1>

$\because x=0.722 \quad y=2.641$ 为有效数字

$\therefore e(x) \le \frac{1}{2}\times10^{-3} \quad e(y) \le \frac{1}{2}\times10^{-3}$

$\therefore e_r(x) = \frac{e(x)}{x} \le 6.925\times10^{-4} \quad e_r(y) = \frac{e(y)}{y} \le 1.893\times10^{-4}$

$\cancel{e_r(f) = \frac{\partial f}{\partial x} e_r(x) \frac{x}{f} + \frac{\partial f}{\partial y} e_r(y) \frac{y}{f}}$

$\cancel{= (e^y - 4x) \frac{x}{}}$

$e(f) = \frac{\partial f}{\partial x} e(x) + \frac{\partial f}{\partial y} e(y) = (e^y - 4x) e(x) + |xe^y \cdot e(y)|$

$\le |e^{2.641} - 4\times0.722| \times \frac{1}{2}\times10^{-3} + |0.722\times e^{2.641} \times \frac{1}{2}\times10^{-3}| \approx 0.0106 = 0.1\times10^{-1} < 0.5\times10^{-1}$

$f(x,y) = 0.722\times e^{2.641} - 2\times0.722^2 \approx 9.085$

$e_r(f) \le 1.167\times10^{-3}$ 具有两位有效数字

2>

$f(x) = x^3 - 2x^2 - 5$

$f'(x) = 3x^2 - 4x = 3x(x - \frac{4}{3})$

$\therefore f(x)$ 在 $(-\infty, 0), (\frac{4}{3}, +\infty)$ 上递增，在 $(0, \frac{4}{3})$ 递减

$\because f(0) = -5 < 0 \quad \therefore f(x) = 0$ 仅有一个实根

$x = \sqrt[3]{2x^2+5} = \varphi(x) \quad x_{k+1} = \sqrt[3]{2x_k^2+5}$

$\because f(2) = -5 \quad f(3) = 4 \quad \therefore$ 实根位于 $(2,3)$

取初始值为 $x_0 = 2.5 \quad x_1 \approx 2.5962 \quad x_2 \approx 2.6439 \quad x_3 \approx 2.6675$

$x_4 \approx 2.6792 \quad x_5 \approx 2.6850 \quad x_6 \approx 2.6878 \quad x_7 \approx 2.6892 \quad x_8 \approx 2.6899$

$x_9 \approx 2.6903 \quad x_{10} \approx 2.6904$

$\because |x_{10} - x_9| = 0.1\times10^{-3} \le 0.5\times10^{-3} \quad \therefore$ 取 $x^* \approx x_{10} = 2.6904$

$\varphi'(x) = \frac{1}{3} \frac{1}{(2x^2+5)^{\frac{2}{3}}}$ 显然 $\varphi'(x)$ 在 $(2,3)$ 上恒小于1

又 $\because$ $\varphi(x)$为增函数 $\varphi(2) \approx 2.351 \quad \varphi(3) \approx 2.844 \quad \therefore$ 当 $x \in [2,3]$ 时，$\varphi(x) \in [2,3]$

$\therefore$ 迭代格式在既定区间 $(2,3)$ 上收敛

3 >

$$(A,b)=\begin{bmatrix}-2&1&0&0&2\\ \frac{1}{2}&4&1&0&-\frac{1}{2}\\ 0&1&-3&-1&-5\\ 0&0&3&-\frac{1}{2}&2\end{bmatrix}\xrightarrow{r_2+\frac{1}{4}r_1}\begin{bmatrix}-2&1&0&0&2\\ 0&\frac{19}{4}&1&0&1\\ 0&1&-3&-1&-5\\ 0&0&3&-\frac{1}{2}&2\end{bmatrix}\xrightarrow{r_3-\frac{4}{19}r_2}$$

$$\begin{bmatrix}-2&1&0&0&2\\ 0&\frac{19}{4}&1&0&1\\ 0&0&-\frac{61}{19}&-1&-\frac{99}{19}\\ 0&0&3&-\frac{1}{2}&2\end{bmatrix}\xrightarrow{r_4+\frac{57}{61}r_3}\begin{bmatrix}-2&1&0&0&2\\ 0&\frac{19}{4}&1&0&1\\ 0&0&-\frac{61}{19}&-1&-\frac{99}{19}\\ 0&0&0&-\frac{175}{122}&-\frac{175}{61}\end{bmatrix}$$

$$\cancel{x_4=\frac{175}{59}\quad x_3=\left(-\frac{99}{19}+\frac{175}{59}\right)\times\left(-\frac{19}{61}\right)=\frac{99}{61}-\frac{175}{59}\times\frac{19}{61}=\frac{99\times59-175\times19}{3599}=\frac{2516}{3599}}$$

$$\cancel{x_2=\left(1-\frac{2516}{3599}\right)\times\frac{4}{19}=\frac{228}{3599}\quad x_1=\left(2-\frac{228}{3599}\right)\times\left(-\frac{1}{2}\right)=-\frac{3485}{3599}}$$

$$\begin{bmatrix}-1\\0\\1\\2\end{bmatrix}\qquad x_4=2\quad x_3=\left(-\frac{99}{19}+2\right)\times\left(-\frac{19}{61}\right)=1$$

$$x_2=(1-1)\times\frac{4}{19}=0\qquad x_1=2\times\left(-\frac{1}{2}\right)=-1$$

4 >

$$x_1^{k+1}=\frac{1-x_2^k-x_3^k}{2}\qquad (L+D)x^{k+1}=B-Ux^k$$

$$x_2^{k+1}=\frac{2-2x_3^k-x_1^{k+1}}{3}$$

$$x_3^{k+1}=\frac{3-2x_2^{k+1}-x_1^{k+1}}{4}$$

$$x^{k+1}=-(L+D)^{-1}Ux^k+(L+D)^{-1}B$$

$$(L+D)=\begin{bmatrix}2&0&0\\1&3&0\\1&2&4\end{bmatrix}\quad (L+D,E)=\begin{bmatrix}2&0&0&1&0&0\\1&3&0&0&1&0\\1&2&4&0&0&1\end{bmatrix}$$

$$\xrightarrow[\substack{r_2-r_1\\ r_3-r_1}]{\frac{1}{2}r_1}\begin{bmatrix}1&0&0&\frac{1}{2}&0&0\\0&3&0&-\frac{1}{2}&1&0\\0&2&4&-\frac{1}{2}&0&1\end{bmatrix}\xrightarrow{\frac{1}{3}r_2}\begin{bmatrix}1&0&0&\frac{1}{2}&0&0\\0&1&0&-\frac{1}{6}&\frac{1}{3}&0\\0&0&4&-\frac{1}{6}&-\frac{2}{3}&1\end{bmatrix}\xrightarrow{\frac{1}{4}r_3}\begin{bmatrix}1&0&0&\frac{1}{2}&0&0\\0&1&0&-\frac{1}{6}&\frac{1}{3}&0\\0&0&1&-\frac{1}{24}&-\frac{1}{6}&\frac{1}{4}\end{bmatrix}$$

$$(L+D)^{-1}=\begin{bmatrix}\frac{1}{2} & 0 & 0\\ -\frac{1}{6} & \frac{1}{3} & 0\\ -\frac{1}{24} & -\frac{1}{6} & \frac{1}{4}\end{bmatrix} \qquad \begin{bmatrix}2 & 0 & 0\\ 1 & 3 & 0\\ 1 & 2 & 4\end{bmatrix}$$

$$(L+D)^{-1}U=\begin{bmatrix}\frac{1}{2} & 0 & 0\\ -\frac{1}{6} & \frac{1}{3} & 0\\ -\frac{1}{24} & -\frac{1}{6} & \frac{1}{4}\end{bmatrix}\begin{bmatrix}0 & \phi & 1\\ 0 & 0 & 2\\ 0 & 0 & 0\end{bmatrix}=\begin{bmatrix}0 & \frac{1}{2} & \frac{1}{2}\\ 0 & -\frac{1}{6} & \frac{1}{2}\\ 0 & -\frac{1}{24} & -\frac{3}{8}\end{bmatrix}$$

$$|\lambda E+(L+D)^{-1}U|=\begin{vmatrix}\lambda & \frac{1}{2} & \frac{1}{2}\\ 0 & \lambda-\frac{1}{6} & \frac{1}{2}\\ 0 & -\frac{1}{24} & \lambda-\frac{3}{8}\end{vmatrix}=\lambda(\lambda-\frac{1}{6})(\lambda-\frac{3}{8})+\frac{1}{48}\lambda$$

$$=\lambda^3-\frac{13}{24}\lambda^2+\frac{\lambda}{16}+\frac{\lambda}{48}$$

$$=\lambda^3-\frac{13}{24}\lambda^2+\frac{\lambda}{12}$$

$$=\lambda(\lambda^2-\frac{13}{24}\lambda+\frac{1}{12})$$

$$\cancel{=\lambda(\lambda-\frac{1}{2})(\lambda-\frac{1}{24})}$$

$$\cancel{\frac{1}{2}+\frac{1}{24}=\frac{13}{24}}$$

$$\cancel{\frac{1}{3}+\frac{1}{4}=}$$

$$\cancel{\frac{13}{24}}\quad \frac{\frac{13}{24}\pm\sqrt{(\frac{13}{24})^2-\frac{1}{3}}}{2}=\frac{13}{48}\pm\frac{\sqrt{23}i}{48}$$

$$13^2+23$$

$$\begin{matrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{matrix}$$

$\rho(-(L+D)^{-1}U)<1$ ∴该迭代格式收敛

∵ A为严格对角占优矩阵 $\Rightarrow$

5 >

$$\begin{array}{cccc} 0 & \frac{1}{2} & -1 & \frac{3}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} & \\ 1 & 1 & & \end{array}$$

$$H_0(x) = \frac{1}{2} - (x-0) + \frac{3}{2}x^2$$

$$= \frac{3}{2}x^2 - x + \frac{1}{2}$$

$$H_0'(x) = 3x - 1 \quad H_0'(1) = \frac{3}{2} - 1 + \frac{1}{2} = 1$$

$$\begin{array}{cccc} 1 & 1 & 1 & -\frac{1}{2} \\ 1 & 1 & \frac{1}{2} & \\ 2 & \frac{3}{2} & & \end{array}$$

$$H_1(x) = 1 + (x-1) - \frac{1}{2}(x-1)^2$$

$$H_1'(x) = 1 - x + 1 = 2 - x \quad H_1'(2) = 0$$

$$\begin{array}{cccc} 2 & \frac{3}{2} & 0 & \frac{1}{2} \\ 2 & \frac{3}{2} & \frac{1}{2} & \\ 3 & 2 & & \end{array}$$

$$H_2(x) = \frac{3}{2} + \frac{1}{2}(x-2)^2$$

$$H_2'(x) = x - 2$$

$$H(x) = \begin{cases} \frac{3}{2}x^2 - x + \frac{1}{2} & [0,1] \\ -\frac{1}{2}x^2 + 2x - \frac{1}{2} & [1,2] \\ \frac{1}{2}x^2 - 2x + \frac{7}{2} & [2,3] \end{cases}$$

6 > $\varphi = \{1, x\}$

$$\int_{-1}^{1} 1^2 dx = 2 \quad \int_{-1}^{1} x\,dx = \frac{x^2}{2}\Big|_{-1}^{1} = 0 \quad \int_{-1}^{1} x^2 dx = \frac{x^3}{3}\Big|_{-1}^{1} = \frac{2}{3}$$

$$\int_{-1}^{1} x^3 dx = 0 \quad \int_{-1}^{1} x^4 dx = \frac{2}{5}$$

$$\begin{bmatrix} 2 & 0 \\ 0 & \frac{2}{3} \end{bmatrix} \begin{bmatrix} C_0 \\ C_1 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{2}{5} \end{bmatrix} \quad \begin{array}{l} C_1 = \frac{3}{5} \\ C_0 = 0 \end{array}$$

$$\min\left\{\int_{-1}^{1}[x^3 - q(x)]^2 dx\right\} = \int_{-1}^{1}\left(x^3 - \frac{3}{5}x\right)^2 dx = \int_{-1}^{1}\left(x^6 - \frac{6}{5}x^4 + \frac{9}{25}x^2\right)dx$$

$$= \frac{2}{7} - \frac{6}{5} \times \frac{2}{5} + \frac{9}{25} \times \frac{2}{3} = \frac{2}{7} - \frac{12}{25} + \frac{6}{25}$$

$$= \frac{8}{175}$$

$\therefore$ 原式 $\geq \frac{8}{175}$

$T \xrightarrow{f(x)=1} \int_{-1}^{1} 1\,dx = 2 = A + B + C$

$f(x)=x \quad \int_{-1}^{1} x\,dx = 0 = -A + C + D + E$

$f(x)=x^2 \quad \int_{-1}^{1} x^2 dx = \frac{2}{3} = A + C - 2D + 2E$

$f(x)=x^3 \quad \int_{-1}^{1} x^3 dx = 0 = -A + C + 2D + 2E$

$f(x)=x^4 \quad \int_{-1}^{1} x^4 dx = \frac{2}{5} = A + C - 3D + 3E$

$D-E = \frac{4}{15}$

$D+E = 0$

$D = \frac{2}{15}$

$E = -\frac{2}{15}$

$C = A$

$2A - 2\times\frac{4}{15} = \frac{2}{3}$

$A = \frac{9}{15} = \frac{3}{5}$

$B = \frac{4}{5}$

$$I_N(f) = \frac{3}{5}f(-1) + \frac{4}{5}f(0) + \frac{3}{5}f(1) + \frac{2}{15}f'(-1) - \frac{2}{15}f'(1)$$

$\because I_N(f)$ 具有5次代数精度

$\therefore$ 可构造 $H_5(x)$ 使得 $H_5(-1)=f(-1)$ $\quad H_5'(-1)=f'(-1)$ $\quad H_5(0)=f(0)$

$H_5(1)=f(1)$ $\quad H_5'(1)=f'(1)$ $\quad H_5'(0)=f'(0)$

$$I(f) - I_N(f) = \int_{-1}^{1}\left[f(x) - H_5(x)\right]dx = \int_{-1}^{1} \frac{f^{(6)}(\xi)}{6!}(x-1)^2(x+1)^2x^2\,dx$$

$$= \frac{f^{(6)}(\eta)}{6!}\int_{-1}^{1}(x-1)^2(x+1)^2x^2\,dx = \frac{f^{(6)}(\eta)}{6!}\left(\frac{2}{7} - 2\times\frac{2}{5} + \frac{2}{3}\right) = \frac{f^{(6)}(\eta)}{4725}$$

$$\int_a^b g(t)\,dt \overset{t=\frac{a+b}{2}+\frac{b-a}{2}x}{=\!=} \int_{-1}^{1} g\left(\frac{a+b}{2} + \frac{b-a}{2}x\right)\frac{b-a}{2}\,dx$$

$$= \frac{b-a}{2}\left(\frac{3}{5}g(a) + \frac{4}{5}g\left(\frac{a+b}{2}\right) + \frac{3}{5}g(b) + \frac{2}{15}g'(a) - \frac{2}{15}g'(b)\right)$$

8 > $k_1 = y'(x_i)$ | $k_2 = f(x_i + \beta h, y(x_i) + \beta h y'(x_i))$

$$= f(x_i, y(x_i)) + \underbrace{\frac{\partial f(x_i, y(x_i))}{\partial x}\beta h + \frac{\partial f(x_i, y(x_i))}{\partial y}\beta h y'(x_i)}_{\triangleq \beta h y''(x_i)}$$

$$+ \underbrace{\frac{1}{2}\left(\frac{\partial f}{\partial x^2}\beta^2 h^2 + 2\frac{\partial f}{\partial x \partial y}\beta^2 h^2 y'(x_i) + \frac{\partial f}{\partial y^2}\beta^2 h^2 y'^2(x_i)\right)}_{\triangleq \frac{\beta^2 h^2}{2}\left(y'''(x_i) - \frac{\partial f}{\partial y} y''(x_i)\right)} + o(h^3)$$

$$y(x_{i+1}) - y_{i+1} = \cancel{y(x_i)} + y'(x_i)h + \frac{1}{2}y''(x_i)h^2 + \frac{y'''(x_i)}{6}h^3 + o(h^4)$$

$$- \cancel{y(x_i)} - \alpha h\left(y'(x_i) + \beta h y''(x_i) + \frac{\beta^2 h^2}{2} y'''(x_i) - \frac{\beta^2 h^2}{2}\frac{\partial f}{\partial y} y''(x_i)\right)$$

$$= (1-\alpha)y'(x_i) + \left(\frac{1}{2} - \alpha\beta\right)h^2 y''(x_i) + \left(\frac{y'''(x_i)}{6} - \frac{\alpha\beta^2 y'''(x_i)}{2} + \frac{\alpha\beta^2}{2}\frac{\partial f}{\partial y} y''(x_i)\right) + o(h^4)$$

$1 - \alpha = 0 \qquad \alpha = 1$

$\frac{1}{2} - \alpha\beta = 0 \qquad \beta = \frac{1}{2}$

$$R_{i+1} = \left(\frac{1}{24} y'''(x_i) + \frac{1}{8}\frac{\partial f}{\partial y} y''(x_i)\right)h^3 + o(h^4)$$

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9 > $$\frac{\partial^2 u}{\partial y^2}(x_i,y_j) = \frac{1}{h^2}\left[u(x_i,y_{j+1}) - 2u(x_i,y_j) + u(x_i,y_{j-1})\right] - \frac{h^2}{12}\frac{\partial^4 u}{\partial y^4}(x_i,\eta_j^i),\ \eta_j^i \in (y_{j-1}, y_{j+1})$$

$$\frac{\partial^2 u}{\partial x^2}(x_i,y_j) = \frac{1}{h^2}\left[u(x_{i+1},y_j) - 2u(x_i,y_j) + u(x_{i-1},y_j)\right] - \frac{h^2}{12}\frac{\partial^4 u}{\partial x^4}(\xi_i^j,y_j),\ \xi_i^j \in (x_{i-1}, x_{i+1})$$

$$-\frac{1}{h^2}\left(u(x_i,y_{j+1}) - 2u(x_i,y_j) + u(x_i,y_{j-1})\right) - \frac{1}{h^2}\left(u(x_{i+1},y_j) - 2u(x_i,y_j) + u(x_{i-1},y_j)\right)$$

$$= f(x_i,y_j) + R_{ij}$$

$$R_{ij} = \frac{h^2}{12}\left(\frac{\partial^4 u}{\partial y^4}(x_i,\eta_j^i) + \frac{\partial^4 u}{\partial x^4}(\xi_i^j,y_j)\right)$$

则 $u_i^{j+1} = 4u_i^j - u_{i-1}^j - u_{i+1}^j - u_i^{j-1} - h^2 f(x_i,y_j)$

$$u_{12} - 4u_{11} + u_{10} + u_{21} + u_{01} = -\frac{1}{9}\times 9\left(\frac{1}{3}+\frac{1}{3}\right)$$

$$u_{13} - 4u_{12} + u_{02} + u_{22} + u_{11} = -\left(\frac{1}{3}+\frac{2}{3}\right)$$

$$u_{22} - 4u_{21} + u_{11} + u_{31} + u_{20} = -\left(\frac{2}{3}+\frac{1}{3}\right)$$

$$u_{23} - 4u_{22} + u_{12} + u_{32} + u_{21} = -\left(\frac{2}{3}+\frac{2}{3}\right)$$

$$-4u_{11} + u_{12} + u_{21} = -\frac{2}{3} \qquad u_{11} = \frac{5}{12}$$

$$u_{11} - 4u_{12} + u_{22} = -1 \qquad u_{12} = \frac{1}{2}$$

$$u_{11} - 4u_{21} + u_{22} = -1 \qquad u_{21} = \frac{1}{2}$$

$$u_{12} + u_{21} - 4u_{22} = -\frac{4}{3} \qquad u_{22} = \frac{7}{12}$$