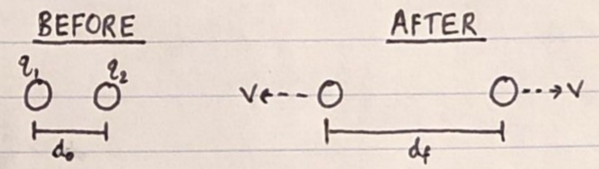
[redacted]

CHAPTER 6 REVIEW PROBLEMS

1. $d_o = 1.2\,m$ $\quad d_f = 2.2\,m$ $\quad m = m_p = 1.67\times10^{-27}\,kg$ $\quad q = q_p = 1.6\times10^{-19}\,C$

$V_f = ?$



Conservation of energy: $\Delta U + \Delta K = 0$

$$U_f - U_o + K_f - K_o = 0$$

$$U_o + \cancel{K_o} = U_f + K_f$$ ← particles initially at rest, so $K_o = 0$

$$U_o = \frac{kq_1q_2}{r} = \frac{kq_p^2}{d_o}$$

$$U_f = \frac{kq_1q_2}{r} = \frac{kq_p^2}{d_f}$$

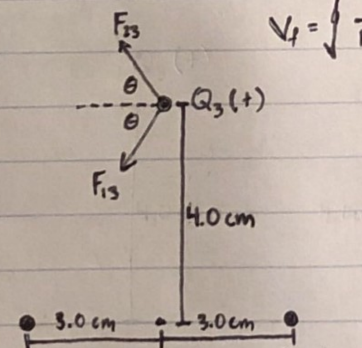
both particles will have the same final speed V_f

$$K_f = \frac{1}{2}m_1V_{1f}^2 + \frac{1}{2}m_2V_{2f}^2 = \frac{1}{2}m_pV_f^2 + \frac{1}{2}m_pV_f^2 = m_pV_f^2$$

$$\Rightarrow \frac{kq_p^2}{d_o} = \frac{kq_p^2}{d_f} + m_pV_f^2$$

$$V_f = \sqrt{\frac{1}{m_p}\left(kq_p^2\left(\frac{1}{d_o} - \frac{1}{d_f}\right)\right)} \quad \overset{\text{plug in \#s}}{\Rightarrow} \quad \boxed{V_f = 0.23\ m/s}$$

2.



$Q_1 = -1.5\,\mu C$ $\quad Q_2 = +1.5\,\mu C$ $\quad Q_3 = +2.5\,\mu C$

$r_{13} = 5.0\,cm$ $\quad r_{12} = 6.0\,cm$ $\quad r_{23} = 5.0\,cm$

a) $F_3 = ?$

$$F_3 = \sqrt{F_{3x}^2 + F_{3y}^2}$$

$$F_{3x} = F_{23x} + F_{13x} = \frac{kQ_2Q_3}{r_{23}^2}\cos\theta + \frac{kQ_1Q_3}{r_{13}^2}\cos\theta$$

$$= kQ_3\cos\theta\left(\frac{|Q_2|}{r_{23}^2} + \frac{|Q_1|}{r_{13}^2}\right)$$

$\cos\theta = \frac{3}{5}$ from the diagram

plug in #s $\Rightarrow$ $F_{3x} = 16.2\,N$

$$F_{3y} = F_{23y} - F_{13y} = \frac{kQ_2Q_3}{r_{23}^2}\sin\theta - \frac{kQ_1Q_3}{r_{13}^2}\sin\theta$$

$$= kQ_3\sin\theta\left(\frac{|Q_2|}{r_{23}^2} - \frac{|Q_1|}{r_{23}^2}\right)$$

plug in #s $\Rightarrow$ $F_{3y} = 0\,N$

$$F_3 = \sqrt{16.2^2 + 0^2} = \boxed{16.2\,N}$$

b) $U = ?$

$U = U_{12} + U_{13} + U_{23}$

$U_{12} = \frac{kQ_1Q_2}{r_{12}} = -0.337\ J$

$U_{13} = \frac{kQ_1Q_3}{r_{13}} = -0.674\ J$

$U_{23} = \frac{kQ_2Q_3}{r_{23}} = +0.674\ J$

$\Rightarrow U = -0.337 - 0.674 + 0.674 = \boxed{-0.337\ J}$

c) $Q_4 = 2.2\ \mu C$

$W_{ext} = ?$

$W_{ext} = \Delta U = U_f - \cancel{U_i}$ ← $U_i = 0$ b/c Q_4 is initially at infinite distance from the system

$W_{ext} = U_f$

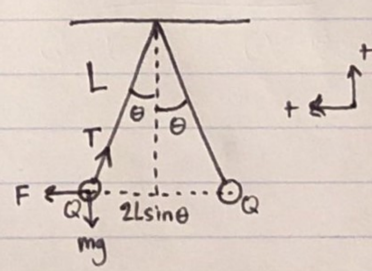
$U_f = U_{14} + U_{24} + U_{34}$

$= \frac{kQ_4Q_1}{r_{14}} + \frac{kQ_4Q_2}{r_{24}} + \frac{kQ_4Q_3}{r_{34}}$

$= kQ_4\left(\frac{Q_1}{r_{14}} + \frac{Q_2}{r_{24}} + \frac{Q_3}{r_{34}}\right)$ where $r_{14} = r_{24} = 3.0\ cm$ and $r_{34} = 4.0\ cm$

plug in #s $\Rightarrow \boxed{1.24\ J}$

3. $m = 2.2\times10^{-4}\ kg$ $Q = 5.25\times10^{-9}\ C$ $\theta = 5°$



$L = ?$

$\Sigma F_x = F - T_x = 0$

$F - T\sin\theta = 0 \Rightarrow F = T\sin\theta$

$\Sigma F_y = T_y - mg = 0$

$T\cos\theta - mg = 0 \Rightarrow mg = T\cos\theta$

] divide these equations to cancel T

$\frac{F}{mg} = \frac{T\sin\theta}{T\cos\theta} = \tan\theta$

$\Rightarrow F = mg\tan\theta$

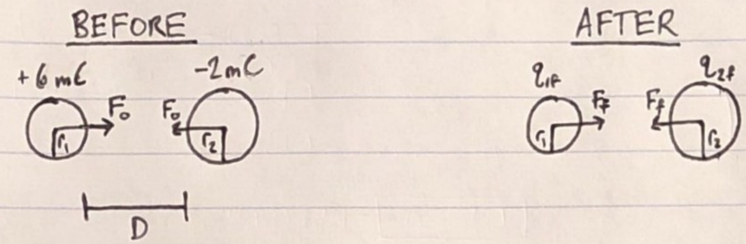
$F = \frac{kQQ}{r^2} = \frac{kQ^2}{(2L\sin\theta)^2} = \frac{kQ^2}{4L^2\sin^2\theta}$ ← Coulomb's Law applies to F

$\Rightarrow mg\tan\theta = \frac{kQ^2}{4L^2\sin^2\theta}$

$L = \sqrt{\frac{kQ^2}{4mg\tan\theta\sin^2\theta}} = \boxed{0.208\ m}$

4. $q_1 = 6\text{ mC}$ $\quad q_2 = -2\text{ mC}$ $\quad F_0 = 20.0\text{ N}$ $\quad r_1 = 3.0\text{ cm}$ $\quad r_2 = 4.0\text{ cm}$

$F_f = ?$



When the spheres come into contact, their combined charges distribute proportionally to their radii.

$$\frac{q_{1f}}{q_{2f}} = \frac{r_1}{r_2}$$

$q_{1f} + q_{2f} = q_1 + q_2 = 4\text{ mC}$ (Conservation of charge)

$q_{1f} = (4\times10^{-3}) - q_{2f}$

$$\frac{4\times10^{-3} - q_{2f}}{q_{2f}} = \frac{r_1}{r_2}$$

plug in #s

$\Rightarrow q_{2f} = 2.29\times10^{-3}\text{ C}$

$q_{1f} = 1.71\times10^{-3}\text{ C}$

$$F_f = \frac{k q_{2f} q_{1f}}{r^2}$$

$r = ?$

$$F_0 = \frac{k q_1 q_2}{r^2}$$

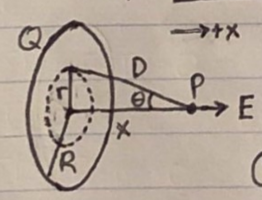
$$r = \sqrt{\frac{k q_1 q_2}{F_0}}$$

$\Rightarrow r = 73.444\text{ m}$

plug in #s

$\Rightarrow F_f = \boxed{6.53\text{ N}}$

5.



G: $R, +Q, \sigma$

E at distance $x = ?$

① Find E of thin ring:

$$E_{eff} = E_x = \int \frac{k\,dq}{r^2}\cos\theta$$

this is the r^2 from the integral, not the diagram

$$r^2 = \left(\sqrt{x^2+r^2}\right)^2 = D^2 \quad \text{(see diagram)}$$

$$\cos\theta = \frac{x}{D}$$

$$E = \int \frac{k\,dq}{D^2}\cdot\frac{x}{D} = \underbrace{\frac{kx}{D^3}}_{\text{constant!}}\int dq = \frac{kxQ}{D^3}$$

$D = \sqrt{x^2+r^2}$, so $D^3 = (x^2+r^2)^{3/2}$

$$\Rightarrow \underbrace{E = \frac{kxQ}{(x^2+r^2)^{3/2}}}$$

this is the electric field due to one thin ring in the disk!

Integrate all the thin rings!

② Find E of entire disk:

$$\int dE = \int_0^R \frac{kx\,dq}{(x^2+r^2)^{3/2}}$$

$$\sigma = \frac{Q}{A} = \frac{dq}{2\pi r\,dr}$$

$dq = \sigma 2\pi r\,dr$

$$E = kx\int_0^R \frac{2\pi\sigma r\,dr}{(x^2+r^2)^{3/2}} = 2kx\pi\sigma\int_0^R \frac{r}{(x^2+r^2)^{3/2}}\,dr$$

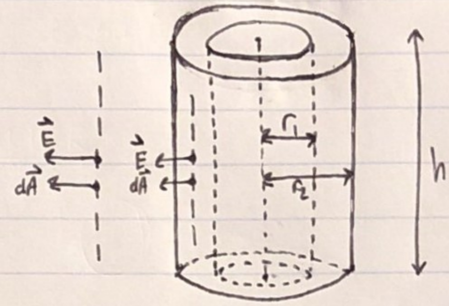
u-sub w/ $u = x^2+r^2$...

$$\Rightarrow E = -2kx\pi\sigma\cdot\frac{1}{\sqrt{x^2+r^2}}\Bigg|_0^R$$

$$= -2kx\pi\sigma\left(\frac{1}{\sqrt{x^2+R^2}} - \frac{1}{x}\right)$$

$$\Rightarrow \boxed{E = 2kx\pi\sigma\left(\frac{1}{x} - \frac{1}{\sqrt{x^2+R^2}}\right), \text{ +x direction}}$$

6. G: ρ, r_1, r_2, h



a) $x < r_1$:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

$\boxed{E = 0}$ b/c $Q_{in} = 0$

b) $r_2 > x > r_1$:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

$\vec{E} \parallel d\vec{A}$, E constant at x

$$EA = \frac{Q_{in}}{\varepsilon_0}$$

$$E \cdot 2\pi x h = \frac{Q_{in}}{\varepsilon_0}$$

$$Q_{in} = \rho \cdot V = \rho(\pi x^2 h - \pi r_1^2 h) = \rho \pi h (x^2 - r_1^2)$$

$$E \cdot 2\cancel{\pi} x \cancel{h} = \frac{\rho \cancel{\pi} \cancel{h} (x^2 - r_1^2)}{\varepsilon_0}$$

$$E \cdot 2x = \frac{\rho(x^2 - r_1^2)}{\varepsilon_0}$$

$$\Rightarrow \boxed{E = \frac{\rho(x^2 - r_1^2)}{2\varepsilon_0 x}}$$

c) $x > r_2$:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$$

$\vec{E} \parallel d\vec{A}$, E constant at x

$$EA = \frac{Q_{in}}{\varepsilon_0}$$

$$E \cdot 2\pi x h = \frac{Q_{in}}{\varepsilon_0}$$

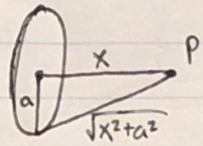
$$Q_{in} = \rho V = \rho(\pi r_2^2 h - \pi r_1^2 h) = \rho \pi h (r_2^2 - r_1^2)$$

$$E \cdot 2\cancel{\pi} x \cancel{h} = \frac{\rho \cancel{\pi} \cancel{h} (r_2^2 - r_1^2)}{\varepsilon_0}$$

$$E \cdot 2x = \frac{\rho(r_2^2 - r_1^2)}{\varepsilon_0}$$

$$\Rightarrow \boxed{E = \frac{\rho(r_2^2 - r_1^2)}{2\varepsilon_0 x}}$$

7. a) G: $r=a$, Q



V at distance $x = ?$

$V = \int \frac{k\,dq}{r}$

$r = \sqrt{x^2+a^2}$ ← constants!

$V = \int \frac{k\,dq}{\sqrt{x^2+a^2}} = \frac{k}{\sqrt{x^2+a^2}} \int dq$

$\Rightarrow \boxed{\frac{kQ}{\sqrt{x^2+a^2}}}$

b) E at distance $x = ?$

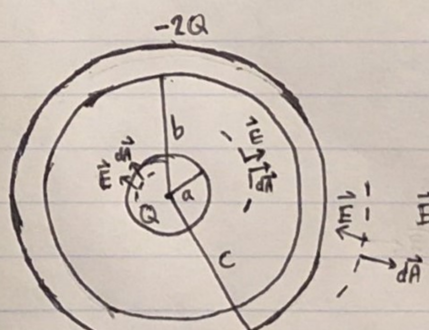
$E = \frac{-d}{dx}(V)$

$= \frac{-d}{dx}\left(\frac{kQ}{\sqrt{x^2+a^2}}\right)$

$= -kQ \cdot \frac{d}{dx}(x^2+a^2)^{-1/2}$

$\Rightarrow \boxed{\frac{kQx}{(x^2+a^2)^{3/2}}}$ Don't forget the Chain Rule!!

8. Let r be the distance from the center.



$\vec{E}$ is inward b/c the Q_{in} is $-2Q+Q=-Q$, which is negative!

a) Region 1: $r<a$:

$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$

$\vec{E} \parallel d\vec{A}$, E constant at r

$EA = \frac{Q_{in}}{\varepsilon_0}$

$E \cdot 4\pi r^2 = \frac{Q_{in}}{\varepsilon_0}$

$\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi a^3} = \frac{Q_{in}}{\frac{4}{3}\pi r^3}$

$\frac{Q}{a^3} = \frac{Q_{in}}{r^3} \Rightarrow Q_{in} = Q\frac{r^3}{a^3}$

$E \cdot 4\pi \cancel{r^2} = \frac{Q r^{\cancel{3}\,1}}{\varepsilon_0 a^3}$

$E \cdot 4\pi = \frac{Qr}{\varepsilon_0 a^3}$

$\Rightarrow \boxed{E = \frac{Qr}{4\pi\varepsilon_0 a^3}}$

b) Region 2: $a<r<b$:

$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$

$\vec{E} \parallel d\vec{A}$, E constant at r

$EA = \frac{Q_{in}}{\varepsilon_0}$

$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$

$\Rightarrow \boxed{E = \frac{Q}{4\pi\varepsilon_0 r^2}}$

d) Region 4: $r>c$:

$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_0}$

$\vec{E} \parallel d\vec{A}$, $\theta = 180°$, E constant at r

$-EA = \frac{Q_{in}}{\varepsilon_0}$

$-E \cdot 4\pi r^2 = \frac{(-2Q+Q)}{\varepsilon_0} = \frac{-Q}{\varepsilon_0}$

$\Rightarrow \boxed{E = \frac{Q}{4\pi\varepsilon_0 r^2}}$ E pointing inwards

c) Region 3: $b<r<c$:

$\boxed{E=0}$ b/c inside conductor!

e) Charge distribution in shell:

$-Q$ on the inside of the shell, $-Q$ on the outside of the shell

↳ The $+Q$ of the insulating sphere attracts $-Q$ to the inside of the shell