \end{bmatrix}, \qquad A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

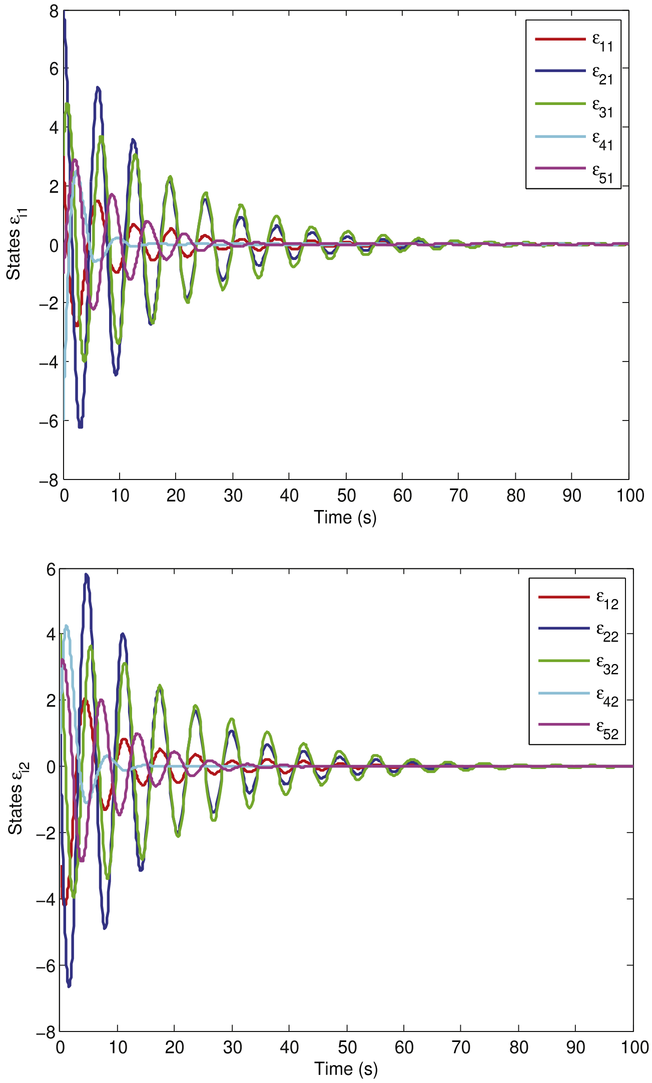

Fig. 4. Trajectories of $\varepsilon_{i1}(t)$ and $\varepsilon_{i2}(t)$ $(i = 1, 2, \ldots, 5)$.

The adjacency matrix is written as

$$A = \begin{bmatrix} 0 & 0 & 1 & -2 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ -2 & 0 & 0 & 0 & 3 \\ 0 & 0 & -1 & 3 & 0 \end{bmatrix},$$

with the leader weight matrices $R = diag(1, 0, 0, 3, 0)$, which means that the information of leader is available to node 1 and node 4. Through simple calculation, the eigenvalues of L_R are 9.5247, 4.8437, 0.6217, 2.5049 + 0.7475i, 2.5049 − 0.7475i. Assume that $u_0(t) = \cos(r_0(t))$. By solving the Riccati equation (6) gives the feedback matrix as $K = [-0.5349 - 4.0644]$. Set $r_0(0) = [3, 6]^T$, $r_1(0) = [0, -3]^T$, $r_2(0) = [5, 7]^T$, $r_3(0) = [-4, 10]^T$, $r_4(0) = [-3, -5]^T$, $r_5(0) = [2, 0]^T$. From Figs. 2–3, it can be mentioned that the agents belonging to subset $\mathcal{V}_1$ converge to the leader's state $r_0(t)$, while the agents belonging to subset $\mathcal{V}_2$ converge to the leader's opposite state $-r_0(t)$, which is in accordance with the partition $\mathcal{V}_1$ and $\mathcal{V}_2$. The trajectories of tracking errors $\varepsilon_i(t) = [\varepsilon_1(t), \varepsilon_2(t)]^T$ between the leader and each agent are presented in Fig. 4. It is observed that the state of the leader is of importance in reaching bipartite tracking consensus.

5. Conclusion

This study dealt with the bipartite tracking consensus problem for a class of MASs with nonlinear dynamics. The distinguishing feature of such protocol is the control inputs of the leader can be nonzero and zero. By using the general Riccati equation, Lyapunov stability theory and so on, the bipartite tracking consensus for MASs with Lipschitz-Type was proven under related assumptions. In addition, two kinds of nonlinear functions are considered to show the organized methods could achieve control requirements. Future work may be challenging in bipartite tracking consensus of MASs with dynamic topology and fully adaptive protocol via event-triggered control.

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