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Bipartite Consensus of Linear Multi-Agent Systems Over Signed Digraphs: An Output Feedback Control Approach ⋆

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Abstract:
This paper studies bipartite consensus problems for multi-agent system over signed directed graphs. We consider general linear agents and design a dynamic output feedback control law for the agents to achieve bipartite consensus. Our results show that structural balance property of the graph and an appropriate consensus error information are two crucial factors of bipartite consensus.

Keywords: bipartite consensus; multi-agent system; output feedback; signed graph

1. INTRODUCTION

In the past two decades, there has been tremendous interest in developing distributed control laws for multi-agent systems with a primary focus on consensus over nonnegative graphs (Jadbabaie et al. [2003], Lin et al. [2005], Olfati-Saber et al. [2007], Ren and Cao [2011], Lewis et al. [2014], etc.). Nonnegative graphs which are defined by positive edges are appropriate for describing collaborative behaviors between agents. When both collaborative and antagonistic interactions coexist within a group of agents, such communication networks can be more suitably represented by signed graphs, in which a positive edge means collaboration and a negative edge represents an antagonistic interaction. Research on collective behaviors over signed graphs finds applications in scenarios of social networks (Wasserman and Faust [1994], Altafini [2012]), predator-prey dynamics (Lee [2006]), and so on.

Altafini [2013] studied bipartite consensus problem over signed graphs. In this work, it was found that two subgroups of nodes are formed during evolution, and consensus is achieved within each subgroup, moving individually towards opposite directions. Altafini [2013] examined only single-integrator dynamics and pointed out that it may be nontrivial to extend the results of (Altafini [2013]) to higher order integrator dynamics. In the authors' recent paper (Zhang and Chen [2014]), we studied bipartite consensus problem of general linear systems, where a distributed control law using full state information was proposed. We showed that for general linear multi-agent systems, bipartite consensus over signed graphs is equivalent to ordinary consensus over nonnegative graphs.

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In this paper, we extend our work to the setting of output feedback control. In particular, we propose a dynamic output feedback control law for linear multi-agent systems over signed directed graphs. We show that bipartite consensus can be achieved when the graph has a spanning tree and is structurally balanced. Moreover, we point out that an appropriate definition of the neighborhood consensus error is also crucial for achieving bipartite consensus.

The paper is organized as follows. Section 2 introduces concepts and preliminary results on signed graphs. In Section 3, an output feedback control law is designed and rigorously analyzed. Simulation examples are provided in Section 4, and Section 5 concludes the paper.

Notations Notations used throughout the paper are rather standard. The empty set is $\varnothing$. For a square matrix A, $\sigma(A)$ is its spectrum. A matrix with entries a_{ij} is denoted as $[a_{ij}]$. A diagonal matrix with entries $\sigma_1, \ldots, \sigma_n$ is $diag(\sigma_1, \ldots, \sigma_n)$. A vector of all ones is denoted as $\mathbf{1}_n \in \mathbb{R}^n$ and the identity matrix is $I_N \in \mathbb{R}^{N\times N}$. The Kronecker product is denoted as $\otimes$.

2. PRELIMINARIES ON SIGNED GRAPHS

Communication network of a multi-agent system can be modeled by a graph, where the nodes/vertices correspond to the agents and the edges correspond to the communication links between agents. A graph $\mathcal{G}$ can be mathematically represented by $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, with $\mathcal{V} = \{v_1, \ldots, v_N\}$ being the set of nodes, and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ the set of edges. Generally, a weight a_{ij} is assigned to an edge (v_j, v_i), which may represent the communication strength. When there is an edge from node j to node i, graphically depicted by an arrow with head at node i and tail at node j, we have $a_{ij} \neq 0$; otherwise $a_{ij} = 0$. When $a_{ij} \neq 0$, node i can get information from node j which is often called a neighbor of node i, and also node j is known as the parent node and i the child node. A positive edge and a negative edge can represent collaborative or antagonistic interactions between agents, respectively. A graph with all edges positive is a nonnegative graph, while a graph having both positive edges and negative edges is a signed graph. An edge may be undirectional (i.e., $a_{ij} = a_{ji}$) or directional (i.e., $a_{ij} \neq a_{ji}$), corresponding respectively to undirected graphs or directed graphs (or digraphs). This paper concerns collective behaviors over signed digraphs. We further assume there is no self-loop, i.e., $a_{ii} = 0$.

Three classes of graph topologies are often encountered when studying the multi-agent systems over digraphs: graph being a spanning tree, graph having a spanning tree, and graph being strongly connected. A graph is a spanning tree if each node, except for one node (called the root node, which only has child node(s)), has only one parent node; a graph has a spanning tree if deleting some edges properly leaves a spanning tree; a graph is strongly connected if for each ordered pair of nodes $[v_i, v_j]$, $i \neq j$, there is a directed path in the form $\{(v_i, v_l), (v_l, v_p), \ldots, (v_q, v_j)\}$.

The adjacency matrix fully captures the topology of a graph and is defined as $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$. Let $\mathcal{G}(\mathcal{A})$ explicitly denote a graph whose adjacency matrix is $\mathcal{A}$. For nonnegative graphs, an important matrix associated with graph $\mathcal{G}(\mathcal{A})$ is the Laplacian matrix (Godsil and Royle [2001], Ren and Beard [2008])

$$L = [l_{ij}] = diag\left(\sum_{j=1}^{N} a_{1j}, \ldots, \sum_{j=1}^{N} a_{Nj}\right) - \mathcal{A}. \tag{1}$$

This Laplacian matrix plays an important role in analyzing collective behaviors of multi-agent systems over nonnegative graphs. It has a simple eigenvalue of zero associated with a right eigenvector $\mathbf{1}_N$ when the graph has a spanning tree (Lin et al. [2005], Ren and Beard [2008]). However, this nice property does not generally hold for signed graphs, which makes this Laplacian matrix less useful. In this paper, we adopt another Laplacian matrix (Altafini [2013]) for signed graphs, defined as

$$L = [l_{ij}] = diag\left(\sum_{j=1}^{N} |a_{1j}|, \ldots, \sum_{j=1}^{N} |a_{Nj}|\right) - \mathcal{A}. \tag{2}$$

We shall differentiate definition (1) from (2) using the attribute "*ordinary*". Thought out the paper, Laplacian matrix for signed graphs is always defined as in (2), unless otherwise indicated.

A new graph property specific to signed graphs is defined as follows.

Definition 1. (Harary [1953], Altafini [2013]) A signed graph $\mathcal{G}(\mathcal{A})$ is structurally balanced if it has a bipartition of the nodes $\mathcal{V}_1$, $\mathcal{V}_2$, i.e., $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$ and $\mathcal{V}_1 \cap \mathcal{V}_2 = \varnothing$, such that $a_{ij} \leq 0$ when nodes i and j are in different subgroups; otherwise, $a_{ij} \geq 0$.

The following several lemmas are instrumental in the analysis of consensus patterns on signed graphs.

Lemma 1. (Altafini [2013]) A spanning tree is always structurally balanced.

Lemma 2. (Zhang and Chen [2014]) Suppose the signed digraph $\mathcal{G}(\mathcal{A})$ *has a spanning tree*. Denote the signature matrices set

$$\mathcal{D} = \{D = diag(\sigma_1, \ldots, \sigma_N) \mid \sigma_i \in \{1, -1\}\}.$$

Then the following statements are equivalent.

a) $\mathcal{G}(\mathcal{A})$ is structurally balanced;
b) $a_{ij}a_{ji} \geq 0$, and the corresponding undirected graph $\mathcal{G}(\mathcal{A}_u)$ is structurally balanced, where $\mathcal{A}_u = \frac{\mathcal{A}+\mathcal{A}^T}{2}$;
c) $\exists D \in \mathcal{D}$, such that $\bar{\mathcal{A}} = [\bar{a}_{ij}] = D\mathcal{A}D$ is a nonnegative matrix, i.e., $\bar{a}_{ij} = |a_{ij}|$.

Lemma 3. (Zhang and Chen [2014]) Suppose the signed digraph $\mathcal{G}(\mathcal{A})$ has a spanning tree. If the graph is structurally balanced, then 0 is a simple eigenvalue of L and all its other eigenvalues have positive real parts; but not vice versa.

Lemma 2 bridges the gap between signed graphs and nonnegative graphs, through the use of the signature matrix D.

3. DESIGN OF OUTPUT FEEDBACK CONTROL

In this section, we propose an output feedback control law to solve the bipartite consensus problem. We also show that appropriate choices of both neighborhood bipartite consensus error and topology of the graph are crucial for the multi-agent systems to achieve a bipartite consensus.

3.1 Problem formulation

Consider a group of agents, each modeled by an identical linear time-invariant (LTI) system

$$\dot{x}_i = Ax_i + Bu_i, \quad y_i = Cx_i, \qquad i = 1, \ldots, N \tag{3}$$

where $x_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^m$, and $y_i \in \mathbb{R}^q$ are the state, input and output, respectively; the triple (A, B, C) is controllable and observable. The underlying communication network is a signed digraph. Our objective is to design a distributed output feedback control law $u_i(y_i, y_{j|j\in N_i})$ for each node i, such that a bipartite consensus is achieved, which is defined as follows.

Definition 2. (Zhang and Chen [2014]) The linear multi-agent system (3) is said to achieve a bipartite consensus if $\lim_{t\to\infty}(x_i(t) - x^*(t)) = 0$, $\forall i \in p$ and $\lim_{t\to\infty}(x_j(t) + x^*(t)) = 0$, $\forall j \in q$ for some nontrivial trajectory $x^*(t)$, where $p \cup q = \{1, \ldots, N\}$ and $p \cap q = \varnothing$.

When either p or q is empty, Definition 2 reduces to the ordinary consensus (Ren and Beard [2008]), where all nodes converge to the same value. When $x^*(t) = 0$, Definition 2 reduces to a trivial consensus in the ordinary sense.

3.2 Controller design

For each node i, define the local output estimation error as

$$\tilde{y}_i = y_i - \hat{y}_i = Cx_i - C\hat{x}_i,$$

where $\hat{y}_i$ and $\hat{x}_i$ are the estimated output and state information, respectively; and define the neighborhood estimated bipartite consensus error as

$$\hat{\epsilon}_i = \sum_{j\in N_i} (a_{ij}\hat{x}_j - |a_{ij}|\hat{x}_i). \tag{4}$$

Modified from [Zhang et al., 2011, Section V.B], the controller and observer are designed here as

$$u_i = cK\hat{\epsilon}_i = cK\sum_{j\in N_i} (a_{ij}\hat{x}_j - |a_{ij}|\hat{x}_i), \tag{5}$$

$$\dot{\hat{x}}_i = A\hat{x}_i + Bu_i - cF\tilde{y}_i, \tag{6}$$

where

- c is a scalar control gain chosen as
$$c \geq \frac{1}{2\min_{i\in\mathcal{I}} Re(\lambda_i)}, \tag{7}$$
where $Re(\lambda_i)$ is the real part of the i-th eigenvalue of the Laplacian matrix L defined in (2), and $\mathcal{I} = \{i \mid Re(\lambda_i) > 0, \quad i \in \{1, \ldots, N\}\}$;

- F is designed such that $(A + cFC)$ is Hurwitz;
- K is designed using the Riccati design approach as follows
$$K = R^{-1}B^T P \tag{8}$$
with P being the unique positive definite solution of the algebraic Riccati equation (ARE)
$$A^T P + PA + Q - PBR^{-1}B^T P = 0,$$
where Q and R are both positive definite design matrices with appropriate dimensions.

It is well known (Lewis and Syrmos [1995]) that (8) is the optimal state feedback control gain of the system (3) with respect to the local performance index
$$J = \int_0^\infty (x_i^T Q x_i + u_i^T R u_i) dt.$$
The next result demonstrates that this local optimal control gain (8) leads to a bipartite consensus over a signed digraph.

Theorem 1. Assume the signed digraph $\mathcal{G}(\mathcal{A})$ has a spanning tree and is structurally balanced. Under the dynamic control law (5)-(6), system (3) achieves a bipartite consensus, and the state estimates converge to their real values, i.e., $\lim_{t\to\infty}(x_i - \hat{x}_i) = 0$ for all i.

Proof. The closed-loop system is
$$\dot{x}_i = Ax_i + cBK \sum_{j\in N_i} (a_{ij}\hat{x}_j - |a_{ij}|\hat{x}_i).$$
Put it in a compact form as
$$\dot{x} = (I_N \otimes A)x - (cL \otimes BK)\hat{x}, \tag{9}$$
where $x = [x_1^T, \ldots, x_N^T]^T$, $\hat{x} = [\hat{x}_1^T, \ldots, \hat{x}_N^T]^T$, L is the Laplacian matrix for signed graph $\mathcal{G}$ defined as (2). The observer dynamics is
$$\dot{\hat{x}} = (I_N \otimes A)\hat{x} - (cL \otimes BK)\hat{x} - (cI_N \otimes FC)(x - \hat{x}). \tag{10}$$

Let the state estimation error be
$$\tilde{x} = x - \hat{x}.$$
Then
$$\dot{\tilde{x}} = (I_N \otimes (A + cFC))\tilde{x}. \tag{11}$$
Since $(A + cFC)$ is Hurwitz, so is $I_N \otimes (A + cFC)$. Then we have
$$\lim_{t\to\infty} \tilde{x}(t) = 0, \tag{12}$$
i.e., $\lim_{t\to\infty}(x_i - \hat{x}_i) = 0$ for all i.

Since graph $\mathcal{G}(\mathcal{A})$ has a spanning tree and is structurally unbalanced, according to Lemma 2, there is a signature matrix $D \in \mathcal{D}$ such that the associated graph $\mathcal{G}(\bar{\mathcal{A}})$ is a nonnegative graph and has a spanning tree, where $\bar{\mathcal{A}} = D\mathcal{A}D$. Let $\bar{L}$ be the ordinary Laplacian matrix (1) of graph $\mathcal{G}(\bar{\mathcal{A}})$. Then we have $\bar{L} = DLD$.

Equation (9) can be written as
$$\dot{x} = (I_N \otimes A - cL \otimes BK)x + (cL \otimes BK)\tilde{x}. \tag{13}$$
Define $z = (D \otimes I_n)x$ and $\tilde{z} = (D \otimes I_n)\tilde{x}$. Note that $D = D^T = D^{-1}$. Then $x = (D \otimes I_n)z$ and $\tilde{x} = (D \otimes I_n)\tilde{z}$. Straightforward computation gives
$$\dot{z} = (I_N \otimes A - c\bar{L} \otimes BK)z + (c\bar{L} \otimes BK)\tilde{z}. \tag{14}$$

Since graph $\mathcal{G}(\bar{\mathcal{A}})$ is nonnegative and has a spanning tree, $\bar{\lambda}_1 = 0$ is a simple eigenvalue of the Laplacian matrix $\bar{L}$, and all other eigenvalues $\bar{\lambda}_i$ have positive real parts (Lin et al. [2005]), i.e, $0 = \bar{\lambda}_1 < Re(\bar{\lambda}_2) \leq \cdots \leq Re(\bar{\lambda}_N)$. Please be noted that in the following development, we shall replace $\bar{\lambda}_i$ with λ_i without changing any result, for $\bar{L} = DLD = D^{-1}LD$. There exists a nonsingular matrix $M = [m_1 \mid m_2 \mid \cdots \mid m_N] \in \mathbb{R}^{N\times N}$, where $m_i \in \mathbb{R}^N$ and $m_1 = \mathbf{1}_N = [1\ 1\ \ldots\ 1]^T \in \mathbb{R}^N$ is the right eigenvector of $\bar{L}$ associated with eigenvalue 0, such that the Jordan form of $\bar{L}$ is
$$J = M^{-1}\bar{L}M = \left[\begin{array}{c|c} 0 & \\ \hline & J_{N-1} \end{array}\right],$$
where $J_{N-1} \in \mathbb{R}^{(N-1)\times(N-1)}$ is itself a Jordan form with nonzero diagonal entries $\lambda_2, \cdots, \lambda_N$.

Define $q = \left[q_1^T \mid q_2^T\ \ldots\ q_N^T\right]^T = \left[q_1^T \mid \mathfrak{q}^T\right]^T = (M^{-1} \otimes I_n)z$ and $\tilde{q} = (M^{-1} \otimes I_n)\tilde{z}$, where $q_i \in \mathbb{R}^n$, $\mathfrak{q} = [q_2^T, \ldots, q_N^T]^T \in \mathbb{R}^{(N-1)n}$. Then
$$\begin{aligned}\dot{q} &= (M^{-1} \otimes I_n)\dot{z} \\ &= (I_N \otimes A - cJ \otimes BK)q + (cJ \otimes BK)\tilde{q},\end{aligned}$$
or equivalently,
$$\dot{q}_1 = Aq_1, \tag{15}$$
$$\dot{\mathfrak{q}} = \bar{A}_c\mathfrak{q} + (c[0 \mid J_{N-1}] \otimes BK)\tilde{q}, \tag{16}$$
where $\bar{A}_c = (I_{N-1} \otimes A - cJ_{N-1} \otimes BK)$.

Matrix $\bar{A}_c$ is a block diagonal or block upper-triangular matrix with diagonal blocks $A - c\lambda_i BK$ $(i = 2, \ldots, N)$. Similar with Theorem 1 in Zhang et al. [2011], we can show that $A - c\lambda_i BK$ are Hurwitz, and thus $\bar{A}_c$ is Hurwitz. Also, by recalling that $\lim_{t\to\infty} \tilde{x}(t) = 0$, we have
$$\lim_{t\to\infty} \tilde{q}(t) = \lim_{t\to\infty} (M^{-1} \otimes I_n)(D \otimes I_n)\tilde{x}(t) = 0.$$
Therefore, solving (15) and (16) yields
$$\lim_{t\to\infty} q_1(t) = e^{At}q_1(0), \tag{17}$$
$$\lim_{t\to\infty} \mathfrak{q}(t) = 0. \tag{18}$$
Since
$$\begin{aligned} z &= (M \otimes I_n)q \\ &= (m_1 \otimes I_n)q_1 + \cdots + (m_N \otimes I_n)q_N,\end{aligned}$$
finally we have $\lim_{t\to\infty} z(t) = (m_1 \otimes I_n)e^{At}q_1(0)$, that is,
$$\lim_{t\to\infty} z_i(t) = e^{At}q_1(0), \quad \forall i = 1, 2, \ldots, N.$$
Then, bipartite consensus of system (3) follows from the fact that $x = (D \otimes I_n)z$. □

Structural balance is a necessary graph property that guarantees the bipartite consensus behaviors. This is shown by the following results.

Theorem 2. Assume the signed digraph $\mathcal{G}(\mathcal{A})$ is strongly connected and structurally unbalanced. Under the dynamic control law (5)-(6), the system (3) achieves a trivial consensus, i.e., $\lim_{t\to\infty} x_i = 0$; and the state estimates converge to their real values, i.e., $\lim_{t\to\infty}(x_i - \hat{x}_i) = 0$ for all i.

Proof. First, it can be seen that derivation of the state estimation error dynamics (11) does not depends on the graph topology. Thus convergence of the observer holds as shown in Theorem 1.

The closed loop system is the same as (13). Let $A_c = I_N \otimes A - cL \otimes BK$. By [Altafini, 2013, Corollary 3], all eigenvalues λ_i of matrix L has positive real parts. Let J be a Jordan form of matrix L. It is trivial to show that matrix A_c is similar to the matrix $(I_N \otimes A - cJ \otimes BK)$, which is a block diagonal or block upper-triangular matrix with diagonal blocks being $A - c\lambda_i BK$. Noting that all matrices $A - c\lambda_i BK$ ($i = 1, \dots, N$) are Hurwitz, which can be shown the same way as in [Zhang et al., 2011, Theorem 1], it is clear that A_c is Hurwitz. Due to the fact that $\lim_{t\to\infty} \tilde{x}(t) = 0$, solving (13) yields $\lim_{t\to\infty} x(t) = 0$. □

When the signed digraph $\mathcal{G}(\mathcal{A})$ has a spanning tree and is structurally unbalanced, interestingly more complex collective behaviors will emerge, depending on specific graph topologies. We shall not attempt to provide a rigorous analysis of this case herein, but only demonstrate this phenomena by a simulation example in Section 4.3.

The neighborhood estimated bipartite consensus error (4) is also vital for bipartite consensus. This is illustrated by the next result.

Theorem 3. If the controller (5) is modified to be

$$u_i = cK \sum_{j \in N_i} |a_{ij}|(\hat{x}_j - \hat{x}_i), \tag{19}$$

then ordinary consensus is achieved, i.e., $\lim_{t\to\infty}(x_i - x_j) = 0$, $\forall i \neq j$, over any signed digraph, either structurally balanced or unbalanced, as long as it has a spanning tree; and the state estimates converge to their real values, i.e., $\lim_{t\to\infty}(x_i - \hat{x}_i) = 0$ for all i.

This can be proved by showing that it is equivalent to an ordinary consensus over graph $\mathcal{G}(\mathcal{A}^*)$, where $\mathcal{A}^* = [a^*_{ij}] \in \mathbb{R}^{N\times N}$ and $a^*_{ij} = |a_{ij}|$. Details are omitted.

4. SIMULATION EXAMPLES

Consider a multi-agent system with 6 nodes and each node is modeled by an LTI system (3) with

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \text{ and } C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

4.1 Example 1

This example illustrates Theorem 1, where the signed graph has a spanning tree and is structurally balanced (see Fig. 1a), and the controller takes the form (5)-(6). Let $x_{i,j}$ be the j-th component of the state vector x_i, i.e., $x_i = [x_{i,1}, \dots, x_{i,n}]^T$. The simulation results are shown by Figs. 2-3. A bipartite consensus is achieved, with two subgroups being $\mathcal{V}_1 = \{v_3, v_4\}$ and $\mathcal{V}_2 = \{v_1, v_2, v_5, v_6\}$ (see Fig. 2). The convergence of the observer is shown by Fig. 3.

4.2 Example 2

This example illustrates Theorem 2, where the signed graph is strongly connected and is structurally unbalanced (see Fig. 1b), the controller is taking the form (5)-(6). In this case, all nodes achieve a trivial consensus, i.e., $\lim_{t\to\infty} x_{i,j} = 0$ for all i and j (see Fig. 4). The observer converges for all nodes (see Fig. 5).

4.3 Example 3

When the signed graph has a spanning tree and is structurally unbalanced, under the controller (5)-(6), neither ordinary consensus nor bipartite consensus is obtained. In this example, the

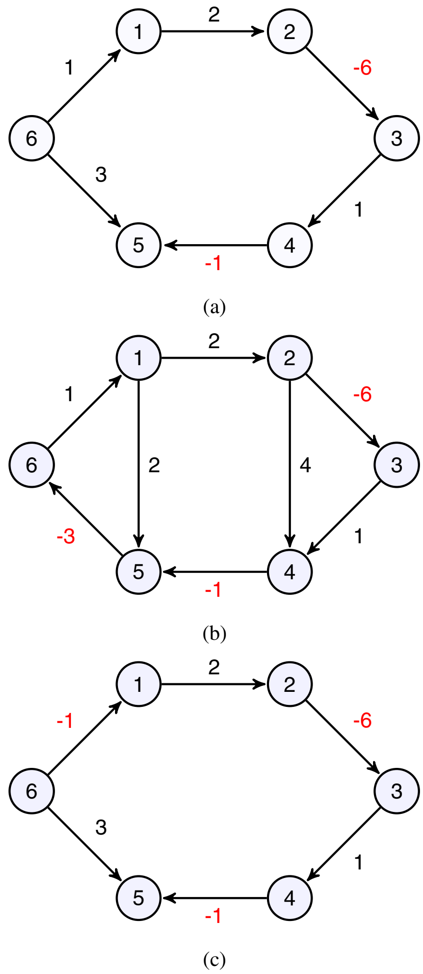

Fig. 1. Three types of graph topologies: (a) A graph which has a spanning tree and is structurally balanced; (b) A graph which is strongly connected and structurally unbalanced; (c) A graph which has a spanning tree and is structurally unbalanced.

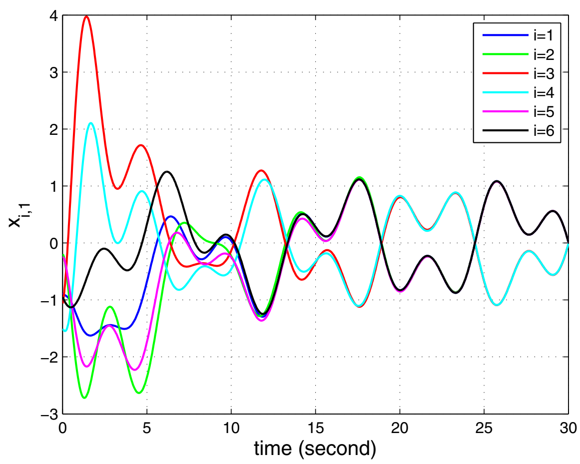

Fig. 2. State trajectories of $x_{i,1}$

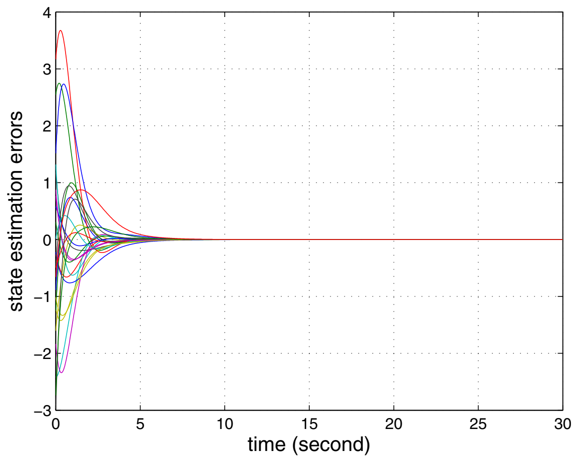

Fig. 3. Trajectories of state estimation errors $\tilde{x}_{i,j}$

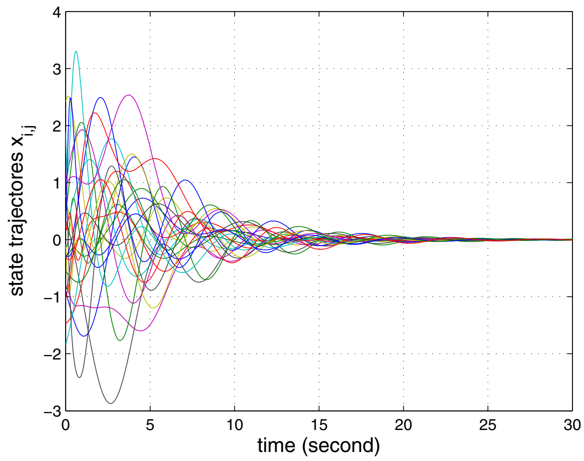

Fig. 4. State trajectories of $x_{i,j}$ for all i and j

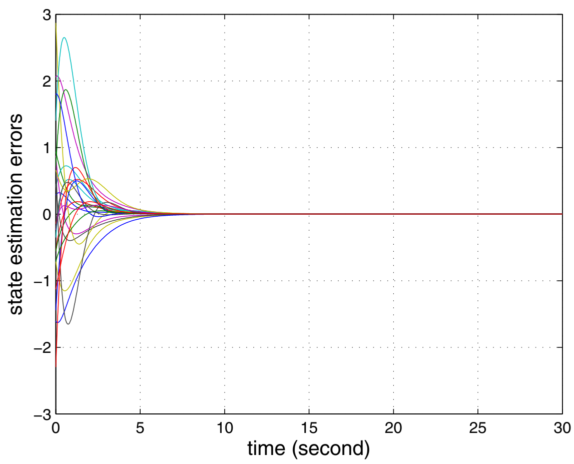

Fig. 5. Trajectories of state estimation errors $\tilde{x}_{i,j}$

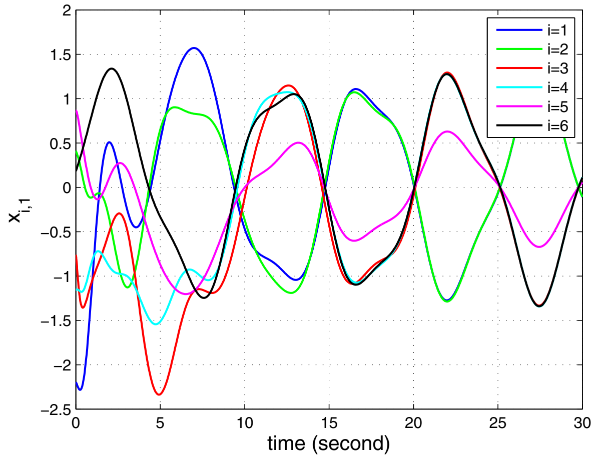

Fig. 6. State trajectories of $x_{i,1}$

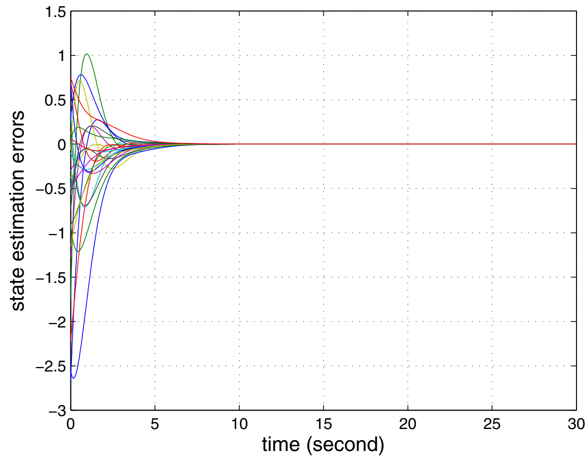

Fig. 7. Trajectories of state estimation errors $\tilde{x}_{i,j}$

nodes are separated into three subgroups $\mathcal{V}_1 = \{v_3, v_4, v_6\}$, $\mathcal{V}_2 = \{v_1, v_2\}$ and $\mathcal{V}_3 = \{v_5\}$. While subgroups $\mathcal{V}_1$ and $\mathcal{V}_2$ achieve a bipartite consensus, subgroup $\mathcal{V}_3$ agrees with neither subgroups (see Fig. 6). The observer still converges as shown in Fig. 7.

5. CONCLUSIONS

This paper investigated bipartite consensus of general linear multi-agent systems over signed digraphs, which may have both positive edges and negative edges. An observer based output feedback control law was proposed. We also showed that two factors are crucial for achieving a bipartite consensus, of which one is an appropriate choice of the neighborhood estimated bipartite consensus error (4), and the other is the appropriate graph topology which requires the signed digraph to have a spanning tree and be structurally balanced.

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