QWORLD

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Basics of quantum system

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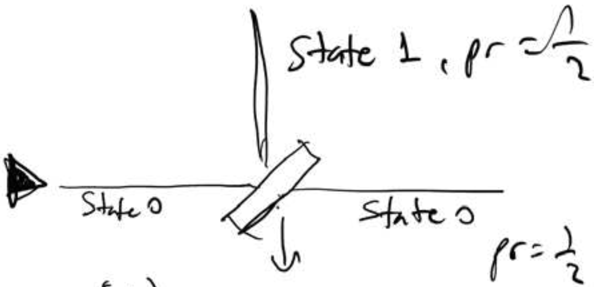
Hadamard operator



$|v\rangle \rightarrow$ ket

$\langle 1| \leftarrow$ bra

$(v^*)^T$

bra-ket notation

$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

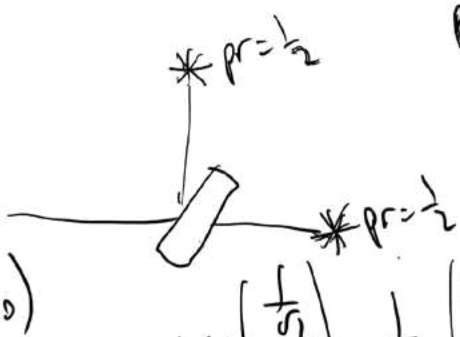
many different operators

↳ Hadamard

$H = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$

$? = H|0\rangle$

Amplitudes and probabilities



probabilities

$\frac{1}{2}$

$\frac{1}{2}$

values

amplitudes

$H \cdot |0\rangle$

$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}}_{H} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

amplitudes

$\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\begin{pmatrix} a \\ b \end{pmatrix} = a|0\rangle + b|1\rangle$

pr $= |a|^2$ pr $= |b|^2$

$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$

$\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}}_{\text{fair coin-flipping}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Observation

$$\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix}$$

$$|+\rangle = H\ |0\rangle$$

we observe

pr. of observing $|0\rangle$ is $\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$, then state is $\begin{pmatrix}1\\ 0\end{pmatrix}$ $|0\rangle$

pr. of observing $|1\rangle$ is $\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$, then state is $\begin{pmatrix}0\\ 1\end{pmatrix}$ $|1\rangle$

The probability summation

My quantum state is $\begin{pmatrix}a\\ b\end{pmatrix} \in \mathbb{R}^2$ $\quad a,b \in \mathbb{R}$

Can $\begin{pmatrix}2\\ -2\end{pmatrix}$ be a quantum state? No.

Because,

$$pr(|0\rangle) = (2)^2 = 4$$
$$pr(|1\rangle) = (-2)^2 = 4$$
$$\overline{8}$$

the probability summation must be 1.

Euclidian length

$|u\rangle = \begin{pmatrix} a \\ b \end{pmatrix} \in \mathbb{R}$, $a, b \in \mathbb{R}$ and $|a|^2 + |b|^2 = 1$

$a^2 + b^2 = 1$

$|u\rangle = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{pmatrix} \in \mathbb{R}^n$, $a_1^2 + a_2^2 + \dots + a_n^2 = 1$

geometrical meaning of this formula?

A unit vector is a vector with length 1 (Euclidean length).

norm

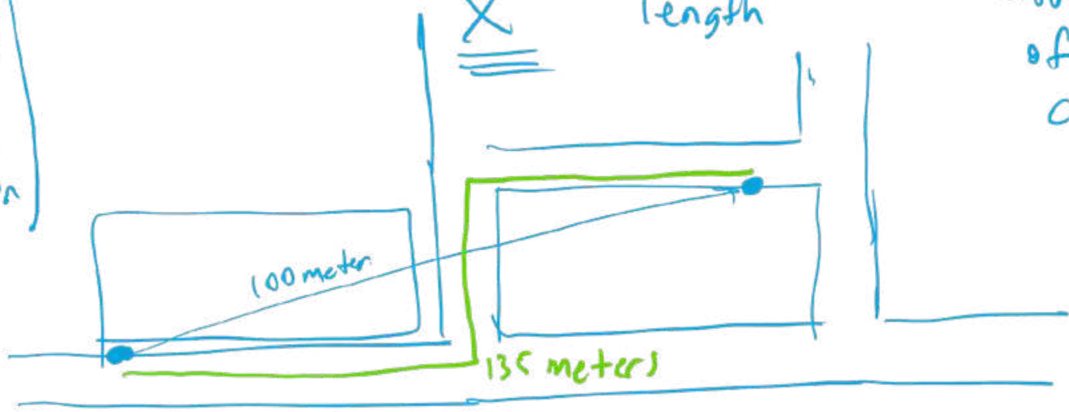
unit, global sphere → length (Euclidean length)

L1 and L2 norms

Euclidean length / norm → how many different types of norms can be defined.

X length

$U = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$



$|v|_1 = |a_1| + |a_2| + \dots + |a_n|$

$|v|_2 = \sqrt{|a_1|^2 + |a_2|^2 + \dots + |a_n|^2}$

l_1-norm l_2-norm ... $l_3, l_4, \dots, l_\infty$

$|v|_m = \left(|a_1|^m + |a_2|^m + \dots + |a_n|^m\right)^{\frac{1}{m}}$

Linear systems

probabilistic system: ℓ_1-norm is preserved

quantum system: ℓ_2-norm is preserved

⋮

No other linear systems.

Quantum operators

$\boxed{\text{A quantum state is a unit vector.}}$

A quantum operator?

A quantum (evolution) operator preserves the length
ℓ_2-norm.

$$\left(\frac{a+b}{\sqrt{2}}\right)^2 + \left(\frac{a-b}{\sqrt{2}}\right)^2$$

$$= \frac{a^2+\cancel{2ab}+b^2}{2} + \frac{a^2-\cancel{2ab}+b^2}{2}$$

$$= \frac{2a^2+2b^2}{2} = a^2+b^2 = 1$$

$$\underbrace{\begin{pmatrix} \frac{a+b}{\sqrt{2}} \\ \frac{a-b}{\sqrt{2}} \end{pmatrix}}_{|v'\rangle} = \underbrace{\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}}_{H} \underbrace{\begin{pmatrix} a \\ b \end{pmatrix}}_{|v\rangle} \in \mathbb{R}^2$$

$a^2+b^2=1$

Measurement

$$\begin{array}{ll} \underline{\text{state}} & \underline{\text{Pr}} \\ |0\rangle, & \frac{1}{2} = \left(\frac{1}{\sqrt{2}}\right)^2 \\ |1\rangle, & \frac{1}{2} = \left(\frac{-1}{\sqrt{2}}\right)^2 \end{array}$$

$$\xLeftarrow{\text{observing}} \; |-\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{(1\rangle)}$$

$$|+\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

<u>measurement</u>

Double Hadamard

$$|+\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \underbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}_{|0\rangle}$$

3rd experiment.

$$? = H|+\rangle$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} + \frac{1}{2} \\ \cancel{\frac{1}{2}} - \cancel{\frac{1}{2}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \\ \underbrace{\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + \left(\frac{-1}{\sqrt{2}}\right) \cdot \frac{1}{\sqrt{2}}}_{\text{there are transitions.}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

Interference

$H|0\rangle = |+\rangle$

$H|1\rangle = |-\rangle$

(1) $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle = H|0\rangle$

(2) $H\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)$

$\frac{1}{\sqrt{2}}H|0\rangle + \frac{1}{\sqrt{2}}H|1\rangle$

because of linearity

$\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right) + \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right)$

After second Hadamard we have four transitions.

Constructive interference $\frac{1}{2}|0\rangle + \cancel{\frac{1}{2}|1\rangle}$ $\qquad \frac{1}{2}|0\rangle - \cancel{\frac{1}{2}|1\rangle}$ destructive interference

$1\cdot|0\rangle + 0\cdot|1\rangle$

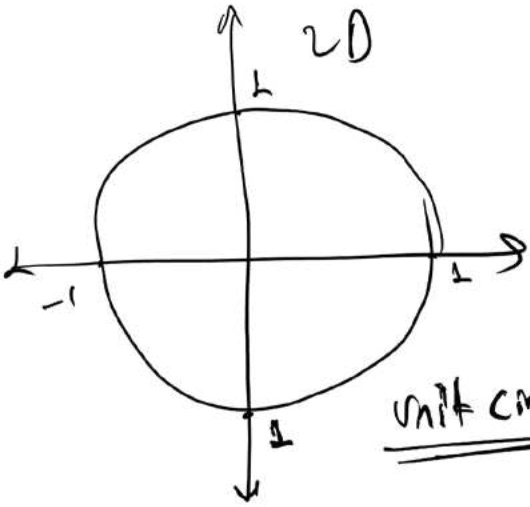
Superposition

$|0\rangle$ or $|1\rangle \xleftarrow{\text{measurement}} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$

superposition of basis states $|0\rangle$ and $|1\rangle$

$\cancel{\begin{pmatrix} .1 \\ .1 \end{pmatrix}}$

Unit circle



Unit circle

$\begin{pmatrix} a \\ b \end{pmatrix} \in \mathbb{R}^2$

$a^2 + b^2 = 1$

Which geometrical object this equation defines?

Summary

→ quantum state, unit vector

→ quantum (evolution) operator, preserving ℓ_2-norm

→ measurement → one of basis states is observed, → and then the state becomes this observed state.

→ Superposition + interference: ↙ destructive, ↘ constructive.

Reversibility

Reversibility

Do you think the quantum evolution operators are reversible?

$\boxed{B = A^{-1}}$

$H^{-1} = ?$

$|y\rangle = A\,|x\rangle$

unit vector

unit vector

$|x\rangle = B\,|y\rangle$