# An Intuition for Propagators

George Wilson

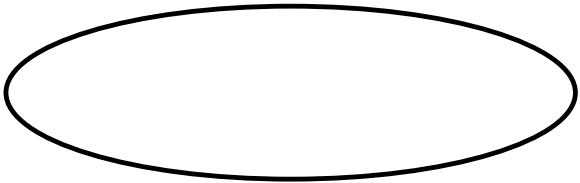
CSIRO's Data61

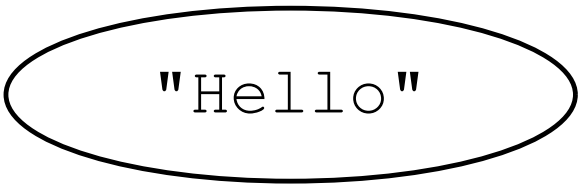
george.wilson@data61.csiro.au

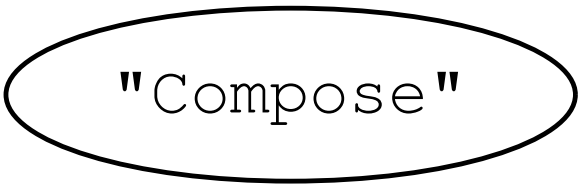
2nd September 2019

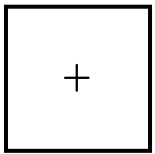

1970s, MIT

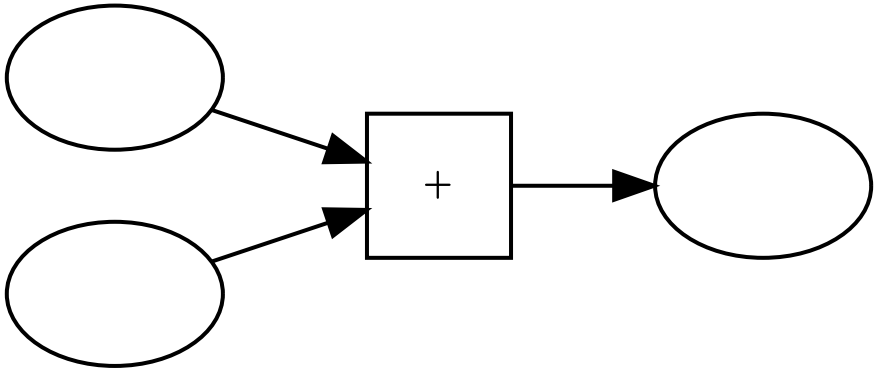
a model of computation for **highly concurrent** machines

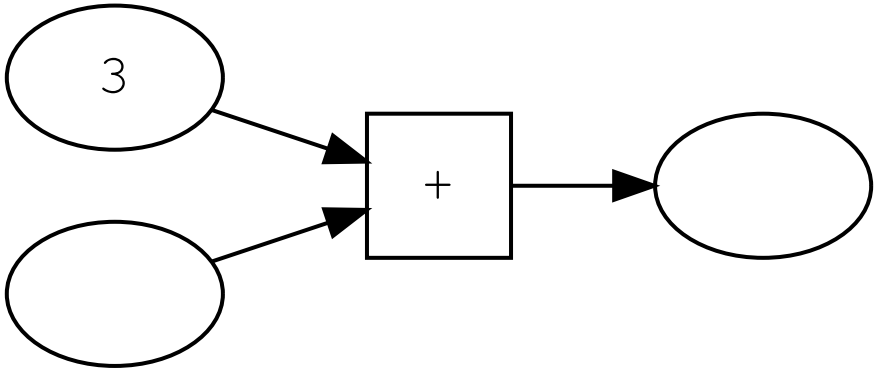


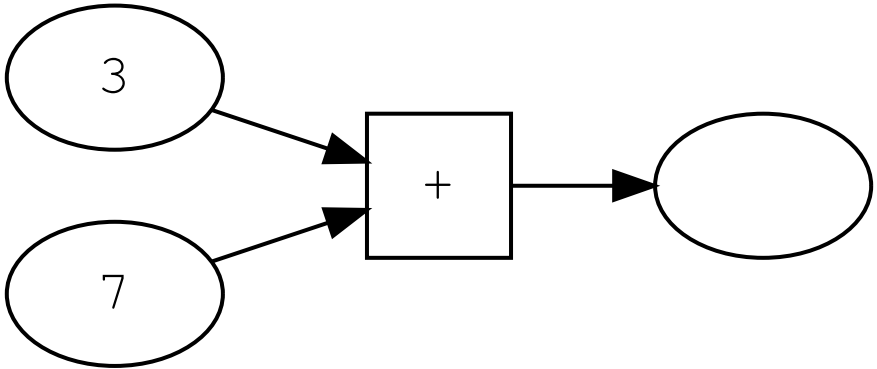

"Hello"

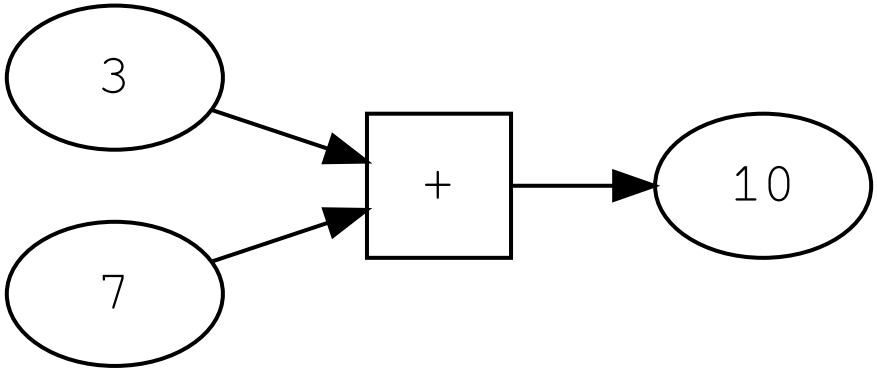

"Compose"

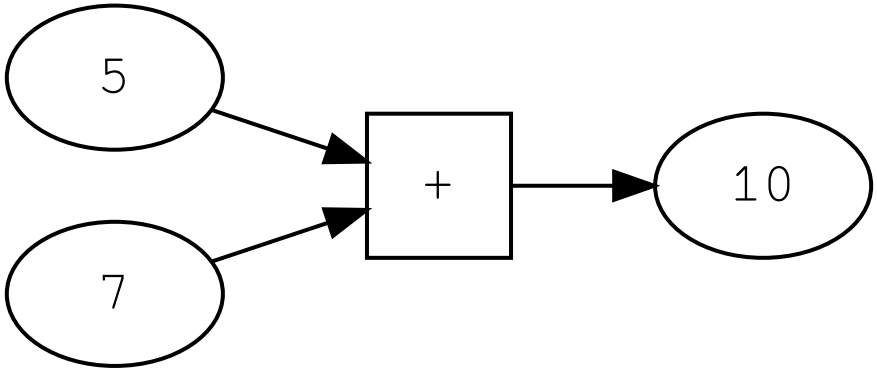


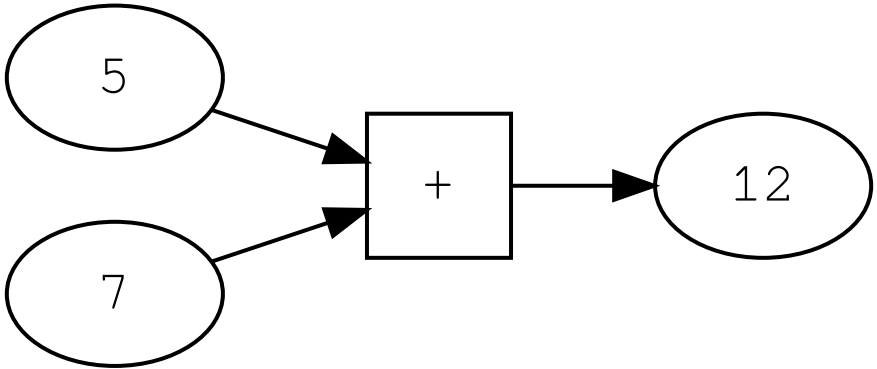



3
+


3
7
+


3
7
+
10


5
7
+
10

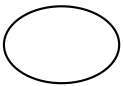

5
7
+
12

```haskell
-- types
data Par a
instance Monad Par

data Cell a
```

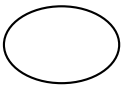
```haskell
-- types
data Par a
instance Monad Par

data Cell a


-- Creating a cell
cell    :: Par (Cell a)
```

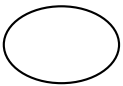
```haskell
-- types
data Par a
instance Monad Par

data Cell a


-- Creating a cell
cell    :: Par (Cell a)


-- Working with Cells
content :: Cell a -> Par (Maybe a)
write   :: Cell a -> a -> Par ()
```

```haskell
-- types
data Par a
instance Monad Par

data Cell a


-- Creating a cell
cell    :: Par (Cell a)


-- Working with Cells
content :: Cell a -> Par (Maybe a)
write   :: Cell a -> a -> Par ()


-- Creating a propagator
watch   :: Cell a -> (a -> Par ()) -> Par ()
```

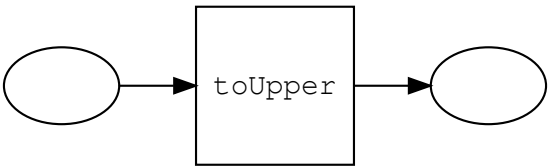


```
do
  input  <- cell
```

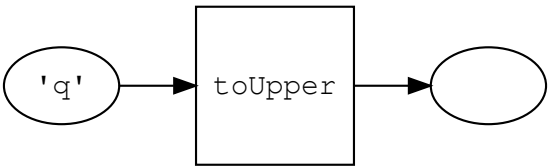




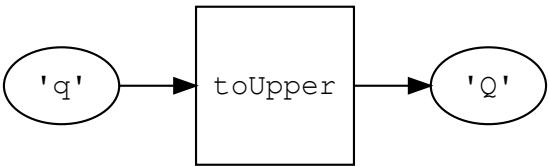
```
do
  input  <- cell
  output <- cell
```



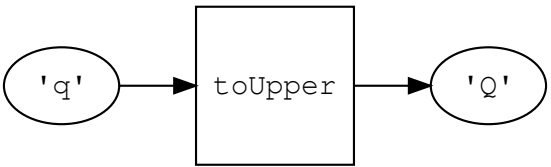

```
do
  input  <- cell
  output <- cell
  watch input (\c ->
    write output (toUpper c))
```




```
do
  input  <- cell
  output <- cell
  watch input (\c ->
    write output (toUpper c))

  write input 'q'
  content output    -- Just 'Q'
```

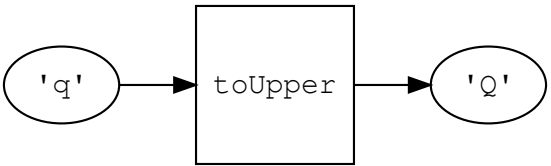



```
do
  input  <- cell
  output <- cell
  watch input (\c ->
    write output (toUpper c))

  write input 'q'
  content output    -- Just 'Q'
```



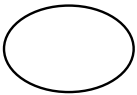

```
do
  input  <- cell
  output <- cell
  watch input (\c ->
    write output (toUpper c))

  write input 'q'
  content output    -- Just 'Q'
```

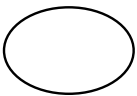
```
lift :: (a -> b) -> Cell a -> Cell b -> Par ()
lift f input output =
  watch input (\a ->
    write output (f a))
```

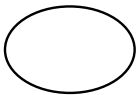


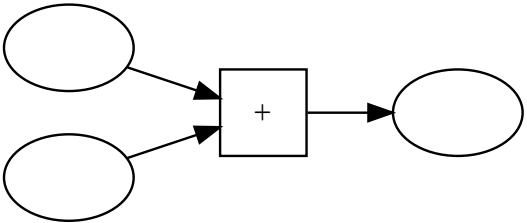

```
do
  input  <- cell
  output <- cell
  lift toUpper input output

  write input 'q'
  content output    -- Just 'Q'
```

```
do
  inL  <- cell
  inR  <- cell
  out  <- cell
```

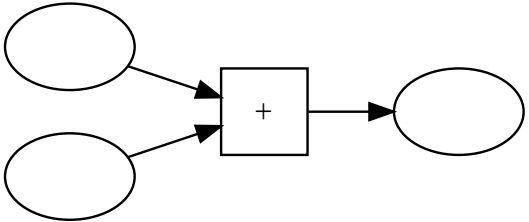






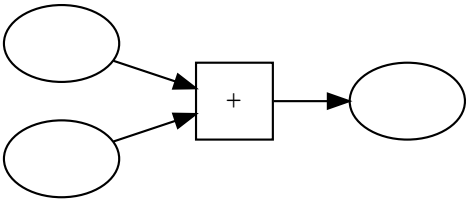



```
do
  inL  <- cell
  inR  <- cell
  out  <- cell

  watch inL (\x -> do
    maybeY <- content inR
    case maybeY of
      Nothing -> pure ()
      Just y  -> write out (x+y)
```

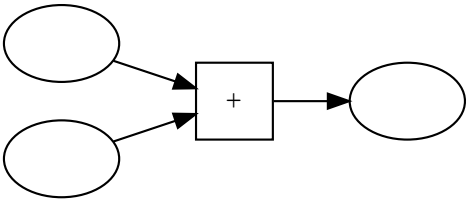



```
do
  inL  <- cell
  inR  <- cell
  out  <- cell

  watch inL (\x -> do
    maybeY <- content inR
    case maybeY of
      Nothing -> pure ()
      Just y  -> write out (x+y)

  watch inR (\y -> do
    maybeX <- content inL
    case maybeX of
      Nothing -> pure ()
      Just x  -> write out (x+y)
```

```
with :: Cell a -> (a -> Par ()) -> Par ()
with theCell callback = do
  maybeA <- content theCell
    case maybeA of
      Nothing -> pure ()
      Just a  -> callback a
```



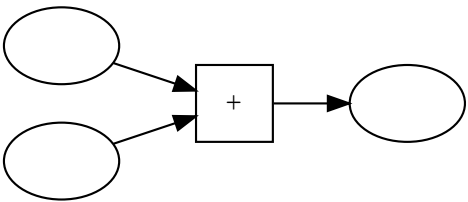

```
do
  inL  <- cell
  inR  <- cell
  out  <- cell

  watch inL (\x ->
    with inR (\y ->
      write out (x+y)
```




```
do
  inL  <- cell
  inR  <- cell
  out  <- cell

  watch inL (\x ->
    with inR (\y ->
      write out (x+y)

  watch inR (\y ->
    with inL (\x ->
      write out (x+y)
```

```
lift2 :: (a -> b -> c)
      -> Cell a -> Cell b -> Cell c
      -> Par ()
```

```
lift2 :: (a -> b -> c)
      -> Cell a -> Cell b -> Cell c
      -> Par ()
lift2 f inL inR out = do
  watch inL (\a ->
    with inR (\b ->
      write out (f a b)))
  watch inR (\b ->
    with inL (\a ->
      write out (f a b)))
```




```
do
  inL  <- cell
  inR  <- cell
  out  <- cell

  adder inL inR out

    where
      adder l r o = do
        lift2 (+) l r o
        lift2 (-) o l r
        lift2 (-) o r l
```

TODO: scary giant propagator network

?!

How can we fix this?

To be deterministic we must

- tolerate **reordering** of writes
- tolerate **grouping** of writes
- ignore **redundancy** of writes

```haskell
data WriteOnce a
  = None
  | Written a
  | TooMany

tryWrite ::           a -> WriteOnce a -> WriteOnce a
tryWrite a w = case w of
  None      -> Written a
  Written b -> TooMany
  TooMany   -> TooMany
```

```haskell
data WriteOnce a
  = None
  | Written a
  | TooMany

tryWrite :: (Eq a) => a -> WriteOnce a -> WriteOnce a
tryWrite a w = case w of
  None      -> Written a
  Written b -> if a == b then Written b else TooMany
  TooMany   -> TooMany
```

Now every network will give a **deterministic answer** in **finite time**

Mutability is **chaos**

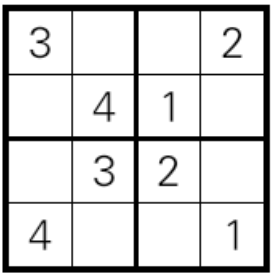
WriteOnce is **rigid**

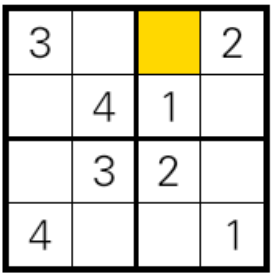
Accumulate information about a value

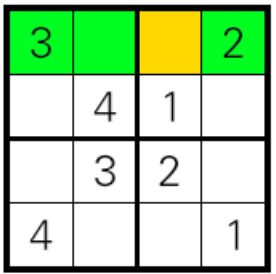
```haskell
data WriteOnce a

  -- I have heard contradictory answers!
  = TooMany

  -- I know the answer exactly
  | Written a

  -- I don't know anything
  | None
```

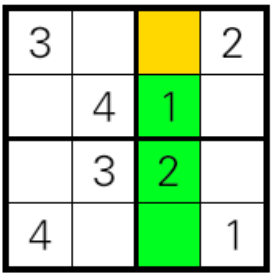
```haskell
data WriteOnce a

  -- I have heard contradictory answers!
  = TooMany

  -- I know the answer exactly
  | Written a

  -- I don't know anything
  | None
```

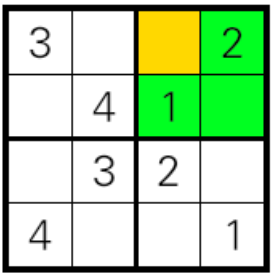
**Monotonic** growth

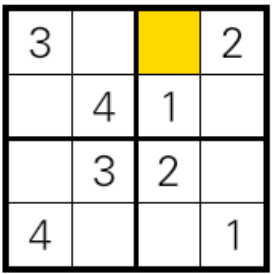

3 2
4 1
3 2
4 1

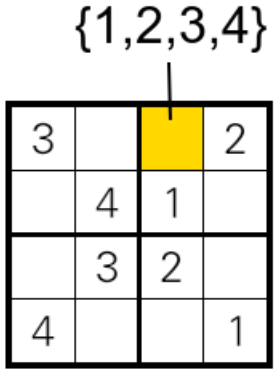

3
2
4
1
3
2
4
1

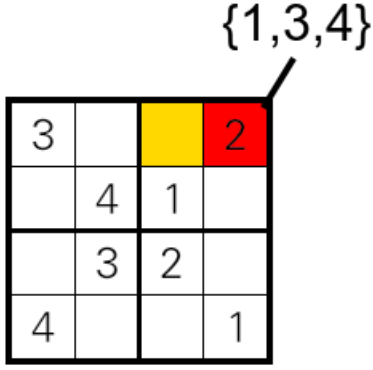

3
2
4
1
3
2
4
1

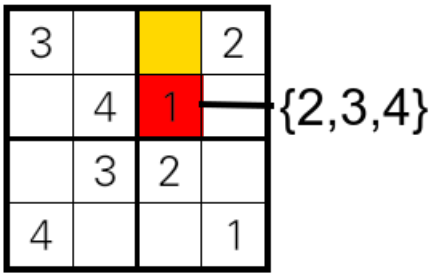

3
2
4
1
3
2
4
1

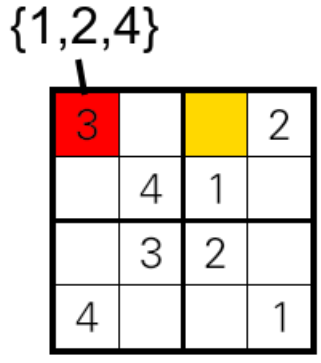

3
2
4
1
3
2
4
1


3
2
4
1
3
2
4
1


{1,2,3,4}
3
2
4
1
3
2
4
1


{1,3,4}
3
2
4
1
3
2
4
1

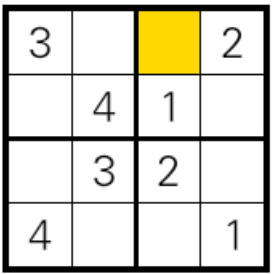

3
2
4
1
{2,3,4}
3
2
4
1

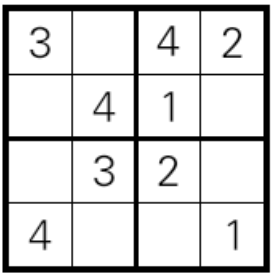

{1,2,4}
3
2
4
1
3
2
4
1

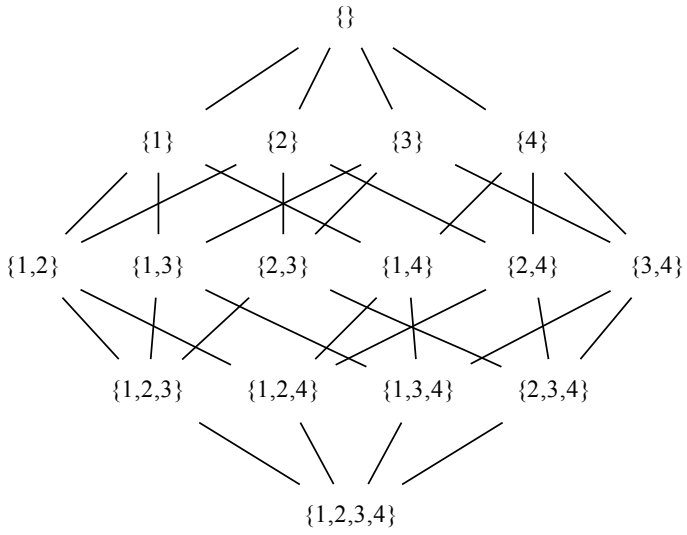
$$\{2,3,4\} \cap \{1,3,4\} \cap \{1,2,4\} \cap \{1,2,3,4\}$$

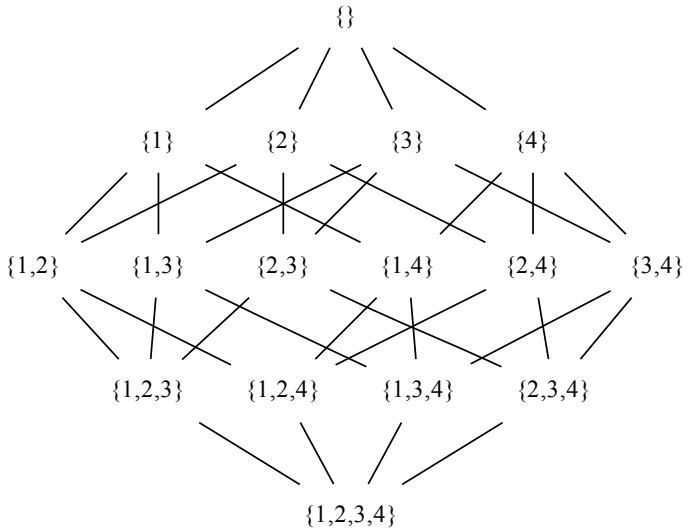
| | | | |
|---|---|---|---|
| 3 | | | 2 |
| | 4 | 1 | |
| | 3 | 2 | |
| 4 | | | 1 |

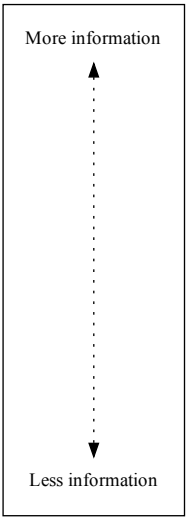
{4}

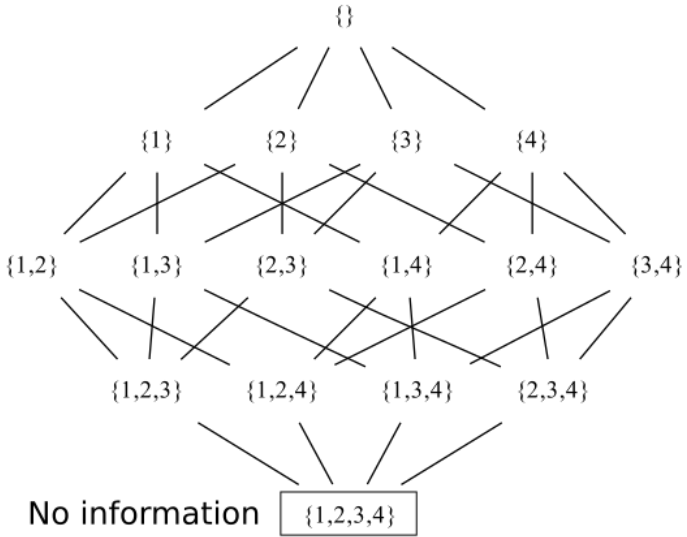


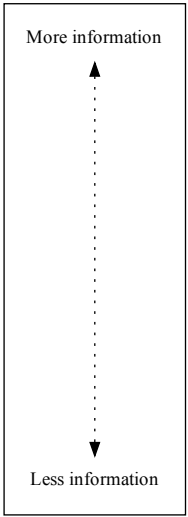

3 4 2
4 1
3 2
4 1

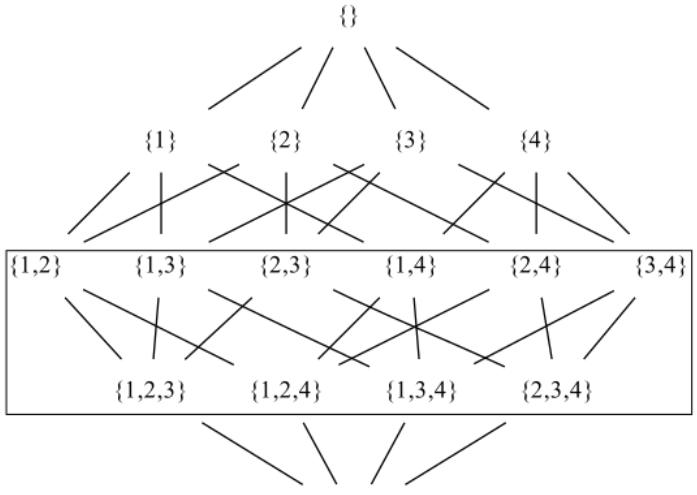

{}
{1}
{2}
{3}
{4}
{1,2}
{1,3}
{2,3}
{1,4}
{2,4}
{3,4}
{1,2,3}
{1,2,4}
{1,3,4}
{2,3,4}
{1,2,3,4}


{}
{1}
{2}
{3}
{4}
{1,2}
{1,3}
{2,3}
{1,4}
{2,4}
{3,4}
{1,2,3}
{1,2,4}
{1,3,4}
{2,3,4}
{1,2,3,4}

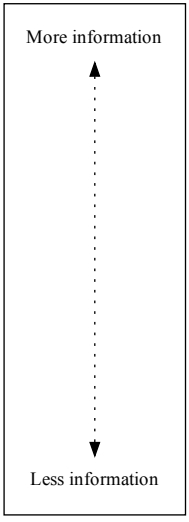

More information
Less information

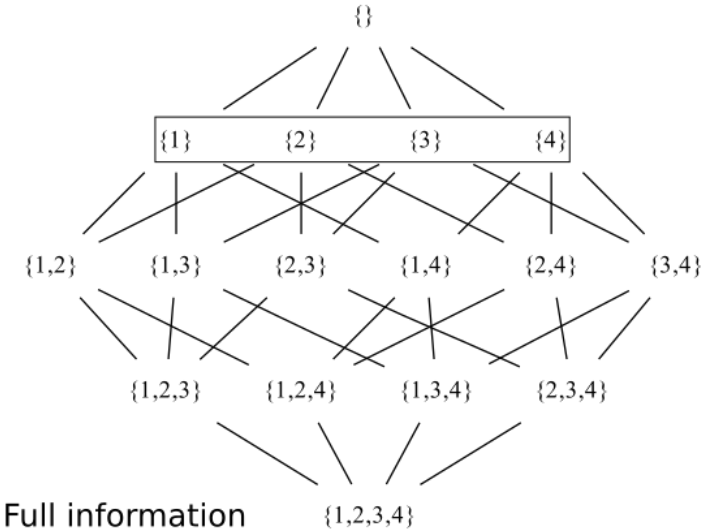

{}
{1}
{2}
{3}
{4}
{1,2}
{1,3}
{2,3}
{1,4}
{2,4}
{3,4}
{1,2,3}
{1,2,4}
{1,3,4}
{2,3,4}
No information
{1,2,3,4}

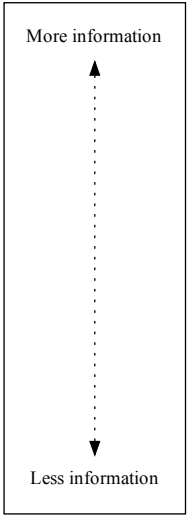

More information
Less information

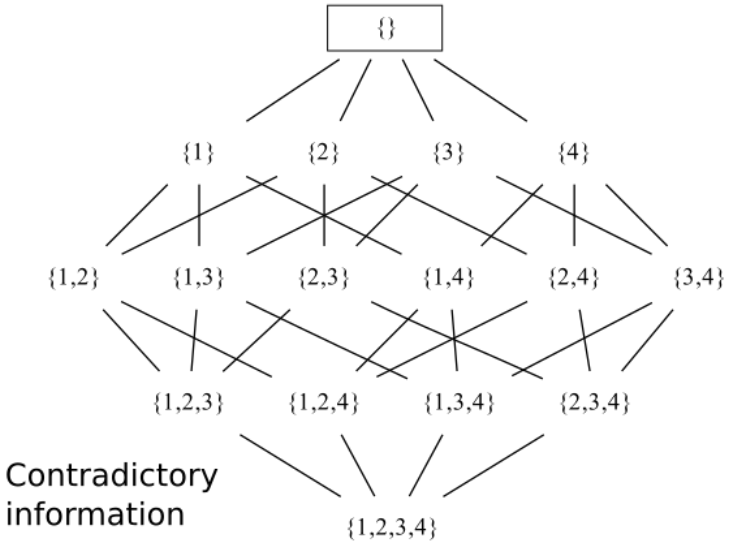


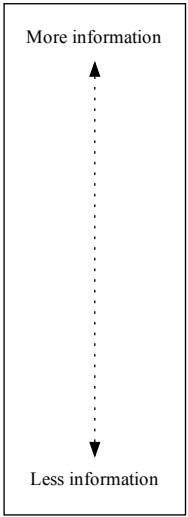

Some information

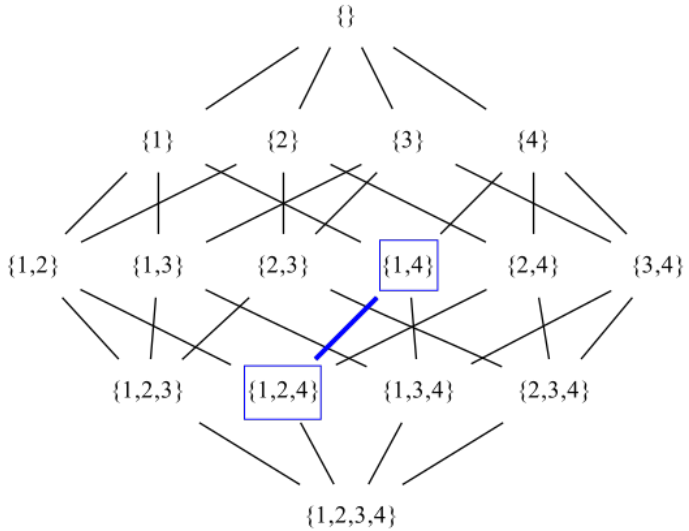


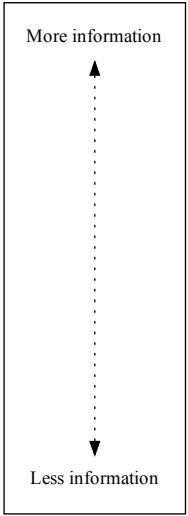

{}
{1}
{2}
{3}
{4}
{1,2}
{1,3}
{2,3}
{1,4}
{2,4}
{3,4}
{1,2,3}
{1,2,4}
{1,3,4}
{2,3,4}
{1,2,3,4}
Full information


More information
Less information

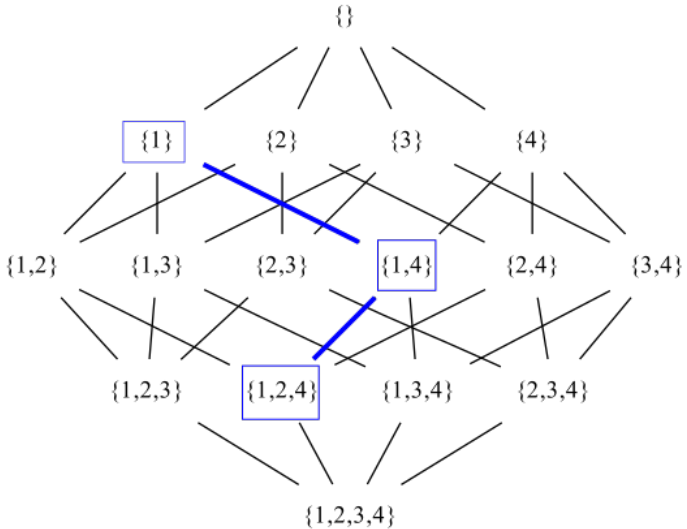

{}
{1}
{2}
{3}
{4}
{1,2}
{1,3}
{2,3}
{1,4}
{2,4}
{3,4}
{1,2,3}
{1,2,4}
{1,3,4}
{2,3,4}
{1,2,3,4}
Contradictory information

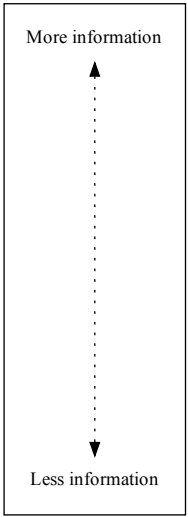

More information
Less information


{}
{1}
{2}
{3}
{4}
{1,2}
{1,3}
{2,3}
{1,4}
{2,4}
{3,4}
{1,2,3}
{1,2,4}
{1,3,4}
{2,3,4}
{1,2,3,4}

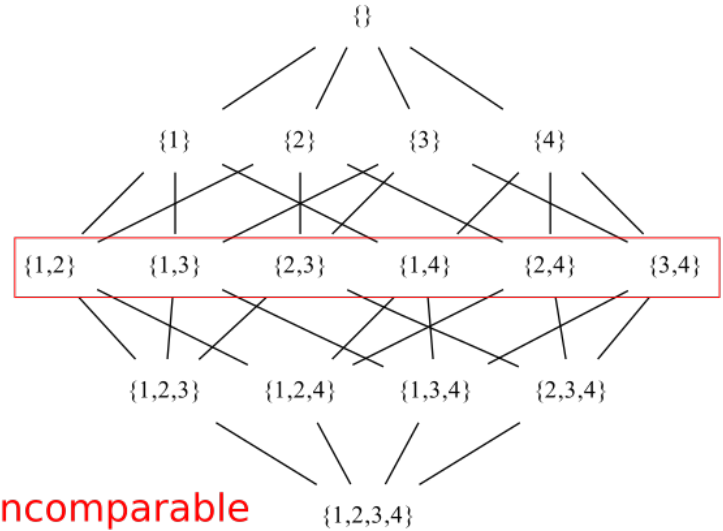

More information
Less information

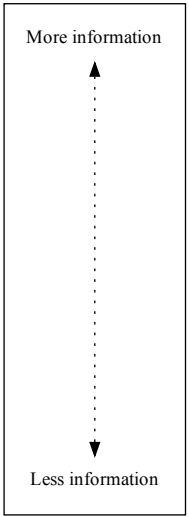
$\{1,2,4\} < \{1,4\}$

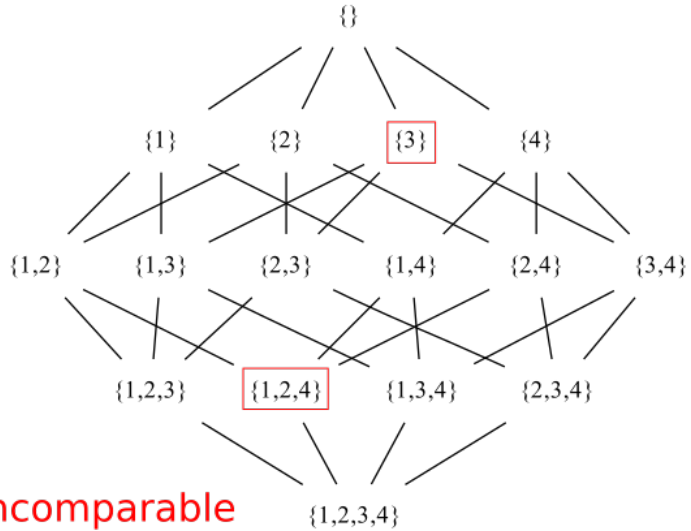

{}
{1}
{2}
{3}
{4}
{1,2}
{1,3}
{2,3}
{1,4}
{2,4}
{3,4}
{1,2,3}
{1,2,4}
{1,3,4}
{2,3,4}
{1,2,3,4}

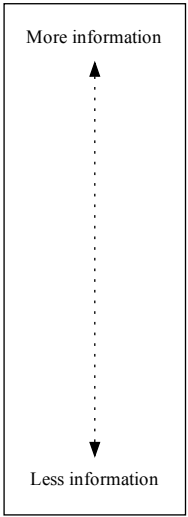

More information
Less information

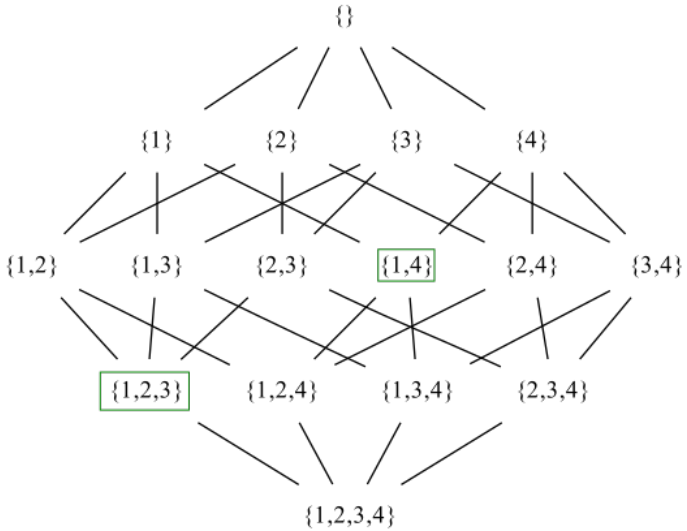
$\{1,2,4\} < \{1,4\} < \{1\}$

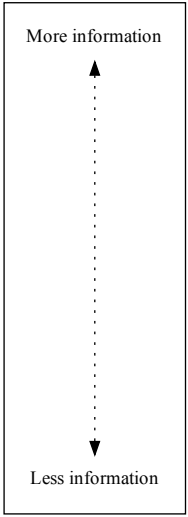

{}
{1}
{2}
{3}
{4}
{1,2}
{1,3}
{2,3}
{1,4}
{2,4}
{3,4}
{1,2,3}
{1,2,4}
{1,3,4}
{2,3,4}
{1,2,3,4}
Incomparable


More information
Less information

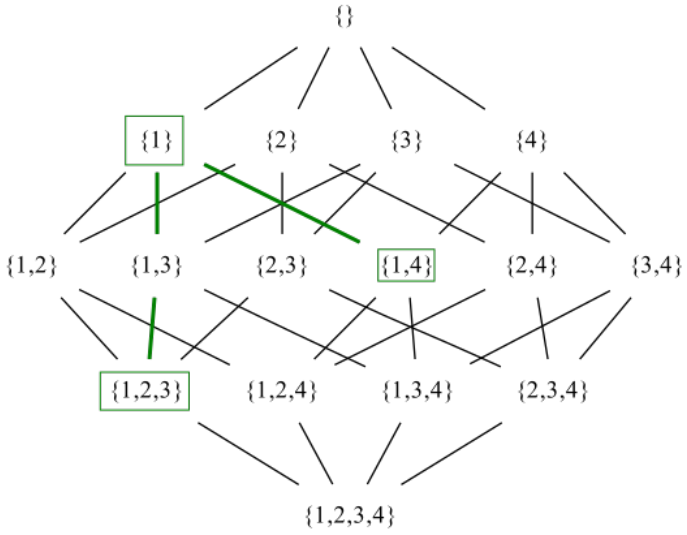

{}
{1}
{2}
{3}
{4}
{1,2}
{1,3}
{2,3}
{1,4}
{2,4}
{3,4}
{1,2,3}
{1,2,4}
{1,3,4}
{2,3,4}
{1,2,3,4}
Incomparable

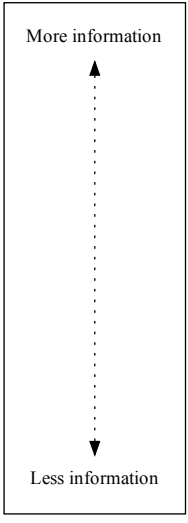

More information
Less information


{}
{1}
{2}
{3}
{4}
{1,2}
{1,3}
{2,3}
{1,4}
{2,4}
{3,4}
{1,2,3}
{1,2,4}
{1,3,4}
{2,3,4}
{1,2,3,4}


More information
Less information

$\{1,2,3\} \vee \{1,4\}$


{}
{1}
{2}
{3}
{4}
{1,2}
{1,3}
{2,3}
{1,4}
{2,4}
{3,4}
{1,2,3}
{1,2,4}
{1,3,4}
{2,3,4}
{1,2,3,4}


More information
Less information

$\{1,2,3\} \vee \{1,4\} = \{1\}$

# Bounded join semilattice

Identity:

$$x \vee bottom = bottom = bottom \vee x$$

Associative:

$$x \vee (y \vee z) = (x \vee y) \vee z$$

Commutative:

$$x \vee y = y \vee x$$

Idempotent:

$$x \vee x = x$$

```haskell
class SemiLattice a where
  (\/)    :: a -> a -> a
  bottom :: a


instance (Eq a) => SemiLattice (WriteOnce a) where
  None       \/ b           = b
  TooMany    \/ x           = TooMany
  Written a \/ None         = Written a
  Written a \/ TooMany      = TooMany
  Written a \/ Written b    = if a == b then Written a else TooMany
```

```haskell
class SemiLattice a where
  (\/)   :: a -> a -> a
  bottom :: a

data SudokuVal = One | Two | Three | Four deriving (Eq, Ord)

data Possibilities = Pos (Set SudokuVal)

instance Semilattice Possibilities where
  Pos p \/ Pos q = Pos (Set.intersection p q)
  bottom = Pos (Set.fromList [One,Two,Three,Four])
```

Monotonicity gives us

# Thanks for listening!

(Real) code for all these examples and more:
`https://github.com/qfpl/propagator-examples`