Theory of Computation

Release Date. Monday, May 9, 2022
Due Date. Monday, May 23, 2022, 23.50 Canberra time
Maximum credit. 50

Exercise 1 **Pessimistic Sat** (20 credits)

A literal in a clause is *negative* if it is of the form $\neg x$ for a variable x, and *positive* (that is, a non-negated variable) otherwise. For example, in the clause $\neg x \vee \neg y \vee z$, the literals $\neg x$ and $\neg y$ are negative, and the literal z is positive.

We call a clause *pessimistic* if it contains at most one positive literal. For example, the clauses $\neg x \vee \neg y \vee z$ and $\neg x \vee \neg y$ as well as the (one-element) clauses x and $\neg y$ are pessimistic.

Consider the following variant PESSIMISTIC SAT of the CSAT problem:

Input: A formula F that is a conjunction of pessimistic clauses.

Problem: Is F satisfiable?

Prove that PESSIMISTIC SAT is in P.

Exercise 2 **NP-Hard but not in NP** (15 credits)

Recall that a problem A is *NP-hard* if every problem in NP is polytime reducible to A.

Give an example of a problem A such that A is NP-hard, but A itself is not in NP. Give proof for both claims.

Exercise 3 **Jailbreak!** (15 credits)

Prisoners at a jail are trying to organise an escape plan. The plan will succeed only if there is a large enough group of conspiring prisoners. More precisely, the plan succeeds if there is a group of prisoners of size at least k such that any two prisoners in this group trust one another.

Consider the decision problem JAILBREAK:

Input: An undirected graph whose vertices represent prisoners, and where an edge between two vertices (prisoners) signifies that the two vertices (prisoners) trust one another, and a natural number $k \geq 0$.

Problem: Does there exists a set of prisoners (vertices) of size $\geq k$ such that any two prisoners (vertices) in that group trust one another (are connected by an edge)?

Prove that JAILBREAK is NP-complete.

Rubric. We expect a level of detail comparable to that of the model answers of the tutorial exercises.