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# Leader-following consensus of multi-agent systems under antagonistic networks

Qianyao Wang [a], Kexin Liu [b], Xiong Wang [c], Lulu Wu [d], Jinhu Lü [b,*]

[a] Key Laboratory of Systems and Control, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China
[b] School of Automation Sciences and Electrical Engineering, State Key Laboratory of Software Development Environment, and Beijing Advanced Innovation Center for Big Data and Brain Computing, Beihang University, Beijing 100191, China
[c] School of Mathematics, Northwest University, Xi'an 710075, China
[d] School of Science, Shandong Jianzhu University, Jinan 250101, China

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ABSTRACT

In this paper, we address the leader-following consensus of multi-agent systems (MASs) under a class of antagonistic networks. Different from the traditional literature, in our model, whether a follower can access the leader depends on a matrix instead of a scalar. For antagonistic and directed networks with structurally balanced properties, sufficient conditions are established under which the MASs will achieve bipartite consensus. Detailed algorithms are proposed to design the gain matrices and the coupling strength. What is more, we design adaptive laws to self-tune the value of the coupling strength. Simulation results are also given to verify the effectiveness of our methods.



## 1. Introduction

In recent years, due to its wide applications in many fields, such as physics, biology, social networks, and engineering sciences [1–4], the consensus problem of multi-agent systems(MASs) has been extensively investigated.

There is a lot of literature focusing on the consensus analysis of MASs, such as [5–7]. We just name a few examples. DeLellis et al. [5] investigated the consensus of MASs with nonlinearly coupled network, where the inner coupling matrix is not with full rank. Yu *et al.*. studied synchronization via pinning control on general complex dynamical networks [7]. Ni and Cheng [8] considered the leader-following consensus of linear MASs under fixed and switching topologies. In [9], discrete-time consensus protocol for a group of agents were proposed which communicated through a class of stochastically switching networks. [10] studied synchronization on dynamical networks whose topology and intrinsic parameters stochastically change. In [11–13], multiplex proportional-integral protocols were proposed to realize the consensus of heterogeneous MASs affected by constant disturbances.

It is noticed that a vast amount of the literature focusing on the consensus of MASs under cooperative networks [14–16]. However, in several real world scenarios, it is more plausible to assume that there both exist cooperative behaviors and competitive behaviors among the agents. Antagonistic interaction in networks are very common in the networked system, such as trust dynamics [17], opinion dynamics [18–21], and social network graphs [22]. The antagonistic networks are with some negative weights, where a positive/negative weight can be associated to an allied/adversary relationship or trust/distrust, like/dislike interaction between the two agents, see [23–25]. In general, collective behaviors of MASs over signed graphs is more challenging than those over nonnegative graphs, because the adjacency matrix of the signed graph ceases to be a nonnegative matrix [22]. Up to now, there have been some reported results studying the consensus of antagonistic networks, for example, Claudio Altafini extended the notion of consensus and its distributed feedback designs to networks containing interactions which are competitive in nature,modeled as negative weights on the communication edges in [26,27] considered the consensus and bipartite consensus in high-order multi-agent dynamical systems with antagonistic interactions.

Most of the previous literature on leader-following consensus of MASs is based on the common assumption, that the followers always share the same input matrix, like [28]. In real practices, due to the constraint of the system configuration or the controlla-

* Corresponding author.
*E-mail addresses:* wangqianyao15@mails.ucas.edu.cn (Q. Wang), skxliu@163.com (K. Liu), xiongwang@amss.ac.cn (X. Wang), wululu1762248@126.com (L. Wu), jhlu@iss.ac.cn (J. Lü).

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bility of the system, the followers are often with different input matrices. Thus, an agent can obtain the information about the leader and depends on the values of a matrix rather than a scalar like [29]. Recently, Liu et al. made an significantly attempt on the leader-following of MASs with heterogeneous input matrices in [15]. However, the communication network among the agents were assumed to be cooperative. As far as we know, there are fewer literatures focusing on the consensus analysis of these kinds of MASs under antagonistic networks.

So, connecting the antagonistic networks that always exist in the realistic world and different input matrices, we analyze the leader-following consensus.

In this paper, we analyze the leader-following consensus of a class of MASs in the presence of different input matrices and antagonistic communication networks. For structurally balanced directed network, conditions are derived to guarantee the existence of relevant parameters such that the MASs achieve bipartite consensus, where all followers converge to a leader which is the same for all in modulus but not in sign. In addition, we also utilize the adaptive regulation method to self-tune the coupling strength of the network.

The remainder of this paper is organized as follows. In Section 2, some preliminaries are presented and the problem we are to study is stated. In Section 3, two theorems on the leader-following of MASs are shown. An adaptive law to self-tune the coupling strength of leader–follower systems is proposed in Section 4. Simulation is given in Section 5. Finally, conclusions are drawn in Section 6.

## 2. Preliminaries

### 2.1. Nonnegative graph

A graph is denoted by $\mathcal{G} = (\{\mathcal{V}, \mathcal{E}\}, \mathcal{A})$, where $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ is the nodes set, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edges set and $\mathcal{A}$ is the corresponding adjacency matrix. $\mathcal{A} = [\alpha_{ij}]$ and $\mathcal{A} \in \mathbb{R}^{N\times N}$ with being the (i,j) th entry, if and only if $(v_j, v_i) \in \mathcal{E}$ , which means $v_i$ can get the information from $v_j$. In this case, $v_j$ is called a neighbor of $v_i$. Using a set $N_i$ to describe the indices of all neighbors of $v_i$, i.e., $N_i = \{j | (v_j, v_i) \in \mathcal{E}\}$. We assume that there is no self-loop in the graph, that is, $(v_j, v_i) \subsetneq \mathcal{E}$ for all $i = 1, 2, \ldots, N$. Define $\mathcal{G}(\mathcal{A})$ as a graph with $\mathcal{A}$ being the associated adjacency matrix. A graph is undirected, if $\alpha_{ij} = \alpha_{ji}$ for all i and j. A graph is directed, known as digraph, if all its edges are directional. If $\alpha_{ij} \geqslant 0$ for all i and j, then graph $\mathcal{G}(\mathcal{A})$ is called nonnegative. The Laplacian matrix $\mathcal{L} \in \mathbb{R}^{N\times N}$ is defined by $\mathcal{L}_{ij} = -\alpha_{ij}, i \neq j$ and $\mathcal{L}_{ii} = \sum_{j=1}^{N} \alpha_{ij}$.

### 2.2. Signed Graph

A (weighted) signed graph $\mathcal{G}$ is denoted by $\mathcal{G} = (\{\mathcal{V}, \mathcal{E}\}, \mathcal{A})$ where $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ is a set of nodes, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of edges, and $\mathcal{A} = [\alpha_{ij}] \in \mathbb{R}^{N\times N}$ is the matrix of the signed weights of $\mathcal{G} : \alpha_{ij} \neq 0 \Longleftrightarrow (v_j, v_i) \in \mathcal{E}$. The adjacency matrix $\mathcal{A}$ alone completely specifies a signed graph. For the signed graph corresponding to $\mathcal{A}$ we shall use the notation $\mathcal{G}(\mathcal{A})$. We will not consider graphs with self-loops: $\alpha_{ii} = 0, i = 1, 2, \ldots, N$.

### 2.3. Problem formulation

Consider the MASs consisting of one leader and N followers. The agents exchange information through the communication network, which is represented by the antagonistic graph mentioned before. Let the leader be labeled as node 0, set $\widetilde{\mathcal{V}} = \mathcal{V} \cup \mathcal{V}_0$, and the graph $\mathcal{G} = \left(\left\{\widetilde{\mathcal{V}}, \widetilde{\mathcal{E}}\right\}, \widetilde{\mathcal{A}}\right)$. The dynamic equations of multi-agent system with N followers and a leader are described by.

The leader:

$$\dot{x}_0 = Ax_0 \tag{1}$$

Followers:

$$\begin{aligned} \dot{x}_i = &\quad Ax_i - \nu\sum_{j=1}^{N}|\alpha_{ij}|(x_i - \mathrm{sgn}(\alpha_{ij})x_j) + B_i\mu_i \\ i = &\quad 1, 2, \ldots, N, \end{aligned} \tag{2}$$

where $A = [a_{ij}] \in \mathbb{R}^{N\times N}, B_i \in \mathbb{R}^{n\times m_i}$are the system matrices, $\nu > 0$ is the coupling strength, and $\mu_i$ will be constructed below.

The MASs considered the different input matrix $B_i$ compared with the traditional models with a common matrix $B$, like [26].

**Definition 1.** [26,30] Consider the linear time-invariant MASs (2) under antagonistic networks $\mathcal{G}$. The network of MASs is said to achieve bipartite consensus if

$$\lim_{t\to\infty}\|d_i x_i - x_0\| = 0, \quad i = 1, 2, \ldots, N, \tag{3}$$

where $d_i \in \{-1, 1\}$.

**Definition 2.** A connected signed digraph $\mathcal{G}$ is said to be structurally balanced if the graph can be divided into two disjoint subgroups $\mathcal{V}_1$ and $\mathcal{V}_2$, with $\mathcal{V}_1 \cap \mathcal{V}_2 = \varnothing$, such that $\alpha_{ij} \geqslant 0$ for all nodes $v_i, v_j$ in the same subgroup; $\alpha_{ij} \leqslant 0$ for nodes in different subgroups.

**Lemma 1.** *(Schur complement [33]) Let*

$$S = \begin{bmatrix} A & B \\ B^T & C \end{bmatrix},$$

*where A and C are symmetric and square. The following two properties are equivalent*

1. *S is negative definite.*
2. *Matrices A and $C - B^T A^{-1} B$ are negative definite.*

**Lemma 2.** *[26] A signed graph $\mathcal{G}$ is structurally balanced if and only if there exists a signature matrix D such that $\bar{\mathcal{A}} = (|\alpha_{ij}| = D\mathcal{A}D)$ is a nonnegative matrix, when $D = \mathrm{diag}\{d_1, d_2, \ldots, d_N\},\ d_i \in \{-1, 1\}$.*

**Lemma 3.** *[26] A connected signed graph $\mathcal{G}$ is structurally balanced if and only if any of the following equivalent conditions holds:*

1. *All cycles of $\mathcal{G}$ are positive;*
2. *$\exists D \in \mathcal{D}, \mathcal{D} = \mathrm{diag}\{d_1, d_2, \ldots, d_i, \ldots, d_N\}$, where $d_i \in \{1, -1\}, i = 1, 2, \ldots, N$, such that $D\mathcal{A}D$ has all nonnegative entries;*
3. *0 is an eigenvalue of its Laplacian matrix.*

## 3. Bipartite consensus over antagonistic networks

This section presents our main results on different networks, which are connected through two kinds networks, strongly connected antagonistic network, and a directed antagonistic network that contains a spanning tree. Both of the networks are structurally balanced.

### 3.1. Bipartite consensus over strongly connected antagonistic network

This subsection focuses on the consensus of leader-following MASs over a strongly connected network $\mathcal{G}$,and the graph $\mathcal{G}$ containing the nodes of followers $x_i, i = 1, 2, \ldots, N$ is strongly connected.

**Lemma 4.** [32] *For a matrix $H \in \mathbb{R}^{N\times N}$ satisfying $\sum_{j=1}^{N} H_{ij} = 0$ and $H_{ij} < 0$ with $i \neq j$, all its eigenvalues of H have nonnegative real parts and one of its eigenvalues is 0 with the corresponding right eigenvector $1_N$. Here, $1_N$ denotes the vector in $\mathbb{R}^N$ with all elements 1.*

In this network, we construct the leader-following MASs in the following form:

$$\begin{aligned} \dot{x}_0 = &\ Ax_0 \\ \dot{x}_i = &\ Ax_i - \nu\sum_{j=1}^{N}|\alpha_{ij}|(x_i - \mathrm{sgn}(\alpha_{ij})x_j) + B_iK_i(d_ix_i - x_0) \\ i = &\ 1, 2, \ldots, N, \end{aligned} \tag{4}$$

where $\mu_i = K_i(d_ix_i - x_0)$ in (2), and $\alpha_{ij}$ is an arbitrary real number.

**Lemma 5.** [7] *Let $\bar{\mathcal{L}}$ be a Laplacian matrix of a strong connected network $\bar{\mathcal{G}}(\bar{\mathcal{A}}) = (|\alpha_{ij}|)$, and $\xi_i = [\xi_1, \xi_2, \ldots, \xi_N]^T$ be the left-eigenvector of $\bar{\mathcal{L}}$ associated with left-eigenvalue 0, where $\xi_i > 0$ for all $(i = 1, 2, \ldots, N)$. In addition, for the positive definite diagonal matrix $\Xi = \mathrm{diag}\{\xi_1, \xi_2, \ldots, \xi_N\}$, the matrix $\widetilde{\mathcal{L}} = \frac{1}{2}(\Xi\bar{\mathcal{L}} + \bar{\mathcal{L}}^T\Xi)$ is symmetric and associated with an undirected and connected network. Without loss of generality, we assume that $\mathbf{1}_N^T\xi = 1$. In addition, we have $\sum_{j=1}^{N}\widetilde{\mathcal{L}}_{ij} = \sum_{j=1}^{N}\widetilde{\mathcal{L}}_{ji} = 0, (i = 1, 2, \ldots, N)$.*

**Remark 1.** According to [32], we can find a positive integer $\eta$, such that $-\bar{\mathcal{L}} + \eta I_N$ is a nonnegative matrix. Since $-\bar{\mathcal{L}} + \eta I_N$ is nonnegative, by Lemma 2.8 in [31], one can find a positive vector $\xi_i = [\xi_1, \xi_2, \ldots, \xi_N]^T$ satisfying $(-\bar{\mathcal{L}} + \eta I_N)\xi = \eta\xi$.

**Assumption 1.** The pair $(A, B)$ is stabilizable, where $B = [B_1, B_2, \ldots, B_N]$.

Based on Assumption 1, we establish the following result.

**Theorem 1.** *Consider system (2) under strongly connected antagonistic networks $\mathcal{G}$. Supposed that Assumption 1 is satisfied. Then, there exist feedback gain matrices $K_i, i = 1, 2, \ldots, N$ and a coupling strength $\nu$ such that the system (4) achieve bipartite consensus.*

**Proof.** Since $(A, B)$ is stabilizable, and $B = [B_1, B_2, \ldots, B_N]$, there exists a matrix $K = \left[\widetilde{K}_1^T, \widetilde{K}_2^T, \ldots, \widetilde{K}_N^T\right]^T$, such that $A + BK = A + \sum_{i=1}^{N} B_i\widetilde{K}_i, i = 1, 2, \ldots, N$ is Hurwitz. Thus, a positive definite matrix $P$ can be found such that the following Lyapunov inequality is satisfied,

$$P\left(A + \sum_{i=1}^{N} B_i\widetilde{K}_i\right) + \left(A + \sum_{i=1}^{N} B_i\widetilde{K}_i\right)^T P < 0, i = 1, 2, \ldots, N \tag{5}$$

Take $e_i = d_ix_i - x_0$ as the tracking error of follower $i$, and $\mathbf{e} = [e_1^T, e_2^T, \ldots, e_N^T]^T, d_i \in \{-1, 1\}$, for all $i = 1, 2, \ldots, N$. Let $\widetilde{x}_i = d_ix_i$, there is $e_i = \widetilde{x}_i - x_0$. Then, we have

$$\begin{aligned} \dot{\widetilde{x}}_i &= d_i\dot{x}_i = d_iAx_i - d_i\nu\sum_{j=1}^{N}|\alpha_{ij}|(x_i - \mathrm{sgn}(\alpha_{ij})x_j) + d_iB_iK_ie_i \\ &= d_iAx_i - \nu\sum_{j=1}^{N}|\alpha_{ij}|(e_i - e_j) + d_iB_iK_ie_i. \end{aligned} \tag{6}$$

By setting $\widetilde{x} = [\widetilde{x}_1^T, \widetilde{x}_2^T, \ldots, \widetilde{x}_N^T]^T$, we obtain $\dot{\widetilde{x}} = (D \otimes A)x - \nu(\bar{\mathcal{L}} \otimes I_n)\mathbf{e} + (D \otimes I_n)\mathfrak{B}\mathfrak{K}\mathbf{e}$, where

$$\mathfrak{B} = \begin{pmatrix} B_1 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & B_N \end{pmatrix}, \ \mathfrak{K} = \begin{pmatrix} K_1 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & K_N \end{pmatrix}, \ D = \begin{pmatrix} d_1 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & d_N \end{pmatrix}. \tag{7}$$

So, we can get

$$\dot{\mathbf{e}} = (I_N \otimes A + (D \otimes I_n)\mathfrak{B}\mathfrak{K})\mathbf{e} - \nu(\bar{\mathcal{L}} \otimes I_n)\mathbf{e} \tag{8}$$

We construct the Lyapunov candidate function

$$V = \frac{1}{2}\mathbf{e}^T(\Xi \otimes P)\mathbf{e},$$

Taking derivative of $V$ along (8), we have

$$\begin{aligned} \dot{V} &= \frac{1}{2}\dot{\mathbf{e}}^T(\Xi \otimes P)\mathbf{e} + \frac{1}{2}\mathbf{e}^T(\Xi \otimes P)\dot{\mathbf{e}} = \frac{1}{2}\mathbf{e}^T\Big((I_N \otimes A + (D \otimes I_n)\mathfrak{B}\mathfrak{K} - \nu\bar{\mathcal{L}} \otimes I_n)^T(\Xi \otimes P) + (\Xi \otimes P)(I_N \otimes A + (D \otimes I_n)\mathfrak{B}\mathfrak{K} - \nu\bar{\mathcal{L}} \otimes I_n)\Big)\mathbf{e} \\ &= \mathbf{e}^T\left\{\frac{1}{2}\mathrm{diag}\Big(\xi_1\left[(A + d_1B_1K_1)^TP + P(A + d_1B_1K_1)\right],\ \xi_2\left[(A + d_2B_2K_2)^TP + P(A + d_2B_2K_2)\right], \ldots,\ \xi_N\left[(A + d_NB_NK_N)^TP + P(A + d_NB_NK_N)\right]\Big) - \frac{1}{2}\nu(\bar{\mathcal{L}}\Xi + \Xi\bar{\mathcal{L}}) \otimes P\right\}\mathbf{e} \\ &= \mathbf{e}^T\left(\Psi - \nu\widetilde{\mathcal{L}} \otimes P\right)\mathbf{e} \end{aligned} \tag{9}$$

where

$$\begin{aligned} &\Psi = \tfrac{1}{2}\mathrm{diag}\{\xi_1\Psi_1, \xi_2\Psi_2, \ldots, \xi_N\Psi_N\} \\ &\Psi_i = (A + d_iB_iK_i)^TP + P(A + d_iB_iK_i) \\ &\widetilde{\mathcal{L}} = \tfrac{1}{2}(\Xi\bar{\mathcal{L}} + \bar{\mathcal{L}}^T\Xi) \end{aligned} \tag{10}$$

Next, we only need to show the matrix $\Psi - \nu\hat{\mathcal{L}} \otimes P$ is negative definite for some $\nu$. Let

$$K_i = \xi_i^{-1}\sum_{i=1}^{N}\xi_i\left(d_i\widetilde{K}_i\right), i = 1, 2, \ldots, N \tag{11}$$

then we can easily prove the following

$$\sum_{i=1}^{N}\xi_i\Psi_i=\left(P(A+BK)+(A+BK)^TP\right)<0. \tag{12}$$

According to Lemma 4 and 5, it is easy to find a unitary matrix $W=\left[1/\sqrt{N}\mathbf{1}_N,Z\right]\in\mathbb{R}^{N\times N}$ with $Z\in\mathbb{R}^{N\times(N-1)}$, such that $W^T\widetilde{\mathcal{L}}W=\widehat{J}$, where $\widehat{J}=\mathrm{diag}\left\{0,\hat{\lambda}_2,\ldots,\hat{\lambda}_N\right\}$, and $0<\hat{\lambda}_2\leqslant\hat{\lambda}_3\leqslant,\ldots,\leqslant\hat{\lambda}_N$ are non-zero eigenvalues of $\widetilde{\mathcal{L}}$.

The negative definiteness of matrix $\left(W^T\otimes I_n\right)\left(\Psi-v\widetilde{\mathcal{L}}\otimes P\right)(W\otimes I_n)$ implies that $\Psi-v\widetilde{\mathcal{L}}\otimes P$ is negative definitive.

Denote $\mathbb{Z}=Z^T\otimes I_n$. $\mathbb{1}_{\mathbb{N}}=\mathbf{1}_{\mathbb{N}}\otimes\mathbb{I}_{\mathbb{n}}$. We have

$$\begin{aligned}&\left(W^T\otimes I_n\right)\left(\Psi-v\left(\widetilde{\mathcal{L}}\otimes P\right)\right)(W\otimes I_n)\\&=\begin{bmatrix}(1/N)\sum_{i=1}^{N}\xi_i\Psi_i & \left(1/\sqrt{N}\right)\mathbb{1}_{\mathbb{N}}^T\Psi\mathbb{Z}\\ (\mathbb{1}/\sqrt{\mathbb{N}})\mathbb{Z}^T\Psi\mathbb{1}_{\mathbb{N}} & \mathbb{Z}^T\Psi\mathbb{Z}-v\left(\mathrm{diag}\left\{\hat{\lambda}_2,\ldots,\hat{\lambda}_{\mathbb{N}}\right\}\otimes\mathbb{P}\right)\end{bmatrix}\end{aligned}$$

Note $(1/N)\sum_{i=1}^{N}\xi_i\Psi_i<0, i=1,2,\ldots,N$. Choose a positive constant $v$ satisfying

$$v>-\frac{\lambda_{\max}\left(\mathbb{Z}^T\left(\Psi-\Psi\mathbb{1}_{\mathbb{N}}\left(\sum_{\mathbb{i}=\mathbb{1}}^{\mathbb{N}}\xi_{\mathbb{i}}\Psi_{\mathbb{i}}\right)^{-\mathbb{1}}\mathbb{1}_{\mathbb{N}}^T\Psi\right)\mathbb{Z}\right)}{\lambda_2\cdot\lambda_{\min}(P)} \tag{13}$$

where $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ are the maximum and minimum eigenvalue of a symmetric matrix, respectively. Thus,

$$\mathbb{Z}^T\left(\Psi-\Psi\mathbb{1}_{\mathbb{N}}\left(\sum_{\mathbb{i}=\mathbb{1}}^{\mathbb{N}}\xi_{\mathbb{i}}\Psi_{\mathbb{i}}\right)^{-\mathbb{1}}\mathbb{1}_{\mathbb{N}}^T\Psi\right)\mathbb{Z}-v\mathrm{diag}\left\{\hat{\lambda}_2,\ldots,\hat{\lambda}_{\mathbb{N}}\right\}\otimes\mathbb{P}<\mathbb{0}.$$

By Lemma 1, matrix $\left(W^T\otimes I_n\right)\left(\Psi-v\widetilde{\mathcal{L}}\otimes P\right)(W\otimes I_n)$ is negative definite, which implies that $\dot{V}<0$. Hence, (8) is asymptotically stable. The MASs (2) can achieve bipartite consensus.

**Remark 2.** If the antagonistic network $\mathcal{G}$ is structurally balanced, then, according to Remark 1, one can choose a matrix $\Xi$ in Lemma 5. Suppose that Assumption 1 holds and matrix $K=\left[\widetilde{K}_1^T,\widetilde{K}_2^T,\ldots,\widetilde{K}_N^T\right]^T$ is found such that $A+BK$ is Hurwitz. Let $K_i=\xi_i^{-1}\sum_{i=1}^{N}\xi_i\left(d_i\widetilde{K}_i\right)$, for all $i=1,2,\ldots,N$. $P>0$ solves Lyapunov inequality (5). Choose $v$ according to (13). Thus, the MASs (2) will achieve bipartite consensus.

*3.2. Bipartite consensus under antagonistic connected network containing a spanning tree*

This subsection focuses on the MASs under an antagonistic network that contains a spanning tree.

**Lemma 6.** *[33] Let $\lambda_i, i=1,2,\ldots,n$, be eigenvalues of matrix $H\in\mathbb{R}^{n\times n}$ and $\mu_j, j=1,2,\ldots,N$ be eigenvalues of matrix $F\in\mathbb{R}^{N\times N}$.*

*Then, $H\oplus F=(I_N\otimes H)+(F\otimes I_n)$ has $nN$ eigenvalues, which are denoted by $\lambda_i+\mu_j, i=1,2,\ldots,n$ and $j=1,2,\ldots,N$.*

Recall a maximal strongly connected subgraph is called a strongly connected component. A directed netwok can be decomposed into the union of some disjoint strongly connected components. And we recall the definition of a source component. Since a network with a spanning tree contains only one source component, we analyze the cooperative observers in this kind of networks with loss of generality.

**Lemma 7.** *[35] If a graph $\mathcal{G}$ contains a spanning tree, then, under suitable permutation, its Laplacian matrix can be rewritten into the following form,*

$$\mathcal{L}=\begin{bmatrix}L_{11} & L_{12} & \cdots & L_{1k}\\ 0 & L_{22} & \cdots & L_{2k}\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & L_{kk}\end{bmatrix}, \tag{14}$$

*where $L_{ii}$ is irreducible for $i=1,\ldots,k-1$ and $L_{kk}$ is irreducible or a scaler 0. Here the matrix $\mathcal{L}$ is the Laplacian matrix of the graph $\mathcal{G}(\bar{\mathcal{A}})=(|\alpha_{ij}|)$ in Lemma 2.*

**Lemma 8.** *[34] For the Laplacian matrix $\mathcal{L}$ in Lemma 7, all eigenvalues of matrix*

$$\hat{\mathcal{L}}=\begin{bmatrix}L_{11} & L_{12} & \cdots & L_{1k-1}\\ 0 & L_{22} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & L_{k-1k-1}\end{bmatrix}, \tag{15}$$

*have positive real parts.*

In the rest of this subsection, we will assume, generally, the indexes of the nodes of followers are permuted in the way such that the Laplacian matrix $\mathcal{L}$ is in the form of (14). Let $\mathbf{S}$ be the dimension of $\hat{\mathcal{L}}$ in Lemma 8. So, the nodes corresponding to $L_{kk}$ can be indexed as $\mathbf{S}+1,\mathbf{S}+2,\ldots,N$. We make the following assumption on these nodes.

**Assumption 2.** The pair $(A,B_s)$ is stabilizable, where $B_s=[B_{\mathbf{S}+1},B_{\mathbf{S}+2},\ldots,B_N]$.

Under this assumption, we establish the following result.

**Theorem 2.** *Consider MASs (2) connected through the antagonistic network which corresponding to the graph $\mathcal{G}$ that contains a spanning tree. Supposed that Assumptions 2 is satisfied. Then, there exist feedback gain matrix $\hat{K}_i$ and a coupling strength $v$, such that the network of MASs (2) achieve bipartite consensus.*

**Proof.** Let $e_i=d_ix_i-x_0, i=1,2,\ldots,N, \mathbf{e}_1=\left[e_1^T,e_2^T,\ldots,e_S^T\right]^T, \mathbf{e}_2=\left[e_{S+1}^T,e_{S+2}^T,\ldots,e_N^T\right]^T$, and $\mathbf{e}=\left[e_1^T,e_2^T,\ldots,e_N^T\right]^T$. Clearly, $\mathbf{e}=\left[\mathbf{e}_1^T,\mathbf{e}_2^T\right]^T$.

For the Laplacian matrix $\mathcal{L}$ is in form of (14), the dynamics of consensus error system corresponding to $\mathbf{e}_2$ is described by

$$\begin{aligned}\dot{\mathbf{e}}_2=&\,\mathrm{diag}\{A+d_{\mathbf{S}+1}B_{\mathbf{S}+1}K_{\mathbf{S}+1},A+d_{\mathbf{S}+2}B_{\mathbf{S}+2}K_{\mathbf{S}+2},\ldots,A+d_NB_NK_N\}\mathbf{e}\\&-v\mathcal{L}_{kk}\otimes I_n\mathbf{e}_2.\end{aligned} \tag{16}$$

which is independent of $\mathbf{e}_1$. And we already know that $\mathcal{L}_{kk}$ is irreducible or a scaler zero.

In the next, we will show the existence of $\widehat{K}_i=d_1K_i, i=\mathbf{S}+1,\mathbf{S}+2,\ldots,N$ , and $v>0$ such that the (17) is asymptotically stable. If $\mathcal{L}_{kk}$ is a scalar zero, then Assumption 2 reduce to $A+B_N\widehat{K}_N$ being stabilizable. So, we can find a $K_N$ such that $A+B_N\widehat{K}_N$ is Hurwitz. So, (17) is certainly asymptotically stable. Note that, the asymptotically stability of the above degenerate dynamics is independent of the choice of $v$ .

If $\mathcal{L}_{kk}$ is irreducible, then the dimension of $\mathcal{L}_{kk}$ which is a Laplacian matrix, is greater than one. Following Remark 2, we choose $\widehat{K}_i, i=\mathbf{S}+1,\mathbf{S}+2,\ldots,N$, and $v_1>0$such that

$$\lim_{t^T o\infty}\|\mathbf{e}_2\| = 0; \mathbf{e}_2(0) \in \mathbb{R}^{(N-\mathbf{S})}. \tag{17}$$

holds for $v \geqslant v_1$.

Next, we will prove the asymptotic stability of the dynamics of $\mathbf{e}_1$. Let $\widehat{K}_i = 0$ for all $i = 1, 2, \ldots, \mathbf{S}$. Then, the dynamic of $\mathbf{e}_1$ is described by

$$\dot{\mathbf{e}}_1 = \left(I_\mathbf{S} \otimes A - v\left(\widetilde{\mathcal{L}} \otimes I_n\right)\right)\mathbf{e}_1 + C\mathbf{e}_2 = \left(A \oplus \left(-v\widetilde{\mathcal{L}}\right)\right)\mathbf{e}_1 + C\mathbf{e}_2$$

where $C = \left[\mathcal{L}_{1k}^T, \mathcal{L}_{2k}^T, \ldots, \mathcal{L}_{kk-1}^T\right]^T$ and $\oplus$ is the Kronecker sum define in Lemma 6. By Lemma 8, $\left(\hat{\mathcal{L}} \otimes I_N\right)$ is Hurwitz. Thus, by Lemma 6, choose a $v_2 > 0$ satisfying

$$v_2 > \frac{max_i\{\mathrm{Re}\lambda_i(A)\}}{\min_i\left\{\mathrm{Re}\lambda_i\left(\widetilde{\mathcal{L}}\right)\right\}} \tag{18}$$

where $\lambda_i(\cdot)$ is the ith eigenvalue of a matrix. In this case, $I_\mathbf{S} \otimes A - v\left(\hat{\mathcal{L}} \otimes I_n\right)$ is Hurwitz. Thus $\mathbf{e}_1$ converges to zero asymptotically as $\mathbf{e}_2$. The proof is completed by letting $v \geqslant \max(v_1, v_2)$.

**Remark 3.** Let the network connecting the MASs (2) be described by a antagonistic directed graph containing a spanning tree. Under Assumption 2, we can select $\widehat{K}_i, i = \mathbf{S}+1, \mathbf{S}+2, \ldots, N$, and $v_1 > 0$ according to (17). Choose a $v_2 > 0$ according to (18). Let $\widehat{K}_i = 0, i = 1, 2, \cdot, \mathbf{S}$ and $v \geqslant \max(v_1, v_2)$. Then the MASs (2) will achieve bipartite consensus.

## 4. Adaptive leader–follower system under antagonistic networks

In the previous section, bipartite consensus of MASs (2) connected through two kinds of antagonistic networks are analyzed. Conditions are established to achieve asymptotic bipartite consensus. The bound of coupling strength $v$ can be estimated by (13). However, for a network with a large number of nodes, the computation burden is significant. To overcome this difficulty, we introduce an adaptive law to self-tune the coupling strength. According to (2), we construct the leader-following MASs over signed graph of network by the following form:

$$\begin{aligned} \dot{x}_0 &= Ax_0 \\ \dot{x}_i &= A - \sum_{j=1}^{N}|\alpha_{ij}|v_{ij}(x_i - \mathrm{sgn}(\alpha_{ij})x_j) + B_iK_i(d_ix_i - x_0) \end{aligned} \tag{19}$$

Since the graph $\mathcal{G}(\bar{\mathcal{A}})$ is structurally balanced, we have

$$\begin{aligned} d_i\dot{x}_i &= d_iAx_i - d_i\sum_{j=1}^{N}|\alpha_{ij}|v_{ij}(x_i - \mathrm{sgn}(\alpha_{ij})x_j) + d_iB_iK_i(d_ix_i - x_0) \\ &= d_iAx_i - \sum_{j=1}^{N}|\alpha_{ij}|v_{ij}(d_ix_i - d_jx_j) + d_iB_iK_i(d_ix_i - x_0) \end{aligned}$$

where

$$\begin{aligned} \dot{v}_{ij} = \quad & k_{ij}(d_ix_i - d_jx_j)^T\Gamma(d_ix_i - d_jx_j) \;\; i,j = 1,2,\ldots,N \\ & k_{ij} = \begin{cases} |\alpha_{ij}|, \alpha_{ij} \neq 0 \\ 0, \alpha_{ij} = 0 \end{cases} \end{aligned} \tag{20}$$

and $\alpha_{ij} = \alpha_{ji}$ for all $i,j = 1,2,\ldots,N$. Here, $v_{ij}$ denotes the time-varying coupling strength between the ith node and jth node with the initial value $v_{ij}(0) = v_{ji}(0) \in \mathbf{R}$, and $\Gamma \in \mathbb{R}^{n\times n}$ are the feedback gain matrix to be designed.

The following theorem focuses on the design of $B_i, K_i$ and $\Gamma$ for the asymptotic consensus of the adaptive leader-following MASs over an undirected and connected graph.

**Theorem 3.** *Consider MASs (19) under antagonistic nondirected connected networks which are structurally balanced. Suppose that Assumption 1 is satisfied. Then, we can find $B_i, K_i$ and $\Gamma$ , such that MASs (19) with adaptive laws (20), will achieve bipartite consensus. Moreover, the time-varying coupling strength $v_ij$ converge to some constants.*

**Proof.** Define the error function as $e_i = d_ix_i - x_0, i = 1,2,\ldots,N$, and $\mathbf{e} = [e_1^T, e_2^T, \ldots, e_N^T]$. Let $M_{ij}(t) \triangleq \alpha_{ij}v_{ij}(t)$. It is trivial to show that $\dot{M}_{ij} = \dot{M}_{ji}$. Noting that $M_{ij}(0) = M_{ji}(0)$, there is $M_{ij}(t) = M_{ji}(t), \forall t \geqslant 0$. Then, the dynamics of consensus errors is given below,

$$\dot{e}_i = d_i\dot{x}_i - \dot{x}_0 = (A + d_iB_iK_i)e_i - \sum_{j=1}^{N}|\alpha_{ij}|v_{ij}(e_i - e_j) \tag{21}$$

$$\dot{v}_{ij} = k_{ij}(e_i - e_j)^T\Gamma(e_i - e_j) \tag{22}$$

There exist feedback gain matrices $K = \left[\widehat{K}_1^T, \widehat{K}_1^T, \ldots, \widehat{K}_N^T\right]^T$, such that $A + BK$ is Hurwitz. Let $\widehat{K}_i = d_iK_i, i = 1,2,\ldots,N$ and $\Gamma = P$. Construct the following Lyapunov function candidate:

$$V = \sum_{i=1}^{N}e_i^TPe_i + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}(v_{ij} - \alpha)^2$$

where $\alpha$ is a positive scalar whose value will be determined in the later.

The derivative of $V$ along the trajectories of (21) is given by:

$$\begin{aligned} \dot{V} = & \sum_{i=1}^{N}\left[e_i^T\left((A + d_iB_iK_i)^TP + P(A + d_iB_iK_i)\right)e_i\right] \\ & - 2\sum_{i=1}^{N}\sum_{j=1}^{N}(|\alpha_{ij}|v_{ij}e_i^TP(e_i - e_j)) \\ & + \sum_{i=1}^{N}\sum_{j=1}^{N}(v_{ij} - \alpha)k_{ij}(e_i - e_j)^T\Gamma(e_i - e_j) \\ = & \sum_{i=1}^{N}e_i^T\Phi_ie_i - 2\sum_{i=1}^{N}\sum_{j=1}^{N}(|\alpha_{ij}|v_{ij}e_i^TP(e_i - e_j)) \\ & + \sum_{i=1}^{N}\sum_{j=1}^{N}(v_{ij} - \alpha)k_{ij}(e_i - e_j)^T\Gamma(e_i - e_j) \end{aligned} \tag{23}$$

where $\Phi_i = (A + d_iB_iK_i)^TP + P(A + d_iB_iK_i)$.

Because $M_{ij}(t) = M_{ji}(t), \forall t \geqslant 0$, we have

$$\begin{aligned} & \sum_{i=1}^{N}\sum_{j=1}^{N}(v_{ij} - \alpha)k_{ij}(e_i - e_j)^T\Gamma(e_i - e_j) \\ & = 2\sum_{i=1}^{N}\sum_{j=1}^{N}v_{ij}k_{ij}e_i^T\Gamma(e_i - e_j) \\ & \quad - \alpha\left[\sum_{i=1}^{N}\sum_{j=1}^{N}k_{ij}e_i^T\Gamma(e_i - e_j) + \sum_{i=1}^{N}\sum_{j=1}^{N}k_{ij}e_j^T\Gamma(e_j - e_i)\right] \end{aligned}$$

Since (20) and the graph $\mathcal{G}$ is nondirected, we get $v_{ij}k_{ij} = k_{ij}v_{ij}$. Therefore, substitution of which into (23) yields

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{N} e_i^T \Phi_i e_i - 2\alpha \mathbf{e}^T (\mathcal{L} \otimes \Gamma)\mathbf{e} \\ &= \mathbf{e}^T (\mathrm{diag}\{\Phi_1, \Phi_2, \ldots, \Phi_N\} - 2\alpha\mathcal{L} \otimes P)\mathbf{e}\end{aligned}$$

where $\mathcal{L}$ is the Laplacian matrix of the graph $\mathcal{G}$, for all $i,j = 1,2,\ldots,N$.

From the proof of Theorem 1, $\mathrm{diag}\{\Phi_1, \Phi_2, \ldots, \Phi_N\} - 2\alpha\mathcal{L} \otimes P$ is negative definite for a large enough $\alpha$. Let such a $\alpha$ be chosen such that $\dot{V}$ is negative semi-definite, which implies that the trajectories of $e_i(t)$ and $v_ij$ remain bounded. From the Barbalat lemma, $e_i(t)$ will converge to zero as time goes to infinity. Therefore, the adaptive leader–follower systems achieve bipartite consensus. Moreover, by (20)), each $v_{ij}(t)$ is monotonously nondecreasing, along with its boundedness, which implies its convergence to some constants.

**Remark 4.** In this paper, we just study the structurally balanced signed graphs and its bipartite consensus problem over different topology networks. For the structurally unbalanced signed graphs, we will solve in the future.

## 5. Simulations

In this section, several examples are given to verify the proposed results. Take the system matrix $A$ as,

$$A = \begin{pmatrix} 0 & 2 & 0 & 0 \\ -3 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & -1 & 0 \end{pmatrix}. \quad (24)$$

and set $B_1 = [1,0,0,0]^T, B_2 = [0,1,0,0]^T, B_,3 = [0,0,1,0]^T, B_,4 = [0,0,0,1]^T$, $B_i = [1,0,0,0]^T$, for $i = 5,6,\ldots,N$.

### 5.1. Strongly connected antagonistic network

Consider the MASs (2) with nine nodes and the graph adjacency matrix is as follow

$$\mathcal{A} = \begin{bmatrix} 0 & 0 & 0 & -2 & 0 & 0 & 1 & 0 & -1 \\ -1 & 0 & 0 & 0 & -3 & 0 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 & 1 & 0 \end{bmatrix}$$

Its trivial to show that the graph $\mathcal{G}((A))$ is structurally balanced with $\mathcal{V}_1 = \{v_1, v_5, v_6, v_7\}$ and $\mathcal{V}_2 = \{v_2, v_3, v_4, v_8, v_9\}$. Set $\widetilde{K}_1 = [-3,0,0,0], \widetilde{K}_3 = [0,-3,0,0], \widetilde{K}_3 = [0,0-3,0], \widetilde{K}_4 = [0,0,0,-3]$, $\widetilde{K}_i = [0,0,0,0]$, for $i = 5,6,\ldots,N$. Thus, $A + BK = A + \sum_{i=1}^{9} B_i\widetilde{K}_i$ is Hurwitz matrix. In this simulation, the initial values of $x_0$ and $x_i, i = 1,2,\ldots,9$ are chosen randomly.

We can design $K_i, i = 1,2,\ldots,9$ as (11), and set the feedback gains $v = \mathbf{30}$ according to (13). Under the conditions in Theorem 1, $x_0$ and $x_i, i = 1,2,\ldots,9$, in MASs (2) will achieve bipartite sign-consensus and the errors $\|e_i\|_2 = \|d_i x_i - x_0\|_2$ will converge to 0, which are verified in Fig. 1 respectively.

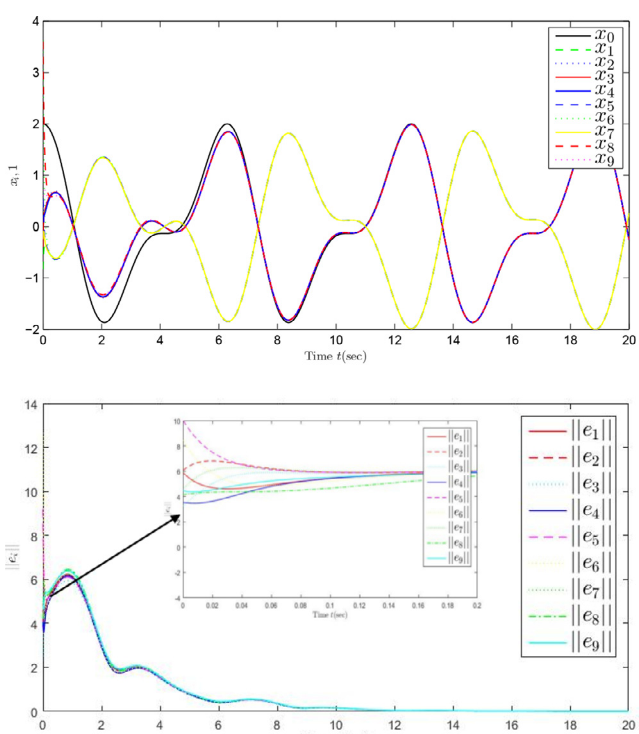


**Fig. 1.** Bipartite consensus of MASs (2) where the network is strongly connected: (a) Trajectories of $x_{i,1}$. (b) evolution of the tracking errors.

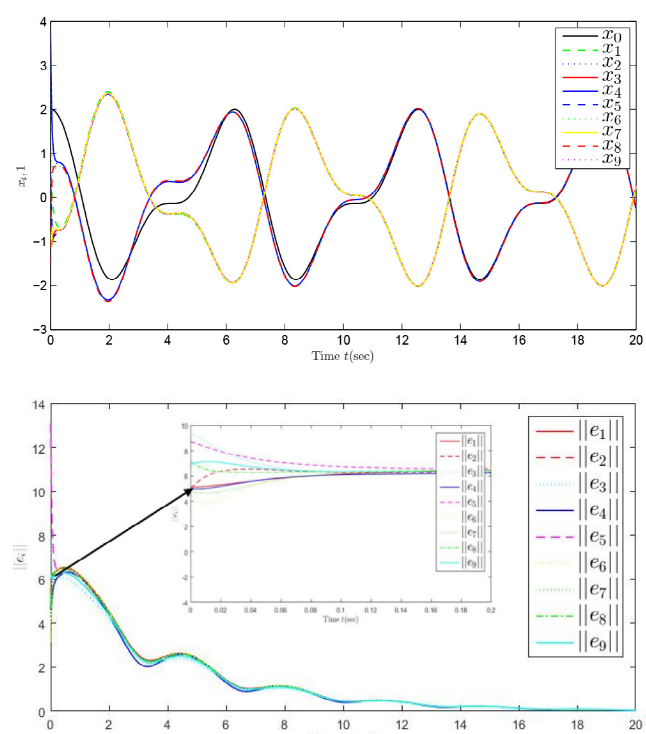


**Fig. 2.** Bipartite consensus of MASs (2) where the network contains a spanning tree: (a) Trajectories of $x_{i,1}$ under Theorem 2. (b) evolution of the tracking errors.

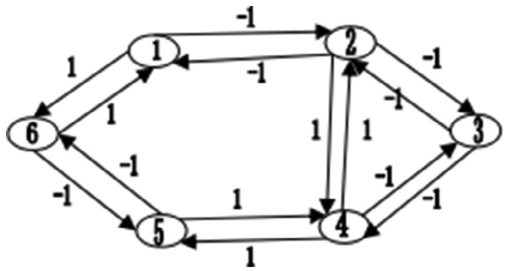


**Fig. 3.** Graph topology which satisfies Theorem 3.

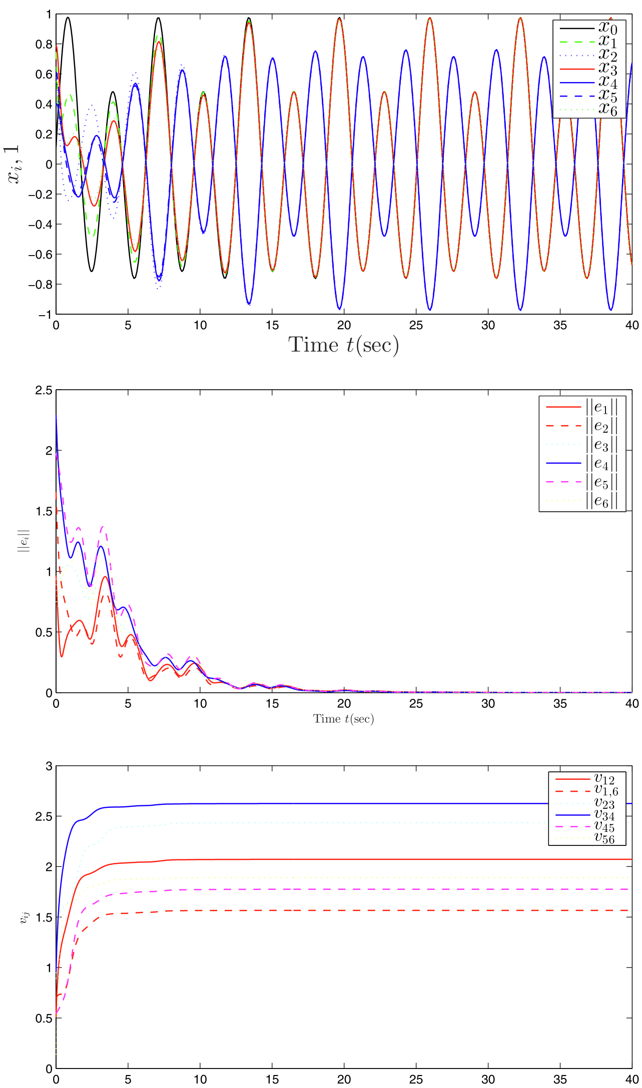


**Fig. 4.** Adaptive regulation of coupling strength for the MASs (12) stabilization: (a) Trajectories of $x_{i,1}$ under Theorem 3. (b) evolution of the errors. (c) evolution of the coupling strengths.

*5.2. Antagonistic network with a spanning tree*

In this section, we consider the graph with a spanning tree, and the graph adjacency matrix is as follow

$$\mathcal{A} = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2 & 0 & -3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ -3 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & -2 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 4 & 0 & 0 & 0 & 2 & -1 & 0 \end{bmatrix}$$

Its trivial to show that the graph $\mathcal{G}((A))$ is structurally balanced with $\mathcal{V}_1 = \{v_1, v_3, v_5, v_7, v_9\}$ and $\mathcal{V}_2 = \{v_2, v_4, v_6, v_8\}$. Under the condition in Theorem 2, $x_0$ and $x_i, i = 1, 2, \ldots, 9$ in MASs (2) will achieve bipartite sign-consensus and the errors $\|e_i\|_2 = \|d_i x_i - x_0\|_2$ will converge to 0, which are verified in Fig. 2, respectively.

*5.3. Adaptive leader–follower system under antagonistic networks*

In this subsection, we assume that there are 6 followers in the MASs (2), and its topology is shown in Fig. 3. Its trivial to show that the graph $\mathcal{G}((A))$ is structurally balanced with $\mathcal{V}_1 = \{v_1, v_3, v_6\}$ and $\mathcal{V}_2 = \{v_2, v_4, v_5\}$, and this graph is symmetric. The matrix $A, B_i$, and $\widehat{K}_i, i = 1, 2, \ldots, 6$ are still chosen as subsection V-5.1. Its obviously that Assumption 1 is satisfied. For the MASs (19), the initial values of $x_0, x_i$, and $v_{ij} > 0, i, j = 1, 2, \ldots, 6$ are chosen randomly. Under the condition in Theorem 3, $x_0$ and $x_i, i = 1, 2, \ldots, 6$ in MASs (19) will achieve bipartite sign-consensus and the errors $\|e_i\|_2 = \|d_i x_i - x_0\|_2$ will converge to 0, which are verified in Fig. 4, respectively. Since $v_{ij} = v_{ji}$, we just need to depict the response of $v_{12}, v_{16}, v_{23}, v_{34}, v_{45}, v_{56}$. The evolution of all $v_{ij}$s are shown in Fig. 4 (c). It shows that they all converge to some positive numbers.

## 6. Conclusion

In this paper, we analyze the leader-following consensus of a class of MASs under structurally balanced antagonistic networks. Different from the traditional literature, in our model, the followers are with heterogeneous dynamics due to the identical input matrices. Sufficient conditions are established under which the MASs will achieve bipartite consensus. Detailed algorithms are proposed to design the gain matrices and the coupling strength. We also design adaptive laws to self-tune the value of the coupling strength. In the future, our research could be added in some other conclusion part, such as event-based control mechanism based on [36,37 and cyber security problem for multi-agent systems based on [38]. Because the different networks, the sufficient conditions that are established to achieve consensus will be different and it need us to do more analysis.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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**Qianyao Wang** received the B.Sc. degree in information and computer science from Shandong Jianzhu University at Jinan, Shandong, China in 2014, and received the M.Sc. degree from the Key Laboratory of Systems and Control, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, and also in University of Chinese Academy of Sciences, Beijing, China in 2018. Her research interests include multi-agent systems, and complex networks.



**Kexin Liu** received the M.Sc. degree in control science and engineering from Shandong University, Jinan, China, in 2013, and the Ph.D. degree in system theory from the Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing, China, in 2016. He is currently an Associate Professor with the School of Automation Science and Electrical Engineering, Beihang University, Beijing. His current research interests include multiagent systems and complex networks.



**Xiong Wang** received the B.Sc. degree in the School of Mathematics and Statistics, Wu han University, Wuhan, China in 2013, and the Ph.D. degree from the Key Laboratory of Systems and Control, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, and also in University of Chinese Academy of Sciences, Beijing, China in 2018. Currently, she is a lecturer with the School of Mathematics, Northwest University. Her research interests include multi-agent systems, system control, complex systems and complex networks.



**LuLu Wu** received the B.Sc. degree in mathematics and applied mathematics from Shandong Normal University at Jinan, Shandong, China in 2013, and received Ph.D. degree from the Key Laboratory of Systems and Control, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, and also in University of Chinese Academy of Sciences, Beijing, China in 2018. Currently, she is a lecturer in Shandong Jianzhu University. Her research interests include biology systems.

**Jinhu Lü** (M'03–SM'06–F'13) received the Ph.D. degree in applied mathematics from the Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing, China, in 2002. He was a Professor with RMIT University, Melbourne, VIC, Australia, and a Visiting Fellow with Princeton University, Princeton, NJ, USA. He is currently the Dean of the School of Automation Science and Electrical Engineering, Beihang University, Beijing. He is also a Professor with the Academy of Mathematics and Systems Science, Chinese Academy of Sciences. He is also the Chief Scientist of the National Key Research and Development Program of China and a Leading Scientist of Innovative Research Groups of the National Natural Science Foundation of China. His current research interests include nonlinear circuits and systems, complex networks, multiagent systems, and big data. Dr. Lü was a recipient of the Prestigious Ho Leung Ho Lee Foundation Award in 2015, the State Natural Science Award thrice from the Chinese Government in 2008, 2012, and 2016, respectively, the Australian Research Council Future Fellowships Award in 2009, the National Natural Science Fund for Distinguished Young Scholars, the Highly Cited Researcher Award in Engineering in 2014–2018, and a Leading Scientist of the Ten Thousand Talents Program of China. He was a member of the Evaluating Committees of the IEEE Circuits and Systems Society, the IEEE Industrial Electronics Society, and the IEEE Computational Intelligence Society. He was the General Co-Chair of the 43rd Annual Conference of the IEEE Industrial Electronics Society in 2017. He was an Editor in various ranks for 15 SCI journals, including seven IEEE TRANSACTIONS journals.