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Controllability Ensured Leader Group Selection on Signed Multiagent Networks

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***Abstract*—Leader–follower controllability on signed multiagent networks is investigated in this paper. Specifically, we consider a dynamic signed multiagent network, where the agents interact via neighbor-based Laplacian feedback and the network allows positive and negative edges to capture cooperative and competitive interactions among agents. The agents are classified as either leaders or followers, thus forming a leader–follower signed network. To enable full control of the leader–follower signed network, controllability ensured leader group selection approaches are investigated in this paper, that is, identifying a small subset of nodes in the signed network, such that the selected nodes are able to drive the network to a desired behavior, even in the presence of antagonistic interactions. In particular, graphical characterizations of the controllability of signed networks are first developed based on the investigation of the interaction between network topology and agent dynamics. Since signed path and cycle graphs are basic building blocks for a variety of networks, the developed topological characterizations are then exploited to develop leader selection methods for signed path and cycle graphs to ensure leader–follower controllability. Along with illustrative examples, heuristic algorithms are also developed showing how leader selection methods developed for path and cycle graphs can be potentially extended to more general signed networks. In contrast to existing results that mainly focus on unsigned networks, this paper characterizes controllability and develops leader selection methods for signed networks. In addition, the developed results are generic, in the sense that they are not only applicable to signed networks but also to unsigned networks, since unsigned networks are a particular case of signed networks that only contain positive edges.**

***Index Terms*—Leader group selection, multiagent system, network controllability, signed graph.**

I. Introduction

LEADER–FOLLOWER multiagent systems that coordinate and cooperate over an information exchange network have been increasingly applied in science and engineering. Typical applications of leader–follower systems include distributed coordination in robotic networks [1]–[4], formation and propagation of opinions in social networks [5]–[7], and analysis of biochemical reaction in biological networks in [8] and [9]. In such applications, agents are classified as either leaders or followers, where leaders are a small subset of the agents tasked to direct the overall network behavior, while the remaining agents, i.e., followers, are influenced by the leaders via the underlying connectivity of the network to perform desired tasks. The success of these applications relies on the capability of driving the network to a desired state by external controls via selected leaders, i.e., network controllability. Consequently, network controllability must be ensured in network design to enable leader–follower control. However, network controllability is deeply coupled with agent dynamics and the underlying network topology whose interactions are still largely unexplored.

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A. Related Work

Based on the interactions among agents, multiagent networks can be classified as either cooperative or competitive networks. Cooperative networks are commonly modeled as unsigned graphs containing only positive edge weights, where positive weights indicate cooperative relationships between agents. Average consensus is a typical example of cooperative networks, where agents positively value information collected from neighboring agents and achieve group consensus via collaboration [10]. If a graph allows to admit negative edge weights, it is called signed graph. Signed graphs are widely used to represent networks with antagonistic interactions [11]. For instance, a positive/negative weight in signed graphs can be used to model friend/adversary relationship in social networks [12] and collaborative/competitive relationship in multiagent systems [13].

Controllability on cooperative networks has been extensively studied in the literature. Leader–follower controllability was considered for the first time in [14], where the network controllability was characterized based on the spectral analysis of the system matrix. Graph theoretic approaches were then explored to provide characterizations of network controllability. For instance, it was established in [15] that symmetry with respect to a single leader can potentially lead to uncontrollability. Graphical and topological characterizations of network controllability were investigated in [16] and [17]. Graph-distance-based lower bounds on the rank of the controllability matrix were developed in [18]. Sufficient and necessary

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conditions for network controllability were developed for tree topologies [19], grid graphs [20], and path and cycle graphs [21]. Other than graphical characterizations of network controllability, structural properties of cooperative networks were also exploited from matrix-theoretical perspectives in the works of [22]–[26] to reveal the connections between network controllability and underlying graphs. Other representative works that investigate network controllability include the results in [27]–[29]. Besides characterizing network controllability, various methods, e.g., combinatorial [30] and heuristic [31] selection methods, were developed to select leaders to ensure controllability of given networks. In [32], leader selection in complex networks was investigated, where, in addition to ensure network controllability, control energy was also taken into account when selecting leaders. In [33], leaders were selected with either minimum number or minimum energy cost to ensure the controllability of dynamic networks. Other representative works regarding leader selection for network controllability include [34]–[36]. However, all of the aforementioned results focus on the characterization of network controllability and leader selection on cooperative networks (i.e., unsigned graphs), without considering networks with potential antagonistic interactions. In addition, due to the existence of negative weights in signed networks, most existing analysis tools (e.g., graph symmetry) and leader selection approaches are no longer applicable to signed networks.

Recent emergence of the control and analysis of signed graphs with applications in social networks [37], brain networks [38], and complex networks [39] motivates the research on controllability problems, where such networks often need to be driven to desirable states by external inputs via selected control nodes within the network. For instance, the controllability of signed graphs were partially studied via structural balance in the works of [40]–[43]. As a variant of controllability, herdability over signed and directed graphs was considered in [44], which investigated the reachability of a specific set, rather than the whole state space as in controllability. A submodular optimization-based leader selection approach was developed in [45] to ensure leader–follower consensus in signed networks. However, network design in terms of leader group selection to ensure the controllability of the signed network remains largely unattended in the literature. Therefore, this paper is particularly motivated to study the leader group selection to ensure controllability of signed networks.

B. Contributions

In this paper, leader–follower controllability on signed networks is investigated. Specifically, we consider signed noncooperative networks, which admit both positive and negative edges. The signed network is capable of representing a variety of practical network applications, such as social network, fault tolerant networks, and secure networks, where the network may have both friendly and adversarial interactions. Motivated by the broad range of potential applications, it is of particular interest in this paper to identify a small subset of nodes in the signed network, such that the selected nodes are able to drive the network to a desired behavior, even in the presence of antagonistic interactions. In other words, this paper focuses on leader selection to ensure the controllability of signed networks. In particular, based on the classic controllability notations in [15], graph-inspired topological characterizations of the leader–follower controllability of signed networks are first developed. Such characterizations investigate the interaction between the underlying network topology and agent dynamics and pave the way for leader selection on signed networks. As the signed path and cycle graphs are basic building blocks for a variety of networks, the revealed topological characterizations are then exploited to develop leader selection methods for signed path and cycle graphs, where topological properties (e.g., structural balance) are explored to extend existing controllability analysis from unsigned to signed networks. Along with illustrative examples, heuristic algorithms are developed showing how leader selection methods developed for path and cycle graphs can be potentially extended for more general signed networks.

The contributions of this paper are multifold. First, controllability ensured leader group selection on signed networks is considered. Despite extensive study of controllability of unsigned networks, relatively few research effort focuses on signed networks. To the best of our knowledge, this paper is one of the first attempts to consider leader group selection on signed networks from graphical perspective. Specifically, we develop leader selection rules for signed path and cycle networks, which provides constructive approaches to select leaders for network controllability. Since most networks can be considered as a combination of path and cycle networks, the developed leader selection rules on path and cycle graphs can be potentially extended to more complex and sophisticated networks.

Second, the developed leader group selection approaches are generic, in the sense that they not only hold for signed graphs but also for unsigned graphs, since unsigned graphs are a particular case of signed graphs that only consider positive edges. For instance, the leader selection approaches developed in this paper are consistent with the results developed for unsigned graphs in [20] and [46], while the leader selection approaches developed for unsigned graphs are not generally applicable to signed graphs.

Third, in contrast to most existing matrix-theoretical approaches to characterize network controllability, graph-inspired understandings of network controllability are realized in this paper. Specifically, we investigate the relationship between the network controllability and the underlying topology and characterize how leader-to-leader and leader-to-follower connections affect the controllability of a signed network with Laplacian dynamics. Such graphical characterizations of network controllability are able to provide more intuitive and effective means in selecting leaders for network controllability. As a result, the leader group selection methods developed for signed path and cycle graphs in this paper can be conveniently employed without requiring any matrix-theoretical analysis.

II. Problem Formulation

Consider a multiagent network represented by an undirected signed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{W})$, where the node set $\mathcal{V} = \{v_1, \ldots, v_n\}$ and the edge set $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ represents the agents and the interactions between pairs of agents, respectively. An undirected edge $(v_i, v_j) \in \mathcal{E}$ indicates that v_i and v_j are able to interact with each other (e.g., mutual information exchange). The potential interactions among agents are captured by the adjacency matrix $\mathcal{W} \in \mathbb{R}^{n\times n}$, where $w_{ij} \neq 0$ if $(v_i, v_j) \in \mathcal{E}$ and $w_{ij} = 0$ otherwise. No self-loop is considered, i.e., $w_{ii} = 0\ \forall i = 1, \ldots, n$. Different from classical unsigned graphs that contain non-negative adjacency matrix, $w_{ij} : \mathcal{E} \rightarrow \{\pm 1\}$ in this paper admits positive or negative weight to capture collaborative or competitive relationships between agents, thus resulting in a signed graph $\mathcal{G}$. Specifically, v_i and v_j are called positive neighbors of each other if $w_{ij} = 1$ and negative neighbors if $w_{ij} = -1$, where positive neighborhood indicates cooperative interactions while negative neighborhood indicates noncooperative interactions between v_i and v_j, respectively.

A path of length $k - 1$ in $\mathcal{G}$ is a concatenation of distinct edges $\{(v_1, v_2), (v_2, v_3), \ldots, (v_{k-1}, v_k)\} \subset \mathcal{E}$. A cycle is a path with identical starting and end node, i.e., $v_1 = v_k$. Graph $\mathcal{G}$ is connected if there exists a path between any pair of nodes in $\mathcal{V}$. The neighbor set of v_i is defined as $\mathcal{N}_i = \{v_j|(v_i, v_j) \in \mathcal{E}\}$, and the degree of v_i, denoted as $d_i \in \mathbb{Z}^+$, is defined as the number of its neighbors, i.e., $d_i = |\mathcal{N}_i| = \sum_{j\in\mathcal{N}_i} \text{abs}(w_{ij})$, where $|\mathcal{N}_i|$ denotes the cardinality of $\mathcal{N}_i$ and $\text{abs}(w_{ij})$ denotes the absolute value of w_{ij}. The signed graph Laplacian of $\mathcal{G}$ is defined as $\mathcal{L}(\mathcal{G}) \triangleq \mathcal{D} - \mathcal{W}$, where $\mathcal{D} \triangleq \text{diag}\{d_1, \ldots, d_n\}$ is a diagonal matrix. Due to the consideration of negative weights, unlike unsigned graphs, $-\mathcal{L}(\mathcal{G})$ in (1) is no longer a Metzler matrix[1] and its row/column sums are not necessary zero.

Let $x(t) = [x_1(t), \ldots, x_n(t)]^T \in \mathbb{R}^n$ denote the stacked system states,[2] where each entry $x_i(t) \in \mathbb{R}$ represents the state of agent v_i. Suppose the system states evolve according to the following Laplacian dynamics:

$$\dot{x}(t) = -\mathcal{L}(\mathcal{G})x(t) \tag{1}$$

where the graph Laplacian $\mathcal{L}(\mathcal{G})$ indicates that each agent v_i updates its state x_i only taking into account the states of its neighboring agents, i.e., $x_j \in \mathcal{N}_i$. Various networked systems feature the Laplacian dynamics described in (1). For instance, formation control [48], flocking [49], and consensus [10], [50], [51] are typical applications of the Laplacian dynamics.

Suppose the agent set $\mathcal{V}$ is classified into a leader set $\mathcal{V}_l \subset \mathcal{V}$ and a follower set $\mathcal{V}_f \subset \mathcal{V}$ with $\mathcal{V}_l \cup \mathcal{V}_f = \mathcal{V}$, thus forming a typical leader–follower network. Without loss of generality, assume that the first m agents form the follower set $\mathcal{V}_f = \{v_1, \ldots, v_m\}$, while the remaining agents form the leader set $\mathcal{V}_l = \{v_{m+1}, \ldots, v_n\}$. Let $x(t) = [x_f^T(t), x_l^T(t)]^T \in \mathbb{R}^n$ be the aggregated system states, where $x_f(t) \in \mathbb{R}^m$ and $x_l(t) \in \mathbb{R}^{n-m}$ represent the aggregated states of followers and leaders, respectively. Similar to [15], the graph Laplacian in (1) can be partitioned as

$$\mathcal{L}(\mathcal{G}) = \left[\begin{array}{c|c} \mathcal{L}_f(\mathcal{G}) & \mathcal{L}_{fl}(\mathcal{G}) \\ \hline \mathcal{L}_{lf}(\mathcal{G}) & \mathcal{L}_l(\mathcal{G}) \end{array}\right] \tag{2}$$

with $\mathcal{L}_f(\mathcal{G}) \in \mathbb{R}^{m\times m}$, $\mathcal{L}_{fl}(\mathcal{G}) = \mathcal{L}_{lf}^T(\mathcal{G}) \in \mathbb{R}^{m\times(n-m)}$, and $\mathcal{L}_l(\mathcal{G}) \in \mathbb{R}^{(n-m)\times(n-m)}$. Based on (1) and (2), the dynamics of the followers become

$$\dot{x}_f(t) = -\mathcal{L}_f(\mathcal{G})x_f - \mathcal{L}_{fl}(\mathcal{G})u(t) \tag{3}$$

where $u(t) \triangleq x_l(t)$ denotes the exogenous control signal dictated by the leaders. In leader–follower networks, leaders are tasked to direct the overall behavior of the network by influencing the followers. The dynamics in (3) signify that the followers are influenced or controlled by the leaders via the connectivity of the network, where the exogenous signal becomes the leader's control input.

Definition 1 (Leader–Follower Controllability): Provided that the leaders are completely controllable and dictated by exogenous input $u(t)$, a leader–follower network with dynamics of (1) is called controllable, if the followers' state $x_f(t)$ in (3) can be driven to any target state by a proper design of $u(t)$. Mathematically, if the controllability matrix

$$\mathcal{C} = \begin{bmatrix} -\mathcal{L}_{fl} & \mathcal{L}_f\mathcal{L}_{fl} & \cdots & (-1)^m\mathcal{L}_f^{m-1}\mathcal{L}_{fl} \end{bmatrix}$$

has full row rank, the leader–follower system in (3) is controllable.

From Definition 1, the leader–follower controllability is dependent on the system matrices $\mathcal{L}_f$ and $\mathcal{L}_{fl}$ in (3). Since $\mathcal{L}(\mathcal{G})$ is determined by the topological structure of $\mathcal{G}$ and the roles of nodes, i.e., leaders or followers, $\mathcal{L}(\mathcal{G})$ can vary significantly with different leader set, resulting either a controllable or uncontrollable network. Therefore, the primary objective of this paper is to characterize the relationship between leader–follower controllability and network topology and identify a subgroup of nodes (i.e., the leader set) in $\mathcal{G}$ such that leader–follower controllability in Definition 1 is ensured.

Remark 1: Different from the dynamics in (3) where the leaders have direct influence on the followers' states, an alternative leader–follower network model employed in the works [40], [41], [46] is

$$\dot{x}(t) = -\mathcal{L}x(t) + Bu(t) \tag{4}$$

where $B = [b_{ij}] \in \mathbb{R}^{n\times(n-m)}$ is a binary matrix with $b_{ij} \neq 0$ if v_i is connected to a leader v_j and $b_{ij} = 0$ otherwise. The model in (4) indicates leaders indirectly influence the followers through both the followers' states and their own dynamics. Despite different representations in (3) and (4), the discussion in [46] indicates the two different models can be equivalently reformulated into each other, thus yielding the same controllability. In other words, if the selected leaders yield a controllable leader–follower network with dynamics (3), the controllability result holds for the same set of leaders on a network with dynamics (4).

[1] Metzler matrices are matrices with non-negative off-diagonal entries [47].

[2] If multidimensional system states (e.g., $x_i \in \mathbb{R}^m$) are considered, the Laplacian dynamics in (1) can be trivially extended to $\dot{x}(t) = -\mathbb{L}(\mathcal{G})x(t)$, where $\mathcal{L}(\mathcal{G})$ is augmented to $\mathbb{L}(\mathcal{G}) \triangleq \mathcal{L}(\mathcal{G}) \otimes I_m$ by the m-dimensional identity I_m and the matrix Kronecker product $\otimes$. Without loss of generality, the subsequent development will focus on the case that $x_i \in \mathbb{R}$ for ease of presentation.

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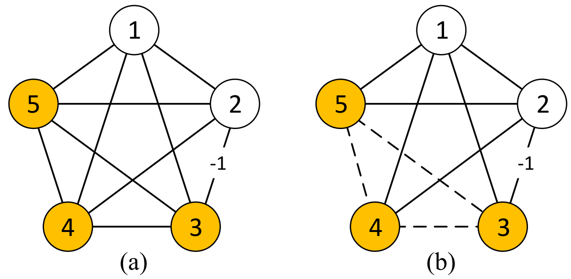

Fig. 1. (a) Controllable network with the leader set $\{3, 4, 5\}$. (b) Network controllability is invariant with respect to arbitrary removal of leader-to-leader connections (i.e., dashed lines).

III. Topological Characterization of Leader–Follower Controllability

Lemma 1 [52]: Consider a linear time-invariant system

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{5}$$

where $A \in \mathbb{R}^{n\times n}$ is the system matrix, $B \in \mathbb{R}^{n\times m}$ is the input matrix, and $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ represent the system states and the control input, respectively. Let $E(\cdot)$ denote the set of left eigenvectors of a matrix. Per the well-known Popov–Belevitch–Hautus test, the system in (5) is uncontrollable if and only if there exists a left eigenvector $\nu \in E(A)$ (i.e., $\nu^T A = \lambda\nu^T$ for some eigenvalue λ) such that $\nu^T B = 0_m$, where 0_m is an m-dimensional vector of all zeros. In other words, the system (5) is controllable if and only if $\nu \notin \ker(B^T)$, $\forall \nu \in E(A)$, where $\ker(\cdot)$ indicates the kernel space.

Proposition 1: Consider a signed leader–follower network $\mathcal{G}$, where the followers evolve according to (3) and the leaders are driven by an exogenous input, i.e., $x_l(t) = u(t)$. Provided that the follower-to-follower and leader-to-follower connections are intact, the leader–follower controllability is invariant to any addition, removal, or change of weight in leader-to-leader connections.

Note that the follower dynamics (3) can be represented using (5), which indicates that the leader–follower controllability is only dependent on the system matrices $\mathcal{L}_f$ and $\mathcal{L}_{fl}$. Since any change to leader-to-leader connections within $\mathcal{G}$ can only affect the structure of $\mathcal{L}_l(\mathcal{G})$ while $\mathcal{L}_{fl}(\mathcal{G})$ and $\mathcal{L}_f(\mathcal{G})$ remain the same, Proposition 1 is an immediate consequence of Definition 1 and Lemma 1.

Example 1: Proposition 1 implies that leader-to-leader connections can be freely altered without affecting the controllability of the original network. To illustrate this idea, consider a controllable signed graph with the leader set $\{3, 4, 5\}$ in Fig. 1(a), where negative edges are labeled with -1 and positive edges are not labeled for the simplicity of presentation. According to Proposition 1, it can be verified that the network in Fig. 1(b) remains controllable if any leader-to-leader connections (i.e., dashed lines) are removed.

As an immediate consequence of Lemma 1, Proposition 1 provides a topological characterization of network controllability, which is instructive in constructing a controllable graph from a set of controllable subgraphs and paves a way to controllability ensured leader selection in the subsequent section. To show how a controllable graph can be constructed, the case of two subgraphs is first considered in Proposition 2, which is then extended to the case of multiple subgraphs in Proposition 3.

First, consider two leader–follower signed graphs $\mathcal{G}_1 = (\mathcal{V}_1, \mathcal{E}_1, \mathcal{W}_1)$ and $\mathcal{G}_2 = (\mathcal{V}_2, \mathcal{E}_2, \mathcal{W}_2)$, where $\mathcal{G}_i$, $i = \{1, 2\}$, has a follower set $\mathcal{V}_{fi} = \{1, \ldots, m_i\}$ and a leader set $\mathcal{V}_{li} = \{m_i + 1, \ldots, n_i\}$ with $\mathcal{V}_{fi} \cup \mathcal{V}_{li} = \mathcal{V}_i$ and $\mathcal{V}_{fi} \cap \mathcal{V}_{li} = \emptyset$, where m_i and n_i denote the cardinality of the follower set and its node set, respectively. The edge set $\mathcal{E}_i$ and weight matrix $\mathcal{W}_i$ indicate the underlying connections among leaders and followers within $\mathcal{G}_i$, $i = \{1, 2\}$.

Proposition 2: Provided that the two signed graphs $\mathcal{G}_1 = (\mathcal{V}_1, \mathcal{E}_1, \mathcal{W}_1)$ and $\mathcal{G}_2 = (\mathcal{V}_2, \mathcal{E}_2, \mathcal{W}_2)$ are controllable and evolve according to (1), $\mathcal{G}_0 = (\mathcal{V}_0, \mathcal{E}_0, \mathcal{W}_0)$ remains controllable if $\mathcal{G}_0$ is constructed such that: 1) $\mathcal{V}_0 = \mathcal{V}_1 \cup \mathcal{V}_2$ with the follower set $\mathcal{V}_{f0} = \{1, \ldots, m_1 + m_2\}$ and the leader set $\mathcal{V}_{l0} = \{m_1 + m_2 + 1, \ldots, n_1 + n_2\}$, where the nodes are reindexed, without loss of generality, as the first m_1+m_2 nodes are followers and the rest nodes are leaders; 2) $\mathcal{E}_0 = \mathcal{E}_1 \cup \mathcal{E}_2 \cup \mathcal{E}'$ where $\mathcal{E}' \subset \mathcal{V}_{l1} \times \mathcal{V}_{l2}$; and 3) $\mathcal{W}_0 = \begin{bmatrix} \mathcal{W}_1 & \bar{\mathcal{W}} \\ \bar{\mathcal{W}}^T & \mathcal{W}_2 \end{bmatrix} \in \mathbb{R}^{(n_1+n_2)\times(n_1+n_2)}$ where $\bar{\mathcal{W}}$ indicates the weights associated with the edges in $\mathcal{E}'$.

Proof: Let $x_{fi} \in \mathbb{R}^{m_i}$ and $u_i \in \mathbb{R}^{n_i - m_i}$, $i = \{1, 2\}$, be the stacked followers' states and the exogenous input by the leaders in $\mathcal{G}_i$, respectively, which evolve according to the following dynamics:

$$\begin{aligned}
\mathcal{G}_1 &: \dot{x}_{f1}(t) = -\mathcal{L}_{f1}(\mathcal{G}_1)x_{f1} - \mathcal{L}_{fl1}(\mathcal{G}_1)u_1(t) \\
\mathcal{G}_2 &: \dot{x}_{f2}(t) = -\mathcal{L}_{f2}(\mathcal{G}_2)x_{f2} - \mathcal{L}_{fl2}(\mathcal{G}_2)u_2(t)
\end{aligned}$$

where $\mathcal{L}_{f1} \in \mathbb{R}^{m_1\times m_1}$, $\mathcal{L}_{fl1} \in \mathbb{R}^{m_1\times(n_1-m_1)}$, $\mathcal{L}_{f2} \in \mathbb{R}^{m_2\times m_2}$, and $\mathcal{L}_{fl2} \in \mathbb{R}^{m_2\times(n_2-m_2)}$are obtained following similar partitions in (2) from $\mathcal{L}_1(\mathcal{G}_1)$ and $\mathcal{L}_2(\mathcal{G}_2)$. Let $E(\mathcal{L}_{f1})$ and $E(\mathcal{L}_{f2})$ be the set of left eigenvectors of $\mathcal{L}_{f1}$ and $\mathcal{L}_{f2}$, respectively. Since $\mathcal{G}_1$ and $\mathcal{G}_2$ are controllable, based on Lemma 1, no eigenvector $\nu \in E(\mathcal{L}_{f1})$ and $\vartheta \in E(\mathcal{L}_{f2})$ are orthogonal to $\mathcal{L}_{fl1}$ and $\mathcal{L}_{fl2}$, respectively.

Let $x_{f0} = [x_{f1}^T, x_{f2}^T]^T$ and $u_0 = [u_1^T, u_2^T]^T$ denote the followers' states and the exogenous input by the leaders in $\mathcal{G}_0$. Based on the definition, $\mathcal{G}_0$ is constructed by connecting $\mathcal{G}_1$ and $\mathcal{G}_2$ in a way that its node set $\mathcal{V}_0$ is formed by the union of $\mathcal{V}_1$ and $\mathcal{V}_2$, and its edge set $\mathcal{E}_0$ keeps $\mathcal{E}_1$ and $\mathcal{E}_2$ while including a new subset $\mathcal{E}'$. Since $\mathcal{E}' \subset \mathcal{V}_{l1} \times \mathcal{V}_{l2}$, the additional edges $\mathcal{E}'$ are restricted to leader-to-leader connections within $\mathcal{V}_{l1}$ and $\mathcal{V}_{l2}$, which only affects the structure of $\mathcal{L}_l(\mathcal{G}_0)$ in $\mathcal{L}(\mathcal{G}_0)$. Therefore, the followers' state x_{f0} evolves according to

$$\dot{x}_{f0}(t) = -\mathcal{L}_{f0}(\mathcal{G}_0)x_{f0} - \mathcal{L}_{fl0}(\mathcal{G}_0)u_0(t)$$

where

$$\mathcal{L}_{f0} = \begin{bmatrix} \mathcal{L}_{f1} & 0_{m_1\times m_2} \\ 0_{m_2\times m_1} & \mathcal{L}_{f2} \end{bmatrix}$$

and

$$\mathcal{L}_{fl0} = \begin{bmatrix} \mathcal{L}_{fl1} & 0_{m_1\times(n_2-m_2)} \\ 0_{m_2\times(n_1-m_1)} & \mathcal{L}_{fl2} \end{bmatrix}.$$

Clearly, for any eigenvector $\nu \in E(\mathcal{L}_{f1})$ and $\vartheta \in E(\mathcal{L}_{f2})$, the vectors $\begin{bmatrix} \nu^T & 0_{m_2}^T \end{bmatrix}^T$ and $\begin{bmatrix} 0_{m_1}^T & \vartheta^T \end{bmatrix}^T$ are the eigenvectors

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of $\mathcal{L}_{f0}$. Since,

$$\begin{bmatrix}\nu^T & 0_{m_2}^T\end{bmatrix}\begin{bmatrix}\mathcal{L}_{f1} & 0_{m_1\times m_2}\\ 0_{m_2\times m_1} & \mathcal{L}_{f2}\end{bmatrix} = \begin{bmatrix}\nu^T\mathcal{L}_{f1} & 0_{m_2}^T\end{bmatrix} \neq 0_{m_1+m_2}$$

and

$$\begin{bmatrix}0_{m_1}^T & \vartheta^T\end{bmatrix}\begin{bmatrix}\mathcal{L}_{f1} & 0_{m_1\times m_2}\\ 0_{m_2\times m_1} & \mathcal{L}_{f2}\end{bmatrix} = \begin{bmatrix}0_{m_1}^T & \vartheta^T\mathcal{L}_{f2}\end{bmatrix} \neq 0_{m_1+m_2}$$

according to Lemma 1, $\mathcal{G}_0$ remains controllable. ∎

To extend Proposition 2 to the case of multiple subgraphs, consider a set of signed graphs $\mathcal{G}_i = (\mathcal{V}_i, \mathcal{E}_i, \mathcal{W}_i)$, $i = \{1, \ldots, n\}$, where each has a follower set $\mathcal{V}_{fi} = \{1, \ldots, m_i\}$ and a leader set $\mathcal{V}_{li} = \{m_i + 1, \ldots, n_i\}$ with m_i and n_i indicating cardinality of the follower set and its node set, respectively.

Proposition 3: Provided a set of controllable signed graphs $\mathcal{G}_i$, $i = \{1, \ldots, n\}$, evolving according to (1), $\mathcal{G}_0 = (\mathcal{V}_0, \mathcal{E}_0, \mathcal{W}_0)$ remains controllable if $\mathcal{G}_0$ is constructed such that: 1) $\mathcal{V}_0 = \bigcup_{i=1}^n \mathcal{V}_i$; 2) $\mathcal{E}_0 = \bigcup_{i=1}^n \mathcal{E}_i \cup \mathcal{E}'$ where $\mathcal{E}' \subset \prod_{i=1}^n \mathcal{V}_{li}$, indicating the additional edges $\mathcal{E}'$ are restricted to leader-to-leader connections within the leader sets $\mathcal{V}_{li}$; and 3) $\mathcal{W}_0$ indicates the weights associated with the edges in $\mathcal{E}_0$.

The proof of Proposition 3 is omitted here, since it can be easily verified following similar procedure as in the proof of Proposition 2.

Remark 2: Proposition 3 provides a constructive topological design approach in generating a combined graph that preserves network controllability from a set of controllable subgraphs. Besides constructing a controllable graph, Proposition 3 also provides insights on leader selection to render network controllability. For instance, if a given graph can be partitioned into a set of connected subgraphs, as long as the selected leaders ensure controllability for each subgraph and are connected following the rules in Proposition 3, the given graph is guaranteed to be controllable by Proposition 3. This idea will be further explored in the subsequent sections. In addition, the results developed in Proposition 1–3 are generic in the sense that they hold for not only signed graphs but also for unsigned graphs.

Example 2: Fig. 2 shows how a controllable graph can be constructed from a set of controllable subgraphs. Fig. 2(a) contains two subgraphs, which become controllable if the nodes $\{1, 5\}$ and $\{2, 3\}$ are selected as leaders, respectively, as shown in Fig. 2(b). The combined graph in Fig. 2(c) is constructed by connecting the two leaders $\{1, 2\}$. It can be verified that the combined graphs in Fig. 2(c) remains controllable, since its construction follows the rules in Proposition 2. It is worth pointing out that only sufficient conditions to preserve network controllability are developed in Proposition 2. There might exist different ways in connecting leaders to preserve network controllability. As a different construction, the combined graph in Fig. 2(d) is constructed by including three new edges [i.e., (1, 2), (1, 3), and (5, 2)], which is also controllable according to Proposition 2.

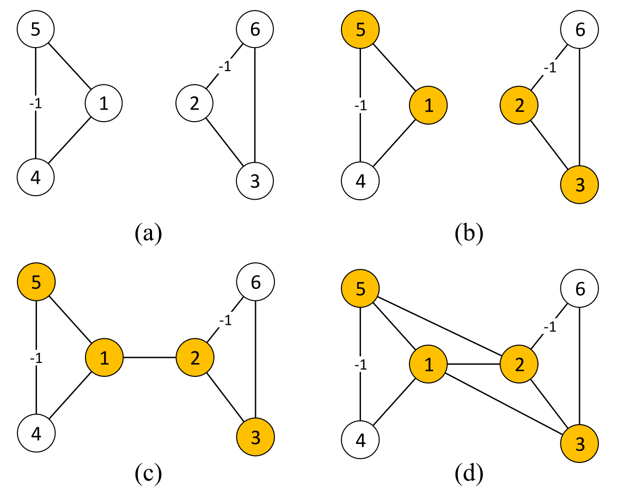

Fig. 2. Examples of constructing a controllable graph from controllable subgraphs. (a) Two subgraphs. (b) Each subgraph is controllable provided that the nodes {1, 5} and {2, 3} are selected as leaders, respectively. (c) Combined graph is controllable if a new leader-to-leader connection (i.e., the edge (1, 2)) is created. (d) Combined graph remains controllable if new leader-to-leader connections [i.e., the edges (1, 2), (1, 3), and (5, 2)] are created.

IV. Leader Selection for Signed Path and Cycle Graphs

Based on the developed topological characterizations of the controllability of signed networks in Section III, this section focuses on developing sufficient conditions on selecting leader nodes for the controllability of signed networks. Specifically, two particular graphs, signed path and cycle graph, are considered first. A *path graph* is a graph where all internal nodes have degree two except that two end nodes have degree one. A *cycle graph* is a graph where all nodes have degree two. Path and cycle graphs are basic building blocks for various sophisticated networks [47]. For instance, grid and lattice networks can be generated by Cartesian products of path graphs [53] while complex circulant networks can be constructed and analyzed based on cycle graphs [54]. Consequently, later in this section, we will show how the developed leader selection rules for path and cycle graphs can be potentially extended to more general signed networks.

As a key tool to study the controllability of signed networks, structural balance is introduced.

Definition 2 (Structural Balance): A signed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{W})$ is structurally balanced if the node set $\mathcal{V}$ can be partitioned into $\mathcal{V}_1$ and $\mathcal{V}_2$ with $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$ and $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$, where $w_{ij} > 0$ if $v_i, v_j \in \mathcal{V}_q$, $q \in \{1, 2\}$, and $w_{ij} < 0$ if $v_i \in \mathcal{V}_q$ and $v_j \in \mathcal{V}_r$, $q \neq r$, and $q, r \in \{1, 2\}$.

Definition 2 indicates that v_i and v_j are positive neighbors if they are from the same subset, i.e., either $\mathcal{V}_1$ or $\mathcal{V}_2$, and negative neighbors if v_i and v_j are from different subset. To characterize structural balance, necessary and sufficient conditions are provided.

Lemma 2 [11]: A connected signed graph $\mathcal{G}$ is structurally balanced if and only if any of the following equivalent conditions holds.

1) All cycles of $\mathcal{G}$ are positive, i.e., the product of edge weights on any cycle is positive.

2) There exists a diagonal matrix $\Xi = \text{diag}\{\sigma_1, \ldots, \sigma_n\}$ with $\sigma_i \in \{\pm 1\}$ such that $\Xi \mathcal{W} \Xi$ has non-negative entries.
3) 0 is an eigenvalue of graph Laplacian $\mathcal{L}(\mathcal{G})$.

Lemma 3: Consider a structurally balanced signed graph $\mathcal{G}$ with nodes partitioned into $\mathcal{V}_1$ and $\mathcal{V}_2$. If leaders are selected from the same subset (i.e., either $\mathcal{V}_1$ or $\mathcal{V}_2$) and followers evolve according to (3), the leader–follower controllability of $\mathcal{G}$ remains the same as its corresponding unsigned graph $\mathcal{G}'$, where $\mathcal{G}' = (\mathcal{V}, \mathcal{E}, \mathcal{W}')$ has the same node and edge set as $\mathcal{G}$ except that $\mathcal{W}' = \Xi \mathcal{W} \Xi = \text{abs}(\mathcal{W})$, where Ξ is defined in Lemma 2 and $\text{abs}(\mathcal{W})$ denotes the entry-wise absolute value of $\mathcal{W}$.

Proof: This lemma is a variant of [40, Th. 3] that accounts for the follower dynamics in (3). See the Appendix for the proof. ■

A. Signed Path Graph

Let $\mathcal{G}_p = (\mathcal{V}, \mathcal{E}, \mathcal{W})$ denote a signed path graph, where $\mathcal{V} = \{1, \ldots, n\}$ and $\mathcal{E} = \{(i, i+1) | i \in \{1, \ldots, n-1\}\}$ represent the node and edge set, respectively, and the weight matrix $\mathcal{W} \in \mathbb{R}^{n \times n}$ indicates associated positive or negative weights in $\mathcal{E}$.

Theorem 1: A signed path graph $\mathcal{G}_p = (\mathcal{V}, \mathcal{E}, \mathcal{W})$ with followers evolving according to (3) is controllable if one of the end nodes (i.e., v_1 or v_n) is selected as leader.

Proof: As indicated in [11, Corollary 1], a spanning tree is always structurally balanced. As a particular case of the spanning tree, the path graph $\mathcal{G}_p$ is thus structurally balanced. Since $\mathcal{G}_p$ is structurally balanced, its node set can be partitioned into $\mathcal{V}_1$ and $\mathcal{V}_2$ as in Definition 2. If one leader is considered, the leader must be selected from either $\mathcal{V}_1$ and $\mathcal{V}_2$, i.e., the same subset. Hence, Lemma 3 indicates that the controllability of $\mathcal{G}_p$ is equivalent to its corresponding unsigned graph $\mathcal{G}'_p = (\mathcal{V}, \mathcal{E}, \mathcal{W}')$, where $\mathcal{W}'$ consists of non-negative edge weights. Given that an unsigned path graph is controllable if an end node is selected as leader in [14] and [15], it can be concluded that the signed path graph $\mathcal{G}_p$ is also controllable if an end node is selected as leader from Lemma 3. ■

Remark 3: As indicated in [15], unsigned path graph is controllable if one of the end nodes is selected as leader. Although the leader selection approach developed in Theorem 1 is similar to that of [15], the inherent analysis is completely different. Specifically, the controllability analysis of unsigned path graph in [15] is based on graph symmetry. Although symmetry with respect to a single leader is sufficient to conclude uncontrollability of unsigned graphs, symmetry alone, as indicated in [41], is in general not sufficient to lead to uncontrollability of signed graphs. Therefore, instead of using graph symmetry, structural balance is exploited in Theorem 1 to characterize the controllability of signed graphs.

The following theorem extends the result in Theorem 1 to multileader selection.

Theorem 2: A signed path graph $\mathcal{G}_p$ with followers evolving according to (3) is controllable if multiple adjacent nodes in $\mathcal{G}_p$ are selected as leaders.

Proof: The case of two leaders is first considered in this proof, which will then be extended to include multiple leaders. Suppose that an arbitrary pair of adjacent nodes v_k and v_{k+1}, $\forall k \in \{1, \ldots, n-1\}$, in $\mathcal{G}_p$ are selected as leaders. Let $\mathcal{G}_{p1} = \{\{v_1, v_2\}, \{v_2, v_3\}, \ldots, \{v_{k-1}, v_k\}\}$ denote the subgraph of $\mathcal{G}_p$ with the leader node v_k and $\mathcal{G}_{p2} = \{\{v_{k+1}, v_{k+2}\}, \ldots, \{v_{n-1}, v_n\}\}$ denote the subgraph of $\mathcal{G}_p$ with the leader node v_{k+1}, respectively. Based on Theorem 1, $\mathcal{G}_{p1}$ and $\mathcal{G}_{p2}$ are both controllable, since v_k and v_{k+1} are the end nodes of the subgraph $\mathcal{G}_{p1}$ and $\mathcal{G}_{p2}$, respectively. Since $\mathcal{G}_p$ can be constructed by connecting the two leaders in $\mathcal{G}_{p1}$ and $\mathcal{G}_{p2}$ while keeping the rest graph intact, Proposition 1 indicates that $\mathcal{G}_p$ will remain controllable.

If n adjacent nodes are selected as leaders, following similar argument above, $\mathcal{G}_p$ can always be partitioned into controllable subpath graphs. Iteratively invoking Proposition 1 indicates $\mathcal{G}_p$ is always controllable if adjacent nodes are selected as leaders. ■

Remark 4: Theorem 2 relaxes the constraint in Lemma 3 that leaders have to be selected from the same partitioned subset. Specifically, Theorem 2 indicates that, for signed path graphs, leaders are allowed to be selected from different partitioned subsets, as long as they are adjacent nodes in $\mathcal{G}_p$. In addition, the developed leader selection approach in Theorem 2 is more convenient in topology design for leader–follower controllability, since no computation for the partitioned subset is required as in Lemma 3.

A signed tree is a particular topology where any two vertices are connected by exactly one simple path and the edges admit negative weights. Networks with tree topology have been broadly applied to model multiagent networks, cyber-physical systems, smart grid, and power networks (see [10], [55] for more applications). Since a tree graph can be naturally partitioned into connected path graphs, the subsequent theorem extends leader selection rules developed in Theorems 1 and 2 to signed trees.

Let $\mathcal{G}_t = (\mathcal{V}, \mathcal{E}, \mathcal{W})$ denote a signed tree, where $\mathcal{V}$, $\mathcal{E}$, and $\mathcal{W}$ represent the node set, edge set, and weight matrix, respectively. Since $\mathcal{G}_t$ is a tree with no cycles, it is well known that $|\mathcal{E}| = |\mathcal{V}| - 1$.

Theorem 3: Suppose that a signed tree $\mathcal{G}_t$ can be partitioned into a set of signed path graphs $\{\mathcal{G}_{pi}\}$, $i \in \{1, \ldots, m\}$, with $\mathcal{G}_t = \cup_{i=1}^{m} \mathcal{G}_{pi}$. The tree $\mathcal{G}_t$ is controllable, if selected leaders ensure the controllability of each path graph in $\{\mathcal{G}_{pi}\}$ and $\mathcal{G}_t$ is reconstructed by only connecting leaders from each path graph.

The proof is omitted here, since Theorem 3 is an immediate consequence of Theorems 1 and 2 and Proposition 1. Example 3 is provided to illustrate Theorem 3.

Example 3: Consider a signed tree shown in Fig. 3(a), where the roles (i.e., leaders or followers) are not assigned yet. To determine leaders that ensure leader–follower controllability, the tree can be first partitioned into five paths, i.e., $\{5, 10, 14\}$, $\{6, 11\}$, $\{3, 7, 12, 15\}$, $\{8, 13\}$, and $\{2, 1, 4, 9\}$. If the leaders are selected as $\{1, 2, 3, 4, 5, 6, 8, 9\}$, then each of the path graphs is controllable by Theorem 1. Since the path graphs are connected in a way that only leaders from each path are connected to form the tree, the tree is controllable based on Theorem 2 and Proposition 1. Therefore, it can be verified

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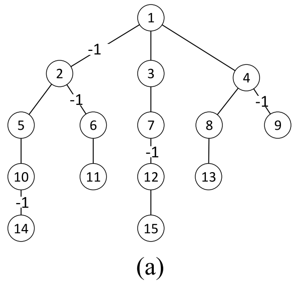

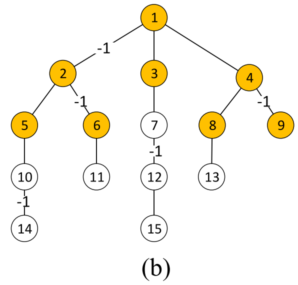

Fig. 3. (a) Signed tree graph. (b) Controllable signed tree graph with the marked nodes $\{1, 2, 3, 4, 5, 6, 8, 9\}$ selected as leaders.

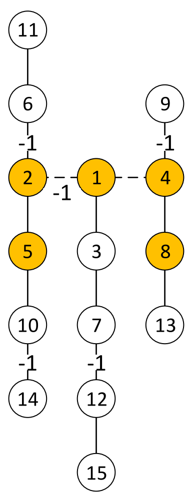

Fig. 4. Same signed tree graph in Fig. 3(a) is considered, where a different set of nodes are selected as leaders to ensure the network controllability.

that the selected leaders ensure leader–follower controllability of the tree graph.

Remark 5: To the best of our knowledge, few existing results consider leader selection from graphical perspective for ensured controllability on signed tree graphs. Theorem 1 provides a sufficient condition for selecting control nodes to ensure the controllability of a signed tree graph. In contrast to matrix-theoretical design and analysis approaches in the literature, the developed leader selection approach is graph-inspired, which is more amenable in topology design.

Since the result developed in Theorem 1 is based on the partition of a tree graph into path graphs, different partitions could result in different sets of leaders. For instance, Fig. 4 considers the same signed tree graph as in Fig. 3(a), where a different set of nodes, i.e., $\{1, 2, 4, 5, 8\}$, is selected as leaders. After reorganizing the nodes, Fig. 4 contains three path graphs, i.e., $\{11, 6, 2, 5, 10, 14\}$,$\{1, 3, 7, 12, 15\}$, and $\{9, 4, 8, 13\}$. Based on Theorem 2, the three path graphs are controllable with respect to the selected leaders. Since the three path graphs are connected such that only leaders are connected, the combined signed tree graph is controllable by Theorem 3. Future research will focus on developing optimal partitions of a tree graph to obtain the smallest set of leaders that ensure controllability.

B. Signed Cycle Graph

A cycle graph $\mathcal{G}_c = (\mathcal{V}, \mathcal{E}, \mathcal{W})$ is defined similar to $\mathcal{G}_p$, except that the edge set in $\mathcal{G}_c$ is defined as $\mathcal{E} = \{(i, i \bmod (n) + 1) | i \in \{1, \ldots, n\}\}$. If $\mathcal{G}_c$ is structurally balanced, based on Lemma 2, there exists a diagonal matrix Ξ such that $\Xi \mathcal{W} \Xi = \mathrm{abs}(W)$ is non-negative. The graph $\mathcal{G}_c' = (\mathcal{V}, \mathcal{E}, \mathrm{abs}(W))$ is then called the unsigned graph of $\mathcal{G}_c$, since $\mathcal{G}_c'$ no longer has negative edge weights.

Lemma 4 [46]: An unsigned cycle graph is controllable if two adjacent nodes are chosen as leaders.

This section focuses on extending the leader selection approach for unsigned cycle graphs developed in Lemma 4 to signed cycle graphs, where the weight matrix $\mathcal{W}$ can take negative entries.

Lemma 5: Consider a signed cycle graph $\mathcal{G}_c = (\mathcal{V}, \mathcal{E}, \mathcal{W})$, where the followers evolve according to (3). The leader–follower controllability of $\mathcal{G}_c$ is equivalent to that of its sign-reversed graph $\bar{\mathcal{G}}_c = (\mathcal{V}, \mathcal{E}, -\mathcal{W})$, where the weight matrix $\mathcal{W}$ in $\mathcal{G}_c$ is replaced by $-\mathcal{W}$ in $\bar{\mathcal{G}}_c$.

Proof: Since every node in a cycle graph has degree two, the degree matrix $\mathcal{D}$ of a cycle graph is $2I_n$, where I_n represents the n-dimensional identity matrix. Without loss of generality, suppose the first m nodes are followers and the rest $n-m$ nodes are leaders. Graph Laplacian of $\mathcal{G}_c$ can then be partitioned as

$$\mathcal{L} = \mathcal{D} - \mathcal{W} = \left[\begin{array}{c|c} 2I_m - \mathcal{W}_f & -\mathcal{W}_{fl} \\ \hline -\mathcal{W}_{fl}^T & 2I_{n-m} - \mathcal{W}_l \end{array}\right] \tag{6}$$

where $\mathcal{W}_f \in \mathbb{R}^{m\times m}$, $\mathcal{W}_{fl} \in \mathbb{R}^{m\times(n-m)}$, and $\mathcal{W}_l \in \mathbb{R}^{(n-m)\times(n-m)}$. By the dynamics (1) and (6), the followers evolve according to

$$\dot{x}_f(t) = -\big(2I_m - \mathcal{W}_f\big)x_f - \big(-\mathcal{W}_{fl}\big)u(t)$$

where $x_f \in \mathbb{R}^m$ and $u(t) \in \mathbb{R}^{n-m}$ denote the followers' states and leaders' input, respectively. Hence, based on Definition 1, the leader–follower controllability of $\mathcal{G}_c$ is completely determined by $2I_n - \mathcal{W}_f$ and $-\mathcal{W}_{fl}$.

Consider the sign reversed cycle graph $\bar{\mathcal{G}}_c$. Similarly, the leader–follower controllability of $\bar{\mathcal{G}}_c$ is determined by $2I_n+\mathcal{W}_f$ and $\mathcal{W}_{fl}$. Note that the set of eigenvectors of $2I_n-\mathcal{W}_f$ is identical to that of $2I_n+\mathcal{W}_f$. In addition, since $2I_n-\mathcal{W}_f$ and $2I_n+\mathcal{W}_f$ are symmetric matrices, their right and left eigenvectors are equal. Let μ be an eigenvector of $2I_n - \mathcal{W}_f$ (or $2I_n + \mathcal{W}_f$). If $\mu \in \ker(-\mathcal{W}_{fl}^T)$, it is always true that $\mu \in \ker(\mathcal{W}_{fl}^T)$. Similarly, if $\mu \notin \ker(-\mathcal{W}_{fl}^T)$, it will also be true that $\mu \notin \ker(\mathcal{W}_{fl}^T)$. Therefore, based on Lemma 1, the controllability of $\mathcal{G}_c$ is equivalent to that of $\bar{\mathcal{G}}_c$. ■

Theorem 4: Provided that all followers evolve according to (3), a signed cycle graph $\mathcal{G}_c$ is controllable if any two adjacent nodes in $\mathcal{G}_c$ are selected as leaders.

Proof: Different from path graphs that are inherently structurally balanced, cycle graphs may not be structurally balanced. In addition, when considering two adjacent nodes as leaders, the sign of interleader edge introduces additional challenge to the analysis of leader–follower controllability. Therefore, based on the topological structure of $\mathcal{G}_c$ and the sign of interleader edge, four cases are discussed.

Case 1: Structurally balanced $\mathcal{G}_c$ with positive interleader edge. Since $\mathcal{G}_c$ is structurally balanced, its node set $\mathcal{V}$ can be partitioned into two subsets $\mathcal{V}_1$ and $\mathcal{V}_2$. Based on Lemma 2, the positive interleader edge indicates that the two leaders are from the same subset, i.e., either $\mathcal{V}_1$ or $\mathcal{V}_2$. In addition, there exists an unsigned cycle graph $\mathcal{G}_c'$, which has the same edge and node set as $\mathcal{G}_c$ except that the edge weights in $\mathcal{G}_c'$ are all positive. Given that leaders are selected from the same subset, Lemma 3 indicates that the controllability of $\mathcal{G}_c$ is equivalent to that of $\mathcal{G}_c'$. Since $\mathcal{G}_c'$ is controllable from Lemma 4 if two adjacent nodes are selected as leaders, it concludes that $\mathcal{G}_c$ is also controllable with similar leader selection approach from Lemma 3.

Case 2: Structurally unbalanced $\mathcal{G}_c$ with negative interleader edge. Since $\mathcal{G}_c$ is structurally unbalanced cycle graph, Lemma 2 indicates that $\mathcal{G}_c$ is a negative cycle, which implies there exists an odd number of negative edges in $\mathcal{G}_c$. Since graph controllability is invariant to changes of interleader edges from Proposition 1, the controllability of $\mathcal{G}_c$ with negative interleader edge will remain the same if the sign of interleader edge flips from negative to positive. After flipping the sign to be positive, $\mathcal{G}_c$ will now contain an even number of negative edges and becomes structurally balanced from Lemma 2, which implies that $\mathcal{G}_c$ is controllable due to the equivalent to case 1.

Case 3: Structurally balanced $\mathcal{G}_c$ with negative interleader edge. Based on Lemma 5, the controllability of $\mathcal{G}_c$ is invariant if all of its signs are flipped. Based on the number of negative edges in $\mathcal{G}_c$, two subcases are further discussed.

1) If $\mathcal{G}_c$ in this case has an even number of edges, it must have an even number of negative edges and an even number of positive edges from Lemma 2. After flipping all of its signs, $\mathcal{G}_c$ remains structurally balanced but with positive interleader edge, which implies that $\mathcal{G}_c$ in this case is controllable due to the equivalence to case 1.
2) If $\mathcal{G}_c$ has an odd number of edges, it must have an even number of negative edges and an odd number of positive edges, due to Lemma 2. After flipping all of its signs, $\mathcal{G}_c$ becomes $\mathcal{G}_c^*$, which is structurally unbalanced with positive interleader edge and equivalent to case 4 below. If the sign of any leader–follower edge in $\mathcal{G}_c^*$ is flipped again, $\mathcal{G}_c^*$ becomes $\mathcal{G}_c^\star$, which is structurally balanced with positive interleader edge due to the change of negative edges from an odd number of to an even number. Following similar procedure as in the proof of Lemma 5, it can be trivially verified that the change of sign of any leader–follower edge will not affect the controllability of the system. In other words, the controllability of $\mathcal{G}_c^\star$ remains the same as $\mathcal{G}_c^*$, which is then the same as $\mathcal{G}_c$ due to Lemma 5. Since $\mathcal{G}_c^\star$ is structurally balanced with positive interleader edge, which is the same as case 1, $\mathcal{G}_c^\star$ is controllable, and hence $\mathcal{G}_c$ is also controllable.

Case 4: Structurally unbalanced $\mathcal{G}_c$ with positive interleader edge. Since this case has been discussed in case 3, $\mathcal{G}_c$ is controllable.

Algorithm 1 Leader Selection for Signed Graph

```
procedure INPUT:(Graph G(V, E, W));
    Output: The set of leaders V_l
  Calculate the node degree d_i for each node v_i ∈ V;
  Select nodes v_i with d_i > 2 to form V_l;
  Use graph partition techniques (e.g., [56]–[58]) to partition G into cycles and
  paths based on the selected high degree nodes in V_l;
  for Each cycle or path do
      if the cycle or path is controllable then
          Keep the selected leaders in V_l;
      else
          Apply Theorems 1-4 to select appropriate nodes as leaders and update V_l;
      end if
  end for
  Update V_l based on Propositions 1-3 to ensure the network controllability;
  Output V_l;
end procedure
```

Based on cases 1–4, $\mathcal{G}_c$ is controllable if adjacent nodes are selected as leaders. ■

Remark 6: To the best of our knowledge, the leader selection rules developed in Theorem 4 is one of the first attempts to consider signed cycle graphs. In addition, the leader selection approach developed in Theorem 4 is valid for all types of signed cycle graphs, no matter it is structurally balanced or not.

The following corollary is an immediate consequence of Theorem 4.

Corollary 1: A signed path graph is controllable if two end nodes are selected as leaders.

Given a controllable signed cycle graph where two adjacent nodes are selected as leaders, if the edge connecting the two leaders is removed, the cycle graph will turn into a path graph with leaders on two ends. Since the removal of leader-to-leader edges will not affect controllability of the system from Proposition 1, the path graph is controllable if two end nodes are selected as leaders. In addition to Theorems 1–3, Corollary 1 provides an alternative way to select leaders for ensured controllability.

C. Extensions and Discussion

This section shows how the leader selection rules developed for signed path and cycle graphs in previous sections can be potentially extended to more general signed networks. Since path and cycle graphs are basic building blocks for general signed graphs, the idea of leader selection in this section is to first partition the general signed graphs into a set of path and cycle graphs, where the results developed in Theorems 1–4 can be applied.

Motivated by this idea, heuristic leader selection rules are developed in Algorithm 1. In Algorithm 1, we start from identifying nodes in a given signed graph whose node degree is greater than two. Since nodes in either path and cycle graphs have degree at most two, the reason to identify nodes with degree more than two is to find out those nodes that potentially connect path or cycle graphs. Those high degree nodes will facilitate the partition of the signed graph into individual path and cycle graphs, where existing graph partition techniques are applicable (e.g., graph partition into paths in [56], cycles in [57], and both paths and cycles in [58]). Once the graph is partitioned into a

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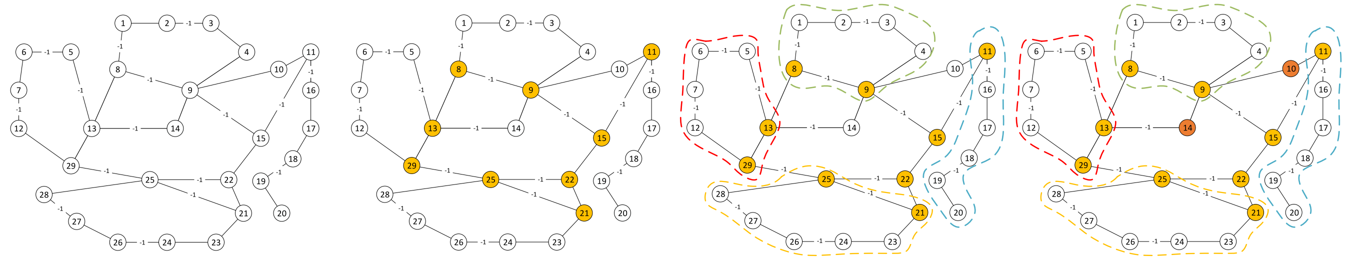


(a) (b) (c) (d)

Fig. 5. (a) Signed graph with 29 nodes. (b) Initial selection of leader nodes (i.e., nodes with degree more than two) are marked. (c) Based on the initially selected leader node, the signed graph is partitioned into a set of path and cycle graphs. (d) Update the leader nodes based on the rules in Theorems 1–4 to ensure the resulting leader–follower network is controllable.

set of path and cycle graphs, the leader selection rules developed in Theorems 1–4 and Propositions 1–3 can be immediately applied to generate a controllable leader–follower network.

To illustrate Algorithm 1, Example 4 is provided.

Example 4: Consider the signed graph in Fig. 5(a), where the objective is to select a set of leaders such that the leader–follower network is controllable. Following rules in Algorithm 1, the leader nodes (i.e., high degree nodes $\{8, 9, 11, 13, 15, 21, 22, 25, 29\}$) are first identified in Fig. 5(b). Based on the selected leader nodes and graph partition techniques in [56]–[58], the signed graph is partitioned into a set of path and cycle graphs, which are shown in dashed lines. The leader selection rules developed in Theorems 1–4 are then applied to ensure that each path and cycle graph are controllable. For instance, the selected nodes $\{8, 9\}$ ensure the controllability of the cycle graph formed by $\{1, 2, 3, 4, 8, 9\}$. The leader set is then updated to include more nodes (i.e., addition leaders $\{10, 14\}$) whenever necessary such that the individual path and cycle graphs are connected satisfying Propositions 1–3, which ensues the controllability of the original signed graph.

Remark 7: Note that Algorithm 1 is based on partitioning a general signed graph into a set of path and cycle graphs. As indicated in [56]–[58], particular classes of graph topologies, such as trees or loosely connected graphs, can be efficiently partitioned into a small number of path and cycle graphs. Other types of graphs, such as densely connected graphs or complete graphs, can be more challenging to be partitioned. Therefore, the developed leader selection rules perform better on graph topologies that are amenable to be partitioned into path and cycle graphs.

Remark 8: The leader selection rules developed in Algorithm 1 are only sufficient conditions to ensure network controllability. In other words, the selected leader set from Algorithm 1 is by no means an optimal set. There may exist other leader group selections that can also ensure network controllability but with fewer leaders. For instance, minimal controllability problems were considered in [30], where a greedy heuristic approach was developed to ensure network controllability while minimizing the number of selected leaders. Robust minimal controllability was investigated in [59], where additional constraints were included in the minimal controllability problem. However, only unsigned graphs were considered in [30] and [59]. Nevertheless, the developed optimization approach and the unraveled fundamental relationship between minimal controllability and network topological sparsity can be potentially helpful in developing leader group selection rules for signed graphs. In particular, the solution of the minimal controllability problem in [30] and [59] is heavily dependent on the network topological sparsity. Such topological properties have been extensively studied on path and cycle graphs in the works of [20] and [46]. Since our approach is based on the partition of a general signed graph into a set of path and cycle graphs, additional research will therefore leverage tools from [20], [30], [46], and [59] to investigate minimal controllability problems over signed graphs.

V. Conclusion

Leader selection on signed multiagent networks for ensured controllability is considered in this paper. We developed graph-inspired topological characterizations of the controllability of signed networks, based on which leader selection methods are developed for signed path and cycle graphs. Heuristic algorithms are also developed showing how leader selection methods developed for path and cycle graphs can be potentially extended to more general signed networks. Although the effectiveness of the developed leader selection rules is demonstrated via examples, there might exist different leader sets that are capable of ensuring network controllability with additional constraints (e.g., minimal leader number). Future research will consider extending the results in this paper taking into account additional constraints.

Appendix

Proof of Lemma 3

Suppose $\mathcal{G}$ contains m followers and $n-m$ leaders. Since $\mathcal{W}' = \Xi \mathcal{W} \Xi$ in the unsigned graph $\mathcal{G}'$, the graph Laplacian of $\mathcal{G}'$ is

$$\mathcal{L}(\mathcal{G}') = \mathcal{D}(\mathcal{G}') - \Xi \mathcal{W} \Xi = \Xi \mathcal{L}(\mathcal{G}) \Xi \tag{7}$$

where the fact that $\mathcal{D}(\mathcal{G}') = \mathcal{D}(\mathcal{G})$ and $\Xi\mathcal{D}(\mathcal{G})\Xi = \mathcal{D}(\mathcal{G})$ are used. Based on (2), the graph Laplacian in (7) can be further expanded as

$$\begin{aligned}\mathcal{L}(\mathcal{G}') &= \left[\begin{array}{c|c}\Xi_f & 0\\ \hline 0 & \Xi_l\end{array}\right]\left[\begin{array}{c|c}\mathcal{L}_f(\mathcal{G}) & \mathcal{L}_{fl}(\mathcal{G})\\ \hline \mathcal{L}_{lf}(\mathcal{G}) & \mathcal{L}_l(\mathcal{G})\end{array}\right]\left[\begin{array}{c|c}\Xi_f & 0\\ \hline 0 & \Xi_l\end{array}\right]\\ &= \left[\begin{array}{c|c}\Xi_f\mathcal{L}_f(\mathcal{G})\Xi_f & \Xi_f\mathcal{L}_{fl}(\mathcal{G})\Xi_l\\ \hline \Xi_l\mathcal{L}_{lf}(\mathcal{G})\Xi_f & \Xi_l\mathcal{L}_l(\mathcal{G})\Xi_l\end{array}\right]\end{aligned} \tag{8}$$

where $\Xi_f \in \mathbb{R}^{m\times m}$ and $\Xi_f \in \mathbb{R}^{(n-m)\times(n-m)}$ are diagonal matrices.

For notional simplicity, let $A \triangleq \Xi_f\mathcal{L}_f(\mathcal{G})\Xi_f$ and $B \triangleq \Xi_f\mathcal{L}_{fl}(\mathcal{G})\Xi_l$. The follower dynamics over $\mathcal{G}'$ can then be written from (1) and (8) as

$$\dot{x}_f(t) = -Ax_f - Bu(t) \tag{9}$$

whose controllability can be verified from

$$\begin{aligned}&\text{rank}\left\{\left[B\ AB\ \cdots A^{m-1}B\right]\right\}\\ &= \text{rank}\left\{\left[\Xi_f\mathcal{L}_{fl}\Xi_l\ \Xi_f\mathcal{L}_f\mathcal{L}_{fl}\Xi_l\ \cdots\ \Xi_f\mathcal{L}_f^{m-1}\mathcal{L}_{fl}\Xi_l\right]\right\}\end{aligned} \tag{10}$$

where $\Xi_f\Xi_f = I_m$ is used. If leaders are selected from the same set (i.e., either $\mathcal{V}_1$ or $\mathcal{V}_2$), then based on the properties of Ξ_f in Lemma 2, we have either $\mathcal{L}_{fl}\Xi_l = \mathcal{L}_{fl}$ or $\mathcal{L}_{fl}\Xi_l = -\mathcal{L}_{fl}$. Hence, the rank of controllability matrix in (10) can be further simplified as

$$\begin{aligned}&\text{rank}\left\{\left[B\ AB\ \cdots\ A^{m-1}B\right]\right\}\\ &= \text{rank}\left\{\pm\Xi_f\left[\mathcal{L}_{fl}\ \mathcal{L}_f\mathcal{L}_{fl}\ \cdots\ \mathcal{L}_f^{m-1}\mathcal{L}_{fl}\right]\right\}\\ &= \text{rank}\left\{\left[\mathcal{L}_{fl}\ \mathcal{L}_f\mathcal{L}_{fl}\ \cdots\ \mathcal{L}_f^{m-1}\mathcal{L}_{fl}\right]\right\}\end{aligned}$$

which indicates that the controllability of (9) remains the same as the controllability of the original system $\dot{x}_f(t) = -\mathcal{L}_f(\mathcal{G})x_f - \mathcal{L}_{fl}(\mathcal{G})u(t)$ over $\mathcal{G}$.

References

[1] G. E. Asimakopoulou, A. L. Dimeas, and N. D. Hatziargyriou, "Leader–follower strategies for energy management of multi-microgrids," *IEEE Trans. Smart Grid*, vol. 4, no. 4, pp. 1909–1916, Dec. 2013.

[2] J. R. Klotz, Z. Kan, J. M. Shea, E. L. Pasiliao, and W. E. Dixon, "Asymptotic synchronization of a leader–follower network of uncertain Euler–Lagrange systems," *IEEE Trans. Control Netw. Syst.*, vol. 2, no. 2, pp. 174–182, Jun. 2015.

[3] J. R. Klotz, S. Obuz, Z. Kan, and W. E. Dixon, "Synchronization of uncertain Euler–Lagrange systems with uncertain time-varying communication delays," *IEEE Trans. Cybern.*, vol. 48, no. 2, pp. 807–817, Feb. 2018.

[4] W. Xu, D. W. Ho, L. Li, and J. Cao, "Event-triggered schemes on leader-following consensus of general linear multiagent systems under different topologies," *IEEE Trans. Cybern.*, vol. 47, no. 1, pp. 212–223, Jan. 2017.

[5] Z. Kan, J. R. Klotz, E. L. Pasiliao, Jr., and W. E. Dixon, "Containment control for a social network with state-dependent connectivity," *Automatica*, vol. 56, pp. 86–92, Jun. 2015.

[6] A. Mirtabatabaei and F. Bullo, "Opinion dynamics in heterogeneous networks: Convergence conjectures and theorems," *SIAM J. Control Optim.*, vol. 50, no. 5, pp. 2763–2785, 2011.

[7] D. Acemoğlu, G. Como, F. Fagnani, and A. Ozdaglar, "Opinion fluctuations and disagreement in social networks," *Math. Oper. Res.*, vol. 38, no. 1, pp. 1–27, 2013.

[8] Y.-Y. Liu, J.-J. Slotine, and A.-L. Barabási, "Observability of complex systems," *Proc. Nat. Acad. Sci. USA*, vol. 110, no. 7, pp. 2460–2465, 2013.

[9] S. Gu *et al.*, "Controllability of structural brain networks," *Nat. Commun.*, vol. 6, Oct. 2015, Art. no. 8414.

[10] W. Ren, R. W. Beard, and E. M. Atkins, "Information consensus in multi-vehicle cooperative control," *IEEE Control Syst. Mag.*, vol. 27, pp. 71–82, Apr. 2007.

[11] C. Altafini, "Consensus problems on networks with antagonistic interactions," *IEEE Trans. Autom. Control*, vol. 58, no. 4, pp. 935–946, Apr. 2013.

[12] G. Facchetti, G. Iacono, and C. Altafini, "Computing global structural balance in large-scale signed social networks," *Proc. Nat. Acad. Sci. USA*, vol. 108, no. 52, pp. 20953–20958, 2011.

[13] L. Tian, Z. Ji, T. Hou, and K. Liu, "Bipartite consensus on coopetition networks with time-varying delays," *IEEE Access*, vol. 6, pp. 10169–10178, 2018.

[14] H. G. Tanner, "On the controllability of nearest neighbor interconnections," in *Proc. IEEE Conf. Decis. Control*, vol. 3, 2004, pp. 2467–2472.

[15] A. Rahmani, M. Ji, M. Mesbahi, and M. Egerstedt, "Controllability of multi-agent systems from a graph-theoretic perspective," *SIAM J. Control Optimiz*, vol. 48, no. 1, pp. 162–186, 2009.

[16] Z. Ji and H. Yu, "A new perspective to graphical characterization of multiagent controllability," *IEEE Trans. Cybern.*, vol. 47, no. 6, pp. 1471–1483, Jun. 2017.

[17] R. Haghighi and C. C. Cheah, "Topology-based controllability problem in network systems," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 47, no. 11, pp. 3077–3088, Nov. 2017.

[18] A. Yazıcıoğlu, W. Abbas, and M. Egerstedt, "Graph distances and controllability of networks," *IEEE Trans. Autom. Control*, vol. 61, no. 12, pp. 4125–4130, Dec. 2016.

[19] Z. Ji, H. Lin, and H. Yu, "Leaders in multi-agent controllability under consensus algorithm and tree topology," *Syst. Control Lett.*, vol. 61, no. 9, pp. 918–925, 2012.

[20] G. Notarstefano and G. Parlangeli, "Controllability and observability of grid graphs via reduction and symmetries," *IEEE Trans. Autom. Control*, vol. 58, no. 7, pp. 1719–1731, Jul. 2013.

[21] X. Liu and Z. Ji, "Controllability of multiagent systems based on path and cycle graphs," *Int. J. Robust Nonlinear Control*, vol. 28, no. 1, pp. 296–309, 2018.

[22] X. Liu, H. Lin, and B. M. Chen, "Structural controllability of switched linear systems," *Automatica*, vol. 49, no. 12, pp. 3531–3537, 2013.

[23] C.-T. Lin, "Structural controllability," *IEEE Trans. Autom. Control*, vol. AC-19, no. 3, pp. 201–208, Jun. 1974.

[24] R. W. Shields and J. B. Pearson, "Structural controliability of multi-input linear systems," *IEEE Trans. Autom. Control*, vol. AC-21, no. 2, pp. 203–212, Apr. 1976.

[25] X. Liu, H. Lin, and B. M. Chen, "Graph-theoretic characterisations of structural controllability for multi-agent system with switching topology," *Int. J. Control*, vol. 86, no. 2, pp. 222–231, 2013.

[26] Y. Tang, Z. Wang, H. Gao, H. Qiao, and J. Kurths, "On controllability of neuronal networks with constraints on the average of control gains," *IEEE Trans. Cybern.*, vol. 44, no. 12, pp. 2670–2681, Dec. 2014.

[27] C. O. Aguilar and B. Gharesifard, "Graph controllability classes for the Laplacian leader–follower dynamics," *IEEE Trans. Autom. Control*, vol. 60, no. 6, pp. 1611–1623, Jun. 2015.

[28] C. Commault and J.-M. Dion, "Input addition and leader selection for the controllability of graph-based systems," *Automatica*, vol. 49, no. 11, pp. 3322–3328, 2013.

[29] Z. Ji, Z. Wang, H. Lin, and Z. Wang, "Interconnection topologies for multi-agent coordination under leader–follower framework," *Automatica*, vol. 45, no. 12, pp. 2857–2863, 2009.

[30] A. Olshevsky, "Minimal controllability problems," *IEEE Trans. Control Netw. Syst.*, vol. 1, no. 3, pp. 249–258, Sep. 2014.

[31] M. Van De Wal and B. De Jager, "A review of methods for input/output selection," *Automatica*, vol. 37, no. 4, pp. 487–510, 2001.

[32] F. Pasqualetti, S. Zampieri, and F. Bullo, "Controllability metrics, limitations and algorithms for complex networks," *IEEE Trans. Control Netw. Syst.*, vol. 1, no. 1, pp. 40–52, Mar. 2014.

[33] G. Li, P. Tang, C. Wen, and Z. Meng, "Boundary constraints for minimum cost control of directed networks," *IEEE Trans. Cybern.*, vol. 47, no. 12, pp. 4196–4207, Dec. 2017.

[34] B. Marx, D. Koenig, and D. Georges, "Optimal sensor and actuator location for descriptor systems using generalized gramians and balanced realizations," in *Proc. Amer. Control Conf.*, vol. 3, 2004, pp. 2729–2734.

[35] H. R. Shaker and M. Tahavori, "Optimal sensor and actuator location for unstable systems," *J. Vib. Control*, vol. 19, no. 12, pp. 1915–1920, 2013.

[36] K. B. Lim, "Method for optimal actuator and sensor placement for large flexible structures," *J. Guid. Control Dyn.*, vol. 15, no. 1, pp. 49–57, 1992.

[37] S. Wasserman and K. Faust, *Social Network Analysis: Methods and Applications*, vol. 8. New York, NY, USA: Cambridge Univ. Press, 1994.

[38] J. Z. Kim *et al.*, "Role of graph architecture in controlling dynamical networks with applications to neural systems," *Nature Phys.*, vol. 14, no. 1, p. 91, 2018.

[39] J. Hu and W. X. Zheng, "Emergent collective behaviors on coopetition networks," *Phys. Lett. A*, vol. 378, nos. 26–27, pp. 1787–1796, 2014.

[40] C. Sun, G. Hu, and L. Xie, "Controllability of multiagent networks with antagonistic interactions," *IEEE Trans. Autom. Control*, vol. 62, no. 10, pp. 5457–5462, Oct. 2017.

[41] S. Alemzadeh, M. H. de Badyn, and M. Mesbahi, "Controllability and stabilizability analysis of signed consensus networks," in *Proc. IEEE Conf. Control Technol. Appl.*, 2017, pp. 55–60.

[42] M. H. de Badyn, S. Alemzadeh, and M. Mesbahi, "Controllability and data-driven identification of bipartite consensus on nonlinear signed networks," *arXiv preprint arXiv:1709.06679*, 2017.

[43] Y. Guan and L. Wang, "Controllability of multi-agent systems with directed and weighted signed networks," *Syst. Control Lett.*, vol. 116, pp. 47–55, Jun. 2018.

[44] S. F. Ruf, M. Egerstedt, and J. S. Shamma, "Herdable systems over signed, directed graphs," *arXiv preprint arXiv:1804.04230*, 2018.

[45] A. Clark, Q. Hou, L. Bushnell, and R. Poovendran, "A submodular optimization approach to leader–follower consensus in networks with negative edges," in *Proc. Amer. Control Conf.*, 2017, pp. 1346–1352.

[46] G. Parlangeli and G. Notarstefano, "On the reachability and observability of path and cycle graphs," *IEEE Trans. Autom. Control*, vol. 57, no. 3, pp. 743–748, Mar. 2012.

[47] C. Godsil and G. F. Royle, *Algebraic Graph Theory* (Graduate Texts in Mathematics). New York, NY, USA: Springer, 2001.

[48] Z. Han, L. Wang, Z. Lin, and R. Zheng, "Formation control with size scaling via a complex Laplacian-based approach," *IEEE Trans. Cybern.*, vol. 46, no. 10, pp. 2348–2359, Oct. 2016.

[49] H. G. Tanner, A. Jadbabaie, and G. J. Pappas, "Flocking in fixed and switching networks," *IEEE Trans. Autom. Control*, vol. 52, no. 5, pp. 863–868, May 2007.

[50] L. Cheng, Y. Wang, W. Ren, Z.-G. Hou, and M. Tan, "Containment control of multi-agent systems with dynamic leaders based on a PI^n-type approach," *IEEE Trans. Cybern.*, vol. 46, no. 12, pp. 3004–3017, Dec. 2016.

[51] Z. Kan, J. M. Shea, and W. E. Dixon, "Leader–follower containment control over directed random graphs," *Automatica*, vol. 66, pp. 56–62, Apr. 2016.

[52] K. Ogata and Y. Yang, *Modern Control Engineering*. Englewood Cliffs, NJ, USA: Prentice-Hall, 1970.

[53] A. Chapman, M. Nabi-Abdolyousefi, and M. Mesbahi, "Controllability and observability of network-of-networks via Cartesian products," *IEEE Trans. Autom. Control*, vol. 59, no. 10, pp. 2668–2679, Oct. 2014.

[54] M. Nabi-Abdolyousefi and M. Mesbahi, "On the controllability properties of circulant networks," *IEEE Trans. Autom. Control*, vol. 58, no. 12, pp. 3179–3184, Dec. 2013.

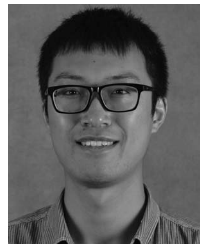
[55] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *Proc. IEEE*, vol. 95, no. 1, pp. 215–233, Jan. 2007.

[56] G. Steiner, "On the *k*-path partition of graphs," *Theor. Comput. Sci.*, vol. 290, no. 3, pp. 2147–2155, 2003.

[57] Z. Hu and H. Li, "Partition of a graph into cycles and vertices," *Discr. Math.*, vol. 307, nos. 11–12, pp. 1436–1440, 2007.

[58] H. Enomoto, "Graph partition problems into cycles and paths," *Discr. Math.*, vol. 233, nos. 1–3, pp. 93–101, 2001.

[59] S. Pequito, G. Ramos, S. Kar, A. P. Aguiar, and J. Ramos, "The robust minimal controllability problem," *Automatica*, vol. 82, pp. 261–268, Aug. 2017.



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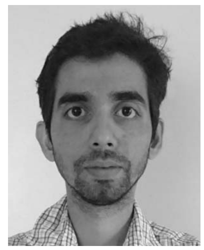

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