Phased Array Radar Resource Management Using Continuous Double Auction

ALEXANDER CHARLISH, Member, IEEE
Fraunhofer FKIE
Wachtberg, Germany

KARL WOODBRIDGE, Senior Member, IEEE
HUGH GRIFFITHS, Fellow, IEEE
University College London
London, U.K.

This paper addresses the problem of allocating resource and selecting operational parameters for multiple tasks for an electronically steered phased array radar system. The continuous double auction parameter selection (CDAPS) algorithm is presented, which controls the operational parameters such that a resource constraint is satisfied. The algorithm optimises the complete multiple task parameter selection problem, in contrast to the common approach of optimising the parameter selection for each single task separately. It is shown that CDAPS produces a near-optimal solution for the single resource discrete parameter selection problem. Simulated scenarios verify that this near-optimum solution significantly improves upon conventional rule based methods. The additional desirable characteristics of the CDAPS algorithm: computational efficiency, scalability, and rapid reaction, are also highlighted.

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Authors' addresses: A. Charlish, Department of Sensor Data and Information Fusion, Fraunhofer Institute for Communication, Information Processing and Ergonomics FKIE, Fraunhoferstraße 20, 53343 Wachtberg, Germany. E-mail: (alexander.charlish@fkie.fraunhofer.de). K. Woodbridge, H. Griffiths, Radar Group in the Department of Electronic and Electrical Engineering, University College London, Roberts Building, Gower Street, London, WC1E 6BT, U.K.

I. INTRODUCTION

Advances in electronic components [1] have enabled the development of agile, multirole sensor systems, such as the multifunction radar (MFR) [2]. The MFR is able to control the execution of numerous tasks which support varied functions. These functions can include searching, target tracking, environmental/situational assessment, communications, weapons control, and calibration [3]. The MFR typically utilises an electronically steered array (ESA) antenna [4, 5], which enables extreme beam agility. Therefore, the radar beam can be steered almost instantaneously [4, 5] and the available radar time/power budget can be dynamically allocated between radar tasks. Consequently, a large number of radar tasks can be executed by a single antenna, multiplexed in time and angle [6]. More generally, an MFR can configure operational parameters, such that the transmitted signal and the receiver processing is optimised to each radar task [7].

It is desired to achieve an overall best possible performance for all tasks, using the finite radar time/power budget which is available. This radar resource management problem can be formulated as a multidimensional parameter selection problem, in which the parameters control such aspects as the task revisit interval time, the task dwell duration, or the transmitted waveform. A critical aspect of the problem is that sequential measurements of a dynamic scenario and environment are received. Therefore, a solution should be available at each measurement instance, based on the information contained in the most recent measurements. To be suitable for real-time operation, this must be achieved with low computational load.

Radar resource management for tracking and volume search has been widely studied in the literature, with the majority of work focussed on tracking. Van Keuk and Blackman (VKB) [8] state rules for selecting the revisit interval time and signal-to-noise ratio (SNR) which minimises the loading of each active tracking task. Active tracking is the process of maintaining tracks using measurements from a series of dwells which are dedicated to each target. Minimising the individual track loading is considered complementary to the objective of maximising the number of targets which can be tracked with the finite resource available. Boers et al. [9] extend this approach by minimising the track loading for a required track accuracy without the need to specify an SNR. This results in a lower transmit energy spent on the maintenance of each track.

Strong scenario dependence led to the introduction of the benchmark problems. The benchmark problems [10] allowed for the comparative assessment of tracking parameter control algorithms for a variety of manoeuvring targets. Some solutions to the benchmark problems [11–13] schedule track updates at the latest possible time, such that a bound on the estimation error is not exceeded. These solutions demonstrate significant reductions in track loading, in comparison to using fixed revisit interval times.

These previous works all optimise track parameter control for single radar tracking tasks independently. Therefore, when used to select parameters for multiple targets, a summation of local optima is produced. An effective tracking parameter control algorithm for multiple targets should consider the possible performances of all of the tracking tasks as well as the resource constraints for the set of tasks. Only then can the best performance for the multiple targets be achieved, with the finite resource available.

Q-RAM (Quality of Service (QoS) resource allocation method) [14–16] is a resource management method that maximises the summation of the individual task utilities whilst satisfying resource constraints. As such it has been applied to the multiple target parameter control problem [17,18]. By satisfying the Karush-Kuhn-Tucker (KKT) conditions [19] a near-optimal solution is produced, whose deviation from the optimum depends on whether the optimal parameter selections lie on the concave majorant [20] and how well the concave majorant is approximated [18]. The Q-RAM algorithm is particularly important as it transitions from the previous methods of optimising radar tasks individually to optimising the complete problem considering all tasks and the global resource availability simultaneously.

As the environment and the scenario are dynamic, and new information is received in each measurement, Q-RAM must repetitively recompute the operational parameters in resource allocation frames. Ideally this would be recomputed after each measurement, however, an allocation frame duration in the order of seconds is more practically realistic. This recomputation is computationally inefficient, as the entire allocation may not need to be recomputed. Additionally, the time taken to compute a new allocation frame imposes a limit on the reaction time of the algorithm. The computational inefficiency of recomputing allocation frames has previously been identified by Lee et al. [21] where the use of "service class" templates is proposed. However, these templates are inevitably coarse representations of the reality and so a suboptimal solution ensues. Wintenby and Krishnamurthy [22] propose a method based on stochastic control, however, the high computational demand for an optimal allocation is again identified.

This paper presents the continuous double auction parameter selection (CDAPS) algorithm [23–25], which utilises a continuous double auction mechanism which settles on a market equilibrium that satisfies the KKT conditions. This enables the solution from the previous time step to be adapted to the current time step without a complete recomputation of the resource allocation, hence reducing computation for dynamic problems such as the radar resource management problem.

Contributions: The primary contribution of this paper is the presentation and evaluation of the CDAPS algorithm, which is described in Section III. A secondary contribution is the reformulation of the VKB strategy into a tracking resource allocation model in Section IV-C.

Structure: Section II gives a general formulation of the radar resource management problem, which is followed by a description of the CDAPS algorithm in Section III. The paper then focuses on the example of allocating radar resource between multiple targets maintained by the active tracking function. As such, an active tracking resource allocation model based on the VKB model is presented in Section IV. Simulated results are presented in Section V, where the benefits of the CDAPS algorithm in comparison with the Q-RAM algorithm are analysed. Finally conclusions are drawn in Section VI.

II. RADAR RESOURCE MANAGEMENT PROBLEM FORMULATION

The general radar resource management and task parameter control problem is formulated in this section as a constrained optimisation problem, similar to the formulation used by the Q-RAM algorithm [17]. A condition for the optimal solution to the problem is given in Section II-B.

A. Problem Formulation

There exists a set of K independent radar tasks $T = \{T_1, T_2, ..., T_K\}$, which must share the finite radar time budget available. It is necessary to select operational parameters for each task, which are the optimisation variables in the optimisation problem. These operational parameter selections represent the current resource management plan for a time horizon extending into the future. It is necessary to have valid operational parameter selections for each measurement instance $t \in T_s$ where T_s is the complete set of measurement instances during the operation of the radar. The resource management time horizon is much larger than the time interval between measurement instances. Examples of operational parameters are the task dwell duration, the task revisit interval time, or the transmitted waveform. The operational parameters for radar task T_k at measurement instance t are denoted υ_{tk} and it is required to select parameters for all radar tasks, to give the set of selections $\upsilon_t = \{\upsilon_{t1}, \upsilon_{t2}, ..., \upsilon_{tK}\}$. The parameter selection $\upsilon_{tk} = \{\upsilon_{tk}^1, \upsilon_{tk}^2, ..., \upsilon_{tk}^L\}$ is itself a set of parameters with L dimensions.

Additionally, there are uncontrollable environmental parameters which impact on the resource loading and the quality achieved by each radar task. Examples of the environmental parameters for a tracking task are the target range, target bearing, or parameters of the target model. The environmental parameters for tracking task T_k at measurement instance t are denoted e_{tk}. In practice these environmental parameters are not known and must be estimated from the received measurements.

The operational parameters selected for each radar task impact on the resource loading of the radar task. The resource loading of radar task T_k for the time horizon starting from measurement instance t is denoted r_{tk}. The calculation of the resource loading is denoted by a resource function that maps the operational and environmental parameters into resource space:

$$r_{tk} = g_k(\upsilon_{tk}, e_{tk}) \tag{1}$$

The radar time budget available to maintain all the radar tasks is constrained. Hence, the total resource available for the time horizon starting from measurement instance t is denoted as $\hat{r}_t$, and so the resource function:

$$g(\upsilon_t) = \left(\sum_{k=1}^{K} g_k(\upsilon_{tk}, e_{tk})\right) - \hat{r}_t \tag{2}$$

must satisfy the constraint:

$$g(\upsilon_t) \leq 0 \tag{3}$$

The operational parameters selected for each radar task at measurement instance t affect the task quality that can be achieved over the future time horizon that starts at measurement instance t. The expected quality of task T_k predicted over the future time horizon but based on the state of the task at measurement instance t is denoted q_{tk}. This calculation is denoted as a quality function that maps operational and environmental parameters into quality space:

$$q_{tk} = q_k(\upsilon_{tk}, e_{tk}) \tag{4}$$

The achieved task quality is to some degree satisfactory, depending on the specific requirement of the radar task, which is dictated by the current mission. The utility of radar task T_k at measurement instance t is denoted u_{tk}. A utility function is defined to map from task quality space into task utility space:

$$u_{tk} = u_k(q_k(\upsilon_{tk}, e_{tk})) \tag{5}$$

In this work the future task quality is approximated using a steady state model, however, this formulation could be adapted such that the quality and utility functions are numerically evaluated by aggregating the quality and utility predicted over the measurement instances within the time horizon. This adaptation would then bear a closer resemblance to receding horizon control applied in nonmyopic control problems such as partially observable Markov decision processes.

The total utility for the time horizon starting at measurement instance t can be found by summing across the individual task utilities to give the objective function:

$$u(\upsilon_t) = \sum_{k=1}^{K} u_k(q_k(\upsilon_{tk}, e_{tk})) \tag{6}$$

The utility of each radar task represents the satisfaction that is associated with its task quality. Therefore, the total utility represents the overall satisfaction or performance across all the tasks maintained by the system.

The radar resource management problem can be formulated as a constrained optimisation problem for measurement instance t:

$$\begin{aligned}
&\underset{\upsilon_t}{\textbf{maximise}}: \ u(\upsilon_t) = \sum_{k=1}^{K} u_k(q_k(\upsilon_{tk}, e_{tk})) \\
&\textbf{subject to}: \ g(\upsilon_t) \leq 0 \\
&\text{where}: \qquad g(\upsilon_t) = \left(\sum_{k=1}^{K} g_k(\upsilon_{tk}, e_{tk})\right) - \hat{r}_t
\end{aligned}$$

The radar resource management problem is dynamic, in that the environmental parameters (as well as their estimates) for each radar task can vary at every measurement instance. Therefore this constrained optimisation problem should be solved for each time horizon starting from all measurement instances $t \in T_s$. As the solution for each time horizon represents a longer time interval than each measurement instance duration, a scheduler uses the updated solutions to decide which task makes a measurement at each measurement instance.

B. Optimal Solution

Consider, that the objective function $u(\upsilon_t)$ is a concave differentiable function and the resource function $g(\upsilon_t)$ is a convex differentiable function and that $\upsilon_t^* = \{\upsilon_{t1}^*, \upsilon_{t2}^*, ..., \upsilon_{tK}^*\}$ is a globally optimal parameter selection set. Then, the KKT conditions [19] are a set of sufficient conditions for the optimal parameter selection set υ_t^*. The KKT conditions for the radar resource management problem described in Section II-A are

$$-\nabla u\left(\upsilon_t^*\right) + \mu \nabla g\left(\upsilon_t^*\right) = 0 \tag{7}$$

$$g\left(\upsilon_t^*\right) \leq 0 \tag{8}$$

$$\mu \geq 0 \tag{9}$$

$$\mu g\left(\upsilon_t^*\right) = 0 \tag{10}$$

where μ is a KKT multiplier. Note that as the problem involves a maximisation, the objective function $u(\upsilon_t)$ must be concave for the KKT conditions to be sufficient instead of just necessary.

Equation (7) has an important interpretation, due to the independence of the radar tasks. First note that υ_{tk}^* is itself a vector given by $\upsilon_{tk}^* = (\upsilon_{tk}^{1*}, \upsilon_{tk}^{2*}, ..., \upsilon_{tk}^{L*})$, namely the set of optimal parameters for radar task T_k at measurement instance t. Therefore we write the gradient components in (7) as

$$\frac{\partial u\left(\upsilon_t^*\right)}{\partial \upsilon_{tk}^*} = \begin{pmatrix} \frac{\partial u\left(\upsilon_t^*\right)}{\partial \upsilon_{tk}^{1*}} \\ \frac{\partial u\left(\upsilon_t^*\right)}{\partial \upsilon_{tk}^{2*}} \\ .... \\ \frac{\partial u\left(\upsilon_t^*\right)}{\partial \upsilon_{tk}^{L*}} \end{pmatrix} \tag{11}$$

Due to the independence of the radar tasks, $u(\upsilon_t)$ in (6) is a sum of independent components and so

$$\frac{\partial u\left(\upsilon_t^*\right)}{\partial \upsilon_{tk}^{l*}} = \frac{\partial \tilde{u}_k\left(\upsilon_{tk}^*\right)}{\partial \upsilon_{tk}^{l*}} \quad \forall l \in \{1, 2, ..., L\} \tag{12}$$

where for notational convenience we define $\tilde{u}_k(\upsilon_{tk}) = u_k(q_k(\upsilon_{tk}, e_{tk}))$. Likewise from (2) $g(\nu)$ is a sum of independent components and so

$$\frac{\partial g\left(\upsilon_t^*\right)}{\partial \upsilon_{tk}^{l*}} = \frac{\partial \tilde{g}_k\left(\upsilon_{tk}^*\right)}{\partial \upsilon_{tk}^{l*}} \quad \forall l \in \{1, 2, ..., L\} \tag{13}$$

where for notational convenience we define $\tilde{g}_k(\upsilon_{tk}) = g_k(\upsilon_{tk}, e_{tk})$. Therefore (7) indicates that

$$\mu = \frac{\partial_l \tilde{u}_k\left(\upsilon_{tk}^*\right)}{\partial_l \tilde{g}_k\left(\upsilon_{tk}^*\right)} \quad \begin{array}{l}\forall k \in \{1, 2, ..., K\} \\ \forall l \in \{1, 2, ..., L\}\end{array} \tag{14}$$

where ∂_l denotes the partial derivative with respect to υ_{tk}^{l*}. This set of conditions implies that the stationarity condition is satisfied when the gradients of resource over utility in all dimensions and for all tasks are equal to a common value, which is the KKT multiplier μ. The optimal solution is found when the stationarity condition is satisfied [(7) and (14)] and the solution is primal feasible (8) and dual feasible (9) and either all the resource has been allocated or there would be no utility increase from further resource allocation (10).

The solution to this problem can be found using convex optimisation techniques, such as the Q-RAM algorithm, which solves this problem for discrete parameter selections. This discretisation is useful for performance models that have no closed form representation or need to be evaluated numerically, as is often encountered in radar resource management. In the discrete case, it is unlikely that the gradients in (14) can be equal, instead the optimum is found when they are as close to equal as possible. The proposed CDAPS algorithm is novel in that it enables the solution from the previous measurement instance to be adapted to the current measurement instance without recalculating the entire resource allocation frame. This enables a benefit in computation, for dynamic problems such as radar resource management, but gives no benefit for static problems.

III. CONTINUOUS DOUBLE AUCTION PARAMETER SELECTION

The primary contribution of this paper is the utilisation of a continuous double auction (CDA) mechanism that settles on a market equilibrium that satisfies the KKT conditions in (7)–(10). The resulting algorithm is referred to as CDAPS. General CDA mechanisms are described in Section III-A. The CDA implementation used in the CDAPS algorithm is described in Section III-B - Section III-D.

A. General Continuous Double Auction Mechanisms

The radar resource management algorithm proposed in this paper relies upon trading in a synthesised market. Markets are populated by participants, referred to throughout as agents, who can buy or sell varying quantities of resources. An agents' complete set of feasible offers to buy or sell resource is referred to as its preferences.

A market can achieve the optimal market efficiency, in terms of maximising agent profit, by optimally setting the transaction price for resource trades. In a centralised market, this can be achieved through a central auctioneer who aggregates the complete preferences of all the agents. However, it is computationally demanding for each agent to calculate its complete preferences and for the auctioneer to determine the resource allocation. Yet this computationally demanding centralised approach can be likened to existing techniques for radar resource management such as Q-RAM.

In contrast, the CDA mechanism [26, 27] governs the exchange of resources between agents in a decentralised manner. The mechanism is double-sided as each agent is able to assume the role of both a buyer or a seller of resource, and it is continuous because trades can occur at any point in time. The central auctioneer no longer aggregates the complete preferences, instead the resource allocation emerges as a competitive market equilibrium that is based on just a small number of offers which are revealed by the agents whilst competing with each other. The revelation of just a small number of offers requires significantly less computation. Despite the partial preference revelation, it has crucially been shown [28] that selfish, profit-driven agents in a CDA still achieve close to the optimal market efficiency of the centralised mechanism.

B. Continuous Double Auction Parameter Selection Algorithm

The objective of the CDAPS algorithm is to utilise a CDA, which settles on a market equilibrium that satisfies the KKT conditions in (7)–(10). The CDAPS algorithm is best thought of as a multiagent system [29]. There exists a set of K task agents $\kappa = \{\kappa_1, \kappa_2, ..., \kappa_K\}$, with a single agent representing a single radar task. Additionally, an auctioneer agent is present, who facilitates the trading in the market. The task agents engage in resource trades with the goal of acquiring as much resource as affordable to maintain their represented radar task.

The CDAPS algorithm implements the following CDA protocol:

1) The resource held by task agent κ_k at measurement instance t, which is denoted r_{tk}, is the allowed radar loading for the radar task that is represented by the agent.

2) The total resource held by all task agents at any time t cannot exceed the radar resource loading available $\hat{r}_t$ for all the radar tasks:

$$\sum_{k=1}^{K} r_{tk} \leq \hat{r}_t \tag{15}$$

3) Each agent may publicly announce offers to trade comprising of a quantity s, unit price p, and the agent identifier κ_k.

4) At any measurement instance each task agent may announce bids to buy resource ($b_n(s, p, k)$) or ask to sell resource ($a_m(s, p, k)$).

5) Each offer remains active until it is cleared or updated by the agent, giving a set of M active asks $A = \{a_1, ..., a_M\}$ and a set of N active bids $B = \{b_1, ..., b_N\}$ at any time.

6) A subset $P \subseteq A$ of I asks, with value

$$V_P = \sum_{i=1}^{I} p_i s_i$$

and quantity

$$S_P = \sum_{i=1}^{I} s_i$$

and a subset $Q \subseteq B$ of J bids, with value

$$V_Q = \sum_{j=1}^{J} p_j s_j$$

and quantity

$$S_Q = \sum_{j=1}^{J} s_j$$

can generate a transaction if $V_P < V_Q$ and $S_P \geq S_Q$. Transactions of equal value do not occur, as an equal value transaction does not increase the total utility.

7) The k-pricing rule [30] is applied, so the transaction price $\hat{p}$ is a weighted average of the highest price in the ask set p_{max} and the lowest price in the bid set q_{min}. A k-pricing value of 0.5 is used as it has been shown [31] to achieve the maximum market efficiency. Therefore, the transaction price is calculated as

$$\hat{p} = 0.5 p_{max} + 0.5 q_{min}$$

Each agent having differing and potentially dynamic valuations of resource and utility, which is calculated using the resource, quality and utility functions, drives the trading in the algorithm. After trading, the market settles in a competitive market equilibrium.

C. Task Agents

The offer a task agent announces is based upon the task resource (1), quality (4), and utility (5) functions. Using these functions, a possible parameter selection for a radar task can be plotted in resource-utility space. This is illustrated in Fig. 1 for an example tracking task, where each cross in the figure represents a possible parameter selection. Differing environmental parameters for different radar tasks lead to each possible parameter selection occupying a different point in resource-utility space. Therefore, each of the multiple radar tasks can have a differing resource-utility plot.

Each offer represents the transition from the current operational parameter selection to a neighbouring parameter selection on the concave majorant [20] in

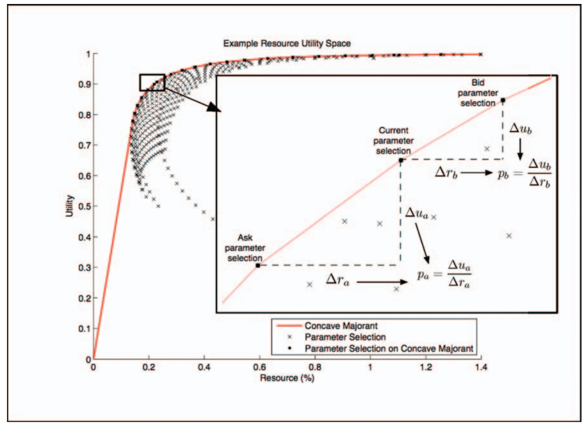


Fig. 1. Parameter selections and concave majorant in resource-utility space for example tracking task. Zoomed-in section of concave majorant around current operational parameter selection is also illustrated. Bid and ask announced are calculated using difference in resource and utility to nearest two parameter selections on concave majorant.

resource-utility space. The concave majorant for the example tracking task is also illustrated in Fig. 1. Reducing the problem to the parameter selections on the concave majorant is useful as it reduces the parameter selections considered while ensuring the objective function is concave.

Fig. 1 illustrates a zoomed in section of the concave majorant. As illustrated, a bid is generated from the nearest parameter selection on the concave majorant with an increase in resource, denoted $\hat{\upsilon}_{tk}$. Likewise, an ask is generated from the nearest parameter selection on the concave majorant with a decrease in resource, denoted $\tilde{\upsilon}_{tk}$.

The transition from the current parameter selection to the neighbouring parameter selection with an increase in resource would result in a change in the task resource loading Δr_b and a change in the task utility Δu_b. Therefore, the true valuation of the bid price p_b^k that task agent κ_k is prepared to trade at is:

$$p_b^k = \frac{\Delta u_b}{\Delta r_b} = \frac{u_k\left(q_k\left(\hat{\upsilon}_{tk}, e_{tk}\right)\right) - u_k\left(q_k\left(\upsilon_{tk}, e_{tk}\right)\right)}{g_k\left(\hat{\upsilon}_{tk}, e_{tk}\right) - g_k\left(\upsilon_{tk}, e_{tk}\right)} \quad (16)$$

Likewise, the transition from the current parameter selection to the neighbouring parameter selection with a decrease in resource would result in a change in the task resource loading Δr_a and a change in the task utility Δu_a. Therefore, the true valuation of the ask price p_a^k that task agent κ_k is prepared to trade at is

$$p_a^k = \frac{\Delta u_a}{\Delta r_a} = \frac{u_k\left(q_k\left(\upsilon_{tk}, e_{tk}\right)\right) - u_k\left(q_k\left(\tilde{\upsilon}_{tk}, e_{tk}\right)\right)}{g_k\left(\upsilon_{tk}, e_{tk}\right) - g_k\left(\tilde{\upsilon}_{tk}, e_{tk}\right)} \quad (17)$$

Neighbouring points on the concave majorant can be found using an adapted version of the first-order traversal technique applied in Q-RAM [18]. The adaptation continues the traversal process until the current parameter selection and a specified number of adjacent offer parameter selections found using traversal form a concave set.

TABLE I
Order Book Able to Clear

Bids (price, quantity, identifier)	Asks (price, quantity, identifier)
b_1 (11, 2, 4)	a_1(3, 3, 6)
b_2 (9, 3, 2)	a_1(6, 2, 3)
b_3 (7, 1, 7)	a_3(8, 2, 8)
b_4 (4, 2, 1)	a_4(10, 1, 5)

The announced bid and asks are bounds on an estimate of the derivative at the current parameter selection:

$$p_b^k \leq \frac{\partial_l u_k \left(q_k \left(\upsilon_{tk}, e_{tk}\right)\right)}{\partial_l g_k \left(\upsilon_{tk}, e_{tk}\right)} \leq p_a^k \quad \forall l \in \{1, 2, ..., L\} \quad (18)$$

Trading in the market drives the algorithm to converge on a competitive market equilibrium price, which is equal to the KKT multiplier μ. Hence the gradients in resource-utility space for all task parameter selections are as close to equal as possible and the global optimum found. Another interpretation is that if the gradients of two tasks in resource-utility space are not equal, then resource can be exchanged which results in an increase in total utility. The CDAPS algorithm continuously moves resource until it is not possible to increase the total utility further, thus producing the optimal solution.

When the concave majorant step is used then the solution is only near optimal, as it cannot be guaranteed that the optimal selection lies on the concave majorant [32]. However, when there are a large number of possible parameter selections then the deviation from the optimal in practice is quite small. Deviations from the optimal solution can also occur if the concave majorant is approximated and not calculated exactly, as in the case with first-order traversal [18] and the traversal method applied in this paper.

D. Auctioneer Agent

An auctioneer agent is also present, who is responsible for facilitating the trading between the task agents. The auctioneer agent maintains a public list of the highest bid prices and the lowest ask prices which have been announced. The number of entries in the order book can be limited as a percentage of the total resource available. After each new offer is announced, the auctioneer attempts to match a set of active bids and asks. If a transaction can be generated, the auctioneer notifies the task agents who subsequently perform the transaction.

An example of an order book is shown in Table I. The order book lists the most competitive active bids and the most competitive active asks. This order book is plotted as a supply and demand curve in Fig. 2. In Table I and Fig. 2 it can be seen that a transaction can be generated by matching the ask transaction set $P = \{a_1, a_2\}$ with the bid transaction set $Q = \{b_1, b_2\}$. The transaction price is $\hat{p} = 7.5$, which as illustrated in Fig. 2 is the point at which supply equals demand. Subsequently, for the ask

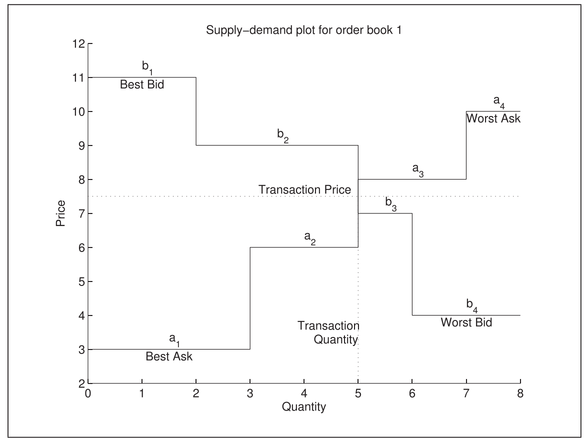


Fig. 2. Order book from Table I which is able to clear resulting in ask transaction set $P = \{a_1, a_2\}$ and bid transaction set $Q = \{b_1, b_2\}$.

TABLE II
Order Book Unable to Clear

Bids (price, quantity, identifier)	Asks (price, quantity, identifier)
b_1 (6, 2, 4)	a_1(7, 3, 6)
b_2 (5, 3, 2)	a_2(8, 2, 3)
b_3 (4, 1, 7)	a_3(9, 2, 8)
b_4 (3, 2, 1)	a_4(10, 1, 5)

transaction set $S_P = 5$ and $V_P = 21$, and for the bid transaction set $S_Q = 5$ and $V_Q = 49$.

It is also possible that a transaction cannot be generated if no bid transaction set has a higher value than an ask transaction set. Such a situation, where the market is in an equilibrium state, is demonstrated in the order book in Table II.

Although the CDAPS algorithm has a central auctioneer, the algorithm is decentralised. This is because the auctioneer compiles just a small subset of all the active offers. The full set of offers, and hence the full information, remains distributed amongst the task agents.

IV. ACTIVE TRACKING RESOURCE ALLOCATION MODELS

The previous sections presented a general problem formulation and the CDAPS algorithm for radar resource management. This section reduces the scope to demonstrate the proposed algorithm on a multiple target active tracking scenario. Consequently, an active tracking resource allocation model is introduced that allows the task resource (1), quality (4), and utility (5) functions to be derived.

The resource allocation model is based on the strategy used by van Keuk and Blackman (VKB). Section IV-A and Section IV-B introduce the model and the strategy used by VKB, respectively. Section IV-C presents an additional contribution of this paper, that is, the reformulation of the VKB strategy into a resource allocation model suitable for optimisation using Q-RAM or CDAPS.

A. Van Keuk and Blackman Model

The VKB model [8] assumes that multiple well-separated targets are maintained by the active tracking function of an electronically steered phased array radar. The VKB model assumes a point target which occupies a single range-Doppler resolution cell. The target dynamics evolve in three-dimensional Cartesian coordinates assuming that the acceleration in a single coordinate is a Gaussian Markov process that is described by the acceleration standard deviation Σ and correlation time Θ.

When a track update is executed, the radar beam is directed towards the estimated position of the target in angle space. A beam positioning power loss occurs when the true target angle is offset from the estimated angle. In the VKB model this power loss is modelled by a Gaussian loss function that is matched to the antenna beamwidth.

It is assumed that the signal is coherently integrated in a range-Doppler filter. This is followed by a square law detector that declares a detection when the signal power exceeds a threshold, which is dependent on the desired probability of false alarm P_f. The target amplitude is assumed to fluctuate according to the Swerling I model, i.e., it is assumed that consecutive dwells are uncorrelated.

Unbiased measurements of the angular position of a target are corrupted by additive Gaussian noise that in high SNR has a standard deviation according to [33]:

$$\sigma = \frac{2 \cdot \theta_B}{k_m \sqrt{2 \cdot \mathrm{SNR}}} \tag{19}$$

where k_m is the slope of the normalised monopulse error curve, which is taken by VKB as $k_m = \sqrt{2}$. SNR is the signal-to-noise ratio encountered in the range-Doppler cell and θ_B is the antenna half beamwidth, i.e., half of the antenna 3 dB beamwidth.

B. Van Keuk and Blackman Strategy

VKB describe a strategy where track updates are scheduled when the angular estimation error, along the major axis of the uncertainty ellipse, equals a fraction of the half beamwidth θ_B. This fraction of the half beamwidth is referred to as the track sharpness ν_0. The strategy minimises the track loading by balancing the trade between using short and long revisit interval times. Short revisit intervals imply a high track update frequency, which exerts a high track load. Long revisit intervals also exert a high track load due to a lowered probability of detection and degraded measurement accuracy. These are encountered because the degraded angular estimation error increases the beam positioning loss.

The minimum track loading can be found by selecting the probability of false alarm P_f, the track sharpness ν_0, and the desired SNR without beam positioning loss, which is denoted SN_0. VKB recommend selecting the probability of false alarm between 10^{-4} and 10^{-5}, the track sharpness to $\nu_0 \approx 0.3$ half beamwidths, and a coherent dwell length, which achieves $\mathrm{SN}_0 \approx 16$ dB.

The track revisit interval time t_r that achieves a specified track sharpness ν_0 is calculated according to [8]

$$t_r = 0.4 \left(\frac{R\sigma\sqrt{\Theta}}{\Sigma} \right)^{0.4} \frac{U^{2.4}}{1 + \frac{1}{2}U^2} \tag{20}$$

where R is the target range and Θ and $\sum$ are the Singer model parameters. This is an approximation to the revisit interval time when the tracking filter is in steady state, given the specified target parameters. The variance reduction ratio U is the ratio of the track estimation error to the measurement error:

$$U = \frac{\theta_B \nu_0}{\sigma} \tag{21}$$

The measurement noise standard deviation σ is calculated using (19). Due to a nonunity probability of detection, it may be necessary to have numerous looks for the target on a single track update. VKB describe a search strategy that minimises the number of looks required at each update. They approximate the expected number of looks n_l using this strategy as

$$n_l = \frac{1}{P_d} \left(1 + \left(\gamma \nu_0^2 \right)^2 \right)^{1/2} \tag{22}$$

where, assuming Swerling I target fluctuations, the probability of detection is calculated as

$$P_d = P_f^{\left(\frac{1}{1+\mathrm{SN}_0} \right)} \tag{23}$$

and they state γ in (22) as

$$\gamma \simeq 1 + 14 \left(\left| \ln P_f \right| / \mathrm{SN}_0 \right)^{1/2} \tag{24}$$

The expected SNR conditioned on the occurrence of a detection using this search strategy is approximated as

$$\mathrm{SNR} = \frac{\mathrm{SN}_0 - \ln P_f}{1 + 2\nu_0^2} \tag{25}$$

this accounts for the beam positioning loss associated with increased track estimation error. This SNR is used in (19) to calculate the measurement noise variance.

This strategy presented by VKB is a rule based method for tracking parameter control. As such, it is used as a basis of comparison in the simulations in Section V, where it is referred to as rule based parameter selection (RBPS).

C. Reformulation of Van Keuk and Blackman Strategy

The VKB strategy contains rules for selecting the track operational parameters to achieve specified track accuracy. However for CDAPS, it is necessary to calculate the track accuracy, track loading, and utility that are achieved for a specific operational parameter selection. This subsection describes how the track quality, loading, and utility can be calculated from the equations given in the VKB strategy.

1) *Operational Parameters*: The operational parameters that can be selected using the VKB model are the coherent dwell length τ_c and the revisit interval time

t_r. Therefore, the operational parameter selection for tracking task T_k at time t is $\upsilon_{tk} = \{\tau_c, t_r\}$.

2) *Environmental Parameters*: The environmental parameters for the active tracking model presented in Section IV are the target range R_t and bearing θ_t at time t, as well as the radar cross section ρ, and the Singer manoeuvre model parameters $\sum$ and Θ. These parameters can be estimated from the received measurements using standard tracking techniques [7]. Therefore, the environmental parameters for tracking task T_k at the time t are $e_{tk} = \{R_t, \theta_t, \rho, \Theta, \Sigma\}$.

3) *Resource Function*: The steady state temporal loading that a track exerts on the radar system can be approximated as

$$r = \frac{n_l \tau_c}{t_r} \tag{26}$$

where the expected number of looks n_l is given by the VKB strategy in (22).

4) *Quality Function*: To calculate the angular estimation error achieved by an operational parameter selection, it is necessary to invert (20). Using (19), (21), and (25), (20) can be rewritten as

$$1 = \frac{\alpha \beta \nu_0^{2.4}}{1 + \left(\frac{\beta}{2} + 2\right) \nu_0^2} \tag{27}$$

where

$$\alpha = \frac{0.4}{t_r} \left(\frac{R_t \theta_B \sqrt{\Theta}}{\Sigma} \right)^{0.4} \tag{28}$$

and

$$\beta = SN_0 - \ln P_f \tag{29}$$

The track sharpness υ_0 can then be calculated by finding the root of the function:

$$1 + \left(\frac{\beta}{2} + 2\right) \nu_0^2 - \alpha \beta \nu_0^{2.4} = 0 \tag{30}$$

this is done numerically using the Newton-Raphson method [34].

The half beamwidth θ_B used in (21) and (28) must account for the beam broadening which occurs when the beam is scanned off the antenna boresight. The broadened half beamwidth is $\theta_B = \frac{\theta_{B0}}{\cos \theta_t}$ where θ_t is the off-boresight scan angle and θ_{B0} is the half beamwidth on the antenna boresight. The SNR without beam positioning loss SN_0 for pulsed radar operation is dependent on the coherent dwell duration τ_c, the target range R_t, and the target radar cross section ρ. The range at which SN_0 is unity can be specified on the antenna boresight with a specific nominal coherent dwell duration τ_c^n and radar cross section ρ^n. Then, SN_0 can be calculated at any range R, for any coherent dwell duration τ_c, target radar cross section ρ, and off-boresight beam steering angle θ using

$$\mathrm{SN}_0 = \left(\frac{R_0}{R_t}\right)^4 \cdot \left(\frac{\rho}{\rho^n}\right) \cdot \left(\frac{\tau_c}{\tau_c^n}\right) \cdot \cos^3 \theta_t \tag{31}$$

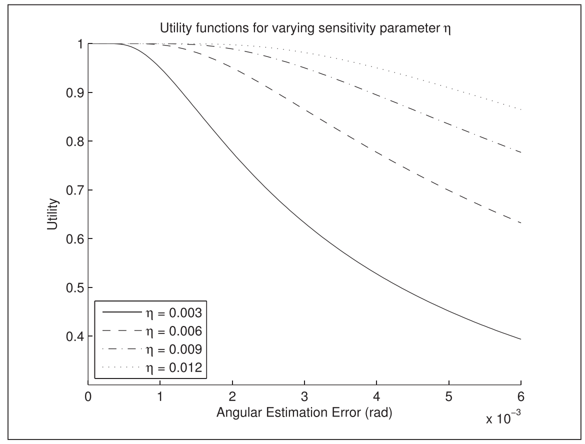


Fig. 3. Exponential utility function of angular estimation error for varying tracking sensitivity parameter η.

where $\cos^3 \theta_t$ is an approximation of the two-way loss of gain resulting from beam steering off the antenna boresight, which in practice depends on the antenna element factor. This is a useful form, as the single R_0 term now incorporates all of the radar parameters.

5) *Utility Function*: The utility function describes the satisfaction that is associated with the achieved track accuracy. A low angular estimation error ϕ, which relates to high track accuracy is desirable. In this work the utility is calculated as

$$u = 1 - \exp\left(\frac{-\eta}{\nu_0 \cdot \theta_B}\right) \tag{32}$$

where η is a sensitivity parameter, which determines the sensitivity to track accuracy. The angular estimation error ϕ is related to the track sharpness according to $\phi = \nu_0 \cdot \theta_B$. Examples of this exponential utility function with different sensitivity parameter η is illustrated in Fig. 3.

This utility function is chosen fairly arbitrarily, and in practice should be chosen to exactly match the requirements of the represented task. In practice this can be challenging, and the construction of accurate utility functions remains an open topic of research.

D. Summary

The track quality, utility, and resource functions presented in the previous subsection can be used by the CDAPS algorithm to optimise tracking operational parameter selection for multiple targets. This is a steady state model, and therefore gives an approximation of the asymptotic tracking accuracy.

The use of the VKB model for these functions imposes limitations. For example, the model assumes well-separated targets embedded in homogeneous Gaussian noise. Inhomogeneities or deviations to these assumptions inevitably degrade resource allocation performance from the best which is theoretically achievable. Additionally, the only quality measure which can be optimised using the VKB model is the angular estimation error. Despite the restrictions of this model, it is

TABLE III
Singer Manoeuvre Parameters for Three Target Types

	Manoeuvre std (m/s^2)	Manoeuvre time (s)
Type 1	20-35	10-20
Type 2	0-5	1-4
Type 3	5-20	30-50

more realistic than assuming an exponential function as in [32].

V. SIMULATIONS

This section presents simulated scenarios that are used to quantify the performance of CDAPS in comparison to Q-RAM and the VKB rule based method. The objective is to demonstrate that the CDAPS algorithm achieves an identical near-optimal solution to Q-RAM, whilst reducing the computation, as the solution from the previous measurement instance can be adapted without entirely recomputing a new allocation frame.

A. Simulation Set-Up

Following the problem formulation in Section II the resource manager must select the set of operational parameters $\upsilon_t = \{\upsilon_{t1}, \upsilon_{t2}, ..., \upsilon_{tK}\}$ at each measurement instance $t \in T_s$ where $\upsilon_{tk} = \{\tau_c, t_r\}$ are the operational parameters for tracking task T_k. The operational parameters selected are time varying, to respond to changes in the environmental parameters. For each tracking task, the coherent dwell duration can be chosen in the range [0.1, 0.2, ..., 10] ms and the revisit interval time can be chosen in the range [0.1, 0.2, ..., 3.5] s.

In the simulation, 200 targets require active tracking. It is assumed that each target is detected by an independent search function that is followed by a confirmation stage to eliminate false targets. Therefore, the number of targets that could be actively tracked is known. The environmental parameters $e_{tk} = \{R_t, \theta_t, \rho, \Theta, \Sigma\}$ for tracking task T_k are the target range R_t, bearing θ_t at time t as well as the radar cross section ρ and Singer manoeuvre model parameters Θ and $\sum$. The target type is chosen randomly from the types listed in Table III. The subsequent Singer manoeuvre parameters are also uniformly distributed random variables in the respective ranges in Table III. The initial Cartesian coordinates of each target are uniformly distributed within the radar field of view, which is between [10, 120] km in range and $[-45, 45]^0$ in bearing. The Cartesian positions of the targets evolve over the simulation time according to a randomly generated Singer trajectory.

The radar parameters result in an instrumental range of $R_0 = 260$ km when a coherent dwell duration of $\tau_c^n = 10ms$ and a $\rho^n = 1$ m^2 radar cross section target on the antenna boresight. The probability of false alarm is $P_f = 10^{-5}$ and the radar 3 dB beamwidth is taken as 1.5°.

In the simulations, the following resource management methods are compared.

1) CDAPS - As described in Section III with the track quality, utility, and resource functions described in Section IV-C. Two tracking sensitivity parameters for the utility function are assessed, $\eta = 0.003$ and $\eta = 0.012$.

2) RBPS 1 - VKB's method described in Section IV-A. The coherent dwell duration is chosen for $SN_0 = 16$ dB and the track sharpness parameter $\nu_0 = 0.3$ half beamwidths. Tracks are dropped randomly when there is insufficient resource available to maintain all tracks.

3) RBPS 2 - Identical to RBPS1, however, the highest loading tracks are dropped first, when insufficient resource is available to maintain all tracks.

4) Q-RAM - Applied with identical quality, utility, and resource functions as CDAPS, which are described in Section IV. It is assumed that Q-RAM is capable of computing one allocation per second.

The set of operational parameters that are selected by each method is passed to an earliest deadline first (EDF) scheduler [35]. The EDF scheduler produces a sequence of track updates for the radar to execute, such that no track updates overlap in time. Additionally, the scheduler has been implemented to synthesise a constraint on the resource available to maintain the tracking tasks. Therefore, the resource available is not necessarily enough to maintain all the targets.

The realised dwell duration and revisit interval time for each task is then extracted from the sequence of track updates generated by the scheduler. The quality and utility of the tracking task, based on these realised values, is calculated according to Section IV-C. Therefore the same model is used for performance assessment as for the resource allocation and no actual tracking is performed. This performance assessment is not exact in practice, as the realised track quality would differ from the allocation model. However, this performance assessment is suitable for this comparison as all methods use the same performance model and therefore no unfair preference is given to any of the proposed methods.

B. Number of Targets Maintained

Fig. 4 plots the number of targets that were chosen to be maintained for each of the four methods, over a range of resource constraints. It can be seen that Q-RAM and CDAPS with $\eta = 0.012$ maintain the greatest number of targets, with RBPS2 maintaining a similar number. RBPS2 maintains a large number of targets as it is designed to select parameter selections with a low resource loading for each track. It can also be seen that RBPS1 maintains significantly less targets when the resource is severely constrained. This is due to RBPS1 randomly dropping targets instead of considering which targets can achieve the best performance with the limited resource. The number of targets maintained using Q-RAM and CDAPS depends on the tracking sensitivity parameter.

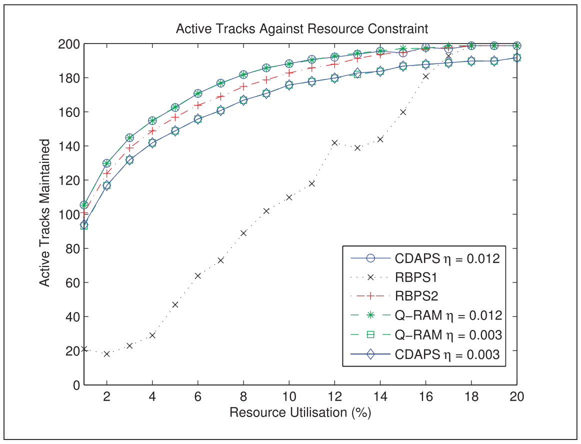


Fig. 4. Number of active tracks maintained against varying resource constraints for RBPS1 and RBPS2, as well as CDAPS and Q-RAM with tracking sensitivity parameter of $\eta = 0.003$ and $\eta = 0.012$. For each resource constraint, number of active targets is averaged over simulation time.

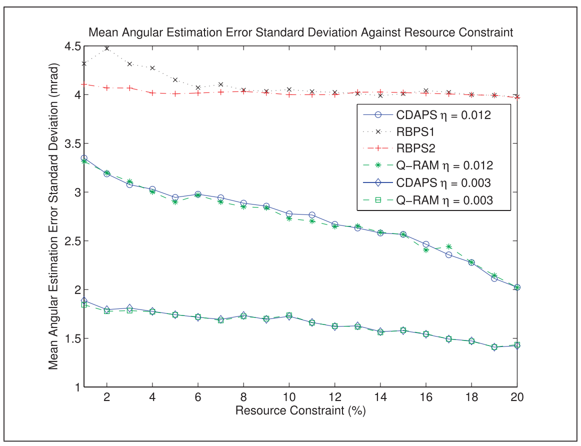


Fig. 5. Average angular estimation error standard deviation against varying resource constraints for RBPS1 and RBPS2 as well as CDAPS and Q-RAM with tracking sensitivity of $\eta = 0.003$ and $\eta = 0.012$.

C. Angular Estimation Error

The average angular estimation error for the four methods is plotted in Fig. 5 against varying resource constraints. It can be seen in Fig. 5 that RBPS1 and RBPS2 maintain an angular accuracy over 4 mrad. Q-RAM and CDAPS achieve angular accuracies that depend on the choice of tracking sensitivity parameter. A sensitivity parameter choice of $\eta = 0.012$ places more importance on the number of tracks than accuracy whereas a sensitivity parameter choice of $\eta = 0.003$ places more importance on track accuracy than the number of tracks. Both Q-RAM and CDAPS significantly outperform RBPS1 and RBPS2 in the angular estimation error.

D. Track Utility

Ultimately the best performance depends on what is actually required, that is, to track more targets or to achieve a better estimation error. However, the utility function describes what is required and can therefore be used as an indicator of the overall performance. The total utility for varying resource constraints is shown in Fig. 6 for the four methods and the two choices of tracking sensitivity parameter for the utility function. In the figure, it can be seen that CDAPS and Q-RAM significantly outperform the rule based approaches.

This result verifies that CDAPS and Q-RAM produce the same near-optimal solution, with equal performance. Both methods have a total utility which significantly improves upon the conventional rule based methods. This performance improvement is attributable to CDAPS and Q-RAM optimising the complete multiple task parameter selection problem, whereas the rule based methods apply rules to each task separately. The trade off between the number of targets and the tracking accuracy can be changed by redefining what is required from the task via the utility function. However, by producing the near-optimum parameter selection set, CDAPS and Q-RAM must always perform equal to or better than the locally optimised rules in terms of the total utility of the resource allocation.

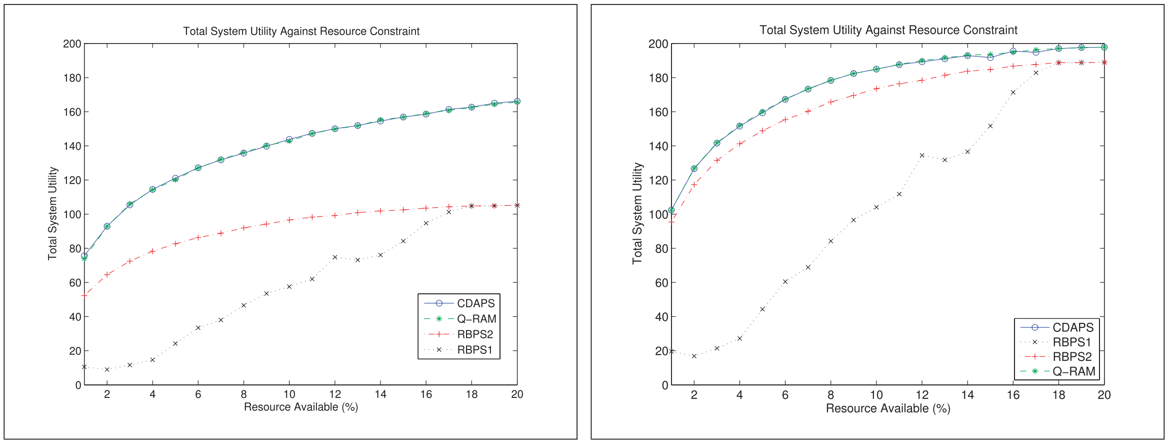
E. Computational Load

CDAPS and Q-RAM produce equivalent solutions, but can be differentiated in terms of computational load. For this simulation, the number of targets increases from 150 to 200 in the first 40 s and decreases from 200 to 150 in the last 40 s. This is a highly contrived scenario, but it is useful for analysing the resulting computation.

The number of parameter selections that are evaluated per second is indicative of the total computation. The number of parameter selections per second with a 15% resource constraint is plotted in Fig. 7. It can be seen that CDAPS evaluates significantly fewer parameter selections per second than Q-RAM. This is due to the ability of CDAPS to adapt the solution from the previous measurement instance to the current measurement instance, without recomputing the entire solution. In contrast, Q-RAM repetitively re-evaluates all of the parameter selections on the concave majorant in fixed allocation frames. As Q-RAM repetitively recomputes the solution in allocation frames, the time for Q-RAM to react to a change in the environmental parameters is limited by the allocation frame time. However, CDAPS is able to react rapidly as it continuously modifies the existing solution when required.

It can also be seen that the number of parameter selections evaluated by Q-RAM has a strong dependence on the number of tasks. CDAPS does not have such a strong dependence, which suggests it is scalable for increases in the number of tasks.

These results for the CDAPS algorithm were produced in real time. This is possible due to the low computation burden of CDAPS combined with the low computational complexity associated with using the VKB strategy equations. The rule based methods cannot be compared in terms of the number of parameter selections evaluated per



(a) Utility function sensitivity $\eta = 0.003$

(b) Utility function sensitivity $\eta = 0.012$

Fig. 6. Total utility against varying resource constraints for CDAPS, RBPS1 and RBPS2, and Q-RAM. For each resource constraint, total utility is averaged over simulation time.

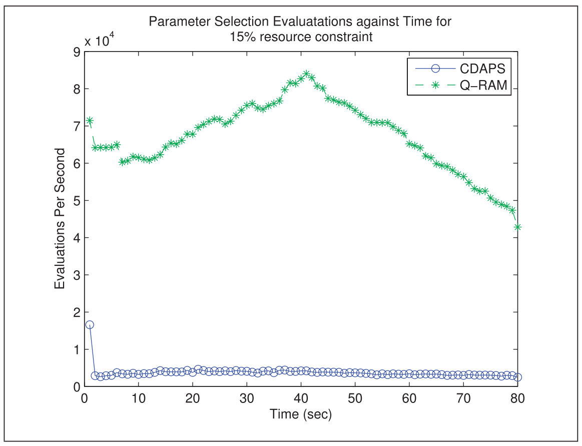


Fig. 7. Number of parameter selections evaluated over simulation time for resource constraint of 15%.

second, but it is assumed their computational complexity is less than CDAPS and Q-RAM.

VI. CONCLUSION

The CDAPS algorithm has been presented as a solution to the radar resource management and task operational parameter selection problem that has been formulated as a QoS constrained optimisation problem. The algorithm generates a near-optimal solution by utilising a continuous double auction mechanism to settle on a competitive market equilibrium that satisfies the KKT conditions. This solution can be achieved using other optimisation methods such as Q-RAM, however, CDAPS has the benefit that the solution from the previous measurement instance can be adapted to the current measurement instance without recomputing the entire allocation. Therefore, CDAPS is rapid to react to changes in environmental parameters and enables a reduction in computation for dynamic problems such as radar resource management. The algorithm has been demonstrated for allocating resource between multiple actively tracked targets using a resource allocation model based on the VKB strategy. It was shown that CDAPS enables a significant improvement in the overall performance of all the radar tasks, which in this work was angular estimation error.

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Alexander Charlish (M'08) obtained his M.Eng. degree from the University of Nottingham in 2006, and received his Ph.D. degree from University College London in 2011.

In 2011 he joined the Sensor Data and Information Fusion Department at the Fraunhofer Institute for Communication, Information Processing and Ergonomics (FKIE), where he now leads the Sensor and Resources Management team. In this role he conducts research on automating the self-optimization of agile/reconfigurable sensors and sensor networks for radar and electronic warfare applications. His general research interests lie in the area of intelligent sensor systems, tracking and data fusion, multiagent systems, and sensor signal processing.

Karl Woodbridge (SM'12) is Professor of Electronic Engineering in the Department of Electronic and Electrical Engineering at University College London. Current RF research activities at UCL include multistatic and netted radar systems, sea clutter, target tracking and classification, and land and air based passive sensing. His research activities have been carried out in investigator and technical consultancy roles for a wide range of customers in the civil and defence sectors.

Dr. Woodbridge has published or presented over 200 journal and conference papers in the areas of his research interests. He is a Fellow of the IET and a Fellow of the UK Institute of Physics.

Hugh Griffiths (M'86—SM'90—F'99) received his M.A. degree in physics from Oxford University in l975. He received a Ph.D. degree in l986 and a D.Sc. (Eng.) degree in 2000, both from University College London.

He holds the THALES/Royal Academy Chair of RF Sensors in the Department of Electronic and Electrical Engineering at University College London, England. From 2006-2008 he was the Principal of the Defence Academy of Management and Technology, Shrivenham. From 2001-2006 he was Head of Department at University College London. His research interests include radar and sonar systems, signal processing (particularly synthetic aperture radar and bistatic and multistatic radar), and antenna measurement techniques. He carried out some of the first experiments in passive bistatic radar.

Dr. Griffiths has published over four hundred papers and technical articles in the fields of radar, antennas, and sonar. In 1996 he received the IEEE AESS Fred Nathanson Award (Radar Systems Panel Award), and in 2013 he received the IET A F Harvey Research Prize for his work on bistatic radar. He has also received the Brabazon Premium of the IERE and the Mountbatten and Maxwell Premium Awards of the IEE. He is a Fellow of the IET, and in 1997, he was elected to Fellowship of the Royal Academy of Engineering. He served as President of the IEEE Aerospace and Electronic Systems Society for 2012-2013. He has been a member of the IEEE AES Radar Systems Panel since 1989, serving as the chair from 2007-2009, and he chaired the working group which revised the IEEE Radar Definitions Standard P686 and reaffirmed the Radar Letter Band Standard in 2008. He is an IEEE AES Distinguished Lecturer.