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Research Article

On the Observability of Leader-Based Multiagent Systems with Fixed Topology

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This paper investigates the observability of first-order, second-order, and high-order leader-based multiagent systems (MASs) with fixed topology, respectively. Some new algebraic and graphical characterizations of the observability for the first-order MASs are established based on agreement protocols. Moreover, under the same leader-following framework with the predefined topology and leader assignment, the observability conditions for systems of double-integrator and high-integrator agents are also obtained. Finally, the effectiveness of the theoretical results is verified by numerical examples and simulations.

1. Introduction

Self-organizing behavior of multiple intelligent dynamic agents is widespread in nature, for example, bird flocking flight [1], fish group patrol [2], microbial collective foraging [3], and ant colony cooperation [4]. This self-organizing behavior of such intelligent and dynamical networked system, through transferring and interaction of information between local agents, as a whole, presenting the ability of effective coordination and the level of advanced intelligence, has led to the wide concern and interest in many fields including applied mathematics, ecology, biology, sociology, physics, computer science, control, and engineering. At present, coordinated control of intelligent systems has become a hot research topic in the field of control and is widely used in smart grids, UAV formation, intelligent transportation systems, artificial satellite networks, and so on [5–20].

Observability is a significant problem and becomes the basis of the design of optimal control and optimal estimation in modern control theory, which was put forward by Kalman in 1960. Intuitively speaking, the so-called system's observability means that the internal states of the system can be correctly known by observing the external variables of the system. In general, for a multiagent network, the observability is the key problem and means that the entire system's state can be completely reconstructed by only observing a small quantity of intelligent dynamic agents (i.e., leaders). Observability plays a fundamental role in state estimation and system control as well as has recently been widely applied in wireless sensor networks, satellite navigation systems, autonomous underwater vehicles, machine games, and so on.

The concepts of controllability and observability were first tried to apply to networked multiagent systems by Tanner in 2004 [21], where all agents are partitioned into leader agents and follower agents, in which the follower agents are only affected by the neighbor interaction protocol and the leader agents are influenced by the external control input. This structure, called leader-follower structure, can make the whole system as a standard linear time-invariant system so that the classical controllability and observability are all applied into networked multiagent systems. Observability is very important to study distributed estimation and intrusion detection problems in distributed sensor networks [22]. Zelazo and Mesbahi [23] investigated the observability of networked systems with homogeneous and heterogeneous dynamics, respectively, as well as provided an analysis for the observability of networked linear systems. In

[24], the study mainly focused on controllability and observability of multiagent networks under a leader-follower framework with the chain and cyclic topologies, respectively. In [25], the authors established a decentralized condition for controllability and observability of networked MASs using decentralized Laplacian spectrum estimation. The observability of a network system was investigated based on average consensus algorithm for path and cycle graphs in [26], graph Cartesian products in [27], regular graphs and distance-regular graphs [28, 29], and equitable partition [30], respectively. Furthermore, the observability of switched linear systems and switched multiagent systems was studied in [31, 32], respectively. Recently, the researchers [33–35] investigated the observability of MASs with time-varying and switching topologies, respectively. In addition, Liu et al. [36] analyzed the observability for complex networks.

It is worthwhile to note that most of the previous research conclusions mainly focused on controllability and observability of first-order networked MASs. However, it is well known that, in control engineering, almost all control systems are high-order systems, that is, higher-order differential/difference equations are used to describe those control systems studied in many areas, such as consensus, controllability, flocking, containment control, and stabilizability. In this paper, the observability of a class of MASs is considered, and a unified framework for the inertia of first-order, second-order, and high-order MASs is given. Most of the existing literatures on the observability of MASs focused only on a single dynamic equation, such as a first-order dynamics, a second-order dynamics, or a high-order dynamics, rather than a unified consideration. In general, it is difficult to bring the three kinds of networks into a unified framework, which is a hard point in modelling and analyzing. This paper aims at the observability of MASs with first-order/second-order/high-order agents based on agreement protocols, respectively. The main contributions of this paper are summarized as follows: (i) Some necessary and sufficient conditions for the observability of first-order MASs are built by Laplacian matrix, which relies only on the information flows within the agents and between the agents from the leaders. (ii) It is proved that the observability of the second-order/high-order MASs is equivalent to that of first-order MASs with the same information of topology structure and the same prescribed leaders. (iii) A unified framework for the observability inertia of first-order, second-order, and high-order MASs is established, which relies only on the information topology of such network.

2. Mathematical Preliminaries

For the convenience of discussing the observability of networked systems, the information communication links among agents can be described by a weighted undirected/directed graph denoted as $\mathscr{G} = (\mathscr{V}, \mathrm{E}, \mathscr{A})$, in which $\mathscr{V} = \{v_1, v_2, \ldots, v_n\}$ is a *vertex set* and $\mathrm{E} = \{(v_i, v_j) : v_i, v_j \in \mathscr{V}\}$ is an *edge set*, respectively. The weighted adjacency matrix of $\mathscr{G}$ is marked as $\mathscr{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ with $a_{ij} \neq 0$ if and only if $(v_i, v_j) \in \mathscr{V}$ and $a_{ij} = 0$, otherwise. Moreover, $\mathscr{N}_i = \{v_j \in \mathscr{V} : (v_i, v_j) \in \mathrm{E}\}$ represents the neighbors' set of agent v_i. The Laplacian matrix is given as $L \triangleq \Delta - \mathscr{A}$, where $\mathscr{A}$ is the weighted adjacency matrix and Δ is the diagonal degree matrix.

For a network with leader-follower structure (see Figure 1), the Laplacian matrix L can be divided into the following block matrices:

$$L = \begin{bmatrix} L_f & L_{fl} \\ L_{lf} & L_l \end{bmatrix}, \tag{1}$$

where L_f and L_l stand for the communication information links within the followers and the leaders, respectively, L_{fl} stands for the communication information links from the leaders to the followers, and L_{lf} stands for the communication information links from the followers to the leaders, where $L_{fl}^T = L_{lf}$ if graph $\mathscr{G}$ is undirected.

3. Observability Analysis

3.1. Observability of First-Order MASs. A first-order MAS consists of $(n + l)$ agents, in which the former n agents are regarded as followers and the remaining l agents are regarded as leaders, and its dynamic equation is described by

$$\begin{gathered} \dot{x}_i = u_i, \\ i \in \mathscr{L}_{n+l}, \end{gathered} \tag{2}$$

where $x_i \in \mathbb{R}^n$ represents the state and $u_i \in \mathbb{R}^q$ is the external input. Interactions among agents of system (2) in this paper are given by the agreement protocol as follows:

$$u_i = \sum_{j\in\mathscr{N}_i} a_{ij}\left(x_j - x_i\right), \quad i \in \mathscr{L}_{n+l}, \tag{3}$$

where $\mathscr{N}_i$ is the neighbor set of agent i and $a_{ij} \geq 0$ for all i, j.

Let $x \triangleq [x_1, \ldots, x_{n+l}]^T$, $x_f \triangleq [x_1, \ldots, x_n]^T$, and $x_l \triangleq [x_{n+1}, \ldots, x_{n+l}]^T$. From the partition of L, the dynamic equations (2) and (3) can be redescribed by

$$\dot{x} = \begin{bmatrix} \dot{x}_f \\ \dot{x}_l \end{bmatrix} = -Lx = -\begin{bmatrix} L_f & L_{fl} \\ L_{fl}^T & L_l \end{bmatrix}\begin{bmatrix} x_f \\ x_l \end{bmatrix}. \tag{4}$$

In the actual control process, because of the limitation of the measuring devices, the state of the system is difficult to be measured directly, but can be determined by observing the output of the system. The observability of MASs refers to the observation of the state of the whole MAS only from a few agents as much as possible. Note that it is congenial with reason that some leaders can measure their neighbors' states and exchange information flows between the other leaders, so we can let y be the output vector measured by the leaders. Supposed the leaders can directly exchange information with the other agents, there is a complete communication graph between the leaders. In other words, each leader can broadcast its measured output to all other leaders and has access to the entire output vector y. Therefore, each leader here can get all the output vectors. According to the partition of Laplace matrix, $y = L_{fl}^T x_f$ is chosen as the output. Therefore, the dynamic equation of the networked MAS can be looked upon as a classical linear time-invariant (LTI) system as follows:

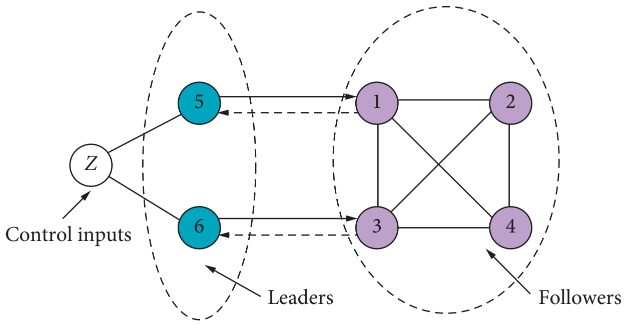


FIGURE 1: A leader-follower framework.

$$\begin{cases} \dot{x}_f = -L_f x_f - L_{fl} x_l, \\ y = -L_{fl}^T x_f, \end{cases} \tag{5}$$

whose observability is equivalent to that of the following system:

$$\begin{cases} \dot{x}_f = L_f x_f + L_{fl} x_l, \\ y = L_{fl}^T x_f, \end{cases} \tag{6}$$

where $y \in \mathbb{R}^n$ represents the output, whose corresponding output matrix is denoted as L_{fl}^T, and contains all the state vectors measured by the leaders. Therefore, the classical concept of observability of a standard LTI system can be used to solve the problem of the observable state variables that are measurable by the leaders.

Observability is a fundamental problem of networked MASs, which focuses on whether the whole system state can be reconstructed by the output-observed agents' evolution behaviors. Then, observability concerns on the identical zero output brought by the nonzero initial states. In other words, such multiagent network is observable if there exists no unobservable state. For the convenience of theoretical demonstration and calculation, we only need to study the observability of system (6) in the following.

Definition 1 (see [35]). Given $x(t_0) = x_0$, a state x_0 of system (6) is unobservable if $y(t) = 0$ for any $t \in [t_0, t_f]$, where finite time $t_f > t_0$.

According to known results in linear time-invariant system theory, we can derive the following proposition.

Proposition 1. *For system (6), the following statements are equivalent:*

(1) *System (6) is observable.*

(2) *Observability matrix* $Q_o = \begin{bmatrix} L_{fl}^T \\ L_{fl}^T L_f \\ \vdots \\ L_{fl}^T (L_f)^{n-1} \end{bmatrix}$ *has full rank.*

(3) *Matrix* $\begin{bmatrix} \lambda I - L_f \\ L_{fl}^T \end{bmatrix}$ *has full rank for* $\forall \lambda \in \mathbb{R}$.

(4) *Matrix L_f has no nonzero right eigenvectors orthogonal to all rows of L_{fl}^T. Namely, for all eigenvalues (λ_i, $i = 1, \ldots, n$) of matrix L_f, if*

$$\begin{aligned} L_f \alpha &= \lambda \alpha, \\ L_{fl}^T \alpha &= 0, \end{aligned} \tag{7}$$

both hold, then the right eigenvector $\alpha = 0$.

Together with some fundamental propositions of the graph Laplacian for an undirected gragh, the following results can be obtained.

Theorem 1. *The first-order MAS (6) is observable iff L and L_f do not share common eigenvalues.*

Proof. Similar proof can be seen from Lemma 2.2 in reference [37], omitted here. □

Remark 1. It is noted from Theorem 1 that the observability of networked MASs relies only on the information links between agents. Theorem 1 provides a simpler and more easily checkable method because it only involves the couplings of the entire network (between all agents) and subnetwork (only among follower agents). It is also easy to compute the eigenvalues of the coupling matrices of the entire network and the subnetwork by MATLAB. In addition, Theorem 1 is the theoretical basis and analytic technique on observability.

Theorem 2. *The first-order MAS (6) is observable iff there exists no eigenvector of L taking zero on the element corresponding to the leader agent.*

Proof (necessity). By contradiction, supposed λ is the eigenvalue of L, whose corresponding eigenvector of L is denoted as $\xi \triangleq \begin{bmatrix} \xi_f \\ \xi_l \end{bmatrix}$, in which the elements corresponding to the leader agent are zero, that is, $\xi_l = 0$, then we can have

$$L\xi = \begin{bmatrix} L_f & L_{fl} \\ L_{fl}^T & L_l \end{bmatrix} \begin{bmatrix} \xi_f \\ 0 \end{bmatrix} = \begin{bmatrix} L_f \xi_f \\ L_{fl}^T \xi_f \end{bmatrix} = \lambda \begin{bmatrix} \xi_f \\ 0 \end{bmatrix}, \tag{8}$$

and then, $L_f \xi_f = \lambda \xi_f$ and $L_{fl}^T \xi_f = 0$. Therefore, system (6) is unobservable from Proposition 1 (6), which is in contradiction with the fact that system (6) is observable. Then, the proof of the necessity is completed. *Sufficiency*. By contradiction, assume that system (6) is unobservable; from Theorem 1, we can know that L and L_f share one common eigenvalue at least, say λ, which corresponds to the eigenvector ξ satisfying

$$\xi = M\xi_f = \begin{bmatrix} \xi_f \\ 0 \end{bmatrix}, \tag{9}$$

where $M \triangleq [I_f \; 0]^T$ and ξ and ξ_f are the eigenvectors of L and L_f corresponding to the common eigenvalue λ, respectively. Moreover, we can know from equation (4) that

$$L\xi = \begin{bmatrix} L_f & L_{fl} \\ L_{fl}^T & L_l \end{bmatrix} \begin{bmatrix} \xi_f \\ 0 \end{bmatrix} = \lambda \begin{bmatrix} \xi_f \\ 0 \end{bmatrix}, \tag{10}$$

which contradicts to the fact there exists no eigenvector of L taking zero on the element corresponding to the leader agent. Thus, the proof of the sufficiency is completed. □

Theorem 3. *The first-order MAS (6) is observable if*

(i) *The eigenvalues of L_f are all distinct;*

(ii) *All the row vectors of L_{fl}^T are not orthogonal to at least one eigenvector of L_f simultaneously.*

Proof. L_f can be shown as $L_f = UDU^{-1}$ if the eigenvalues of L_f are all distinct, where $D \triangleq \mathrm{diag}(\lambda_1, \ldots, \lambda_n)$ with λ_i being the eigenvalues of L_f for $i = 1, \ldots, n$, as well as U consists of the eigenvectors of L_f. The observability matrix Q_o can be presented as

$$Q_o = \begin{bmatrix} L_{fl}^T \\ L_{fl}^T L_f \\ \vdots \\ L_{fl}^T (L_f)^{n-1} \end{bmatrix} = \begin{bmatrix} L_{fl}^T U U^{-1} \\ L_{fl}^T U D U^{-1} \\ \vdots \\ L_{fl}^T (UDU^{-1})^{n-1} \end{bmatrix} = \begin{bmatrix} L_{fl}^T U U^{-1} \\ L_{fl}^T U D U^{-1} \\ \vdots \\ L_{fl}^T U D^{n-1} U^{-1} \end{bmatrix} = \begin{bmatrix} L_{fl}^T U \\ L_{fl}^T U D \\ \vdots \\ L_{fl}^T U D^{n-1} \end{bmatrix} U^{-1}. \tag{11}$$

□

Because U^{-1} is nonsingular, we only consider the matrix

$$\widetilde{Q}_o^T = \left[L_{fl}^T U \vdots L_{fl}^T U D \vdots \cdots \vdots L_{fl}^T U D^{n-1} \right], \tag{12}$$

whose rank is completely determinated by matrix $L_{fl}^T U$ since the diagonal matrix D is nonsingular. If all the row vectors of L_{fl}^T are not orthogonal to at least one eigenvector of L_f simultaneously, then all the elements of at least one column of $L_{fl}^T U$ are nonzero so that observability matrix Q_o has full rank. Therefore, the first-order multiagent system (6) is observable.

From Theorem 3, we have obtained the following result immediately.

Theorem 4. *The first-order MAS (6) is observable if*

(i) *The eigenvalues of L_f are all distinct;*

(ii) *Every column of $L_{fl}^T U$ has at least one nonzero element, where U consists of the eigenvectors of L_f.*

Proof. From condition (i) and Theorem 3, we can have

$$\mathrm{rank}\begin{bmatrix} \lambda_i I_n - D \\ L_{fl}^T U \end{bmatrix} = \mathrm{rank}\begin{bmatrix} \lambda_i - \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_i - \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_i - \lambda_n \\ b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{21} & \cdots & b_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{l1} & b_{l2} & \cdots & b_{\mathrm{ln}} \end{bmatrix}, \tag{13}$$

where $D = \mathrm{diag}(\lambda_1, \lambda_1, \ldots, \lambda_n)$ with λ_i $(i = 1, 2, \ldots, n)$ being the eigenvalues of L_f and $L_{fl}^T U = [b_{ij}] \in \mathbb{R}^{l \times n}$. Since $\lambda_i \neq \lambda_j$ for $i \neq j$, and every column of $L_{fl}^T U$ has at least one nonzero element, then

$$\mathrm{rank}\begin{bmatrix} \lambda_i I_n - D \\ L_{fl}^T U \end{bmatrix} = n. \tag{14}$$

According to Proposition 1 (3), this conclusion holds. □

Remark 2. It is noted from Proposition 1 (2) the observability of first-order MASs relies only on the information links between agents, that is, (L_f, L_{fl}^T). Theorem 1 mainly illustrates the influence of the relationship between the entire network among all agents (from leader agents to follower agents and among follower agents) and the subnetwork among follower agents on the observability of the system. Theorem 2 shows how to choose the leaders to make the whole system observable. Theorem 3 and Theorem 4 explain the topological relations in Theorem 1 in more detail. Therefore, Theorem 1 is the theoretical basis and analytic technique on observability of Theorems 2–4.

In fact, the observability of first-order MASs cannot be directly extended to the second-order or higher-order MASs. It is of physical and theoretical significance to study the observability for MASs of second-order integrator or high-order integrator agents since first-order system is too simple to depict the dynamic characteristics accurately in the actual physical systems. Therefore, the model with second-order integrator or high-order integrator agents is beneficial for the research of cooperation in real world. However, for the observability of MASs with different orders, the observability problem brings new characteristics and research difficulties, involving in how to model, design the control protocols, select the number of leaders, and locate the leaders, as well as define the observability and so on. Specially, the leaders' positions and speeds have important impacts on observability. These factors make the observability of MASs of second-order integrator or high-order integrator agents a nontrivial new problem. Due to lack of research tools, there are very few results for the observability of MASs with second-order or high-order being available in the existing literatures. Therefore, the results for the first-order case cannot be directly extended to the second-order or high-order MASs.

3.2. Observability of Second-Order MASs. A second-order MAS consists of $(n+l)$ agents, in which the former n agents are regarded as followers and the remaining l agents are regarded as leaders, and its dynamic equation is depicted by

$$\begin{aligned} \dot{x}_i &= v_i, \\ \dot{v}_i &= u_i, \\ i &\in \mathscr{L}_{n+l}, \end{aligned} \tag{15}$$

with

$$u_i = c_1 \sum_{j\in\mathcal{N}_i} a_{ij}\left(x_j - x_i\right) + c_2 v_i, \tag{16}$$

where $x_i \in \mathbb{R}^n$ is the position state and $v_i \in \mathbb{R}^n$ is the velocity state of agent i, respectively; $u_i \in \mathbb{R}^q$ is the input; c_1 and c_2 are nonzero feedback gains.

Let $x_f \triangleq [x_1,\ldots,x_n]^T$, $x_l \triangleq [x_{n+1},\ldots,x_{n+l}]^T$, $v_f \triangleq [v_1,\ldots,v_n]^T$, and $v_l \triangleq [v_{n+1},\ldots,v_{n+l}]^T$. Based on the partition of follower agents and leader agents, the dynamics (15) and (16) can be rewritten as follows:

$$\begin{bmatrix} \dot{x}_f \\ \dot{x}_l \\ \dot{v}_f \\ \dot{v}_l \end{bmatrix} = \begin{bmatrix} 0 & 0 & I_n & 0 \\ 0 & 0 & 0 & I_n \\ c_1 L_f & c_1 L_{fl} & c_2 I_n & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_f \\ x_l \\ v_f \\ v_l \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ u_{\text{ext}} \end{bmatrix}, \tag{17}$$

where $u_{\text{ext}} = [u_{\text{ext}}^1 \ldots, u_{\text{ext}}^l]^T \in \mathbb{R}^l$ is the external control input. Specially, the dynamic equation (17) of followers can be described as

$$\begin{cases} \dot{\overline{x}}_f = \begin{bmatrix} \dot{x}_f \\ \dot{v}_f \end{bmatrix} = \begin{bmatrix} 0 & I_n \\ c_1 L_f & c_2 I_n \end{bmatrix} \begin{bmatrix} x_f \\ v_f \end{bmatrix} + \begin{bmatrix} 0 \\ c_1 L_{fl} \end{bmatrix} x_l \triangleq \overline{L}_f \overline{x}_f + \overline{L}_{fl} x_l, \\ y = \overline{L}_{fl}^T \begin{bmatrix} x_f \\ v_f \end{bmatrix} \triangleq \overline{L}_{fl}^T \overline{x}_f, \end{cases} \tag{18}$$

where $\overline{x}_f \triangleq [x_f, v_f]^T$, $\overline{L}_f \triangleq \begin{bmatrix} 0 & I_n \\ c_1 L_f & c_2 I_n \end{bmatrix}$, and $\overline{L}_{fl} \triangleq \begin{bmatrix} 0 \\ c_1 L_{fl} \end{bmatrix}$.

For the observability of second-order multiagent systems, we can obtain the following results.

Theorem 5. *The second-order MAS (18) is observable iff (L_f, L_{fl}^T) is observable.*

Proof. Supposed λ is the eigenvalue of matrix $\overline{L}_f$, which corresponds to the right eigenvector of $\overline{L}_f$ denoted as $\beta = [\beta_1\, \beta_2]^T$, then we can have $\overline{L}_f [\beta_1\, \beta_2]^T = \lambda [\beta_1\, \beta_2]^T$, and then

$$\begin{cases} \lambda\beta_1 = \beta_2, \\ \lambda\beta_2 = c_1 L_f \beta_1 + c_2 \beta_2. \end{cases} \tag{19}$$

□

Consequently, $L_f\beta_1 = (\lambda^2 - \lambda c_2/c_1) \triangleq u\beta_1$ (since $c_1 \neq 0$), where u is the eigenvalue of L_f with the corresponding right eigenvector β_1, which is the root of equation $s^2 - c_2 s - c_1 u = 0$ with $\beta_2 = s\beta_1$. In the following, we will show the fact that $(\overline{L}_f, \overline{L}_{fl}^T)$ is observable iff (L_f, L_{fl}^T) is observable.

3.2.1. Sufficiency. Supposed (L_f, L_{fl}^T) is unobservable, then there must exist an eigenvalue u of L_f, whose corresponding right eigenvector is $\beta_1 \neq 0$, such that $L_f\beta_1 = u\beta_1$ and $L_{fl}^T\beta_1 = 0$. Therefore, we can assert that the eigenvalue of $\overline{L}_f$ satisfies equation $s^2 - c_2 s - c_1 u = 0$, and the corresponding right eigenvector of $\overline{L}_f$ is $[\beta_1\, \beta_2]^T$ with $\beta_2 = s\beta_1$. Then, $\overline{L}_{fl}^T [\beta_1\, \beta_2]^T = 0$, which contradicts that $(\overline{L}_f, \overline{L}_{fl}^T)$ is observable. The proof of the sufficiency is completed.

3.2.2. Necessity. Supposed $(\overline{L}_f, \overline{L}_{fl}^T)$ is unobservable, then there must exist an eigenvalue λ of $\overline{L}_f$, whose corresponding nonzero right eigenvector is $[\beta_1\, \beta_2]^T$, such that $\overline{L}_f [\beta_1\, \beta_2]^T = \lambda [\beta_1\, \beta_2]^T$ and $\overline{L}_{fl}^T [\beta_1\, \beta_2]^T = 0$. Therefore, we can also assert that L_f has an eigenvalue $u = (\lambda^2 - \lambda c_2/c_1)$ with the corresponding nonzero right eigenvector β_1. Then, $L_f\beta_1 = u\beta_1$ and $L_{fl}^T\beta_1 = 0$, which contradicts that (L_f, L_{fl}^T) is observable. The proof of the necessity is completed.

Note that, from Theorem 5, the nonzero feedback gains c_1 and c_2 can be designed to ensure that equation $s^2 - c_2 s - c_1 u = 0$ has roots. Therefore, we can obtain the following result.

Theorem 6. *The second-order MAS (18) is observable if system (18) satisfies*

(i) $\operatorname{rank}[(\lambda_i^2 - \lambda_i c_2) I_n - c_1 L_f] = n$,

(ii) $\operatorname{rank}[\lambda_i c_1 L_{fl}^T] = n$, or

(iii) $\operatorname{rank}\begin{bmatrix} (\lambda_i^2 - \lambda_i c_2) I_n - c_1 L_f \\ \lambda_i c_1 L_{fl}^T \end{bmatrix} = n.$

Proof. From Proposition 1 (4), we can have

$$\begin{aligned} \operatorname{rank}\begin{bmatrix} \lambda_i I_{2n} - \overline{L}_f \\ \overline{L}_{fl}^T \end{bmatrix} &= \operatorname{rank}\begin{bmatrix} \lambda_i I_n & -I_n \\ -c_1 L_f & (\lambda_i - c_2) I_n \\ 0 & c_1 L_{fl}^T \end{bmatrix} \\ &= \operatorname{rank}\begin{bmatrix} 0 & -I_n \\ \left(\lambda_i^2 - \lambda_i c_2\right) I_n - c_1 L_f & (\lambda_i - c_2) I_n \\ \lambda_i c_1 L_{fl}^T & c_1 L_{fl}^T \end{bmatrix}, \end{aligned} \tag{20}$$

where λ_i $(i = 1, 2, \ldots, 2n)$ are the eigenvalues of $\overline{L}_f$. Obviously, system (18) is observable if system (18) satisfies one of the conditions (i), (ii), and (iii). □

Remark 3. Notice from Theorem 5 that the observability of the second-order MAS depends on the system couplings (L_f, L_{fl}^T). However, from Theorem 6, we can select appropriate nonzero feedback gains c_1 and c_2 so that the observability of the second-order MAS depends only on the information topology between followers or the information topology from leaders to followers. This makes it easier to study the observability through designing the topological relationship of such system.

3.3. Observability of High-Order MASs. A high-order MAS consists of $(n+l)$ agents, in which the former n agents are regarded as followers and the remaining l agents are regarded as leaders, and its dynamic equation is described by

$$\dot{x}_i^{(1)} = x_i^{(2)}, \ldots, \dot{x}_i^{(m-1)} = x_i^{(m)}, \dot{x}_i^{(m)} = u_i, \quad i \in \mathcal{L}_{n+l}, \tag{21}$$

with

$$u_i = \sum_{j\in\mathcal{N}_i} a_{ij} \sum_{k=0}^{m-1} r_k \left(x_j^{(k+1)} - x_i^{(k+1)}\right), \tag{22}$$

where $x_i^{(k+1)}$ $(i = 1, \ldots, n+l)$ is the k-th order of $x_i^{(1)}$, m is the order number, and $r_0, \ldots, r_{m-1}$ are nonzero feedback gains.

Let $\tilde{x}_f \triangleq [x_1^{(1)}, \ldots, x_n^{(1)}, \ldots, x_1^{(m)}, \ldots, x_n^{(m)}]^T$ and $\tilde{x}_l \triangleq [x_{n+1}^{(1)}, \ldots, x_{n+l}^{(1)}, \ldots, x_1^{(m)}, \ldots, x_{n+1}^{(m)}]^T$. Based on the partition of follower agents and leader agents, the dynamics (21) and (22) can be rewritten as follows:

$$\begin{cases} \dot{\tilde{x}}_f = \tilde{L}_f \tilde{x}_f + \tilde{L}_{fl} \tilde{x}_l, \\ y = \tilde{L}_{fl}^T \tilde{x}_f, \end{cases} \tag{23}$$

where

$$\tilde{L}_f = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix}_{m\times m} \otimes I_n + \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \\ r_0 & r_1 & \cdots & r_{m-1} \end{bmatrix}_{m\times m} \otimes L_f,$$

$$\tilde{L}_{fl} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \\ r_0 & r_1 & \cdots & r_{m-1} \end{bmatrix}_{m\times m} \otimes L_{fl}. \tag{24}$$

For the observability of high-order MASs, we can have the following result.

Theorem 7. *The high-order MAS (23) is observable iff (L_f, L_{fl}^T) is observable.*

Proof. Supposed λ is the eigenvalue of $\tilde{L}_f$, whose corresponding right eigenvector is denoted as $\beta = [\beta_1 \beta_2 \ldots \beta_m]^T$, then $\tilde{L}_f [\beta_1, \beta_2, \ldots, \beta_m]^T = \lambda [\beta_1, \beta_2, \ldots, \beta_m]^T$, which follows that

$$\begin{cases} \beta_2 = \lambda\beta_1, \\ \beta_3 = \lambda\beta_2, \\ \vdots \\ \beta_m = \lambda\beta_{m-1}, \\ r_0 L_f \beta_1 + r_1 L_f \beta_2 + \cdots + r_{m-1} L_f \beta_m = \lambda\beta_m. \end{cases} \tag{25}$$

Consequently,

$$\begin{cases} \beta_2 = \lambda\beta_1, \\ \beta_3 = \lambda^2\beta_1, \\ \vdots \\ \beta_m = \lambda^{m-1}\beta_1, \\ r_0 L_f \beta_1 + r_1 L_f \lambda\beta_1 + \cdots + r_{m-1} L_f \lambda^{m-1} \beta_1 = \lambda^m \beta_1, \end{cases} \tag{26}$$
$$\left(r_0 + r_1\lambda + \cdots + r_{m-1}\lambda^{m-1}\right) L_f \beta_1 = \lambda^m \beta_1.$$

Therefore, we can have the polynomial $f(\lambda) = r_0 + r_1\lambda + \cdots + r_{m-1}\lambda^{m-1} \neq 0$. Otherwise, if $f(\lambda) = 0$, it implies that $\beta_1 = 0$ or $\lambda = 0$. If $\beta_1 = 0$, then $\beta_1 = \beta_2 = \beta_3 = \cdots = \beta_N = 0$; and if $\lambda = 0$, then $r_0 = 0$ which contradicts the fact that r_0 is a nonzero gain. Let $u = \lambda^m / r_0 + r_1\lambda + \cdots + r_{m-1}\lambda^{m-1}$, then $L_f \beta_1 = u\beta_1$, where u is the root of equation $s^m - ur_{m-1}s^{m-1} - \cdots - ur_0 = 0$ with

$$\begin{cases} \beta_2 = s\beta_1, \\ \beta_3 = s^2\beta_1, \\ \vdots \\ \beta_m = s^{m-1}\beta_1. \end{cases} \tag{27}$$

Next, we will show the fact that $(\tilde{L}_f, \tilde{L}_{fl}^T)$ is observable iff (L_f, L_{fl}^T) is observable. The following proof is similar to that of Theorem 5, omitted here. □

Remark 4. This paper has investigated not only the observability of first-order MASs but also the observability of second-order and high-order cases. Higher-order systems have more complex and higher dimensional system matrices, so it is more difficult to establish the conditions on the observability and more complex to calculate using observability criteria.

4. Simulations

4.1. Example 1. A five-agent first-order MAS is considered, where agents 1–3 and agents 4–5 are regarded as followers and leaders, respectively. The network topology is described in Figure 2. The system matrices are as follows:

$$L_f = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix},$$
$$L_{fl}^T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}. \tag{28}$$

By MATLAB, we can compute $\lambda(L_f) = \{0.5858, 2.0000, 3.4142\}$ and $L_{fl}^T U = \begin{bmatrix} -0.5000 & 0.7071 & 0.5000 \\ -0.5000 & -0.7071 & 0.5000 \end{bmatrix}$. Obviously, it is easy to find that $\lambda(L_f)$ are all distinct, and all elements of every column of $L_{fl}^T U$ are nonzero. From Theorem 4, such system is observable.

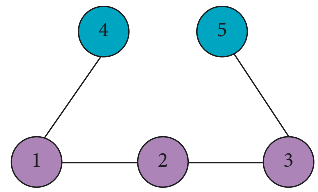


Figure 2: The topology with 5 agents.

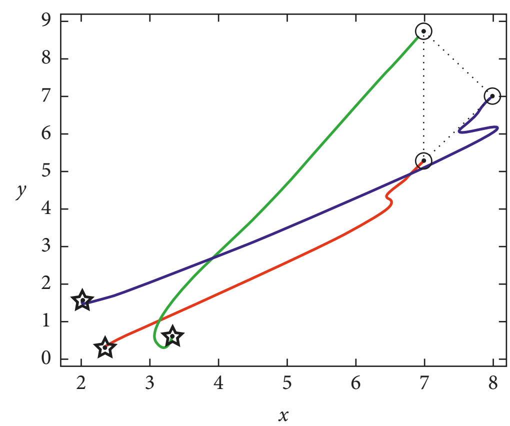


Figure 3: A regular triangle array.

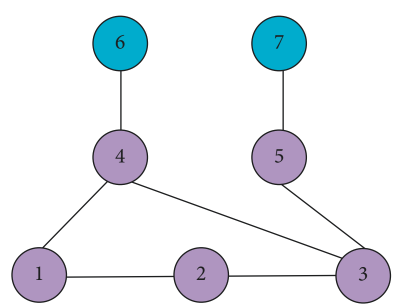


Figure 4: The topology with 7 agents.

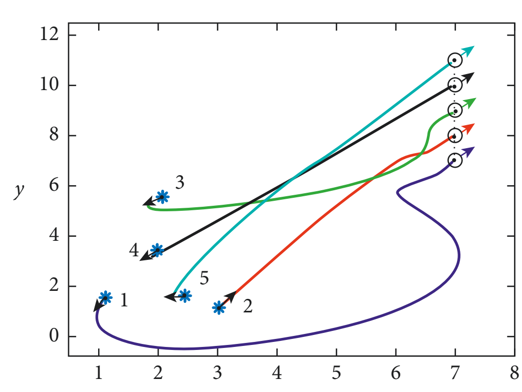


Figure 5: A straight line array.

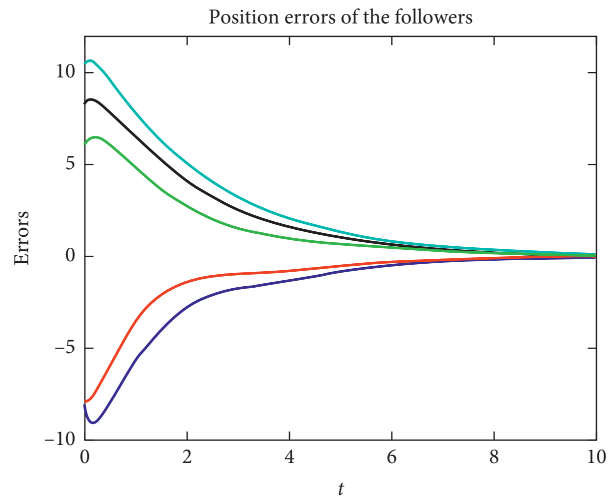
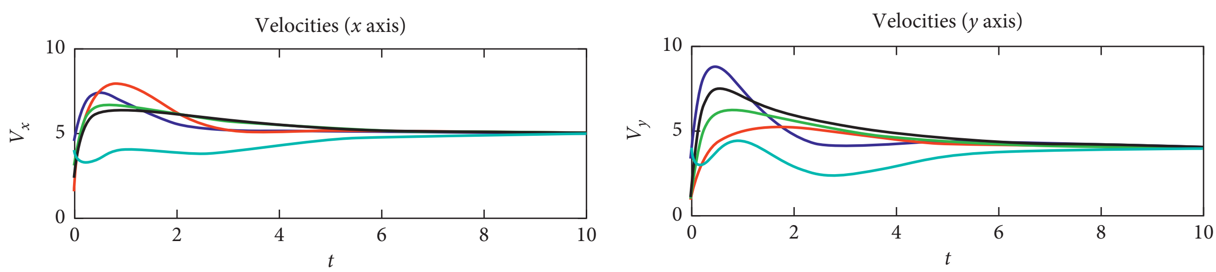


Figure 6: Position errors.



(a) (b)

Figure 7: Velocities.

Figure 3 describes the initial states, trajectories, and final states of the followers, where the random initial states of all followers (the black stars) are to be finally steered to a regular triangle array (the black circles).

4.2. Example 2. A seven-agent second-order MAS is considered, where agents 1–5 and agents 6–7 are regarded as followers and leaders, respectively. The network topology is described in Figure 4. Let $c_1 = c_2 = 1$ for the sake of simplicity. The system matrices are as follows:

$$\bar{L}_f = \begin{bmatrix} 0 & I_5 \\ L_f & I_5 \end{bmatrix} = \left[\begin{array}{ccccc|c} \multicolumn{5}{c|}{0} & I_5 \\ \hline 2 & -1 & -1 & 0 & 0 & \\ -1 & 3 & 0 & -1 & 0 & \\ -1 & 0 & 2 & -1 & 0 & I_5 \\ 0 & -1 & -1 & 3 & -1 & \\ 0 & 0 & 0 & -1 & 2 & \end{array}\right], \tag{29}$$

$$\bar{L}_{fl}^T = \begin{bmatrix} 0 & L_{fl}^T \end{bmatrix} = \left[\begin{array}{c|ccccc} 0 & 0 & -1 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & -1 \end{array}\right].$$

By computing,

$$\lambda\left(\bar{L}_f\right) = \{-1.7389, -1.3028, -1.1391, -0.8292, -0.2308, 1.2308, 2.7389, 1.8292, 2.3028, 2.1391\}. \tag{30}$$

For each λ, we have $\operatorname{rank}[(\lambda^2 + \lambda)I_5 - L_f] = 5$. According to Theorem 6, the second-order multiagent system (18) is observable.

Since the second-order multiagent system (18) is observable, the followers finally achieve the expected states. Figure 5 describes the initial states, trajectories, and final states of the followers, where the random initial states of all followers (the black stars) are to be finally steered to a straight line array (the black circles). Arrows represent the direction of each agent's movement. Figures 6 and 7 are the position errors and velocities of the followers, respectively.

5. Conclusion

This paper has studied the observability of first-order/second-order/high-order MASs with relative protocol on fixed topology, respectively, and brings the three kinds of networks into a unified framework. It has proved that the observability of MASs is depended only on structural pair (L_f, L_{fl}^T). However, for second-order or high-order MASs, it is very hard to deal with the observability because of complexity of the topology structure and lack of theoretical research tools. Our main result shows an advantage of a unified scheme for the observability of MASs and increasing the theoretical studying on the observability concept for MASs. Future research studies are directed on observability of MASs with switching topology, time delays, and observability of general linear discrete-time and continuous-time MASs, switched MASs and heterogeneous MASs.

Data Availability

Our paper does not use the data information.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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