QWORLD

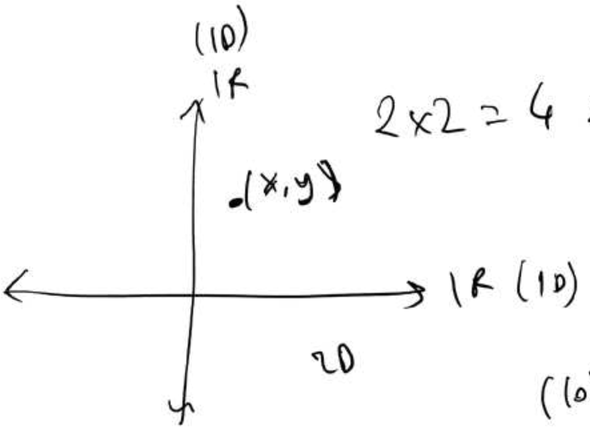
Saturday, October 23, 2021

Entanglement and basic protocols

Two qubits

a single qubit $|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$, $|1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$

two qubits $\Rightarrow$ a quantum register system



$2 \times 2 = 4$ states. $\begin{Bmatrix}|0\rangle\\|1\rangle\end{Bmatrix} \times \begin{Bmatrix}|0\rangle\\|1\rangle\end{Bmatrix}$

$\Rightarrow |0\rangle|0\rangle$, $|0\rangle|1\rangle$, $|1\rangle|0\rangle$, $|1\rangle|1\rangle$

$\Rightarrow \underline{|00\rangle}$, $\underline{|01\rangle}$, $\underline{|10\rangle}$, $\underline{|11\rangle}$

$(|0\rangle, |0\rangle)$
$\Rightarrow |00\rangle$

vector form $\Rightarrow$ tensor product

Basis states

$$\underline{|10\rangle} = |1\rangle \otimes |0\rangle = \begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0 \cdot \begin{pmatrix}1\\0\end{pmatrix}\\ 1 \begin{pmatrix}1\\0\end{pmatrix}\end{pmatrix} = \begin{pmatrix}0\\0\\1\\0\end{pmatrix}$$

3 qubits

$|000\rangle$

$\vdots$

$|111\rangle$

$|00\rangle = |0\rangle \otimes |0\rangle$

order is important $\underline{|01\rangle} = |0\rangle \otimes |1\rangle$

$|101\rangle = |1\rangle \otimes |0\rangle \otimes |1\rangle$ $|11\rangle = |1\rangle \otimes |1\rangle$

$\begin{pmatrix}0\\1\end{pmatrix} \otimes \begin{pmatrix}1\\0\end{pmatrix} \otimes \begin{pmatrix}0\\1\end{pmatrix}$

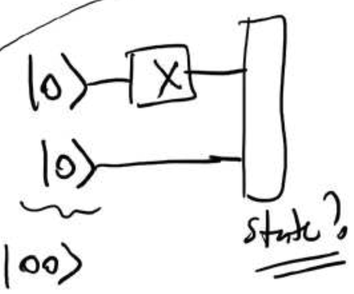
Ordering basis states

3 qubits

$|000\rangle \rightarrow$
001
010
$\vdots$
$|111\rangle$

$$\underset{|000\rangle}{\begin{pmatrix}1\\0\\ \vdots\\0\end{pmatrix}},\ \underset{|001\rangle}{\begin{pmatrix}0\\1\\0\\ \vdots\\0\end{pmatrix}},\ \underset{|010\rangle}{\begin{pmatrix}0\\0\\1\\0\\ \vdots\\0\end{pmatrix}},\ \underset{|011\rangle}{\cdots}\ \cdots$$

Tensoring 1



$$\left.\begin{array}{l}|0\rangle \xrightarrow{X} |1\rangle\\ |0\rangle \xrightarrow{I} |0\rangle\end{array}\right\} \quad |1\rangle \otimes |0\rangle = \begin{pmatrix}0\\0\\1\\0\end{pmatrix}$$

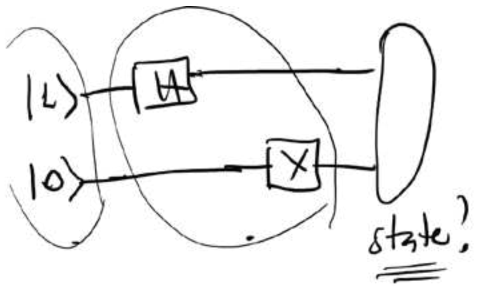
$$|v_1\rangle = M \cdot |v_0\rangle = \begin{pmatrix}0\\0\\1\\0\end{pmatrix}$$

$$\overset{M}{\begin{pmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{pmatrix}} \cdot \underset{|v_0\rangle}{\begin{pmatrix}1\\0\\0\\0\end{pmatrix}}$$

$$M = X \otimes I = \begin{pmatrix}0&1\\1&0\end{pmatrix} \otimes \begin{pmatrix}1&0\\0&1\end{pmatrix} = \begin{pmatrix}0\begin{pmatrix}1&0\\0&1\end{pmatrix} & 1\begin{pmatrix}1&0\\0&1\end{pmatrix}\\ 1\begin{pmatrix}1&0\\0&1\end{pmatrix} & 0\begin{pmatrix}1&0\\0&1\end{pmatrix}\end{pmatrix}$$

4×4

Tensoring 2



$$\begin{pmatrix}0\\ \frac{1}{\sqrt{2}}\\ 0\\ 0\end{pmatrix} + \begin{pmatrix}0\\ 0\\ 0\\ -\frac{1}{\sqrt{2}}\end{pmatrix} = \begin{pmatrix}0\\ \frac{1}{\sqrt{2}}\\ 0\\ -\frac{1}{\sqrt{2}}\end{pmatrix}$$

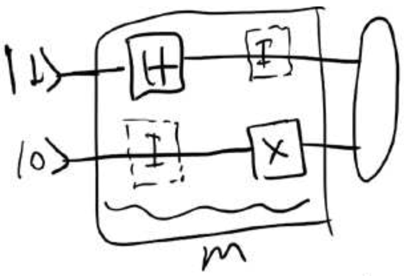
$H|1\rangle = |-\rangle$
$X|0\rangle = |1\rangle$ $\Rightarrow |-\rangle \otimes |1\rangle$

$$\Rightarrow \begin{pmatrix}\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}}\end{pmatrix} \otimes \begin{pmatrix}0\\ 1\end{pmatrix} = \begin{pmatrix}0\\ \frac{1}{\sqrt{2}}\\ 0\\ -\frac{1}{\sqrt{2}}\end{pmatrix}$$

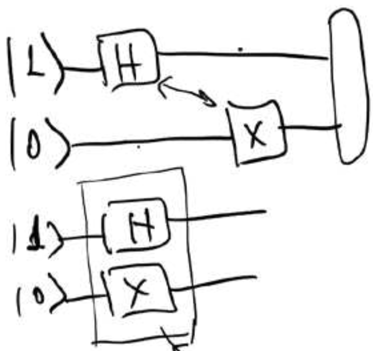
$$\left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right) \otimes |1\rangle$$

$$\Rightarrow = \frac{1}{\sqrt{2}} \underbrace{|0\rangle \otimes |1\rangle}_{\substack{|01\rangle \\ 2^{nd}\text{ entry}}} - \frac{1}{\sqrt{2}} \underbrace{|1\rangle \otimes |1\rangle}_{\substack{|11\rangle \\ 4^{th}\text{ entry}}} = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ 1\\ 0\\ 0\end{pmatrix} - \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ 0\\ 0\\ 1\end{pmatrix}$$

Tensoring 3



$$|10\rangle = \begin{pmatrix}0\\ 0\\ 1\\ 0\end{pmatrix} \qquad m|10\rangle$$



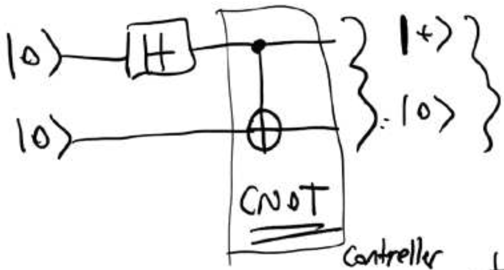
$$\underbrace{\underset{4\times 4}{(H \otimes I)} \cdot \underset{4\times 4}{(I \otimes X)}}_{M} = (HI \otimes I \cdot X) \Leftarrow \text{verify}$$

$$= H \otimes X$$

$$m = H \otimes X = \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{pmatrix} \otimes \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} = \begin{pmatrix}0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0\\ 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0\end{pmatrix}$$

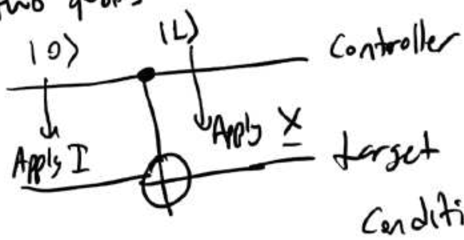
$$m|10\rangle = \begin{pmatrix}0\\ \frac{1}{\sqrt{2}}\\ 0\\ -\frac{1}{\sqrt{2}}\end{pmatrix}$$

CNOT



$$\left(\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|10\rangle\right)$$

Controlled -NOT operates on two qubits



Conditionals in programming

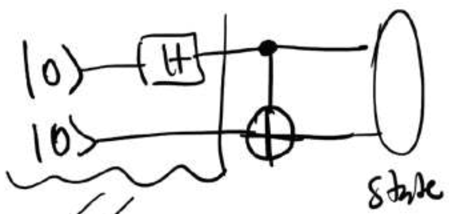
controller

target

$$\begin{aligned} CNOT|00\rangle &= |00\rangle \\ CNOT|01\rangle &= |01\rangle \\ CNOT|10\rangle &= |11\rangle \\ CNOT|11\rangle &= |10\rangle \end{aligned}$$

$$\begin{array}{cc} \begin{array}{cccc} |00\rangle & |01\rangle & |10\rangle & |11\rangle \end{array} & \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} & \begin{array}{c} |00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle \end{array} \end{array}$$

Bell states

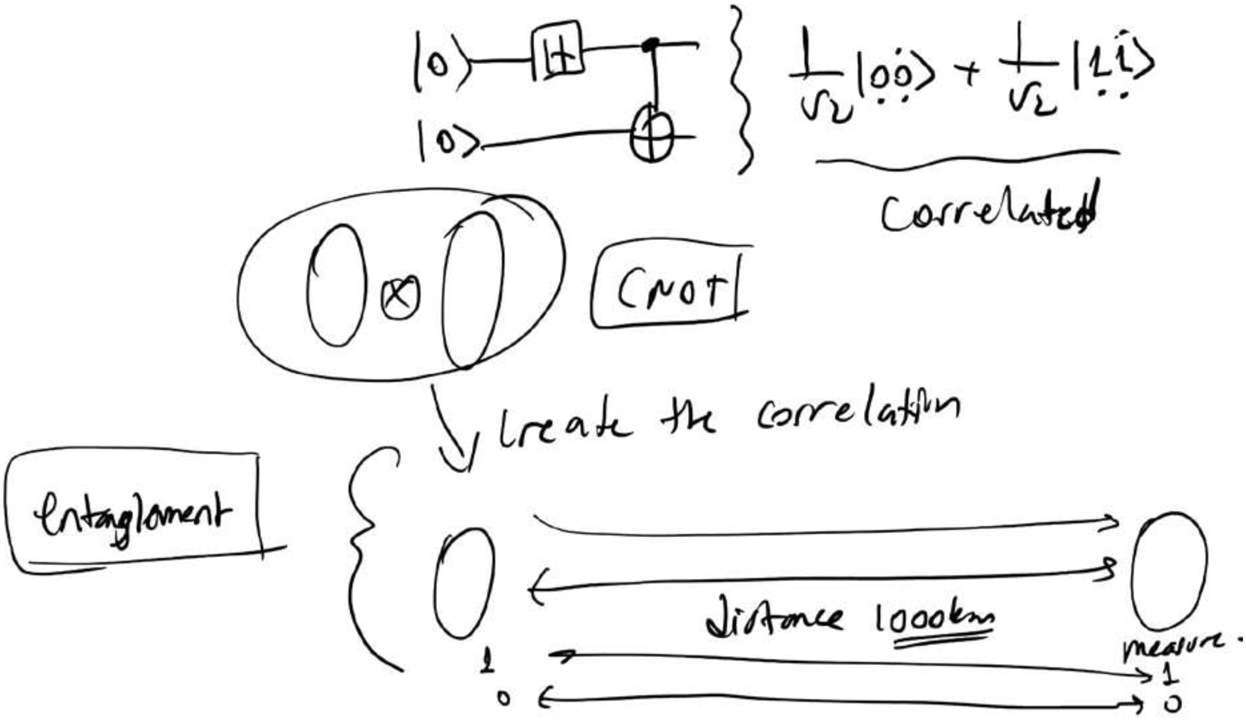


$$CNOT \cdot \left(\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|10\rangle\right) = \frac{1}{\sqrt{2}} CNOT|00\rangle + \frac{1}{\sqrt{2}} CNOT|10\rangle$$

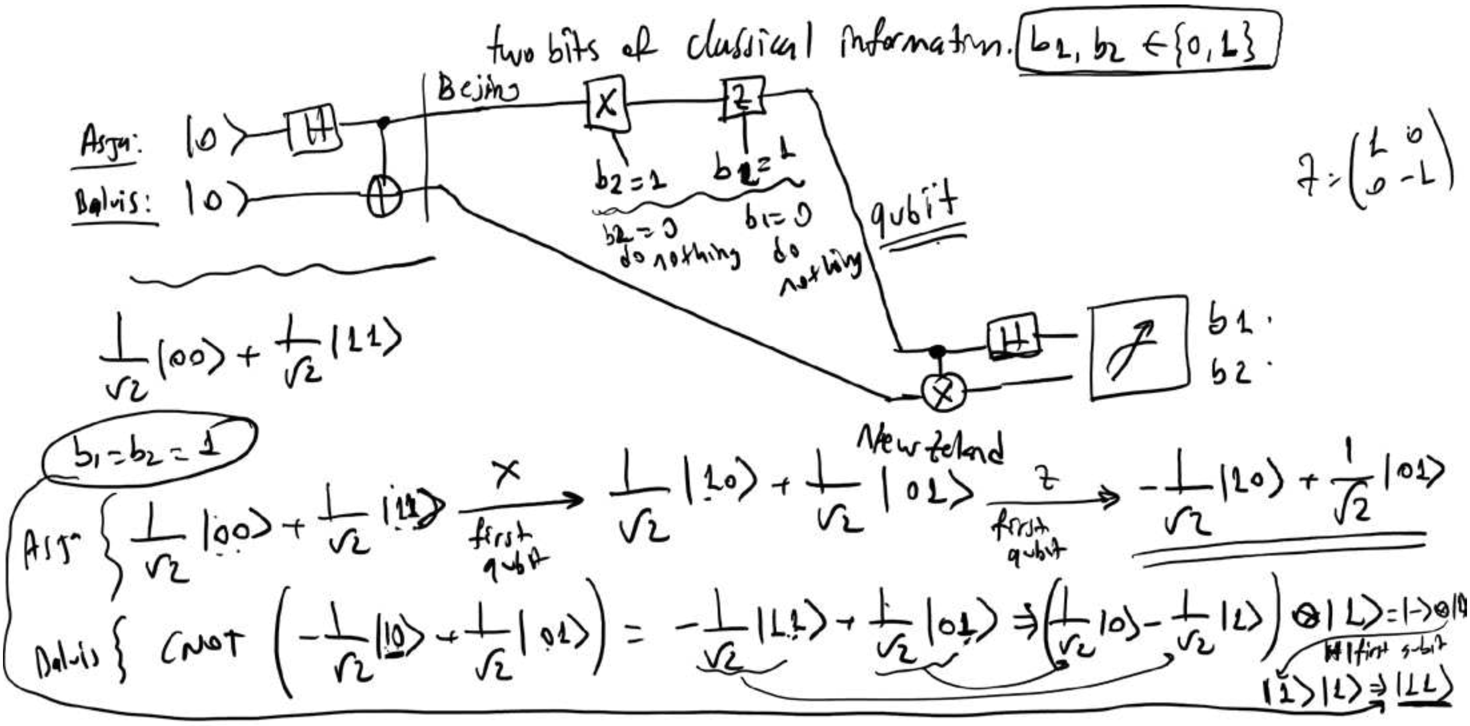
$$= \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

Entanglement



Superdense coding

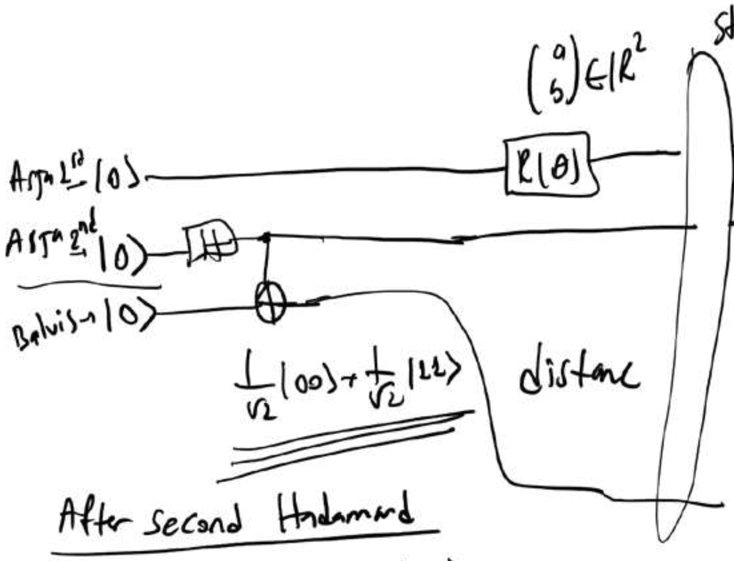


Exercise

$b_2 = b_1 = 1$ } previous screen

$b_1 = 0, b_2 = 0$
$b_1 = 1, b_2 = 0$
$b_1 = 0$ $b_2 = 1$ } do by yourself

Quantum teleportation 1



$(a|0\rangle + b|1\rangle) \otimes \left(\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle\right)$

H Asja's first qubit ⊗ Asja's 2nd qubit ⊗ Balvis' qubit

$\Rightarrow \frac{a}{\sqrt{2}}|000\rangle + \frac{a}{\sqrt{2}}|011\rangle + \frac{b}{\sqrt{2}}|100\rangle + \frac{b}{\sqrt{2}}|111\rangle$

After second CNOT:

$\frac{a}{\sqrt{2}}|000\rangle + \frac{a}{\sqrt{2}}|011\rangle + \frac{b}{\sqrt{2}}|110\rangle + \frac{b}{\sqrt{2}}|101\rangle$

After second Hadamard

$\underbrace{|000\rangle}_{H} \Rightarrow |0\rangle \otimes |00\rangle \Rightarrow H|0\rangle \otimes |00\rangle = \left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) \otimes |00\rangle$
$= \frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|100\rangle$

Quantum teleportation 2

$$\frac{a}{\sqrt{2}}|000\rangle + \frac{a}{\sqrt{2}}|011\rangle + \frac{b}{\sqrt{2}}|110\rangle + \frac{b}{\sqrt{2}}|101\rangle$$

Apply H to the first qubit:

$$\frac{a}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|000\rangle + \frac{1}{\sqrt{2}}|100\rangle\right) + \frac{a}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|011\rangle + \frac{1}{\sqrt{2}}|111\rangle\right) + \frac{b}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|010\rangle - \frac{1}{\sqrt{2}}|110\rangle\right) + \frac{b}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|001\rangle - \frac{1}{\sqrt{2}}|101\rangle\right)$$

$$\frac{a}{2}|000\rangle + \frac{a}{2}|100\rangle + \frac{a}{2}|011\rangle + \frac{a}{2}|111\rangle + \frac{b}{2}|010\rangle - \frac{b}{2}|110\rangle + \frac{b}{2}|001\rangle - \frac{b}{2}|101\rangle$$

Re-write

$$|00\rangle \otimes \left(\frac{a}{2}|0\rangle + \frac{b}{2}|1\rangle\right) + |01\rangle \otimes \left(\frac{a}{2}|1\rangle + \frac{b}{2}|0\rangle\right) + |10\rangle \otimes \left(\frac{a}{2}|0\rangle - \frac{b}{2}|1\rangle\right) + |11\rangle \otimes \left(\frac{a}{2}|1\rangle - \frac{b}{2}|0\rangle\right)$$

Quantum teleportation 3

$$|00\rangle \otimes \left(\frac{a}{2}|0\rangle + \frac{b}{2}|1\rangle\right) + |01\rangle \otimes \left(\frac{a}{2}|1\rangle + \frac{b}{2}|0\rangle\right) + |10\rangle \otimes \left(\frac{a}{2}|0\rangle - \frac{b}{2}|1\rangle\right) + |11\rangle \otimes \left(\frac{a}{2}|1\rangle - \frac{b}{2}|0\rangle\right)$$

If Asja measures her qubits. | m | first qubit, second qubit

normalized

$pr = \frac{1}{4}$, "00", $\begin{pmatrix} a \\ b \end{pmatrix}$, $pr = \frac{1}{4}$, "01", $\begin{pmatrix} b \\ a \end{pmatrix}$, $pr = \frac{1}{4}$, "10", $\begin{pmatrix} a \\ -b \end{pmatrix}$, $pr = \frac{1}{4}$, "11", $\begin{pmatrix} -b \\ a \end{pmatrix}$

Asja sends her measurement outcomes

how can Balvis set his qubit to $\begin{pmatrix} a \\ b \end{pmatrix}$

Balvis

$00 \to I \to \begin{pmatrix} a \\ b \end{pmatrix}$

$01 \to X \to X\begin{pmatrix} b \\ a \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$

$10 \to \begin{pmatrix} a \\ -b \end{pmatrix} \xrightarrow{Z} \begin{pmatrix} a \\ b \end{pmatrix}$

$11 \to \begin{pmatrix} -b \\ a \end{pmatrix} \xrightarrow{X} \begin{pmatrix} a \\ -b \end{pmatrix} \xrightarrow{Z} \begin{pmatrix} a \\ b \end{pmatrix}$

"post processing"