Chapter 2 WS #2 Solutions

CQ2.3 Yes. If a car is travelling eastward and slowing down, its acceleration is opposite to the direction of travel: its acceleration is westward.

CQ2.9 No: Car A might have greater acceleration than B, but they might both have zero acceleration, or otherwise equal accelerations; or the driver of B might have tramped hard on the gas pedal in the recent past to give car B greater acceleration just then.

P2.1 The average velocity is the slope, not necessarily of the graph line itself, but of a secant line cutting across the graph between specified points. The slope of the graph line itself is the instantaneous velocity, found, for example, in Problem 6 part (b). On this graph, we can tell positions to two significant figures:

(a) $x = 0$ at $t = 0$ and $x = 10$ m at $t = 2$ s:

$$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{10\text{ m} - 0}{2\text{ s} - 0} = \boxed{5.0\text{ m/s}}$$

(b) $x = 5.0$ m at $t = 4$ s:

$$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{5.0\text{ m} - 0}{4\text{ s} - 0} = \boxed{1.2\text{ m/s}}$$

(c) $$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{5.0\text{ m} - 10\text{ m}}{4\text{ s} - 2\text{ s}} = \boxed{-2.5\text{ m/s}}$$

(d) $$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{-5.0\text{ m} - 5.0\text{ m}}{7\text{ s} - 4\text{ s}} = \boxed{-3.3\text{ m/s}}$$

(e) $$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{0.0\text{ m} - 0.0\text{ m}}{8\text{ s} - 0\text{ s}} = \boxed{0\text{ m/s}}$$

P2.3 Speed is positive whenever motion occurs, so the average speed must be positive. For the velocity, we take as positive for motion to the right and negative for motion to the left, so its average value can be positive, negative, or zero.

(a) The average speed during any time interval is equal to the total distance of travel divided by the total time:

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}} = \frac{d_{AB} + d_{BA}}{t_{AB} + t_{BA}}$$

But $d_{AB} = d_{BA}$, $t_{AB} = d/v_{AB}$, and $t_{BA} = d/v_{BA}$

so $$\text{average speed} = \frac{d+d}{(d/v_{AB})+(d/v_{BA})} = \frac{2(v_{AB})(v_{BA})}{v_{AB}+v_{BA}}$$

and

$$\text{average speed} = 2\left[\frac{(5.00\ \text{m/s})(3.00\ \text{m/s})}{5.00\ \text{m/s}+3.00\ \text{m/s}}\right] = \boxed{3.75\ \text{m/s}}$$

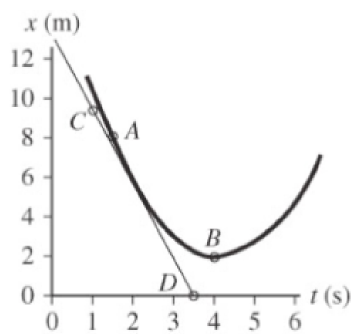
(b) The average velocity during any time interval equals total displacement divided by elapsed time.

$$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t}$$

Since the walker returns to the starting point, $\Delta x = 0$ and $\boxed{v_{x,\text{avg}} = 0}$.

P2.7 For average velocity, we find the slope of a secant line running across the graph between the 1.5-s and 4-s points. Then for instantaneous velocities we think of slopes of tangent lines, which means the slope of the graph itself at a point.

We place two points on the curve: Point A, at $t = 1.5$ s, and Point B, at $t = 4.0$ s, and read the corresponding values of x.



ANS. FIG. P2.7

(a) At $t_i = 1.5$ s, $x_i = 8.0$ m (Point A)

At $t_f = 4.0$ s, $x_f = 2.0$ m (Point B)

$$v_{\text{avg}} = \frac{x_f - x_i}{t_f - t_i} = \frac{(2.0-8.0)\ \text{m}}{(4.0-1.5)\ \text{s}}$$
$$= -\frac{6.0\ \text{m}}{2.5\ \text{s}} = \boxed{-2.4\ \text{m/s}}$$

(b) The slope of the tangent line can be found from points C and D. ($t_C = 1.0$ s, $x_C = 9.5$ m) and ($t_D = 3.5$ s, $x_D = 0$),

$$v \approx \boxed{-3.8\ \text{m/s}}$$

The negative sign shows that the **direction** of v_x is along the negative x direction.

(c) The velocity will be zero when the slope of the tangent line is zero. This occurs for the point on the graph where x has its minimum value. This is at $t \approx \boxed{4.0\ \text{s}}$.

P2.12 The trip has two parts: first the car travels at constant speed v_1 for distance d, then it travels at constant speed v_2 for distance d. The first part takes the time interval $\Delta t_1 = d/v_1$, and the second part takes the time interval $\Delta t_2 = d/v_2$.

(a) By definition, the average velocity for the entire trip is $v_{\text{avg}} = \Delta x / \Delta t$, where $\Delta x = \Delta x_1 + \Delta x_2 = 2d$, and

$\Delta t = \Delta t_1 + \Delta t_2 = d / v_1 + d / v_2$. Putting these together, we have

$$v_{\text{avg}} = \left(\frac{\Delta d}{\Delta t}\right) = \left(\frac{\Delta x_1 + \Delta x_2}{\Delta t_1 + \Delta t_2}\right) = \left(\frac{2d}{d/v_1 + d/v_2}\right) = \left(\frac{2v_1 v_2}{v_1 + v_2}\right)$$

We know $v_{\text{avg}} = 30$ mi/h and $v_1 = 60$ mi/h.

Solving for v_2 gives

$$(v_1 + v_2)v_{avg} = 2v_1 v_2 \rightarrow v_2 = \left(\frac{v_1 v_{avg}}{2v_1 - v_{avg}}\right).$$

$$v_2 = \left[\frac{(30 \text{ mi/h})(60 \text{ mi/h})}{2(60 \text{ mi/h}) - (30 \text{ mi/h})}\right] = \boxed{20 \text{ mi/h}}$$

(b) The average velocity for this trip is $v_{\text{avg}} = \Delta x / \Delta t$, where $\Delta x = \Delta x_1 + \Delta x_2 = d + (-d) = 0$; so, $v_{\text{avg}} = \boxed{0}$.

(c) The average speed for this trip is $v_{\text{avg}} = d / \Delta t$, where $d = d_1 + d_2 = d + d = 2d$ and $\Delta t = \Delta t_1 + \Delta t_2 = d / v_1 + d / v_2$; so, the average speed is the same as in part (a): $v_{\text{avg}} = \boxed{30 \text{ mi/h.}}$

P2.19 (a) The area under a graph of a vs. t is equal to the change in velocity, Δv. We can use Figure P2.19 to find the change in velocity during specific time intervals.

The area under the curve for the time interval 0 to 10 s has the shape of a rectangle. Its area is

$$\Delta v = (2\ \text{m/s}^2)(10\ \text{s}) = 20\ \text{m/s}$$

The particle starts from rest, $v_0 = 0$, so its velocity at the end of the 10-s time interval is

$$v = v_0 + \Delta v = 0 + 20\ \text{m/s} = \boxed{20\ \text{m/s}}$$

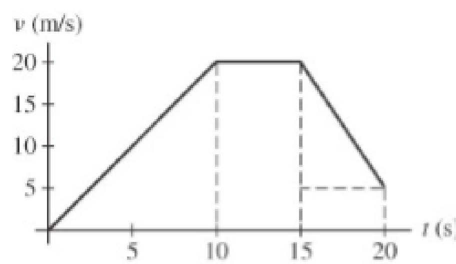
Between $t = 10$ s and $t = 15$ s, the area is zero: $\Delta v = 0$ m/s.

Between $t = 15$ s and $t = 20$ s, the area is a rectangle: $\Delta v = (-3\ \text{m/s}^2)(5\ \text{s}) = -15\ \text{m/s}$.

So, between $t = 0$ s and $t = 20$ s, the total area is $\Delta v = (20\ \text{m/s}) + (0\ \text{m/s}) + (-15\ \text{m/s}) = 5\ \text{m/s}$, and the velocity at $t = 20$ s is $\boxed{5\ \text{m/s.}}$

(b) We can use the information we derived in part (a) to construct a graph of x vs. t; the area under such a graph is equal to the displacement, Δx, of the particle.

From (a), we have these points (t, v) = (0 s, 0 m/s), (10 s, 20 m/s), (15 s, 20 m/s), and (20 s, 5 m/s). The graph appears below.



The displacements are:

0 to 10 s (area of triangle): $\Delta x = (1/2)(20\ \text{m/s})(10\ \text{s}) = 100\ \text{m}$

10 to 15 s (area of rectangle): $\Delta x = (20\ \text{m/s})(5\ \text{s}) = 100\ \text{m}$

15 to 20 s (area of triangle and rectangle):

$$\Delta x = (1/2)[(20 - 5)\ \text{m/s}](5\ \text{s}) + (5\ \text{m/s})(5\ \text{s})$$
$$= 37.5\ \text{m} + 25\ \text{m} = 62.5\ \text{m}$$

Total displacement over the first 20.0 s:

$$\Delta x = 100\ \text{m} + 100\ \text{m} + 62.5\ \text{m} = 262.5\ \text{m} = \boxed{263\ \text{m}}$$

P2.21 To find position we simply evaluate the given expression. To find velocity we differentiate it. To find acceleration we take a second derivative.

With the position given by $x = 2.00 + 3.00t - t^2$, we can use the rules for differentiation to write expressions for the velocity and acceleration as functions of time:

$$v_x = \frac{dx}{dt} = \frac{d}{dt}\left(2 + 3t - t^2\right) = 3 - 2t \text{ and } a_x = \frac{dv}{dt} = \frac{d}{dt}(3 - 2t) = -2$$

Now we can evaluate x, v, and a at $t = 3.00$ s.

(a) $x = (2.00 + 9.00 - 9.00)\text{ m} = \boxed{2.00\text{ m}}$

(b) $v = (3.00 - 6.00)\text{ m/s} = \boxed{-3.00\text{ m/s}}$

(c) $a = \boxed{-2.00\text{ m/s}^2}$

P2.28 (a) We use Equation 2.15:

$x_f - x_i = \frac{1}{2}\left(v_i + v_f\right)t$ becomes $40.0\text{ m} = \frac{1}{2}\left(v_i + 2.80\text{ m/s}\right)(8.50\text{ s})$,

which yields $v_i = \boxed{6.61\text{ m/s}}$.

(b) From Equation 2.13,

$$a = \frac{v_f - v_i}{t} = \frac{2.80\text{ m/s} - 6.61\text{ m/s}}{8.50\text{ s}} = \boxed{-0.448\text{ m/s}^2}$$

P2.35 Since we don't know the initial and final velocities of the car, we will need to use two equations simultaneously to find the speed with which the car strikes the tree. From Equation 2.13, we have

$$v_{xf} = v_{xi} + a_x t = v_{xi} + (-5.60\text{ m/s}^2)(4.20\text{ s})$$

$$v_{xi} = v_{xf} + (5.60\text{ m/s}^2)(4.20\text{ s}) \qquad [1]$$

and from Equation 2.15,

$$x_f - x_i = \frac{1}{2}\left(v_{xi} + v_{xf}\right)t$$

$$62.4\text{ m} = \frac{1}{2}\left(v_{xi} + v_{xf}\right)(4.20\text{ s}) \qquad [2]$$

Substituting for v_{xi} in [2] from [1] gives

$$62.4\text{ m} = \frac{1}{2}\left[v_{xf} + \left(5.60\text{ m/s}^2\right)(4.20\text{ s}) + v_{xf}\right](4.20\text{ s})$$

$$14.9\text{ m/s} = v_{xf} + \frac{1}{2}\left(5.60\text{ m/s}^2\right)(4.20\text{ s})$$

Thus, $v_{xf} = \boxed{3.10\text{ m/s}}$

P2.38 (a) Compare the position equation $x = 2.00 + 3.00t - 4.00t^2$ to the general form

$$x_f = x_i + v_i t + \frac{1}{2}at^2$$

to recognize that $x_i = 2.00$ m, $v_i = 3.00$ m/s, and $a = -8.00\text{ m/s}^2$. The velocity equation, $v_f = v_i + at$, is then

$$v_f = 3.00\text{ m/s} - (8.00\text{ m/s}^2)t$$

The particle changes direction when $v_f = 0$, which occurs at $t = \frac{3}{8}$ s. The position at this time is

$$x = 2.00\text{ m} + (3.00\text{ m/s})\left(\frac{3}{8}\text{ s}\right) - \left(4.00\text{ m/s}^2\right)\left(\frac{3}{8}\text{ s}\right)^2$$

$$= \boxed{2.56\text{ m}}$$

(b) From $x_f = x_i + v_i t + \frac{1}{2}at^2$, observe that when $x_f = x_i$, the time is

given by $t = -\frac{2v_i}{a}$. Thus, when the particle returns to its initial position, the time is

$$t = \frac{-2(3.00\text{ m/s})}{-8.00\text{ m/s}^2} = \frac{3}{4}\text{ s}$$

and the velocity is

$$v_f = 3.00\text{ m/s} - \left(8.00\text{ m/s}^2\right)\left(\frac{3}{4}\text{ s}\right) = \boxed{-3.00\text{ m/s}}$$