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Controllability of Leader-Follower Networks with Antagonistic Interactions

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Abstract: Signed networks have been the concern in the network control community as they allow studying antagonistic interactions in multi-agent systems(MASs). In this paper, we mainly address the controllability of a class of signed complete graphs with antagonistic interactions based on the leader-follower structure. The research shows that the leader-follower structure proposed by Tanner (2004) is also applicable to the MASs with antagonistic interactions. We accordingly provide two necessary conditions for the controllability of the system. Particularly, we provide an analysis about the controllability problems of Peterson graphs with antagonistic interactions under the partition of partite graph and give a general conclusion about the controllable subspace. Lastly, some simulations are given to verify the result.

Key Words: Antagonistic Interactions; Multi-Agent Systems; Signed Complete Graphs; Leader-Follower Structure; Generalized Equitable Partition; Peterson Graph

1 Introduction

In recent years, cooperative control of MASs has been widely studied, and simultaneously the research outputs have been applied to many areas such as engineering, ecology, biology, sociology, computer communication. Based on cooperative control of MASs many basic important issues has been studied, including consensus [1], uncontrollable topologies construction [2], switching topology [3], [4], etc.

Controllability is a fundamental research topic in the cooperative control of MASs. The controllability of MASs was first presented by Tanner in [5] where one of the agents was appointed as a leader. Based on nearest neighbor interconnections, a necessary and sufficient condition was presented for the controllability in terms of the Laplacian matrix under a fixed time-invariant topology. According to those, some sufficient/necessary conditions are given for the controllability of MAS. Based on this, another algebraic condition was proposed in [6]. Afterwards, researchers investigated the controllability of MASs from the graphic point of view [7], [8]. Specifically, various concepts and properties of graph partitions were employed to study MASs controllability, such as equitable partition [9], relaxed equitable partition [10], etc. Recently, the controllability of MASs on some special graphs was studied as well. Examples include the path and cycle graphs [11], star graphs [12], and so on.

Therefore, this paper is devoted to solve the MASs controllable problem of antagonistic interaction. At first, Leader-Follower structure is introduced into the antagonistic network, which validates the necessary and sufficient conditions of the system proposed by Tanner(2004). The result shows that the controllable conditions under the cooperative network are not completely appropriate for the antagonistic network. At last, we studied the controllability of a complete graph under a class of antagonistic interactions. The rest of this article is arranged as follows. Section 2 presents the notations and the system model. In Section 3, the leader-follower structure is introduced. Section 4 and Section 5 give specific discussion. Section 6 prevents the simulations. Finally, Section 7 shows the conclusion.

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2 PRELIMINARIES

In this section we introduce the algebraic graph theory and the system model. In order to make the paper self-contained we will recall some existing definitions in algebraic graph theory.

2.1 Algebraic Graph Theory

Let $G = (V, E, A)$ represent a undirected graph with antagonistic interactions. An MAS consists of n single integrator dynamics agents, which are labeled by the vertex set $V = \{v_1, v_2, \cdots, v_n\}$. The edge set $E = \{(v_i, v_j) : v_i, v_j \in V\}$ denotes the connection between two nodes, if agent j can directly get information from agent i, i.e., $(v_i, v_j) \in E$, the edge is an ordered pair with the nodes in set V. When $(v_i, v_j) \in E \Leftrightarrow (v_j, v_i) \in E$, signed graph G is called an undirected graph, otherwise directed graph. Assuming there is no self-loop in the graph, i.e., all edges of E are in accord with $i \neq j$. The $n \times n$ signed adjacency matrix is expressed as $A = \{a_{ij}\}$. $a_{ij} \in \{0, 1, -1\}$, if $v_i, v_j \in V(v_i, v_j) \in E$, agent v_i is called a neighbor of agent v_j. If $a_{ij} = 1$, agent j is called a positive neighbor of agent i, $e_{ij} \in E_+$; If $a_{ij} = -1$, agent j is called a negative neighbor of agent i, $e_{ij} \in E_-$, else $v_i, v_j \in V(v_i, v_j) \notin E$, then $a_{ij} = 0$, agent j is not a neighbor of agent i. The edge set $E = E_+ \cup E_-$,

E_+ and E_- denote the sets of positive and negative edges, respectively. In particular, any $a_{ii} = 0$ means that each diagonal elements of A is 0. $N_i = \{v_j \in V : (v_i, v_j) \in E\}$ represents the neighborhood set of vertex i. The number of neighbors is the degree of the node. The degree of vertex v_i is defined as $d_i = \sum_{j=1}^{n} |a_{ij}|$. The valency matrix of a graph G is a diagonal matrix which is defined as $\triangle(G) = diag(d_i) \in \mathbb{R}^{n\times n}$. A graph with antagonistic interactions is called an signed complete graph if any two nodes are neighbors. Let

$$L(G) = \triangle(G) - A(G) \in \mathbb{R}^{n\times n} \tag{1}$$

be the signed Laplacian matrix of signed graph G. If signed graph G was given precisely, it can be abbreviated as L. If graph G is an undirected graph, the signed Laplacian matrix L and the signed adjacency matrix A are symmetrical.

Hence the entries of the matrix L can be written as

$$[L]_{ij} = \begin{cases} \sum\limits_{j\in N_{i_j}} |a_{ij}| & i = j \\ -a_{ij} & i \neq j, j \in N_{i_j} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

2.2 System Description

Consider a multi-agent network with vertex set $V = \{v_1, v_2, \cdots, v_n\}$ which consists of n single integrator dynamics agents. We suppose that $m(m \leq n)$ agents in the network are treated as leaders, and others are followers. Furthermore, assume that the first m agents are selected as leaders, i.e., the set of leaders and followers can be separately expressed as V_L=$\{v_1, \cdots, v_m\}$ and $V_F = V \backslash V_\mathrm{L}$.

The dynamics of each follower $i \in V_F$ is restricted by the updating law

$$\dot{x}_i = -d_i x_i + \sum_{j\in N_{i_j}} a_{ij} x_j \tag{3}$$

where $x_i \in \mathbb{R}$ is the state of agent i and $d_i = \sum_{j=1}^{n} |a_{ij}|$ is the degree of agent i. Each leader $i \in V_L$ is assigned to a control input u_i, and governed by the updating law

$$\dot{x}_i = -d_i x_i + \sum_{j\in N_{i_j}} a_{ij} x_j + u_i \tag{4}$$

The dynamics of the antagonistic network can be expressed as

$$\dot{x} = -Lx + Mu \tag{5}$$

where $x = [x_1, \cdots, x_n]^T \in \mathbb{R}^n$, $u = [u_1, \cdots, u_m]^T \in \mathbb{R}^m$, $L \in \mathbb{R}^{n\times n}$ is the signed Laplacian matrix of interaction topology and $M = \begin{bmatrix} I_m \\ 0_{(n-m)\times m} \end{bmatrix}$.

3 LEADER-FOLLOWER STRUCTURE

An MAS consists of n single integrator dynamics agents, assuming there are two different kinds of agents: some strictly follow the consensus protocol and others can be controlled externally; the former is called followers, while the latter is called leaders. The system (5) can be partitioned as

$$\begin{bmatrix} \dot{x_l} \\ \dot{x_f} \end{bmatrix} = -\begin{bmatrix} L_l & 0 \\ l_{fl} & L_f \end{bmatrix} \begin{bmatrix} x_l \\ x_f \end{bmatrix} + \begin{bmatrix} u \\ 0 \end{bmatrix} \tag{6}$$

L_l and L_f correspond to the serial number of leaders and followers respectively, and l_{fl} expresses the communication connection from leaders to followers; u denotes the control input vector, x_l and x_f are the aggregated state vectors of leaders and followers separately. According to the leader-follower structural division system, (6) can be rewritten as follows

$$\begin{cases} \dot{x_l} = u \\ \dot{x_f} = -L_f x_f - l_{fl} x_l \end{cases} \tag{7}$$

The controllability problem of system (5) is equivalent to the problem of leaders' control over followers, i.e., the controllability problem of the following system

$$\dot{x_f} = -L_f x_f - l_{fl} x_l \tag{8}$$

For a certain topology graph G, system (8) is a linear time invariant system, with the system state matrix $-L_f \in \mathbb{R}^{(n-m)\times(n-m)}$, and the input matrix $-l_{fl} \in \mathbb{R}^{(n-m)\times m}$. Both of the matrices are in connection with the signed Laplacian matrix L. If the leader is single ($m = 1$), matrix $-l_{fl}$ is going to be reduced to a column vector. The controllability matrix of system (8) is

$$C = \left[\, -l_{fl}, L_f l_{fl}, -{L_f}^2 l_{fl}, \cdots, (-1)^n {L_f}^{n-1} l_{fl} \,\right]$$

The above analysis shows that $Tanner$'s leader-follower structure is completely appropriate for the controllability problem of the network with antagonistic interactions. The following discussion will continue to quote the definition by Tanner(2004).

$$\dot{x} = -Fy - rz \tag{9}$$

where $-F$ is the system state matrix; $-r$ is the control input matrix. Then the controllability matrix can be written as

$$C = \left[\, -r, Fr, -F^2 r, \cdots, (-1)^n F^{n-1} r \,\right]$$

Matrix F is symmetric since L is symmetric. Hence there exists a matrix U such that $F = UDU^T$, where the columns of matrix U contain the orthonormal eigenvectors of matrix F, and matrix D is the diagonal matrix of the eigenvalues of matrix F. Thus matrix C can be rewritten as

$$C = \Big[-r, UDU^T r, -\big(UDU^T\big)^2 r, \cdots \\ (-1)^n \big(UDU^T\big)^{n-1} r \Big]$$

It can be simplified as

$$\begin{aligned} C = \Big[-r, UDU^T r, -\big(UDU^T\big)^2 r, \cdots \\ (-1)^n \big(UDU^T\big)^{n-1} r \Big] \\ = U \left[\, -U^T r, DU^T r, -D^2 U^T r, \cdots \right. \\ \left. (-1)^n D^{n-1} U^T r \,\right] \end{aligned}$$

Because U is nonsingular, we can focus on the rank of the above controllability matrix as

$$\left[\, -U^T r, DU^T r, -D^2 U^T r, \cdots, (-1)^n D^{n-1} U^T r \,\right] \tag{10}$$

which is the controllability matrix of the system $\dot{q} = -Dq - U^T rz$. Since matrix D is diagonal and nonsingular, the multiplying of vector by D implies a scaling along each of the vector's dimensions. It's clear that a necessary condition for (10) to be full row rank is that: each row of $U^T r$ is nonzero, i.e., any column of U is not orthogonal to r simultaneously, or else a whole row in (10) will be zero. System $(-F, -r)$ has the same controllability as $\left(-D, -U^T r\right)$. Considering

$$[\, s_i I + D, -U^T r \,] \tag{11}$$

When (11) is full row rank for all eigenvalues of matrix $-D$, i.e., $rank[s_i I + D \;\; -U^T r] = n - m$, it is controllable. S-ince matrix U is invertible and full rank, the column vectors of U are linearly independent, which means

$$rank\left(U^T r\right) = rank\,(r)$$

Assume that the rank of $U^T r$ is p. When the multiplicity of any s_i is greater than p, the number of zero rows of $U^T r$ is greater than p, thus $[\, sI + D, -U^T r \,]$ is not full row rank, which means that the multiplicity of s_i is no more than the rank of r for the system to be controllable.The above arguments result in the following result.

Theorem 1 For an antagonistic network described by $\dot{x} = -Fy - rz$, if the system is controllable, then the following conditions are holded:

- The algebraic multiplicity of each eigenvalue of F is not greater than the rank of r;
- Each of eigenvector of F is not simultaneously orthogonal to all columns of r.

4 PETERSON GRAPH WITH ANTAGONISTIC INTERACTIONS

Peterson graph Fig.1(a) is an undirected graph consisting of 10 vertices and 15 edges, which is also a 3 partite graph. That is the vertices set V of Peterson graph can be divided into 3 non-orthogonal sets of two non-intersecting ones, and the two endpoints of any one edge are not in the same subset. Based on the 3 partite graph, a class of partition can be given

$$\pi = \{\ \{1,4,9\}, \{3,6,7\}, \{2,5,8,10\}\ \}$$

The Laplacian matrix can be written as

$$L = \begin{bmatrix} 3 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 \\ -1 & 3 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 3 & -1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 3 & -1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 3 & -1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & -1 & 3 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 & 3 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 & 0 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 3 & -1 \\ -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 3 \end{bmatrix}$$

For the partition π, where $V_1 = \{1,4,9\}$, $V_2 = \{3,6,7\}$, $V_3 = \{2,5,8,10\}$. Matrix C represents the controllability matrix of leader-follower structure. The controllable matrix of the corresponding full-positive network is C_E.

$$C_E = [\, -r_E, Fr_E, -F^2 r_E, \cdots, (-1)^n F^{n-1} r_E \,]$$

As shown in Fig.1(b), all vertexes of V_1 are treated as leaders. Then

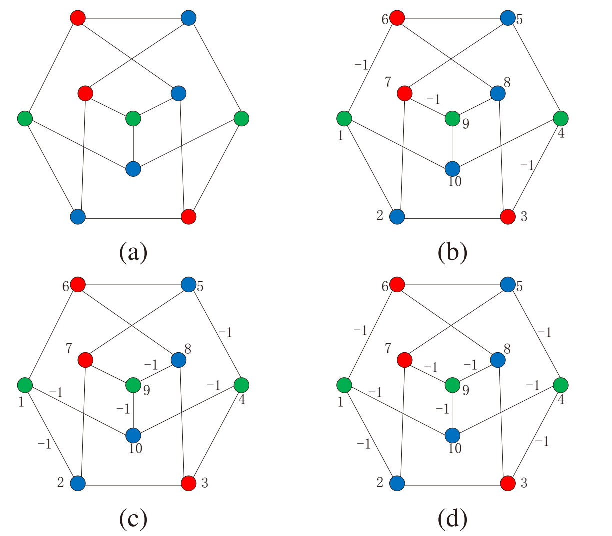


Fig. 1: (a) Peterson graph, (b) $E_- = \{e_{16}, e_{34}, e_{79}\}$, (c) $E_- = \{e_{12}, e_{1,10}, e_{45}, e_{4,10}, e_{98}, e_{9,10}\}$, (d) $E_- = \{e_{16}, e_{34}, e_{79}, e_{12}, e_{1,10}, e_{45}, e_{4,10}, e_{98}, e_{9,10}\}$.

$$F = \begin{bmatrix} 3 & -1 & 0 & 0 & -1 & 0 & 0 \\ -1 & 3 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 3 & -1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 3 & 0 & -1 & 0 \\ -1 & 0 & -1 & 0 & 3 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}, r = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

The ranks of C and C_E are

$$rank\,(C) = 4, rank\,(C_E) = 4$$

Similarly, as shown in Fig.1(c), when all vertexes of V_1 are treated as leaders, we can get $rank\,(C) = 4, rank\,(C_E) = 4$. When all vertexes of V_3 are treated as leaders, the result is the same as the above.

Lastly, as shown in the Fig.1(d), when all vertexes of V_1 are treated as leaders, we have the same observation $rank\,(C) = 4, rank\,(C_E) = 4$. When all vertexes of V_3 are treated as leaders, the result is the same as the above. This proves Lemma 1. When the vertex set V of a signed graph can be partitioned into two non-intersecting subsets V_1 and V_2, so as to every edges between V_1 and V_2 is negative and all the others is positive, then the graph is structural balance.

Lemma 1 [13] Suppose that the interconnected antagonistic network is structurally balanced. If the leaders are chosen from the same subset, then the controllability of (F, r) is equivalent to that of (F, r_E).

The above discussions together with Lemma 1 can result in the following Theorem.

Theorem 2 The controllability of Peterson graph with negative edges selected as the follow procedure is the same as that of the corresponding full-positive with the above assignment of negative edges and the selection of leaders, the controllability of the Peterson graph is invariant with respect to the full-positive network.

- All the edges between V_1 and V_2/V_3 are assigned as negative; Selecting all nodes in V_1 or V_2/V_3 as leaders.

- All the edges between V_1 and V_2, V_1 and V_3 as well, are assigned as negative; Selecting all nodes in V_1, V_2 or V_3 as leaders.

Theorem 2 also applies to the general 3 partite graph.

5 SIGNED COMPLETE GRAPHS

Consider an antagonistic network consists of n agents. Let the first m agents be selected as leaders, and others be followers, where $m \leq [\frac{n}{2}]$. $[x]$ is the intpart of x. There are no negative edges among the followers. Since the all-positive complete graph consisting of n nodes, there are $[\frac{n}{2}]$ non-adjacent edges at most. Each of the non-adjacent edges is only connected to one leader, i.e., there are $m \leq [\frac{n}{2}]$ leaders. The Laplacian matrix L can be expressed as

$$L = \begin{bmatrix} n-1 & -1 & \cdots & -1 \\ -1 & n-1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & -1 \\ -1 & \cdots & -1 & n-1 \end{bmatrix} \in \mathbb{R}^{n \times n}$$

Under the leader-follower structure, matrix F is

$$F = \begin{bmatrix} n-1 & -1 & \cdots & -1 \\ -1 & n-1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & -1 \\ -1 & \cdots & -1 & n-1 \end{bmatrix} \in \mathbb{R}^{(n-m) \times (n-m)}$$

The elements of matrix $r \in \mathbb{R}^{(n-m) \times m}$ are all -1. Since $F = UDU^T$, matrix D can be written as

$$D = diag \begin{pmatrix} m & \underbrace{n \cdots n}_{n-m-1} \end{pmatrix}$$

According to Theorem 1, $n - m - 1 \leq rank\,(r)$ is the necessary condition for the controllability of system (9).

5.1 Single Negative Edge

Consider a signed complete graph consisting of n nodes with $m \leq [\frac{n}{2}]$ non-adjacent negative edges. Each of these edges takes a node as a leader, and the rest are the followers.

Example 1 Consider the signed complete graph shown in Fig.2, e_{13} is the only negative edge. If the nodes 1 and 4 are

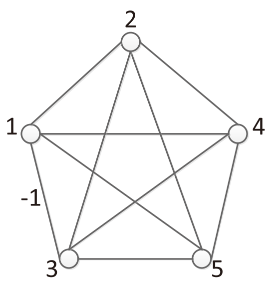


Fig. 2: A signed complete graph with a single negative edge.

specified as leaders, matrix F and r are written as

$$F = \begin{bmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{bmatrix}, r = \begin{bmatrix} -1 & -1 \\ 1 & -1 \\ -1 & -1 \end{bmatrix}$$

The rank of the controllable matrix of (9) is

$$rank\,(C) = 2, rank\,(C_E) = 1$$

Remark 1 For any signed complete graph that contains a single negative edge, the dimension of controllable subspace is larger than the corresponding full-positive network.

Lemma 2 For a signed complete graph, there are two kinds of leaders: some cooperative with all of the followers, called the cooperative leaders; some have antagonistic interactions with the followers, called the antagonistic leaders. The cooperative leaders does not affect the dimension of controllable subspace when the antagonistic leaders exists.

proof : If we assign a cooperative leader to a signed complete graph, matrix r will reduce one row in which all elements is -1 and add one column in which all elements is -1. In the controllable matrix C, it is shown that a linearly dependent row is reduced, and thus the rank of C is not affected. ■

Theorem 3 For a signed complete graph, each addition of a non-adjacent negative edge implies that the rank of matrix C increases by 1 accordingly. System (F, r) is controlled if and only if there exists $m = [\frac{n}{2}]$ edges which are not adjacent, and the dimensions of the controllability matrix is conform to $rank(C) = n - m$.

proof : For a signed complete graph, according to Lemma 2, let us select the antagonistic leaders to constitute the matrix r. If this graph has a single negative edge, there will be a row that is linearly independent with the rest rows in the controllable matrix C, i.e., $rank(C) = 2$. If a non-adjacent negative edge is added, a linear unrelated column vector will be added in the matrix r, and there are two rows that are linearly independent and the other rows in the matrix C are linearly independent, i.e., $rank(C) = 3$. Thus, the rank of the controllability matrix is increased by 1 for each additional negative edge that is not adjacent to each other. Thus the system is controllable if and only if it is increased to m non-adjacent negative edges. ■

Example 2 Based on Example 1, e_{13}, e_{24} are chosen as negative edges, and 1, 2 are selected as leaders. Then

$$F = \begin{bmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{bmatrix}, r = \begin{bmatrix} -1 & -1 \\ 1 & -1 \\ -1 & 1 \end{bmatrix}$$

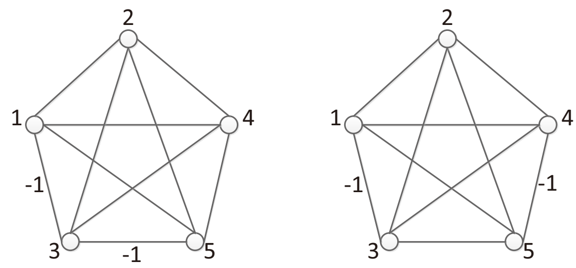
The rank of C is

$$rank \left[-r, Fr, -F^2 r \right] = 3$$

The controllable matrix is full row rank, i.e., the system is controllable.

5.2 Two Negative Edges

For a further discussion, we consider a signed complete graph consisting of n nodes, $n \geq 5$. There are two negative edges, and each negative edge has only one vertex which is selected as leaders. There are two cases: One is that two negative edges are non-adjacent to each other, e.g. Fig.3(a); The other is that two negative edges share a common node, e.g. Fig.3(b). In Fig.3(a), if we choose nodes 1, 5 as leaders,



(a) (b)

Fig. 3: (a)$E_- = \{e_{13}, e_{35}\}$, (b)$E_- = \{e_{13}, e_{45}\}$.

matrix F and matrix r can be written as

$$F = \begin{bmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{bmatrix}, r = \begin{bmatrix} -1 & -1 \\ 1 & 1 \\ -1 & -1 \end{bmatrix}$$

The rank of C is

$$\begin{aligned} rank \left[-r, Fr, -F^2r \right] = 2 \\ rank \left[-r_E, Fr_E, -F^2r_E \right] = 1 \end{aligned}$$

If we select node 3 as leader, the followers are symmetrical relative to the leader,

$$F = \begin{bmatrix} 4 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 \\ -1 & -1 & 4 & -1 \\ -1 & -1 & -1 & 4 \end{bmatrix}, r = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$

It follows that

$$\begin{aligned} rank \left[-r, Fr, -F^2r, F^3r \right] = 1 \\ rank \left[-r_E, Fr_E, -F^2r_E, F^3r_E \right] = 1 \end{aligned}$$

Lemma 3 [14] A system is uncontrollable if it is leader symmetric.

Lemma 4 If a signed complete graph has $n - m = 2n_0$ followers and n_0 followers have negative edges with each leader, while the rest followers have full-positive edges. Then the controllability of the antagonistic network is equivalent to that of the corresponding full-positive network, and $rank(C) = 1$.

proof : Matrix r can be written as

$$r = \begin{bmatrix} 1_{n_0 \times m} \\ -1_{n_0 \times m} \end{bmatrix}$$

There are n_0 identical row vectors which can be expressed as α_{n_0} that are linearly dependent of other n_0 identical row vectors expressed as $-\alpha_{n_0}$. Thus $rank(C) = 1$. ■

In Fig.3(b), if nodes 1, 5 are choose as leaders,

$$F = \begin{bmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{bmatrix}, r = \begin{bmatrix} -1 & -1 \\ 1 & -1 \\ -1 & 1 \end{bmatrix}$$

We can get

$$\begin{aligned} rank \left[-r, Fr, -F^2r, F^3r, -F^4r \right] = 3 \\ rank \left[-r_E, Fr_E, -F^2r_E, F^3r_E, -F^4r_E \right] = 1 \end{aligned}$$

Remark 2 According to Theorem 3, if there are two non-adjacent negative edges, the dimension of controllable subspace is increased by 2 compared with the corresponding full-positive network. The above discussions can be summarized in the following Theorem.

Theorem 4 A signed complete graph with two negative edges. If two negative edges are non-adjacent to each other, the dimension of controllable subspace increases by 2; if two negative edges share a common node, the dimension of controllable subspace increases by 1. The above conclusion holds if and only if the following two conditions are satisfied:

- There are $n - m = 2n_0 - 1$ followers, $n \geq 5$;
- There are $n - m = 2n_0$ followers, $n > 5$.

6 SIMULATION

Example 3 Consider a multi-agent network with antagonistic interactions shown in Fig.4.

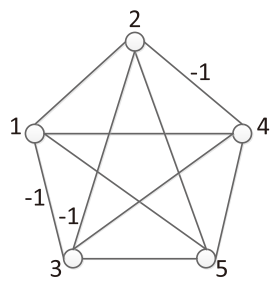


Fig. 4: $E_- = \{e_{13}, e_{23}, e_{24}\}$.

If agents 1 and 2 are specified as leaders, the Laplacian matrix L and the control matrix M are

$$L = \begin{bmatrix} 4 & -1 & 1 & -1 & -1 \\ -1 & 4 & 1 & 1 & -1 \\ 1 & 1 & 4 & -1 & -1 \\ -1 & 1 & -1 & 4 & -1 \\ -1 & -1 & -1 & -1 & 4 \end{bmatrix}, M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

According to the system (8), the control matrix and the input matrix are

$$L_f = \begin{bmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{bmatrix}, l_{fl} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ -1 & -1 \end{bmatrix}$$

The controllability matrix in system (5) and (8) can be respectively written as

$$\begin{aligned} rank \left[M, LM, L^2M, L^3M, L^4M \right] = 5 \\ rank \left[-l_{fl}, L_f l_{fl}, -{L_f}^2 l_{fl} \right] = 3 \end{aligned}$$

It shows that the leader-follower structure is also applicable to the antagonistic network.

Example 4 For a signed complete graph consisting of 13 nodes, matrix F can be written as

$$F = \begin{bmatrix} 12 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & 12 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & 12 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & 12 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & 12 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & 12 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & 12 \end{bmatrix}$$

Matrix r is

$$r = \begin{bmatrix} 1 & -1 & -1 & -1 & -1 & 1 \\ -1 & 1 & -1 & -1 & -1 & -1 \\ -1 & -1 & 1 & -1 & -1 & 1 \\ -1 & -1 & -1 & 1 & -1 & 1 \\ -1 & -1 & -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 \end{bmatrix}$$

We get $rank\,(r) = 6$. It can be verified that F and r are in accord with Theorem 1. The rank of controllability matrix is

$$rank \left[-r, Fr, -F^2r, F^3r, -F^4r, F^5r, -F^6r \right] = 6$$

It's uncontrollable. It can be seen that the controllable conditions in regard to F and r between antagonistic and cooperative interactions is not in agreement.

7 CONCLUSION

In this paper, the MASs controllability with antagonistic interactions based on leader-follower systems has been studied. We have introduced the leader-follower structure under signed graphs with antagonistic interactions, are studied the controllability of the signed complete graphs and partite graphs and discussed the relationship between the controllability of antagonistic and cooperative interactions. The result shows that the controllability of the signed complete graphs conditions is equivalent to that of full-positive network. The derivation of the result provides a quantitative analysis for controllability. In future, more focus will be given on the controllability of general graph with antagonistic interactions.

References

[1] Kaien Liu, Zhijian Ji, and Wei Ren. Necessary and sufficient conditions for consensus of second-order multi-agent systems under directed topologies without global gain dependency, IEEE Transactions on Cybernetics, 47(8): 2089-2098, 2017.

[2] Zhijian Ji, Hai Lin, and Haisheng Yu. Protocols design and uncontrollable topologies construction for multi-agent networks. IEEE Transactions on Automatic Control, 60(3): 781-786, 2015.

[3] Ji Z, Lin H, Tong H L. Controllability of Multi-agent Systems with Switching Topology[C] Robotics, Automation and Mechatronics, 2008 IEEE Conference on. IEEE, 421-426, 2008.

[4] Yongqiang Guan, Zhijian Ji, Lin Zhang, Long Wang, Decentralized stabilizability of multi-agent systems under fixed and switching topologies, Systems and Control Letters (IF: 1.886), vol.62, no.5, pp.438-446, 2013.

[5] Tanner H G. On the controllability of nearest neighbor interconnections[C] Decision and Control, 2004. Cdc. IEEE Conference on. IEEE, Vol.3, 2467-2472, 2004.

[6] Ji Z, Wang Z, Lin H, et al. Interconnection topologies for multi-agent coordination under leaderfollower framework [J]. Automatica, Vol.45, No.12, 2857-2863, 2009.

[7] Ji Z, Lin H, Tong H L. A graph theory based characterization of controllability for multi-agent systems with fixed topology[C] Decision and Control, 2008. Cdc 2008. IEEE Conference on. IEEE, 5262-5267, 2008.

[8] Guan Y, Ji Z, Zhang L, et al. Structural controllability of higher-order multi-agent systems under absolute and relative protocols[C] Control Conference. IEEE, 6832-6837, 2013.

[9] Rahmani A, Ji M, Mesbahi M, et al. Controllability of Multi-Agent Systems from a Graph-Theoretic Perspective[J]. Siam Journal on Control Optimization, Vol.48, No.1, 162-186, 2009.

[10] Zhang S, Cao M, Camlibel M K. Upper and Lower Bounds for Controllable Subspaces of Networks of Diffusively Coupled Agents[J]. IEEE Transactions on Automatic Control, Vol.59, No.3, 745-750, 2014.

[11] Xianzhu Liu, and Zhijian Ji. Controllability of multi-agent systems based on path and cycle graphs, International Journal of Robust and Nonlinear Control; 1-15. DOI: 10.1002/rnc.3870, 2017.

[12] Yongcui Chao, and Zhijian Ji. Necessary and sufficient conditions for multi-agent controllability of path and star topologies by exploring the information of second-order neighbors, IMA Journal of Mathematical Control and Information, July 15, DOI: 10.1093/imamci/dnw013, 2016.

[13] Sun C, Hu G, Xie L. Controllability of Multi-agent Networks with Antagonistic Interactions[J]. IEEE Transactions on Automatic Control, PP(99): 1-1, 2017.

[14] Rahmani A, Ji M, Mesbahi M, et al. Controllability of Multi-Agent Systems from a Graph-Theoretic Perspective[J]. Siam Journal on Control optimization, 48(1): 162-186, 2009.