AN EXAMPLE ARTICLE*

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Abstract. This is an example SIAM LaTeX article. This can be used as a template for new articles. Abstracts must be able to stand alone and so cannot contain citations to the paper's references, equations, etc. An abstract must consist of a single paragraph and be concise. Because of online formatting, abstracts must appear as plain as possible. Any equations should be inline.

Key words. example, LaTeX

MSC codes. 68Q25, 68R10, 68U05

1. Introduction.

The introduction introduces the context and summarizes the manuscript. It is importantly to clearly state the contributions of this piece of work.

For $\Omega = (0,1)$, $1 < \alpha < 2$, suppose $f \in C^2(\Omega)$

$$\begin{cases} (-\Delta)^{\frac{\alpha}{2}} u(x) = f(x), & x \in \Omega \\ u(x) = 0, & x \in \mathbb{R} \setminus \Omega \end{cases} \tag{1.1}$$

where

$$(-\Delta)^{\frac{\alpha}{2}} u(x) = -\frac{\partial^\alpha u}{\partial |x|^\alpha} = -\kappa_\alpha \frac{d^2}{dx^2} \int_\Omega \frac{u(y)}{|x-y|^{\alpha-1}} dy \tag{1.2}$$

$$\kappa_\alpha = -\frac{1}{2\cos(\alpha\pi/2)\Gamma(2-\alpha)} > 0 \tag{1.3}$$

The paper is organized as follows. Our main results are in section 3, experimental results are in section 5, and the conclusions follow in section 7.

2. Numeric Format.

$$x_i = \begin{cases} \frac{1}{2}\left(\frac{i}{N}\right)^r, & 0 \le i \le N \\ 1 - \frac{1}{2}\left(\frac{2N-i}{N}\right)^r, & N \le i \le 2N \end{cases} \tag{2.1}$$

where $r \geq 1$. And let

$$h_j = x_j - x_{j-1}, \quad 1 \le j \le 2N \tag{2.2}$$

Let $\{\phi_j(x)\}_{j=1}^{2N-1}$ be standard hat functions, which are basis of the piecewise linear function space.

$$\phi_j(x) = \begin{cases} \frac{1}{h_j}(x - x_{j-1}), & x_{j-1} \le x \le x_j \\ \frac{1}{h_{j+1}}(x_{j+1} - x), & x_j \le x \le x_{j+1} \\ 0, & \text{otherwise} \end{cases} \tag{2.3}$$

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And then, we can approximate $u(x)$ with

$$u_h(x) := \sum_{j=1}^{2N-1} u(x_j)\phi_j(x) \tag{2.4}$$

For convience, we denote

$$I_h(x) := \int_\Omega |x-y|^{1-\alpha} u_h(y)dy \tag{2.5}$$

And now, we can approximate the operator (1.2) at x_i with

$$D_h^\alpha u_h(x_i) := -\kappa_\alpha \frac{2}{h_i+h_{i+1}} \left(\frac{1}{h_i} I_h(x_{i-1}) - \left(\frac{1}{h_i}+\frac{1}{h_{i+1}}\right) I_h(x_i) + \frac{1}{h_{i+1}} I_h(x_{i+1}) \right) \tag{2.6}$$

Finally, we approximate the equation (1.1) with

$$D_h^\alpha u_h(x_i) = f(x_i), \quad 1 \le i \le 2N-1 \tag{2.7}$$

The discrete equation (2.7) can be written in matrix form

$$AU = F \tag{2.8}$$

where U is unknown, $F = (f(x_1), \cdots, f(x_{2N-1}))$. The matrix A is constructed as follows: Since

$$\begin{aligned} I_h(x_i) &= \int_\Omega |x_i-y|^{1-\alpha} u_h(y)dy = \sum_{j=1}^{2N-1} \int_\Omega |x_i-y|^{1-\alpha} u(x_j)\phi_j(y)dy \\ &= \sum_{j=1}^{2N-1} u(x_j) \int_{x_{j-1}}^{x_{j+1}} |x_i-y|^{1-\alpha}\phi_j(y)dy \\ &= \sum_{j=1}^{2N-1} \frac{u(x_j)}{(2-\alpha)(3-\alpha)} \left(\frac{|x_i-x_{j-1}|^{3-\alpha}}{h_j} - \frac{h_j+h_{j+1}}{h_j h_{j+1}}|x_i-x_j|^{3-\alpha} + \frac{|x_i-x_{j+1}|^{3-\alpha}}{h_{j+1}} \right) \\ &=: \sum_{j=1}^{2N-1} \tilde{a}_{ij}\, u(x_j), \quad 0 \le i \le 2N \end{aligned} \tag{2.9}$$

Then, substitute in (2.6), we have

$$D_h^\alpha u_h(x_i) = \sum_{j=1}^{2N-1} a_{ij}\, u(x_j) \tag{2.10}$$

where

$$a_{ij} = -\kappa_\alpha \frac{2}{h_i+h_{i+1}} \left(\frac{1}{h_i}\tilde{a}_{i-1,j} - \left(\frac{1}{h_i}+\frac{1}{h_{i+1}}\right)\tilde{a}_{i,j} + \frac{1}{h_{i+1}}\tilde{a}_{i+1,j} \right) \tag{2.11}$$

3. Main results.

Here we state our main results; the proof is deferred to **??**.

THEOREM 3.1 (truncation error). *If $f \in C^2(\Omega)$ and $\alpha \in (1,2)$, then there exists a constant $C = C(\alpha, r, \|f\|_{C^2(\Omega)})$, such that the truncation error of the discrete format*

THEOREM 3.2 (Mean Value Theorem). *Suppose f is a function that is continuous on the closed interval $[a, b]$. and differentiable on the open interval (a, b). Then there exists a number c such that $a < c < b$ and*

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

In other words,

$$f(b) - f(a) = f'(c)(b - a).$$

COROLLARY 3.3. *Let $f(x)$ be continuous and differentiable everywhere. If $f(x)$ has at least two roots, then $f'(x)$ must have at least one root.*

Proof. Let a and b be two distinct roots of f. By Theorem 3.2, there exists a number c such that

$$f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{0 - 0}{b - a} = 0.$$ ∎

Note that it may require two LaTeX compilations for the proof marks to show.

Display matrices can be rendered using environments from `amsmath`:

$$S = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad C = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{3.1}$$

Equation (3.1) shows some example matrices.

We calculate the Fréchet derivative of F as follows:

$$\begin{aligned} F'(U,V)(H,K) &= \langle R(U,V), H\Sigma V^T + U\Sigma K^T - P(H\Sigma V^T + U\Sigma K^T)\rangle \quad (3.2a) \\ &= \langle R(U,V), H\Sigma V^T + U\Sigma K^T\rangle \\ &= \langle R(U,V)V\Sigma^T, H\rangle + \langle \Sigma^T U^T R(U,V), K^T\rangle. \quad (3.2b) \end{aligned}$$

Equation (3.2a) is the first line, and (3.2b) is the last line.

4. Algorithm.

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Our analysis leads to the algorithm in Algorithm 4.1.

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Algorithm 4.1 Build tree

Define $P := T := \{\{1\}, \ldots, \{d\}\}$
while $\#P > 1$ **do**
  Choose $C' \in \mathcal{C}_p(P)$ with $C' := \operatorname{argmin}_{C \in \mathcal{C}_p(P)} \varrho(C)$
  Find an optimal partition tree $T_{C'}$
  Update $P := (P \setminus C') \cup \{\bigcup_{t \in C'} t\}$
  Update $T := T \cup \{\bigcup_{t \in \tau} t : \tau \in T_{C'} \setminus \mathcal{L}(T_{C'})\}$
end while
return T

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5. Experimental results. Quisque facilisis auctor sapien. Pellentesque gravida hendrerit lectus. Mauris rutrum sodales sapien. Fusce hendrerit sem vel lorem. Integer pellentesque massa vel augue. Integer elit tortor, feugiat quis, sagittis et, ornare non, lacus. Vestibulum posuere pellentesque eros. Quisque venenatis ipsum dictum nulla. Aliquam quis quam non metus eleifend interdum. Nam eget sapien ac mauris malesuada adipiscing. Etiam eleifend neque sed quam. Nulla facilisi. Proin a ligula. Sed id dui eu nibh egestas tincidunt. Suspendisse arcu.

Figure 1 shows some example results. Additional results are available in the supplement in Table 1.

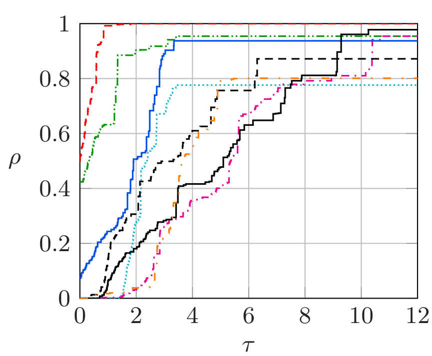

Fig. 1. *Example figure using external image files.*

Table 1 shows additional supporting evidence.

Table 1
Example table.

Species	**Mean**	**Std. Dev.**
1	3.4	1.2
2	5.4	0.6
3	7.4	2.4
4	9.4	1.8

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7. Conclusions. Some conclusions here.

Appendix A. An example appendix. Aenean tincidunt laoreet dui. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Integer ipsum lectus, fermentum ac, malesuada in, eleifend ut, lorem. Vivamus ipsum turpis, elementum vel, hendrerit ut, semper at, metus. Vivamus sapien tortor, eleifend id, dapibus in, egestas et, pede. Pellentesque faucibus. Praesent lorem neque, dignissim in, facilisis nec, hendrerit vel, odio. Nam at diam ac neque aliquet viverra. Morbi dapibus ligula sagittis magna. In lobortis. Donec aliquet ultricies libero. Nunc dictum vulputate purus. Morbi varius. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In tempor. Phasellus commodo porttitor magna. Curabitur vehicula odio vel dolor.

LEMMA A.1. *Test Lemma.*

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