ТЧ

$\varphi \sim \rho(x,\theta) = \frac{\theta}{2}\{(-1,1)\setminus\{0\}\} + \frac{1-\theta}{2}\{0\} + \frac{1-\theta}{2}\{2\}$

$\alpha_1 = M[\varphi] = \int\limits_{-\infty}^{+\infty} x\rho(x,\theta)dx = \int\limits_{-1}^{1}\frac{\theta}{2}x\,dx + \frac{1-\theta}{2}\cdot 0 + \frac{1-\theta}{2}\cdot 2 = 1-\theta$

$\alpha_2 = M[\varphi^2] = \frac{\theta}{2}\int\limits_{-1}^{1}x^2dx + \frac{1-\theta}{2}\cdot 2^2 = \frac{\theta}{3} + 2(1-\theta) = 2 - \frac{5}{3}\theta$

$\mu_2 = \alpha_2 - \alpha_1^2 = D[\varphi] = 2 - \frac{5}{3}\theta - \theta^2 + 2\theta - 1 =$

$= 1 + \frac{\theta}{3} - \theta^2$

1. а) ОММ: α_1:

$\alpha_1(\theta) = \overline{\alpha}_1 = \overline{x} \Rightarrow 1-\theta = \overline{x} \Rightarrow \tilde{\theta}_1 = 1-\overline{x}$

б) Несмещ.:

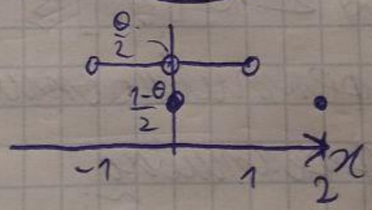
$M[\tilde{\theta}_1] = M[1-\overline{x}] = 1 - M[\varphi] = 1-1+\theta = \theta \Rightarrow$ несм.

Состоят.:

$D[\tilde{\theta}_1] = D[1-\overline{x}] = \frac{1}{n}D[\varphi] \to 0, n\to\infty$

$\Rightarrow$ состоят. по ДУ.

2 а) ОМП:



$$L = \left(\frac{\theta}{2}\right)^{n-m_1-m_2}\left(\frac{1-\theta}{2}\right)^{m_1+m_2}$$

$$\ln L = (n-m_1-m_2)\ln\frac{\theta}{2} + (m_1+m_2)\ln\left(\frac{1-\theta}{2}\right)$$

$$(\ln L)' = 0 \Rightarrow n\theta - n + m_1 + m_2 = 0 \Rightarrow \theta = 1 - \nu_1 - \nu_2$$

$$(\ln L)'' = \frac{m_1+m_2-n}{\theta^2} - \frac{m_1+m_2}{(\theta-1)^2} = \frac{(m_1+m_2-n)(\theta-1)^2 - (m_1+m_2)\theta^2}{\theta^2(\theta-1)^2} =$$

$$= \{\theta = 1-\nu_1-\nu_2\} = \frac{(\nu_1+\nu_2)(\nu_1+\nu_2-1)}{n\,\theta^2(\theta-1)^2} < 0 \Rightarrow$$

$\Rightarrow \theta = 1-\nu_1-\nu_2$ — т. макс. $\Rightarrow \tilde{\theta}_2 = 1-\nu_1-\nu_2$

б) несмещ.:

$$M[\tilde{\theta}_2] = 1 - M[\nu_1] - M[\nu_2] = 1 - \frac{1-\theta}{2} - \frac{1-\theta}{2} = \theta$$

$\Rightarrow$ несмещ. $\quad \frac{(1-\theta)\theta}{n} \to 0,\ n\to\infty \Rightarrow$ **состоят.**

Состоят.:

$D[\tilde{\theta}_2] = D[\nu_1+\nu_2]$ "

c) Проверка модели на регулярность:

1) $\rho(x,\theta) \in C^\infty_\theta((0,1))$

2) $\frac{\partial}{\partial\theta}\int_{-\infty}^{\infty}\rho(x,\theta)dx = \int_{-\infty}^{\infty}\frac{\partial}{\partial\theta}\rho(x,\theta)\,dx \,(=0)$ на $(0,1)$

$$\int_{-\infty}^{\infty}\frac{\partial\rho(x,\theta)}{\partial\theta} = \frac{1}{2}\int_{-1}^{1}dx + \left(\frac{1-\theta}{2}\right)'_\theta + \left(\frac{1-\theta}{2}\right)'_\theta = 1 - \frac{1}{2} - \frac{1}{2} = 0$$

3) $I(\theta) \in C(0,1)$, $I(\theta) > 0$ на $(0,1)$:

$$\frac{\partial \ln p(x,\theta)}{\partial \theta} = \frac{\partial}{\partial \theta}\left(\ln \frac{\theta}{2}\right) = \frac{1}{\theta}$$

$$\frac{\partial \ln P_{\theta 2}}{\partial \theta} = \frac{\partial}{\partial \theta} \cdot \left(\ln\left(\frac{1-\theta}{2}\right)\right) = \frac{1}{\theta - 1}$$

$$I(\theta) = \int_{-1}^{1} \frac{1}{2\theta}\,dx + \frac{1}{(\theta-1)^2} \cdot \left(\frac{1-\theta}{2}\right) \cdot 2 =$$

$$= \frac{1}{\theta} + \frac{1}{1-\theta} = \frac{1}{\theta(1-\theta)} \in C(0,1), > 0 \text{ на } (0,1)$$

$\Rightarrow$ модель регулярна

$\tilde{\theta}_1 = 1 - \bar{x}$ – несм. $D[\tilde{\theta}_1]$ – огр. на $\forall$ компакте из $(0,1)$

$\Rightarrow \tilde{\theta}_1$ – регулярна

$\tilde{\theta}_2 = 1 - \nu_2 - \tilde{\nu}_1$ – несмещ. $D[\tilde{\theta}_2]$ – огр на $\forall$ ком из $(0,1)$ $\Rightarrow \tilde{\theta}_2$ – регулярна

$\Rightarrow$ используем нер-во Крамера-Рао

$\tilde{\theta}_2$: $D[\tilde{\theta}_2] \geq \frac{1}{nI(\theta)} = \frac{\theta(1-\theta)}{n}$

$\frac{\theta(1-\theta)}{n} \geq \frac{\theta(1-\theta)}{n} \Rightarrow$ по дост. усл-ю эффект.

$\tilde{\theta}_2$ эффект $\Rightarrow$

$\Rightarrow \tilde{\theta}_1$ – не эффект.