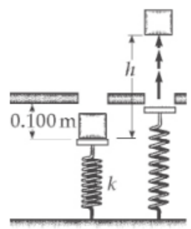
P8.3 From conservation of energy for the block-spring-Earth system,

$$U_{gf} = U_{si}$$

or

$$(0.250 \text{ kg})(9.80 \text{ m/s}^2)h = \left(\frac{1}{2}\right)(5\,000 \text{ N/m})(0.100 \text{ m})^2$$



ANS. FIG. P8.3

This gives a maximum height, $h = \boxed{10.2 \text{ m}}$.

P8.4 (a) $\Delta K + \Delta U = 0 \rightarrow \Delta K = -\Delta U$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -(mgy_f - mgy_i)$$

$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv_i^2 + mgy_f$$

We use the Pythagorean theorem to express the original kinetic energy in terms of the velocity components (kinetic energy itself does not have components):

$$\left(\frac{1}{2}mv_{xi}^2 + \frac{1}{2}mv_{yi}^2\right) = \left(\frac{1}{2}mv_{xf}^2 + 0\right) + mgy_f$$

$$\frac{1}{2}mv_{xi}^2 + \frac{1}{2}mv_{yi}^2 = \frac{1}{2}mv_{xf}^2 + mgy_f$$

Because $v_{xi} = v_{xf}$, we have

$$\frac{1}{2}mv_{yi}^2 = mgy_f \rightarrow y_f = \frac{v_{yi}^2}{2g}$$

so for the first ball:

$$y_f = \frac{v_{yi}^2}{2g} = \frac{[(1\,000 \text{ m/s})\sin 37.0°]^2}{2(9.80 \text{ m/s}^2)} = \boxed{1.85 \times 10^4 \text{ m}}$$

and for the second,

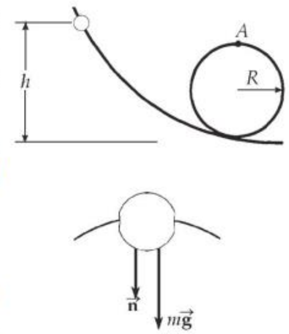
$$y_f = \frac{(1\,000 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{5.10 \times 10^4 \text{ m}}$$

(b) The total energy of each ball-Earth system is constant with value

$$E_{\text{mech}} = K_i + U_i = K_i + 0$$

$$E_{\text{mech}} = \frac{1}{2}(20.0 \text{ kg})(1\,000 \text{ m/s})^2 = \boxed{1.00 \times 10^7 \text{ J}}$$

P8.5 The speed at the top can be found from the conservation of energy for the bead-track-Earth system, and the normal force can be found from Newton's second law.



ANS. FIG. P8.5

(a) We define the bottom of the loop as the zero level for the gravitational potential energy.

Since $v_i = 0$,

$$E_i = K_i + U_i = 0 + mgh = mg(3.50R)$$

The total energy of the bead at point Ⓐ can be written as

$$E_A = K_A + U_A = \frac{1}{2}mv_A^2 + mg(2R)$$

Since mechanical energy is conserved, $E_i = E_A$, we get

$$mg(3.50R) = \frac{1}{2}mv_A^2 + mg(2R)$$

simplifying,

$$v_A^2 = 3.00\,gR$$

$$\boxed{v_A = \sqrt{3.00gR}}$$

(b) To find the normal force at the top, we construct a force diagram as shown, where we assume that n is downward, like mg. Newton's second law gives $\sum F = ma_c$, where a_c is the centripetal acceleration.

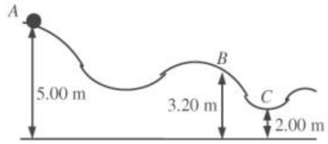
$$\sum F_y = ma_y: \quad n + mg = \frac{mv^2}{r}$$

$$n = m\left[\frac{v^2}{R} - g\right] = m\left[\frac{3.00gR}{R} - g\right] = 2.00mg$$

$$n = 2.00(5.00 \times 10^{-3} \text{ kg})(9.80 \text{ m/s}^2)$$

$$= \boxed{0.098\,0 \text{ N downward}}$$

P8.6 (a) Define the system as the block and the Earth.



ANS. FIG. P8.6

$$\Delta K + \Delta U = 0$$

$$\left(\frac{1}{2}mv_B^2 - 0\right) + \left(mgh_B - mgh_A\right) = 0$$

$$\frac{1}{2}mv_B^2 = mg\left(h_A - h_B\right)$$

$$v_B = \sqrt{2g\left(h_A - h_B\right)}$$

$$v_B = \sqrt{2\left(9.80\text{ m/s}^2\right)\left(5.00\text{ m} - 3.20\text{ m}\right)} = \boxed{5.94\text{ m/s}}$$

Similarly,

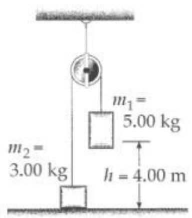
$$v_C = \sqrt{2g\left(h_A - h_C\right)}$$

$$v_C = \sqrt{2g\left(5.00 - 2.00\right)} = \boxed{7.67\text{ m/s}}$$

(b) Treating the block as the system,

$$W_g\Big|_{A\to C} = \Delta K = \frac{1}{2}mv_C^2 - 0 = \frac{1}{2}(5.00\text{ kg})(7.67\text{ m/s})^2 = \boxed{147\text{ J}}$$

P8.7 We assign height $y = 0$ to the table top. Using conservation of energy for the system of the Earth and the two objects:



ANS. FIG. P8.7

(a) Choose the initial point before release and the final point, which we code with the subscript *fa*, just before the larger object hits the floor. No external forces do work on the system and no friction acts within the system. Then total mechanical energy of the system remains constant and the energy version of the isolated system model gives

$$(K_A + K_B + U_g)_i = (K_A + K_B + U_g)_{fa}$$

At the initial point, K_{Ai} and K_{Bi} are zero and we define the gravitational potential energy of the system as zero. Thus the total initial energy is zero, and we have

$$0 = \frac{1}{2}(m_1 + m_2)v_{fa}^2 + m_2gh + m_1g(-h)$$

Here we have used the fact that because the cord does not stretch, the two blocks have the same speed. The heavier mass moves down, losing gravitational potential energy, as the lighter mass moves up, gaining gravitational potential energy. Simplifying,

$$(m_1 - m_2)gh = \frac{1}{2}(m_1 + m_2)v_{fa}^2$$

$$v_{fa} = \sqrt{\frac{2(m_1 - m_2)gh}{(m_1 + m_2)}} = \sqrt{\frac{2(5.00\text{ kg} - 3.00\text{ kg})g(4.00\text{ m})}{(5.00\text{ kg} + 3.00\text{ kg})}}$$

$$= \sqrt{19.6}\text{ m/s} = \boxed{4.43\text{ m/s}}$$

(b) Now we apply conservation of energy for the system of the 3.00-kg object and the Earth during the time interval between the instant when the string goes slack and the instant at which the 3.00-kg object reaches its highest position in its free fall.

$$\Delta K + \Delta U = 0 \quad \to \quad \Delta K = -\Delta U$$

$$0 - \frac{1}{2}m_2v^2 = -m_2g\Delta y \to \Delta y = \frac{v^2}{2g}$$

$$\Delta y = 1.00\text{ m}$$

$$y_{\max} = 4.00\text{ m} + \Delta y = \boxed{5.00\text{ m}}$$

P8.11 When block B moves up by 1 cm, block A moves down by 2 cm and the separation becomes 3 cm. We then choose the final point to be when B has moved up by $\frac{h}{3}$ and has speed $\frac{v_A}{2}$. Then A has moved down $\frac{2h}{3}$ and has speed v_A:

$$\Delta K + \Delta U = 0$$

$$\left(K_A + K_B + U_g\right)_f - \left(K_A + K_B + U_g\right)_i = 0$$

$$\left(K_A + K_B + U_g\right)_i = \left(K_A + K_B + U_g\right)_f$$

$$0 + 0 + 0 = \frac{1}{2}mv_A^2 + \frac{1}{2}m\left(\frac{v_A}{2}\right)^2 + \frac{mgh}{3} - \frac{mg2h}{3}$$

$$\frac{mgh}{3} = \frac{5}{8}mv_A^2$$

$$\boxed{v_A = \sqrt{\frac{8gh}{15}}}$$

P8.45 Taking $y = 0$ at ground level, and using conservation of energy from when the boy starts from rest ($v_i = 0$) at the top of the slide ($y_i = H$) to the instant he leaves the lower end ($y_f = h$) of the frictionless slide at speed v, where his velocity is horizontal ($v_{xf} = v$, $v_{yf} = 0$), we have

$$E_0 = E_{top} \rightarrow \frac{1}{2}mv^2 + mgh = 0 + mgH$$

or $$v^2 = 2g(H-h) \qquad \textbf{[1]}$$

Considering his flight as a projectile after leaving the end of the slide,

$$\Delta y = v_{yi}t + \frac{1}{2}a_y t^2$$

gives the time to drop distance h to the ground as

$$-h = 0 + \frac{1}{2}(-g)t^2 \quad \text{or} \quad t = \sqrt{\frac{2h}{g}}$$

The horizontal distance traveled (at constant horizontal velocity) during this time is d, so

$$d = vt = v\sqrt{\frac{2h}{g}} \quad \text{and} \quad v = d\sqrt{\frac{g}{2h}} = \sqrt{\frac{gd^2}{2h}}$$

Substituting this expression for v into equation [1] above gives

$$\frac{gd^2}{2h} = 2g(H-h) \quad \text{or} \quad \boxed{H = h + \frac{d^2}{4h}}$$

P8.50 (a) Simplified, the equation is

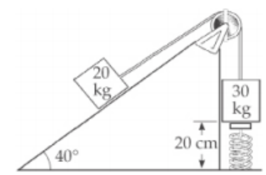
$$0 = (9700\ \text{N/m})x^2 - (450.8\ \text{N})x - 1395\ \text{N}\cdot\text{m}$$

Then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{450.8\ \text{N} \pm \sqrt{(450.8\ \text{N})^2 - 4(9700\ \text{N/m})(-1395\ \text{N}\cdot\text{m})}}{2(9700\ \text{N/m})}$$
$$= \frac{450.8\ \text{N} \pm 7370\ \text{N}}{19\,400\ \text{N/m}} = \boxed{0.403\ \text{m or } -0.357\ \text{m}}$$

(b) From a perch at a height of 2.80 m above the top of a pile of mattresses, a 46.0-kg child jumps upward at 2.40 m/s. The mattresses behave as a linear spring with force constant 19.4 kN/m. Find the maximum amount by which they are compressed when the child lands on them.

P8.64 We choose the zero configuration of potential energy for the 30.0-kg block to be at the unstretched position of the spring, and for the 20.0-kg block to be at its lowest point on the incline, just before the system is released from rest. From conservation of energy, we have



ANS. FIG. P8.64

$$(K+U)_i = (K+U)_f$$

$$0 + (30.0\ \text{kg})(9.80\ \text{m/s}^2)(0.200\ \text{m}) + \frac{1}{2}(250\ \text{N/m})(0.200\ \text{m})^2$$
$$= \frac{1}{2}(20.0\ \text{kg} + 30.0\ \text{kg})v^2$$
$$+ (20.0\ \text{kg})(9.80\ \text{m/s}^2)(0.200\ \text{m})\sin 40.0°$$

$$58.8\ \text{J} + 5.00\ \text{J} = (25.0\ \text{kg})v^2 + 25.2\ \text{J}$$

$$\boxed{v = 1.24\ \text{m/s}}$$