Propagators: An Introduction

George Wilson

Data61/CSIRO

george.wilson@data61.csiro.au

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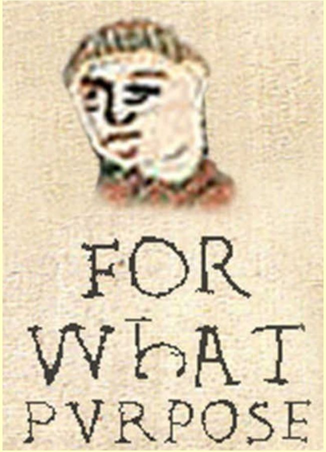
What?



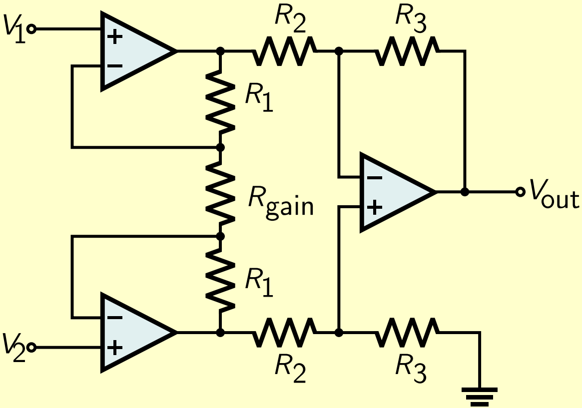
Why?

Beginnings as early as the 1970's at MIT

- Guy L. Steele Jr.
- Gerald J. Sussman
- Richard Stallman

More recently:

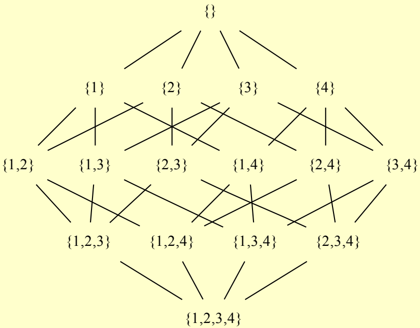
- Alexey Radul



```
(define (map f xs)
  (cond ((null? xs) '())
        (else (cons (f (car xs))
                    (map f (cdr xs))))))
```

And then

- Edward Kmett



$$x \le y \implies f(x) \le f(y)$$

Propagators

The *propagator model* is a model of computation
We model computations as *propagator networks*

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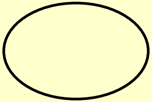
We model computations as *propagator networks*

Propagator networks:

- are extremely expressive
- lend themselves to parallel and distributed evaluation
- allow different strategies of problem-solving to seamlessly cooperate

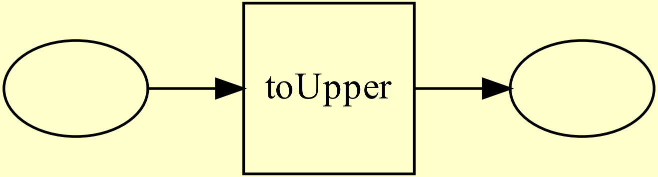
A propagator network comprises

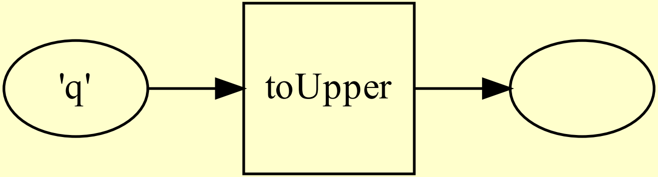
- cells
- propagators
- connections between cells and propagators

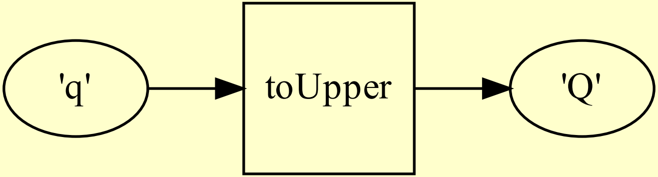


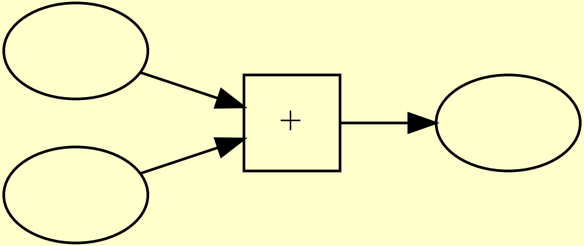
3

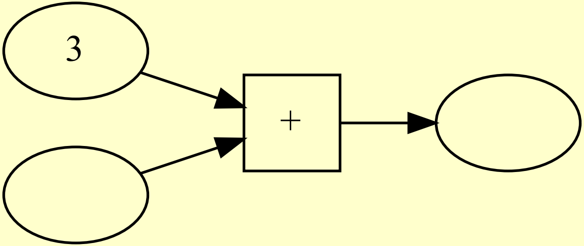
toUpper

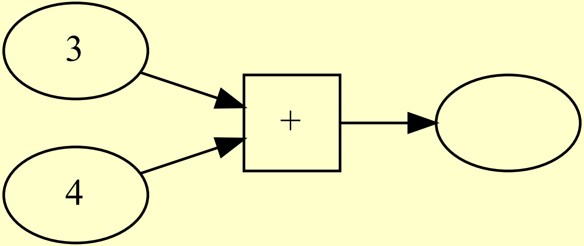

toUpper

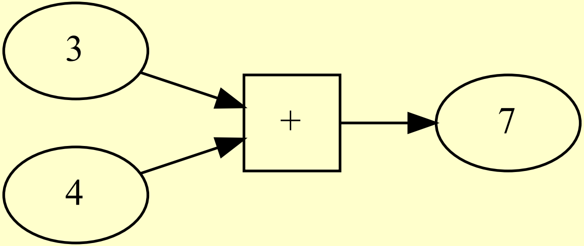

'q'
toUpper


'q'
toUpper
'Q'




3
+


3
4
+


3
4
+
7

$$z \leftarrow x + y$$

$$z = x + y$$

$$7 = x + 4$$

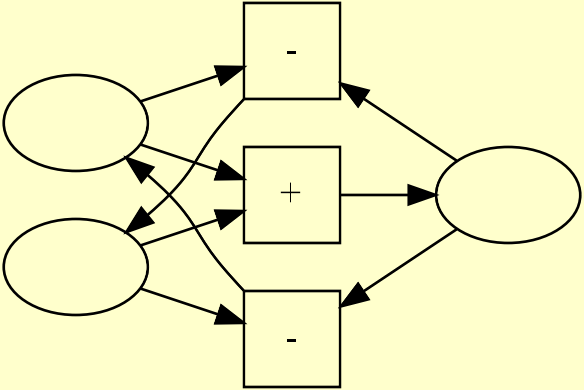
$$7 = 3 + 4$$

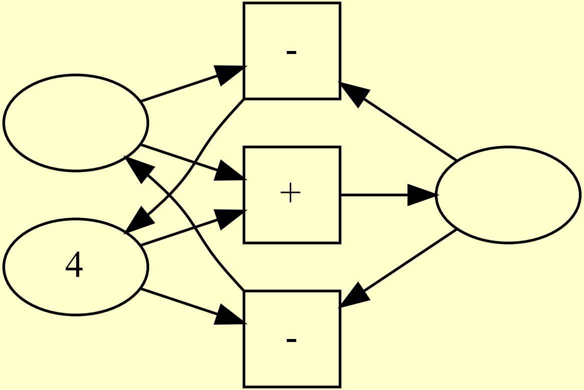
$$z = x + y$$

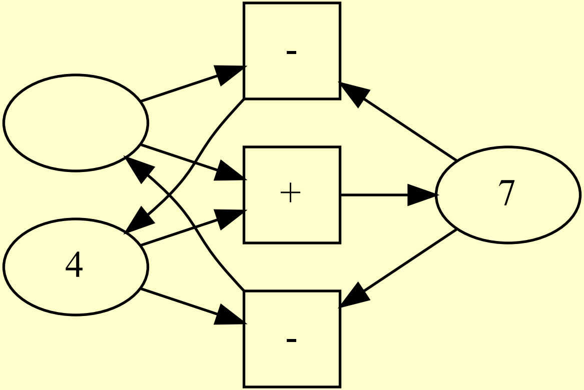
$$z \leftarrow x + y$$

$$x \leftarrow z - y$$

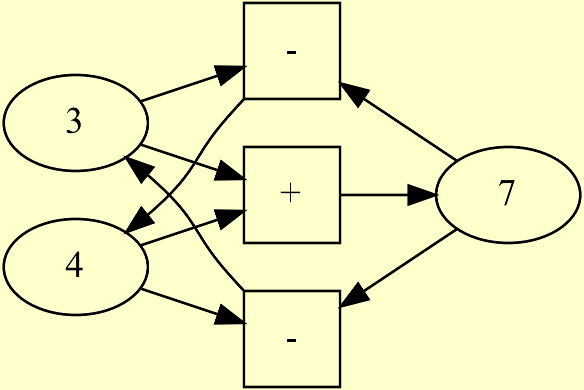
$$y \leftarrow z - x$$




-
+
4
-



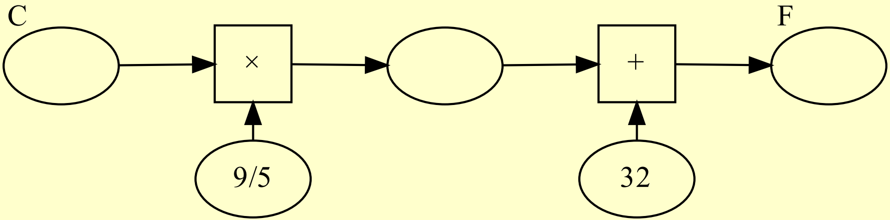
-
+
-
4
7



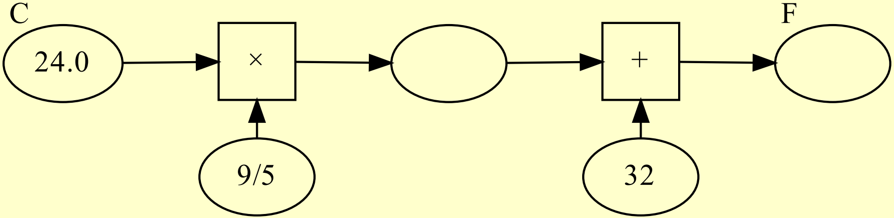
-
3
+
7
4
-

Propagators let us express multi-directional relationships!

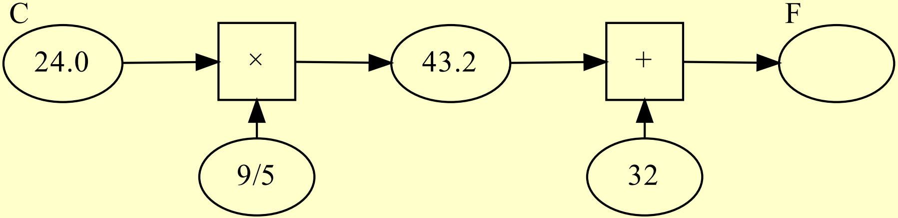
$$°F = °C \times \frac{9}{5} + 32$$



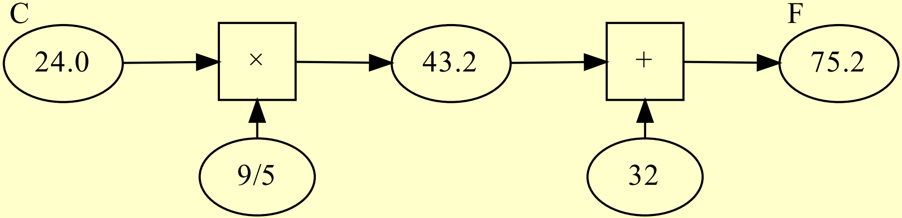
$$°F = °C \times \frac{9}{5} + 32$$



$$°F = °C \times \frac{9}{5} + 32$$

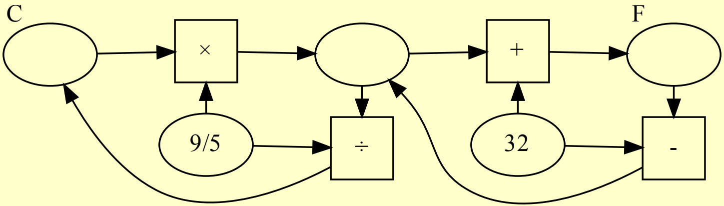


$$°F = °C \times \frac{9}{5} + 32$$



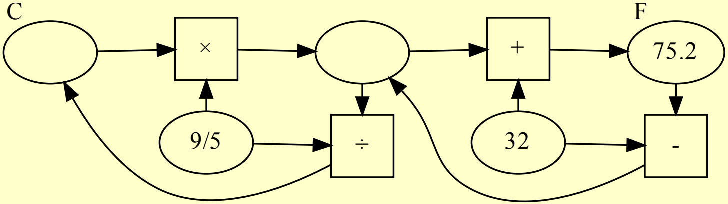
$$^\circ F = {}^\circ C \times \frac{9}{5} + 32$$

$$^\circ C = ({}^\circ F - 32) \div \frac{9}{5}$$



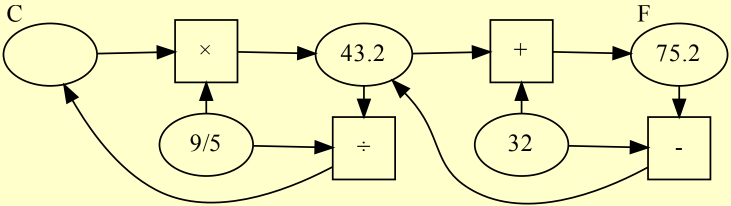
$$^\circ F = {}^\circ C \times \frac{9}{5} + 32$$

$$^\circ C = ({}^\circ F - 32) \div \frac{9}{5}$$



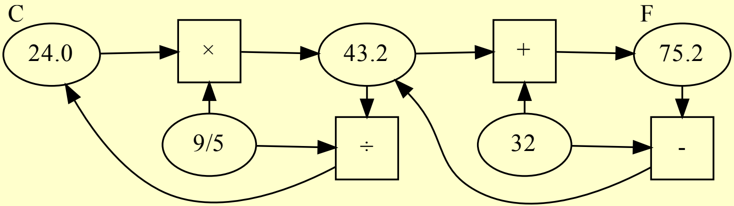
$$^\circ F = {}^\circ C \times \frac{9}{5} + 32$$

$$^\circ C = ({}^\circ F - 32) \div \frac{9}{5}$$



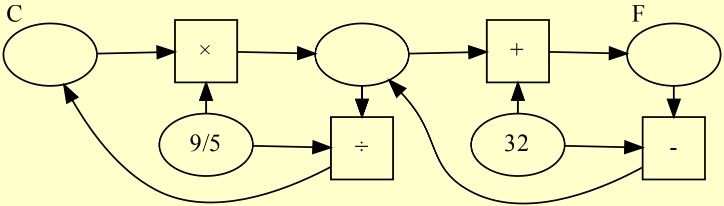
$$°F = °C \times \frac{9}{5} + 32$$

$$°C = (°F - 32) \div \frac{9}{5}$$



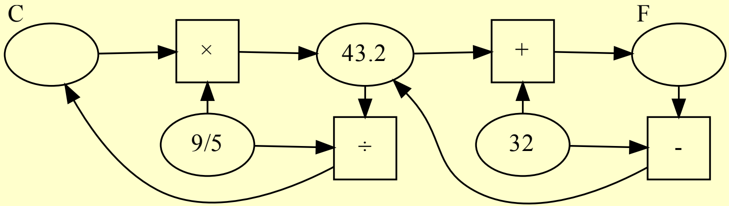
$$°F = °C \times \frac{9}{5} + 32$$

$$°C = (°F - 32) \div \frac{9}{5}$$



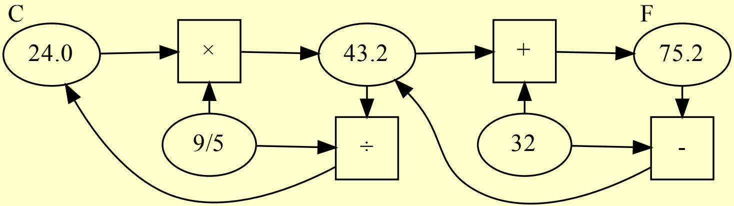
$$^\circ F = {^\circ C} \times \frac{9}{5} + 32$$

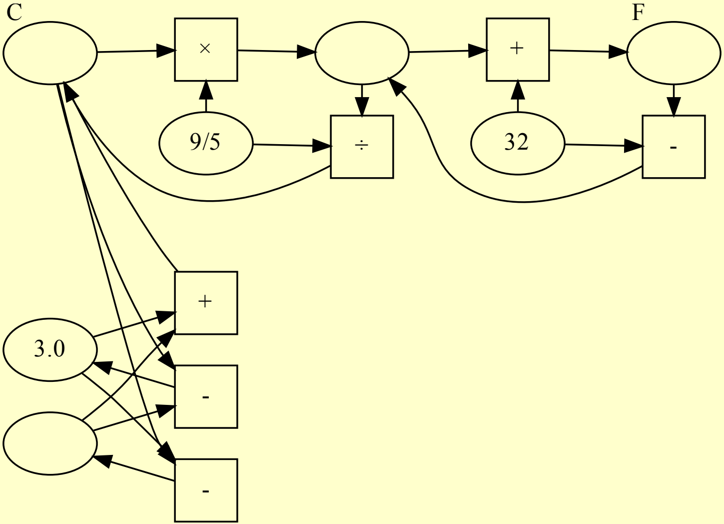
$$^\circ C = ({^\circ F} - 32) \div \frac{9}{5}$$

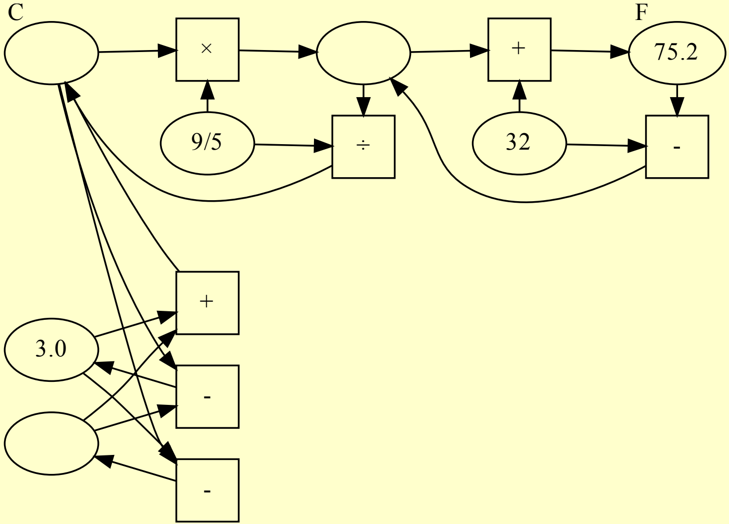


$$°F = °C \times \frac{9}{5} + 32$$

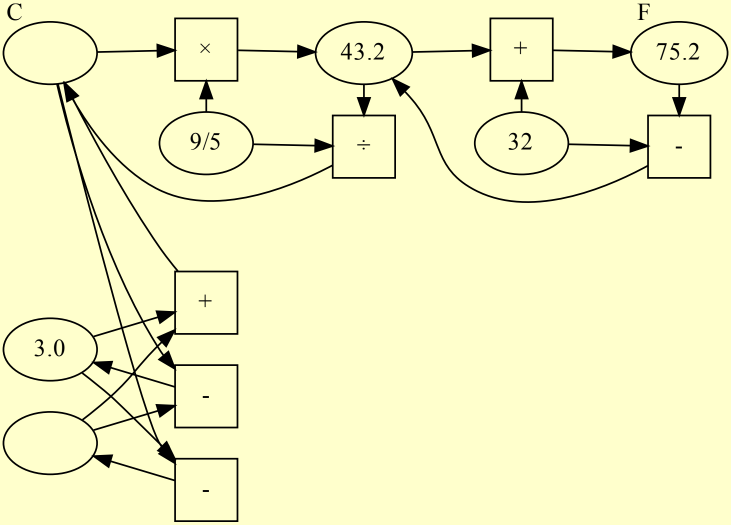
$$°C = (°F - 32) \div \frac{9}{5}$$

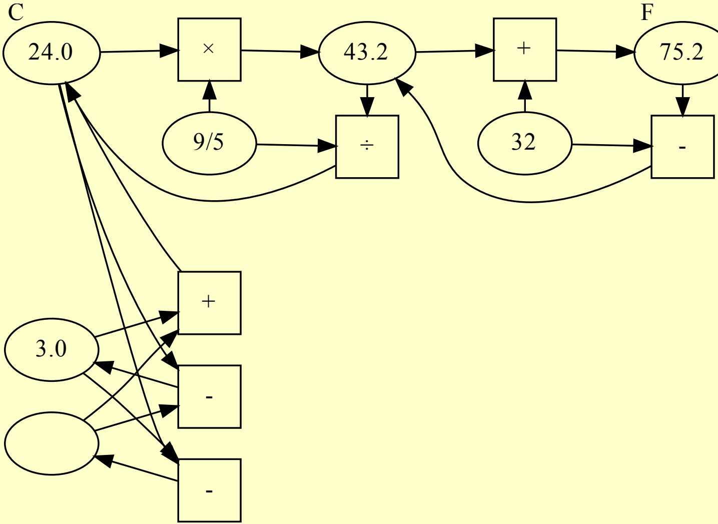



C
×
+
F
9/5
÷
32
-
+
3.0
-
-

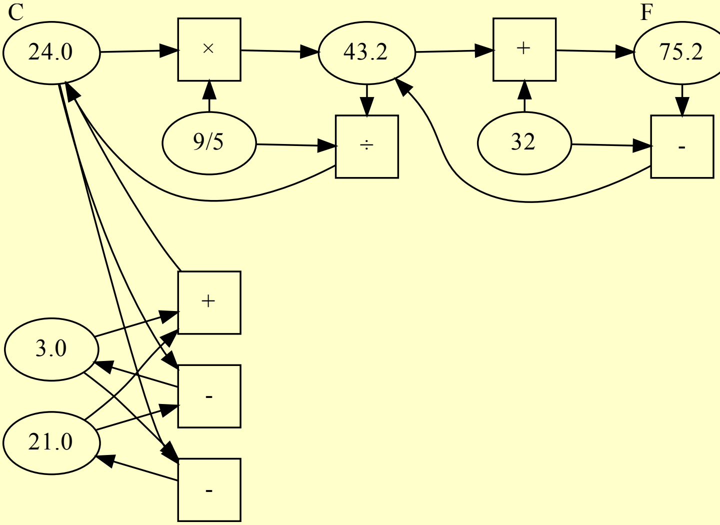


C
×
F
+
75.2
9/5
÷
32
-
+
3.0
-
-


C
×
43.2
+
F
75.2
9/5
÷
32
-
+
3.0
-
-



C
24.0
×
43.2
+
F
75.2
9/5
÷
32
-
+
3.0
-
-


C
24.0
×
43.2
+
F
75.2
9/5
÷
32
-
+
3.0
-
21.0
-

We can combine networks into larger networks!

?

Cells *accumulate information* about a value

3			2
	4	1	
	3	2	
4			1

3			2
	4	1	
	3	2	
4			1

3			2
	4	1	
	3	2	
4			1

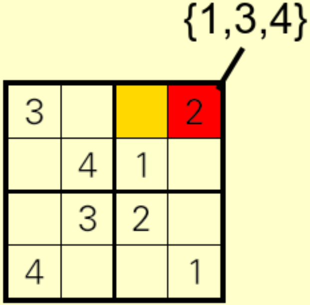
3			2
	4	1	
	3	2	
4			1

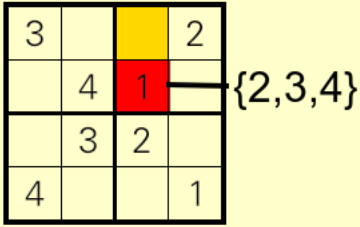
3			2
	4	1	
	3	2	
4			1

3			2
	4	1	
	3	2	
4			1

{1,2,3,4}

3			2
	4	1	
	3	2	
4			1


{1,3,4}


3 2
4 1
3 2
4 1
{2,3,4}

{1,2,4}

3			2
	4	1	
	3	2	
4			1

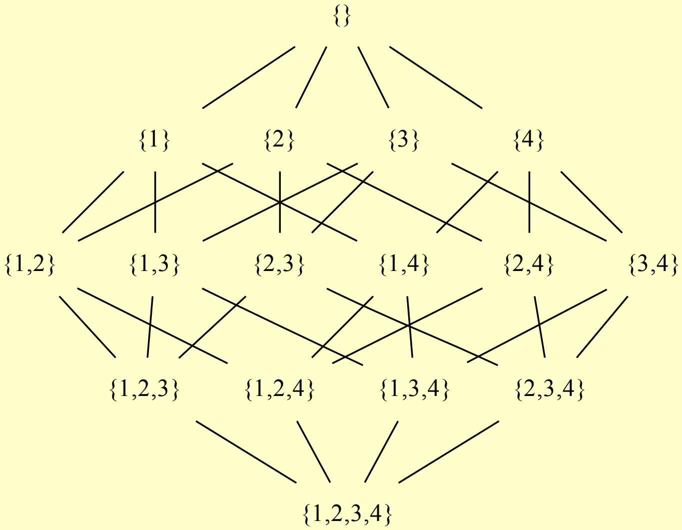
$$\{2,3,4\} \cap \{1,3,4\} \cap \{1,2,4\} \cap \{1,2,3\}$$

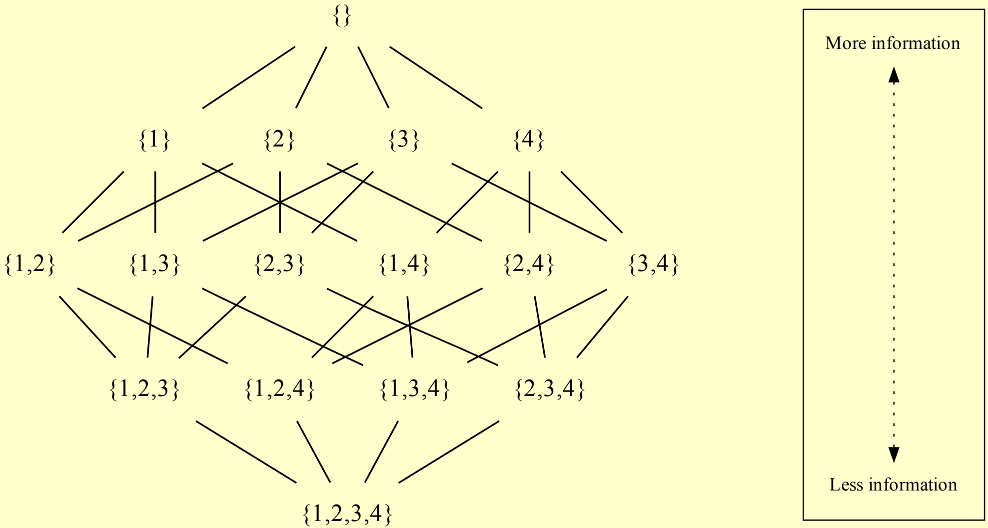
3			2
	4	1	
	3	2	
4			1

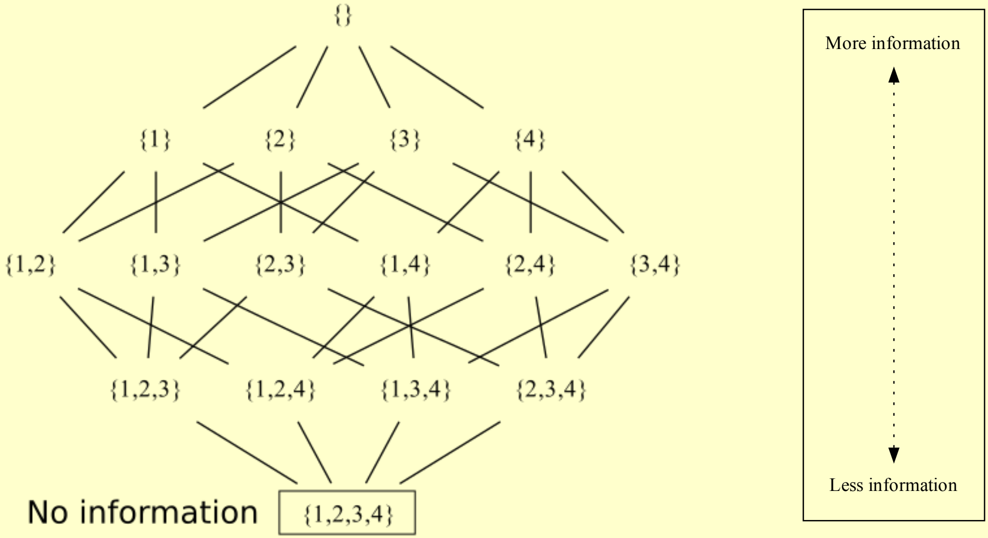
{4}

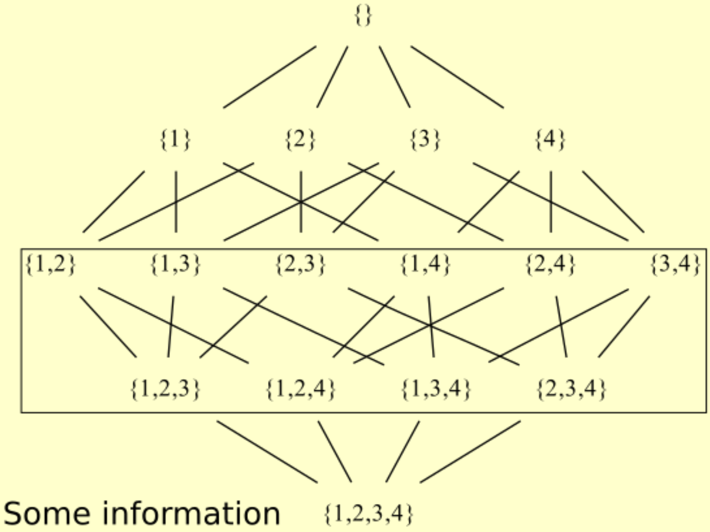
3			2
	4	1	
	3	2	
4			1

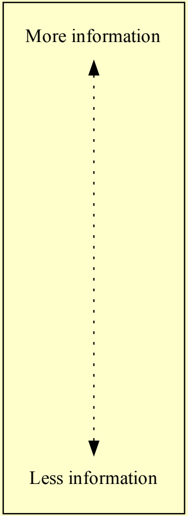
3		4	2
	4	1	
	3	2	
4			1

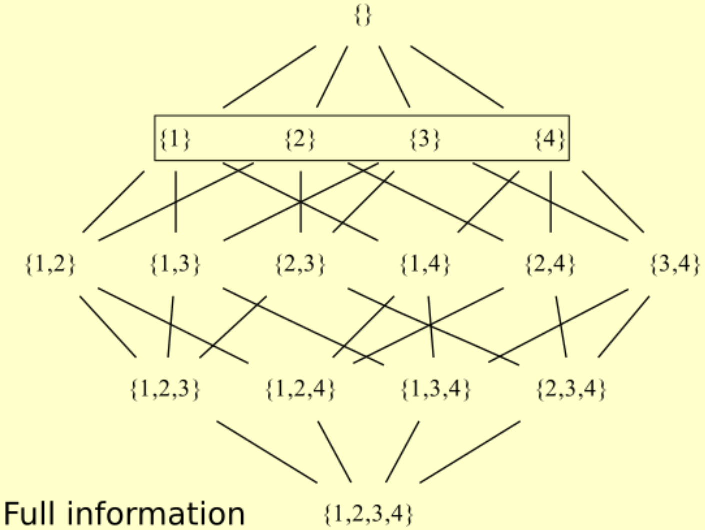

{}
{1}
{2}
{3}
{4}
{1,2}
{1,3}
{2,3}
{1,4}
{2,4}
{3,4}
{1,2,3}
{1,2,4}
{1,3,4}
{2,3,4}
{1,2,3,4}

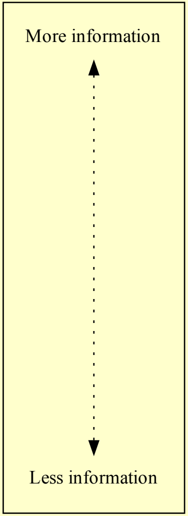

{}
{1}
{2}
{3}
{4}
{1,2}
{1,3}
{2,3}
{1,4}
{2,4}
{3,4}
{1,2,3}
{1,2,4}
{1,3,4}
{2,3,4}
{1,2,3,4}
More information
Less information

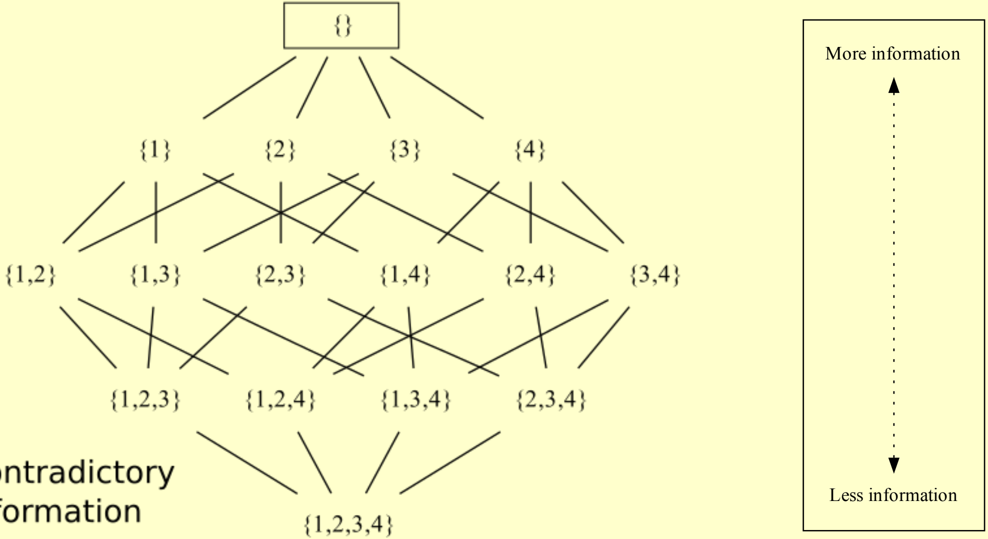

{}
{1}
{2}
{3}
{4}
{1,2}
{1,3}
{2,3}
{1,4}
{2,4}
{3,4}
{1,2,3}
{1,2,4}
{1,3,4}
{2,3,4}
No information
{1,2,3,4}
More information
Less information


{}
{1}
{2}
{3}
{4}
{1,2}
{1,3}
{2,3}
{1,4}
{2,4}
{3,4}
{1,2,3}
{1,2,4}
{1,3,4}
{2,3,4}
{1,2,3,4}
Some information


More information
Less information


{}
{1}
{2}
{3}
{4}
{1,2}
{1,3}
{2,3}
{1,4}
{2,4}
{3,4}
{1,2,3}
{1,2,4}
{1,3,4}
{2,3,4}
Full information
{1,2,3,4}


More information
Less information



Contradictory information

Cells accumulate information in a *bounded join-semilattice*

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A bounded join-semilattice is:

- A *partially ordered set*
- with a least element
- such that any set of elements has a *least upper bound*

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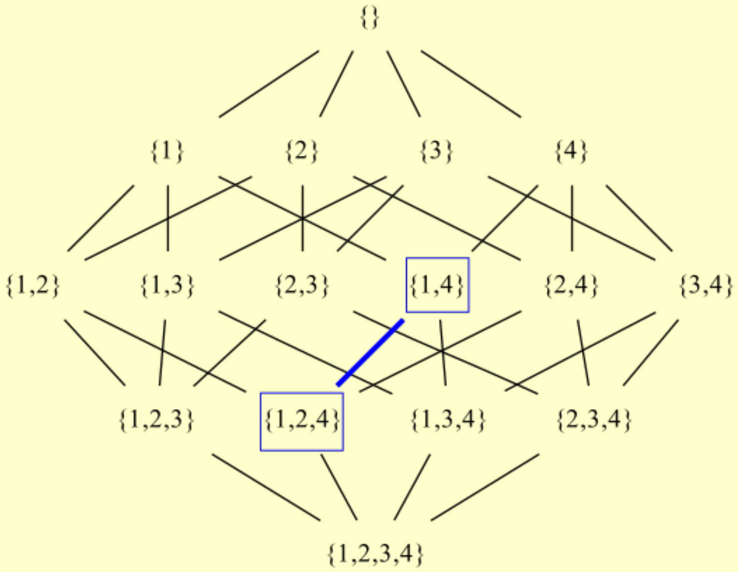
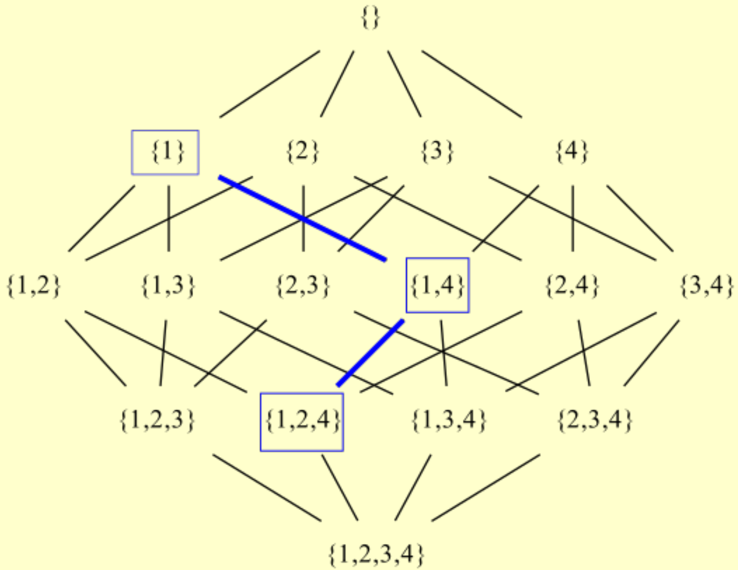
- such that any set of elements has a *least upper bound*

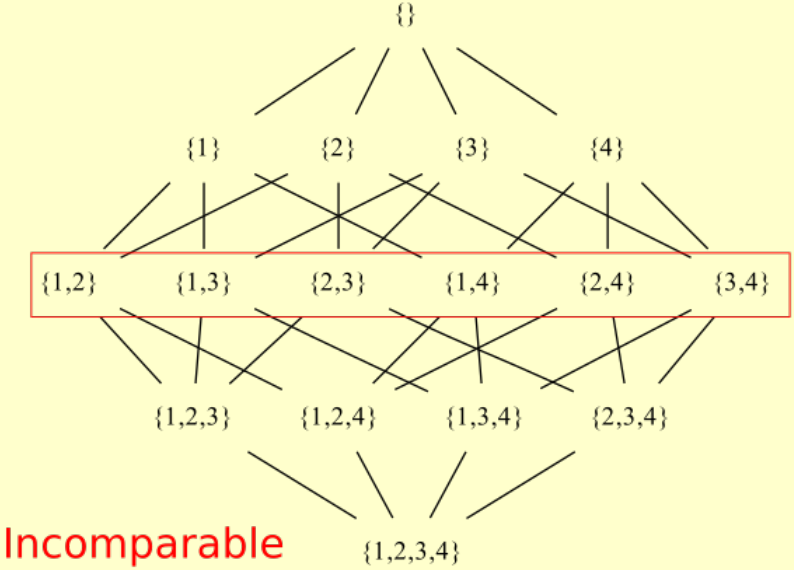
"Least upper bound" is denoted as $\vee$ and is usually pronounced "join"

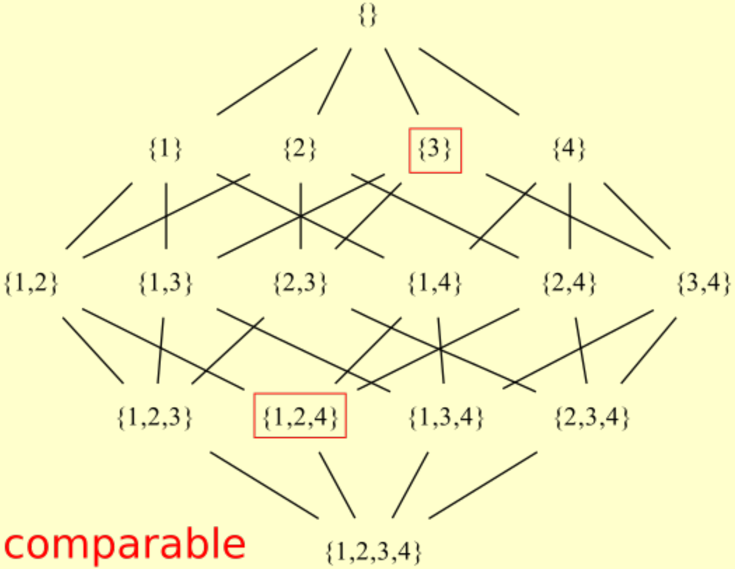


{1,2,4} < {1,4}

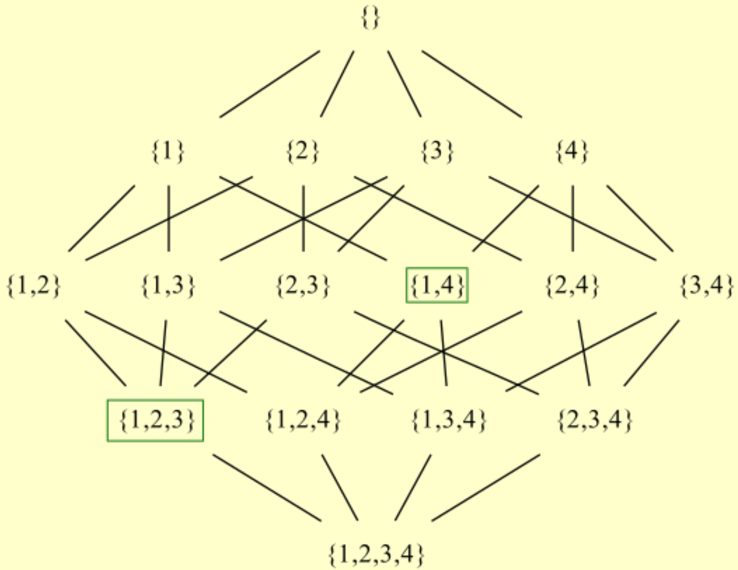


{1,2,4} < {1,4} < {1}

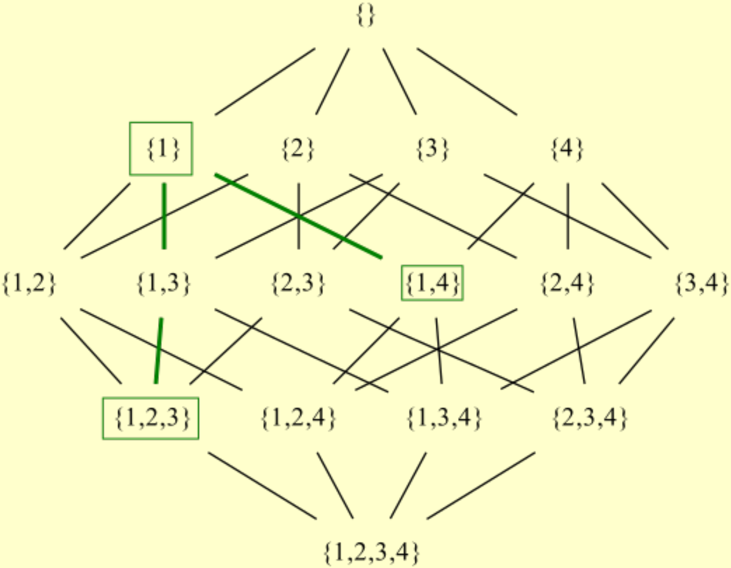

{}
{1}
{2}
{3}
{4}
{1,2}
{1,3}
{2,3}
{1,4}
{2,4}
{3,4}
{1,2,3}
{1,2,4}
{1,3,4}
{2,3,4}
{1,2,3,4}
Incomparable



Incomparable



$\{1,2,3\} \vee \{1,4\}$



$\{1,2,3\} \vee \{1,4\} = \{1\}$

$\vee$ has useful algebraic properties. It is:

- A monoid
- that's commutative
- and idempotent

Left identity

$$\epsilon \vee x = x$$

Right identity

$$x \vee \epsilon = x$$

Associativity

$$(x \vee y) \vee z = x \vee (y \vee z)$$

Commutative

$$x \vee y = y \vee x$$

Idempotent

$$x \vee x = x$$

We don't write values directly to cells
Instead we *join information in*

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This makes our propagators *monotone*, meaning that as the input cells gain information, the output cells gain information (or don't change)

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This makes our propagators *monotone*, meaning that as the input cells gain information, the output cells gain information (or don't change)

A function $f : A \to B$ where A and B are partially ordered sets is **monotone** if and only if, for all $x, y \in A.\ x \leq y \implies f(x) \leq f(y)$

The bounded join-semilattice laws and monotonicity make propagator networks deterministic, even in the face of parallelism and distribution

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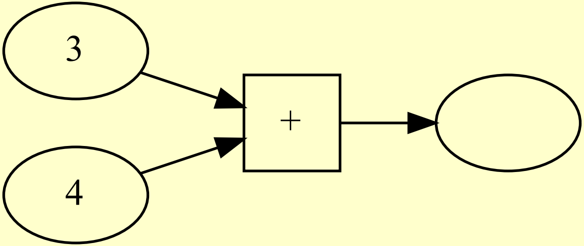
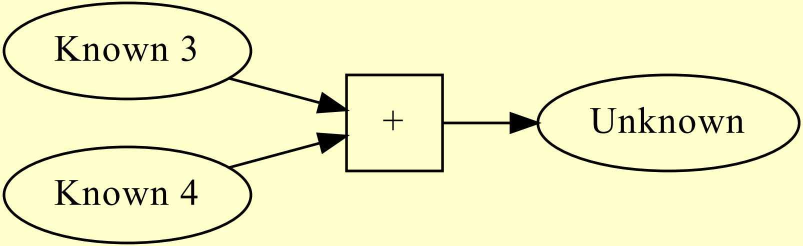
Bounded join-semilattices are already popular in the distributed systems world
See: Conflict Free Replicated Datatypes

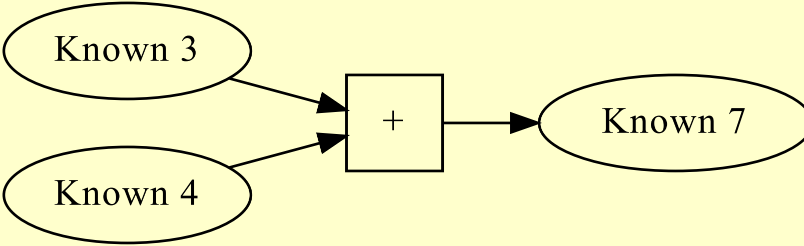
?

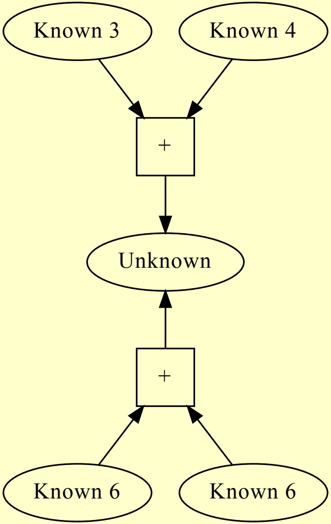

3
4
+

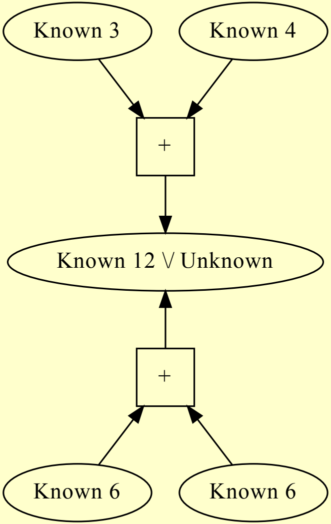
```
data Perhaps a = Unknown | Known a | Contradiction
```

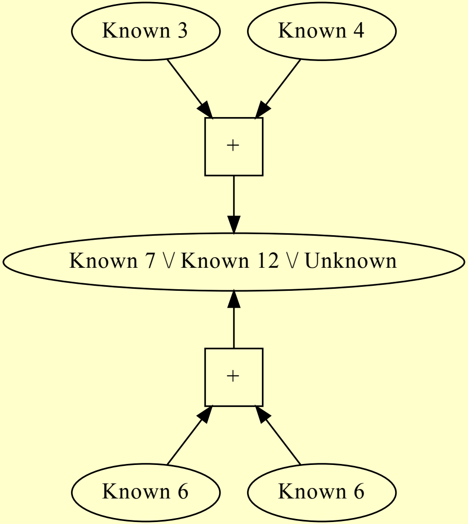
```haskell
data Perhaps a = Unknown | Known a | Contradiction

instance Eq a => BoundedJoinSemiLattice (Perhaps a) where
  bottom = Unknown
  (\/) Unknown x             = x
  (\/) x       Unknown       = x
  (\/) Contradiction _       = Contradiction
  (\/) _     Contradiction   = Contradiction
  (\/) (Known a) (Known b) =
    if a == b
      then Known a
      else Contradiction
```

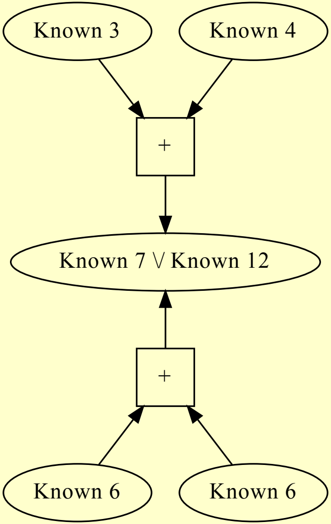

Known 3
Known 4
+
Unknown


Known 3
Known 4
+
Known 7

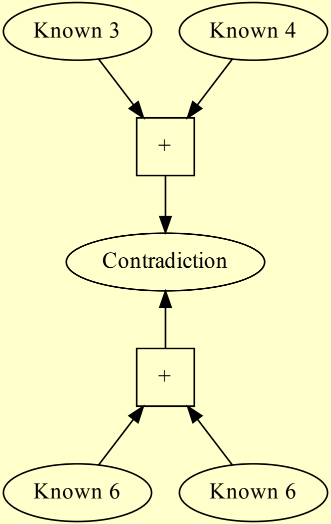

Known 3
Known 4
+
Unknown
+
Known 6
Known 6

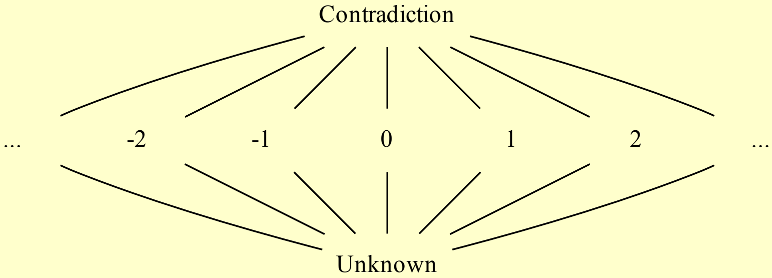

Known 3
Known 4
+
Known 12 ∨ Unknown
+
Known 6
Known 6

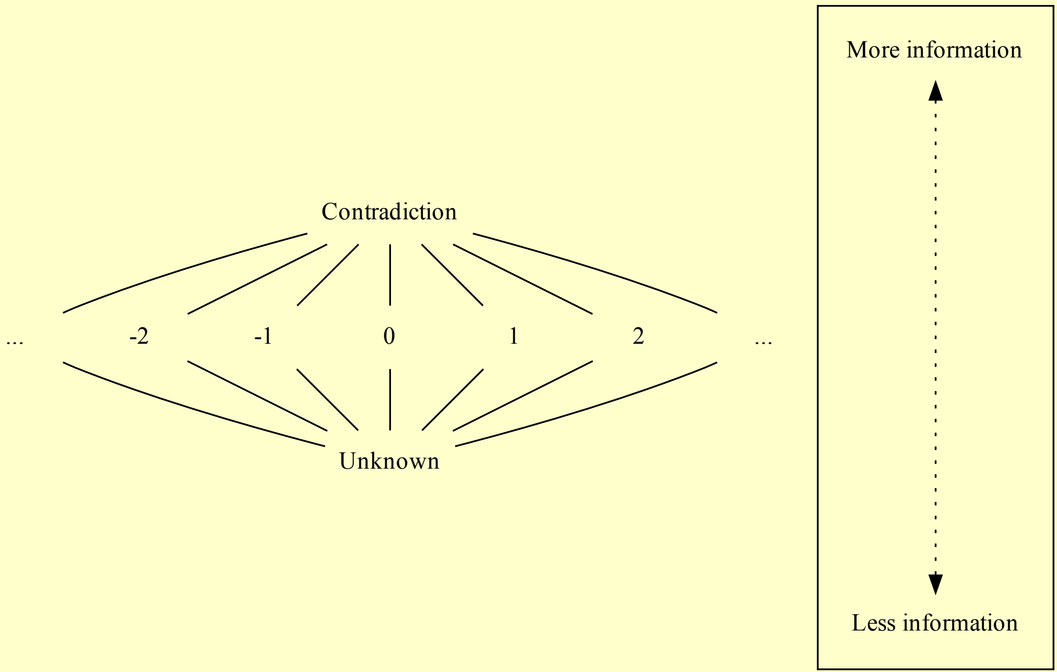

Known 3
Known 4
+
Known 7 \/ Known 12 \/ Unknown
+
Known 6
Known 6



Known 3
Known 4
+
Known 7 \/ Known 12
+
Known 6
Known 6


Known 3
Known 4
+
Contradiction
+
Known 6
Known 6


Contradiction
... -2 -1 0 1 2 ...
Unknown


Contradiction
...
-2
-1
0
1
2
...
Unknown
More information
Less information

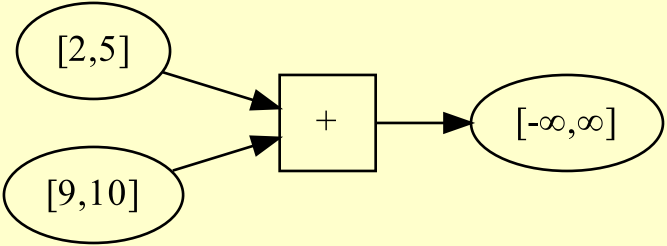
There are loads of other bounded join-semilattices too!

$$[1, 5]$$

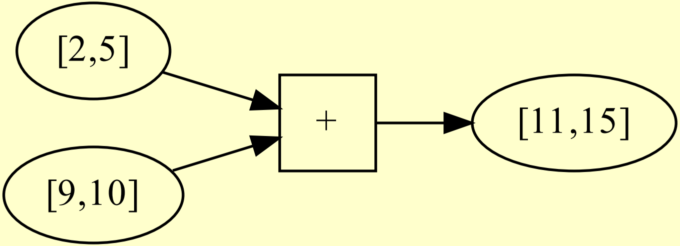
$$[1, 5] \cup [2, 7] = [2, 5]$$

$$[1, 5] \cup [2, 7] = [2, 5]$$

$$[2, 5] + [9, 10] = [11, 15]$$

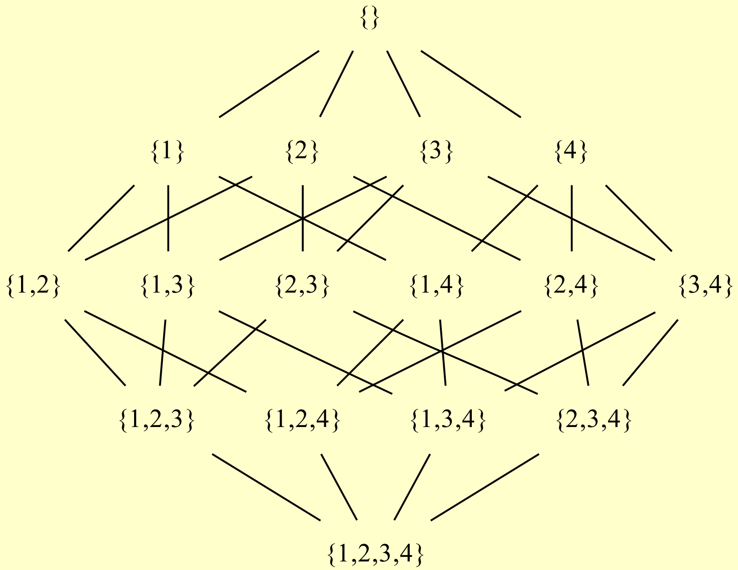


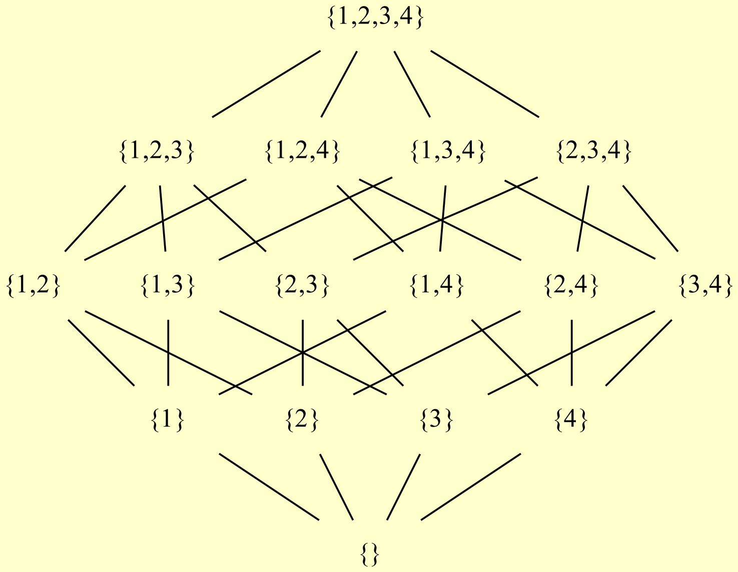
[2,5]
[9,10]
+
[-∞,∞]


[2,5]
[9,10]
+
[11,15]

We can use this to combine multiple imprecise measurements

What other bounded join-semilattices are there?


{}
{1}
{2}
{3}
{4}
{1,2}
{1,3}
{2,3}
{1,4}
{2,4}
{3,4}
{1,2,3}
{1,2,4}
{1,3,4}
{2,3,4}
{1,2,3,4}


{1,2,3,4}
{1,2,3}
{1,2,4}
{1,3,4}
{2,3,4}
{1,2}
{1,3}
{2,3}
{1,4}
{2,4}
{3,4}
{1}
{2}
{3}
{4}
{}

And so many more!

(Talk about truth-management system machinery)

Using truth management systems, we can do logic programming (TODO expand)

Alexey Radul's work on propagators:

- Art of the Propagator
  `http://web.mit.edu/~axch/www/art.pdf`
- Propagation Networks: A Flexible and Expressive Substrate for Computation
  `http://web.mit.edu/~axch/www/phd-thesis.pdf`

Lindsey Kuper's work on LVars is closely related, and works today:

- Lattice-Based Data Structures for Deterministic Parallel and Distributed Programming
  `https://www.cs.indiana.edu/~lkuper/papers/lindsey-kuper-dissertation.pdf`
- lvish library
  `https://hackage.haskell.org/package/lvish`

Edward Kmett has worked on:

- Making propagators go fast
- Scheduling strategies and garbage collection
- Relaxing requirements (Eg. not requiring a full join-semilattice, admitting non-monotone functions)

Ed's stuff:

- `http://github.com/ekmett/propagators`
- `http://github.com/ekmett/concurrent`
- Lambda Jam talk (Easy mode): `https://www.youtube.com/watch?v=acZkF6Q2XKs`
- Boston Haskell talk (Hard mode): `https://www.youtube.com/watch?v=DyPzPeOPgUE`

In conclusion, propagator networks:

- Admit any Haskell function you can write today . . .
- . . . and more functions!
- compute bidirectionally
- give us constraint solving and search
- parallelise and distribute

Thanks for listening!