Propagators: An Introduction

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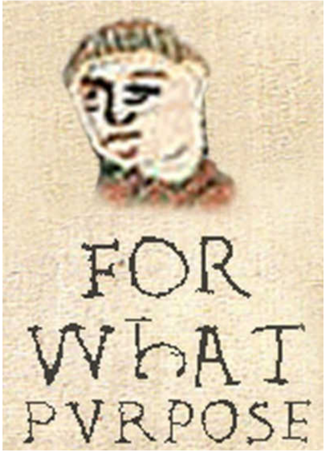
What?



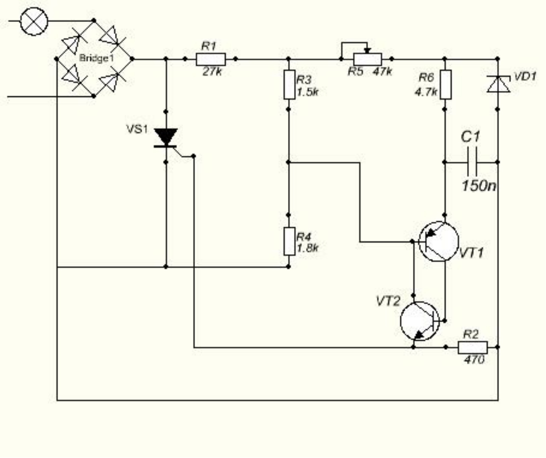
Why?

Roots as early as the 1970's at MIT

- Guy L. Steele Jr.
- Gerald J. Sussman
- Richard Stallman

More recently:

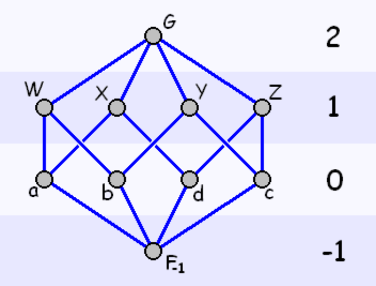
- Alexey Radul



```
(define (map f xs)
  (cond ((null? xs) '())
        (else (cons (f (car xs))
                    (map f (cdr xs))))))
```

And then

- Edward Kmett



$$x \leq y \implies f(x) \leq f(y)$$

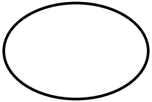
Propagators

The *propagator model* is a model of computation

We model computations as *propagator networks*

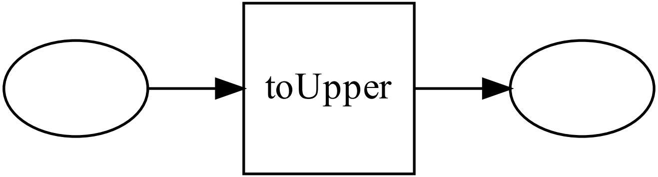
A propagator network comprises

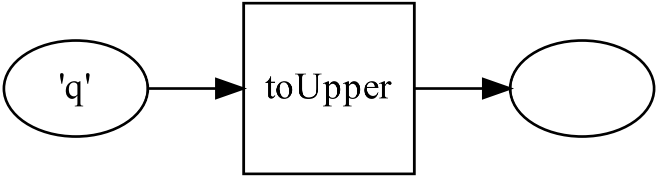
- cells
- propagators
- connections between cells and propagators

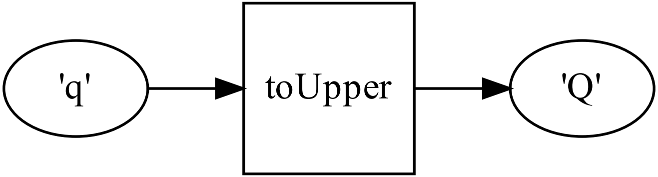


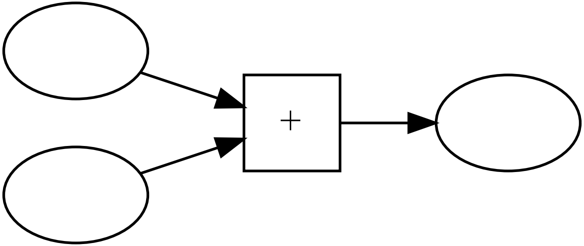
3

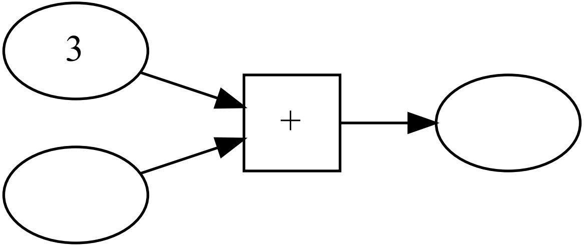
toUpper

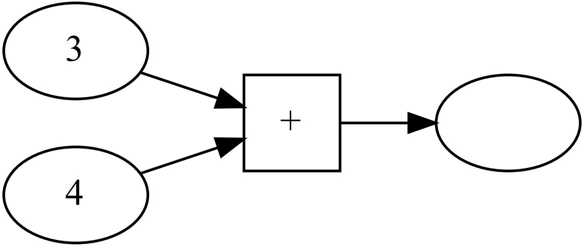

toUpper

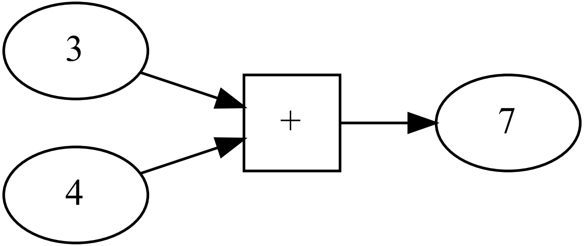

'q'
toUpper


'q'
toUpper
'Q'




3
+


3
4
+


3
4
+
7

$$z \leftarrow x + y$$

$$z = x + y$$

$$7 = x + 4$$

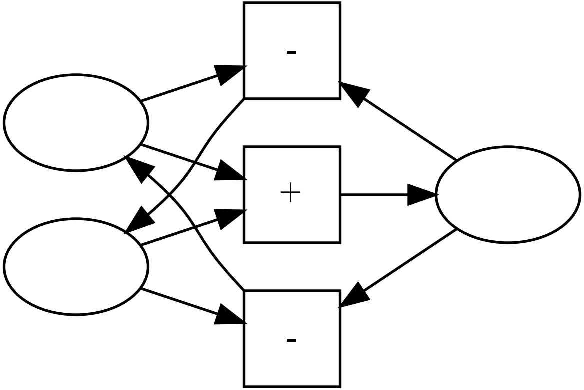
$$7 = 3 + 4$$

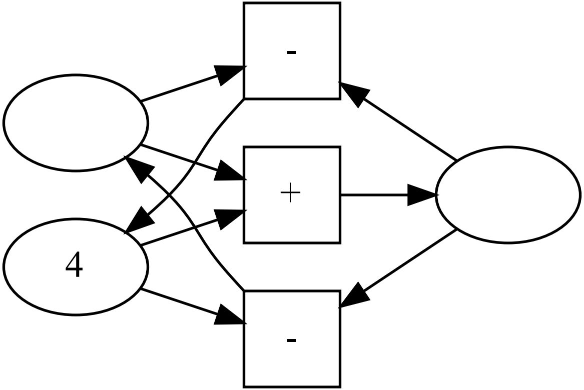
$$z = x + y$$

$$z \leftarrow x + y$$

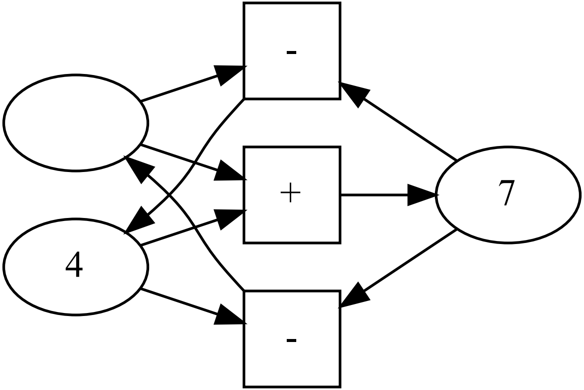
$$x \leftarrow z - y$$

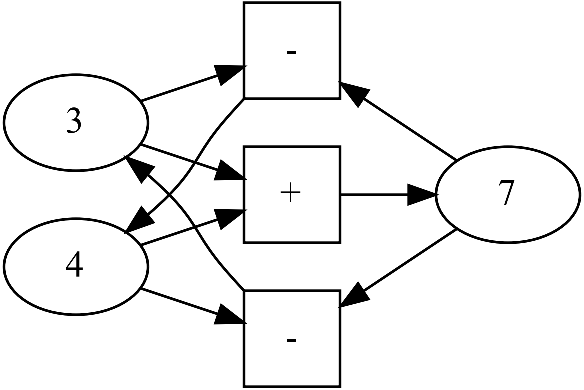
$$y \leftarrow z - x$$





-
+
4
-

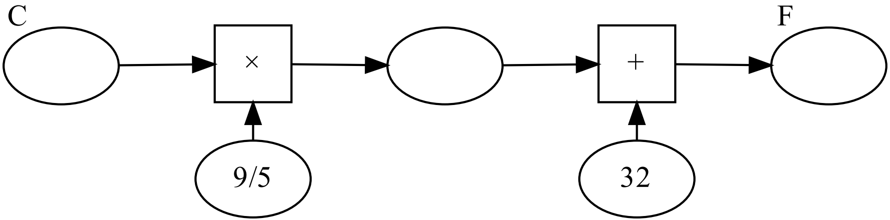

-
+
-
4
7



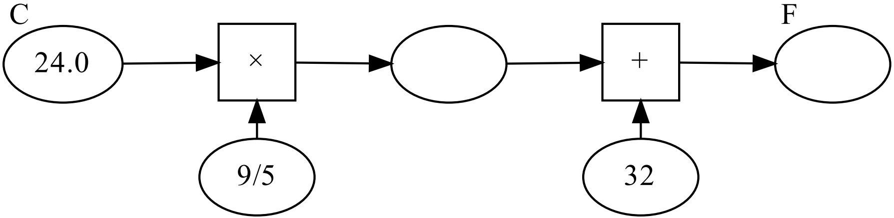
-
3
+
7
4
-

Propagators let us express multidirectional relationships!

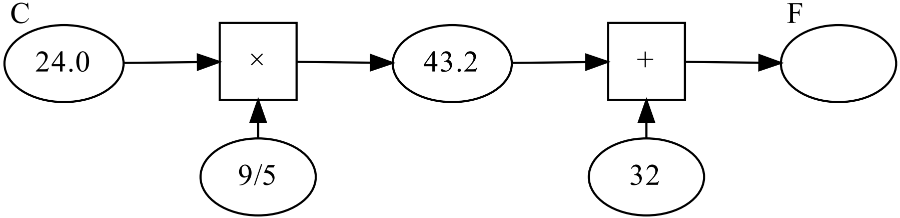
$$°F = °C \times \frac{9}{5} + 32$$



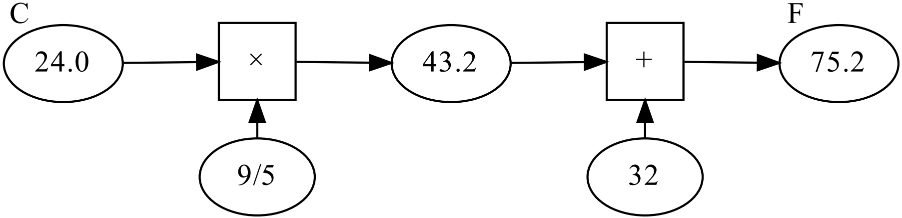
$$°F = °C \times \frac{9}{5} + 32$$



$$^\circ F = {^\circ C} \times \frac{9}{5} + 32$$

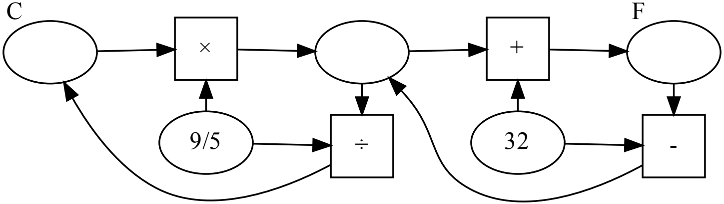


$$^\circ F = {}^\circ C \times \frac{9}{5} + 32$$



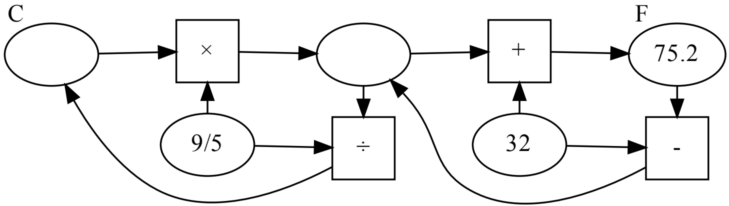
$$°F = °C \times \frac{9}{5} + 32$$

$$°C = (°F - 32) \div \frac{9}{5}$$



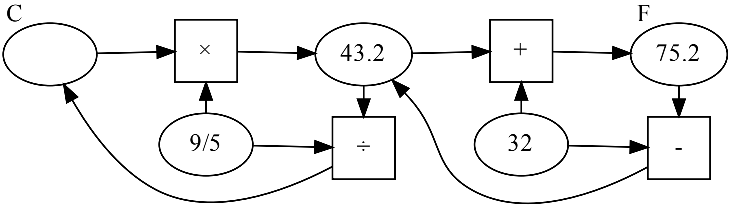
$$^\circ F = {}^\circ C \times \frac{9}{5} + 32$$

$$^\circ C = ({}^\circ F - 32) \div \frac{9}{5}$$



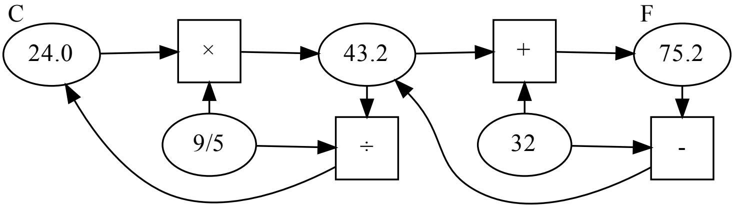
$$^{\circ}F = {}^{\circ}C \times \frac{9}{5} + 32$$

$$^{\circ}C = ({}^{\circ}F - 32) \div \frac{9}{5}$$



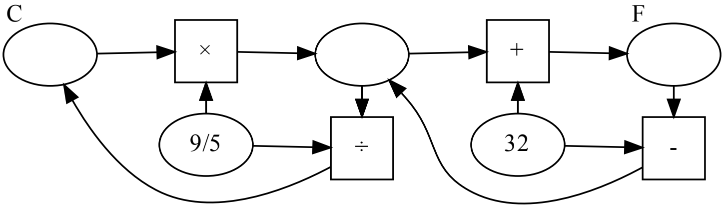
$$°F = °C \times \frac{9}{5} + 32$$

$$°C = (°F - 32) \div \frac{9}{5}$$



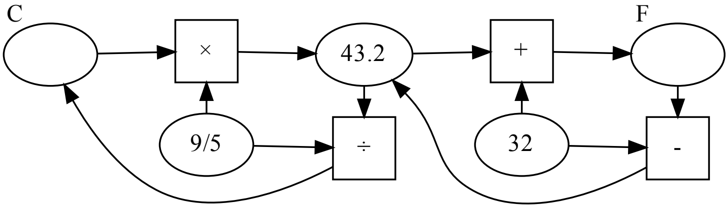
$$^\circ F = {^\circ C} \times \frac{9}{5} + 32$$

$$^\circ C = ({^\circ F} - 32) \div \frac{9}{5}$$



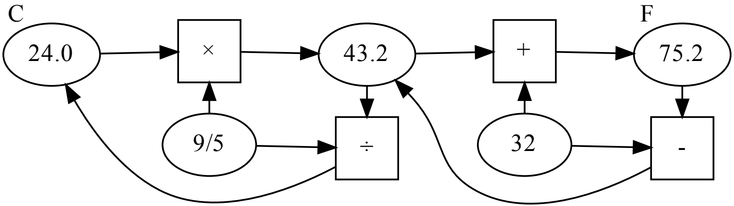
$$^\circ F = {^\circ C} \times \frac{9}{5} + 32$$

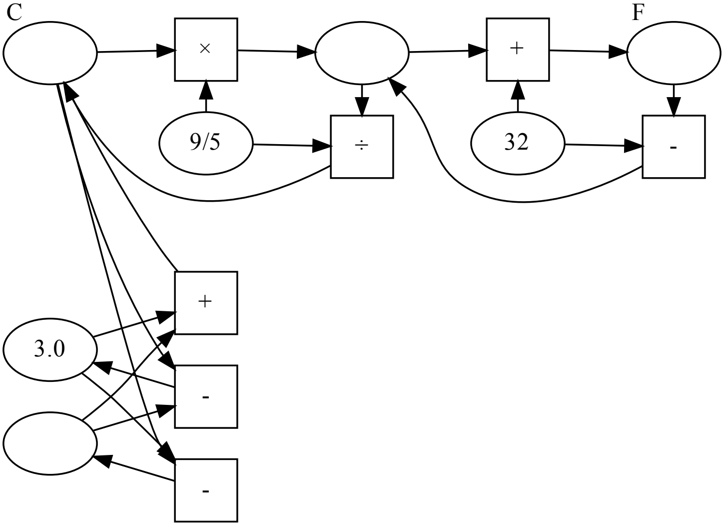
$$^\circ C = ({^\circ F} - 32) \div \frac{9}{5}$$

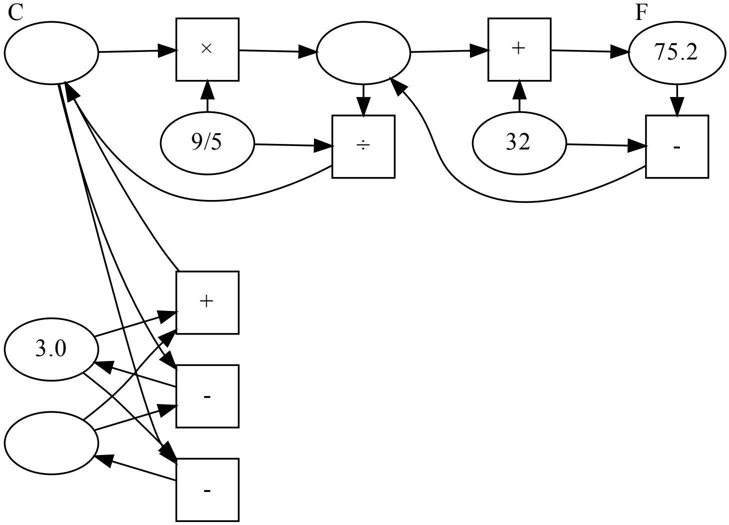


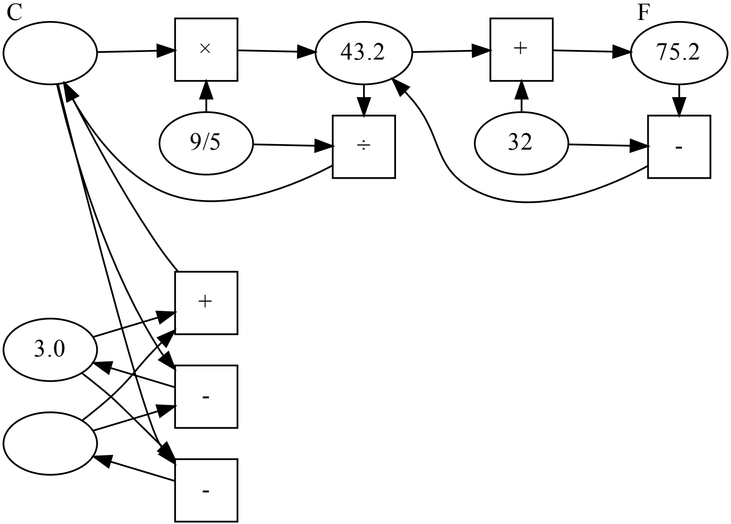
$$^\circ F = {^\circ C} \times \frac{9}{5} + 32$$

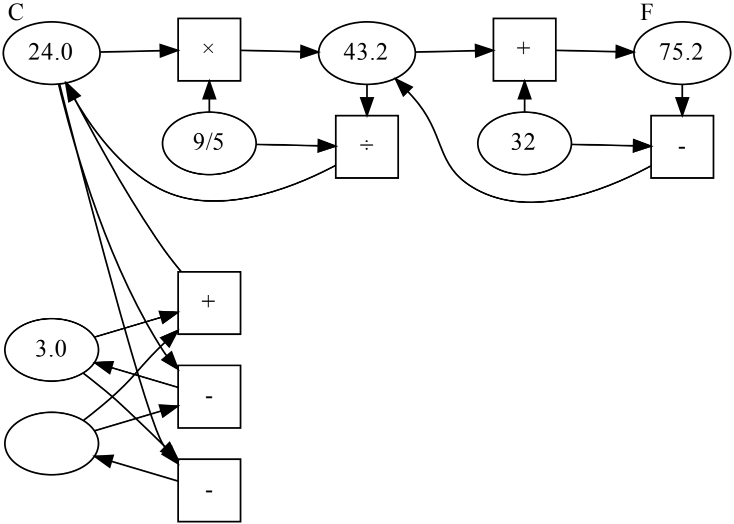
$$^\circ C = ({^\circ F} - 32) \div \frac{9}{5}$$

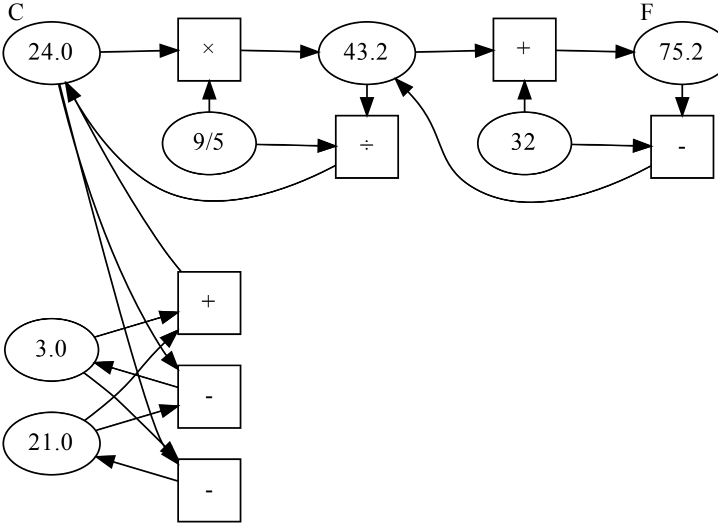



C
×
+
F
9/5
÷
32
-
+
3.0
-
-


C
×
+
F
75.2
9/5
÷
32
-
+
3.0
-
-


C
×
43.2
+
F
75.2
9/5
÷
32
-
+
3.0
-
-

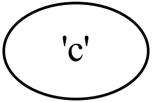

C
24.0
×
43.2
+
F
75.2
9/5
÷
32
-
+
3.0
-
-


C
24.0
×
43.2
+
F
75.2
9/5
÷
32
-
+
3.0
-
21.0
-

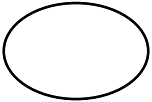
We can combine networks into larger networks!

?

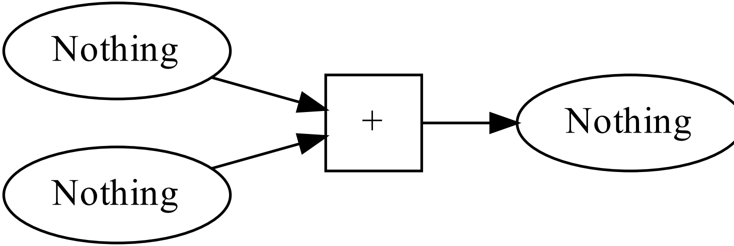
What types are the values of the cells?

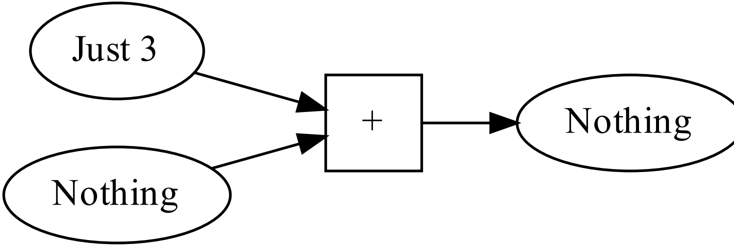

'c'

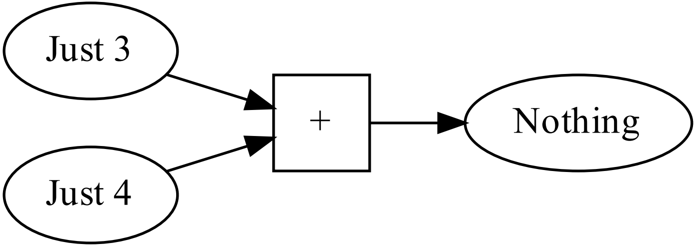
3

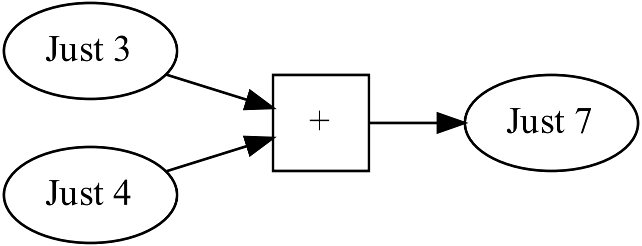


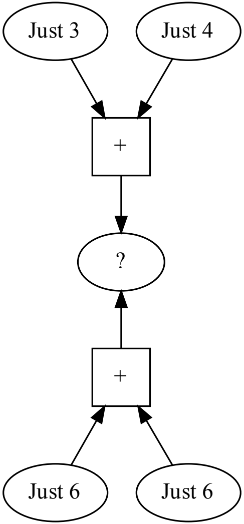
```haskell
data Maybe a = Nothing | Just a
```


Nothing
Nothing
+
Nothing


Just 3
Nothing
+
Nothing


Just 3
Just 4
+
Nothing


Just 3
Just 4
+
Just 7

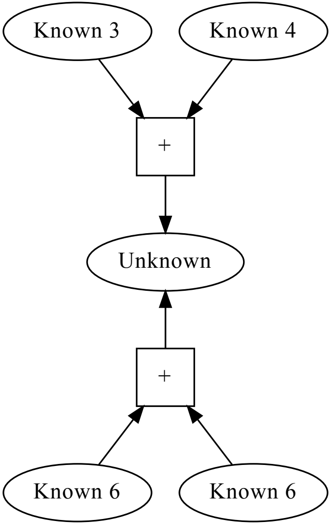

Just 3
Just 4
+
?
+
Just 6
Just 6

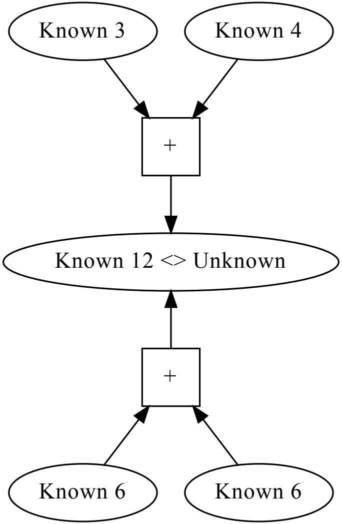
:(

Contradiction

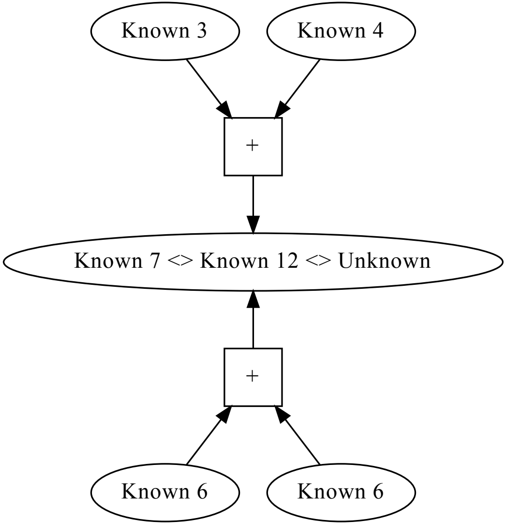
```
data Perhaps a = Unknown | Known a | Contradiction
```

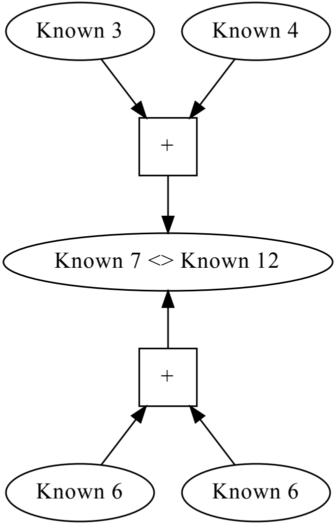
```haskell
data Perhaps a = Unknown | Known a | Contradiction

instance Eq a => Monoid (Perhaps a) where
  mempty = Unknown

  mappend Unknown x             = x
  mappend x       Unknown       = x
  mappend Contradiction _       = Contradiction
  mappend _     Contradiction = Contradiction
  mappend (Known a) (Known b) =
    if a == b
      then Known a
      else Contradiction
```


Known 3
Known 4
+
Unknown
+
Known 6
Known 6

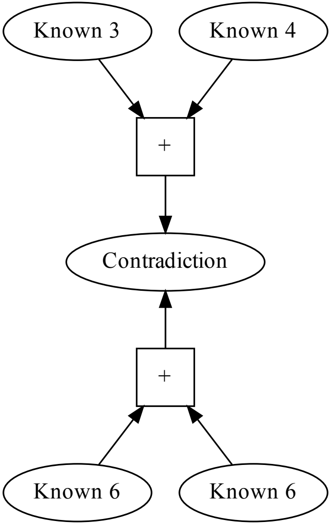


Known 3
Known 4
+
Known 12 <> Unknown
+
Known 6
Known 6


Known 3
Known 4
+
Known 7 <> Known 12 <> Unknown
+
Known 6
Known 6



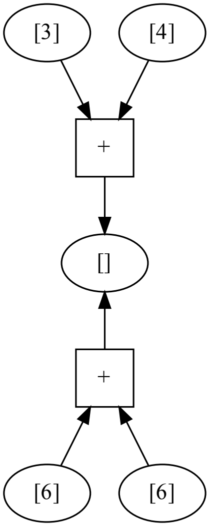
Known 3
Known 4
+
Known 7 <> Known 12
+
Known 6
Known 6


Known 3
Known 4
+
Contradiction
+
Known 6
Known 6

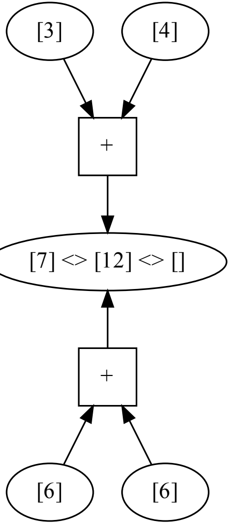
Will other monoids work?

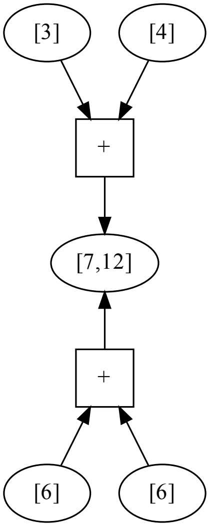
Will other monoids work?

What about List?

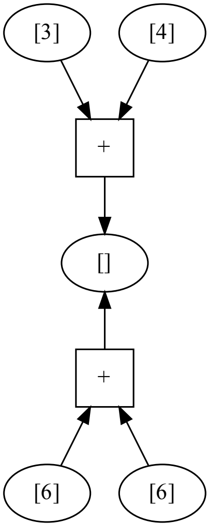


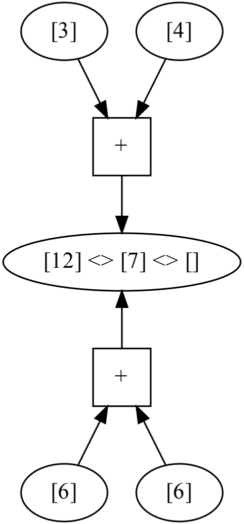
[3]
[4]
+
[]
+
[6]
[6]

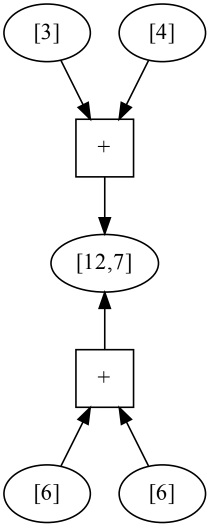

[3]
[4]
+
[7] <> [12] <> []
+
[6]
[6]

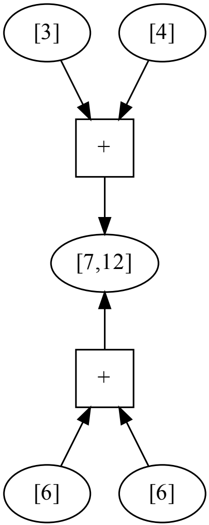

[3]
[4]
+
[7,12]
+
[6]
[6]

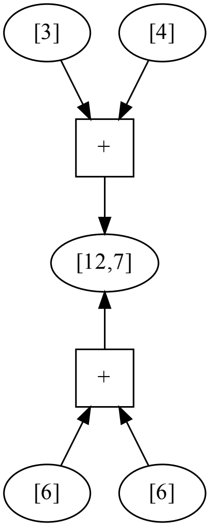
Looking good?


[3]
[4]
+
[]
+
[6]
[6]


[3]
[4]
+
[12] <> [7] <> []
+
[6]
[6]


[3]
[4]
+
[12,7]
+
[6]
[6]


[3]
[4]
+
[7,12]
+
[6]
[6]


[3]
[4]
+
[12,7]
+
[6]
[6]

We need commutativity!

$$x \oplus y = y \oplus x$$

We need commutativity!

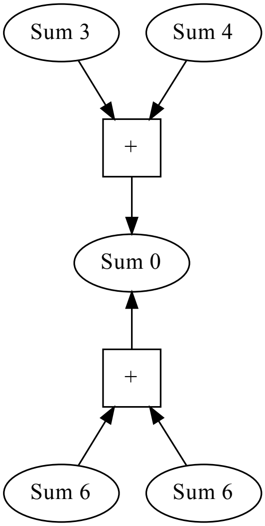
$$x \oplus y = y \oplus x$$

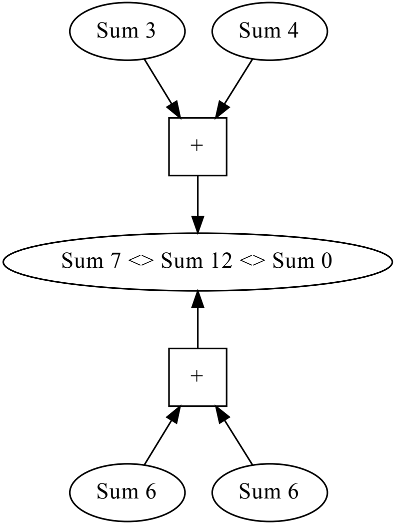
List append is not commutative!

```
[1,2,3] <> [4,5,6] == [1,2,3,4,5,6]
[4,5,6] <> [1,2,3] == [4,5,6,1,2,3]
```

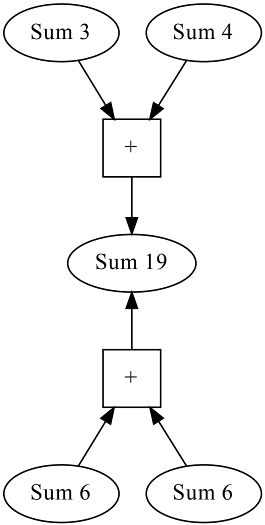
We need a commutative monoid
What about addition?

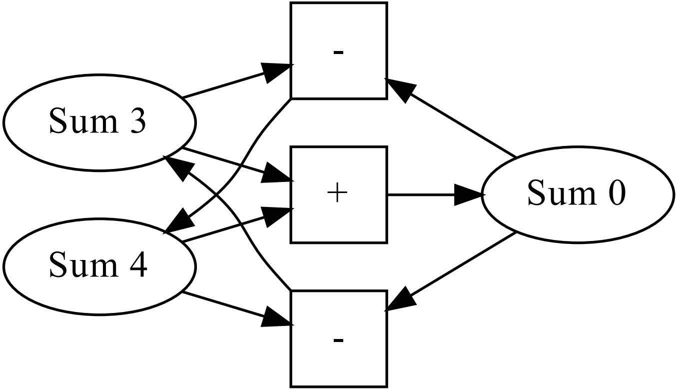
$$x + y = y + x$$

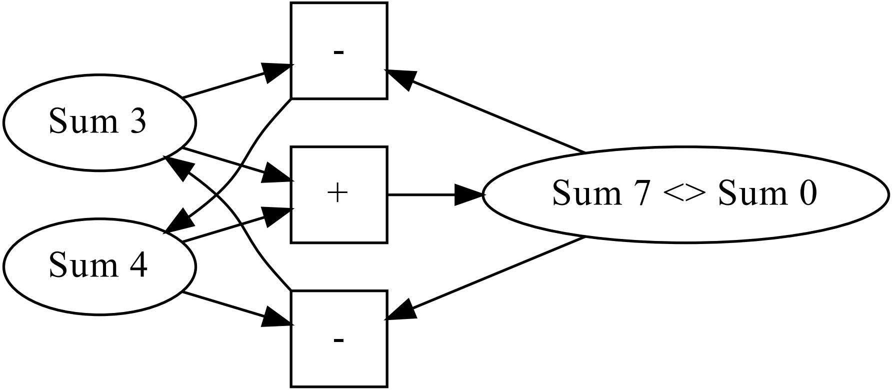

Sum 3
Sum 4
+
Sum 0
+
Sum 6
Sum 6



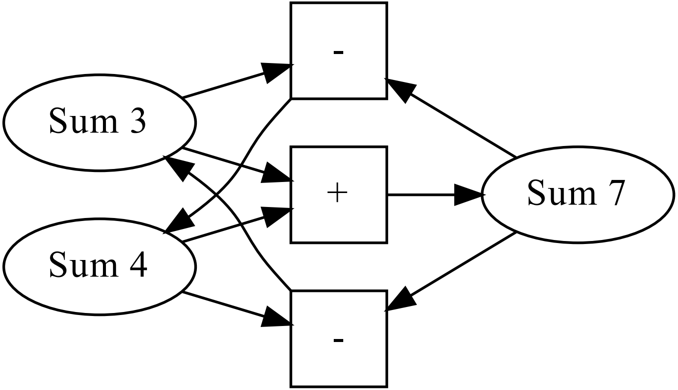
Sum 3
Sum 4
+
Sum 7 <> Sum 12 <> Sum 0
+
Sum 6
Sum 6

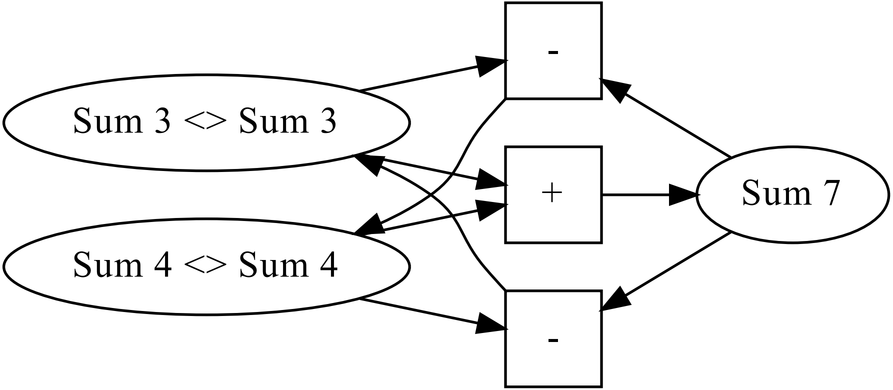

Sum 3
Sum 4
+
Sum 19
+
Sum 6
Sum 6


Sum 3
Sum 4
-
+
-
Sum 0

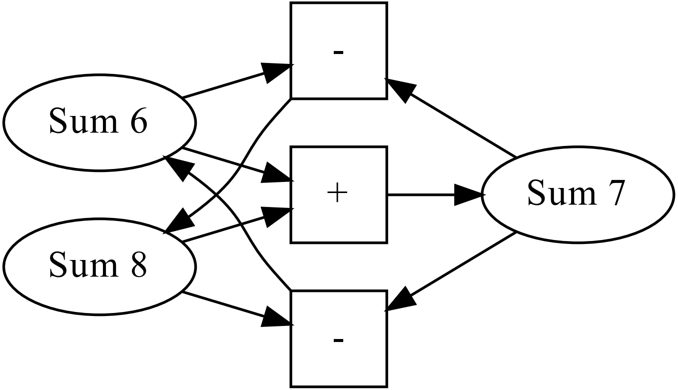


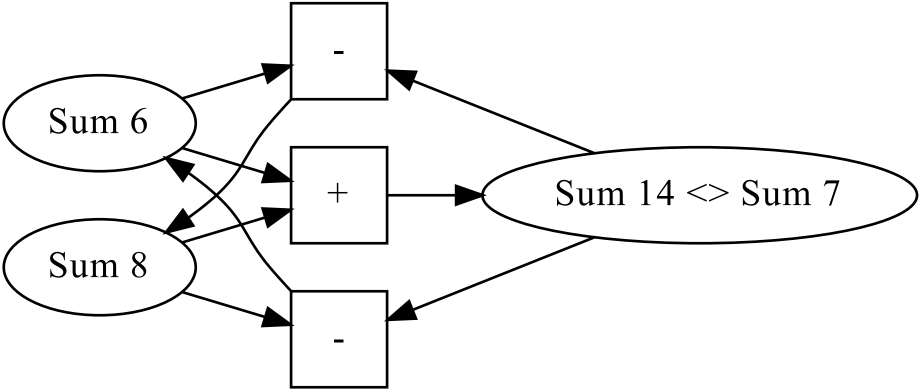
Sum 3
Sum 4
-
+
-
Sum 7 <> Sum 0

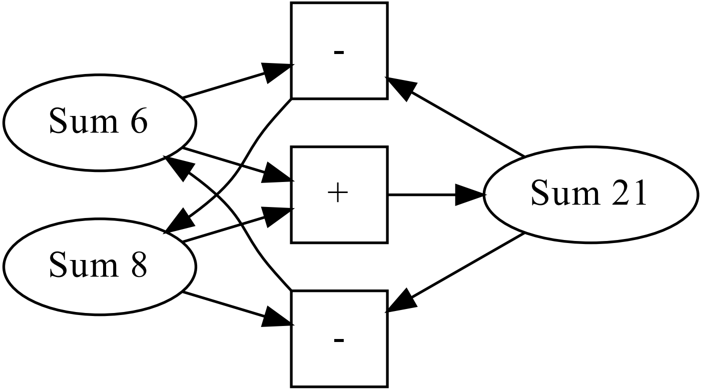

Sum 3
Sum 4
-
+
-
Sum 7



Sum 3 <> Sum 3
Sum 4 <> Sum 4
-
+
-
Sum 7


-
Sum 6
+
Sum 7
Sum 8
-


-
Sum 6
+
Sum 14 <> Sum 7
Sum 8
-


-
Sum 6
+
Sum 21
Sum 8
-

We need idempotence!

$$x \oplus x = x$$

We need an idempotent, commutative monoid.
This structure is called a *join-semilattice*

Associativity

$$(x \vee y) \vee z = x \vee (y \vee z)$$

Commutativity

$$x \vee y = y \vee x$$

Idempotence

$$x \vee x = x$$

Partial information that supports merging!

Other examples?

$$[1, 5]$$

$$[1,5] <> [2,7] = [2,5]$$

$$\{True,\ False\}$$

TODO set intersection examples