

Propagators: An Introduction

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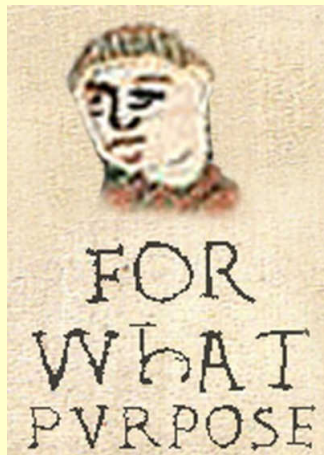
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What?



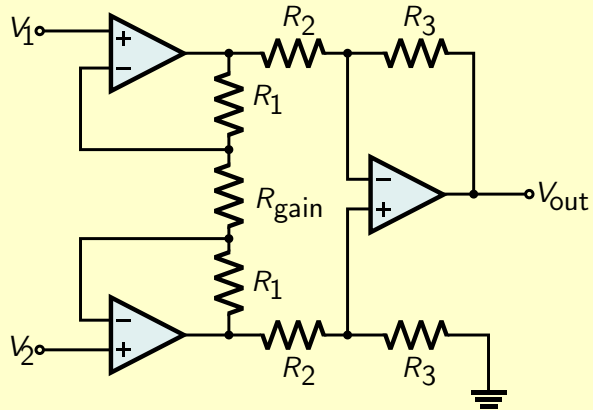
Why?

Beginnings as early as the 1970's at MIT

- Guy L. Steele Jr.
- Gerald J. Sussman
- Richard Stallman

More recently:

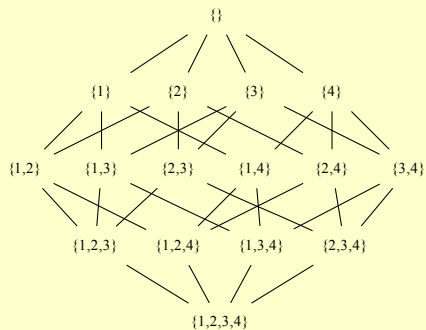
- Alexey Radul



```
(define (map f xs)
  (cond ((null? xs) '())
        (else (cons (f (car xs))
                      (map f (cdr xs)))))))
```

And then

- Edward Kmett



$$x \leq y \implies f(x) \leq f(y)$$

Propagators

The *propagator model* is a model of computation
We model computations as *propagator networks*

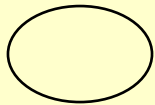
The *propagator model* is a model of computation
We model computations as *propagator networks*

Propagator networks:

- are extremely expressive
- lend themselves to parallel and distributed evaluation
- allow different strategies of problem-solving to seamlessly cooperate

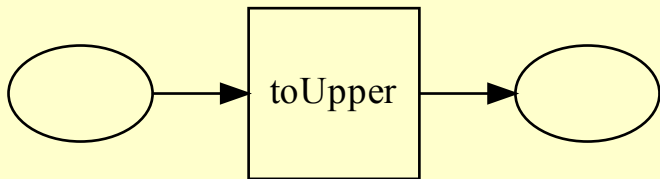
A propagator network comprises

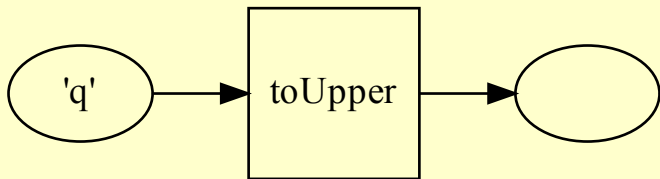
- cells
- propagators
- connections between cells and propagators

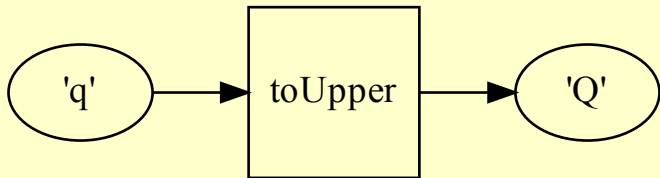


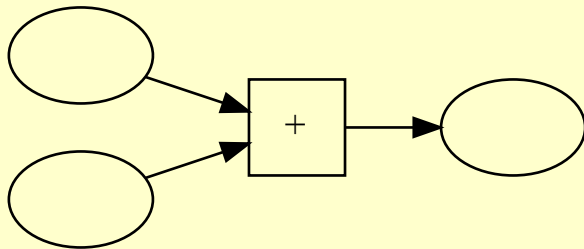
3

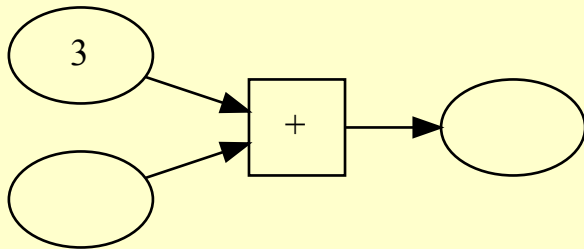
toUpper

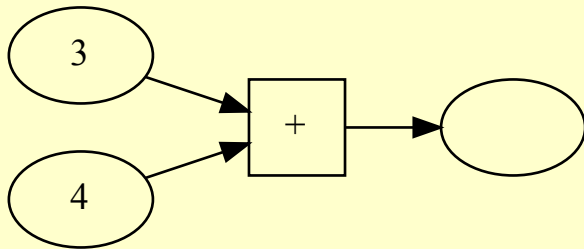


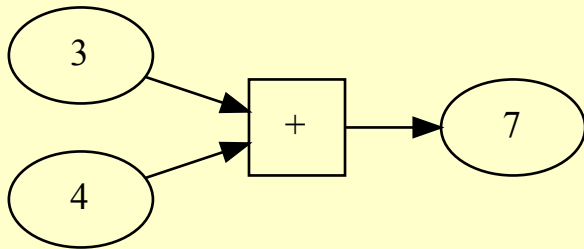












$$z \leftarrow x + y$$

$$z = x + y$$

$$7 = x + 4$$

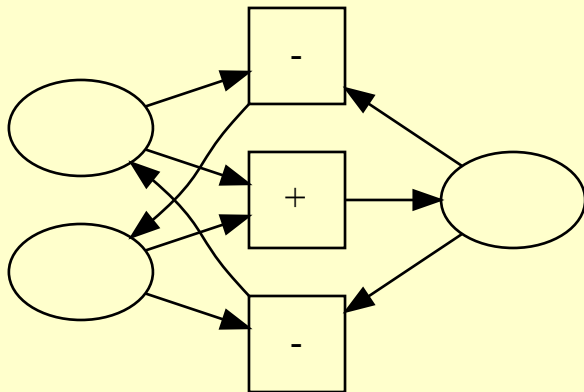
$$7 = 3 + 4$$

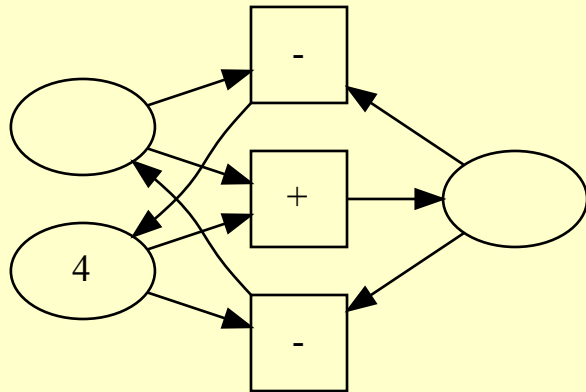
$$z = x + y$$

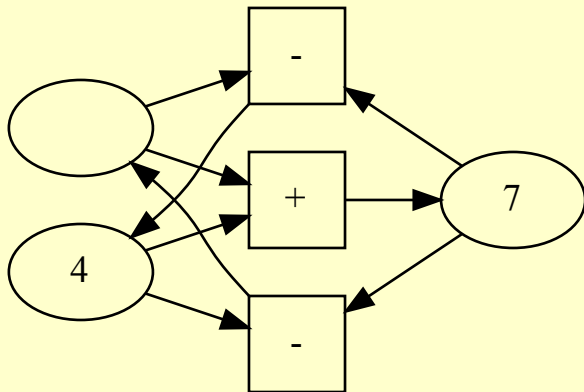
$$z \leftarrow x + y$$

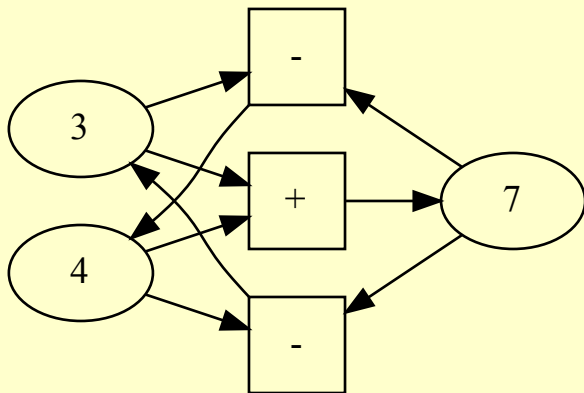
$$x \leftarrow z - y$$

$$y \leftarrow z - x$$



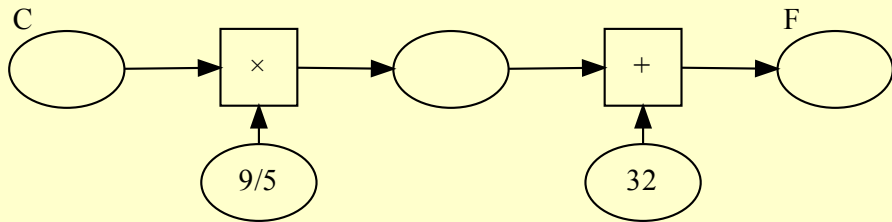




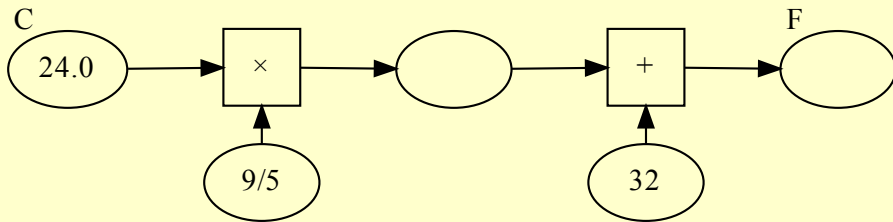


Propagators let us express multi-directional relationships!

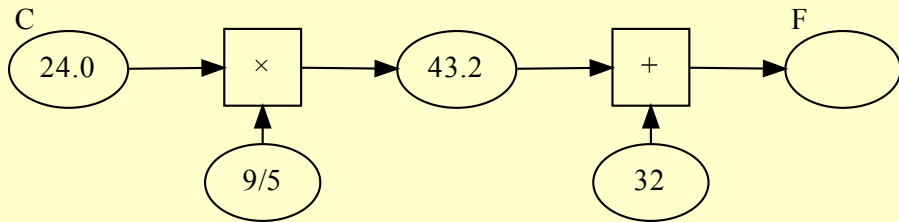
$$^{\circ}F = ^{\circ}C \times \frac{9}{5} + 32$$



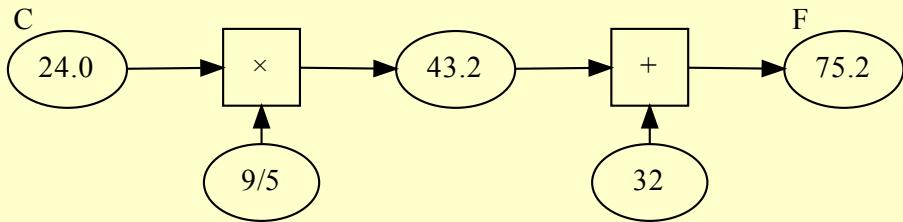
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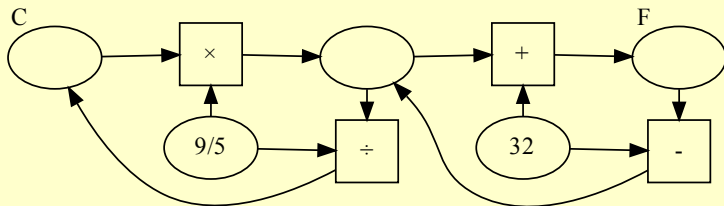


$$^{\circ}F = ^{\circ}C \times \frac{9}{5} + 32$$



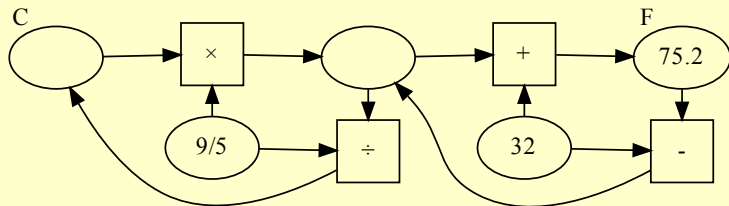
$$^{\circ}F = ^{\circ}C \times \frac{9}{5} + 32$$

$$^{\circ}C = (^{\circ}F - 32) \div \frac{9}{5}$$



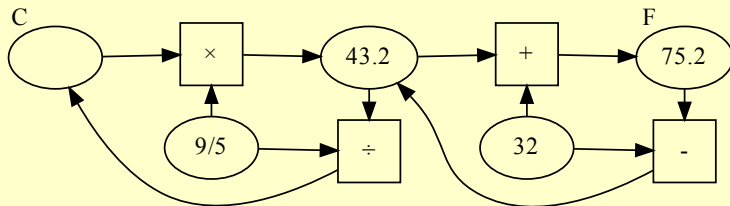
$$^{\circ}F = ^{\circ}C \times \frac{9}{5} + 32$$

$$^{\circ}C = (^{\circ}F - 32) \div \frac{9}{5}$$



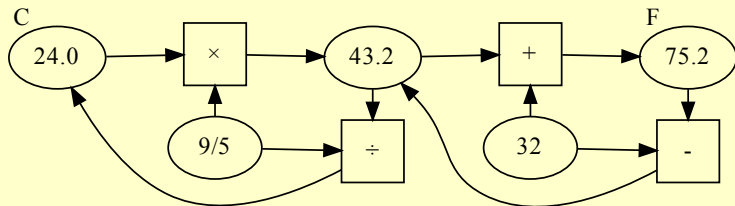
$$^{\circ}F = ^{\circ}C \times \frac{9}{5} + 32$$

$$^{\circ}C = (^{\circ}F - 32) \div \frac{9}{5}$$



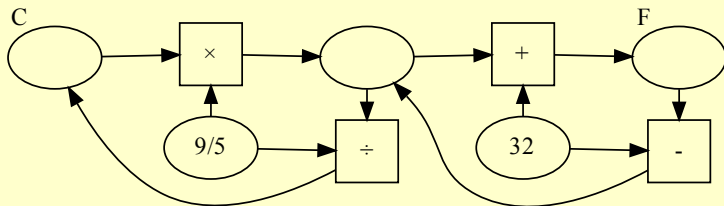
$$^{\circ}F = ^{\circ}C \times \frac{9}{5} + 32$$

$$^{\circ}C = (^{\circ}F - 32) \div \frac{9}{5}$$



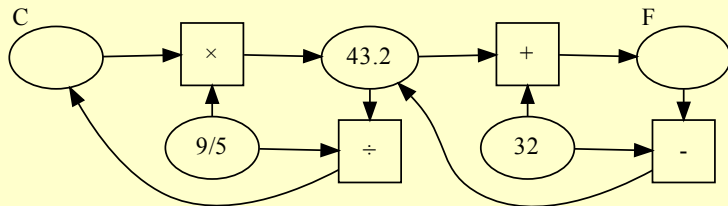
$$^{\circ}F = ^{\circ}C \times \frac{9}{5} + 32$$

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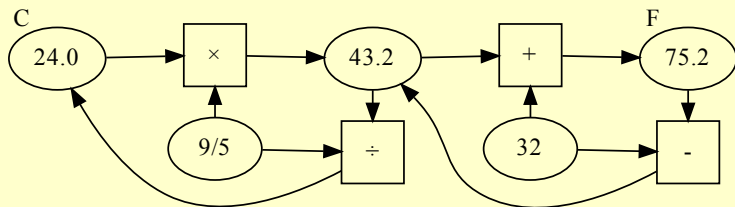
$$^{\circ}F = ^{\circ}C \times \frac{9}{5} + 32$$

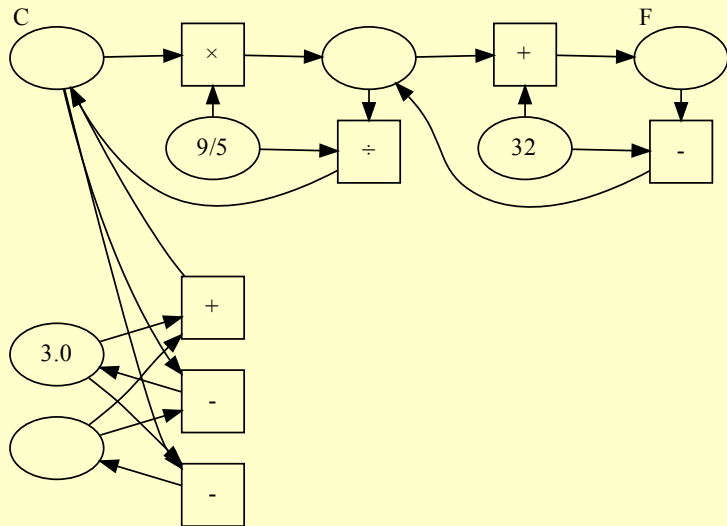
$$^{\circ}C = (^{\circ}F - 32) \div \frac{9}{5}$$

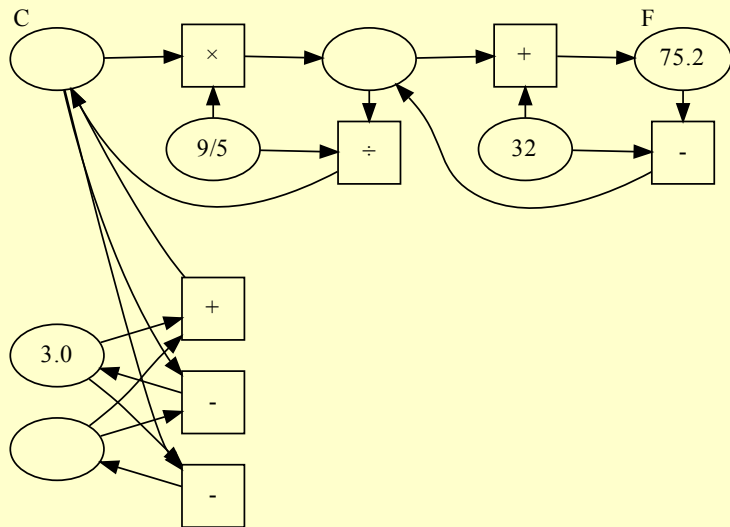


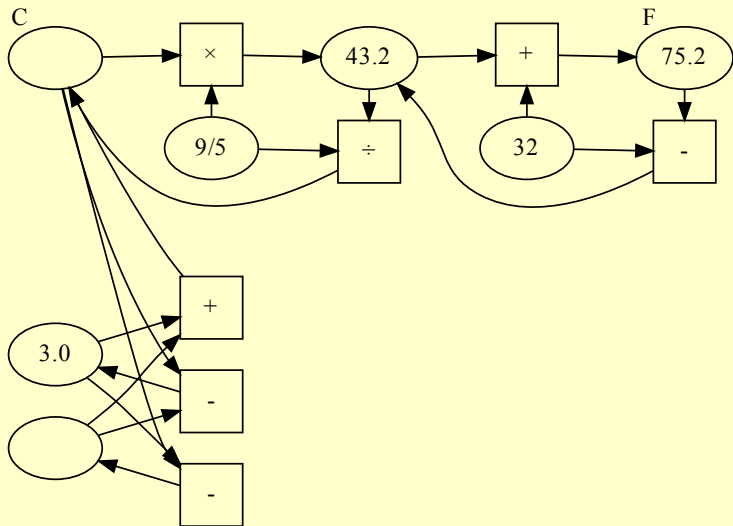
$$^{\circ}F = ^{\circ}C \times \frac{9}{5} + 32$$

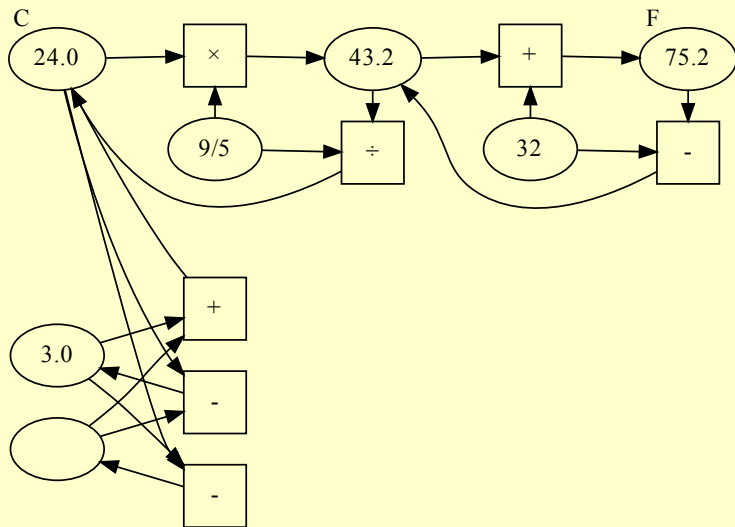
$$^{\circ}C = (^{\circ}F - 32) \div \frac{9}{5}$$

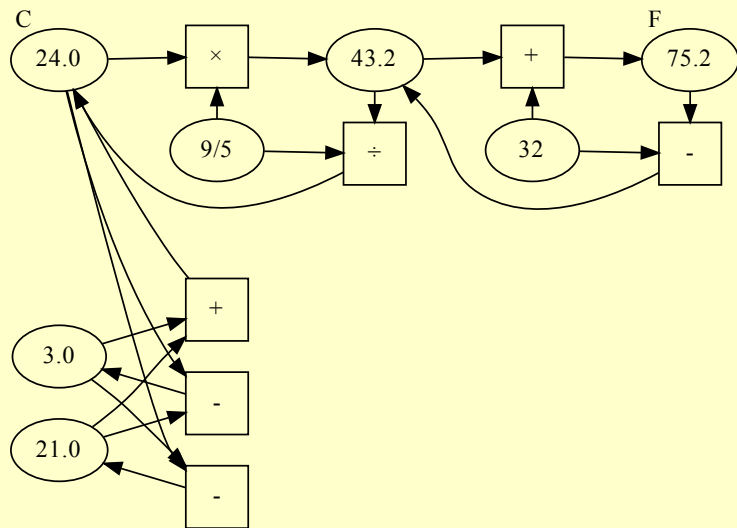












We can combine networks into larger networks!

?

Cells *accumulate information* about a value

3			2
	4	1	
	3	2	
4			1

3			2
	4	1	
	3	2	
4			1

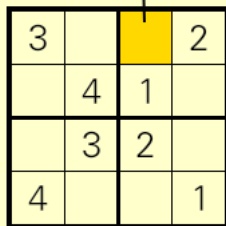
3			2
	4	1	
	3	2	
4			1

3			2
	4	1	
	3	2	
4			1

3			2
	4	1	
	3	2	
4			1

3			2
	4	1	
	3	2	
4			1

$\{1,2,3,4\}$



A 4x4 grid with a yellow cell at (1,3) and a pointer from the set {1,2,3,4} to it.

3			2
	4	1	
	3	2	
4			1

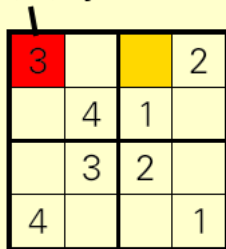
$\{1,3,4\}$

3			2
	4	1	
	3	2	
4			1

3			2
	4	1	
	3	2	
4			1

$\{2,3,4\}$

$\{1,2,4\}$



A 4x4 grid with a red cell containing the number 3 and a yellow cell. An arrow points from the set $\{1,2,4\}$ to the red cell.

3			2
	4	1	
	3	2	
4			1

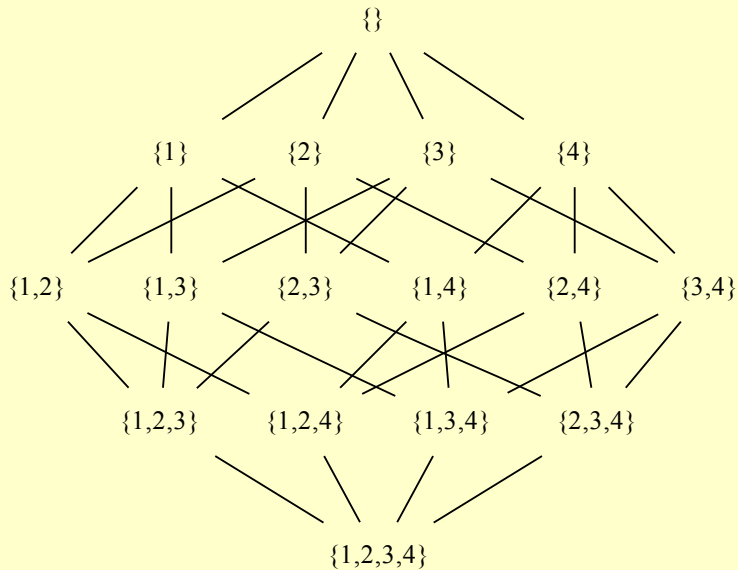
$$\{2,3,4\} \cap \{1,3,4\} \cap \\ \{1,2,4\} \cap \{1,2,3\}$$

3			2
	4	1	
	3	2	
4			1

{4}

3			2
	4	1	
	3	2	
4			1

3		4	2
	4	1	
	3	2	
4			1

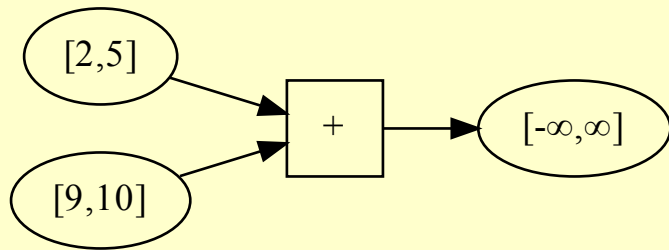


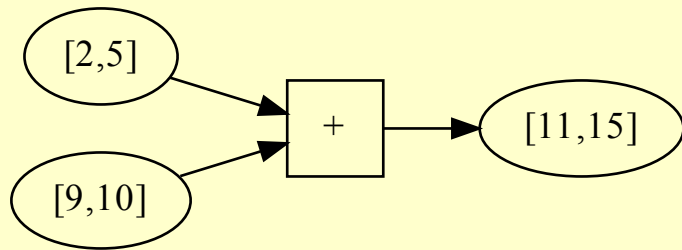
$$[1, 5]$$

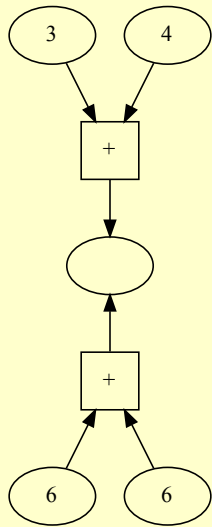
$$[1, 5] \cup [2, 7] = [2, 5]$$

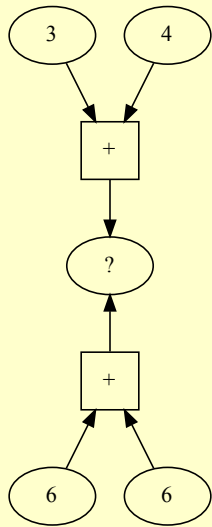
$$[1, 5] \cup [2, 7] = [2, 5]$$

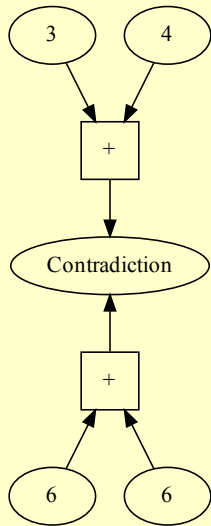
$$[2, 5] + [9, 10] = [11, 15]$$



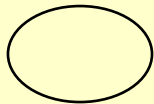








What types are the values of the cells?



3

'c'

Contradiction

data Perhaps a = Unknown | Known a | Contradiction

```
data Perhaps a = Unknown | Known a | Contradiction
```

```
instance Eq a => Monoid (Perhaps a) where
```

```
    mempty = Unknown
```

```
    mappend Unknown x           = x
```

```
    mappend x      Unknown      = x
```

```
    mappend Contradiction _      = Contradiction
```

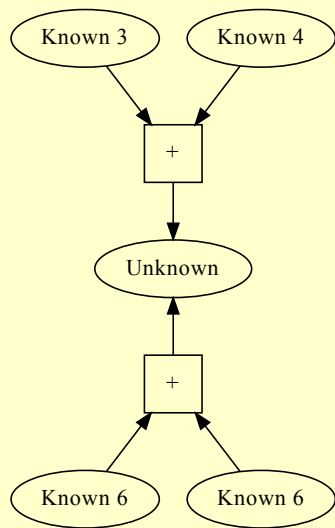
```
    mappend _      Contradiction = Contradiction
```

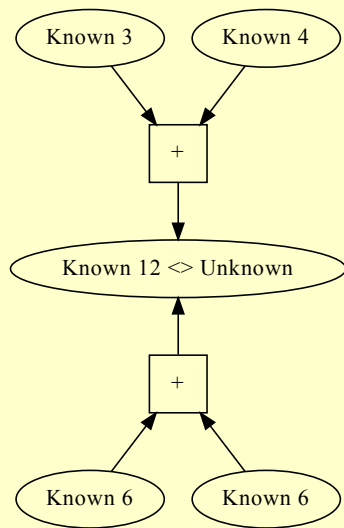
```
    mappend (Known a) (Known b) =
```

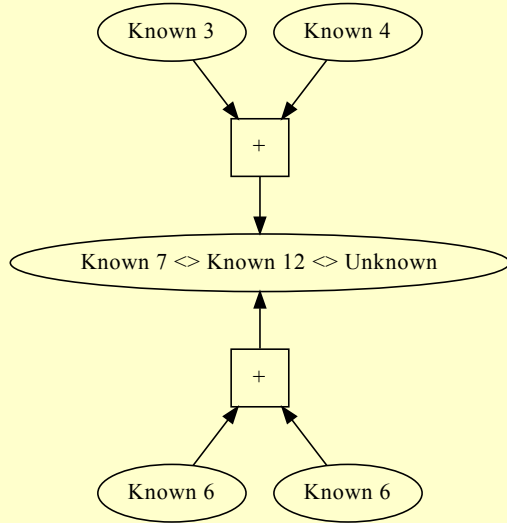
```
        if a == b
```

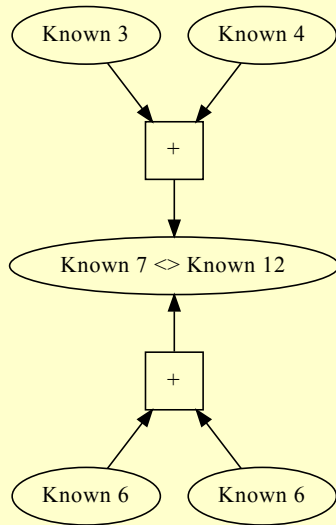
```
        then Known a
```

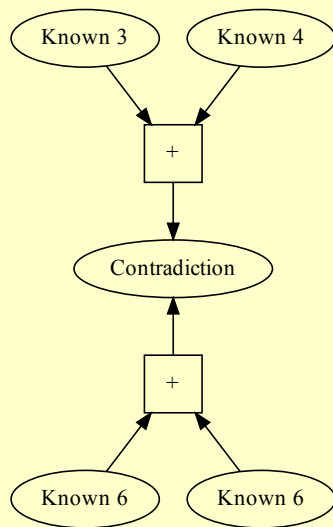
```
        else Contradiction
```







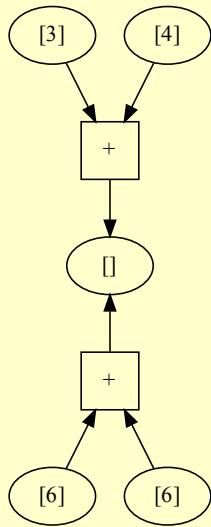


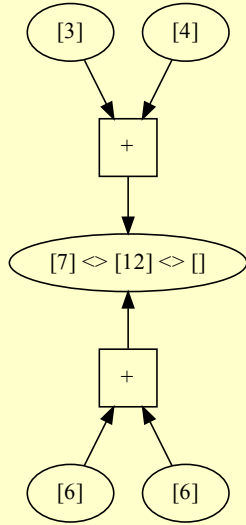


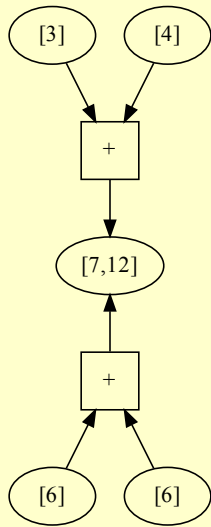
Is this the only type propagator cells can contain?
Will other monoids work?

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Will other monoids work?

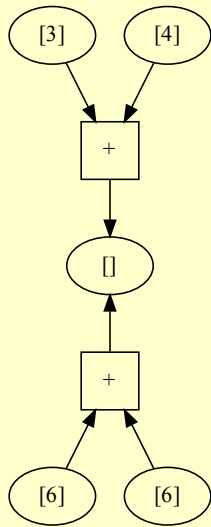
What about List?

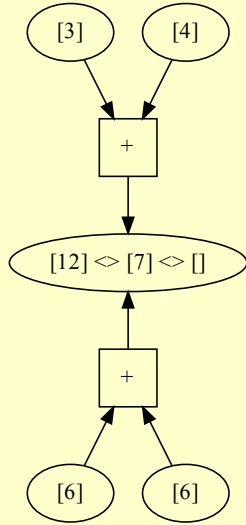


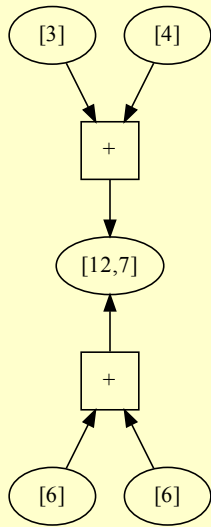


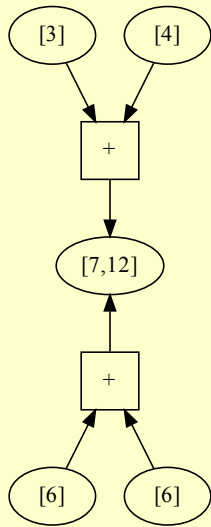


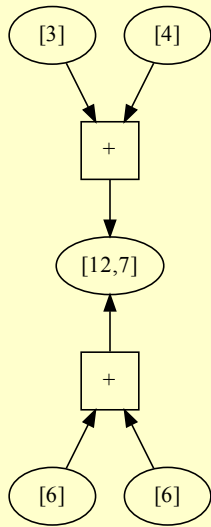
Looking good?











We need commutativity!

$$x \oplus y = y \oplus x$$

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$$x \oplus y = y \oplus x$$

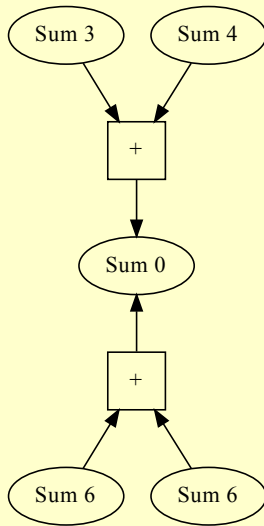
List append is not commutative!

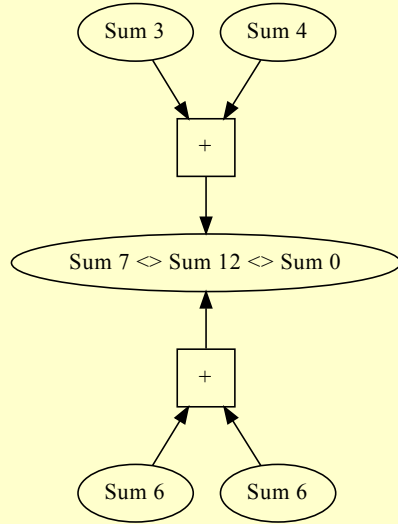
`[1, 2, 3] <> [4, 5, 6] == [1, 2, 3, 4, 5, 6]`

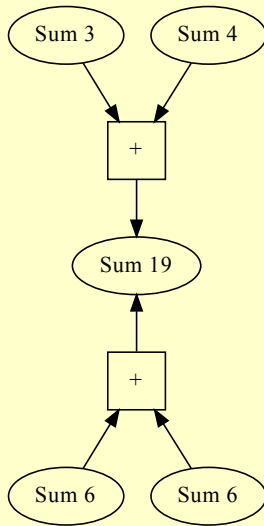
`[4, 5, 6] <> [1, 2, 3] == [4, 5, 6, 1, 2, 3]`

We need a commutative monoid
What about addition?

$$x + y = y + x$$







We need idempotence!

$$x \oplus x = x$$

We need an idempotent, commutative monoid.

This structure is called a *join-semilattice*

Associativity

$$(x \vee y) \vee z = x \vee (y \vee z)$$

Commutativity

$$x \vee y = y \vee x$$

Idempotence

$$x \vee x = x$$

Partial information that supports merging!

Other examples?