QWORLD

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Basics of classical system

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A single bit

⇒ Single bit

∃ many different operators we can define on a single bit?

operator

0 → {

1 →

2
- Identity
- NOT
- ⋮

four

Bit — **2 states**

yes	no
on	off
→ 1	0
true	false
Accept	Reject
⋮	⋮

0   1

update

0
↓
0 ← **Operator**
↓
1
↓
1

Reversibility

$0 \to \{0,1\} \Rightarrow 2$

$1 \to \{0,1\} \to 2$

4

$\left.\begin{array}{l} 0 \to 0 \\ 1 \to 1 \end{array}\right\}$ identity

$\left.\begin{array}{l} 0 \to 1 \\ 1 \to 0 \end{array}\right\}$ NOT

$\left.\begin{array}{l} 0 \to 0 \\ 1 \to 0 \end{array}\right\}$ ZERO

$\left.\begin{array}{l} 0 \to 1 \\ 1 \to 1 \end{array}\right\}$ ONE

Reversible

I, NOT

Not reversible

ZERO, ONE

Two bits

two bits ← composite system

0	0
1	1

00
01
10
11

1st bit, 2nd bit

1st, 2nd: 10.

00 → {00, 01, 10, 11} 4
01 → { '' } 4
10 → { '' } 4
11 → { '' } 4

4^4 ← # of operators

how many of them are reversible?

Transition table

0 → 1
1 → 0

final states

	00	01	…
11	1	1	0	0

initial states: two bits

final states	00	01	10	11
00	0	1	1	0
01	0	0	0	1
10	1	0	0	0
11	0	0	0	0

The table for transitions

there is a single 1 in each column.

0 → on → exists
1 → off → doesn't exist

of operators

$B_1 \quad B_2 \quad B_3 = B$

0 0 0

1 1 1

$B \to 8$ states

000 → 0
001 → 1
010 → 2
⋮
111 → 8

	00	01	10	11
00	0	0	1	0
01	1	0	0	0
10	0	1	0	0
11	0	0	0	1

0 1 2 ... 4 3 2 1 = 4!

	000	001	...
000			
001			...

$n \to$ bits
# of states

0 0 0 ... 0
1 1 1 ... 1

$2^n = N$

$N^N \to$ # of ope.

$N! \to$ reversible

Probabilistic bit

probabilistic bit ?

0, 1 } deterministic states } basis state

$Pr = \frac{1}{3}, \ 0$
$Pr = \frac{2}{3}, \ 1$

Vectors table with a single column

← Pr of being state 0

← pr of being state 1

state $0 \Rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

state $1 \Rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

are these probabilistic state or not?

$\begin{pmatrix} \frac{1}{3} \\ \frac{2}{3} \end{pmatrix}$

probabilistic state.

Linear state

$$\begin{pmatrix} 0.3 \\ 0.7 \end{pmatrix} \to \text{probabilistic state}$$

$\begin{pmatrix} 0.3 \\ 0.7 \end{pmatrix}$ $\to (0.3, 0)$, $\to (0.7, 1)$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ co-efficients

$$\begin{pmatrix} 0.3 \\ 0.7 \end{pmatrix} = 0.3\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 0.7\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.3 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0.7 \end{pmatrix} = \begin{pmatrix} 0.3 \\ 0.7 \end{pmatrix}$$

$\begin{pmatrix} 0.1 \\ 0.3 \\ 0.4 \\ 0.2 \end{pmatrix}$ $(0.1, s_1)$, $(0.3, s_2)$, $(0.4, s_3)$, $(0.2, s_4)$

$$= 0.1 \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + 0.3\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + 0.4\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} + 0.2\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

a linear combination of basis state.

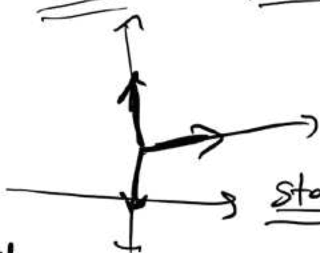
Coefficients

probabilistic state

n-state ⇒ n deterministic states
basis states.

$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$p_1\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + p_2\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + p_3\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}$$



stochastic vector

linear combination of basis states,
where the coefficients are probabilities.

$0 \leq p_1, p_2, p_3 \leq 1$ and $p_1 + p_2 + p_3 = 1$

$\begin{pmatrix} \\ \end{pmatrix} \geq 0$, 1

Stochastic vector

Probabilistic state $\longleftrightarrow$ Stochastic vector.

$$\begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{pmatrix} \geq 0, \quad \sum = 1 \qquad \equiv \qquad \begin{pmatrix} \\ \\ \end{pmatrix} \geq 0, \quad \sum = 1$$

probabilistic operator?

Probabilistic operator

$$\begin{pmatrix} & & & \\ & \geq 0 & & \\ & & & \end{pmatrix}, \quad \sum: \; 1 \quad 1 \quad 1 \quad 1$$

	00	01	10	11
00	0.2	0.5	0	0
01	0.3	0	0.4	0
10	0	0	0.3	0
11	0.5	0.5	0.3	1
$\sum$	1	1	1	1

$\begin{pmatrix} 0.3 \\ 0.2 \\ 0.1 \\ 0.4 \end{pmatrix}$

this table is enough to represent a probabilistic operator?

2nd one
$\Rightarrow 0.3 \cdot 0.3 + 0.2 \cdot 0 + 0.1 \cdot 0.4 + 0.4 \cdot 0$
$= 0.09 + 0 + 0.04 + 0$
$= 0.13$

$\rightarrow$ 0.2 | 0.3 | 0.5 $\equiv 1$
00 | 10 | 11

deterministic

0 1

there is no transition

there is a transition

Transition matrix

probabilistic operator ⟷ stochastic matrix

$$\underbrace{\begin{pmatrix} \\ \\ \end{pmatrix}}_{\text{new state}} = A \cdot v = \underbrace{\begin{pmatrix} & & \\ & A & \\ & & \end{pmatrix}}_{\text{transition matrix}} \cdot \underbrace{\begin{pmatrix} \\ v \\ \end{pmatrix}}_{\text{current state}}$$

$$\boxed{v' = A \cdot v}$$

$$\begin{pmatrix} \geq 0 \end{pmatrix}$$

$1 \cdots\cdots 1$

we will follow this convention.

(left) stochastic matrix

$$\boxed{\begin{pmatrix} & \end{pmatrix} = \underbrace{\begin{pmatrix} & \end{pmatrix}}_{\text{probabilistic state}} \begin{pmatrix} & \end{pmatrix} \quad \text{(right) stochastic matrix}}$$

Coin-flipping

Coin flipping

toss a coin $\xrightarrow{\text{outcomes}}$ $\left.\begin{array}{l} \text{Heads} \equiv 0 \\ \text{Tails} \equiv 1 \end{array}\right\}$ probabilistic bit

Fair coin

$$\begin{array}{c|cc} & 0 & 1 \\ \hline 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & \frac{1}{2} & \frac{1}{2} \end{array}$$

Biased coins

biased coin

$$\begin{array}{c|cc} & 0 & 1 \\ \hline 0 & 0.4 & 0.35 \\ 1 & 0.6 & 0.65 \end{array}$$

coin toss

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \xrightarrow{\text{coin toss}} \begin{pmatrix} 0.4 \\ 0.6 \end{pmatrix} \xrightarrow{\text{coin toss}}$$

$$\begin{pmatrix} \ \end{pmatrix} = \begin{pmatrix} 0.4 & 0.35 \\ 0.6 & 0.65 \end{pmatrix} \begin{pmatrix} 0.4 \\ 0.6 \end{pmatrix} \ldots$$

Finding bias

$$\bigcirc \xrightarrow{0.4} \text{Heads} \qquad \bigcirc \xrightarrow{0.6} \text{Tails}$$

biased coin

$$\begin{pmatrix} p & p \\ 1-p & 1-p \end{pmatrix}$$

Can you guess/estimate the value of p?

more more

$p = 0.5000000000001$

$0.500 \ldots 0 \ldots$

$$\bigcirc \xrightarrow{p} \text{Heads} \qquad \bigcirc \xrightarrow{1-p} \text{Tails}$$

many coin-flipping
collect statistics

$$10000 \qquad \frac{H}{10000} = \tilde{p} \approx p$$

Can you determine whether a given coin is a fair or biased coin?

0.5001 ← fair or biased
0.4999

Correlation

Correlation

father → fair coin H T

partan salad

partan salad

my lunch box

mine

my lunch box

my sister's

CNOT

State 1 $\equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\boxed{\begin{matrix} S_1 \equiv 0 \\ S_2 \equiv 1 \end{matrix}}$

two bits

B_1 $\quad$ B_2

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad$ $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ State 0

fair coin flip

$$\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{array}{c|c} \frac{1}{2} & 00 \\ 0 & 01 \\ \frac{1}{2} & 10 \\ 0 & 11 \end{array}$$

CNOT = (controller, target)

else 0 → do nothing

if 1 → apply not

conditional programming

if

else

Correlated bits

CNOT

$$\begin{array}{c|cccc} & 00 & 01 & 10 & 11 \\ \hline 00 & & & & \\ 01 & & & & \\ 10 & & & & \\ 11 & & & & \end{array} \quad \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} I & \\ & NOT \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{2} \\ 0 \\ 0 \\ \frac{1}{2} \end{pmatrix} \qquad \begin{pmatrix} 0 \\ \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix} \begin{matrix} \to 01 \\ \to 10 \end{matrix}$$

00 → 00
01 → 01
10 → 11
11 → 10

$$\begin{matrix} 00 \leftarrow \\ \\ 11 \leftarrow \end{matrix} \begin{pmatrix} \frac{1}{2} \\ 0 \\ 0 \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ 0 \\ \frac{1}{2} \\ 0 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} \frac{1}{2} \\ 0 \\ 0 \\ \frac{1}{2} \end{pmatrix}}_{\text{correlated}} \neq \underbrace{\begin{pmatrix} a \\ b \end{pmatrix} \otimes \begin{pmatrix} c \\ d \end{pmatrix}}_{=\text{not correlated}}$$