CHSH Game

QIA 2025 Hackathon Challenge

The team should implement the CHSH (Clauser-Horne-Shimony-Holt) game using SquidASM. The CHSH game is a two-player cooperative game involving Alice and Bob, who are spatially separated. The players can agree on a strategy for the game ahead of time, but cannot communicate during the game. A referee sends each player a random bit (x to Alice, y to Bob), and each player must respond with an output bit (a from Alice, b from Bob). The players win if their outputs satisfy the winning condition: $a \oplus b = x \wedge y$ (where $\oplus$ is XOR and $\wedge$ is AND).

In classical strategies, the maximum winning probability is 75%. However, using entanglement, players can achieve a winning probability of approximately 85.4% (specifically, $(2+\sqrt{2})/4$), violating Bell's inequality and demonstrating non-locality [1–3].

The Protocol

1. **Preparation phase**: Alice and Bob share a maximally entangled Bell pair $|\Phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$
2. **Input phase**: The referee sends random bits $x \in \{0, 1\}$ to Alice and $y \in \{0, 1\}$ to Bob
3. **Measurement phase**:
   - Alice measures her qubit in basis $\theta_A = x \cdot \pi/4$ (0° for $x = 0$, 45° for $x = 1$)
   - Bob measures his qubit in basis $\theta_B = -y \cdot \pi/4 + \pi/8$ (22.5° for $y = 0$, -22.5° for $y = 1$)
4. **Output phase**: Alice outputs measurement result a, Bob outputs result b
5. **Winning condition**: Players win if $a \oplus b = x \wedge y$

Steps of the challenge:

- Create a Bell pair between Alice and Bob using SquidASM
- Implement the optimal quantum measurement strategies for both players
- Simulate multiple rounds of the CHSH game as described above and calculate the winning probability
- Verify that you achieve the theoretical maximum of ∼85.4%

Classical Strategy Analysis

Implement and test classical strategies to understand their limitations:

- Code up a few different classical strategies for the CHSH game (e.g., deterministic lookup tables based on inputs)
- Verify they can't exceed 75% winning probability
- Implement one strategy using shared randomness. Does it exceed 75% winning probability?
- Compare the performance directly with your quantum implementation

References

[1] John F Clauser, Michael A Horne, Abner Shimony, and Richard A Holt. Proposed experiment to test local hidden-variable theories. *Physical review letters*, 23(15):880, 1969.

[2] John S Bell. On the einstein podolsky rosen paradox. *Physics Physique Fizika*, 1(3):195, 1964.

[3] Nicolas Brunner, Daniel Cavalcanti, Stefano Pironio, Valerio Scarani, and Stephanie Wehner. Bell nonlocality. *Reviews of modern physics*, 86(2):419–478, 2014.