The Australian National University
Research School of Computer Science

Semester 1, 2022
Assignment 2 of 3

Theory of Computation

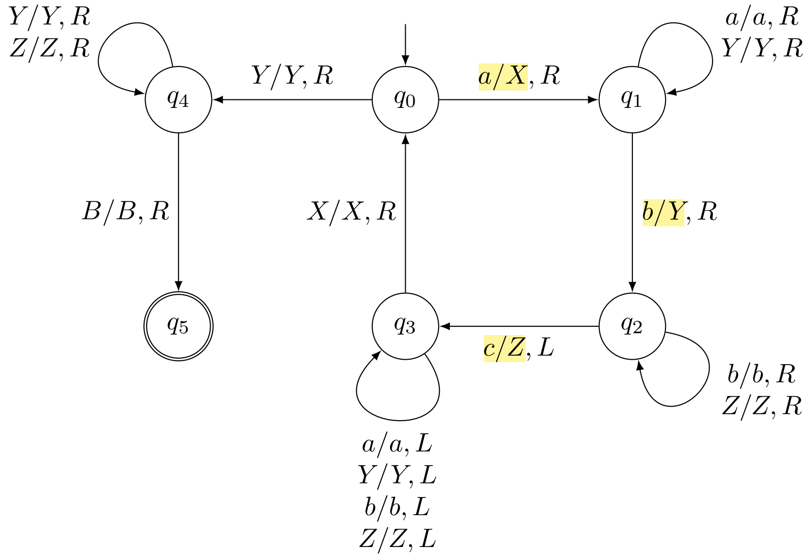
Release Date. Tuesday April 19 2022
Due Date. Tuesday May 3, 2022, 23.50 Canberra time
Maximum credit. 50

Exercise 1 — The language of a Turing machine (4 + 5 + 5 credits)

Consider the Turing machine $\mathcal{M} = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$, where:

- $Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$;
- $\Sigma = \{a, b, c\}$;
- $\Gamma = \{a, b, c, X, Y, Z, B\}$;
- $F = \{q_5\}$; and

and δ is given by the below diagram.



The goal of the exercise is to show that the language accepted by the above Turing machine is precisely

$$\mathsf{L} = \{a^k b^k c^k \mid k \geq 1\}.$$

(a) Show that for all $i \in \mathbb{N}$ and $j \in \mathbb{N}$ with $j \geq 1$

$$X^i q_0 a^j Y^i b^j Z^i c^j \vdash^*_{\mathcal{M}} X^{i+1} q_0 a^{j-1} Y^{i+1} b^{j-1} Z^{i+1} c^{j-1}.$$

(b) Prove that all strings in L are accepted by $\mathcal{M}$.

(c) Prove that every string accepted by $\mathcal{M}$ is in L.

Hint. A possible approach is using the contrapositive: show that every string that is *not* in L, is *not* accepted by $\mathcal{M}$.

for every pair of M and w, need to find M' s.t. M accepts w <=> M' accepts some x of length < 10 in >=100 steps

Exercise 2 **Decidability** (6 + 6 + 6 + 6 credits)

For each of the following problems, state whether it is decidable. If it is decidable, outline a decision procedure for it. If not, prove that it is undecidable.

In each case, the input is the code $\langle M \rangle$ of a Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$.

{<M> : there is x of length >= 10 s.t. M halts on x < 100 steps}

(a) Is there an input $x \in \Sigma^*$ of length < 10 such that M halts in < 100 steps when run on x?

(b) Is there an input $x \in \Sigma^*$ of length ≥ 10 such that M halts in < 100 steps when run on x?

(c) Is there an input $x \in \Sigma^*$ of length < 10 such that M halts in ≥ 100 on input x?

if decidable, then exists T such that T accepts <M'> iff M' accepts some x of length < 10 in >= 100 steps

(d) Is there an input $x \in \Sigma^*$ of length ≥ 10 such that M halts in ≥ 100 steps when run on x?

Exercise 3 **Recursive Enumerability** (6 + 6 credits)

The problems here are the same as in Part (a) and (b) of the previous exercise, but the goal is to show that they are *recursively enumerable*. As in the previous exercise, the input is the code $\langle M \rangle$ of a Turing machine M.

(a) Is there an input $x \in \Sigma^*$ of length < 10 such that M halts in ≥ 100 steps on input x?

(b) Is there an input $x \in \Sigma^*$ of length ≥ 10 such that M halts in ≥ 100 steps when run on x?

Rubric. For Exercise 1 we expect arguments that use the definition of acceptance of a string in a Turing machine, and IDs. For Exercises 2 and 3 we expect a level of detail comparable to that of the lecture slides.