AN EXAMPLE ARTICLE*

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Abstract. This is an example SIAM LaTeX article. This can be used as a template for new articles. Abstracts must be able to stand alone and so cannot contain citations to the paper's references, equations, etc. An abstract must consist of a single paragraph and be concise. Because of online formatting, abstracts must appear as plain as possible. Any equations should be inline.

Key words. example, LaTeX

MSC codes. 68Q25, 68R10, 68U05

1. Introduction.

The introduction introduces the context and summarizes the manuscript. It is importantly to clearly state the contributions of this piece of work.

For $\Omega = (0,1)$, $1 < \alpha < 2$, suppose $f \in C^2(\Omega)$

$$\begin{cases} (-\Delta)^{\frac{\alpha}{2}} u(x) = f(x), & x \in \Omega \\ u(x) = 0, & x \in \mathbb{R} \setminus \Omega \end{cases} \tag{1.1}$$

where

$$(-\Delta)^{\frac{\alpha}{2}} u(x) = -\frac{\partial^\alpha u}{\partial |x|^\alpha} = -\kappa_\alpha \frac{d^2}{dx^2} \int_\Omega \frac{u(y)}{|x-y|^{\alpha-1}} dy \tag{1.2}$$

$$\kappa_\alpha = -\frac{1}{2\cos(\alpha\pi/2)\Gamma(2-\alpha)} > 0 \tag{1.3}$$

The paper is organized as follows. Our main results are in section 3, experimental results are in section 5, and the conclusions follow in section 7.

2. Numeric Format.

$$x_i = \begin{cases} \frac{1}{2}\left(\frac{i}{N}\right)^r, & 0 \le i \le N \\ 1 - \frac{1}{2}\left(\frac{2N-i}{N}\right)^r, & N \le i \le 2N \end{cases} \tag{2.1}$$

where $r \geq 1$. And let

$$h_j = x_j - x_{j-1}, \quad 1 \le j \le 2N \tag{2.2}$$

Let $\{\phi_j(x)\}_{j=1}^{2N-1}$ be standard hat functions, which are basis of the piecewise linear function space.

$$\phi_j(x) = \begin{cases} \frac{1}{h_j}(x - x_{j-1}), & x_{j-1} \le x \le x_j \\ \frac{1}{h_{j+1}}(x_{j+1} - x), & x_j \le x \le x_{j+1} \\ 0, & \text{otherwise} \end{cases} \tag{2.3}$$

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And then, we can approximate $u(x)$ with

$$u_h(x) := \sum_{j=1}^{2N-1} u(x_j)\phi_j(x) \tag{2.4}$$

For convience, we denote

$$I_h(x) := \int_\Omega |x-y|^{1-\alpha} u_h(y)dy \tag{2.5}$$

And now, we can approximate the operator (1.2) at x_i with

$$D_h^\alpha u_h(x_i) := -\kappa_\alpha \frac{2}{h_i + h_{i+1}} \left(\frac{1}{h_i} I_h(x_{i-1}) - \left(\frac{1}{h_i} + \frac{1}{h_{i+1}} \right) I_h(x_i) + \frac{1}{h_{i+1}} I_h(x_{i+1}) \right) \tag{2.6}$$

Finally, we approximate the equation (1.1) with

$$D_h^\alpha u_h(x_i) = f(x_i), \quad 1 \le i \le 2N-1 \tag{2.7}$$

The discrete equation (2.7) can be written in matrix form

$$AU = F \tag{2.8}$$

where U is unknown, $F = (f(x_1), \cdots, f(x_{2N-1}))$. The matrix A is constructed as follows: Since

$$\begin{aligned} I_h(x_i) &= \int_\Omega |x_i - y|^{1-\alpha} u_h(y)dy = \sum_{j=1}^{2N-1} \int_\Omega |x_i - y|^{1-\alpha} u(x_j)\phi_j(y)dy \\ &= \sum_{j=1}^{2N-1} u(x_j) \int_{x_{j-1}}^{x_{j+1}} |x_i - y|^{1-\alpha} \phi_j(y)dy \\ &= \sum_{j=1}^{2N-1} \frac{u(x_j)}{(2-\alpha)(3-\alpha)} \left(\frac{|x_i - x_{j-1}|^{3-\alpha}}{h_j} - \frac{h_j + h_{j+1}}{h_j h_{j+1}} |x_i - x_j|^{3-\alpha} + \frac{|x_i - x_{j+1}|^{3-\alpha}}{h_{j+1}} \right) \\ &=: \sum_{j=1}^{2N-1} \tilde{a}_{ij}\, u(x_j), \quad 0 \le i \le 2N \end{aligned} \tag{2.9}$$

Then, substitute in (2.6), we have

$$D_h^\alpha u_h(x_i) = \sum_{j=1}^{2N-1} a_{ij}\, u(x_j) \tag{2.10}$$

where

$$a_{ij} = -\kappa_\alpha \frac{2}{h_i + h_{i+1}} \left(\frac{1}{h_i} \tilde{a}_{i-1,j} - \left(\frac{1}{h_i} + \frac{1}{h_{i+1}} \right) \tilde{a}_{i,j} + \frac{1}{h_{i+1}} \tilde{a}_{i+1,j} \right) \tag{2.11}$$

3. Main results. Here we state our main results; the proof is deferred to **??**. Let's denote $h = \frac{1}{N}$, we have

THEOREM 3.1 (Truncation Error). *If $f \in C^2(\Omega)$ and $\alpha \in (1,2)$, and $u(x)$ is a solution of the equation (1.1), then there exists a constant $C = C(\alpha, r, \|f\|_{C^2(\Omega)})$, such that the truncation error of the discrete format satisfies*

$$
\begin{aligned}
|D_h^\alpha u_h(x_i) - f(x_i)| \le & C(h^{r\alpha/2+r}(x_i^{-1-\alpha} + (1-x_i)^{-1-\alpha})) \\
& + h^2(x_i^{-\alpha/2-2/r} + (1-x_i)^{-\alpha/2-2/r}) \\
& + h^2 \begin{cases} |\frac{1}{2} - x_{i-1}|^{1-\alpha}, & 1 \le i \le N \\ |\frac{1}{2} - x_{i+1}|^{1-\alpha}, & N < i \le 2N-1 \end{cases} \quad)
\end{aligned}
\tag{3.1}
$$

THEOREM 3.2 (Convergence). *The discrete equation (2.7) has sulotion U, and there exists a positive constant $C = C(\alpha, r, \|f\|_{C^2(\Omega)})$ such that the error between the numerial solution U with the exact solution $u(x_i)$ satisfies*

$$
\max_{1 \le i \le 2N-1} |U_i - x(x_i)| \le C h^{\min\{\frac{r\alpha}{2}, 2\}}
\tag{3.2}
$$

That means the numerial method has convergence order $\min\{\frac{r\alpha}{2}, 2\}$.

4. Algorithm. Sed gravida lectus ut purus. Morbi laoreet magna. Pellentesque eu wisi. Proin turpis. Integer sollicitudin augue nec dui. Fusce lectus. Vivamus faucibus nulla nec lacus. Integer diam. Pellentesque sodales, enim feugiat cursus volutpat, sem mauris dignissim mauris, quis consequat sem est fermentum ligula. Nullam justo lectus, condimentum sit amet, posuere a, fringilla mollis, felis. Morbi nulla nibh, pellentesque at, nonummy eu, sollicitudin nec, ipsum. Cras neque. Nunc augue. Nullam vitae quam id quam pulvinar blandit. Nunc sit amet orci. Aliquam erat elit, pharetra nec, aliquet a, gravida in, mi. Quisque urna enim, viverra quis, suscipit quis, tincidunt ut, sapien. Cras placerat consequat sem. Curabitur ac diam. Curabitur diam tortor, mollis et, viverra ac, tempus vel, metus.

Our analysis leads to the algorithm in Algorithm 4.1.

Algorithm 4.1 Build tree

Define $P := T := \{\{1\}, \ldots, \{d\}\}$
while $\#P > 1$ **do**
  Choose $C' \in \mathcal{C}_p(P)$ with $C' := \operatorname{argmin}_{C \in \mathcal{C}_p(P)} \varrho(C)$
  Find an optimal partition tree $T_{C'}$
  Update $P := (P \backslash C') \cup \{\bigcup_{t \in C'} t\}$
  Update $T := T \cup \{\bigcup_{t \in \tau} t : \tau \in T_{C'} \backslash \mathcal{L}(T_{C'})\}$
end while
return T

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5. Experimental results. Quisque facilisis auctor sapien. Pellentesque gravida hendrerit lectus. Mauris rutrum sodales sapien. Fusce hendrerit sem vel lorem. Integer pellentesque massa vel augue. Integer elit tortor, feugiat quis, sagittis et, ornare non, lacus. Vestibulum posuere pellentesque eros. Quisque venenatis ipsum dictum nulla. Aliquam quis quam non metus eleifend interdum. Nam eget sapien ac mauris malesuada adipiscing. Etiam eleifend neque sed quam. Nulla facilisi. Proin a ligula. Sed id dui eu nibh egestas tincidunt. Suspendisse arcu.

Figure 1 shows some example results. Additional results are available in the supplement in Table 1.

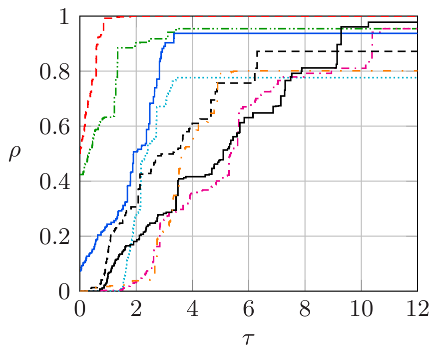

Fig. 1. *Example figure using external image files.*

Table 1 shows additional supporting evidence.

Table 1
Example table.

Species	**Mean**	**Std. Dev.**
1	3.4	1.2
2	5.4	0.6
3	7.4	2.4
4	9.4	1.8

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7. Conclusions.
Some conclusions here.

Appendix A. An example appendix.
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LEMMA A.1. *Test Lemma.*

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