

# AN $\alpha$ -ROBUST DIFFERENCE QUADRATURE METHOD FOR SPACE-TIME CAPUTO-RIESZ FRACTIONAL DIFFUSION EQUATION \*

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**Abstract.** This is an example SIAM L<sup>A</sup>T<sub>E</sub>X article. This can be used as a template for new articles. Abstracts must be able to stand alone and so cannot contain citations to the paper's references, equations, etc. An abstract must consist of a single paragraph and be concise. Because of online formatting, abstracts must appear as plain as possible. Any equations should be inline.

**Key words.** example, L<sup>A</sup>T<sub>E</sub>X

**MSC codes.** 68Q25, 68R10, 68U05

**1. Introduction.** The introduction introduces the context and summarizes the manuscript. It is importantly to clearly state the contributions of this piece of work. The next two paragraphs are text filler, generated by the `lipsum` package.

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The paper is organized as follows. Our main results are in section 2, our new algorithm is in section 3, experimental results are in section 4, and the conclusions follow in section 6.

**2. Main results.** We interleave text filler with some example theorems and theorem-like items.

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Here we state our main result as Theorem 2.1; the proof is deferred to ??.

**THEOREM 2.1** ( $LDL^T$  Factorization [1]). *If  $A \in \mathbb{R}^{n \times n}$  is symmetric and the*

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principal submatrix  $A(1 : k, 1 : k)$  is nonsingular for  $k = 1 : n - 1$ , then there exists a unit lower triangular matrix  $L$  and a diagonal matrix

$$D = \text{diag}(d_1, \dots, d_n)$$

such that  $A = LDL^T$ . The factorization is unique.

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**THEOREM 2.2** (Mean Value Theorem). *Suppose  $f$  is a function that is continuous on the closed interval  $[a, b]$ . and differentiable on the open interval  $(a, b)$ . Then there exists a number  $c$  such that  $a < c < b$  and*

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

In other words,

$$f(b) - f(a) = f'(c)(b - a).$$

Observe that Theorems 2.1 and 2.2 and Corollary 2.3 correctly mix references to multiple labels.

**COROLLARY 2.3.** *Let  $f(x)$  be continuous and differentiable everywhere. If  $f(x)$  has at least two roots, then  $f'(x)$  must have at least one root.*

*Proof.* Let  $a$  and  $b$  be two distinct roots of  $f$ . By Theorem 2.2, there exists a number  $c$  such that

$$f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{0 - 0}{b - a} = 0. \quad \square$$

Note that it may require two L<sup>A</sup>T<sub>E</sub>X compilations for the proof marks to show.

Display matrices can be rendered using environments from `amsmath`:

$$(2.1) \quad S = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad C = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Equation (2.1) shows some example matrices.

We calculate the Fréchet derivative of  $F$  as follows:

$$(2.2a) \quad F'(U, V)(H, K) = \langle R(U, V), H\Sigma V^T + U\Sigma K^T - P(H\Sigma V^T + U\Sigma K^T) \rangle \\ = \langle R(U, V), H\Sigma V^T + U\Sigma K^T \rangle$$

$$(2.2b) \quad = \langle R(U, V)V\Sigma^T, H \rangle + \langle \Sigma^T U^T R(U, V), K^T \rangle.$$

Equation (2.2a) is the first line, and (2.2b) is the last line.

**3. Algorithm.** Sed gravida lectus ut purus. Morbi laoreet magna. Pellentesque eu wisi. Proin turpis. Integer sollicitudin augue nec dui. Fusce lectus. Vivamus faucibus nulla nec lacus. Integer diam. Pellentesque sodales, enim feugiat cursus volutpat, sem mauris dignissim mauris, quis consequat sem est fermentum ligula. Nullam justo lectus, condimentum sit amet, posuere a, fringilla mollis, felis. Morbi nulla nibh, pellentesque at, nonummy eu, sollicitudin nec, ipsum. Cras neque. Nunc augue. Nullam vitae quam id quam pulvinar blandit. Nunc sit amet orci. Aliquam erat elit, pharetra nec, aliquet a, gravida in, mi. Quisque urna enim, viverra quis, suscipit quis, tincidunt ut, sapien. Cras placerat consequat sem. Curabitur ac diam. Curabitur diam tortor, mollis et, viverra ac, tempus vel, metus.

Our analysis leads to the algorithm in Algorithm 3.1.

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**Algorithm 3.1** Build tree

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Define  $P := T := \{\{1\}, \dots, \{d\}\}$ 
while  $\#P > 1$  do
  Choose  $C' \in \mathcal{C}_p(P)$  with  $C' := \operatorname{argmin}_{C \in \mathcal{C}_p(P)} \varrho(C)$ 
  Find an optimal partition tree  $T_{C'}$ 
  Update  $P := (P \setminus C') \cup \{\bigcup_{t \in C'} t\}$ 
  Update  $T := T \cup \{\bigcup_{t \in \tau} t : \tau \in T_{C'} \setminus \mathcal{L}(T_{C'})\}$ 
end while
return  $T$ 

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**4. Experimental results.** Quisque facilisis auctor sapien. Pellentesque gravida hendrerit lectus. Mauris rutrum sodales sapien. Fusce hendrerit sem vel lorem. Integer pellentesque massa vel augue. Integer elit tortor, feugiat quis, sagittis et, ornare non, lacus. Vestibulum posuere pellentesque eros. Quisque venenatis ipsum dictum nulla. Aliquam quis quam non metus eleifend interdum. Nam eget sapien ac mauris malesuada adipiscing. Etiam eleifend neque sed quam. Nulla facilisi. Proin a ligula. Sed id dui eu nibh egestas tincidunt. Suspendisse arcu.

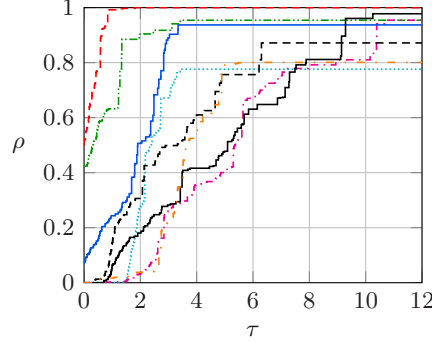
Figure 1 shows some example results. Additional results are available in the supplement in Table 1.

Table 1 shows additional supporting evidence.

TABLE 1  
Example table.

Species	Mean	Std. Dev.
1	3.4	1.2
2	5.4	0.6
3	7.4	2.4
4	9.4	1.8

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FIG. 1. *Example figure using external image files.*

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**5. Discussion of  $Z = X \cup Y$ .** Curabitur nunc magna, posuere eget, venenatis eu, vehicula ac, velit. Aenean ornare, massa a accumsan pulvinar, quam lorem laoreet purus, eu sodales magna risus molestie lorem. Nunc erat velit, hendrerit quis, malesuada ut, aliquam vitae, wisi. Sed posuere. Suspendisse ipsum arcu, scelerisque nec, aliquam eu, molestie tincidunt, justo. Phasellus iaculis. Sed posuere lorem non ipsum. Pellentesque dapibus. Suspendisse quam libero, laoreet a, tincidunt eget, consequat at, est. Nullam ut lectus non enim consequat facilisis. Mauris leo. Quisque pede ligula, auctor vel, pellentesque vel, posuere id, turpis. Cras ipsum sem, cursus et, facilisis ut, tempus euismod, quam. Suspendisse tristique dolor eu orci. Mauris mattis. Aenean semper. Vivamus tortor magna, facilisis id, varius mattis, hendrerit in, justo. Integer purus.

**6. Conclusions.** Some conclusions here.

**Appendix A. An example appendix.** Aenean tincidunt laoreet dui. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Integer ipsum lectus, fermentum ac, malesuada in, eleifend ut, lorem. Vivamus ipsum turpis, elementum vel, hendrerit ut, semper at, metus. Vivamus sapien tortor, eleifend id, dapibus in, egestas et, pede. Pellentesque faucibus. Praesent lorem neque, dignissim in, facilisis nec, hendrerit vel, odio. Nam at diam ac neque aliquet viverra. Morbi dapibus ligula sagittis magna. In lobortis. Donec aliquet ultricies libero. Nunc dictum vulputate purus. Morbi varius. Lorem ipsum dolor sit amet, consectetur adipiscing elit. In tempor. Phasellus commodo porttitor magna. Curabitur vehicula odio vel dolor.

LEMMA A.1. *Test Lemma.*

**Acknowledgments.** We would like to acknowledge the assistance of volunteers in putting together this example manuscript and supplement.

138

## REFERENCES

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