1> $\because$ $x=1.345$ $y=0.2067$ 均为有效数字

$\therefore$ $e(x) \le \frac{1}{2}\times 10^{-3}$ $e(y) \le \frac{1}{2}\times 10^{-4}$

$$|e(f)| = \left|\frac{\partial f}{\partial x}e(x) + \frac{\partial f}{\partial y}e(y)\right| \le |2x-\sin y|\,e(x) + |-x\cos y|\,e(y)$$

1.308×10^{-3}

$$\le (2\times 1.345 - \sin 0.2067)\times\frac{1}{2}\times 10^{-3} + 1.345\times\cos 0.2067\times\frac{1}{2}\times 10^{-4} \approx 1.177\times 10^{-3}$$

$$f(1.345, 0.2067) = 1.345^2 - 1.345\times\sin 0.2067 \approx 1.533$$

$\because$ $e(f) = 0.1177\times 10^{-2} \le 0.5\times 10^{-2}$ $\therefore$ $f(x,y)$有3位有效数字

$$\frac{e(f)}{f} \le \frac{1.378\times 10^{-3}}{1.533} \approx 7.678\times 10^{-4}$$

0.8532×10^{-3}

2> $f(x) = x^5 - 20x^2 - 2 = 0$

$f'(x) = 5x^4 - 40x = 5x(x^3-8) = 5x(x-2)^3 > 0$

$x<0$ 或 $x>2$

$\therefore$ $f(x)$在$(-\infty,0)(2,+\infty)$递增，在$(0,2)$递减

$f(0) = -2 < 0$ $\therefore$ $f(x)$只有唯一正根

$f(2) = 2^5 - 20\times 2^2 - 2 = -50$ $f(3) = 61$

$$\varphi(x) = x - \frac{f(x)}{f'(x)} = x - \frac{x^5-20x^2-2}{5x^4-40x} = \frac{4x^5-20x^2+2}{5x^4-40x}$$

取 $x_0 = 2.5$ $x_1 \approx 2.8079$ $x_2 \approx 2.7330$ $x_3 \approx 2.7266$ $x_4 \approx 2.7265$

$\because$ $x_3 - x_4 = 0.1\times 10^{-3}$ $\therefore$ $x^* \approx x_4 = 2.7265$

$x^* = 2.727$

3 > $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $(A, b) = \begin{bmatrix} 1 & 4 & 2 & -3 \\ 2 & 3 & -1 & -2 \\ 4 & 0 & 5 & 9 \end{bmatrix} \xleftrightarrow{r_1 \leftrightarrow r_3} \begin{bmatrix} 4 & 0 & 5 & 9 \\ 2 & 3 & -1 & -2 \\ -1 & 4 & 2 & -3 \end{bmatrix}$

$$\xrightarrow[r_3 + \frac{1}{4}r_1]{r_2 - \frac{1}{2}r_1} \begin{bmatrix} 4 & 0 & 5 & 9 \\ 0 & 3 & -\frac{7}{2} & -\frac{13}{2} \\ 0 & 4 & \frac{13}{4} & -\frac{3}{4} \end{bmatrix} \xrightarrow{r_2 \leftrightarrow r_3} \begin{bmatrix} 4 & 0 & 5 & 9 \\ 0 & 4 & \frac{13}{4} & -\frac{3}{4} \\ 0 & 3 & -\frac{7}{2} & -\frac{13}{2} \end{bmatrix} \xrightarrow{r_3 - \frac{3}{4}r_2} \begin{bmatrix} 4 & 0 & 5 & 9 \\ 0 & 4 & \frac{13}{4} & -\frac{3}{4} \\ 0 & 0 & -\frac{95}{16} & -\frac{95}{16} \end{bmatrix}$$

$x_3 = 1 \quad x_2 = \dfrac{-\frac{3}{4} - \frac{13}{4}}{4} = -1 \quad x_1 = \dfrac{9-5}{4} = 1$

4 > Jacobi: $Dx^{k+1} = -(L+U)x^k + B$

Gauss-Seidel: $(L+D)x^{k+1} = -Ux^k + B$

$$\cancel{-D^{-1}(L+U) = -\begin{bmatrix} \frac{1}{\alpha} & & \\ & \frac{1}{\alpha} & \\ & & 2 \end{bmatrix} \begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & \frac{2}{\alpha} & \frac{1}{\alpha} \\ \frac{2}{\alpha} & 0 & -\frac{1}{\alpha} \\ 2 & 2 & 0 \end{bmatrix}}$$

$$|\lambda E + D^{-1}(L+U)| = |D^{-1}[\lambda D + (L+U)]| = |D^{-1}| \begin{vmatrix} \lambda\alpha & 2 & 1 \\ 2 & \lambda\alpha & -1 \\ 1 & 1 & \frac{\lambda}{2} \end{vmatrix} = \frac{\alpha^2}{2}\lambda^3 \cancel{-2} \cancel{+2} \cancel{-\lambda\alpha} - 2\lambda \cancel{+\alpha\lambda}$$

$$= \frac{\alpha^2}{2}\lambda^3 - 2\lambda$$

$$= \lambda\left(\frac{\alpha^2}{2}\lambda^2 - 2\right) = 0$$

$\lambda = 0 \quad \lambda = \pm\frac{2}{\alpha}$

$\left|\frac{2}{\alpha}\right| < 1$

$$|\lambda E + (L+D)^{-1}U| = |(L+D)^{-1}(\lambda(L+D) + U)|$$

$$= |(L+D)^{-1}| \begin{vmatrix} \alpha\lambda & 2 & 1 \\ 2\lambda & \alpha\lambda & -1 \\ \lambda & \lambda & \frac{\lambda}{2} \end{vmatrix} = \frac{\alpha^2}{2}\lambda^3 - 2\lambda \cancel{+2\lambda^2} \cancel{-\alpha\lambda^2} \cancel{+\alpha\lambda^2} \cancel{-2\lambda^2}$$

$$= \lambda\left(\frac{\alpha^2}{2}\lambda^2 - 2\right) = 0$$

$\lambda = 0 \quad \lambda = \pm\frac{2}{\alpha}$

$\therefore \alpha > 2$ 或 $\alpha < -2$

5 >

$$\begin{array}{llll} a & f(a) & f'(a) & \frac{f(b)-f(a)}{(b-a)^2}-\frac{f'(a)}{b-a} \quad \frac{f'(b)+f'(a)}{(b-a)^2}-2\frac{f(b)-f(a)}{(b-a)^3} \\ a & f(a) & \frac{f(b)-f(a)}{b-a} & \frac{f'(b)}{b-a}-\frac{f(b)-f(a)}{(b-a)^2} \\ b & f(b) & f'(b) & \\ b & f(b) & & \end{array}$$

$$H_3(x)=f(a)+f'(a)(x-a)+\left(\frac{f(b)-f(a)}{(b-a)^2}-\frac{f'(a)}{b-a}\right)(x-a)^2+\left(\frac{f'(b)+f'(a)}{(b-a)^2}-2\frac{f(b)-f(a)}{(b-a)^3}\right)(x-a)^2(x-b)$$

$$H_4(x)=H_3(x)+\lambda(x-a)^2(x-b)^2$$

$$H_4'(x)\Big|_{x=\frac{a+b}{2}}=f'(a)+\cancel{2\left(\frac{f(b)-f(a)}{(b-a)^2}-\frac{f'(a)}{b-a}\right)(x-a)}+\frac{f(b)-f(a)}{b-a}-f'(a)\,0-\frac{f'(b)+f'(a)}{2}+\frac{f(b)-f(a)}{b-a}$$

$$+\lambda\left(\frac{(b-a)^3}{4}\cancel{+}-\frac{(b-a)^3}{4}\right)=f'\left(\frac{a+b}{2}\right)$$

当 $2\frac{f(b)-f(a)}{b-a}-\frac{f'(b)+f'(a)}{2}=f'\left(\frac{a+b}{2}\right)$ 时，λ可取任意值满足下列条件

$\neq$ ... 时，不存在λ使得条件成立

6 >

$$\max_{0\le x\le 1}\left|\frac{1}{1+x}-P_1(x)\right|\ge\frac{3}{4}-\frac{\sqrt{2}}{2}$$

$\triangleq f(x)$

$$f(0)-P_1(0)=1-b=-(f(x_0)-P_1(x_0))=f(1)-P_1(1)=\frac{1}{2}-a-b$$

$f'(x_0)=P_1'(x_0)$

$1-b=\frac{1}{2}-a-b \qquad a=-\frac{1}{2}$

$1-b=-\left(\frac{1}{1+x_0}-ax_0-b\right) \qquad b-1-\frac{x_0}{2}+b=\frac{1}{1+x_0}$

$-\frac{1}{(1+x_0)^2}=a=-\frac{1}{2} \qquad 2b=\frac{1}{\sqrt{2}}+1+\frac{\sqrt{2}}{2}-\frac{1}{2}$

$=\sqrt{2}+\frac{1}{2}$

$x_0=\sqrt{2}-1$

$b=\frac{\sqrt{2}}{2}+\frac{1}{4}$

$$\max_{0\le x\le 1}|f(x)-P_1(x)|=\frac{3}{4}-\frac{\sqrt{2}}{2}$$

7 > $T_n(f) = \sum_{i=0}^{n-1}\left[\frac{h}{2}(f(x_{i+1})+f(x_i))\right] = \frac{h}{2}(f(a)+f(b)) + h\sum_{i=1}^{n-1} f(x_i)$

$$I(f) - T_n(f) = \sum_{i=0}^{n-1} -\frac{h^3}{12} f''(\xi_i) \qquad \xi_i = (x_i, x_{i+1})$$

$$\cancel{= -\frac{h^3}{12}\sum_{i=0}^{n-1} f''(\xi_i) = -\frac{h^3}{12}\, n f''(\eta)}$$

$$\frac{I(f)-T_n(f)}{h^2} = \sum_{i=0}^{n-1} -\frac{h}{12} f''(\xi_i) \overset{h\to 0}{=} \int_a^b -\frac{f''(x)}{12}\,dx = -\frac{1}{12}\left[f'(b) - f'(a)\right]$$

$h \to 0$ ℓI

$$\frac{I(f) - T_{2n}(f)}{\frac{h^2}{4}} = -\frac{1}{12}\left(f'(b) - f'(a)\right) \approx \frac{I(f) - T_n(f)}{h^2}$$

$$I(f) \approx \frac{4}{3} T_{2n}(f) - \frac{1}{3} T_n(f)$$

8 > $R_{i+1} = y(x_{i+1}) - y(x_i) - Ahf(x_i, y(x_i)) - (1-A)hf(x_i + Bh, y(x_i) + \frac{4}{5}hy'(x_i))$

$= \cancel{y(x_i)} + y'(x_i)h + \frac{y''(x_i)}{2}h^2 + \frac{y'''(x_i)}{6}h^3 + O(h^4) - \cancel{y(x_i)} - Ahy'(x_i)$

$- (1-A)h\Big[f(x_i, y(x_i)) + \frac{\partial f(x_i)}{\partial x}Bh + \frac{\partial f}{\partial y}\frac{4}{5}hy'(x_i) + \frac{1}{2}\Big(\frac{\partial^2 f}{\partial x^2}B^2h^2 + 2\frac{\partial^2 f}{\partial x \partial y}\frac{4B}{5}h^2 y'(x_i)$

$+ \frac{\partial^2 f}{\partial y^2}\frac{16}{25}h^2 y'^2(x_i)\Big) + O(h^3)\Big]$

$= (1 - A - 1 + A)hy'(x_i) + \cancel{h^2(\frac{1}{2} - (1-A)B}\ h^3\left[\left(\frac{1}{6} - \frac{5}{8}\times\frac{1}{2}\times\frac{16}{25}\right)y'''(x_i) + \frac{\partial f}{\partial y}y''(x_i)\right] + O(h^4)$

$B = \frac{4}{5} \qquad \frac{1}{2} - (1-A)\frac{4}{5} = 0 \qquad A = \frac{5}{4}\left(\frac{4}{5} - \frac{1}{2}\right) = \frac{3}{10}\times\frac{5}{4} = \frac{3}{8}$

9 >

$$\frac{\partial u}{\partial t} = \frac{1}{\tau}\left[u(x_i,t_k) + u(x_i,t_{k-1})\right] + \frac{\tau}{2}\frac{\partial^2 u}{\partial t^2}(x_i,\eta_i^k) \quad t_{k-1} < \eta_i^k < t_k$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{h^2}\left(u(x_{i+1},t_k) - 2u(x_i,t_k) + u(x_{i-1},t_k)\right) + \frac{h^2}{12}\frac{\partial^4 u}{\partial x^4}(\xi_i^k,t_k) \quad x_{i-1} < \xi_i^k < x_{i+1}$$

$$\frac{1}{\tau}\left(u_i^k + u_i^{k-1}\right) - \frac{2}{h^2}\left(u_{i+1}^k - 2u_i^k + u_{i-1}^k\right) + u_i^k = f(x_i,t_k) \underbrace{- \frac{\tau}{2}\frac{\partial^2 u}{\partial t^2}(x_i,\eta_i^k) + \frac{h^2}{12}\frac{\partial^4 u}{\partial x^4}(\xi_i^k,t_k)}_{= R_{i+1}}$$