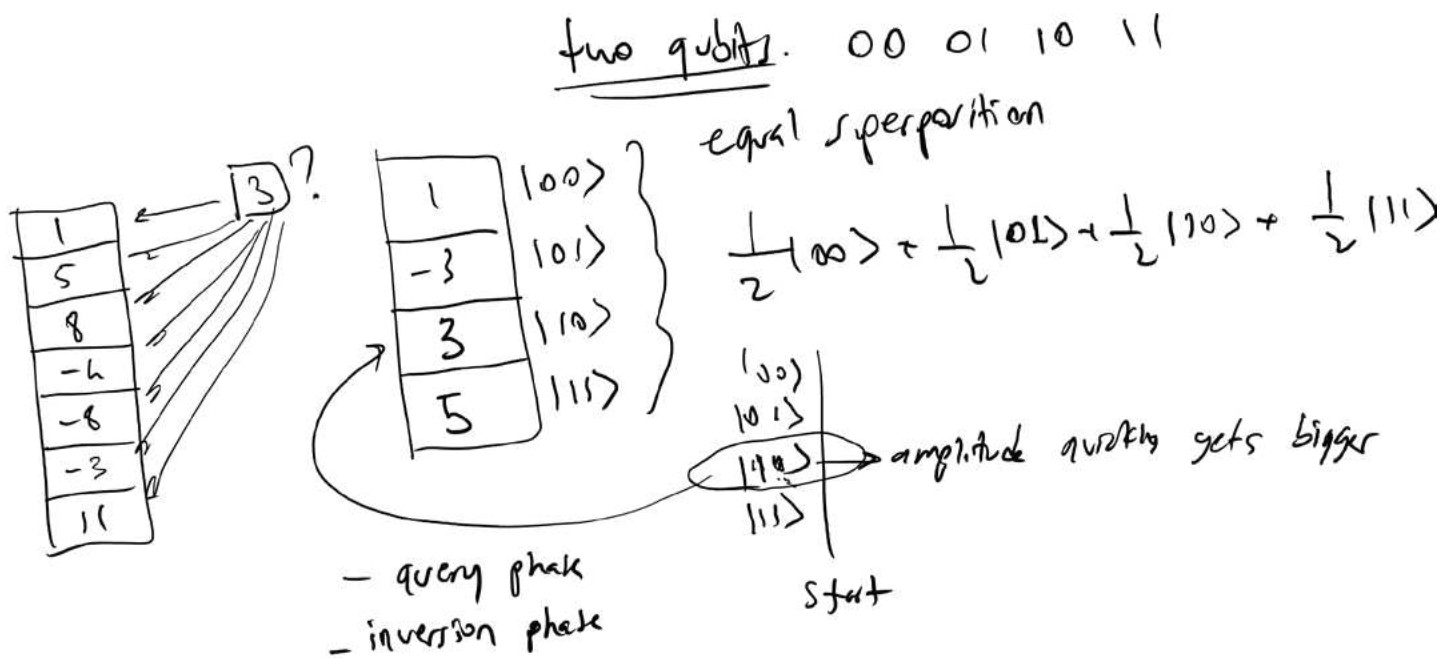


Saturday, October 29, 2021

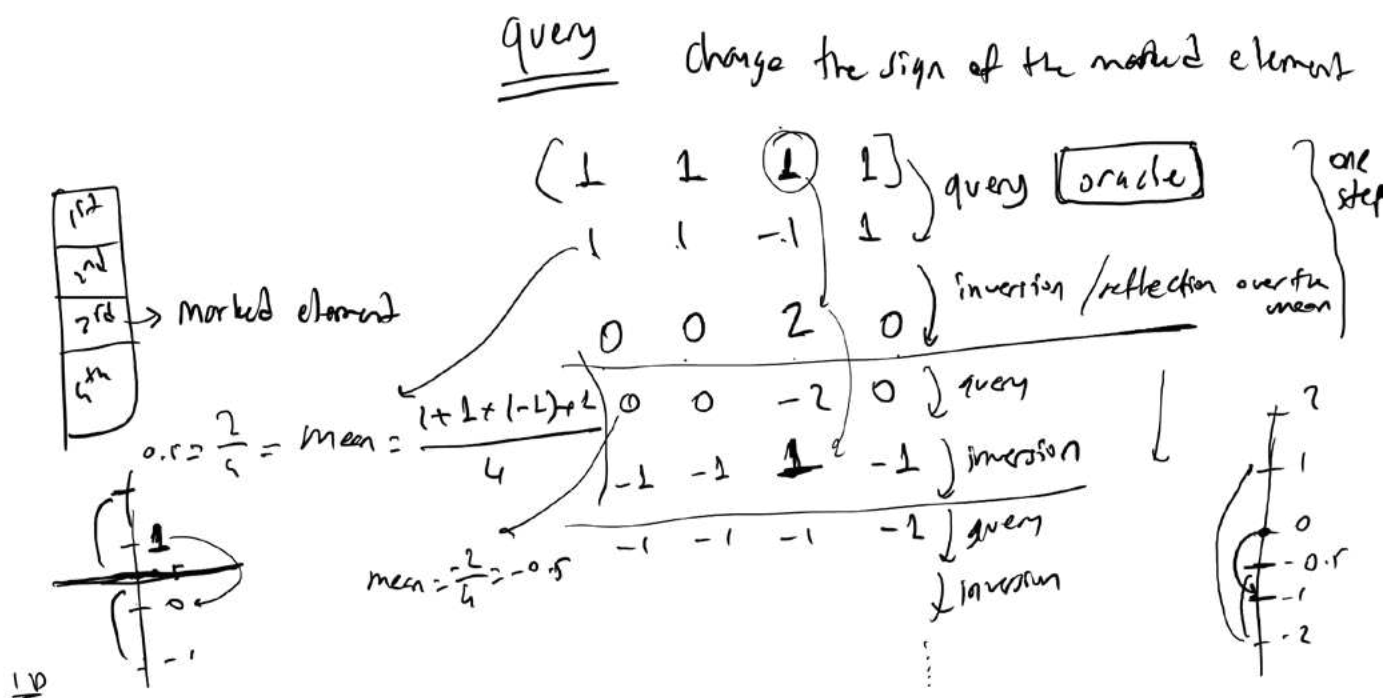
Grover's search algorithm



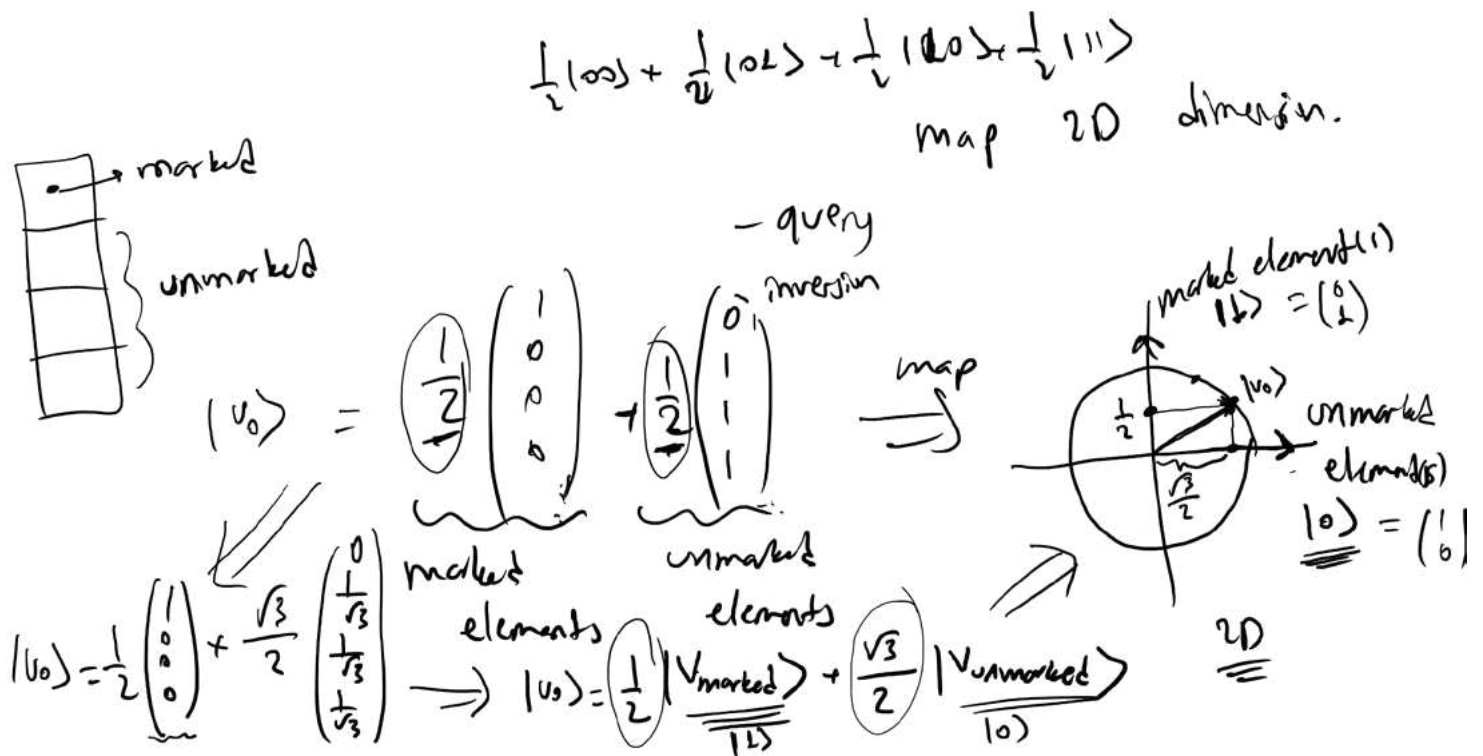
■ Searching on a list



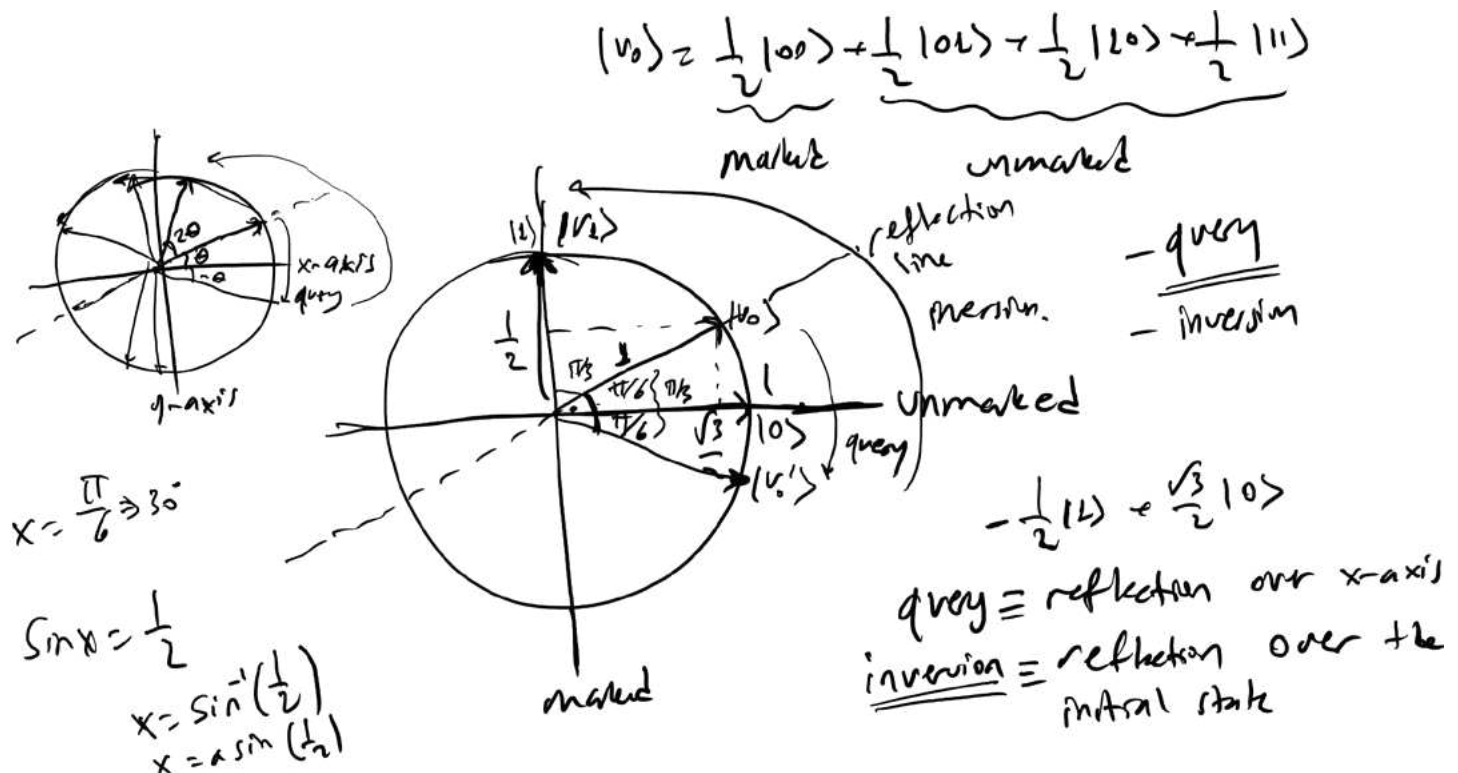
■ Query and inversion



■ 2D representation



■ Reflection lines



■ Another example

8 elements

1. 000 } marked
2. 001 }
3. 010 }
...
8. 111

$|u_0\rangle = \frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|001\rangle + \dots + \frac{1}{\sqrt{2}}|111\rangle$ $\sqrt{2} = \sqrt{4}$

reflection line for inversion

reflection line for every

$\Rightarrow \sqrt{\frac{3}{8}}$

$\sqrt{\frac{7}{8}}$

$|1\rangle$

$|0\rangle$

■ Rotations

128 of elements
L is marked.

N queries \leftarrow classical

\sqrt{N} queries

quantum

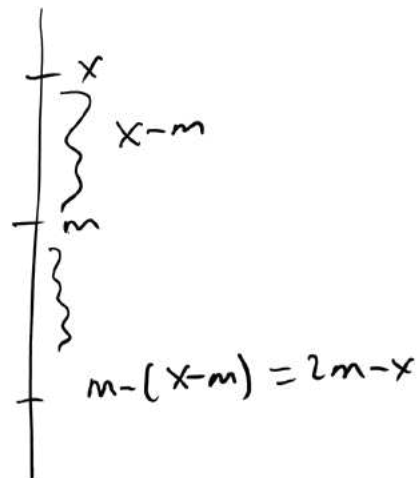
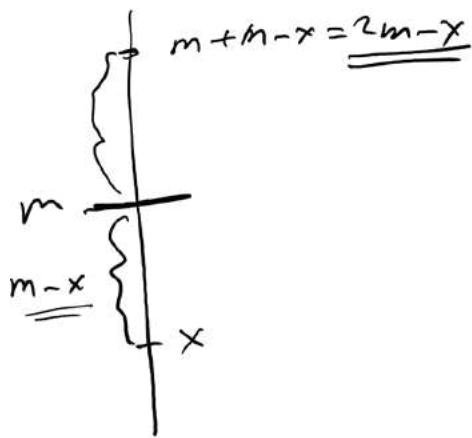
no pre-knowledge on the marked elements.

$k=1, \underline{\hspace{2cm}}$
 $k=2, \underline{\hspace{2cm}}$
 $k=4, \underline{\hspace{2cm}}$
 $k=8, \underline{\hspace{2cm}}$

$O(\sqrt{N})$

based on θ ,
we can calculate the
optimal number of steps.

■ Inversion about the mean



$$x \mapsto 2m - x$$

■ Inversion matrix

$$2m - a_i = \frac{1}{N} \begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & 1 & 1 & \dots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \dots & 1 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_i \\ \vdots \\ a_n \end{pmatrix}$$

$$m = \frac{a_1 + a_2 + \dots + a_n}{N}$$

$$a_i \rightarrow 2m - a_i$$

- Reflected state

$$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ \vdots \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$$

- Reflection over the initial state

$$= \left(2|u_0\rangle\langle u_0| - I \right) |v\rangle$$

