Probability Theory

In progress

Contents

Chapter 1 Probability Space

Note *Pre-requisite is Real Analysis chapter 2: Measure Theory.*

1.1 Motivation of using measure theory

1.2 Introduction

Definition 1.1 (Probability triple)

A **probability triple** is $(\Omega, \mathcal{F}, \mathbb{P})$ where:

- The **sample space** Ω is any non-empty set.
- The **σ-algebra** $\mathcal{F}$ of Ω.
- The **probability measure** $\mathbb{P} : \mathcal{F} \to [0, 1]$ is a measure defined on $(\Omega, \mathcal{F})$ s.t. $\mathbb{P}(\Omega) = 1$.

Remark $\mathbb{P}(\varnothing) = 0$ is redundant for the definition of probability measure, but necessary for the definition of measure.

Corollary 1.1

- $\mathcal{F}$ is closed under the countable intersections.
- $\mathbb{P}(A^C) = 1 - \mathbb{P}(A)$.
- (**Monotonicity**) $\mathbb{P}(A) \leq \mathbb{P}(B)$ whenever $A \subseteq B$.
- (**Principle of inclusion-exclusion**) $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$.
- (**Subadditivity**) For a countable collection $\{A_i\} \subseteq \mathcal{F}$, $\mathbb{P}(\bigcup_i A_i) \leq \sum\limits_i \mathbb{P}(A_i)$.

Theorem 1.1

Let

- Ω be a finite or countable non-empty set.
- $p : \Omega \to [0, 1]$ be ab function satisfying $\sum\limits_{\omega \in \Omega} p(\omega) = 1$,

then there is a probability triple $(\Omega, \mathcal{F}, \mathbb{P})$ where $\mathcal{F} = \mathbb{P}(\Omega)$, and for $A \in \mathcal{F}$, $\mathbb{P}(A) = \sum\limits_{\omega \in A} p(\omega)$.