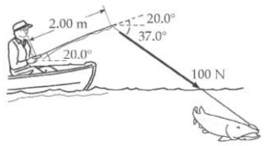
.28 We resolve the 100-N force into components perpendicular to and parallel to the rod, as

$$F_{\text{par}} = (100\text{ N})\cos 57.0° = 54.5\text{ N}$$

and

$$F_{\text{perp}} = (100\text{ N})\sin 57.0° = 83.9\text{ N}$$



ANS. FIG. P10.28

The torque of F_{par} is zero since its line of action passes through the pivot point.

The torque of F_{perp} is

$$\tau = (83.9\text{ N})(2.00\text{ m}) = \boxed{168\text{ N}\cdot\text{m}} \text{ (clockwise)}$$

.29 The flywheel is a solid disk of mass M and radius R with axis through its center.

$$\left.\begin{aligned}\sum\tau &= I\alpha\\ I &= \frac{1}{2}MR^2\end{aligned}\right\} -T_u r + T_b r = \frac{1}{2}MR^2\alpha \to T_b = T_u + \frac{MR^2\alpha}{2r}$$

$$T_b = 135\text{ N} + \frac{(80.0\text{ kg})(0.625\text{ m})^2(-1.67\text{ rad/s}^2)}{2(0.230\text{ m})} = \boxed{21.5\text{ N}}$$

.32 (a) See ANS. FIG. P10.32 below for the force diagrams. For m_1, $\sum F_y = ma_y$ gives

$$+n - m_1 g = 0$$

$$n_1 = m_1 g$$

with $f_{k1} = \mu_k n_1$.

$\sum F_x = ma_x$ gives

$$-f_{k1} + T_1 = m_1 a$$

For the pulley, $\sum\tau = I\alpha$ gives

$$-T_1 R + T_2 R = \frac{1}{2}MR^2\left(\frac{a}{R}\right)$$

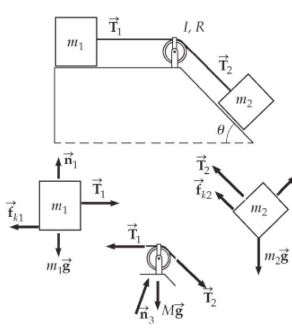
or $\quad -T_1 + T_2 = \frac{1}{2}MR\left(\frac{a}{R}\right) \to -T_1 + T_2 = \frac{1}{2}Ma$

For m_2,

$$+n_2 - m_2 g\cos\theta = 0 \to n_2 = m_2 g\cos\theta$$

$$f_{k2} = \mu_k n_2$$

$$-f_{k2} - T_2 + m_2 g\sin\theta = m_2 a$$



(b) Add equations [1], [2], and [3] and substitute the expressions for f_{k1} and n_1, and $-f_{k2}$ and n_2:

$$-f_{k1} + T_1 + (-T_1 + T_2) - f_{k2} - T_2 + m_2 g\sin\theta = m_1 a + \frac{1}{2}Ma + m_2 a$$

$$-f_{k1} - f_{k2} + m_2 g\sin\theta = \left(m_1 + m_2 + \frac{1}{2}M\right)a$$

$$-\mu_k m_1 g - \mu_k m_2 g\cos\theta + m_2 g\sin\theta = \left(m_1 + m_2 + \frac{1}{2}M\right)a$$

$$a = \frac{m_2(\sin\theta - \mu_k\cos\theta) - \mu_k m_1}{m_1 + m_2 + \frac{1}{2}M}g$$

$$a = \frac{(6.00\text{ kg})(\sin 30.0° - 0.360\cos 30.0°) - 0.360(2.00\text{ kg})}{(2.00\text{ kg}) + (6.00\text{ kg}) + \frac{1}{2}(10.0\text{ kg})}g$$

$$a = \boxed{0.309\text{ m/s}^2}$$

(c) From equation [1]:

$$-f_{k1} + T_1 = m_1 a \to T_1 = 2.00\text{ kg}(0.309\text{ m/s}^2) + 7.06\text{ N} = \boxed{7.67\text{ N}}$$

From equation [2]:

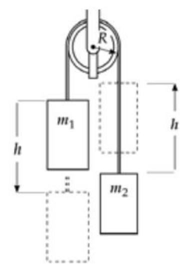
$$-T_1 + T_2 = \frac{1}{2}Ma \to T_2 = 7.67\text{ N} + 5.00\text{ kg}(0.309\text{ m/s}^2)$$

$$= \boxed{9.22\text{ N}}$$

.50 Take the two objects, pulley, and Earth as the system. If we neglect friction in the system, then mechanical energy is conserved and we can state that the increase in kinetic energy of the system equals the decrease in potential energy. Since $K_i = 0$ (the system is initially at rest), we have

$$\Delta K = K_f - K_i$$

$$= \frac{1}{2}m_1 v^2 + \frac{1}{2}m_2 v^2 + \frac{1}{2}I\omega^2$$



ANS. FIG. P10.50

where m_1 and m_2 have a common speed. But $v = R\omega$ so that $\Delta K = \frac{1}{2}\left(m_1 + m_2 + \frac{I}{R^2}\right)v^2$.

From ANS. FIG. P10.50, we see that the system loses potential energy because of the motion of m_1 and gains potential energy because of the motion of m_2. Applying the law of conservation of energy, $\Delta K + \Delta U = 0$, gives

$$\frac{1}{2}\left(m_1 + m_2 + \frac{I}{R^2}\right)v^2 + m_2 gh - m_1 gh = 0$$

$$\boxed{v = \sqrt{\frac{2(m_1 - m_2)gh}{m_1 + m_2 + \frac{I}{R^2}}}}$$

Since $v = R\omega$, the angular speed of the pulley at this instant is given by

$$\boxed{\omega = \frac{v}{R} = \sqrt{\frac{2(m_1 - m_2)gh}{m_1 R^2 + m_2 R^2 + I}}}$$

.51 For the nonisolated system of the top,

$$W = \Delta K \quad\to\quad F\Delta x = \left(\frac{1}{2}I\omega^2 - 0\right)$$

$$\to\quad \omega = \sqrt{\frac{2F\Delta x}{I}} = \sqrt{\frac{2(5.57\text{ N})(0.800\text{ m})}{4\times 10^{-4}\text{ kg}\cdot\text{m}^2}} = \boxed{149\text{ rad/s}}$$

.54 (a) For the isolated rod-ball-Earth system,

$$\Delta K + \Delta U = 0 \quad\to\quad (K_f - 0) + (0 - U_i) = 0 \quad\to\quad K_f = U_i$$

$$K_f = m_{\text{rod}}\,g\,y_{\text{CM, rod}} + m_{\text{ball}}\,g\,y_{\text{CM, ball}}$$

$$= (m_{\text{rod}}\,y_{\text{CM, rod}} + m_{\text{ball}}\,y_{\text{CM, ball}})g$$

$$= [(1.20\text{ kg})(0.120\text{ m}) + (2.00\text{ kg})(0.280\text{ m})](9.80\text{ m/s}^2)$$

$$= \boxed{6.90\text{ J}}$$

(b) We assume the rod is thin. For the compound object

$$I = \frac{1}{3}M_{\text{rod}}L^2 + \left[\frac{2}{5}m_{\text{ball}}R^2 + M_{\text{ball}}D^2\right]$$

$$= \frac{1}{3}(1.20\text{ kg})(0.240\text{ m})^2$$

$$+ \frac{2}{5}(2.00\text{ kg})(4.00\times 10^{-2}\text{ m})^2 + (2.00\text{ kg})(0.280\text{ m})^2$$

$$I = 0.181\text{ kg}\cdot\text{m}^2$$

$$K_f = \frac{1}{2}\omega^2 \quad\to\quad \omega = \sqrt{\frac{2K_f}{I}} = \sqrt{\frac{2(6.90\text{ J})}{0.181\text{ kg}\cdot\text{m}^2}} = \boxed{8.73\text{ rad/s}}$$

(c) $v = r\omega = (0.280\text{ m})(8.73\text{ rad/s}) = \boxed{2.44\text{ m/s}}$

(d) $v_f^2 = v_i^2 + 2a(y_f - y_i)$

$$v_f = \sqrt{0 + 2(9.80\text{ m/s}^2)(0.280\text{ m})} = 2.34\text{ m/s}$$

$\boxed{\text{The speed it attains in swinging is greater by}}$

$$\frac{2.44}{2.34} = \boxed{1.0432\text{ times}}$$

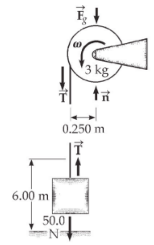
.55 The gravitational force exerted on the reel is

$$mg = (5.10\text{ kg})(9.80\text{ m/s}^2) = 50.0\text{ N down}$$

We use $\sum \tau = I\alpha$ to find T and a.

First find I for the reel, which we know is a uniform disk.

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(3.00\text{ kg})(0.250\text{ m})^2$$
$$= 0.093\ 8\text{ kg}\cdot\text{m}^2$$



ANS. FIG. P10.55

The forces on the reel are shown in ANS. FIG. P10.55, including a normal force exerted by its axle. From the diagram, we can see that the tension is the only force that produces a torque causing the reel to rotate.

$\sum \tau = I\alpha$ becomes

$$n(0) + F_{gp}(0) + T(0.250\text{ m}) = (0.093\ 8\text{ kg}\cdot\text{m}^2)(a/0.250\text{ m}) \quad \textbf{[1]}$$

where we have applied $a_t = r\alpha$ to the point of contact between string and reel. For the object that moves down,

$$\sum F_y = ma_y \quad \text{becomes} \quad 50.0\text{ N} - T = (5.10\text{ kg})a \quad \textbf{[2]}$$

Note that we have defined downwards to be positive, so that positive linear acceleration of the object corresponds to positive angular acceleration of the reel. We now have our two equations in the unknowns T and a for the two connected objects. Substituting T from equation [2] into equation [1], we have

$$[50.0\text{ N} - (5.10\text{ kg})a](0.250\text{ m}) = (0.093\ 8\text{ kg}\cdot\text{m}^2)\left(\frac{a}{0.250\text{ m}}\right)$$

(b) Solving for a from above gives

$$50.0\text{ N} - (5.10\text{ kg})a = (1.50\text{ kg})a$$
$$a = \frac{50.0\text{ N}}{6.60\text{ kg}} = \boxed{7.57\text{ m/s}^2}$$

Because we eliminated T in solving the simultaneous equations, the answer for a, required for part (b), emerged first. No matter—we can now substitute back to get the answer to part (a).

(a) $T = 50.0\text{ N} - 5.10\text{ kg }(7.57\text{ m/s}^2) = \boxed{11.4\text{ N}}$

(c) For the motion of the hanging weight,

$$v_f^2 = v_i^2 + 2a(y_f - y_i) = 0^2 + 2(7.57\text{ m/s}^2)(6.00\text{ m})$$
$$v_f = 9.53\text{ m/s (down)}$$

(d) The isolated-system energy model can take account of multiple objects more easily than Newton's second law. Like your bratty cousins, the equation for conservation of energy grows between visits. Now it reads for the counterweight-reel-Earth system:

$$(K_1 + K_2 + U_g)_i = (K_1 + K_2 + U_g)_f$$

where K_1 is the translational kinetic energy of the falling object and K_2 is the rotational kinetic energy of the reel.

$$0 + 0 + m_1 g y_{1i} = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}I_2\omega_{2f}^2 + 0$$

Now note that $\omega = v/r$ as the string unwinds from the reel.

$$mgy_i = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

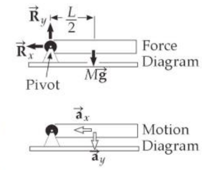
$$2mgy_i = mv^2 + I\left(\frac{v^2}{R^2}\right) = v^2\left(m + \frac{I}{R^2}\right)$$

$$v = \sqrt{\frac{2mgy_i}{m + (I/R^2)}} = \sqrt{\frac{2(5.10\text{ kg})(9.80\text{ m/s}^2)(6.00\text{ m})}{5.10\text{ kg} + \dfrac{0.093\ 8\text{ kg}\cdot\text{m}^2}{(0.250\text{ m})^2}}}$$

$$= \boxed{9.53\text{ m/s}}$$

.73 (a) Since only conservative forces are acting on the bar, we have conservation of energy of the bar-Earth system:

$$K_i + U_i = K_f + U_f$$



ANS. FIG. P10.73

For evaluation of the gravitational energy of the system, a rigid body can be modeled as a particle at its center of mass. Take the zero configuration for potential energy for the bar-Earth system with the bar horizontal.

Under these conditions, $U_f = 0$ and $U_i = MgL/2$.

Using the conservation of energy equation above,

$$0 + \frac{1}{2}MgL = \frac{1}{2}I\omega_f^2 \quad \text{and} \quad \omega_f = \sqrt{MgL/I}$$

For a bar rotating about an axis through one end, $I = ML^2/3$.

Therefore,

$$\omega_f = \sqrt{\frac{MgL}{\frac{1}{3}ML^2}} = \boxed{\sqrt{\frac{3g}{L}}}$$

Note that we have chosen clockwise rotation as positive.

(b) $\sum \tau = I\alpha$: $Mg\left(\frac{L}{2}\right) = \left(\frac{1}{3}ML^2\right)\alpha$ and $\alpha = \boxed{\frac{3g}{2L}}$

(c) $a_x = -a_c = -r\omega_f^2 = -\left(\frac{L}{2}\right)\left(\frac{3g}{L}\right) = -\frac{3g}{2}$

Since this is **centripetal** acceleration, it is directed along the **negative** horizontal.

$$a_y = -a_t = -r\alpha = \frac{L}{2}\alpha = -\frac{3g}{4}$$

$$\boxed{\vec{\mathbf{a}} = -\frac{3}{2}g\hat{\mathbf{i}} - \frac{3}{4}g\hat{\mathbf{j}}}$$

(d) The pivot exerts a force $\vec{\mathbf{F}}$ on the rod. Using Newton's second law, we find

$$F_x = Ma_x = -\frac{3}{2}Mg$$

$$F_y - Mg = Ma_y = -\frac{3}{4}Mg \rightarrow F_y = Mg - \frac{3}{4}Mg = \frac{1}{4}Mg$$

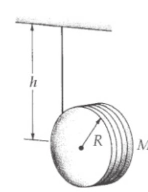
$$\boxed{\vec{\mathbf{F}} = M\vec{\mathbf{a}} = -\frac{3}{2}Mg\hat{\mathbf{i}} + \frac{1}{4}Mg\hat{\mathbf{j}}}$$

.78 Choosing positive linear quantities to be downwards and positive angular quantities to be clockwise, $\sum F_y = ma_y$ yields

$$\sum F = Mg - TM = a \quad \text{or} \quad a = \frac{Mg - T}{M}$$

$\sum \tau = I\alpha$ then becomes

$$\sum \tau = TR = I\alpha = \frac{1}{2}MR^2\left(\frac{a}{R}\right) \quad \text{so} \quad a = \frac{2T}{M}$$



ANS. FIG. P10.78

(a) Setting these two expressions equal,

$$\frac{Mg-T}{M}=\frac{2T}{M} \quad \text{and} \quad T=\boxed{Mg/3}$$

(b) Substituting back,

$$a=\frac{2T}{M}=\frac{2Mg}{3M} \quad \text{or} \quad a=\boxed{\frac{2}{3}g}$$

(c) Since $v_i=0$ and $a=\frac{2}{3}g$, $v_f^2=v_i^2+2ah$ gives us $v_f^2=0+2\left(\frac{2}{3}g\right)h$,

or $v_f=\boxed{\sqrt{4gh/3}}$

(d) Now we verify this answer. Requiring conservation of mechanical energy for the disk-Earth system, we have

$$U_i+K_{\text{rot},\,i}+K_{\text{trans},\,i}=U_f+K_{\text{rot},\,f}+K_{\text{trans},\,f}$$

$$mgh+0+0=0+\frac{1}{2}I\omega^2+\frac{1}{2}mv^2$$

$$mgh=\frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2+\frac{1}{2}Mv^2$$

When there is no slipping, $\omega=\frac{v}{R}$ and $v=\sqrt{\frac{4gh}{3}}$.

$\boxed{\text{The answer is the same.}}$