Propagators: An Introduction

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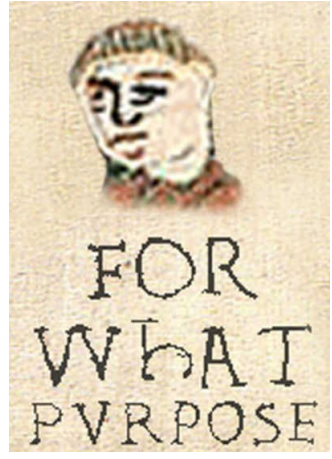
What?



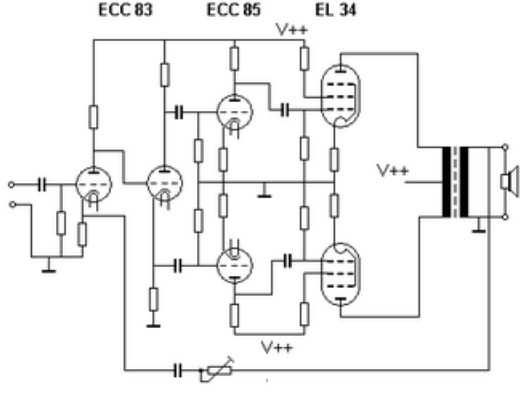
Why?

Roots as early as the 1970's at MIT

- Guy L. Steele Jr.
- Gerald J. Sussman
- Richard Stallman

More recently:

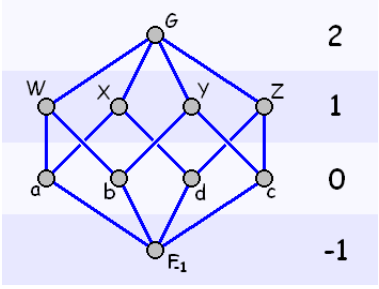
- Alexey Radul



```
(define (map f xs)
  (cond ((null? xs) '())
        (else (cons (f (car xs))
                    (map f (cdr xs)))))))
```

And then

- Edward Kmett



$$x \leq y \implies f(x) \leq f(y)$$

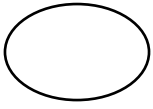
Propagators

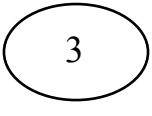
The *propagator model* is a model of computation

We model computations as *propagator networks*

A propagator network comprises

- cells
- propagators
- connections between cells and propagators

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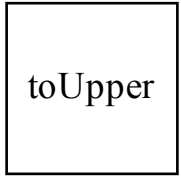
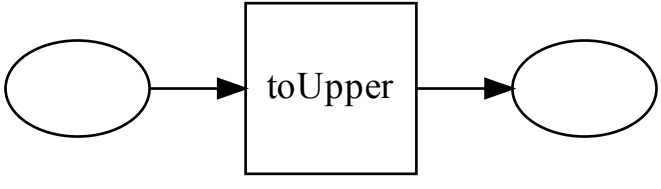
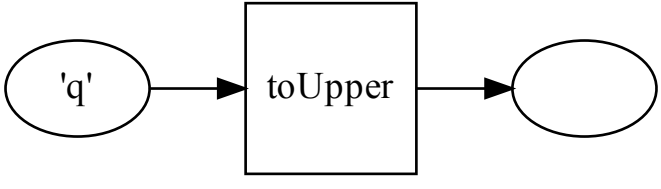
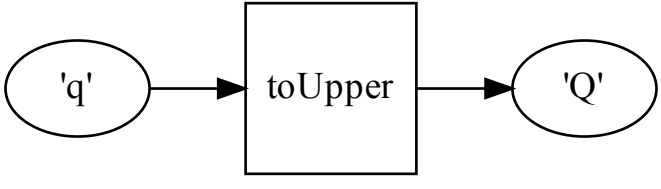
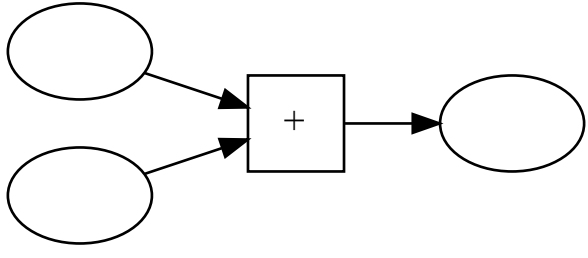
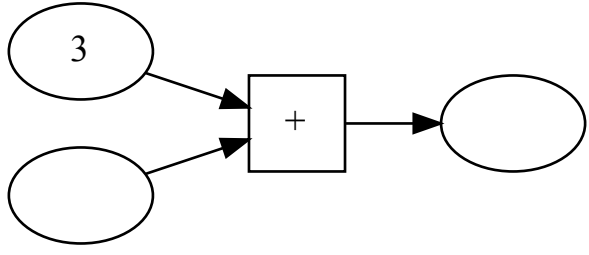

toUpper

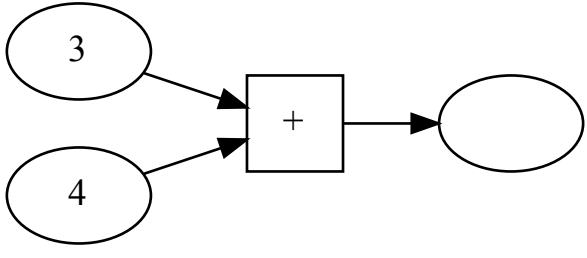

toUpper


'q'
toUpper


'q'
toUpper
'Q'




3
+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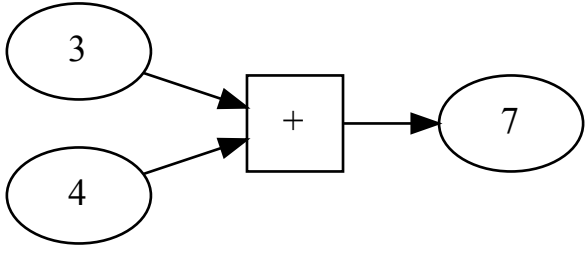

3
4
+
7

$$z \leftarrow x + y$$

$$z = x + y$$

$$7 = x + 4$$

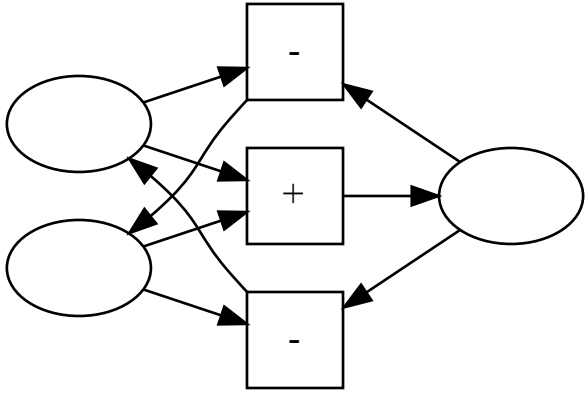
$$7 = 3 + 4$$

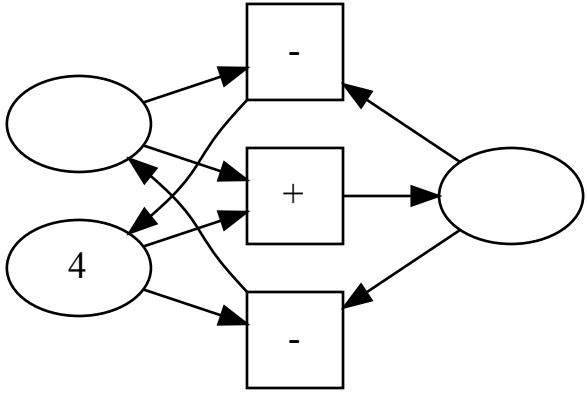
$$z = x + y$$

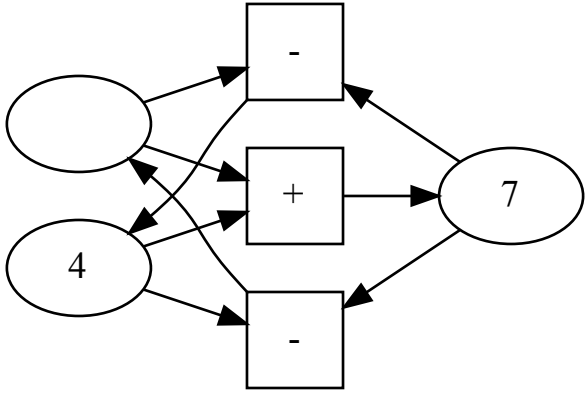
$$z \leftarrow x + y$$

$$x \leftarrow z - y$$

$$y \leftarrow z - x$$




-
+
4
-


-
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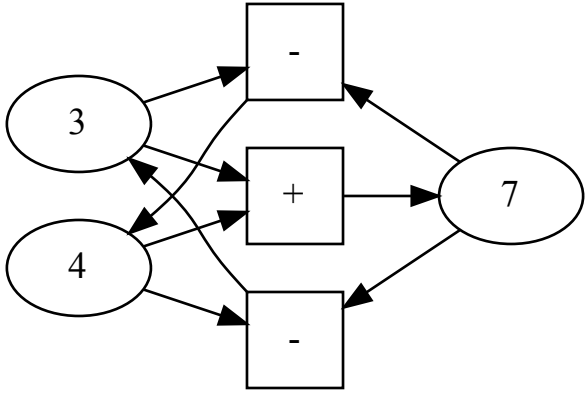
-
3
+
7
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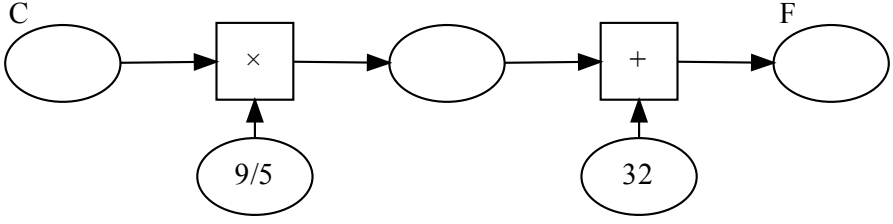
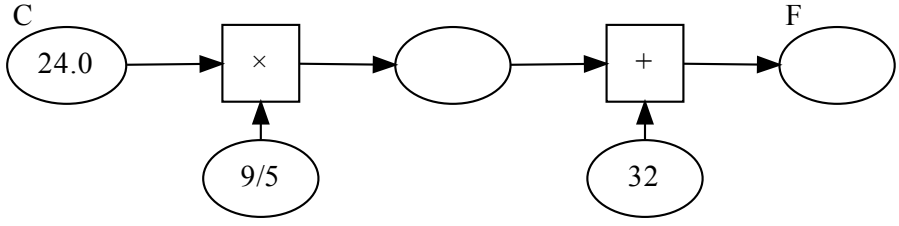
Propagators let us express multidirectional relationships!

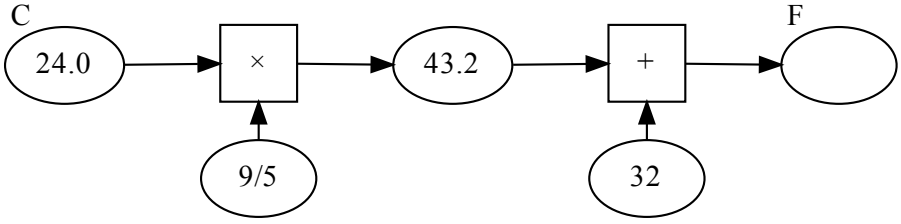
$$^\circ F = {}^\circ C \times \frac{9}{5} + 32$$



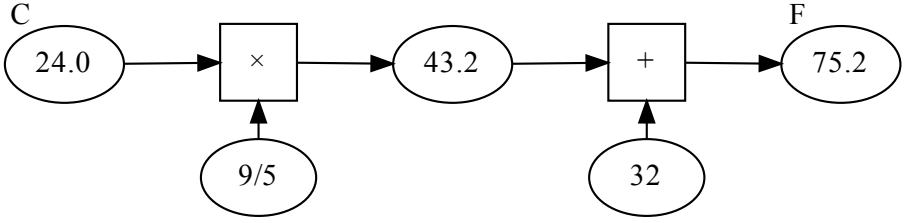
$$^\circ F = {}^\circ C \times \frac{9}{5} + 32$$



$$^\circ F = ^\circ C \times \frac{9}{5} + 32$$

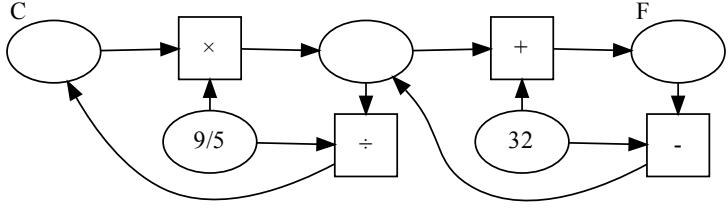


$$^\circ F = ^\circ C \times \frac{9}{5} + 32$$



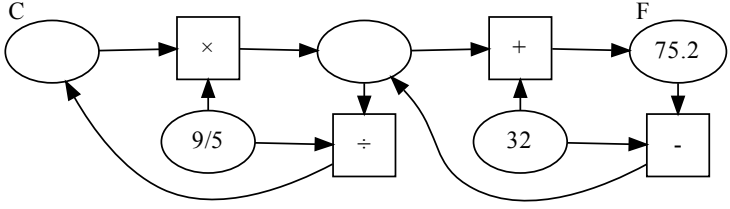
$$^\circ F = {}^\circ C \times \frac{9}{5} + 32$$

$$^\circ C = ({}^\circ F - 32) \div \frac{9}{5}$$



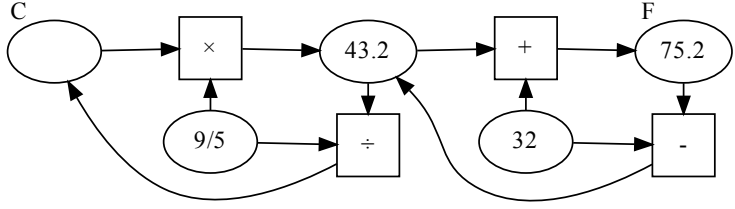
$$^\circ F = {}^\circ C \times \frac{9}{5} + 32$$

$$^\circ C = ({}^\circ F - 32) \div \frac{9}{5}$$



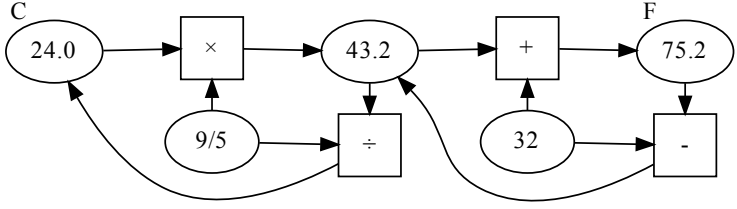
$$^\circ F = {}^\circ C \times \frac{9}{5} + 32$$

$$^\circ C = (^\circ F - 32) \div \frac{9}{5}$$



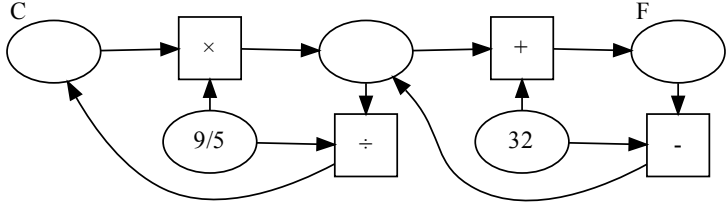
$$^\circ F = {}^\circ C \times \frac{9}{5} + 32$$

$$^\circ C = ({}^\circ F - 32) \div \frac{9}{5}$$



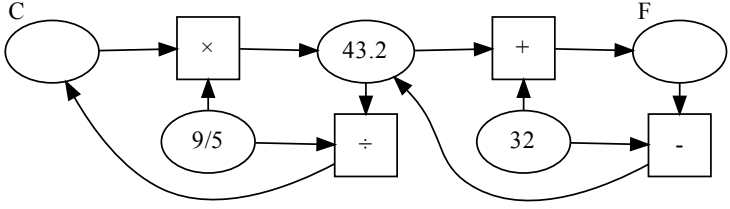
$$^\circ F = {}^\circ C \times \frac{9}{5} + 32$$

$$^\circ C = ({}^\circ F - 32) \div \frac{9}{5}$$



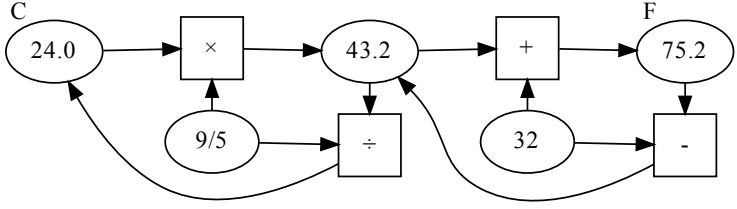
$$^\circ F = {}^\circ C \times \frac{9}{5} + 32$$

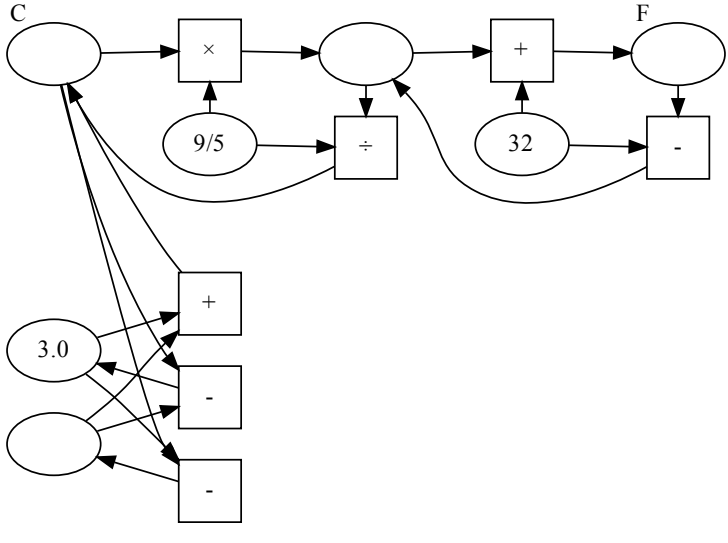
$$^\circ C = (^\circ F - 32) \div \frac{9}{5}$$

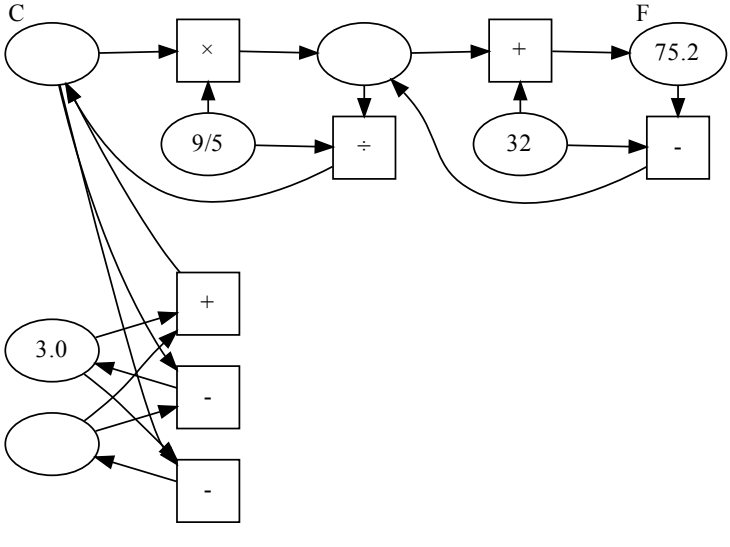


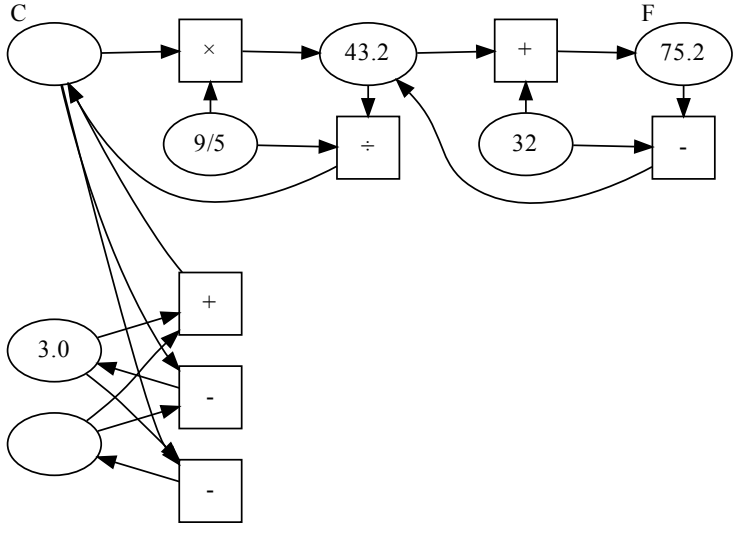
$$^\circ F = {}^\circ C \times \frac{9}{5} + 32$$

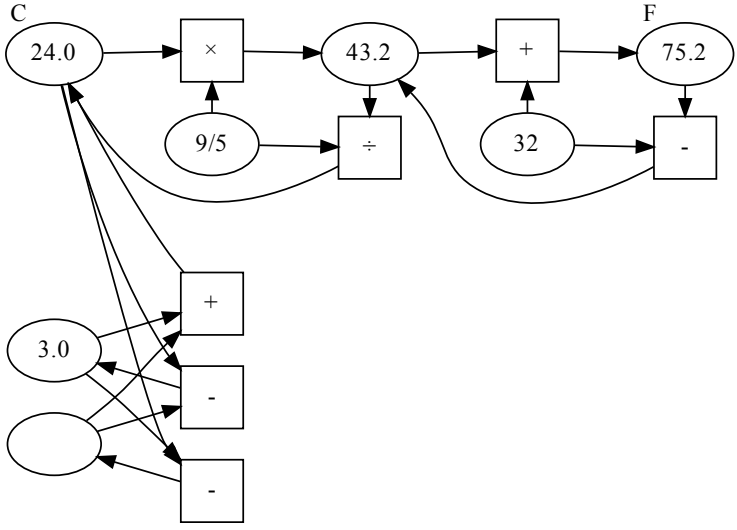
$$^\circ C = ({}^\circ F - 32) \div \frac{9}{5}$$

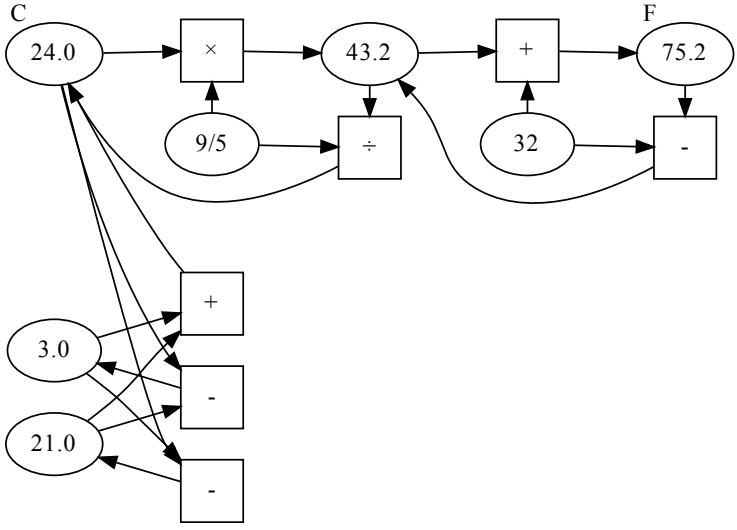



C
×
+
F
9/5
÷
32
-
+
3.0
-
-


C
×
+
F
75.2
9/5
÷
32
-
+
3.0
-
-

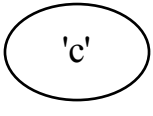

C
×
43.2
+
F
75.2
9/5
÷
32
-
+
3.0
-
-

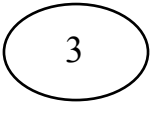

C
24.0
×
43.2
+
F
75.2
9/5
÷
32
-
+
3.0
-
-

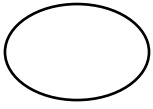

C
24.0
×
43.2
+
F
75.2
9/5
÷
32
-
+
3.0
-
21.0
-

We can combine networks into larger networks!

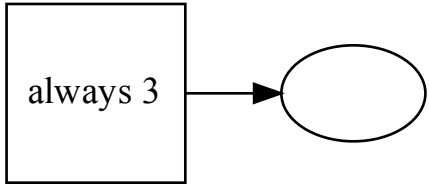
What types are the values of the cells?

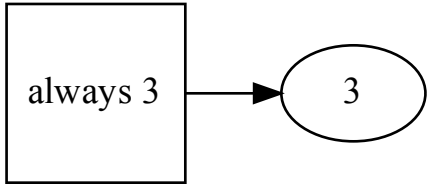

'c'

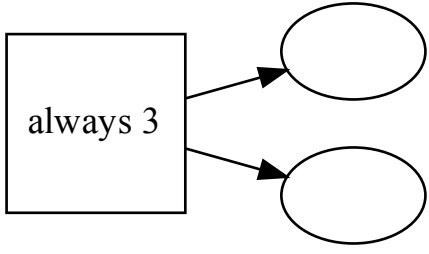

3

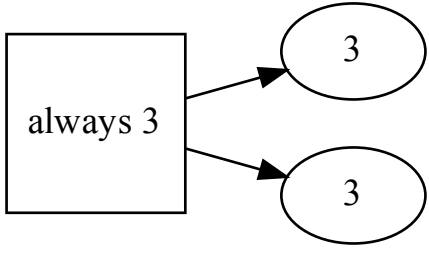


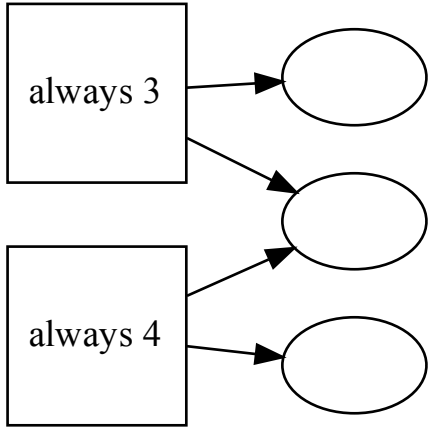
```
data Maybe a = Nothing | Just a
```

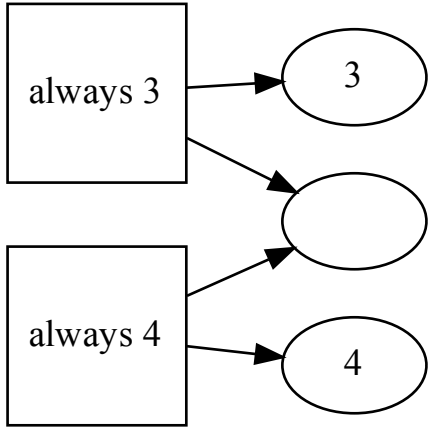

always 3

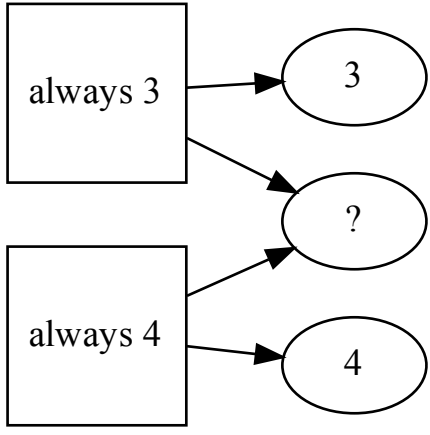

always 3
3


always 3


always 3
3
3


always 3
always 4


always 3
3
always 4
4


always 3
3
?
always 4
4

:(

Contradiction

```haskell
data Perhaps a = Unknown | Known a | Contradiction
```

```haskell
data Perhaps a = Unknown | Known a | Contradiction


instance Eq a => Monoid (Perhaps a) where
  mempty = Unknown

  mappend Unknown x             = x
  mappend x       Unknown       = x
  mappend Contradiction _       = Contradiction
  mappend _     Contradiction = Contradiction
  mappend (Known a) (Known b) =
    if a == b
      then Known a
      else Contradiction
```

Partial information!

?

$[1, 5]$

$$[1,5] <> [2,7] = [2,5]$$

$$\{True,\ False\}$$

TODO set intersection examples