Information Theory and Coding Techniques
Midterm Exam

National Taiwan University

Submit in class: 14:20 Monday, April 22. Total: 100 points

The late submission will be deducted 30% each day. Students are encouraged to discuss the midterm exam, but you must complete the assignment by yourself. Any form of cheating or plagiarism will not be tolerated.

1. 請證明以下定理
   (a) (10 points) $D(p(x,y)||q(x,y)) = D(p(x)||q(x)) + D(p(y|x)||q(y|x))$.
   (b) (10 points) If $f(\cdot)$ is a concave function and X is a random variable, then $E[f(X)] \leq f(EX)$, where E denotes the expectation operator.
   (c) (10 points) The mutual information $I(X;Y)$ is a convex function of $p(Y|X)$, for fixed $p(X)$.
   (d) (10 points) For any estimator $\hat{X}$ such that $X \to Y \to \hat{X}$ form Markov Chain, and define $P_e = P_r(X \neq \hat{X})$, prove that $P_e \geq \dfrac{H(X|Y) - 1}{\log |X|}$.

2. 請證明以下定理
   (a) (10 points) Uniform distribution of finite discrete sources gives Maximal Entropy.
   (b) (10 points) Normal distribution of infinite continuous sources gives Maximal Differential Entropy.

3. (15 points) Consider codes that satisfy the suffix condition, which says no codeword is a suffix of any other codeword. Show that a suffix condition code is uniquely decodable, and show that the minimum average length over all codes satisfying the suffix condition is the same as the average length of the Huffman code for that random variable. (不一定要嚴格的證明，可以敘述的方式說明你/妳的想法)

4. For a finite alphabet source with fixed source symbol distribution;
   (a) (5 points) try to find an algorithm which maps "variable number of source symbols" to "fixed-length codewords" such that the average code length will be asymptotically optimum (minimum).
   (b) (5 points) Take $P_A = 0.7, P_B = 0.2$ and $P_C = 0.1$ as an example, constructing the code-tree according to your algorithm.
   (c) (5 points) 試簡述這種 "variable source symbol"-to-"fixed length codeword" 編碼相較於 Huffman "variable codeword length"-to-"fixed source symbol" 編碼法之優缺點。

5. (a) (5 points) 簡述至目前為止，這門課讓你/妳覺得最有幫助的部分是？為什麼？
   (b) (5 points) 在往後的課程中，你/妳最希望聽到的內容是？為什麼？