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Controllability of discrete-time heterogeneous multi-agent systems with time-delay

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Abstract: In this paper, we investigate the controllability of discrete-time heterogeneous multi-agent systems with time-delay and design a class of neighbor-based control protocols. It is established that the rank test and PBH rank test for the discrete-time heterogeneous multi-agent systems with time-delay under fixed topology. Finally, example and numerical simulation are given to demonstrate the effectiveness of the theoretical results.

Key Words: Controllability, heterogeneous multi-agent system, time-delay.

1 Introduction

In recent years, the coordination control of multi-agent systems has been paid more and more attention, which has become a hot topic and is an extremely important and challenging subject in networked science.

There are three main factors to affect the multi-agent system and its behavior including: the dynamics of agent itself, the communication network topology of information exchange between agents and the control protocols. At present, most of the researches on coordinated control of multi-agent systems regard the agents in multi-agent systems as individuals with the same characteristics, for example, agents have the same dynamics or the same observational abilities. In many aspects, such as fixed network topology, switching network topology, random network topology, linear control protocol and nonlinear control protocol, there are very deep research results in this field [1]–[7].

The controllability of multi-agent systems is a basic problem and plays an important role in dealing with a lot of practical problems, which refers to control all followers from any initial states to desired final states by regulating some leaders. The controllability problem of multi-agent systems was first put forward by Tanner [8], an algebraic necessary and sufficient condition was given. After that, the controllability is studied from the viewpoints of graph theory and higher algebra. Liu et al [9]–[11] studied the controllability of first-order and second-order the discrete-time/ continuous-time multi-agent systems, respectively. Su and Liu [12]–[13] investigated the group controllability of first-order and second-order the discrete-time/ continuous-time multi-agent systems, respectively. Guan et al [14] addressed the controllability of continuous-time heterogeneous multi-agent systems directed and weighted topology. The researchers [15]–[34] studied the controllability of the continuous-time multi-agent systems from the viewpoint of graph theory and obtained some interesting results.

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However, in practice, heterogeneity exists widely in all kinds of networked systems. For example, sensor's nodes have different measurements of target state due to various environmental factors and manufacturing processes in wireless sensor networks; each person has different ages, occupations and hobbies in human social networks; as well as aircraft and tanks in joint operations have different models of their own dynamics in military combat networks, etc. For coordinated control of multi-agent systems, individual heterogeneity often seriously affects the cooperative performance of such system, and even leads to instability of such system. Therefore, we should take individual heterogeneity into account when designing cooperative control algorithms for multi-agent systems.

This paper considers the controllability of discrete-time heterogeneous multi-agent systems with time-delay and discusses the effects of dynamics of leaders on the controllability.

The remainder of the paper is organized as follows. Section 2 presents some mathematical preliminaries and the model. Section 3 gives the main results. Section 4 contains some numerical simulations and the last section makes the conclusion.

2 Problem formulation and preliminaries

Consider the controllability of discrete-time heterogeneous multi-agent systems consisting of $n+l$ first-order and second-order integrator agents, labeled from 1 to $n+l$, where the number of first-order integrator agents is $m(m<n+l)$. Each agent is described as

$$\begin{cases} x_i(k+1)=x_i(k)+u_i(k) & i\in\mathscr{I}_m \\ x_i(k+1)=x_i(k)+v_i(k), v_i(k+1)=v_i(k)+u_i(k) & i\in\mathscr{I}_{n+l}/\mathscr{I}_m \end{cases} \tag{1}$$

where $x_i(k)\in\mathbb{R}, u_i(k)\in\mathbb{R}$ and $v_i(k)\in\mathbb{R}$ are the position-like, control input and velocity-like of agent i at time k, respectively.

The information topology among agents are shown by an undirected graph $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{A})$ consisting of a vertex set $\mathscr{V} = \{v_1, v_2, \cdots, v_N\}$ and an edge set $\mathscr{E} = \{(v_i, v_j) : v_i, v_j \in \mathscr{V}\}$. The neighbor set of agent i is denoted by $\mathscr{N}_i = \{j \in \mathscr{V} \mid (j,i) \in \mathscr{E}\}$. The Laplacian is given by $L = \triangle - \mathscr{A}$, where $\triangle$ is the diagonal degree matrix and $\mathscr{A}$ is the adjacency matrix, $L = [l_{ij}] \in \mathbb{R}^{N \times N}$ with

$$l_{ij} = \begin{cases} -a_{ij}, & i \neq j \\ \sum\limits_{j \in \mathscr{N}_i} a_{ij}, & i = j \end{cases}$$

Let $\mathscr{G}_i$ $(i = 1,2,3,4)$ be subgraphs of $\mathscr{G}$, which contains all the first-order integrator followers, all the second-order integrator followers, all the first-order integrator leaders, all the second-order integrator leaders, respectively. For such system, the Laplacian matrix can be partitioned as

$$L = \begin{bmatrix} L_f & L_{fl} \\ L_{lf} & L_l \end{bmatrix} = \begin{bmatrix} F_1 & R_{12} & R_{13} & R_{14} \\ R_{21} & F_2 & R_{23} & R_{24} \\ R_{31} & R_{32} & F_3 & R_{34} \\ R_{41} & R_{42} & R_{43} & F_4 \end{bmatrix},$$

where L_f, L_l, L_{fl} and L_{lf} are the indices of followers, leaders, from leaders to the followers and from followers to the leaders, respectively; F_i corresponds to $\mathscr{G}_i$, $(i = 1,2,3,4)$, $F \triangleq L_f = \begin{bmatrix} F_1 & R_{12} \\ R_{21} & F_2 \end{bmatrix}$, $R \triangleq L_{fl} = \begin{bmatrix} R_{13} & R_{14} \\ R_{23} & R_{24} \end{bmatrix}$. For the discrete-time heterogeneous multi-agent system, suppose that there are l leaders and n followers with $l = l_1 + l_2 \geq 1$, l_1 and l_2 being the numbers of leaders with first-order and second-order integrator, respectively.

3 Main results

Definition 1 *[14] A multi-agent system is said to be controllable if followers can be steered to proper positions to make up any desirable configuration, in a finite time, by regulating the moving of leaders.*

The control protocol is designed as

$$u_i(k) = \begin{cases} \sum\limits_{j \in \mathscr{N}_i} a_{ij}(x_j(k) - x_i(k)), \quad i \in \mathscr{I}_m \\ \sum\limits_{j \in \mathscr{N}_i} a_{ij}(x_j(k) - x_i(k)) + k_1 \sum\limits_{j \in \mathscr{N}_i} a_{ij}(v_j(k-h) - v_i(k-h)), \\ i \in \mathscr{I}_{n+l} / \mathscr{I}_m \end{cases} \qquad (2)$$

where $a_{ij} > 0$ is the weight from agent j to agent i, $h > 0$ is the positive integer, and $k_1 \neq 0$ is the feedback gain.

From the partition of leaders and followers, system (1) under protocol (2) can be rewritten as

$$\begin{bmatrix} y_1(k+1) \\ z_1(k+1) \\ y_2(k+1) \\ z_2(k+1) \\ v_{y_2}(k+1) \\ v_{y_2}(k) \\ v_{y_2}(k-1) \\ v_{y_2}(k-2) \\ \vdots \\ v_{y_2}(k-h+1) \\ v_{z_2}(k+1) \\ v_{z_2}(k) \\ v_{z_2}(k-1) \\ v_{z_2}(k-2) \\ \vdots \\ v_{z_2}(k-h+1) \end{bmatrix} = \begin{bmatrix} I^{(3)}-F_1 & -R_{13} & -R_{12} & -R_{14} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ -R_{31} & I^{(3)}-F_3 & -R_{32} & -R_{42} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & I^{(1)} & 0 & I^{(1)} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & I^{(2)} & 0 & 0 & 0 & \cdots & 0 & 0 & I^{(2)} & 0 & 0 & \cdots & 0 & 0 \\ -R_{21} & -R_{23} & -F_2 & -R_{24} & I^{(1)} & 0 & 0 & \cdots & 0 & -k_1(\widetilde{L}+\widetilde{R}) & 0 & 0 & 0 & \cdots & 0 & k_1\widetilde{P} \\ 0 & 0 & 0 & 0 & I^{(1)} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I^{(1)} & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I^{(1)} & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & I^{(1)} & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ -R_{41} & -R_{43} & -R_{42} & -F_4 & 0 & 0 & 0 & \cdots & 0 & k_1\widetilde{P}^T & I^{(2)} & 0 & 0 & \cdots & 0 & -k_1\widehat{R} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & I^{(2)} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & I^{(2)} & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & I^{(2)} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & I^{(2)} & 0 \end{bmatrix} \begin{bmatrix} y_1(k) \\ z_1(k) \\ y_2(k) \\ z_2(k) \\ v_{y_2}(k) \\ v_{y_2}(k-1) \\ v_{y_2}(k-2) \\ v_{y_2}(k-3) \\ \vdots \\ v_{y_2}(k-h) \\ v_{z_2}(k) \\ v_{z_2}(k-1) \\ v_{z_2}(k-2) \\ v_{z_2}(k-3) \\ \vdots \\ v_{z_2}(k-h) \end{bmatrix} + \begin{bmatrix} 0 \\ u_1(k) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ u_2(k) \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

where $y_1 = (x_1, \ldots, x_{m-l_1})^T$, $z_1 = (x_{m-l_1+1}, \ldots, x_m)^T$, $y_2 = (x_{m+1}, \ldots, x_{n+l_1})^T$, $z_2 = (x_{n+l_1+1}, \ldots, x_{n+l})^T$, $v_{y_2} = (v_{m+1}, \ldots, v_{n+l_1})^T$, $v_{z_2} = (v_{n+l_1+1}, \ldots, v_{n+l})^T$;

$$\widetilde{R} = diag\Big(\sum_{p=1}^{l_2} a_{1p}, \cdots, \sum_{p=1}^{l_2} a_{(n-m+l_1)p}\Big) \in \mathbb{R}^{(n-m+l_1)\times(n-m+l_1)},$$

$$\widehat{R} = diag\Big(\sum_{p=1}^{n-m+l_1} a_{1p}, \cdots, \sum_{p=1}^{n-m+l_1} a_{l_2 p}\Big) \in \mathbb{R}^{l_2 \times l_2},$$

$$\widetilde{P} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1l_2} \\ a_{21} & a_{22} & \cdots & a_{2l_2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{(n-m+l_1)1} & a_{(n-m+l_1)2} & \cdots & a_{(n-m+l_1)l_2} \end{bmatrix} \in \mathbb{R}^{(n-m+l_1)\times l_2},$$

and $\widetilde{L} = [l_{ij}] \in \mathbb{R}^{(n-m+l_1)\times(n-m+l_1)}$ is Laplacian matrix with followers under second-order integrators. By analytic justification, an equivalent augmented system is introduced as follows:

$$\begin{cases} y_1(k+1) = (I^{(3)} - F_1)y_1(k) - R_{12}y_2(k) \\ y_2(k+1) = y_2(k) + v_{y_2}(k) \\ v_{y_2}(k+1) = -R_{21}y_1(k) - F_2 y_2(k) + v_{y_2}(k) \\ \qquad - k_1(\widetilde{L} + \widetilde{R})v_{y_2}(k-h) - R_{23}z_1(k) - R_{24}z_2(k) \\ \qquad + k_1\widetilde{P}v_{z_2}(k-h) \\ v_{y_2}(k) = v_{y_2}(k) \\ \qquad \vdots \\ v_{y_2}(k-h+1) = v_{y_2}(k-h+1) \end{cases},$$

then we can have the following form:

$$
\begin{bmatrix} y_1(k+1) \\ y_2(k+1) \\ v_{y_2}(k+1) \\ v_{y_2}(k) \\ v_{y_2}(k-1) \\ v_{y_2}(k-2) \\ \vdots \\ v_{y_2}(k-h+1) \end{bmatrix} = \begin{bmatrix} I^{(3)}-F_1 & -R_{12} & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & I^{(1)} & I^{(1)} & 0 & 0 & \cdots & 0 & 0 \\ -R_{21} & -F_2 & I^{(1)} & 0 & 0 & \cdots & 0 & -k_1(\widetilde{L}+\widetilde{R}) \\ 0 & 0 & I^{(1)} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & I^{(1)} & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & I^{(1)} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & I^{(1)} & 0 \end{bmatrix}
$$

$$
\cdot \begin{bmatrix} y_1(k) \\ y_2(k) \\ v_{y_2}(k) \\ v_{y_2}(k-1) \\ v_{y_2}(k-2) \\ v_{y_2}(k-3) \\ \vdots \\ v_{y_2}(k-h) \end{bmatrix} + \begin{bmatrix} -R_{13} & -R_{14} & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ -R_{23} & -R_{24} & 0 & \cdots & k_1\widetilde{P} \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} z_1(k) \\ z_2(k) \\ v_{z_2}(k) \\ v_{z_2}(k-1) \\ v_{z_2}(k-2) \\ v_{z_2}(k-3) \\ \vdots \\ v_{z_2}(k-h) \end{bmatrix}
$$

Define $z(k) \triangleq (y_1(k)^T, y_2(k)^T, v_{y_2}(k)^T, v_{y_2}(k-1)^T, \cdots, v_{y_2}(k-h)^T)^T$, $y(k) \triangleq (z_1(k)^T, z_2(k)^T, v_{z_2}(k)^T, v_{z_2}(k-1)^T, \cdots, v_{z_2}(k-h)^T)^T$, then we have

$$
z(k+1) = \mathscr{F} z(k) + \mathscr{P} y(k),\ k \in J_k, \tag{3}
$$

where

$$
\mathscr{F} = \begin{bmatrix} I^{(3)}-F_1 & -R_{12} & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & I^{(1)} & I^{(1)} & 0 & 0 & \cdots & 0 & 0 \\ -R_{21} & -F_2 & I^{(1)} & 0 & 0 & \cdots & 0 & -k_1(\widetilde{L}+\widetilde{R}) \\ 0 & 0 & I^{(1)} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & I^{(1)} & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & I^{(1)} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & I^{(1)} & 0 \end{bmatrix},
$$

and

$$
\mathscr{P} = \begin{bmatrix} -R_{13} & -R_{14} & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ -R_{23} & -R_{24} & 0 & \cdots & k_1\widetilde{P} \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}
$$

with $\mathscr{F} \in \mathbb{R}^{(n+(n-m+l_1)(h+1))\times(n+(n-m+l_1)(h+1))}$, $\mathscr{P} \in \mathbb{R}^{(n+(n-m+l_1)(h+1))\times(l+l_2(h+1))}$.

Note that system (3) can be regarded as a new system without time-delay. Therefore we can have the following result immediately.

Theorem 1: System (3) is controllable iff controllability matrix $\mathscr{C}$ has full row rank, where $\mathscr{C} \triangleq \left[\mathscr{P} \vdots \mathscr{F}\mathscr{P} \vdots \mathscr{F}^2\mathscr{P} \vdots \cdots \vdots \mathscr{F}^{n-1}\mathscr{P}\right]$.

The proof is obvious, here omitted. Matrix $\mathscr{C}$ is so complex that it is difficult to study the controllability of discrete-time heterogeneous multi-agent system (3) by rank test.

Theorem 2. System (3) is controllable iff one of the following conditions holds

$(i) rank(sI-\mathscr{F},\mathscr{P}) = n+(n-m+l_1)(h+1), \forall s \in \mathbb{C}$;

$(ii) rank(\lambda_i I-\mathscr{F},\mathscr{P}) = n+(n-m+l_1)(h+1)$, where $\lambda_i (\forall i = 1,2,\cdots,n+(n-m+l_1)(h+1))$ is the eigenvalue of matrix $\mathscr{F}$.

In order to obtain a more simple method of discrimination, we will discuss the problem for different cases.

Case I ($l_1 = l, l_2 = 0$)

From the partition of leaders and followers, system (1) under protocol (2) can also be rewritten as

$$
\begin{bmatrix} y(k+1) \\ v_y(k+1) \\ v_y(k) \\ \vdots \\ v_y(k-h+1) \\ \widetilde{z}(k+1) \end{bmatrix} = \begin{bmatrix} I_n & I_n & \cdots & 0 & 0 & 0 \\ -F & I_n & \cdots & 0 & -\widetilde{L} & -R \\ 0 & I_n & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & I_n & 0 & 0 \\ -L_{lf} & 0 & \cdots & 0 & 0 & I_l-L_l \end{bmatrix} \begin{bmatrix} y(k) \\ v_y(k) \\ v_y(k-1) \\ \vdots \\ v_y(k-h) \\ \widetilde{z}(k) \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & u_{ext}(k) \end{bmatrix}^T,
$$

where $y(k) \triangleq [x_1(k)\cdots x_n(k)]^T, v_y(k) \triangleq [v_1(k)\cdots v_n(k)]^T, \widetilde{z}(k) \triangleq [x_{n+1}(k)\cdots x_{n+l}(k)]^T$ are the stacked vectors of followers positions, followers velocities and leaders positions, respectively. $u_{ext}(k)$ is the external control inputs. Then the dynamics of the followers can be given as

$$
\begin{bmatrix} y(k+1) \\ v_y(k+1) \\ v_y(k) \\ \vdots \\ v_y(k-h+1) \end{bmatrix} = \begin{bmatrix} I_n & I_n & \cdots & 0 & 0 \\ -F & I_n & \cdots & 0 & -\widetilde{L} \\ 0 & I_n & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_n & 0 \end{bmatrix} \begin{bmatrix} y(k) \\ v_y(k) \\ v_y(k-1) \\ \vdots \\ v_y(k-h) \end{bmatrix} + \begin{bmatrix} 0 \\ -R \\ 0 \\ \vdots \\ 0 \end{bmatrix} \widetilde{z}(k)
$$

$$
\triangleq \Phi \begin{bmatrix} y(k) \\ v_y(k) \\ v_y(k-1) \\ \vdots \\ v_y(k-h) \end{bmatrix} + \Gamma \widetilde{z}(k), \tag{4}
$$

where

$$
\Phi \triangleq \begin{bmatrix} I_n & I_n & \cdots & 0 & 0 \\ -F & I_n & \cdots & 0 & -\widetilde{L} \\ 0 & I_n & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_n & 0 \end{bmatrix}, \Gamma \triangleq \begin{bmatrix} 0 \\ -R \\ 0 \\ \vdots \\ 0 \end{bmatrix}
$$

Case II (l_1 = 0, $l_2 = l$)

From the partition of leaders and followers, system (1) under protocol (2) can also be rewritten as

$$
\begin{bmatrix} y(k+1) \\ \widetilde{z}(k+1) \\ v_z(k+1) \\ v_z(k) \\ v_z(k-1) \\ \vdots \\ v_z(k-h+1) \end{bmatrix} = \begin{bmatrix} I_n-F & -R & 0 & 0 & \cdots & 0 & 0 \\ 0 & I_l & I_l & 0 & \cdots & 0 & 0 \\ -L_{lf} & -L_l & I^{(2)} & 0 & \cdots & 0 & -k_1\widetilde{P} \\ 0 & 0 & I^{(2)} & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & I^{(2)} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & I^{(2)} & 0 \end{bmatrix} \begin{bmatrix} y(k) \\ \widetilde{z}(k) \\ v_z(k) \\ v_z(k-1) \\ v_z(k-2) \\ \vdots \\ v_z(k-h) \end{bmatrix} + \begin{bmatrix} 0 & 0 & u_{ext}(k) & 0 & 0 & \cdots & 0 \end{bmatrix}^T
$$

where $y(k) \triangleq [x_1(k)\cdots x_n(k)]^T$, $\widetilde{z}(k) \triangleq [x_{n+1}(k)\cdots x_{n+l}(k)]^T$, $v_z(k) \triangleq [v_{n+1}(k)\cdots v_{n+l}(k)]^T$ are vectors of followers positions, leaders positions and leaders velocities, respectively.

Then the dynamics of the followers is given as the following form

$$y(k+1) = (I_n - F)y(k) - R\widetilde{z}(k). \tag{5}$$

Theorem 3. For system (1) with $l(= l_1 + l_2)$ leaders and n followers, the following assertions hold:

i) System (1) under protocol (2) is controllable if (F,R) is controllable;

ii) Suppose $l_1 = l, l_2 = 0$, then system (1) under protocol (2) is controllable iff (F,R) is controllable.

iii) Suppose $l_1 = 0$, $l_2 = l$, then system (1) under protocol (2) is controllable iff (F,R) is controllable.

Proof. i). If matrix pair (F,R) is controllable, then $(-F,-R)$ is also controllable. Then $rank[sI+F \quad -R] = n$, for $\forall s \in \mathbb{C}$, then we have

$$rank \begin{bmatrix} sI+F_1 & R_{12} & -R_{13} & -R_{14} \\ R_{21} & sI+F_2 & -R_{23} & -R_{24} \end{bmatrix} = n. \tag{6}$$

Besides, it is easy to find that from equation (6)

$$\begin{aligned}
& rank[\mathscr{F} - \lambda I \ \mathscr{P}] \\
=& rank \begin{bmatrix}
(1-\lambda)I^{(3)}-F_1 & -R_{12} & 0 & 0 & 0 & \cdots & 0 & 0 & -R_{13} & -R_{14} & 0 & \cdots & 0 \\
0 & (1-\lambda)I^{(1)} & I^{(1)} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
-R_{21} & -F_2 & (1-\lambda)I^{(1)} & 0 & 0 & \cdots & 0 & -k_1(\widetilde{L}+\widetilde{R}) & -R_{23} & -R_{24} & 0 & \cdots & k_1\widetilde{P} \\
0 & 0 & I^{(1)} & -\lambda I^{(1)} & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & I^{(1)} & -\lambda I^{(1)} & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & I^{(1)} & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\
0 & 0 & 0 & 0 & 0 & \cdots & I^{(1)} & -\lambda I^{(1)} & 0 & 0 & 0 & \cdots & 0
\end{bmatrix} \\
=& rank \begin{bmatrix}
(1-\lambda)I^{(3)}-F_1 & -R_{12} & -R_{13} & -R_{14} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
-R_{21} & -F_2 & -R_{23} & -R_{24} & 0 & 0 & 0 & \cdots & 0 & \lambda^h(1-\lambda)I^{(1)}-k_1(\widetilde{L}+\widetilde{R}) & 0 & \cdots & k_1\widetilde{P} \\
0 & 0 & 0 & 0 & I^{(1)} & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & 0 & I^{(1)} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & I^{(1)} & \cdots & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \cdots & \vdots \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & I^{(1)} & 0 & 0 & \cdots & 0 \\
0 & (1-\lambda)I^{(1)} & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & \lambda^h I^{(1)} & 0 & \cdots & 0
\end{bmatrix}
\end{aligned} \tag{7}$$

When $\lambda = 0$, then

$$rank[\mathscr{F} - \lambda I \ \mathscr{P}] = rank \begin{bmatrix} I^{(3)}-F_1 & -R_{12} & -R_{13} & -R_{14} \\ -R_{21} & -F_2 & -R_{23} & -R_{24} \end{bmatrix}$$
$$+ (n-m+l_1)h + (n-m+l_1) = n + (n-m+l_1)(h+1);$$

when $\lambda = 1$, then

$$rank[\mathscr{F} - \lambda I \ \mathscr{P}] = rank \begin{bmatrix} -F_1 & -R_{12} & -R_{13} & -R_{14} \\ -R_{21} & -F_2 & -R_{23} & -R_{24} \end{bmatrix}$$
$$+ (n-m+l_1)(h+1) = n + (n-m+l_1)(h+1);$$

when $\lambda \neq 0, \lambda \neq 1$, then

$$rank[\mathscr{F} - \lambda I \ \mathscr{P}] = rank \begin{bmatrix} (1-\lambda)I^{(3)}-F_1 & -R_{12} & -R_{13} & -R_{14} \\ -R_{21} & -\widetilde{R}_{22} & -R_{23} & -R_{24} \end{bmatrix}$$
$$+ (n-m+l_1)(h+1) = n + (n-m+l_1)(h+1),$$

where $\widetilde{R}_{22} = \lambda(\lambda-1)^2 I^{(1)} + \frac{\lambda-1}{\lambda^h} k_1(\widetilde{L}+R) + F_2$. Suppose that (F,R) is controllable, then we can find that $rank[\mathscr{F} - \lambda I] = n + (n-m+l_1)(h+1)$, and then matrix pair $(\mathscr{F}, \mathscr{P})$ is controllable.

ii). If $l_1 = l, l_2 = 0$, then the dynamics of the followers under protocol (2) is given in (4). We will prove that (Φ, Γ) is controllable.

$$\begin{aligned}
& rank[\Phi - \lambda I \ \ \Gamma] \\
=& rank \begin{bmatrix}
I_n - \lambda I_n & I_n & 0 & \cdots & 0 & 0 & 0 \\
F & I_n - \lambda I_n & 0 & \cdots & 0 & -\widetilde{L} & -R \\
0 & I_n & -\lambda I_n & \cdots & 0 & 0 & 0 \\
0 & 0 & I_n & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & I_n & -\lambda I_n & 0
\end{bmatrix} \\
=& rank \begin{bmatrix}
I_n - \lambda I_n & 0 & 0 & 0 & \cdots & 0 & \lambda^h I_n \\
F & -R & 0 & 0 & \cdots & 0 & -\widetilde{L} + \lambda^h I_n - \lambda^{h+1} I_n \\
0 & 0 & I_n & 0 & \cdots & 0 & 0 \\
0 & 0 & 0 & I_n & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & I_n & 0
\end{bmatrix}
\end{aligned}$$

We can easily find that matrix $[\Phi - \lambda I \ \ \Gamma]$ has full row rank for $\forall \lambda \in \mathbb{C}$.

iii). When $l_1 = 0, l_2 = l$, the proof is similar to that of condition ii), omitted here.

Corollary 1. The roots of $det[(-1)^h(((1-\lambda)I^{(3)} - F_1)(((-1)^{(n+(n-m+l_1+1)h+1)}(\lambda^h(1-\lambda)I^{(1)} - k_1(\widetilde{L}+\widetilde{R}))(1-\lambda)I^{(1)}) + \lambda^h I^{(1)} F_2) + \lambda^h I^{(1)} R_{21} R_{12})] = 0$ are some of the eigenvalues of matrix $\mathscr{F}$.

Proof. The eigenvalues of $\mathscr{F}$ are the roots of

$$
\begin{aligned}
& det(\mathscr{F}-\lambda I) \\
&= det\begin{bmatrix}
(1-\lambda)I^{(3)}-F_1 & -R_{12} & 0 & 0 & \cdots & 0 & 0 \\
0 & (1-\lambda)I^{(1)} & I^{(1)} & 0 & \cdots & 0 & 0 \\
-R_{21} & -F_2 & (1-\lambda)I^{(1)} & 0 & \cdots & 0 & -k_1(\widetilde{L}+\widetilde{R}) \\
0 & 0 & I^{(1)} & -\lambda I^{(1)} & \cdots & 0 & 0 \\
0 & 0 & 0 & I^{(1)} & \cdots & 0 & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & I^{(1)} & -\lambda I^{(1)}
\end{bmatrix} \\
&= (-1)^h \cdot det\begin{bmatrix}
(1-\lambda)I^{(3)}-F_1 & -R_{12} & 0 & \cdots & 0 & 0 \\
0 & (1-\lambda)I^{(1)} & 0 & \cdots & 0 & \lambda^h I^{(1)} \\
0 & 0 & I^{(1)} & \cdots & 0 & 0 \\
0 & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & I^{(1)} & 0 \\
-R_{21} & -F_2 & 0 & \cdots & 0 & \lambda^h(1-\lambda)I^{(1)}-k_1(\widetilde{L}+\widetilde{R})
\end{bmatrix} \\
&= det[(-1)^h(((1-\lambda)I^{(3)}-F_1)(((-1)^{(n+(n-m+l_1+1)h+1)} \\
& (\lambda^h(1-\lambda)I^{(1)}-k_1(\widetilde{L}+\widetilde{R}))(1-\lambda)I^{(1)})+\lambda^h I^{(1)}F_2) \\
& +\lambda^h I^{(1)}R_{21}R_{12})] \\
&= 0.
\end{aligned}
$$

Then we can find the roots of $det[(-1)^h(((1-\lambda)I^{(3)}-F_1)(((-1)^{(n+(n-m+l_1+1)h+1)}(\lambda^h(1-\lambda)I^{(1)}-k_1(\widetilde{L}+\widetilde{R}))(1-\lambda)I^{(1)})+\lambda^h I^{(1)}F_2)+\lambda^h I^{(1)}R_{21}R_{12})]=0$ are some of the eigenvalues of matrix $\mathscr{F}$.

4 Simulations

Consider a five-agent network with agents 1-3 as followers and 4-5 as leaders described by Fig. 1 and system (3) is defined by

$$
F=\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}, R=\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}
$$

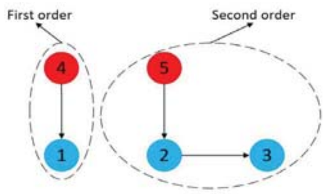

Fig. 1: Topology.

It can be easy to compute rank $[R, FR, F^2R]=3$. Therefore matrix pair (F,R) is controllable. From Theorem 3, such system is controllable. Figs. 2-3 depict the trajectories of such discrete-time heterogeneous multi-agent system with time-delay $h=1$. The pentagram and asterisk denotes the initial state and the final desired configuration, respectively.

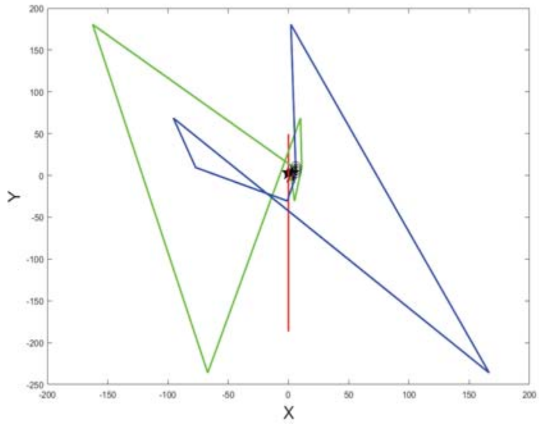

Fig. 2: A straight line.

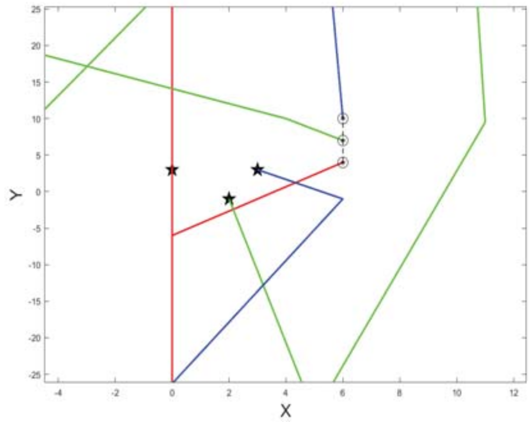

Fig. 3: A magnification of Fig. 2.

5 Conclusion

In this paper, we have investigated the controllability of discrete-time heterogeneous multi-agent systems with time-delay. We have obtained the controllable conditions for discrete-time heterogeneous multi-agent systems via the PB-H test. Specially, we have proved that such discrete-time heterogeneous multi-agent system with time-delay is equivalent to that of matrix pair (F,R) of system.

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