An Intuition for Propagators

George Wilson

CSIRO's Data61

george.wilson@data61.csiro.au

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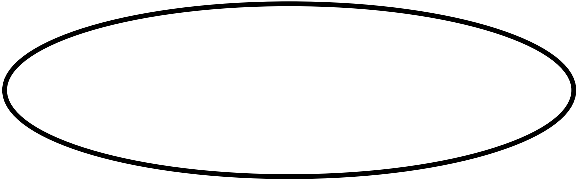
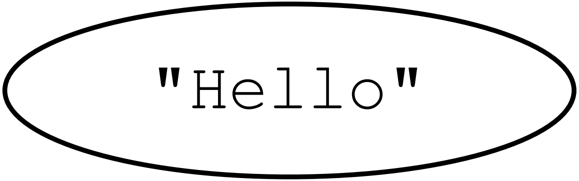
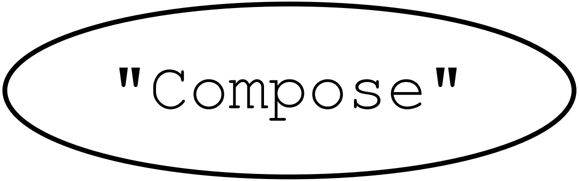
1970s, MIT

a model of computation for **highly parallel** machines

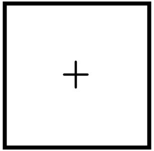


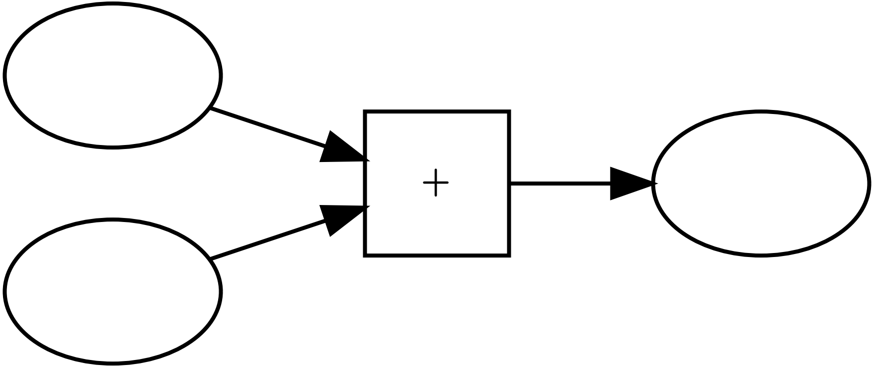


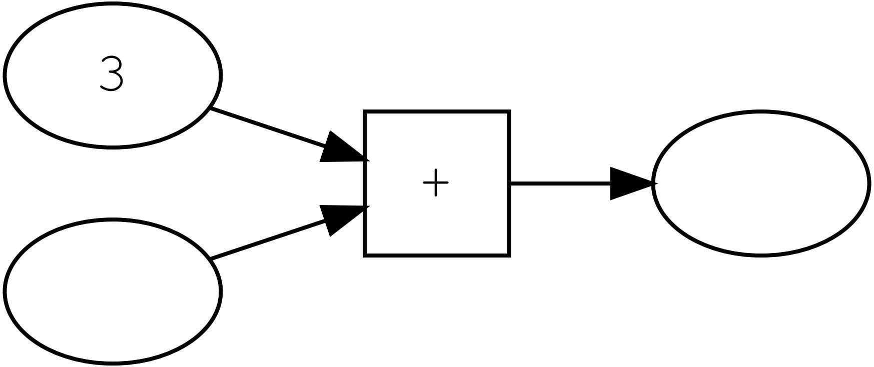
"Hello"

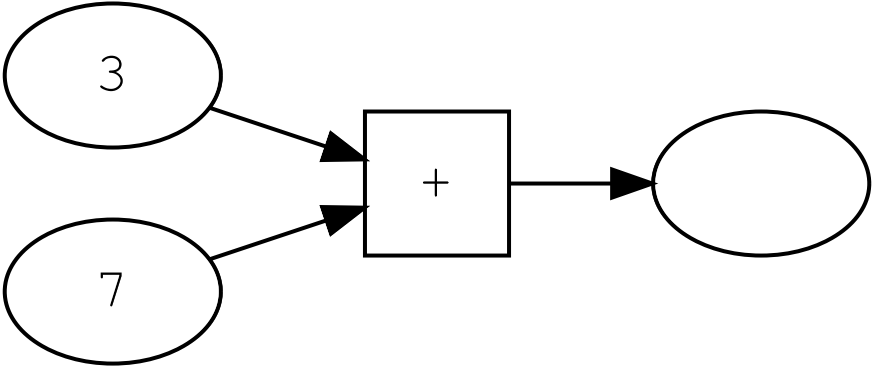


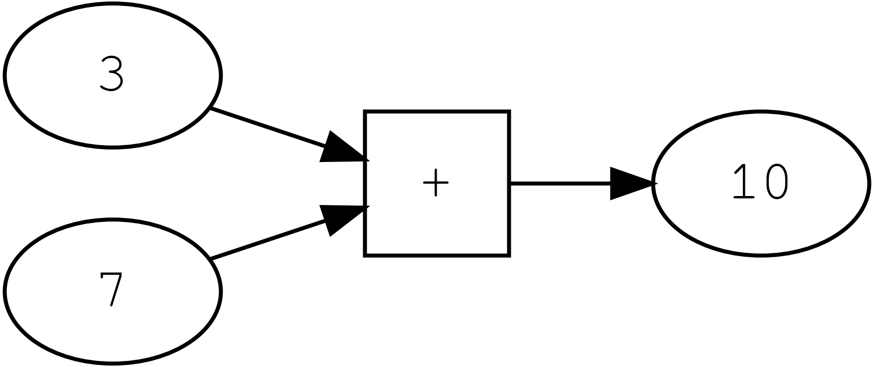
"Compose"

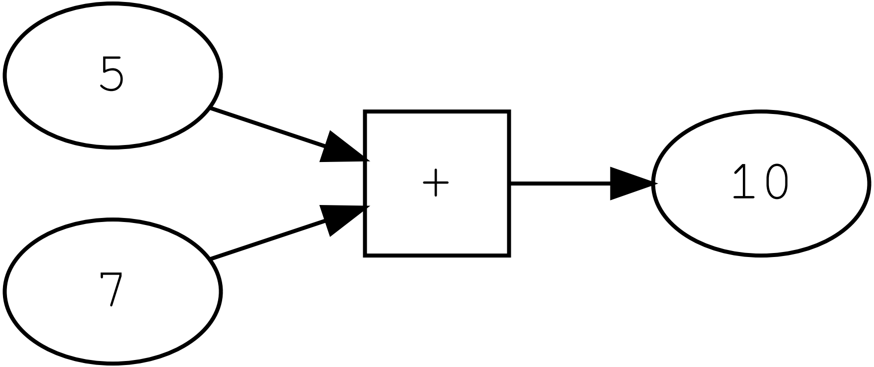


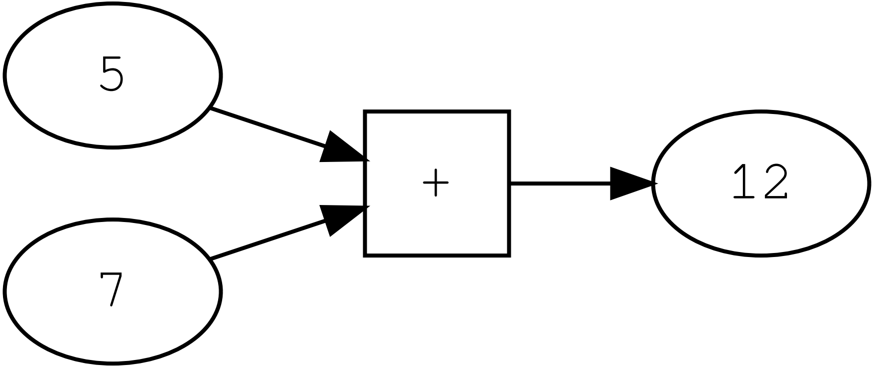



3
+


3
7
+


3
7
+
10


5
7
+
10

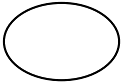

5
7
+
12

```haskell
-- types
data Par a
instance Monad Par

data Cell a
```

```haskell
-- types
data Par a
instance Monad Par

data Cell a


-- Creating a cell
cell    :: Par (Cell a)
```

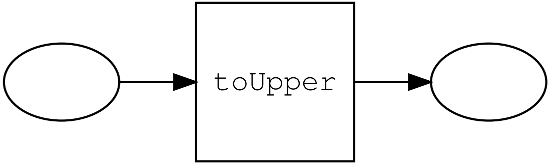
```haskell
-- types
data Par a
instance Monad Par

data Cell a


-- Creating a cell
cell    :: Par (Cell a)


-- Working with Cells
content :: Cell a -> Par (Maybe a)
write   :: Cell a -> a -> Par ()
```

```haskell
-- types
data Par a
instance Monad Par

data Cell a


-- Creating a cell
cell    :: Par (Cell a)


-- Working with Cells
content :: Cell a -> Par (Maybe a)
write   :: Cell a -> a -> Par ()


-- Creating a propagator
watch   :: Cell a -> (a -> Par ()) -> Par ()
```

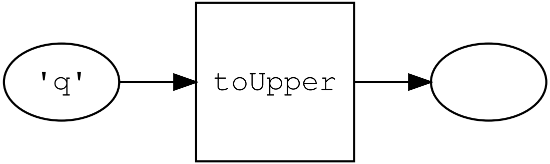


```
do
  input  <- cell
```

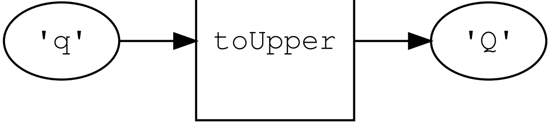
```
do
  input  <- cell
  output <- cell
```



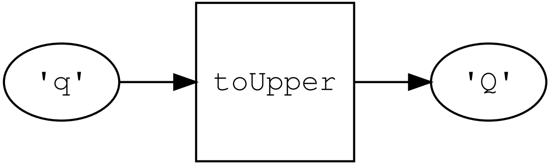
```
do
  input  <- cell
  output <- cell
  watch input (\c ->
    write output (toUpper c))
```



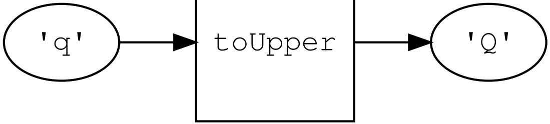
```
do
  input  <- cell
  output <- cell
  watch input (\c ->
    write output (toUpper c))
  -- run the network
  write input 'q'
  content output   -- Just 'Q'
```



```
do
  input  <- cell
  output <- cell
  watch input (\c ->
    write output (toUpper c))
  -- run the network
  write input 'q'
  content output   -- Just 'Q'
```

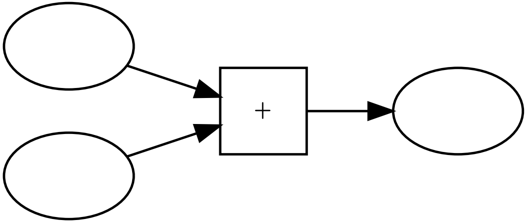


```
do
  input  <- cell
  output <- cell
  watch input (\c ->
    write output (toUpper c))
  -- run the network
  write input 'q'
  content output   -- Just 'Q'
```

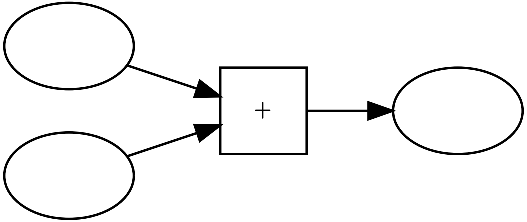


```
do
  input  <- cell
  output <- cell
  lift toUpper input output

  -- run the network
  write input 'q'
  content output   -- Just 'Q'
```

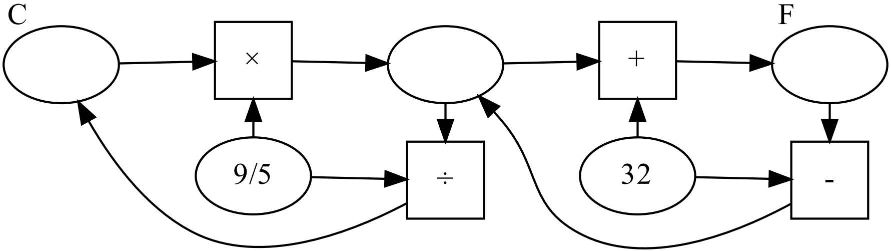
```
do
  inL  <- cell
  inR  <- cell
  out  <- cell
```

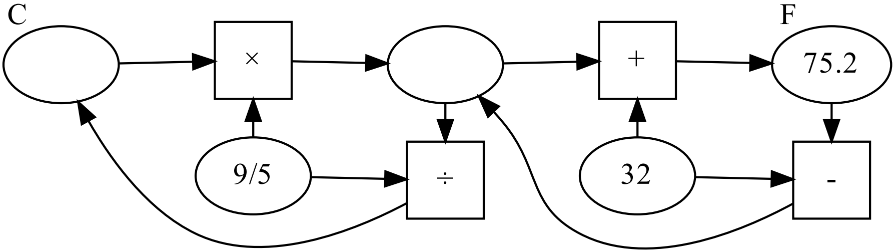


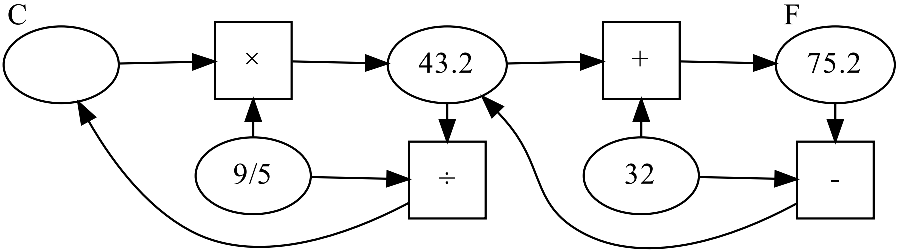
```
do
  inL  <- cell
  inR  <- cell
  out  <- cell

  adder inL inR out

    where
      adder l r o = do
        lift2 (+) l r o
```

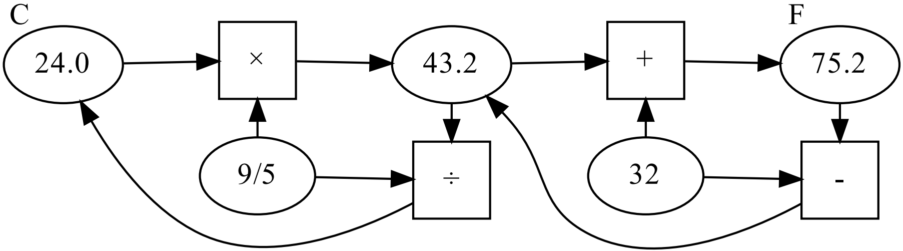


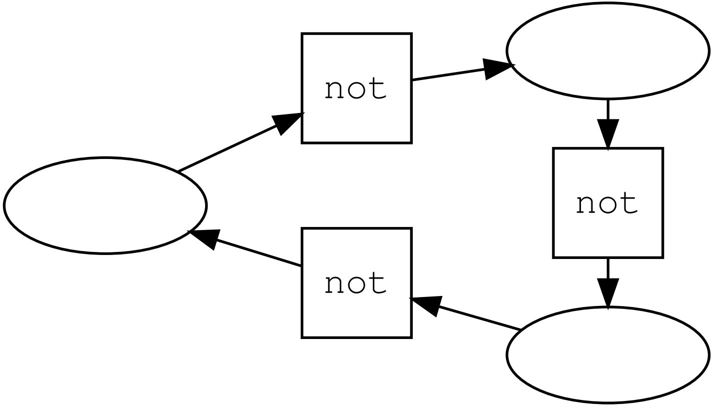
```
do
  inL  <- cell
  inR  <- cell
  out  <- cell

  adder inL inR out

    where
      adder l r o = do
        lift2 (+) l r o
        lift2 (-) o l r
        lift2 (-) o r l
```

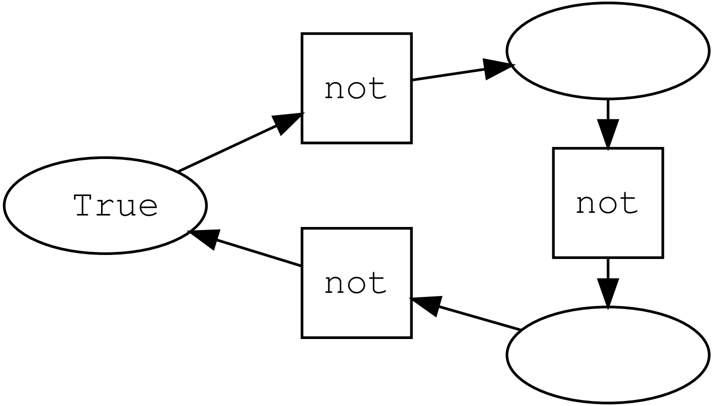

C
×
+
F
9/5
÷
32
-

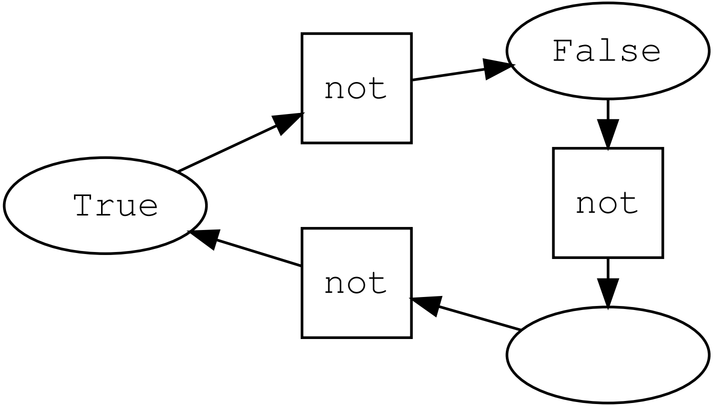

C
F
×
+
75.2
9/5
÷
32
-

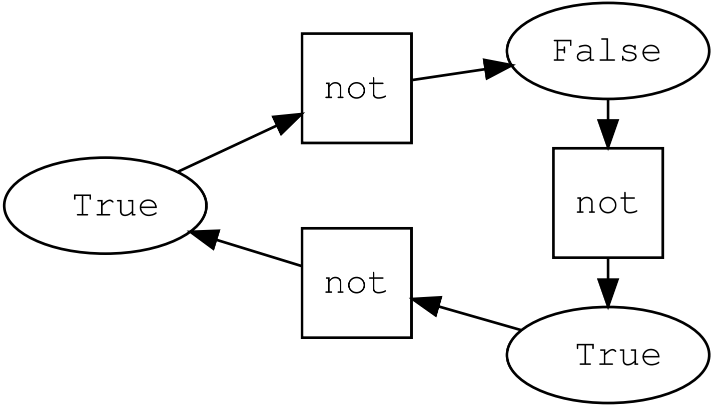

C
×
43.2
+
F
75.2
9/5
÷
32
-

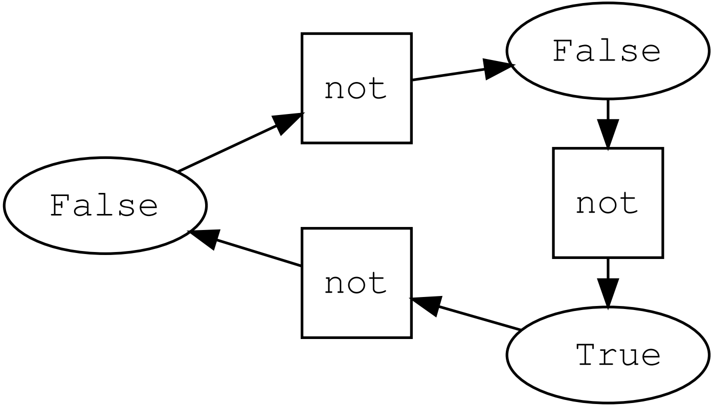

C
24.0
×
43.2
+
F
75.2
9/5
÷
32
-

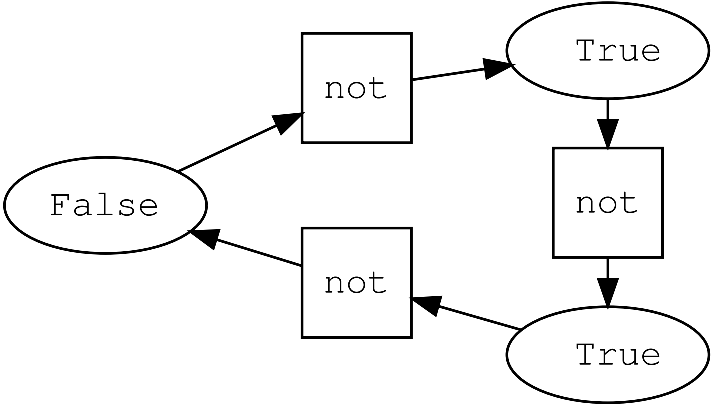

not
not
not

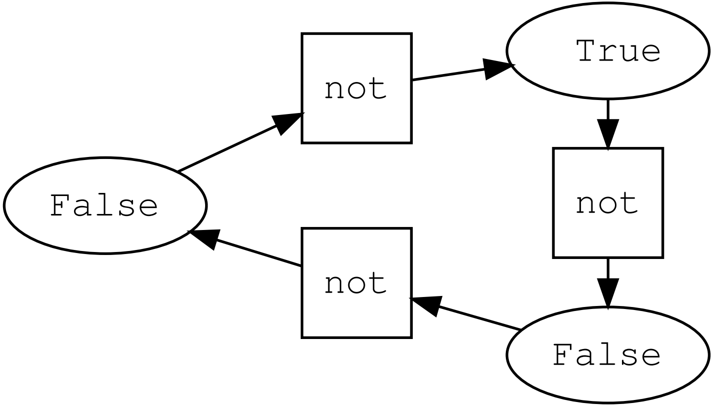

True
not
not
not

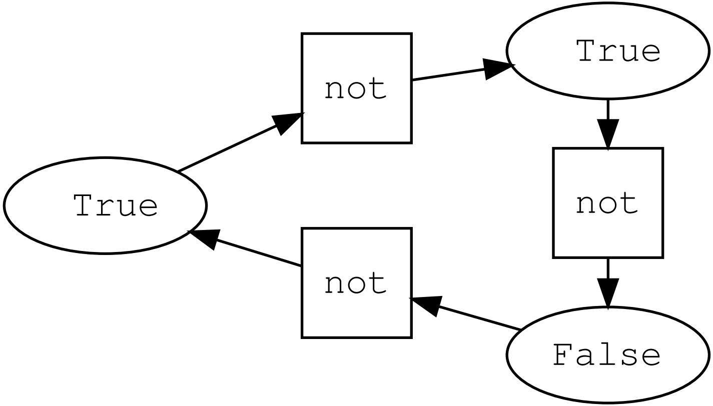

not
False
True
not
not

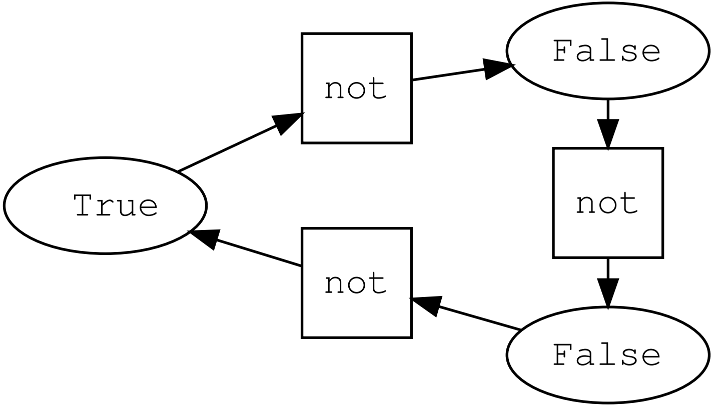

False
not
not
True
not
True


False
not
False
not
True
not


not
True
False
not
not
True


False
not
True
not
False
not


not
True
True
not
not
False



False
not
True
not
not
False

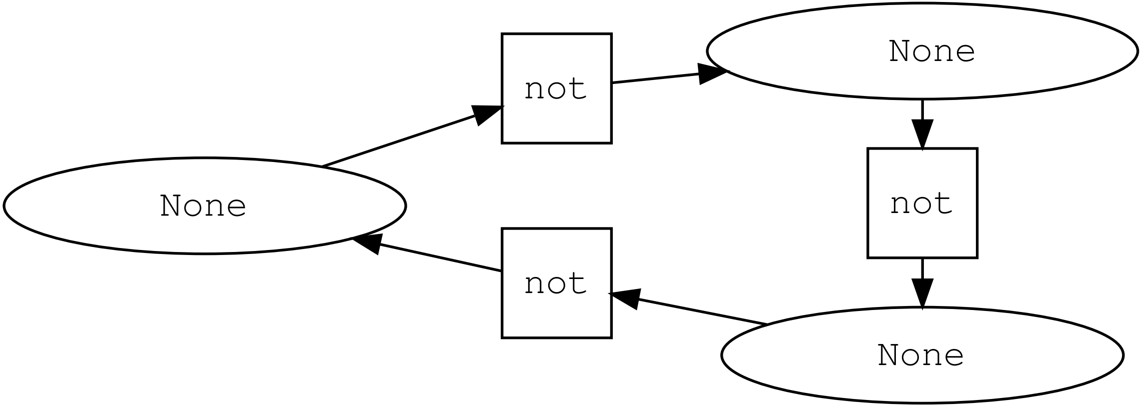
?!

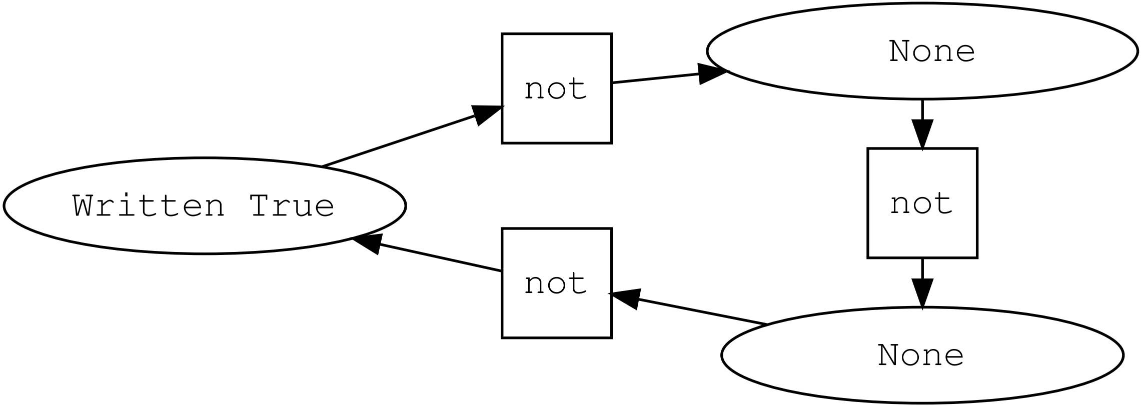
How can we fix this?

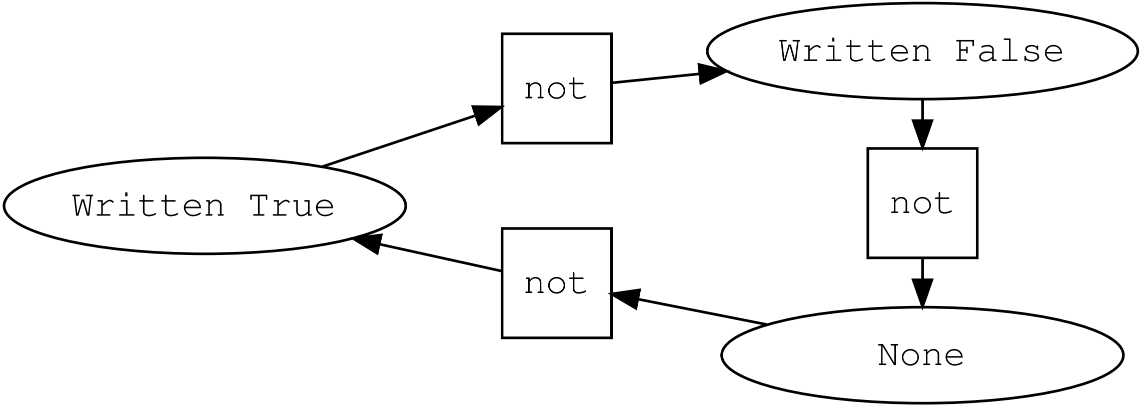
```haskell
data WriteOnce a
  = None
  | Written a
  | TooMany
```

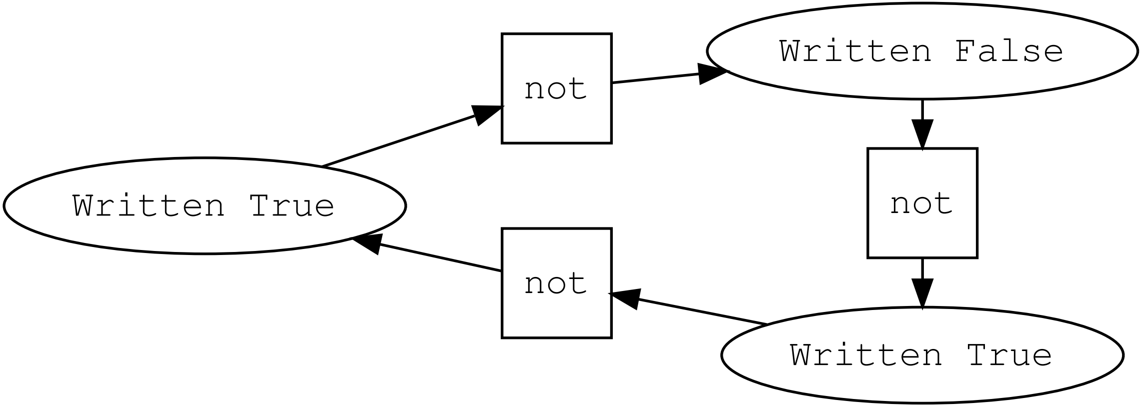
```haskell
data WriteOnce a
  = None
  | Written a
  | TooMany

tryWrite ::           a -> WriteOnce a -> WriteOnce a
tryWrite a w = case w of
  None       -> Written a
  Written b  -> TooMany
  TooMany    -> TooMany
```

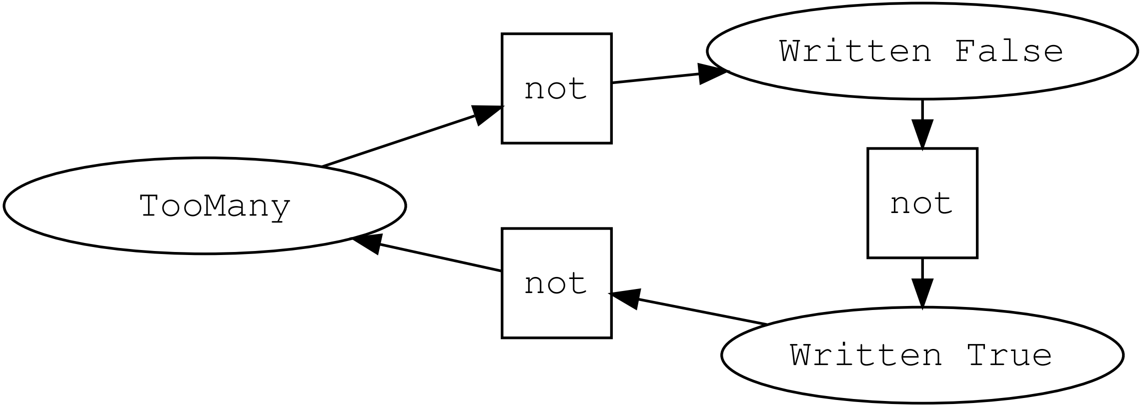
```haskell
data WriteOnce a
  = None
  | Written a
  | TooMany

tryWrite :: (Eq a) => a -> WriteOnce a -> WriteOnce a
tryWrite a w = case w of
  None       -> Written a
  Written b -> if a == b then Written b else TooMany
  TooMany    -> TooMany
```

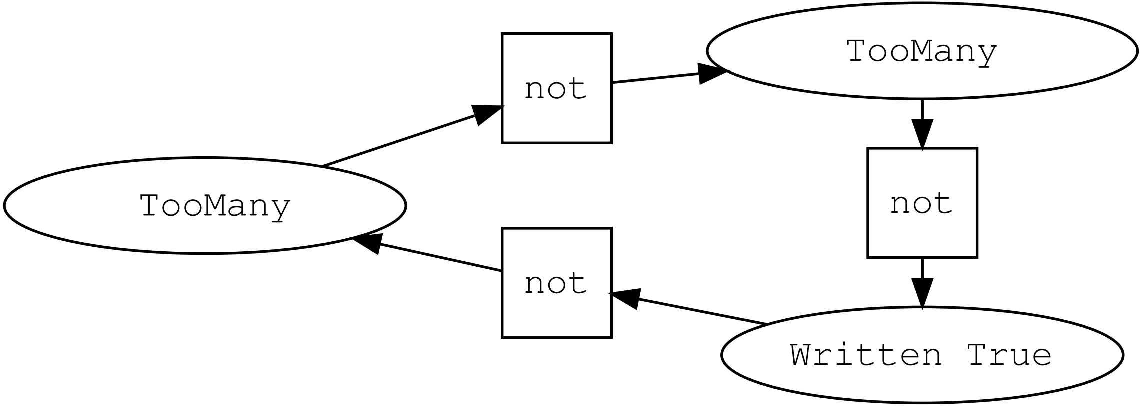

None
not
None
not
not
None

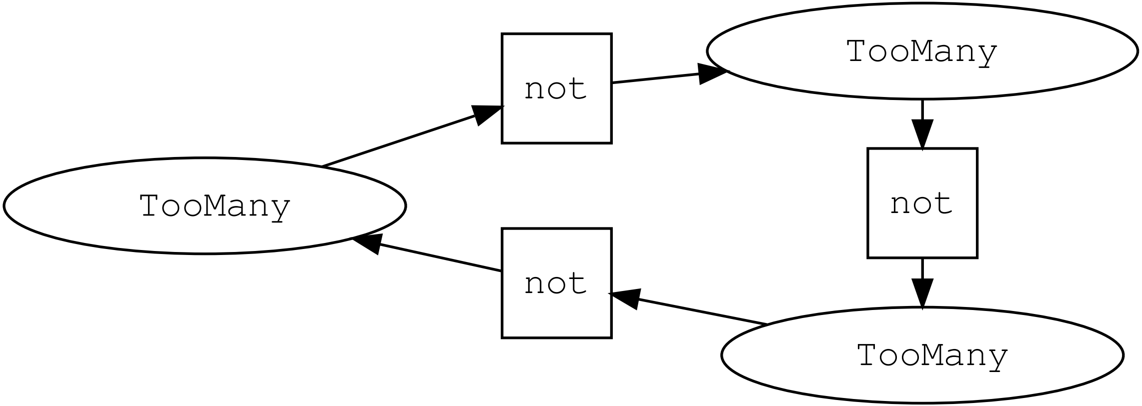

Written True
not
None
not
None
not


not
Written False
Written True
not
not
None


Written True
not
Written False
not
Written True
not


Written False
not
TooMany
not
not
Written True


TooMany
not
TooMany
not
TooMany
not
Written True


not
TooMany
TooMany
not
not
TooMany

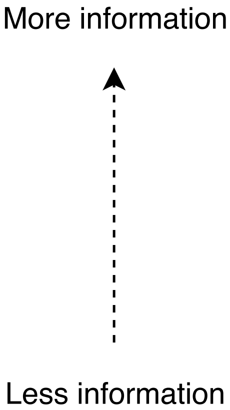
Mutability is **chaos**

WriteOnce is **rigid**

Accumulate information about a value

Accumulate information about a value

monotonically



More information

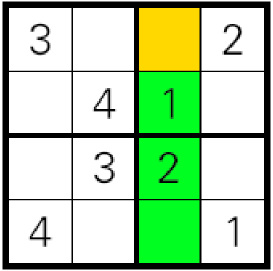
Less information

```
-- I have heard contradictory answers!
  TooMany

-- I know the answer exactly
  Written a

-- I don't know anything
  None
```

3			2
	4	1	
	3	2	
4			1

3		🟨	2
	4	1	
	3	2	
4			1

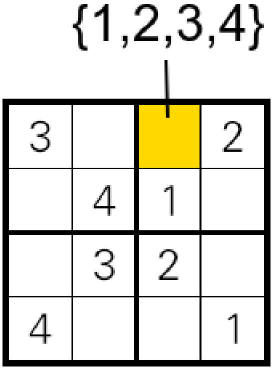
3			2
	4	1	
	3	2	
4			1



3			2
	4	1	
	3	2	
4			1

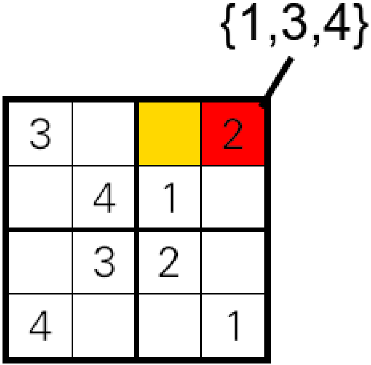
3			2
	4	1	
	3	2	
4			1

3		🟨	2
	4	1	
	3	2	
4			1



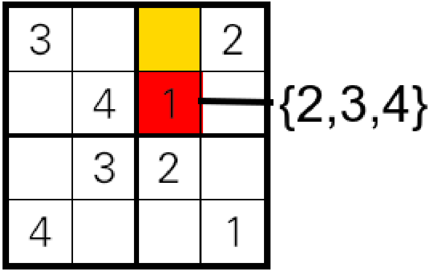
{1,2,3,4}

3			2
	4	1	
	3	2	
4			1



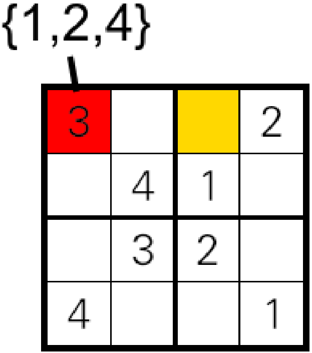
{1,3,4}

3			2
	4	1	
	3	2	
4			1



3			2
	4	1	
	3	2	
4			1

{2,3,4}



{1,2,4}

🟥 3		🟨	2
	4	1	
	3	2	
4			1

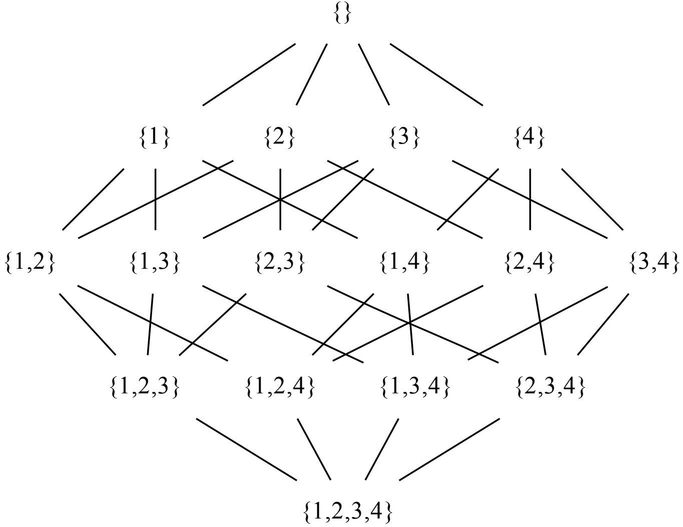
$\{2,3,4\} \cap \{1,3,4\} \cap$
$\{1,2,4\} \cap \{1,2,3,4\}$

3		🟨	2
	4	1	
	3	2	
4			1

{4}

3		🟨	2
	4	1	
	3	2	
4			1

3		4	2
	4	1	
	3	2	
4			1


{}
{1}
{2}
{3}
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{1,2}
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{2,3}
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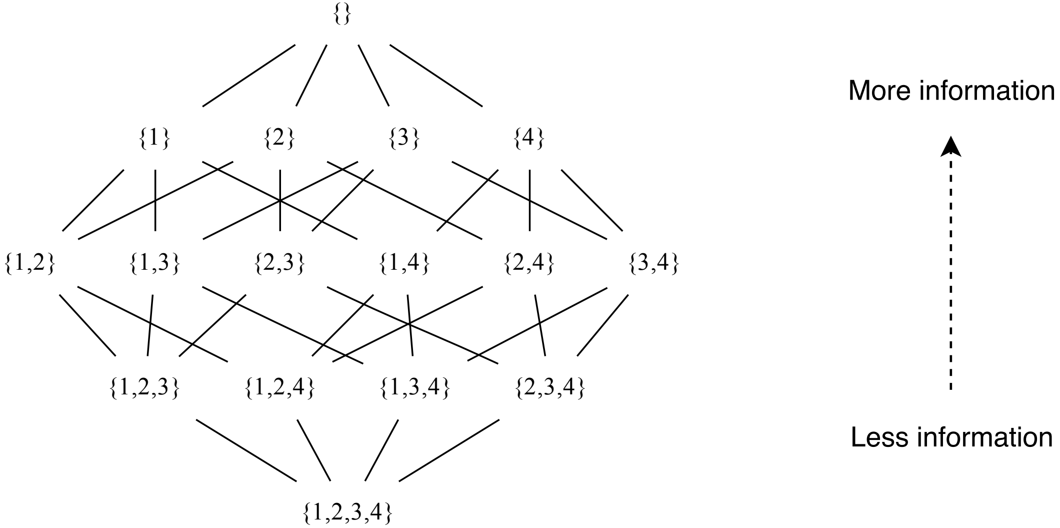
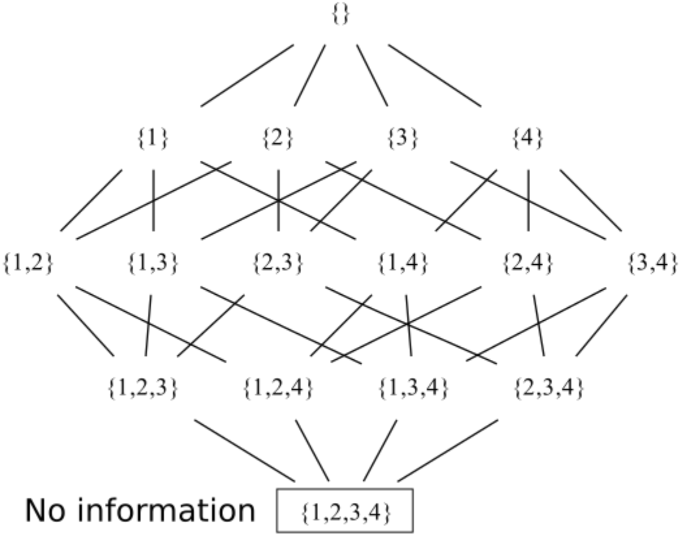
{2,4}
{3,4}
{1,2,3}
{1,2,4}
{1,3,4}
{2,3,4}
{1,2,3,4}

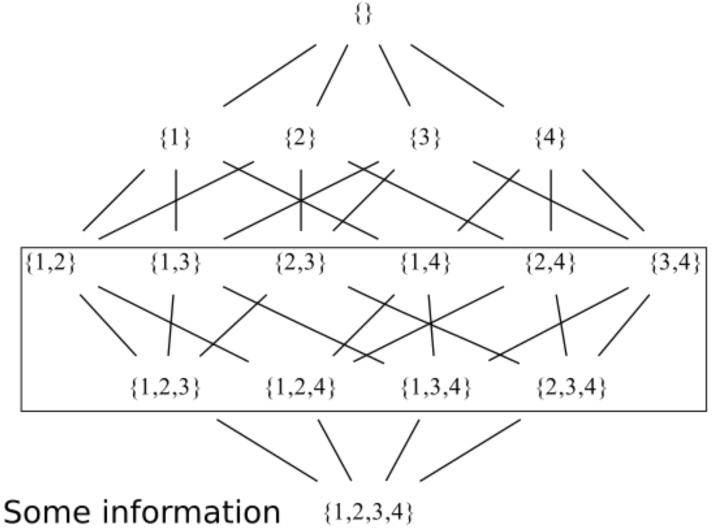

{}
{1}
{2}
{3}
{4}
{1,2}
{1,3}
{2,3}
{1,4}
{2,4}
{3,4}
{1,2,3}
{1,2,4}
{1,3,4}
{2,3,4}
{1,2,3,4}
More information
Less information

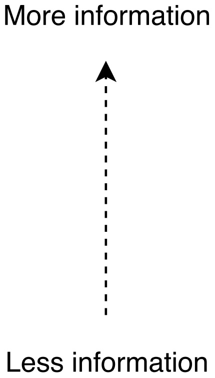


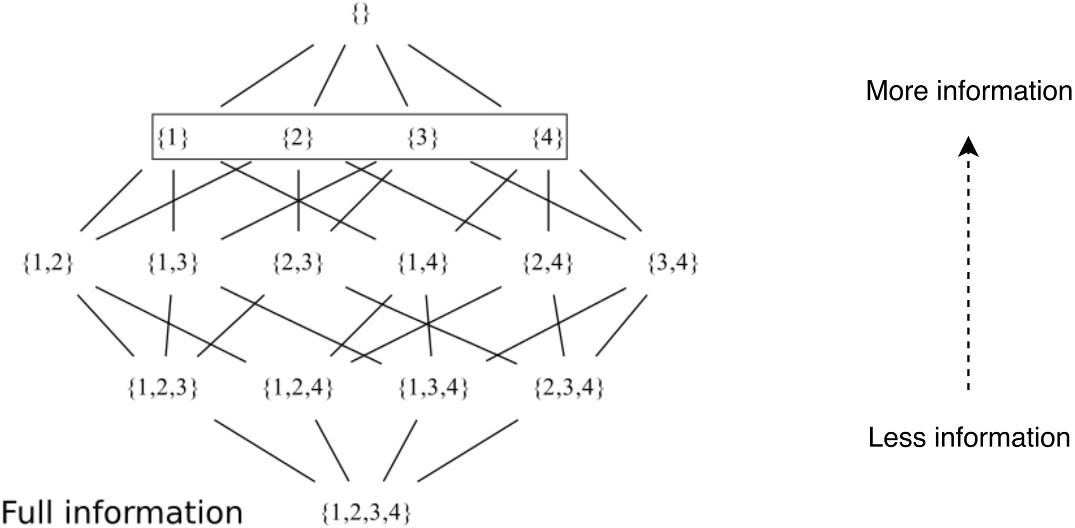
No information

More information

Less information


{}
{1}
{2}
{3}
{4}
{1,2}
{1,3}
{2,3}
{1,4}
{2,4}
{3,4}
{1,2,3}
{1,2,4}
{1,3,4}
{2,3,4}
Some information
{1,2,3,4}

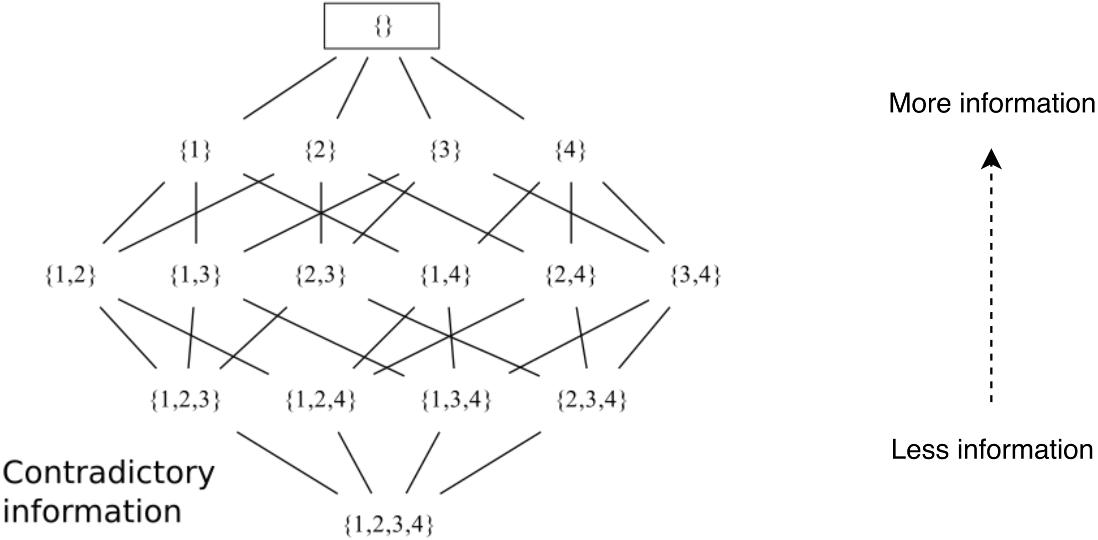

More information
Less information



More information

Less information

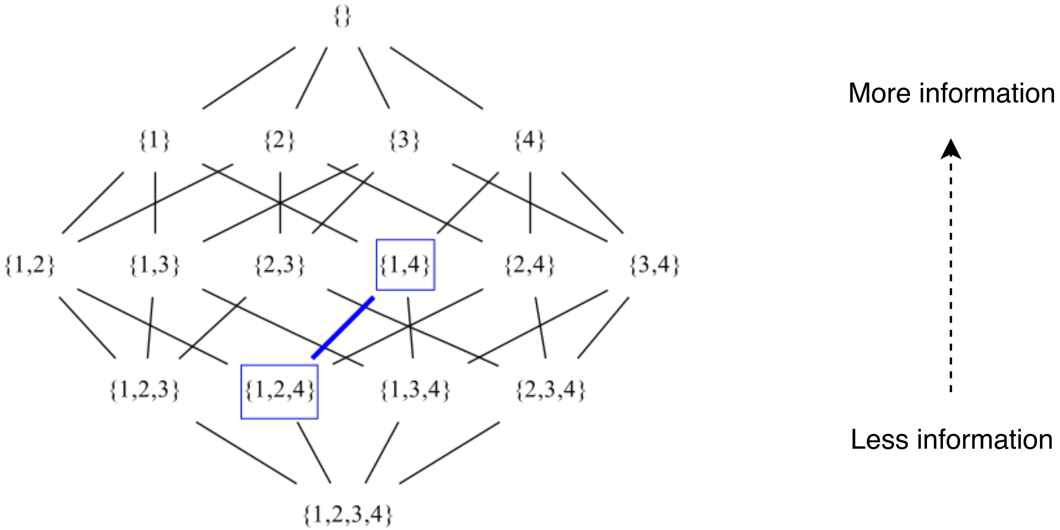
Full information



More information

Less information

Contradictory information



{}
{1}
{2}
{3}
{4}
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{1,4}
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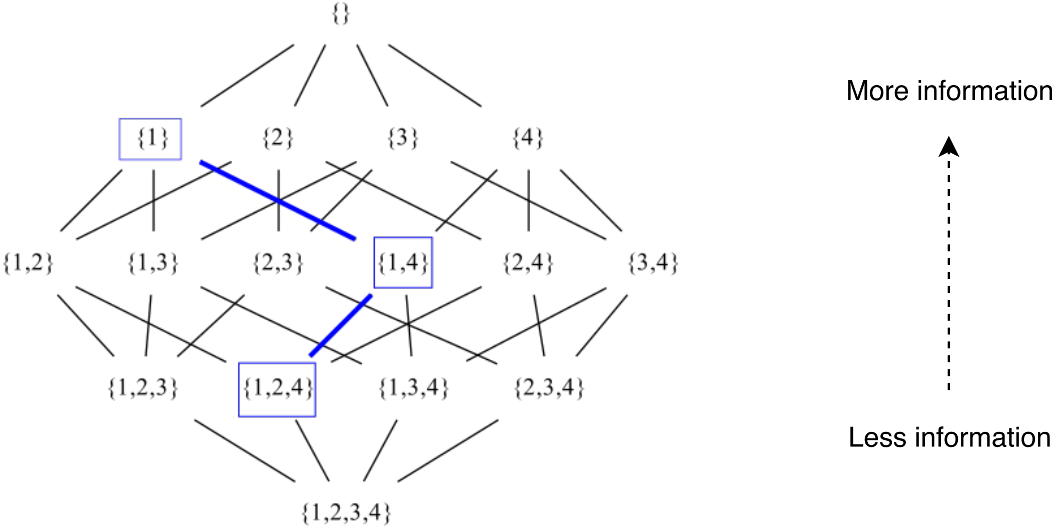
{3,4}
{1,2,3}
{1,2,4}
{1,3,4}
{2,3,4}
{1,2,3,4}
More information
Less information

$\{1,2,4\} < \{1,4\}$


{}
{1}
{2}
{3}
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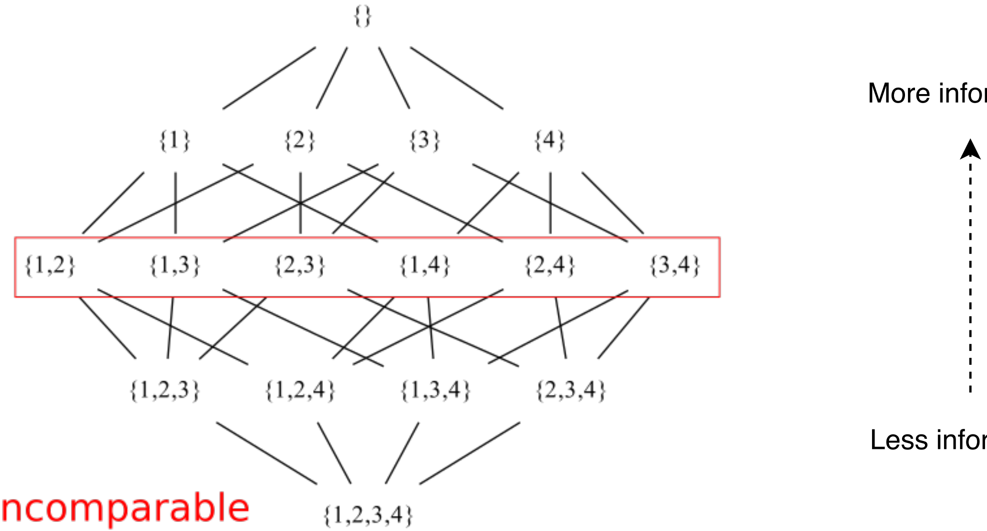
{1,2}
{1,3}
{2,3}
{1,4}
{2,4}
{3,4}
{1,2,3}
{1,2,4}
{1,3,4}
{2,3,4}
{1,2,3,4}
More information
Less information

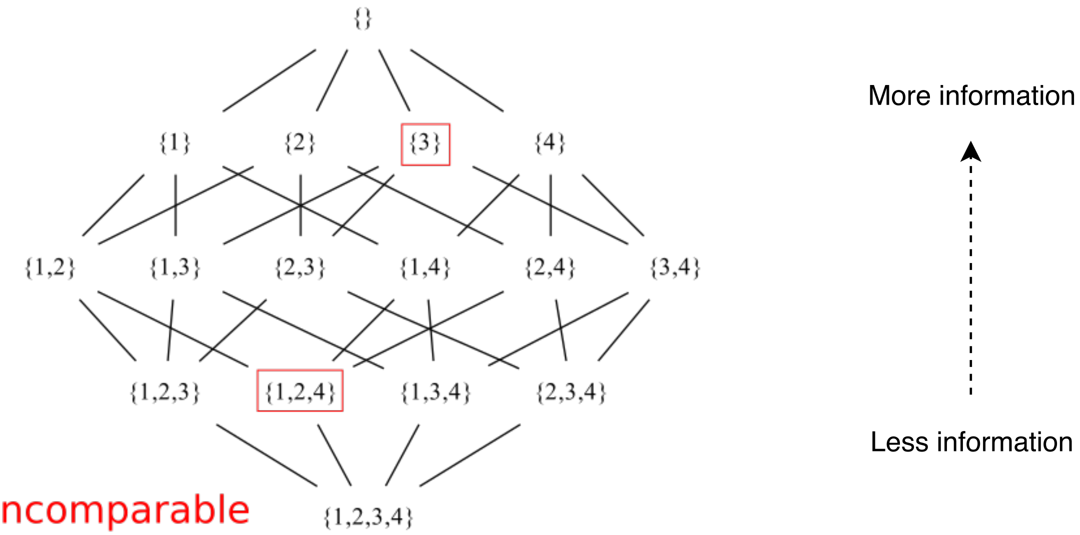
$\{1,2,4\} < \{1,4\} < \{1\}$



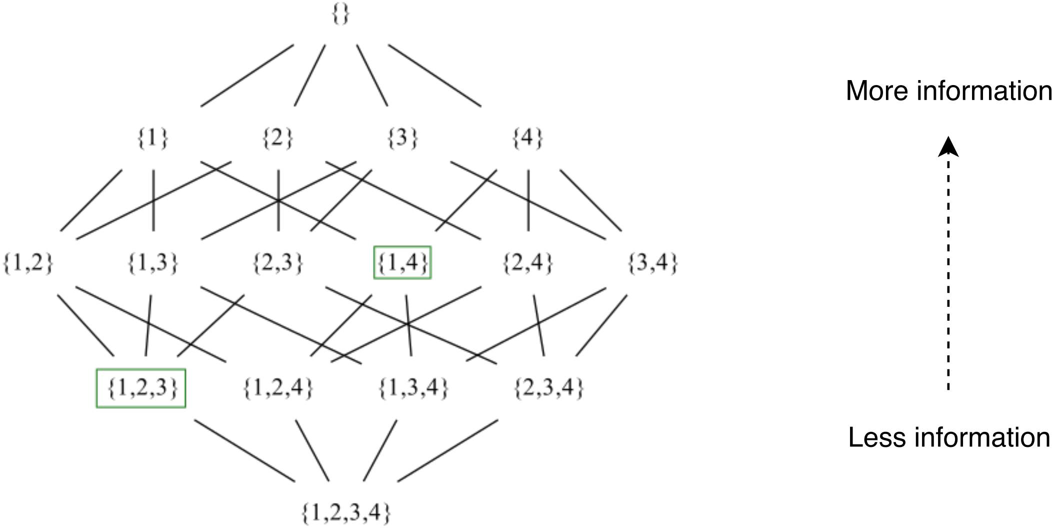
More information

Less information

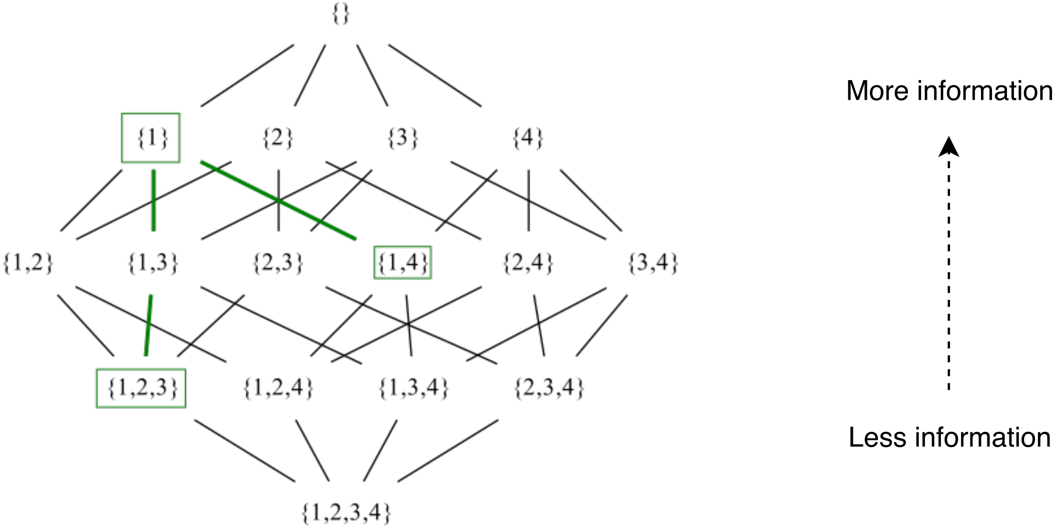
Incomparable



Incomparable


{}
{1} {2} {3} {4}
{1,2} {1,3} {2,3} {1,4} {2,4} {3,4}
{1,2,3} {1,2,4} {1,3,4} {2,3,4}
{1,2,3,4}
More information
Less information

$\{1,2,3\} \vee \{1,4\}$

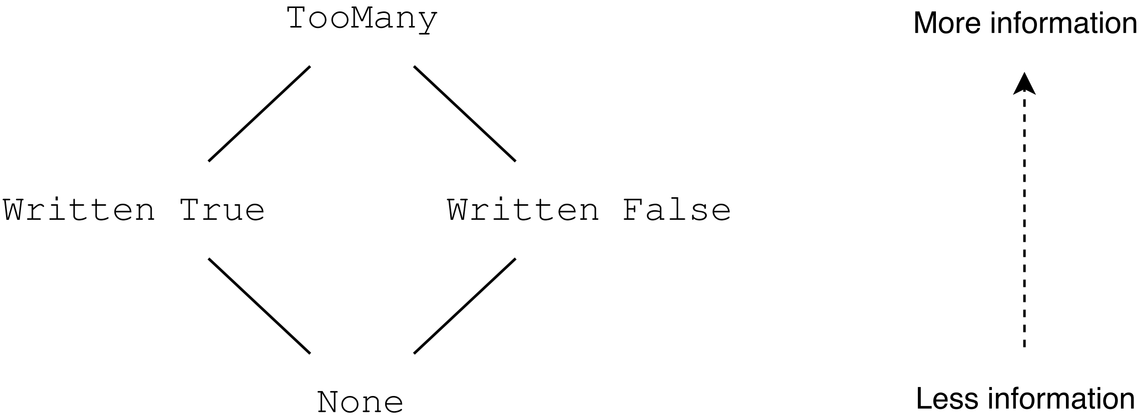

{}
{1}
{2}
{3}
{4}
{1,2}
{1,3}
{2,3}
{1,4}
{2,4}
{3,4}
{1,2,3}
{1,2,4}
{1,3,4}
{2,3,4}
{1,2,3,4}
More information
Less information

{1,2,3} V {1,4} = {1}

```
class SemiLattice a where
  (\/)   :: a -> a -> a
  bottom :: a
```

```
class SemiLattice a where
  (\/)   :: a -> a -> a
  bottom :: a

instance (Eq a) => SemiLattice (WriteOnce a) where
  None      \/ b            = b
  TooMany   \/ x            = TooMany
  Written a \/ None         = Written a
  Written a \/ TooMany      = TooMany
  Written a \/ Written b    = if a == b then Written a else TooMany
```

```haskell
class SemiLattice a where
  (\/)   :: a -> a -> a
  bottom :: a

data SudokuVal = One | Two | Three | Four deriving (Eq, Ord)

data Possibilities = Pos (Set SudokuVal)

instance Semilattice Possibilities where
  Pos p \/ Pos q = Pos (Set.intersection p q)
  bottom = Pos (Set.fromList [One,Two,Three,Four])
```



TooMany

Written True

Written False

None

More information

Less information

Bounded join semilattice

Identity:

$$x \vee bottom = bottom = bottom \vee x$$

Associative:

$$x \vee (y \vee z) = (x \vee y) \vee z$$

Commutative:

$$x \vee y = y \vee x$$

Idempotent:

$$x \vee x = x$$

Cells hold semilattices

Propagators always join information in

WriteOnce (aka IVar, Promise)

Sets (intersection or union)

Intervals

Search

many many more

Thanks for listening!

(Real) code for all these examples and more:
`https://github.com/qfpl/propagator-examples`