WS#5 Solutions

7.45 In the following integrals, remember that

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{i}}=\hat{\mathbf{j}}\cdot\hat{\mathbf{j}}=1 \quad\text{and}\quad \hat{\mathbf{i}}\cdot\hat{\mathbf{j}}=0$$

(a) The work done on the particle in its first section of motion is

$$W_{OA}=\int_0^{5.00\text{ m}} dx\hat{\mathbf{i}}\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right)=\int_0^{5.00\text{ m}} 2y\,dx$$

and since along this path, $y=0$, that means $W_{OA}=0$.

In the next part of its path,

$$W_{AC}=\int_0^{5.00\text{ m}} dy\hat{\mathbf{j}}\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right)=\int_0^{5.00\text{ m}} x^2\,dy$$

For $x=5.00$ m, $W_{AC}=125$ J

and $W_{OAC}=0+125=\boxed{125\text{ J}}$.

(b) Following the same steps,

$$W_{OB}=\int_0^{5.00\text{ m}} dy\hat{\mathbf{j}}\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right)=\int_0^{5.00\text{ m}} x^2\,dy$$

Since along this path, $x=0$, that means $W_{OB}=0$.

$$W_{BC}=\int_0^{5.00\text{ m}} dx\hat{\mathbf{i}}\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right)=\int_0^{5.00\text{ m}} 2y\,dx$$

Since $y=5.00$ m, $W_{BC}=50.0$ J.

$$W_{OAC}=0+125=\boxed{125\text{ J}}$$

(c) $W_{OC}=\int\left(dx\hat{\mathbf{i}}+dy\hat{\mathbf{j}}\right)\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right)=\int\left(2y\,dx+x^2dy\right)$

Since $x=y$ along OC, $W_{OC}=\int_0^{5.00\text{ m}}\left(2x+x^2\right)dx=\boxed{66.7\text{ J}}$

(d) $\boxed{F\text{ is nonconservative.}}$

(e) $\boxed{\text{The work done on the particle depends on the path followed by the particle.}}$

7.47 We use the relation of force to potential energy as the force is the negative derivative of the potential energy with respect to distance:

$$U(r)=\frac{A}{r}$$

$$F_r=-\frac{\partial U}{\partial r}=-\frac{d}{dr}\left(\frac{A}{r}\right)=\boxed{\frac{A}{r^2}}$$

If A is positive, the positive value of radial force indicates a force of repulsion.

.51 (a) For a particle moving along the x axis, the definition of work by a variable force is

$$W_F=\int_{x_i}^{x_f}F_x\,dx$$

Here $F_x=(2x+4)$ N, $x_i=1.00$ m, and $x_f=5.00$ m.

So

$$W_F=\int_{1.00\text{ m}}^{5.00\text{ m}}(2x+4)dx\text{ N}\cdot\text{m}=x^2+4x\Big]_{1.00\text{ m}}^{5.00\text{ m}}\text{ N}\cdot\text{m}$$
$$=\left(5^2+20-1-4\right)\text{ J}=\boxed{40.0\text{ J}}$$

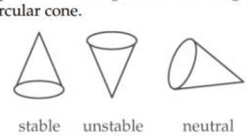
(b) The change in potential energy of the system is the negative of the internal work done by the conservative force on the particle:

$\Delta U=-W_{\text{int}}=\boxed{-40.0\text{ J}}$

(c) From $\Delta K=K_f-\dfrac{mv_1^2}{2}$, we obtain

$$K_f=\Delta K+\frac{mv_1^2}{2}=40.0\text{ J}+\frac{(5.00\text{ kg})(3.00\text{ m/s})^2}{2}=\boxed{62.5\text{ J}}$$

P7.53 The figure below shows the three equilibrium configurations for a right circular cone.



8.21 Use Equation 8.16: $\Delta E_{\text{mech}}=\Delta K+\Delta U=-f_kd$

$$\left(K_f-K_i\right)+\left(U_f-U_i\right)=-f_kd$$
$$K_i+U_i-f_kd=K_f+U_f$$

(a) $K_i+U_i-f_kd=K_f+U_f$

$$0+\frac{1}{2}kx^2-f\Delta x=\frac{1}{2}mv^2+0$$

$$\frac{1}{2}(8.00\text{ N/m})\left(5.00\times10^{-2}\text{ m}\right)^2-\left(3.20\times10^{-2}\text{ N}\right)(0.150\text{ m})=\frac{1}{2}\left(5.30\times10^{-3}\text{ kg}\right)v^2$$

$$v=\sqrt{\frac{2\left(5.20\times10^{-3}\text{ J}\right)}{5.30\times10^{-3}\text{ kg}}}=\boxed{1.40\text{ m/s}}$$

(b) When the spring force just equals the friction force, the ball will stop speeding up. Here $\left|\vec{\mathrm{F}}_s\right|=kx;$ the spring is compressed by

$$\frac{3.20\times10^{-2}\text{ N}}{8.00\text{ N/m}}=0.400\text{ cm}$$

and the ball has moved

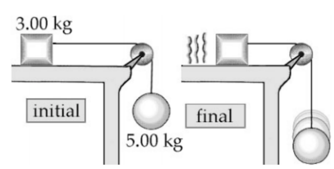
$$5.00\text{ cm}-0.400\text{ cm}=\boxed{4.60\text{ cm from the start}}$$

(c) Between start and maximum speed points,

$$\frac{1}{2}kx_i^2-f\Delta x=\frac{1}{2}mv^2+\frac{1}{2}kx_f^2$$
$$\frac{1}{2}(8.00\text{ N/m})\left(5.00\times10^{-2}\text{ m}\right)^2-\left(3.20\times10^{-2}\text{ N}\right)\left(4.60\times10^{-2}\text{ m}\right)$$
$$=\frac{1}{2}\left(5.30\times10^{-3}\text{ kg}\right)v^2+\frac{1}{2}(8.00\text{ N/m})\left(4.00\times10^{-3}\text{ m}\right)^2$$
$$v=\boxed{1.79\text{ m/s}}$$

.22 For the Earth plus objects 1 (block) and 2 (ball), we write the energy model equation as

$$(K_1+K_2+U_1+U_2)_f-(K_1+K_2+U_1+U_2)_i=\sum W_{\text{other forces}}-f_kd$$



ANS. FIG. P8.22

Choose the initial point before release and the final point after each block has moved 1.50 m. Choose $U=0$ with the 3.00-kg block on the tabletop and the 5.00-kg block in its final position.

So $K_{1i}=K_{2i}=U_{1i}=U_{1f}=U_{2f}=0$

We have chosen to include the Earth in our system, so gravitation is an internal force. Because the only external forces are friction and normal forces exerted by the table and the pulley at right angles to the motion,

$$\sum W_{\text{other forces}}=0$$

We now have

$$\frac{1}{2}m_1v_f^2+\frac{1}{2}m_2v_f^2+0+0-0-0-0-m_2gy_{2i}=0-f_kd$$

where the friction force is

$$f_k=\mu_kn=\mu_km_Ag$$

The friction force causes a negative change in mechanical energy because the force opposes the motion. Since all of the variables are known except for v_f, we can substitute and solve for the final speed.

$$\frac{1}{2}m_1v_f^2+\frac{1}{2}m_2v_f^2-m_2gy_{2i}=-f_kd$$

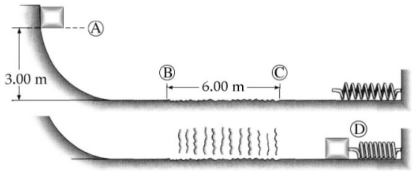
$$v^2=\frac{2gh\left(m_2-\mu_km_1\right)}{m_1+m_2}$$

$$v=\sqrt{\frac{2\left(9.80\text{ m/s}^2\right)(1.50\text{ m})\left[5.00\text{ kg}-0.400(3.00\text{ kg})\right]}{8.00\text{ kg}}}$$

$$=\boxed{3.74\text{ m/s}}$$

WS#5 Solutions

'8.63 The easiest way to solve this problem about a chain-reaction process is by considering the energy changes experienced by the block between the point of release (initial) and the point of full compression of the spring (final). Recall that the change in potential energy (gravitational and elastic) plus the change in kinetic energy must equal the work done on the block by non-conservative forces. We choose the gravitational potential energy to be zero along the flat portion of the track.



There is zero spring potential energy in situation Ⓐ and zero gravitational potential energy in situation Ⓓ. Putting the energy equation into symbols:

$$K_D - K_A - U_{gA} + U_{sD} = -f_k d_{BC}$$

Expanding into specific variables:

$$0 - 0 - mgy_A + \frac{1}{2}kx_s^2 = -f_k d_{BC}$$

The friction force is $f_k = \mu_k mg$, so

$$mgy_A - \frac{1}{2}kx^2 = \mu_k mgd$$

Solving for the unknown variable μ_k gives

$$\mu_k = \frac{y_A}{d} - \frac{kx^2}{2mgd}$$

$$= \frac{3.00 \text{ m}}{6.00 \text{ m}} - \frac{(2\,250 \text{ N/m})(0.300 \text{ m})^2}{2(10.0 \text{ kg})(9.80 \text{ m/s}^2)(6.00 \text{ m})} = \boxed{0.328}$$

8.65 (a) For the isolated spring-block system,

$$\Delta K + \Delta U = 0$$

$$\left(\frac{1}{2}mv^2 - 0\right) + \left(0 - \frac{1}{2}kx^2\right) = 0$$

$$x = \sqrt{\frac{m}{k}}v = \sqrt{\frac{0.500 \text{ kg}}{450 \text{ N/m}}}(12.0 \text{ m/s})$$

$$x = \boxed{0.400 \text{ m}}$$

(b) $\Delta K + \Delta U + \Delta E_{\text{int}} = 0$

$$\left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) + (2mgR - 0) + f_k(\pi R) = 0$$

$$v_f = \sqrt{v_i^2 - 4gR - \frac{2\pi f_k R}{m}}$$

$$= \sqrt{(12.0 \text{ m/s})^2 - 4(9.80 \text{ m/s}^2)(1.00 \text{ m}) - \frac{2\pi(7.00 \text{ N})(1.00 \text{ m})}{0.500 \text{ kg}}}$$

$$v_f = \boxed{4.10 \text{ m/s}}$$

(c) Does the block fall off at or before the top of the track? The block falls if $a_c < g$.

$$a_c = \frac{v_T^2}{R} = \frac{(4.10 \text{ m/s})^2}{1.00 \text{ m}} = 16.8 \text{ m/s}^2$$

Therefore $a_c > g$ and the $\boxed{\text{block stays on the track}}$.