T5

a) $\varphi \sim R(\theta, 2\theta)$ $\quad \rho(x,\theta) = \frac{1}{\theta}\{(\theta, 2\theta)\}$

ОММ:

$$\alpha_1 = M[\varphi] = \int_{-\infty}^{\infty} x\rho(x,\theta)dx = \frac{1}{\theta}\int_{0}^{2\theta} x dx = \frac{3\theta}{2}$$

b_1) $\alpha_1(\theta) = \bar{\alpha}_1 = \bar{x} \Rightarrow \frac{3\theta}{2} = \bar{x} \Rightarrow \tilde{\theta} = \frac{2}{3}\bar{x}$

Несмещ.:

$M[\tilde{\theta}] = M[\frac{2}{3}x] = \frac{2}{3}\cdot\frac{3}{2}\theta = \theta \Rightarrow$ несмещ.

Состоят.:

$$\alpha_2 = M[\varphi^2] = \int_{\theta}^{2\theta} \frac{1}{\theta}x^2 dx = \frac{1}{\theta}\frac{7\theta^3}{3} = \frac{7}{3}\theta^2$$

$$\mu_2 = D[\varphi] = \alpha_2 - \alpha_1^2 = \frac{7}{3}\theta^2 - \frac{9}{4}\theta^2 = \frac{\theta^2}{12}$$

$$D[\tilde{\theta}] = D[\frac{2}{3}\bar{x}] = \frac{4}{9n}D[\varphi] = \frac{4}{9n}\frac{\theta^2}{12} \to 0, n \to \infty$$

$\Rightarrow$ состоят. по ДУ

a_2) ОМП $\quad L = \frac{1}{\theta^n}\{\theta \le x_i \le 2\theta \ \forall x_i\}$

$$x_{max} = 2\theta \Rightarrow \tilde{\theta} = \frac{x_{max}}{2} \qquad \theta < x < 2\theta$$

b_2) $\Phi(x) = (F(x))^n = \left(\int_0^x \frac{1}{\theta}dx\right)^n = \left(\frac{x}{\theta} - 1\right)^n$

$$M[\tilde{\theta}] = M[x_{max}/2] = \int_{\theta}^{2\theta} \frac{n}{2\theta}\left(\frac{x}{\theta} - 1\right)^{n-1} x dx =$$

$$= \frac{2\theta n + \theta}{2(n+1)} = \theta \cdot \frac{2n+1}{2(n+1)}$$

$$\tilde{\theta}^* = \frac{2(n+1)}{2n+1}\tilde{\theta} = \frac{2(n+1)}{2n+1} \cdot \frac{x_{max}}{2}$$

$M[\tilde{\theta}^*] = \theta$ — испр. оценка

$$D[\tilde{\theta}^*] = D\left[\frac{2(n+1)}{2n+1} \cdot \frac{x_{max}}{2}\right] = \left(\frac{n+1}{2n+1}\right)^2 D[x_{max}] =$$

$$= \left(\frac{n+1}{2n+1}\right)^2 \left(M[x_{max}^2] - M^2[x_{max}]\right)$$

$$M[x_{max}^2] = \int_{\theta}^{2\theta} x^2 \frac{n}{\theta}\left(\frac{x}{\theta} - 1\right)^{n-1} dx = 2\theta^2 \cdot \frac{2n^2+4n+1}{(n+2)(n+1)}$$

$$D[\tilde{\theta}^*] = \left(\frac{n+1}{2n+1}\right)^2 \frac{4n^2+8n+2}{(n+1)(n+2)}\theta^2 - \theta^2 =$$

$$= \frac{n\theta^2}{(2n+1)^2(n+2)} \xrightarrow[n\to\infty]{} 0$$

$\tilde{\theta}^*$ — состоят. по дост. усл.

$D[\tilde{\theta}_1] = \frac{\theta^2}{27n}$ $\quad \exists N \ \forall n \geq N \Leftrightarrow$

$$D[\tilde{\theta}_1] > D[\tilde{\theta}_2^*]$$

$\tilde{\theta}_2^*$ — более эффект., чем $\tilde{\theta}_1$

$x_i \in [\theta, 2\theta]$

$\frac{x_i}{\theta} \in [1, 2]$

$$\varphi(x_{max}) = (F(x))^n = \left(\int_1^x dx\right)^n = (x-1)^n$$

$x = \theta$

$$\sqrt[n]{0,025} + 1 < x < \sqrt[n]{0,975} + 1$$

$$\sqrt[n]{0,025} + 1 < \frac{x_{max}}{\theta} < \sqrt[n]{0,975} + 1$$

$$\frac{x_{max}}{\sqrt[n]{0,975} + 1} < \theta < \frac{x_{max}}{\sqrt[n]{0,025} + 1}$$

По ОММ

$$\sqrt{n} \cdot \frac{g(\tilde{\alpha}) - g(\alpha)}{\sigma(\alpha)} \rightsquigarrow N(0,1)$$

$$\sigma(\alpha) = \sqrt{\nabla^T g \, K \, \nabla g} = \frac{2}{3}\sqrt{\alpha_2 - \alpha_1^2}$$

$$\frac{\sqrt{n} \cdot \frac{2}{3}\tilde{\alpha} - \frac{2}{3}\alpha_1}{\frac{2}{3}\sqrt{\alpha_2 - \alpha_1^2}} = \underbrace{\sqrt{n}\,\frac{\tilde{\theta} - \theta}{\frac{2}{3}\sqrt{\alpha_2 - \alpha_1^2}}}_{\text{по л. Слуцкого}} \cdot \underbrace{\frac{\frac{2}{3}\sqrt{\alpha_2 - \alpha_1^2}}{\frac{2}{3}\sqrt{\tilde{\alpha}_2 - \tilde{\alpha}_1^2}}}_{\to 1} \rightsquigarrow N(0,1)$$

$\rightsquigarrow N(0,1)$

$$-1,95 < \sqrt{n}\,\frac{\tilde{\theta} - \theta}{\frac{2}{3}\sqrt{\tilde{\alpha}_2 - \tilde{\alpha}_1^2}} < 1,95$$

$$-\frac{1,95}{\sqrt{n}} \cdot \frac{2}{3}\sqrt{\tilde{\alpha}_2 - \tilde{\alpha}_1^2} + \tilde{\theta} < \theta < \frac{1,95}{\sqrt{n}} \cdot \frac{2}{3}\sqrt{\tilde{\alpha}_2 - \tilde{\alpha}_1^2} + \tilde{\theta}$$

T6

$$p(x) = \begin{cases} \frac{\theta - 1}{x}, & x \geq 1 \\ 0, & x < 1 \end{cases}, \quad \theta > 1$$