.45 The x coordinate of the center of mass is

$$x_{CM} = \frac{\sum m_i x_i}{\sum m_i} = \frac{0+0+0+0}{2.00\text{ kg}+3.00\text{ kg}+2.50\text{ kg}+4.00\text{ kg}} = 0$$

and the y coordinate of the center of mass is

$$y_{CM} = \frac{\sum m_i y_i}{\sum m_i}$$

$$= \left(\frac{1}{2.00\text{ kg}+3.00\text{ kg}+2.50\text{ kg}+4.00\text{ kg}}\right) \times[(2.00\text{ kg})(3.00\text{ m})+(3.00\text{ kg})(2.50\text{ m}) +(2.50\text{ kg})(0)+(4.00\text{ kg})(-0.500\text{ m})]$$

$$y_{CM} = 1.00\text{ m}$$

Then $\boxed{\vec{\mathbf{r}}_{CM} = (0\hat{\mathbf{i}}+1.00\hat{\mathbf{j}})\text{ m}}$

.49 This object can be made by wrapping tape around a light, stiff, uniform rod.

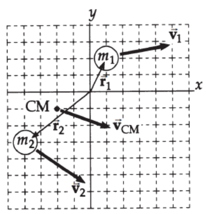
(a) $M = \int_0^{0.300\text{ m}} \lambda\,dx = \int_0^{0.300\text{ m}} [50.0+20.0x]dx$

$$M = \left[50.0x+10.0x^2\right]_0^{0.300\text{ m}} = \boxed{15.9\text{ g}}$$

(b) $x_{CM} = \dfrac{\int_{\text{all mass}} x\,dm}{M} = \dfrac{1}{M}\int_0^{0.300\text{ m}} \lambda x\,dx = \dfrac{1}{M}\int_0^{0.300\text{ m}} \left[50.0x+20.0x^2\right]dx$

$$x_{CM} = \frac{1}{15.9\text{ g}}\left[25.0x^2+\frac{20x^3}{3}\right]_0^{0.300\text{ m}} = \boxed{0.153\text{ m}}$$

52 (a) ANS. FIG. P9.52 shows the position vectors and velocities of the particles.



ANS. FIG. P9.52

(b) Using the definition of the position vector at the center of mass,

$$\vec{\mathbf{r}}_{CM} = \frac{m_1\vec{\mathbf{r}}_1+m_2\vec{\mathbf{r}}_2}{m_1+m_2}$$

$$\vec{\mathbf{r}}_{CM} = \left(\frac{1}{2.00\text{ kg}+3.00\text{ kg}}\right)[(2.00\text{ kg})(1.00\text{ m, }2.00\text{ m}) +(3.00\text{ kg})(-4.00\text{ m, }-3.00\text{ m})]$$

$$\vec{\mathbf{r}}_{CM} = \boxed{(-2.00\hat{\mathbf{i}}-1.00\hat{\mathbf{j}})\text{ m}}$$

(c) The velocity of the center of mass is

$$\vec{\mathbf{v}}_{CM} = \frac{\vec{\mathbf{P}}}{M} = \frac{m_1\vec{\mathbf{v}}_1+m_2\vec{\mathbf{v}}_2}{m_1+m_2}$$

$$= \left(\frac{1}{2.00\text{ kg}+3.00\text{ kg}}\right)[(2.00\text{ kg})(3.00\text{ m/s, }0.50\text{ m/s}) +(3.00\text{ kg})(3.00\text{ m/s, }-2.00\text{ m/s})]$$

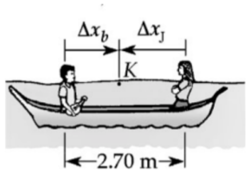
$$\vec{\mathbf{v}}_{CM} = \boxed{(3.00\hat{\mathbf{i}}-1.00\hat{\mathbf{j}})\text{ m/s}}$$

(d) The total linear momentum of the system can be calculated as $\vec{\mathbf{P}} = M\vec{\mathbf{v}}_{CM}$ or as $\vec{\mathbf{P}} = m_1\vec{\mathbf{v}}_1+m_2\vec{\mathbf{v}}_2$. Either gives

$$\vec{\mathbf{P}} = \boxed{(15.0\hat{\mathbf{i}}-5.00\hat{\mathbf{j}})\text{ kg}\cdot\text{m/s}}$$

.53 No outside forces act on the boat-plus-lovers system, so its momentum is conserved at zero and the center of mass of the boat-passengers system stays fixed:

$$x_{CM,i} = x_{CM,f}$$



ANS. FIG. P9.53

Define K to be the point where they kiss, and Δx_J and Δx_b as shown in the figure. Since Romeo moves with the boat (and thus $\Delta x_{\text{Romeo}} = \Delta x_b$), let m_b be the combined mass of Romeo and the boat. The front of the boat and the shore are to the right in this picture, and we take the positive x direction to the right. Then,

$$m_J\Delta x_J + m_b\Delta x_b = 0$$

Choosing the x axis to point toward the shore,

$$(55.0\text{ kg})\Delta x_J + (77.0\text{ kg}+80.0\text{ kg})\Delta x_b = 0$$

and $\Delta x_J = -2.85\Delta x_b$

As Juliet moves away from shore, the boat and Romeo glide toward the shore until the original 2.70-m gap between them is closed. We describe the relative motion with the equation

$$|\Delta x_J| + \Delta x_b = 2.70\text{ m}$$

Here the first term needs absolute value signs because Juliet's change in position is toward the left. An equivalent equation is then

$$-\Delta x_J + \Delta x_b = 2.70\text{ m}$$

Substituting, we find

$$+2.85\Delta x_b + \Delta x_b = 2.70\text{ m}$$

so $\Delta x_b = \dfrac{2.70\text{ m}}{3.85} = \boxed{0.700\text{ m}}$ towards the shore

.60 (a) The fuel burns at a rate given by

$$\frac{dM}{dt} = \frac{12.7\text{ g}}{1.90\text{ s}} = 6.68\times10^{-3}\text{ kg/s}$$

From the rocket thrust equation,

$$\text{Thrust} = v_e\frac{dM}{dt}:\ 5.26\text{ N} = v_e\left(6.68\times10^{-3}\text{ kg/s}\right)$$

$$v_e = \boxed{787\text{ m/s}}$$

(b) $v_f - v_i = v_e\ln\left(\dfrac{M_i}{M_f}\right)$:

$$v_f - 0 = (787\text{ m/s})\ln\left(\frac{53.5\text{ g}+25.5\text{ g}}{53.5\text{ g}+25.5\text{ g}-12.7\text{ g}}\right)$$

$$v_f = \boxed{138\text{ m/s}}$$

.61 The force exerted on the water by the hose is

$$F = \frac{\Delta p_{\text{water}}}{\Delta t} = \frac{mv_f - mv_i}{\Delta t} = \frac{(0.600\text{ kg})(25.0\text{ m/s})-0}{1.00\text{ s}} = \boxed{15.0\text{ N}}$$

According to Newton's third law, the water exerts a force of equal magnitude back on the hose. Thus, the gardener must apply a 15.0-N force (in the direction of the velocity of the exiting water stream) to hold the hose stationary.

Or (dm/dt)v