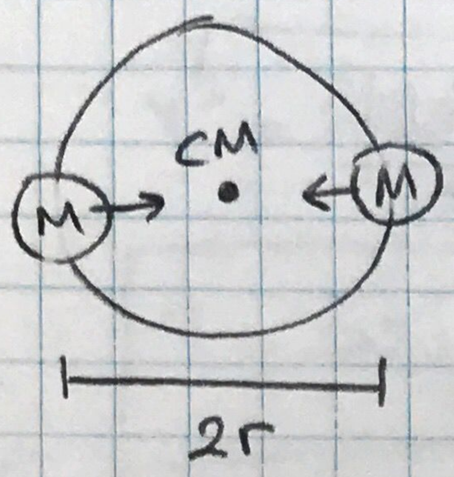
WS #3 CH13 Pr 21, 30, 31, 38, 43, 64

21) $m_1 = m_2 = M = ?$

$V = 220\ km/s \times \frac{1000m}{km}$

$T = 14.4\ day \times 24hr \times 60 \times 60$



$F_g = F_c$

$G\frac{M^2}{4r^2} = \frac{MV^2}{r}$

$V = \frac{2\pi R}{T}$

$R = \frac{VT}{2\pi}$

$M = \frac{4rV^2}{G}$

$= \frac{4V^3T}{2\pi G}$

$= \frac{2(220\times10^3)^3(14.4d)(86400s/d)}{\pi \cdot 6.67\times10^{-11}}$

$\boxed{M = 1.26\times10^{32}\ kg}$

30) $m = 100kg$

$r = r_e + 2\times10^6 m$

$U = ?$

$F_g = ?$

$M_e = 5.98\times10^{24}\ kg$

$r_e = 6.37\times10^6 m$

a) $U = -G\frac{M_e m}{r}$

$= -\frac{(6.67\times10^{-11})(5.98\times10^{24})(100)}{(8.37\times10^6)}$

$\boxed{U = -4.77\times10^9 J}$

b) $F_g = -U\cdot\frac{1}{r} = \frac{(4.77\times10^9)}{8.37\times10^6} = \boxed{569.89\ N}$

31) $M_{moon} = 7.36\times10^{22}\ kg$

$m = 1\times10^3\ kg$

$r_{moon} = 1.74\times10^6 m$

$W = ?$

$W = -\Delta U = U_i - U_f$

$U_i = 0$ @ infinity

$= - - G\frac{M_{moon}\cdot m}{r}$

$= \frac{(6.67\times10^{-11})(7.36\times10^{22})(1\times10^3)}{1.74\times10^6 m}$

$\boxed{W = 2.82\times10^9 J}$

38) $F_g = F_c$

$$G\frac{Mm}{r^2} = \frac{mv^2}{r}$$

$$V_{orbit} = \sqrt{\frac{GM}{r}}$$

$E_o = E_f$ ← @ infinity

$K + U = 0$

$$\frac{1}{2}mv_{escape}^2 - G\frac{Mm}{r} = 0$$

$$V_{escape} = \sqrt{\frac{2GM}{r}}$$

$$\therefore \boxed{V_{escape} = \sqrt{2}\, V_{orbit}}$$

43) $m = 100\,kg$

$r = r_e + 1000\,km$

a) $W_{lift} = ?$

b) $W_{orbit} = ?$

a) $W = \Delta U$ (+ b/c work is done by an outside force)

$= U_f - U_i$

$$= -\frac{GMm}{r_f} - -\frac{GMm}{r_i}$$

$$= GMm\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

$$= (6.67\times10^{-11})(5.98\times10^{24})(100)\left(\frac{1}{6.37\times10^{6}} - \frac{1}{7.37\times10^{6}}\right)$$

$$\boxed{W = 850\times10^{6}\,J}$$

b)

$U_{orbit} = K + U$

↑ Find the K to be in orbit

↑ Already there

$F_g = F_c$

$$\frac{1}{2}\frac{GMm}{r^2} = \frac{mv^2}{r}\cdot\frac{1}{2} \Rightarrow K = \frac{1}{2}mv^2 = \frac{GMm}{2r} = \frac{(6.67\times10^{-11})(5.98\times10^{24})(100)}{2\cdot 7.34\times10^{6}\,m}$$

$$\boxed{K = 2.71\times10^{9}\,J = W}$$

64) $m = 1\times10^{4}\,kg$

$r_i = r_e + 5\times10^{5}\,m = 6.87\times10^{6}\,m$ ↑ perige

$r_f = 2\times10^{7}\,m$ ↑ apogee

$W = ?$

$$a = \frac{6.87\times10^{6} + 2\times10^{7}}{2}$$

$$E = K + U = -\frac{GMm}{2r}$$

$$E_o = -\frac{GMm}{2r}$$

$$E_o = -\frac{(6.67\times10^{-11})(5.98\times10^{24})(10^{4})}{2(6.87\times10^{6}\,m)} = -2.90\times10^{11}\,J$$

$$E_f = -\frac{GMm}{2a} = \frac{(-667\times10^{-11})(5.98\times10^{24})(10^{4})}{2.687\times10^{7}} = -1.48\times10^{11}\,J$$

$W = E_f - E_i = -1.48\times10^{11} + 2.90\times10^{11} = \boxed{1.42\times10^{11}\,J}$