$\sqrt{n}$

T6

$$p(x) = \begin{cases} \frac{\theta-1}{x}, & x \geq 1 \\ 0, & x < 1 \end{cases}, \quad \theta > 1$$

a) $L(x, \theta) = \dfrac{(\theta-1)^n}{\prod\limits_{i=1}^{n} x_i^{\theta}}$

$$\ln L = n \ln(\theta - 1) - \theta \sum_{i=1}^{n} \ln x_i$$

$$(\ln L)'_\theta = \frac{n}{\theta-1} - \sum_{i=1}^{n} \ln x_i = 0$$

$$\tilde{\theta} = 1 + \frac{1}{\sqrt{\ln x_i}}$$

$$(\ln L)''_{\theta\theta} = -\frac{n}{(\theta-1)^2} < 0 \Rightarrow \theta = 1 + \frac{1}{\sqrt{\ln x_i}} - \text{Т.м}$$

в) дов. интервал для медианы:

$$\left(\int \frac{\partial}{\partial \theta} \left(\frac{\theta - 1}{x^{\theta}} \right) dx \right)'_\theta = x^{1-\theta} \ln x$$

$$\int\limits_x \frac{\partial}{\partial\theta}\left(\frac{\theta-1}{x^{\theta}}\right)dx \overset{\shortparallel}{=} x^{1-\theta}\ln x$$ => модель сильно регулярна

$\hat{\theta}$ $$\int\limits_1^{\hat{x}} \frac{\theta-1}{x^{\theta}}dx = -\frac{1}{\hat{x}^{\theta-1}}+1=\frac{1}{2} \quad \hat{x}=2^{\frac{1}{\theta-1}}$$

$$g(\tilde{\theta})=2^{\frac{1}{\tilde{\theta}-1}}$$

$$\sqrt{n}\cdot\frac{g(\tilde{\theta})-g(\theta)}{\sigma(\theta)} \rightsquigarrow N(0,1)$$

$$\sigma(\theta) \to \sigma(\hat{\theta})$$

$$\sigma(\tilde{\theta})=\sqrt{\nabla^T g(\hat{\theta})\, I^{-1}(\tilde{\theta})\nabla g(\theta)}$$

$$I(\tilde{\theta})=M\left[\left((\ln p)'_{\theta}\right)^2\right]=M\left[\left(\frac{1}{\theta-1}-\ln x\right)^2\right]=$$

$$=\int\limits_1^{+\infty}\left(\frac{1}{\theta-1}-\ln x\right)^2 p(x,\theta)\,dx=\int\limits_1^{\infty}\left(\frac{1}{\theta-1}-\ln x\right)^2\frac{\theta-1}{x^2}dx=$$

$$=\frac{1}{(\theta-1)^2}$$ — непр. на $\theta>1$

$$\nabla g(\tilde{\theta})=-\frac{\ln 2\cdot 2^{\frac{1}{\tilde{\theta}-1}}}{\tilde{\theta}-1} \qquad \sigma(\tilde{\theta})=-\frac{\ln 2\cdot 2^{\frac{1}{\tilde{\theta}-1}}}{\tilde{\theta}-1}$$

$$\sqrt{n}\;\frac{g(\tilde{\theta})-g(\theta)}{\sigma(\tilde{\theta})} \rightsquigarrow N(0,1)$$

$$\frac{1{,}95 \cdot \sigma(\tilde{\theta})}{\sqrt{n}} + g(\tilde{\theta}) < g(\theta) < -\frac{1{,}95 \cdot \sigma(\tilde{\theta})}{\sqrt{n}} + g(\hat{\theta})$$

c)

$$\sqrt{n}\,\frac{\tilde{\theta}-\theta}{\sigma(\theta)} \rightsquigarrow N(0,1)$$

$$\sigma(\theta) \xrightarrow{P} \sigma(\tilde{\theta})$$

$$\sigma(\tilde{\theta}) = \theta - 1 \Rightarrow \sqrt{n}\,\frac{\tilde{\theta}-\theta}{\theta-1} \rightsquigarrow N(0,1)$$

$$-\frac{1{,}95(\theta-1)}{\sqrt{n}} + 1 + \frac{1}{\ln x_i} < \theta < \frac{1{,}95(\theta-1)}{\sqrt{n}} + 1 + \frac{1}{\ln x_i}$$