Proceedings of the 39th Chinese Control Conference
July 27-29, 2020, Shenyang, China

Robust H_∞ Consensus Control for a Class of Nonlinear Multi-agent Systems with Similar Structure

Yongqing Fan, Yue An

School of Automation, Xi'an Key Laboratory of Advanced Control and Intelligent Process, Xi'an University of Posts and Telecommunications, Xi'an, 710121, China.

E-mail: fanyongqing@xupt.edu.cn

E-mail: anyue329@163.com

Abstract: A novel robust H_∞ consensus control for nonlinear multi-agent systems with similar structure is proposed in this paper. Similar structure of multi-agent systems is first introduced, the similar parameters are utilized to synthesize consensus control for a class of multi-agent systems with nonidentical dimension. Based on linear matrix inequality (LMI), sufficient conditions can be derived to ensure that all states in follower systems asymptotically converge to the states of leader with a given prescribed H_∞ performance. The salient advantage of the designing control approach is that the consensus of different dimension in multi-agent systems with similar structure also can be guaranteed. Finally, examples of multi-agent systems with identical and nonidentical dimensions are provided to demonstrate the validity of the proposed theoretical results.

Key Words: Consensus, Nonlinear Multi-agent Systems, Similar Structure, Robust H_∞ Control.

1 Introduction

Consensus of the multi-agent systems due to its extensive applications has been recently researched by more researchers [1-3]. The purpose of consensus is to design appropriate protocol to ensure that all controlled objects tend to the coherent behavior of multi-agent systems. The consensus algorithm is an interaction criterion that describes how individuals exchange information among all neighbors in a complex network, which is a popular topic of the leader-follower consensus [4, 5], in which the leader is an isolated individual who can guide other followers to reach consensus agreement in the entire network, then all states in the followers tend toward the states of the leader.

Many important consensus control approaches of linear, nonlinear and fractional-order multi-agent systems have been proposed in [6-9]. However, in many real application projects, the measurement of state is not very accurate due to various complex communication topologies, especially multiple nonlinearities in the systems, so many researchers are incline to study the consistency problem of nonlinear multi-agent systems with external interference, such as literatures [10-16]. For the consistency of multi-agent systems with Lipschitz being satisfied, a distributed protocol algorithms was explored in [10]. By using sampled-data information, the exponential consensus control was provided for general multi-agent systems with Lipschitz nonlinear functions in [11]. Different from [10, 11], the nonlinear part of multi-agent systems was processed by unilateral Lipschitz condition, and the external interference was eliminated with H_∞ index, the consistency of multi-agent systems was achieved by the distributed control designed in [12]. Robust consensus protocol design approaches were addressed for a class of nonlinear multi-agent systems with external disturbance in [13-15].

These existing results were focused on the consistency of identical multi-agent systems, which means that the controls were only valid for the consensus of each agent with same dynamical behaviors [4-15]. To overcome this limitation of works, consensus control algorithms were proposed for nonidentical multi-agent systems. For example, a novel distributed adaptive fuzzy control was investigated for a class of heterogeneous second-order multi-agent systems in [16]. By using the provided fuzzy adaptive control in [17], the output consensus of heterogeneous multi-agent systems can be guaranteed. For the consensus of essential heterogeneous multi-agent systems with external disturbance, a robust consensus protocol scheme was designed in [18]. In [19], a distributed proportional integral control combined with given sufficient conditions was developed to ensure the consensus of the agents. It is noted that the heterogeneous multi-agent systems in the abovementioned literature [16-19] were all dealt with linear systems, and the dimensions of multi-agent systems were identical. Consequently, the rich achievements about consensus control schemes in [16-19] were invalid for some class of heterogeneous multi-agent systems with different dimensions. Therefore, it is necessary to explore some neoteric consensus control, which are not only applicable to heterogeneous multi-agent systems with same dimension, but also to agents with different dimensions.

Motivated by the analyses of above literatures, this paper attempts to fill in the research gap the consensus control protocols between multi-agent systems with same and different dimensions. From the perspective of the connection structure of the multi-agent workspace, the design concept of consensus control draws on the experience of the similar characteristic of large-scale systems in [20-25]. In a network or large-scale systems, each agent can be abbreviated as one node in a topology graph, where the spatial dimension of each node is same or different. In this paper, similar properties of multi-agent

* This work was supported by the National Natural Science Foundation of China under Grant (51875457, 61903298), Shaanxi Provincial Department of Science and Technology key project in the field of industry (2018ZDXM-GY-039), National Natural Science Foundation of Shaanxi under Grant 2019JQ-341

systems are defined as similar nodes with nonidentical dimensions, an originality consensus control based on the information of every agent with different dimensions is constructed.

Compared with the latest consensus research works on heterogeneous multi-agent systems, the main contributions of the control are summed up in two aspects: Firstly, the proposed control with similar information of each agent can be utilized to realize the consensus of nonlinear multi-agent systems with identical and nonidentical dimensions. Secondly, the nonlinear terms are no longer restricted by the condition satisfying with Lipschitz or one-sided Lipschitz, and the control gain can be easily obtained by solving proposed LMI.

The remainder of this paper is summarized as follows: In Section II, some preliminaries and system formulations are provided. In Section III, a H_∞ control scheme for heterogeneous nonlinear multi-agent systems is investigated. In Section IV, a numerical example with different dimensions is presented to illustrate the theoretical result. Finally, conclusion is given in Section VI.

2 Problem formulations

For a network graph of leader-follower system, the communication graph is represented by a graph $\bar{G}$ with the vertex set $\bar{V}=\{v_0\}\cup V$. If the followers have no influence on the leader, then $a_{i0}=0, i=1,2,\ldots,N$. In addition, if follower i can communicate directly with the leader, then $a_{i0}>0$. Defined the matrices $A_0=diag\{a_{10},a_{20},\ldots,a_{N0}\}$ and $\bar{L}=L+A_0$, the following lemma is presented.

Lemma 1[26]: Matrix $\bar{L}$ is symmetric and positive definite, if the communication graph G among the follower agent system is undirected and there exists a directed path from the leader to each follower in $\bar{G}$.

The dynamic model of a leader system can be described as follows:

$$\dot{x}_0(t)=A_0x_0(t)+B_0u_0(t)+f_0(t,x_0(t)) \tag{1}$$

where $A_0\in\mathbb{R}^{n_0\times n_0}$, $B_0\in\mathbb{R}^{n_0\times m_0}$ are the constant matrices that denote the linear component of the leader. $x_0(t)\in\mathbb{R}^{n_0\times 1}$ denotes the state vector of the leader, the control input of the leader is denoted as $u_0(t)\in\mathbb{R}^{m_0\times 1}$. $f_0(t,x_0(t))\in\mathbb{R}^{n_0\times 1}$ represents the nonlinear function in the leader systems.

The follower systems consisting of N agents are considered as the following dynamical model

$$\begin{cases}\dot{x}_i(t)=A_ix_i(t)+B_iu_i(t)+f_i(t,x_i(t))+\omega_i(t)\\ y_i(t)=C_ix_i(t), \quad i=1,2,\ldots,N\end{cases} \tag{2}$$

where $x_i(t)\in\mathbb{R}^{n_i\times 1}$ denotes the state vector of the agent i th, the control input of the i th agent can be denoted as $u_i(t)\in\mathbb{R}^{m_i\times 1}$, $y_i(t)\in\mathbb{R}^{p_i\times 1}$ is the output of the agent i th and $\omega_i(t)\in\mathbb{R}^{n_i\times 1}$ stands for the external disturbance. The constant matrices $A_i\in\mathbb{R}^{n_i\times n_i}$, $B_i\in\mathbb{R}^{n_i\times m_i}$ and $C_i\in\mathbb{R}^{p_i\times n_i}$ are the linear component of the follower agent systems. The nonlinear function of the i th agent system is written as $f_i(t,x_i(t))\in\mathbb{R}^{n_i\times 1}$.

Definition 1[25]: Consider N follower agent systems as shown in (2), if there exist N matrices $F_i\in\mathbb{R}^{n_0\times n_i}$ $(F_i\neq 0)$, $K_0\in\mathbb{R}^{m_0\times n_0}$, and N invertible matrices $K_i\in\mathbb{R}^{m_i\times n_i}$ satisfy the condition as the following (3), the follower agent system (2) is called a similar composite with the leader (1).

$$\begin{cases}F_i(A_i+B_iK_i)=(A_0+B_0K_0)F_i\\ F_iB_i=B_0\end{cases} \tag{3}$$

Remark 1: Definition 1 ensures that the matrices $A_i+B_iK_i$ and $A_0+B_0K_0$ possess some common eigenvalues. Thus, it implies that leader (1) and follower (2) contain certain similarly inner dynamical behaviors and their state dimensions are not limited to be identical. In this paper, F_i and K_i are defined as similar vectors.

Lemma 2[27]: For a given symmetric matrix M with the block matrix form $M=[M_{ij}]$, $M_{11}\in\mathbb{R}^{r\times r}$, $M_{12}\in\mathbb{R}^{r\times(n-r)}$, $M_{22}\in\mathbb{R}^{(n-r)\times(n-r)}$. Then $M=\begin{bmatrix}M_{11} & M_{12}\\ M_{12}^T & M_{22}\end{bmatrix}<0$, $M_{11}<0$, $M_{22}<0$, if and only if one of the following conditions holds

$$\text{(a) } M_{22}-M_{12}^TM_{11}^{-1}M_{12}<0, \quad \text{(b) } M_{11}-M_{12}M_{22}^{-1}M_{12}^T<0 \tag{4}$$

Lemma 3[28]: For any given vector $x,y\in\mathbb{R}^{q\times 1}$ and positive scalar z, then the following inequality be satisfied

$$2x^Ty\le z^{-1}x^Tx+zy^Ty \tag{5}$$

The protocol u_i is said to solve the consensus control problem, which means that all states in follower systems can converge to the states of leader for any given initial state, that is, $\lim_{t\to\infty}\|F_ix_i(t)-F_0x_0(t)\|=0$ can be guaranteed.

Because of the influence of external disturbances, it is difficult to achieve the accurate containment. Therefore, in order to quantitatively analysis the effect of external disturbances on the consensus control problem, a controlled output function is proposed as $e(t)=[e_1^T(t),e_2^T(t),\ldots,e_N^T(t)]^T$

$$e_i(t)=x_i(t)-y_i(t), i=1,2,\ldots,N \tag{6}$$

if $\lim_{t\to+\infty}e(t)=0$, the consensus control problem of the leader-follower system (1)-(2) can be solved. Thus, the attenuating ability of the multi-agent system against external disturbances can be quantitatively measured by

$$\|T_{e\omega}(t)\|_\infty=\sup_{\omega(t)\neq 0}\frac{\|e(t)\|_2}{\|\omega(t)\|_2} \tag{7}$$

where $\omega(t)=[\omega_1^T(t),\omega_2^T(t),\ldots,\omega_N^T(t)]^T$, $T_{e\omega}(t)$ represents the closed-loop transfer function matrix from the external disturbance to the controlled output.

In terms of the above discussions, the H_∞ control goal is proposed as follows:

Control Goal: Protocol u_i is said to solve the H_∞ control problem, if the following two conditions are satisfied simultaneously for all admissible uncertainties:

(1) All followers without external disturbances asymptotically converge to the leader, i.e., $\lim_{t\to+\infty} e(t)=0$.

(2) Under zero-valued initial condition, the control u_i makes the following transfer function hold

$$\left\|T_{e\omega}(t)\right\|_\infty < \gamma \tag{8}$$

where $\gamma > 0$ is a given H_∞ performance index.

3 Main results

In this section, based on the robust H_∞ control theory, the following protocol is designed to solve the consensus control problem of the multi-agent system (1)-(2).

$$u_i(t) = cT\sum_{j=1}^{N} a_{ij}[F_i x_i(t) - F_j x_j(t)] + K_i x_i(t) + \bar{K}\left(F_i x_i(t) - F_0 x_0(t)\right) \tag{9}$$

where $c \in \mathbb{R}$ represents the coupling weight.

Let $\tilde{x}_i(t) = F_i x_i(t)$, and consider the protocol (9), by using Definition 1, it can be rewritten as

$$\dot{\tilde{x}}_i(t) = (A_0 + B_0 K_0)F_i x_i(t) + B_0\bar{K}e_i(t) + B_0 cT \sum_{j=1}^{N} a_{ij}\left[F_i x_i(t) - F_j x_j(t)\right] + F_i f_i(t, x_i(t)) + F_i\omega_i(t) \tag{10}$$

Now, we consider the leader system (1), and let $\tilde{x}_0(t) = F_0 x_0(t)$ with the similarity condition in Definition 1, F_0 is the unit matrix. So the following equation is truth

$$\begin{aligned}\dot{\tilde{x}}_0(t) &= A_0 x_0(t) + B_0 K_0 x_0(t) + f_0(t, x_0(t)) \\ &= (A_0 + B_0 K_0)x_0(t) + f_0(t, x_0(t))\end{aligned} \tag{11}$$

The consensus error is defined as

$$e_i(t) = \tilde{x}_i(t) - \tilde{x}_0(t) = F_i x_i(t) - F_0 x_0(t) \tag{12}$$

By employing the time-derivative operator and using (10)-(11), let $\tilde{f}_i(t) = F_i f_i(t, x_i(t)) - f_0(t, x_0(t))$, error dynamical equation (12) can be further derived as

$$\dot{e}_i(t) = \left(A_0 + B_0 K_0 + B_0\bar{K}\right)e_i(t) + B_0 cT\sum_{j=1}^{N} a_{ij}\left[e_i(t) - e_j(t)\right] + \tilde{f}_i(t) + F_i\omega_i(t) \tag{13}$$

Theorem 1: Consider the nonlinear the leader system (1) and multi-agent follower system (2) with satisfying Definition 1. In the absence of external disturbances $\omega_i(t)=0$. There exist symmetric matrix $Q, P \in \mathbb{R}^{n\times n}$ and positive scalar k_1 and k_2 such that the linear matrix inequalities (LMIs) hold.

$$Q > 0, \lambda > 0, k_1 > 0, k_2 > 0 \tag{14}$$

$$\begin{bmatrix} \Theta & I & I & I & I \\ * & -2k_1 & 0 & 0 & 0 \\ * & * & -2k_2 & 0 & 0 \\ * & * & * & -2k_1^{-1} & 0 \\ * & * & * & * & -2k_2^{-1} \end{bmatrix} < 0 \tag{15}$$

where $\Theta = A_0 Q + QA_0^T + \bar{B}Q + Q\bar{B}^T + B_0 M + MB_0^T - \lambda B_0 B_0^T$, $\bar{B} = B_0 K_0$, $M = \bar{K}Q$, $Q^{-1} = P$. Then, there exists a control protocol (9) such that the nonlinear multi-agent systems in (1) and (2) achieve consensus when $t \to \infty$ and the consensus error e_i, for all $i = 1, 2, \ldots, N$ converge to the origin asymptotically. The controller gain designed by $T = -0.5B_0^T P$.

Proof: Considering the following Lyapunov function:

$$V(t) = \sum_{i=1}^{N} e_i^T(t)Pe_i(t) \tag{16}$$

Firstly, the stability problem of the system (13) without external disturbance is discussed, i.e., $\omega_i(t)=0$. By employing the time-derivative operator, it follows

$$\dot{V}(t) = 2\sum_{i=1}^{N} e_i^T(t)P\left(A_0 + B_0 K_0 + B_0\bar{K}\right)e_i(t) + 2\sum_{i=1}^{N} e_i^T(t) PB_0 cT\sum_{j=1}^{N} a_{ij}\left[e_i(t) - e_j(t)\right] + 2\sum_{i=1}^{N} e_i^T(t)P\tilde{f}_i(t) \tag{17}$$

Considering the following transformations

$$\begin{aligned}2e_i^T(t)P\tilde{f}_i(t) &= e_i^T(t)PF_i f_i(t, x_i(t)) - e_i^T(t)Pf_0(t, x_0(t)) \\ &+ e_i^T(t)PF_i f_i(t, x_i(t)) - e_i^T(t)Pf_0(t, x_0(t))\end{aligned} \tag{18}$$

According to Lemma 2, it gets

$$e_i^T(t)PF_i f_i(t, x_i(t)) \le \frac{1}{2k_1}e_i^T(t)P^T Pe_i(t) + \frac{1}{2}k_1 f_i^T(t, x_i(t))F_i^T F_i f_i(t, x_i(t)) \tag{19}$$

$$-e_i^T(t)Pf_0(t, x_0(t)) \le \frac{1}{2k_2}e_i^T(t)P^T Pe_i(t) + \frac{1}{2}k_2 f_0^T(t, x_0(t))f_0(t, x_0(t)) \tag{20}$$

substituting (18)-(20) into (17), denote $B_0 cT\sum_{j=1}^{N} a_{ij}[e_i(t) - e_j(t)] = c\sum_{i=1}^{N} L_{ij}B_0 Te_j$. Designing $T = -0.5B_0^T P$, it holds

$$\begin{aligned}2\sum_{i=1}^{N} e_i^T(t)Pc\sum_{i=1}^{N} L_{ij}B_0 Te_j &= 2\sum_{i=1}^{N} ce_i^T(t)PL_{ij}B_0\left(-0.5B_0^T P\right)e_j \\ &= -\zeta^T(t)\left[0.5c\left(L + L^T\right)\otimes PB_0 B_0^T P\right]\zeta(t)\end{aligned} \tag{21}$$

Noting

$$-\zeta^T(t)c\left[\left(L + L^T\right)\otimes I\right]\zeta(t) \le -2\alpha(L)c\zeta^T(t)(I\otimes I)\zeta(t) \tag{22}$$

Since $\lambda \le c\alpha(L)$, the result is $-c\alpha(L) \le -\lambda$, then (17) equal to the following inequality

$$\begin{aligned}\dot{V}(t) \le{}& \zeta^T(t)(I\otimes\Phi)\zeta(t) + \zeta^T(t)(I\otimes P)\psi_1(t) \\ &- \zeta^T(t)(I\otimes P)\psi_2(t) + \psi_1^T(t)\left(I\otimes\frac{1}{2}k_1\right)\psi_1(t) \\ &+ \psi_2^T(t)\left(I\otimes\frac{1}{2}k_2\right)\psi_2(t)\end{aligned} \tag{23}$$

where $\zeta^T(t) = \left[e_1^T(t), e_2^T(t), \ldots, e_n^T(t)\right]$, $\psi_1^T(t) = \left[f_1^T(t, x_1(t))F_1^T, f_2^T(t, x_2(t))F_2^T, \ldots, f_n^T(t, x_n(t))F_n^T\right]$, $\psi_2^T(t) = \left[f_0^T(t, x_0(t)), f_0^T(t, x_0(t)), \ldots, f_0^T(t, x_0(t))\right]$,

$$\Phi = P(A_0 + B_0 K_0 + B_0 \bar{K}) + (A_0 + B_0 K_0 + B_0 \bar{K})^T P + \frac{1}{2} P^T k_1^{-1} P$$

$$+\frac{1}{2} P^T k_2^{-1} P - \lambda P B_0 B_0^T P .$$

Then, the inequality (23)<0 can be obtained as follows:

$$\begin{bmatrix} \Phi & P & P \\ * & -2k_1^{-1} & 0 \\ * & * & -2k_2^{-1} \end{bmatrix} < 0 \quad (24)$$

By application of congruence transformation using $diag(P^{-1}, I, I)$ and substituting $Q = P^{-1}$, after employing Schur' Lemma 2, it reveals (15), which completes the proof.

Theorem 2: Consider the leader system (1) and follower nonlinear multi-agent (2) with satisfying Definition 1. If there exist symmetric matrix $Q, P \in \mathbb{R}^{n \times n}$ and a positive scalar γ such that the following LMIs hold, hence the protocol (9) solves the asymptotic consensus control problem with $\|T_{e\omega}(t)\|_\infty < \gamma$ being satisfied.

$$Q > 0, \lambda > 0, k_1 > 0, k_2 > 0 \quad (25)$$

$$\begin{bmatrix} \Theta & I & I & I & I & I & I \\ * & -1 & 0 & 0 & 0 & 0 & 0 \\ * & * & -2k_1 & 0 & 0 & 0 & 0 \\ * & * & * & -2k_2 & 0 & 0 & 0 \\ * & * & * & * & -2k_1^{-1} & 0 & 0 \\ * & * & * & * & * & -2k_2^{-1} & 0 \\ * & * & * & * & * & * & -\gamma^2 \end{bmatrix} < 0 \quad (26)$$

where $\Theta = A_0 Q + Q A_0^T + \bar{B} Q + Q \bar{B}^T + B_0 M + M B_0^T - \lambda B_0 B_0^T$, $\bar{B} = B_0 K_0$, $\bar{K} = M Q^{-1}$, $Q^{-1} = P$.

Proof: Defining the following function

$$J(t, e_i, \tilde{\omega}_i) = \dot{V}(t) + \sum_{i=1}^{N} e_i^T(t) e_i(t) - \sum_{i=1}^{N} \gamma^2 \tilde{\omega}_i^T(t) \tilde{\omega}_i(t) \quad (27)$$

According to (23), equation (27) satisfies the following inequality

$$J(t, e_i, \omega_i) \le \zeta^T(t)(I \otimes \Xi)\zeta(t) + \zeta^T(t)(I \otimes P)\psi_1(t)$$
$$-\zeta^T(t)(I \otimes P)\psi_2(t) + \psi_1^T(t)\left(I \otimes \frac{1}{2}k_1\right)\psi_1(t)$$
$$+\psi_2^T(t)\left(I \otimes \frac{1}{2}k_2\right)\psi_2(t) + \zeta^T(t)(I \otimes I)\zeta(t)$$
$$-\gamma^2 \tilde{\omega}^T(t)(I \otimes I)\tilde{\omega}(t) \quad (28)$$

In order to make sure that $J(t, e_i, \omega_i) < 0$, it is necessary to require matrix $\Xi = P(A_0 + B_0 K_0 + B_0 \bar{K}) + (A_0 + B_0 K_0 + B_0 \bar{K})^T P + \frac{1}{2} P^T k_1^{-1} P + \frac{1}{2} P^T k_2^{-1} P - \lambda P B_0 B_0^T P \le 0$. Multiplying by $diag(P^{-1}, I, I)$ on the both side of this inequality, let $Q = P^{-1}$, by employing the Schur's Lemma, LMI (26) can be hold.

4 Simulation Results

Consider a communication topology with eight nodes as shown in Figure 1.

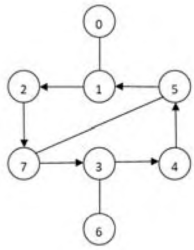

Fig. 1. Communication topology graph

Assuming the leader's nodes are stable, the leader's system is described as

$$\dot{x}_0(t) = A_0 x_0(t) + B_0 u_0(t) + f_0(t, x_0(t)) + \omega_0(t) \quad (29)$$

where $A_0 = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, $B_0 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $f_0 = \begin{bmatrix} -x_{01}(x_{01}^2 + x_{02}^2) \\ -x_{02}(x_{01}^2 + x_{02}^2) \end{bmatrix}$.

The initial condition can be selected as $x_0^0 = (5.5 \quad -2.8)^T$.

Example A: In this subsection, the identical follower multi-agent systems are considered, and the equation is described as

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + f_i(t, x_i(t)) + \omega_i(t), i = 1, 2, \ldots, 7 \quad (30)$$

where $A_i = A_0$, $B_i = B_0$, $f_i = f_0$.

Case (1): External disturbances are assumed as $\omega_i = O_{2\times1}$, the initial values can be taken as the reference [12], parameter $\lambda = 287.5963$ can be obtained by solving the LMI in Theorem 1, and the coupling weight is selected as $c = 490.9462$. The states of leader-follower multi-agent systems are plotted as shown in Figure 2 and 3.

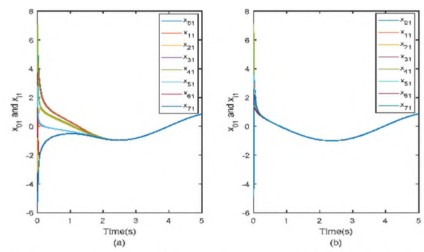

Fig. 2 (a) Consensus time response of x_{01} and x_{i1} in [12]. (b) Consensus time response of x_{01} and x_{i1}.

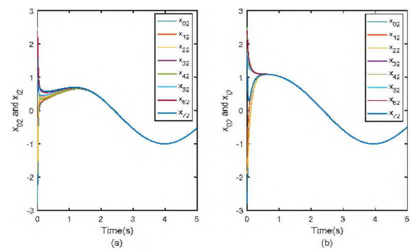

Fig. 3 (a) Consensus time response of x_{02} and x_{i2} in [12]. (b) Consensus time response of x_{02} and x_{i2}.

It can be seen from Figure 2 and Figure 3, if the control was employed in [12], it is easy to know that the consensus of follower system (30) reach to leader (29) at 1.5 second. In addition, comparing with the control in [12], the sates of follower system (30) by utilizing the proposed control (9) shown as (b) in Figure 2 and Figure 3 can reach consensus the states of leader (29) at 0.5 second.

Case (2): Let the external disturbances as shown in reference [12], the consensus of multi-agent systems (29)-(30) are displayed as

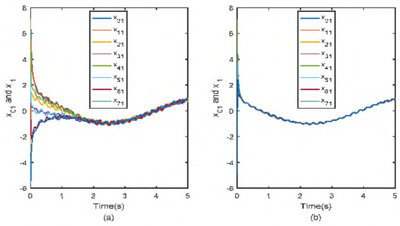

Fig. 4 (a) Consensus time response of x_{01} and x_{i1} in [12].
(b) Consensus time response of x_{01} and x_{i1} .

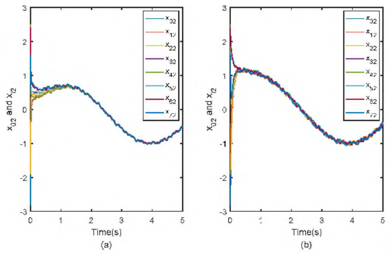

Fig. 5 (a) Consensus time response of x_{02} and x_{i2} in [12].
(b) Consensus time response of x_{02} and x_{i2} .

Two states comparison diagrams of the consensus control implemented by different controllers are shown in Figure 4 and Figure 5. The conclusion of (a) in Figure 4 shows that x_{01} and x_{i1} by the control (9) have a higher speed reach consensus than the proposed control in [12]. Similarly, the states x_{02} and x_{i2} with the control (9) also have faster consensus than the control in [12].

Example B: Different from example A, the dimension of every agent system is different, which are described as

$$A_1=\begin{bmatrix}1&-1&4\\1&1&2\\0&0&-3\end{bmatrix},B_1=\begin{bmatrix}2\\1\\0\end{bmatrix},A_2=\begin{bmatrix}1&-1&4&-2\\1&1&2&-1\\0&0&-3&0\\0&0&0&-5\end{bmatrix}_{4\times4},$$

$$B_2=\begin{bmatrix}2\\1\\0\\0\end{bmatrix},A_3=\begin{bmatrix}1&-1&4&-2&6\\1&1&2&-1&3\\0&0&-3&0&0\\0&0&0&-5&0\\0&0&0&0&-7\end{bmatrix}_{5\times5},B_3=\begin{bmatrix}2\\1\\0\\0\\0\end{bmatrix},$$

$$A_4=\begin{bmatrix}1&-1&4&-2&6&-4\\1&1&2&-1&3&-2\\0&0&-3&0&0&0\\0&0&0&-5&0&0\\0&0&0&0&-7&0\\0&0&0&0&0&-6\end{bmatrix}_{6\times6},B_4=\begin{bmatrix}2\\1\\0\\0\\0\\0\end{bmatrix}^T,$$

$$A_5=\begin{bmatrix}1&-1&4&-2&6&-4&8\\1&1&2&-1&3&-2&4\\0&0&-3&0&0&0&0\\0&0&0&-5&0&0&0\\0&0&0&0&-7&0&0\\0&0&0&0&0&-6&0\\0&0&0&0&0&0&-9\end{bmatrix}_{7\times7},B_5=\begin{bmatrix}2\\1\\0\\0\\0\\0\\0\end{bmatrix},$$

$$A_6=\begin{bmatrix}1&-1&4&-2&6&-4&8&-10\\1&1&2&-1&3&-2&4&-5\\0&0&-3&0&0&0&0&0\\0&0&0&-5&0&0&0&0\\0&0&0&0&-7&0&0&0\\0&0&0&0&0&-6&0&0\\0&0&0&0&0&0&-9&0\\0&0&0&0&0&0&0&-8\end{bmatrix}_{8\times8},B_6=\begin{bmatrix}2\\1\\0\\0\\0\\0\\0\\0\end{bmatrix},$$

$$A_7=\begin{bmatrix}1&-1&4&-2&6&-4&8&-10&-6\\1&1&2&-1&3&-2&4&-5&-3\\0&0&-3&0&0&0&0&0&0\\0&0&0&-5&0&0&0&0&0\\0&0&0&0&-7&0&0&0&0\\0&0&0&0&0&-6&0&0&0\\0&0&0&0&0&0&-9&0&0\\0&0&0&0&0&0&0&-8&0\\0&0&0&0&0&0&0&0&-4\end{bmatrix}_{9\times9},$$

$B_7=\begin{bmatrix}2&1&O_{1\times7}\end{bmatrix}^T$.

The nonlinear function in follower systems can be defined as

$$f_i\left(t,x_i\left(t\right)\right)=[-x_{i1}\left(x_{i1}^2+x_{i2}^2\right),-x_{i2}\left(x_{i1}^2+x_{i2}^2\right),O_{(n_i-2)\times1}]^T \quad (31)$$

The external disturbances are chosen by white noise.

By solving the LMI in Theorem 2, it can get positive matrix $P=\begin{bmatrix}0.0173&-0.0113\\-0.0113&0.1762\end{bmatrix}$, parameter $c=61.0259$ for the L_2 gain γ=0.5 to achieve a robust consensus, the control gain $\bar{K}=\begin{bmatrix}-1.0057&-1.0372\end{bmatrix}$, and the feedback gain in the consensus protocol is $T=\begin{bmatrix}-0.0117&-0.0767\end{bmatrix}$.

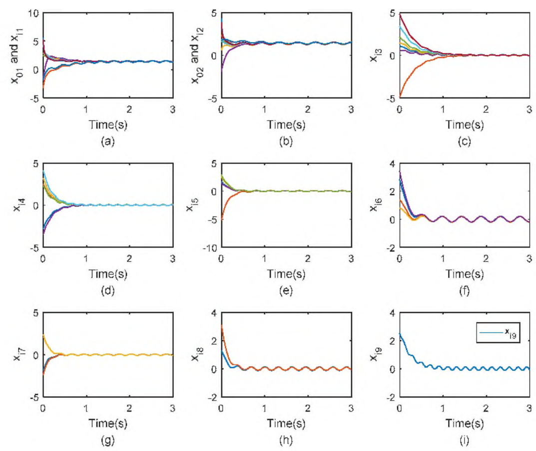

Fig. 6 Consensus of states x_0 and x_i in leader-follower agent systems

As shown in the consistency results in Figure 6, for multi-agent (29)-(30) with different dimensions, the control (9) can be utilized to realize the consensus. Compared with the control designed in [12], the proposed control scheme (9) can not only reach a consensus on multi-agent with identical dimension, but also be effective on nonidentical dimensions.

5 Conclusion

A robust H_∞ consistency control is designed for a class of nonlinear multi-agent systems with the identical and nonidentical dimensions. The proposed control is planed with similar parameters, and the control gain matrix is easily obtained by solving LMIs and Lyapunov theory. Compared with other existing works, the simulation example shows that the control has very good consistency.

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