Propagators: An Introduction

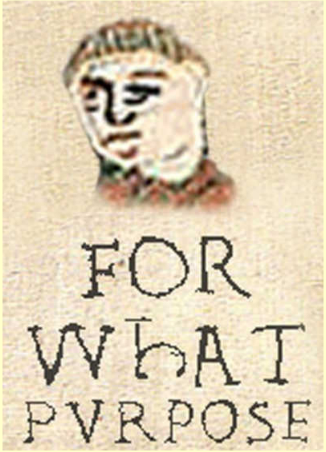
George Wilson

Data61/CSIRO

george.wilson@data61.csiro.au

November 11, 2017

What?



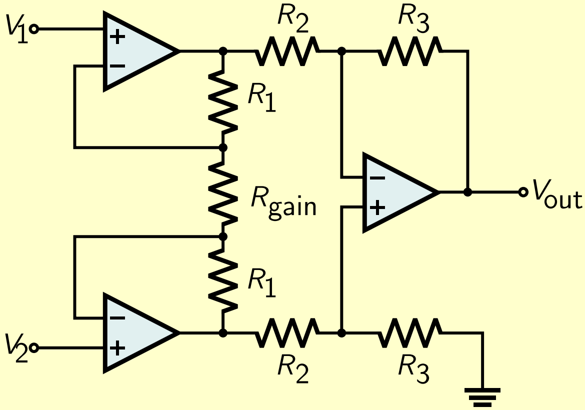
Why?

Beginnings as early as the 1970's at MIT

- Guy L. Steele Jr.
- Gerald J. Sussman
- Richard Stallman

More recently:

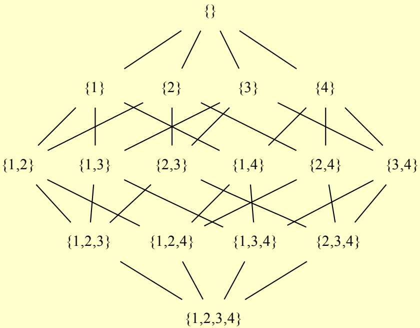
- Alexey Radul



```
(define (map f xs)
  (cond ((null? xs) '())
        (else (cons (f (car xs))
                    (map f (cdr xs))))))
```

And then

- Edward Kmett



$$x \leq y \implies f(x) \leq f(y)$$

Propagators

The *propagator model* is a model of computation
We model computations as *propagator networks*

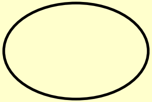
The *propagator model* is a model of computation
We model computations as *propagator networks*

Propagator networks:

- are extremely expressive
- lend themselves to parallel and distributed evaluation
- allow different strategies of problem-solving to seamlessly cooperate

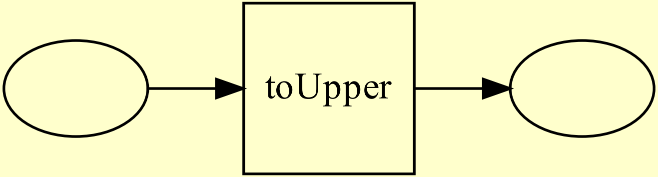
A propagator network comprises

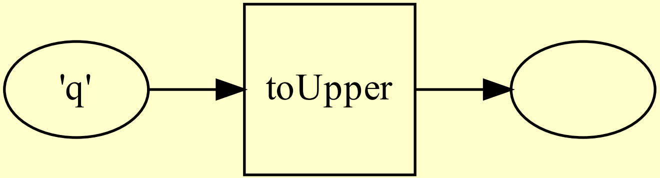
- cells
- propagators
- connections between cells and propagators

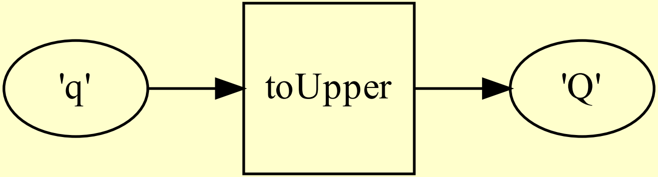


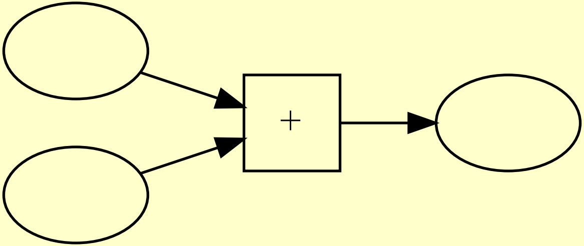
3

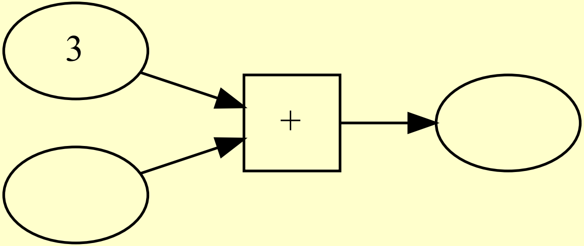
toUpper

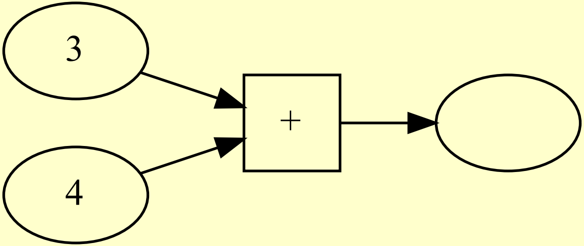

toUpper

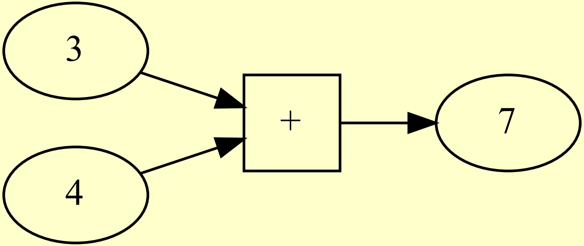

'q'
toUpper


'q'
toUpper
'Q'




3
+


3
4
+


3
4
+
7

$$z \leftarrow x + y$$

$$z = x + y$$

$$7 = x + 4$$

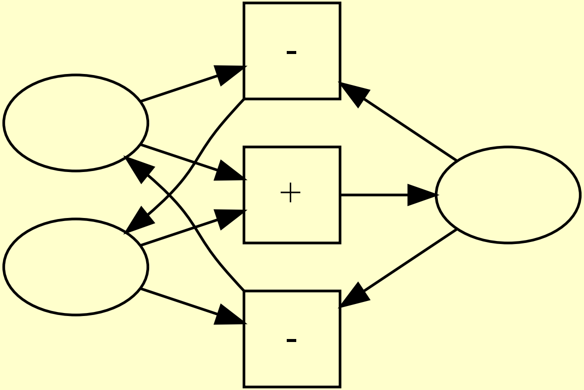
$$7 = 3 + 4$$

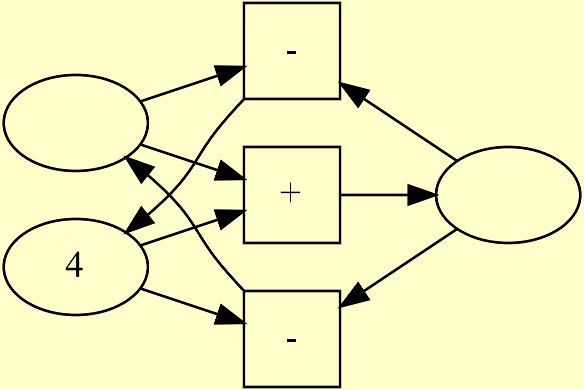
$$z = x + y$$

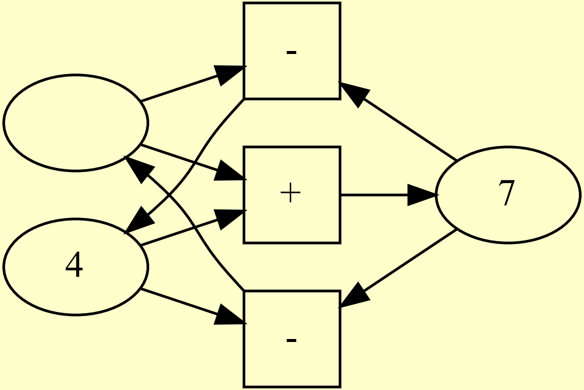
$$z \leftarrow x + y$$

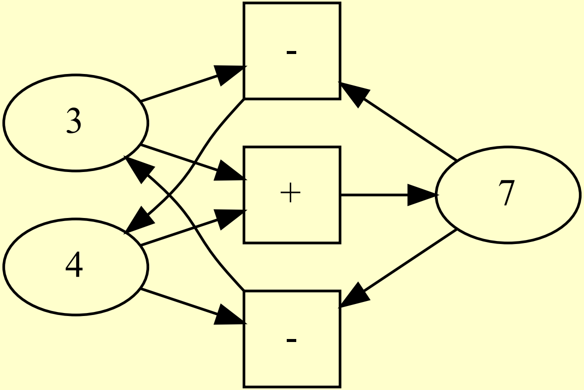
$$x \leftarrow z - y$$

$$y \leftarrow z - x$$



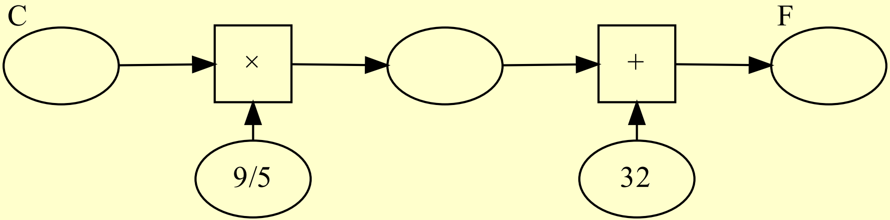

-
+
4
-


-
+
-
4
7

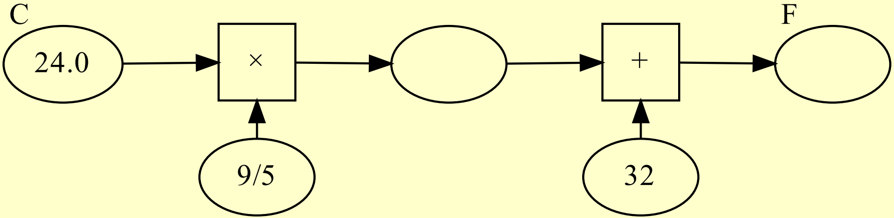

3
4
-
+
-
7

Propagators let us express multi-directional relationships!

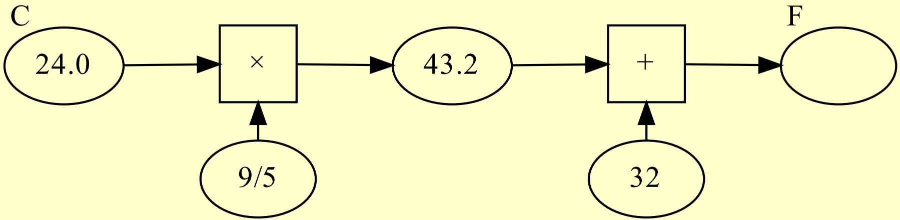
$$^\circ F = {}^\circ C \times \frac{9}{5} + 32$$



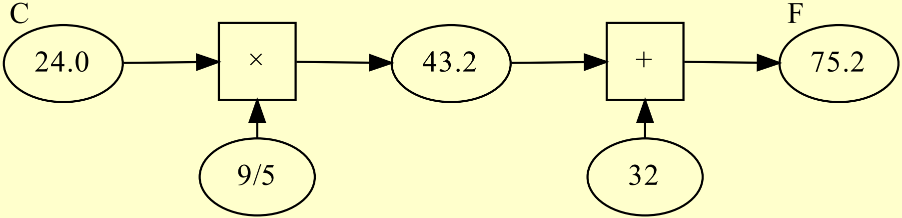
$$^{\circ}F = {^{\circ}C} \times \frac{9}{5} + 32$$



$$^\circ F = {^\circ C} \times \frac{9}{5} + 32$$

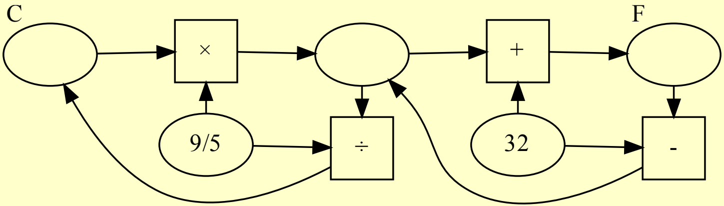


$$^\circ F = {^\circ C} \times \frac{9}{5} + 32$$



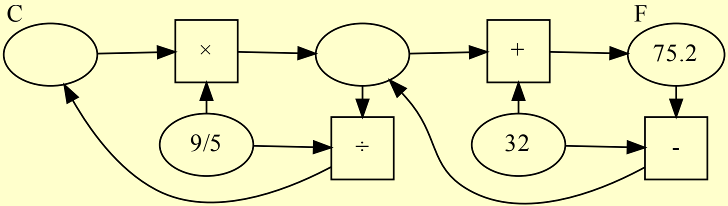
$$^\circ F = {^\circ C} \times \frac{9}{5} + 32$$

$$^\circ C = ({^\circ F} - 32) \div \frac{9}{5}$$



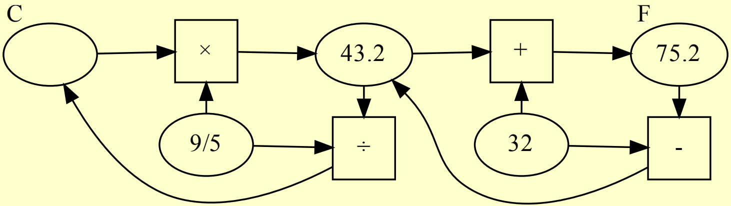
$$^\circ F = {}^\circ C \times \frac{9}{5} + 32$$

$$^\circ C = ({}^\circ F - 32) \div \frac{9}{5}$$



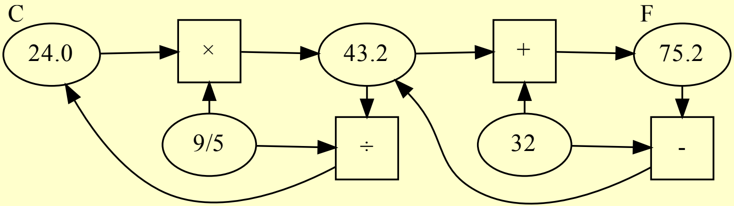
$$°F = °C \times \frac{9}{5} + 32$$

$$°C = (°F - 32) \div \frac{9}{5}$$



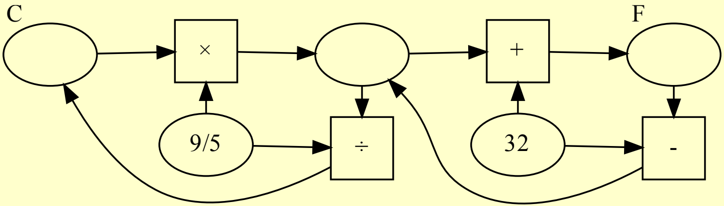
$$^\circ F = {^\circ C} \times \frac{9}{5} + 32$$

$$^\circ C = ({^\circ F} - 32) \div \frac{9}{5}$$



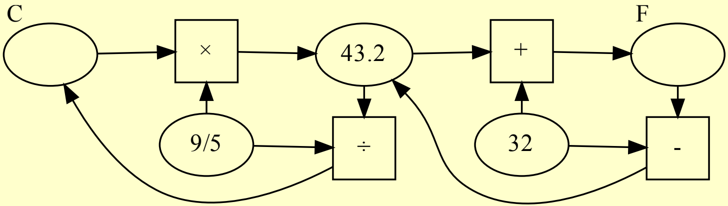
$$°F = °C \times \frac{9}{5} + 32$$

$$°C = (°F - 32) \div \frac{9}{5}$$



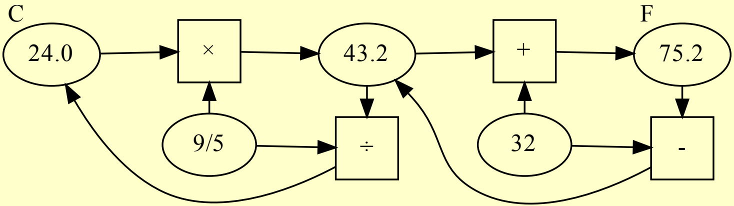
$$°F = °C \times \frac{9}{5} + 32$$

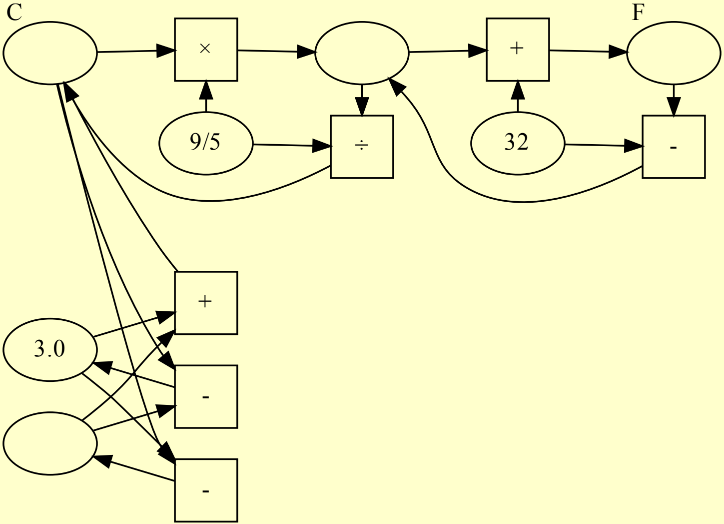
$$°C = (°F - 32) \div \frac{9}{5}$$

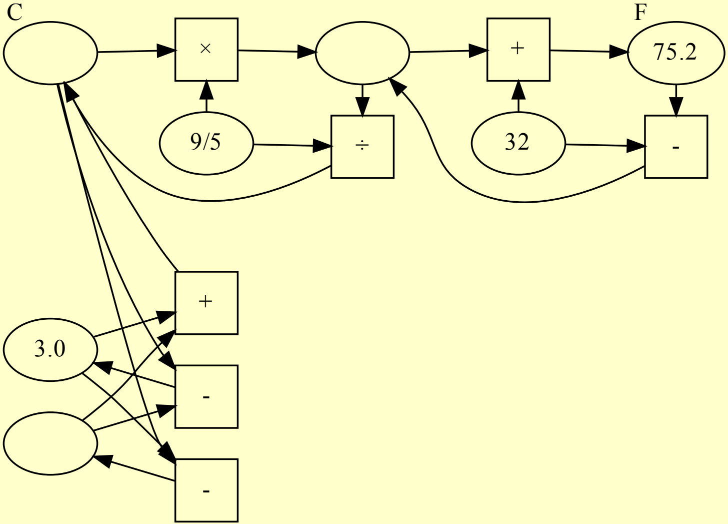


$$°F = °C \times \frac{9}{5} + 32$$

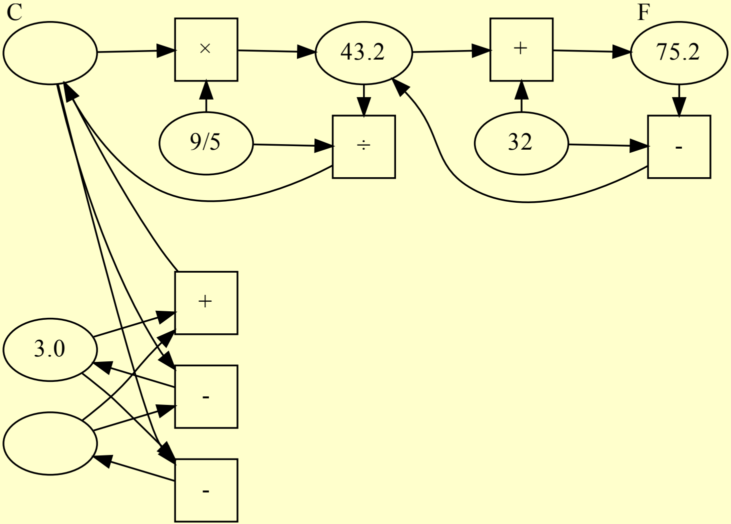
$$°C = (°F - 32) \div \frac{9}{5}$$



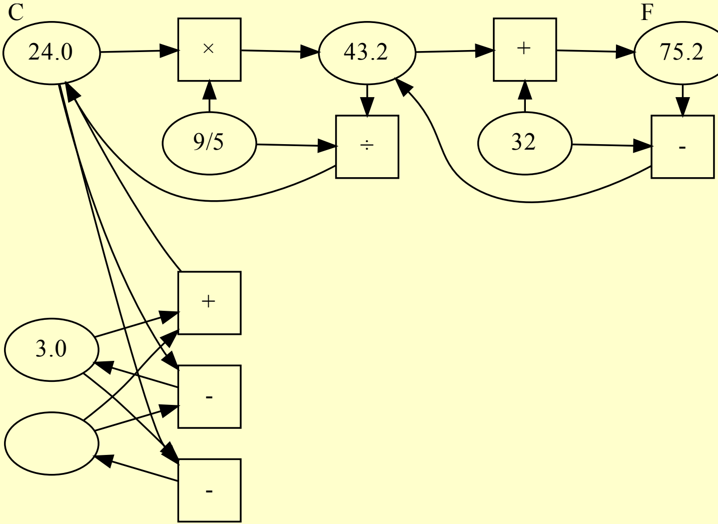

C
×
+
F
9/5
÷
32
-
+
3.0
-
-

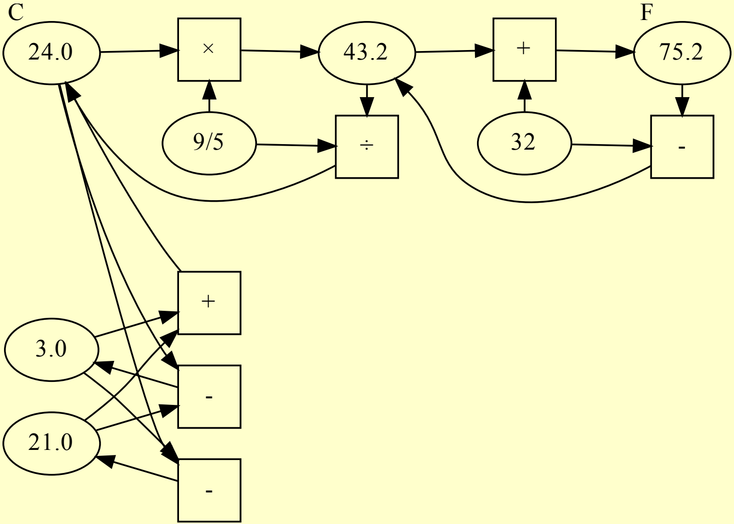


C
×
+
F
75.2
9/5
÷
32
-
+
3.0
-
-



C
×
43.2
+
F
75.2
9/5
÷
32
-
+
3.0
-
-


C
24.0
×
43.2
+
F
75.2
9/5
÷
32
-
+
3.0
-
-



C
24.0
×
43.2
+
F
75.2
9/5
÷
32
-
+
3.0
-
21.0
-

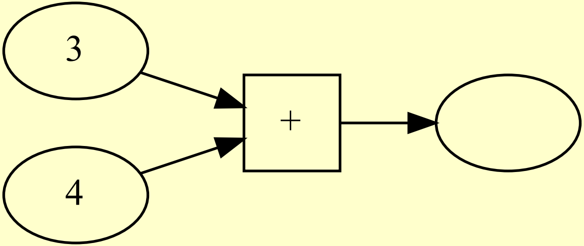
We can combine networks into larger networks!

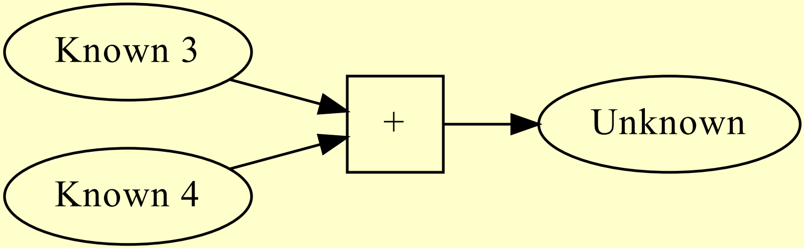
?

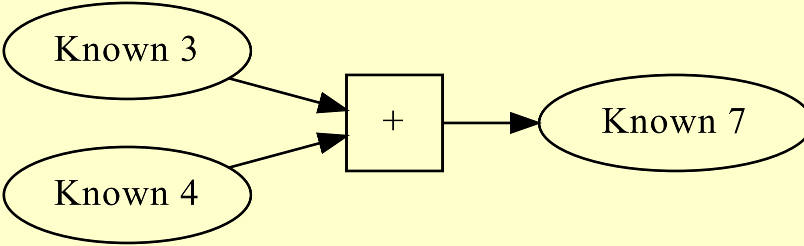
Cells *accumulate information* about a value

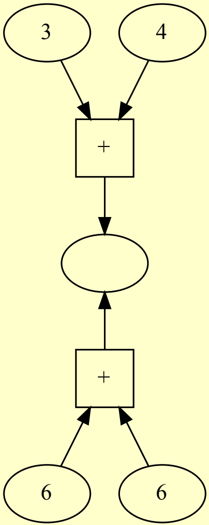
```
data Perhaps a = Unknown | Known a | Contradiction
```

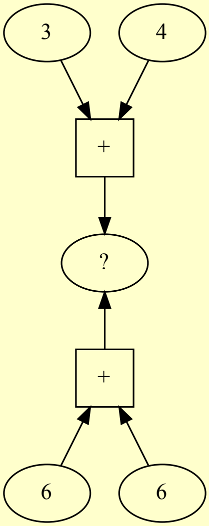
```haskell
data Perhaps a = Unknown | Known a | Contradiction

instance Eq a => Monoid (Perhaps a) where
  mempty = Unknown

  mappend Unknown x             = x
  mappend x       Unknown       = x
  mappend Contradiction _       = Contradiction
  mappend _     Contradiction = Contradiction
  mappend (Known a) (Known b) =
    if a == b
      then Known a
      else Contradiction
```

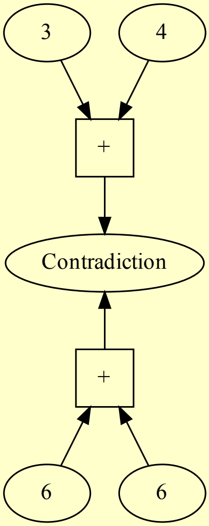

3
4
+

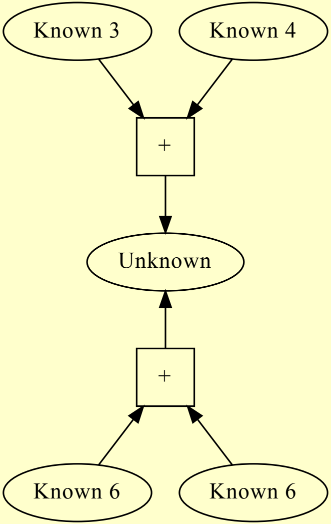

Known 3
Known 4
+
Unknown

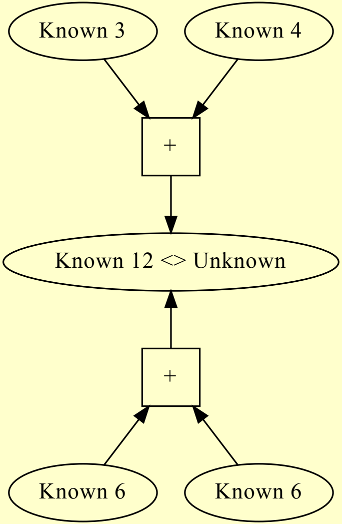

Known 3
Known 4
+
Known 7

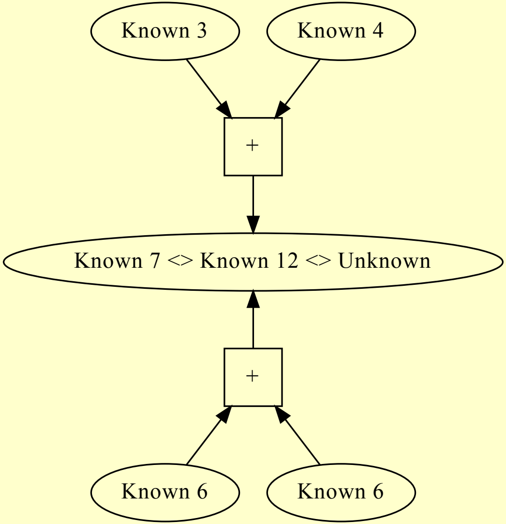

3
4
+
+
6
6

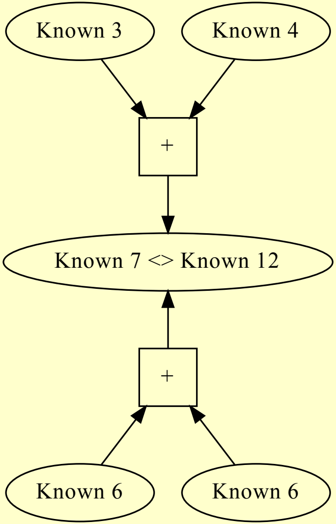

3
4
+
?
+
6
6

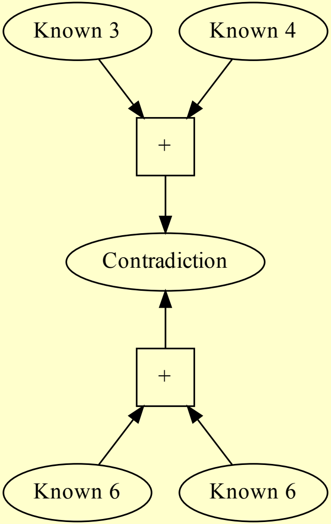

3
4
+
Contradiction
+
6
6

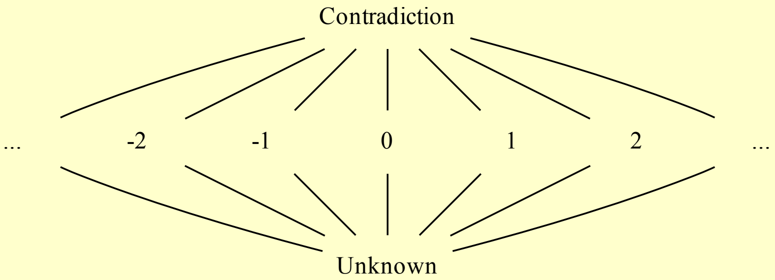

Known 3
Known 4
+
Unknown
+
Known 6
Known 6

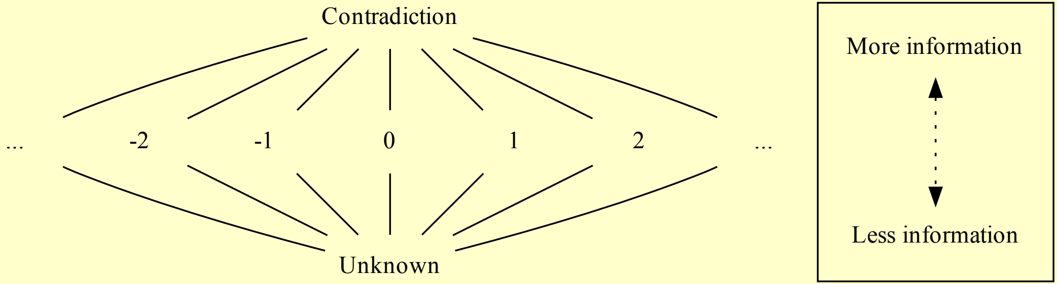

Known 3
Known 4
+
Known 12 <> Unknown
+
Known 6
Known 6


Known 3
Known 4
+
Known 7 <> Known 12 <> Unknown
+
Known 6
Known 6


Known 3
Known 4
+
Known 7 <> Known 12
+
Known 6
Known 6


Known 3
Known 4
+
Contradiction
+
Known 6
Known 6


Contradiction
...
-2
-1
0
1
2
...
Unknown


Contradiction
...
-2
-1
0
1
2
...
Unknown
More information
Less information

3			2
	4	1	
	3	2	
4			1

3			2
	4	1	
	3	2	
4			1

3			2
	4	1	
	3	2	
4			1

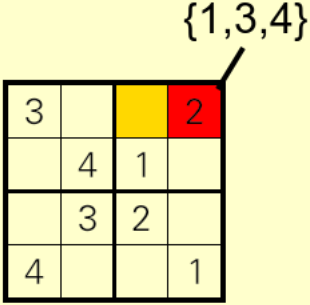
3			2
	4	1	
	3	2	
4			1

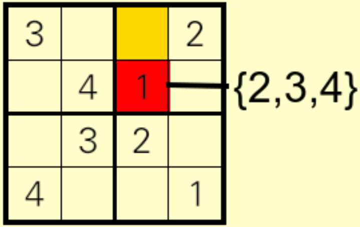
3			2
	4	1	
	3	2	
4			1

3			2
	4	1	
	3	2	
4			1

{1,2,3,4}

3			2
	4	1	
	3	2	
4			1


{1,3,4}
3
2
4
1
3
2
4
1


3 2
4 1
3 2
4 1
{2,3,4}

{1,2,4}

3			2
	4	1	
	3	2	
4			1

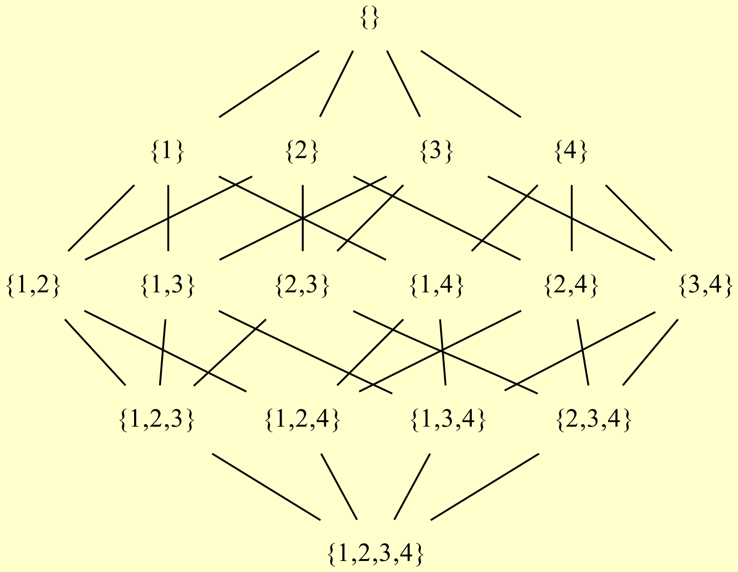
$$\{2,3,4\} \cap \{1,3,4\} \cap \{1,2,4\} \cap \{1,2,3\}$$

3			2
	4	1	
	3	2	
4			1

{4}

3			2
	4	1	
	3	2	
4			1

3		4	2
	4	1	
	3	2	
4			1

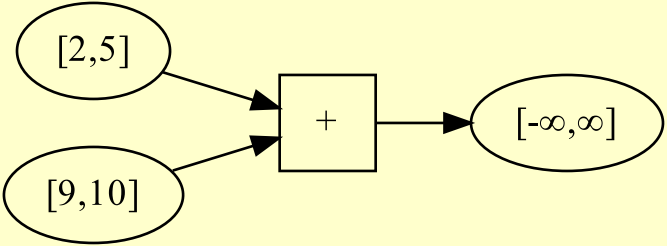

{}
{1}
{2}
{3}
{4}
{1,2}
{1,3}
{2,3}
{1,4}
{2,4}
{3,4}
{1,2,3}
{1,2,4}
{1,3,4}
{2,3,4}
{1,2,3,4}

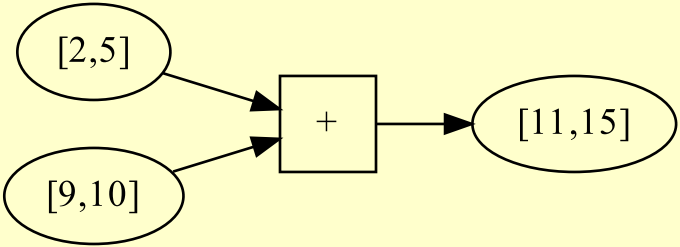
$$[1, 5]$$

$$[1, 5] \cup [2, 7] = [2, 5]$$

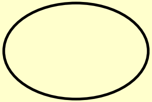
$$[1, 5] \cup [2, 7] = [2, 5]$$

$$[2, 5] + [9, 10] = [11, 15]$$

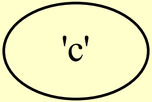

[2,5]
[9,10]
+
[-∞,∞]


[2,5]
[9,10]
+
[11,15]

What types are the values of the cells?



3

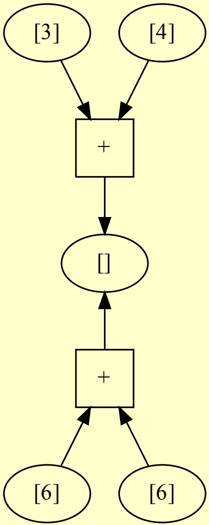

'c'

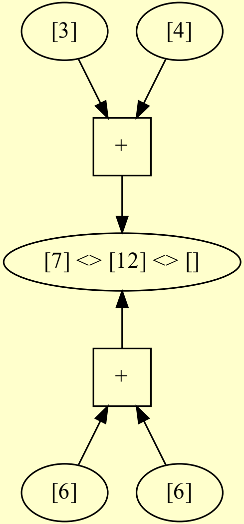
Contradiction

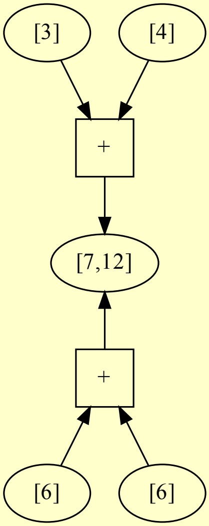
Is this the only type propagator cells can contain?
Will other monoids work?

Is this the only type propagator cells can contain?
Will other monoids work?

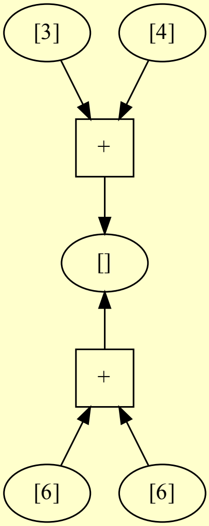
What about List?


[3]
[4]
+
[]
+
[6]
[6]

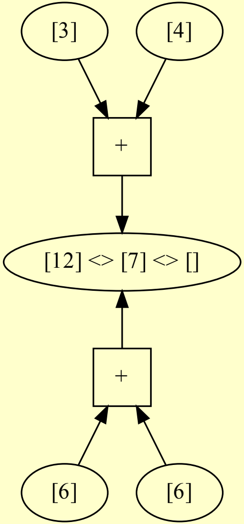

[3]
[4]
+
[7] <> [12] <> []
+
[6]
[6]

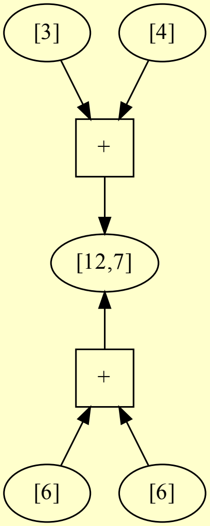

[3]
[4]
+
[7,12]
+
[6]
[6]

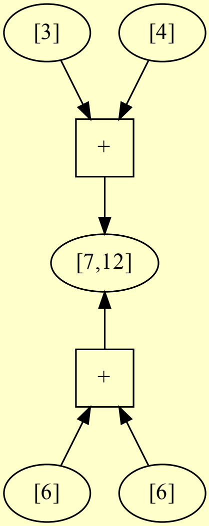
Looking good?

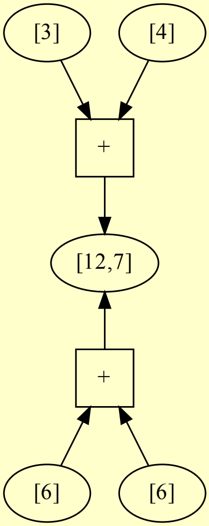


[3]
[4]
+
[]
+
[6]
[6]


[3]
[4]
+
[12] <> [7] <> []
+
[6]
[6]


[3]
[4]
+
[12,7]
+
[6]
[6]


[3]
[4]
+
[7,12]
+
[6]
[6]


[3]
[4]
+
[12,7]
+
[6]
[6]

We need commutativity!

$$x \oplus y = y \oplus x$$

We need commutativity!

$$x \oplus y = y \oplus x$$

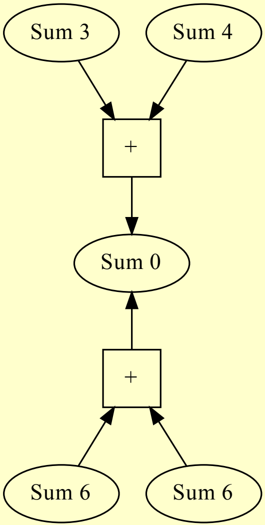
List append is not commutative!

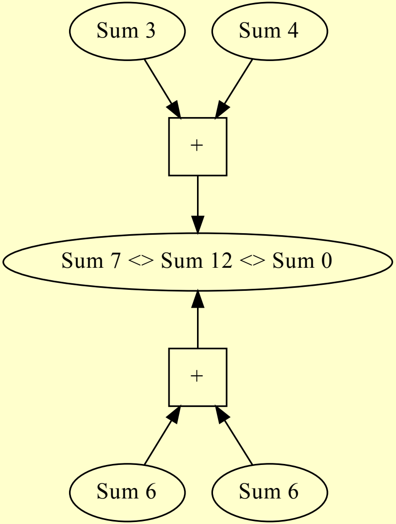
```
[1,2,3] <> [4,5,6] == [1,2,3,4,5,6]
[4,5,6] <> [1,2,3] == [4,5,6,1,2,3]
```

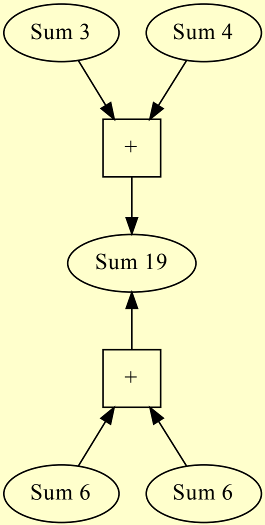
We need a commutative monoid

What about addition?

$$x + y = y + x$$


Sum 3
Sum 4
+
Sum 0
+
Sum 6
Sum 6


Sum 3
Sum 4
+
Sum 7 <> Sum 12 <> Sum 0
+
Sum 6
Sum 6



Sum 3
Sum 4
+
Sum 19
+
Sum 6
Sum 6

We need idempotence!

$$x \oplus x = x$$

We need an idempotent, commutative monoid.
This structure is called a *join-semilattice*

Associativity

$$(x \vee y) \vee z = x \vee (y \vee z)$$

Commutativity

$$x \vee y = y \vee x$$

Idempotence

$$x \vee x = x$$

Other examples?

TODO point to Alexey's stuff here

Edward Kmett has worked on:

- Making it go fast
- Scheduling strategies and garbage collection
- Relaxing requirements (Eg. not requiring a full join-semilattice, admitting non-monotone functions)

TODO finish this slide

TODO expand this
In conclusion, propagator networks:

- Are parallelisable and distributable
- Admit any Haskell function you can write today ...
- ...and more functions!
- compute bidirectionally
- give us constraint solving

TODO expand on all this

Things we could build with propagators:

The best spreadsheet