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The Paillier Cryptosystem

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RAPPERSWIL

Agenda

- Some mathematical properties
- Encryption and decryption
- Additive homomorphic properties
- Zero knowledge proof for n-th powers
- Paillier e-voting simulator
- Non-interactive ZKP using the Fiat-Shamir heuristic
- Damgård-Jurik Cryptosystem (Generalized Paillier)
- Damgård-Jurik JavaScript e-voting client
- Threshold decryption schemes

The Paillier Cryptosystem I

Proposed by Pascal Paillier in 1999:

- Choose two large prime numbers ***p*** and ***q*** and form the modulus
  $n = pq$
- Euler's totient function gives the number of elements in Z_n^*
  $\varphi(n) = (p-1)(q-1)$
- The number of elements in $Z_{n^2}^*$ is
  $\varphi(n^2) = n\varphi(n)$
- The private key λ is determined using Carmichael's function
  $\lambda(n) = \mathrm{lcm}(p-1, q-1)$
- Due to Carmichael's theorem, for every element $\omega \in Z_{n^2}^*$

$$\begin{cases} \omega^{\lambda} = 1 \mod n \\ \omega^{n\lambda} = 1 \mod n^2 \end{cases}$$

The Paillier Cryptosystem II

- The hard problem: Deciding n-th composite residuosity!
  $z = y^n \bmod n^2$
- The set of n-th residues is a multiplicative subgroup of $Z^*_{n^2}$ of order $\varphi(n)$
- Each n-th residue z has exactly n roots of degree n, among which exactly one is strictly smaller than n, namely
  $r = \sqrt[n]{z} \bmod n\,, \quad r \in Z^*_n$
- The n-th roots of unity are the numbers of the form
  $(1+n)^m = 1 + mn \bmod n^2\,, \quad m \in Z_n$
- Generate the multiplicative subgroup $Z^*_{n^2}$ as $Z_n \times Z^*_n \mapsto Z^*_{n^2}$

$$\boxed{(m, r) \;\; \mapsto \;\; g^m \cdot r^n \bmod n^2 = c}$$

Paillier Encryption

m: plaintext message, r: random number for semantic security

Example: Multiplicative Subgroup $Z^*_{15^2}$

		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	m
		1	16	31	46	61	76	91	106	121	136	151	166	181	196	211	g^m
1	1	1	16	31	46	61	76	91	106	121	136	151	166	181	196	211	
2	143	143	38	158	53	173	68	188	83	203	98	218	113	8	128	23	
4	199	199	34	94	154	214	49	109	169	4	64	124	184	19	79	139	
7	118	118	88	58	28	223	193	163	133	103	73	43	13	208	178	148	
8	107	107	137	167	197	2	32	62	92	122	152	182	212	17	47	77	
11	26	26	191	131	71	11	176	116	56	221	161	101	41	206	146	86	
13	82	82	187	67	172	52	157	37	142	22	127	7	112	217	97	202	
14	224	224	209	194	179	164	149	134	119	104	89	74	59	44	29	14	
r	r^n																

p = 3, q = 5, n = 15, n^2 = 225

φ(n) = 8, λ(n) = lcm(2, 4) = 4

Generator in most general form:

$$\boxed{g = (1+\alpha\cdot n)\beta^n\,, \quad \alpha,\beta \in Z_n^*}$$

Paillier Decryption

$$\boxed{m = \frac{L(c^{\lambda} \bmod n^{2})}{L(g^{\lambda} \bmod n^{2})} \quad \bmod n}$$ with $$L(x) = \frac{x-1}{n}$$

- Apply the private key λ and use Carmichael‘s theorem

  $$c^{\lambda} = (g^{m} \cdot r^{n})^{\lambda} = g^{m\lambda} \cdot r^{n\lambda} = g^{\lambda m}$$

- Make use of the relationship $(1+n)^{x} = 1 + xn \bmod n^{2}$

  $$g^{\lambda m} = ((1+n)^{\alpha} \beta^{n})^{\lambda m} = (1+n)^{\alpha\lambda m} \beta^{n\lambda m} = (1+\alpha\lambda mn) \bmod n^{2}$$

- Apply the $L(x)$ function

  $$\frac{L(1+\alpha\lambda mn)}{L(1+\alpha\lambda n)} = \frac{\alpha\lambda m}{\alpha\lambda} \quad \bmod n = m$$

Additive Homomorphic Properties

$$\boxed{D(E(m_1) \cdot E(m_2) \bmod n^2) = m_1 + m_2 \bmod n}$$

- Verification

$$E(m_1) \cdot E(m_2) = g^{m_1} r_1^n \cdot g^{m_2} r_2^n = g^{m_1+m_2} \cdot r_1^n r_2^n \bmod n^2$$

$$\boxed{D(E(m)^k \bmod n^2) = k \cdot m \bmod n}$$

- Use in e-voting systems with homomorphic tallying:

  The additive homomorphic property directly returning the tally is the biggest advantage of the Paillier Cryptosystem over the El Gamal Cryptosystem which has an intrinsically multiplicative homomorphic property requiring the computation of a discrete logarithm over a bounded range to extract the tally.

Validity Proof of Ballot (Case: k = i)

- K valid voting messages (e.g. vote for one out of K candidates)

  $m_1, m_2, \cdots, m_k, \cdots, m_K$

- Zero knowledge proof : Prove that u_k is an n-th power

  $$u_k = \frac{c}{g^{m_k}} = \frac{g^{m_i} \cdot r^n}{g^{m_k}} \bmod n^2 = r^n \quad \text{only if} \quad m_i = m_k$$

- Commitment: Prover chooses a random number ω

  $a_i = \omega^n \bmod n^2 \; , \quad \omega \in Z_n^*$

- Challenge: Verifier chooses a random bit string e_i of length b

  $e_i < 2^b \; , \quad 2^b < \min(p, q)$

- Response: Prover computes z_i

  $z_i = \omega \cdot r^{e_i} \bmod n$

- Verification: $\boxed{z_i^n = a_i \cdot u_i^{e_i} \bmod n^2}$

  $$z_i^n = (\omega \cdot r^{e_i})^n = \omega^n \cdot r^{n \cdot e_i} \bmod n^2$$

Validity Proof of Ballot (Cases: k ≠ i)

- Preparation: Prover chooses z_k and bit string e_k randomly

  $$z_k \in Z_n^* \ , \quad e_k < 2^b \ , \quad 2^b < \min(p,q)$$

- Commitment: Prover computes a_k so that it passes verification

  $$a_k = \frac{z_k^n}{u_k^{e_k}} \bmod n^2$$

- Challenge: Verifier chooses a random bit string e of length b

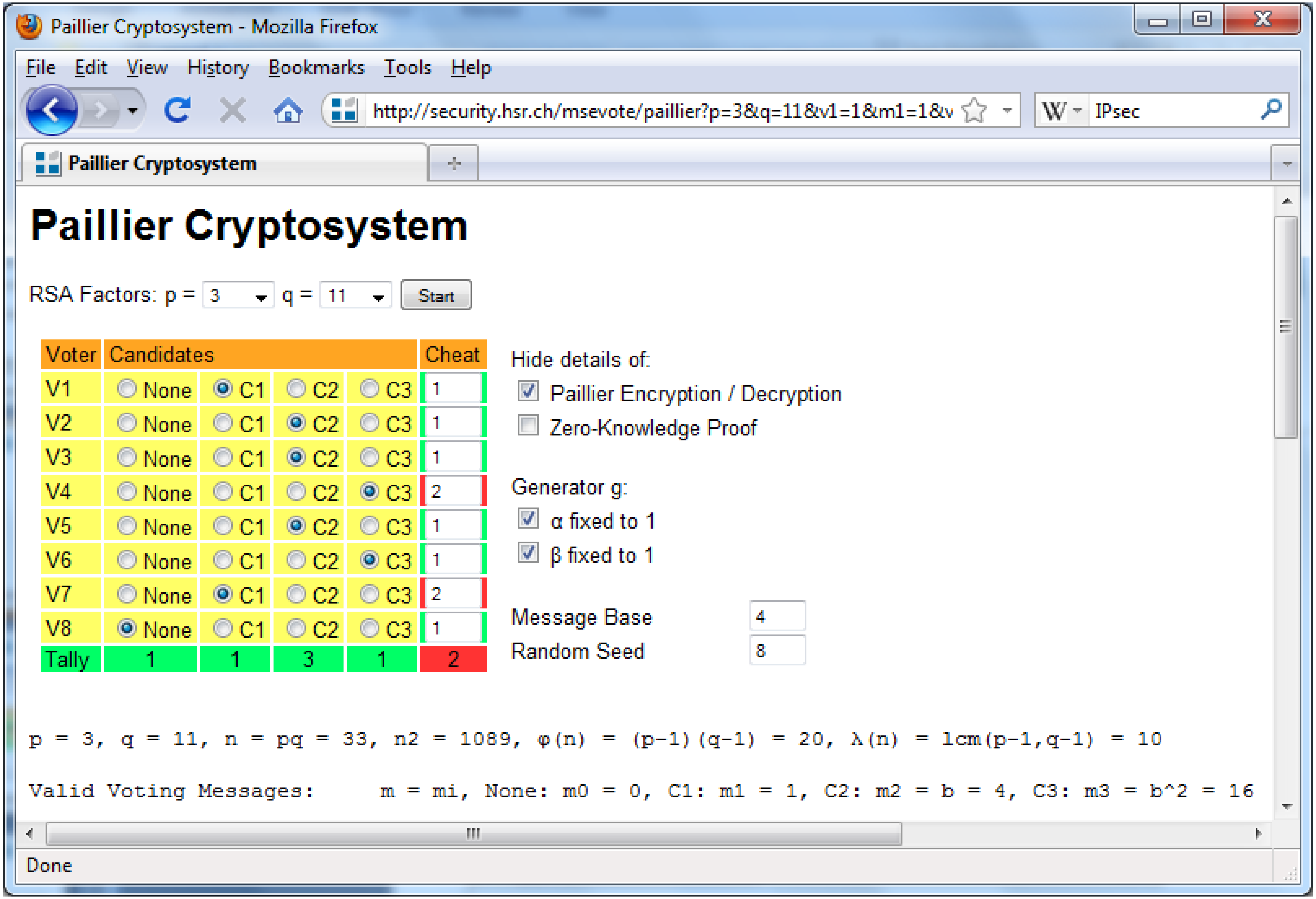
  $$e < 2^b \ , \quad 2^b < \min(p,q)$$

- Response: Prover sends prepared z_k and e_k
- Verification: $\boxed{z_k^n = a_k \cdot u_k^{e_k} \bmod n^2}$

$$\boxed{\sum_{k=1}^{K} e_k = e \quad \bmod 2^b}$$

Prover can preselect all e_k for $k \neq i$ but is bound by e for the choice of e_i.

Paillier E-Voting Simulator


Paillier Cryptosystem - Mozilla Firefox
File Edit View History Bookmarks Tools Help
http://security.hsr.ch/msevote/paillier?p=3&q=11&v1=1&m1=1&v
IPsec
Paillier Cryptosystem
Done

Paillier Cryptosystem

RSA Factors: p = 3 q = 11 Start

Voter	Candidates				Cheat
V1	○ None	● C1	○ C2	○ C3	1
V2	○ None	○ C1	● C2	○ C3	1
V3	○ None	○ C1	● C2	○ C3	1
V4	○ None	○ C1	○ C2	● C3	2
V5	○ None	○ C1	● C2	○ C3	1
V6	○ None	○ C1	○ C2	● C3	1
V7	○ None	● C1	○ C2	○ C3	2
V8	● None	○ C1	○ C2	○ C3	1
Tally	1	1	3	1	2

Hide details of:

☑ Paillier Encryption / Decryption

☐ Zero-Knowledge Proof

Generator g:

☑ α fixed to 1

☑ β fixed to 1

Message Base 4

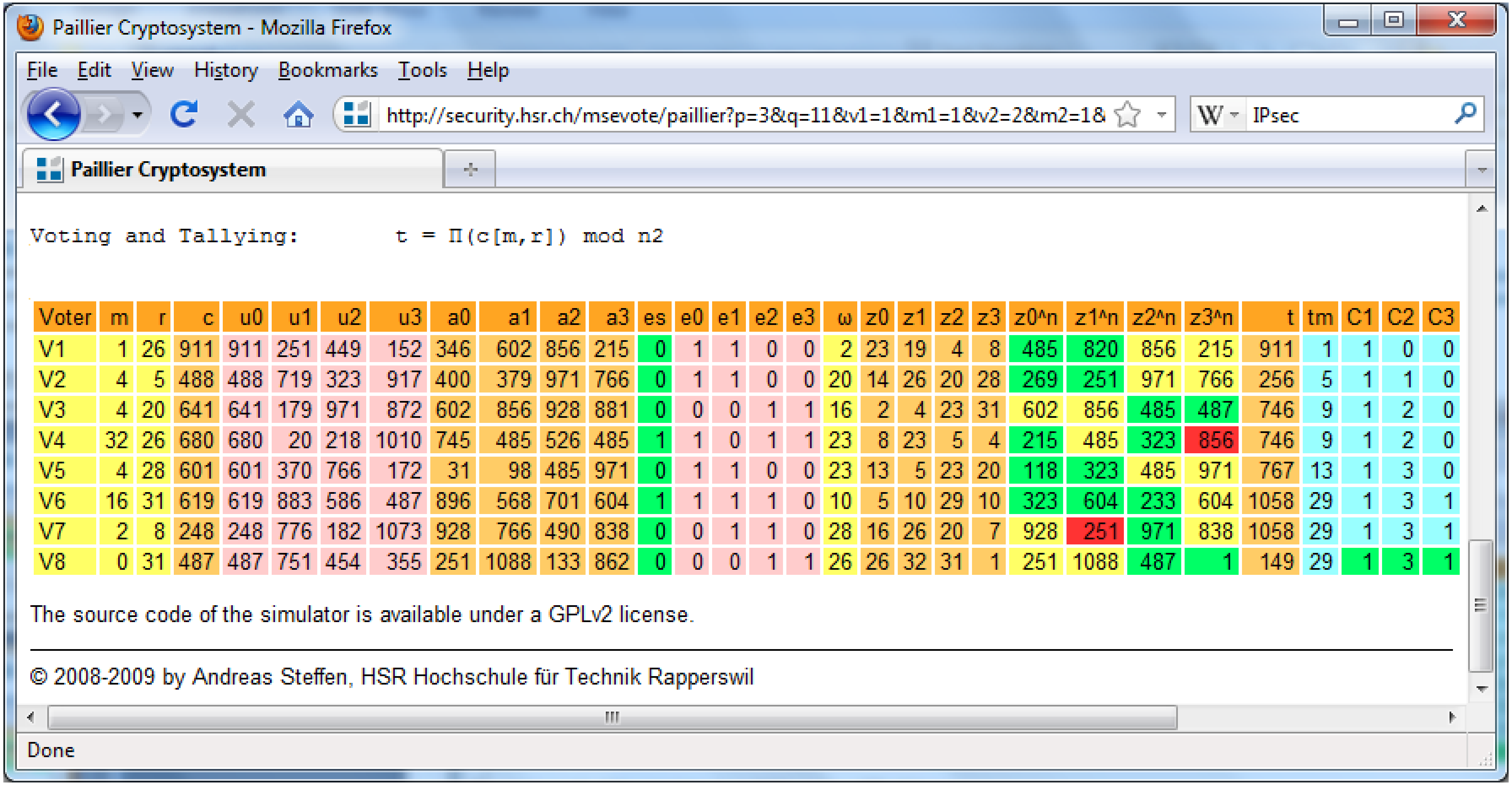
Random Seed 8

```
p = 3, q = 11, n = pq = 33, n2 = 1089, φ(n) = (p-1)(q-1) = 20, λ(n) = lcm(p-1,q-1) = 10

Valid Voting Messages:     m = mi, None: m0 = 0, C1: m1 = 1, C2: m2 = b = 4, C3: m3 = b^2 = 16
```

- http://security.hsr.ch/msevote/paillier

E-Voting Simulator – Tallying with ZKPs


Paillier Cryptosystem - Mozilla Firefox
File Edit View History Bookmarks Tools Help
http://security.hsr.ch/msevote/paillier?p=3&q=11&v1=1&m1=1&v2=2&m2=1&
IPsec
Paillier Cryptosystem
Done

```
Voting and Tallying:      t = Π(c[m,r]) mod n2
```

Voter	m	r	c	u0	u1	u2	u3	a0	a1	a2	a3	es	e0	e1	e2	e3	ω	z0	z1	z2	z3	z0^n	z1^n	z2^n	z3^n	t	tm	C1	C2	C3
V1	1	26	911	911	251	449	152	346	602	856	215	0	1	1	0	0	2	23	19	4	8	485	820	856	215	911	1	1	0	0
V2	4	5	488	488	719	323	917	400	379	971	766	0	1	1	0	0	20	14	26	20	28	269	251	971	766	256	5	1	1	0
V3	4	20	641	641	179	971	872	602	856	928	881	0	0	0	1	1	16	2	4	23	31	602	856	485	487	746	9	1	2	0
V4	32	26	680	680	20	218	1010	745	485	526	485	1	1	0	1	1	23	8	23	5	4	215	485	323	856	746	9	1	2	0
V5	4	28	601	601	370	766	172	31	98	485	971	0	1	1	0	0	23	13	5	23	20	118	323	485	971	767	13	1	3	0
V6	16	31	619	619	883	586	487	896	568	701	604	1	1	1	1	0	10	5	10	29	10	323	604	233	604	1058	29	1	3	1
V7	2	8	248	248	776	182	1073	928	766	490	838	0	0	1	1	0	28	16	26	20	7	928	251	971	838	1058	29	1	3	1
V8	0	31	487	487	751	454	355	251	1088	133	862	0	0	0	1	1	26	26	32	31	1	251	1088	487	1	149	29	1	3	1

The source code of the simulator is available under a GPLv2 license.

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Non-Interactive ZKP (Fiat-Shamir Heuristic)

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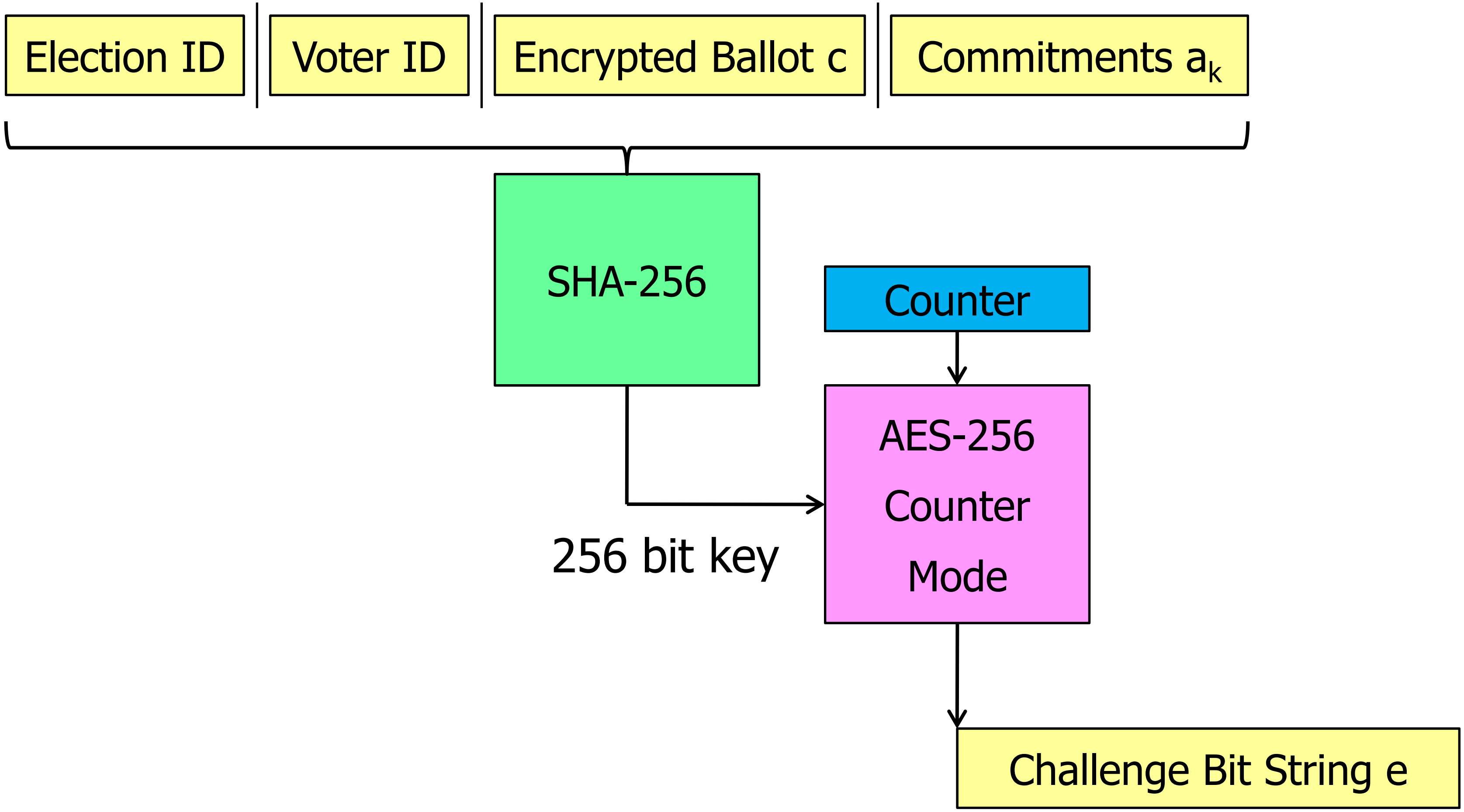

The Damgård-Jurik Cryptosystem

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- Additional parameter s (Paillier: s = 1)

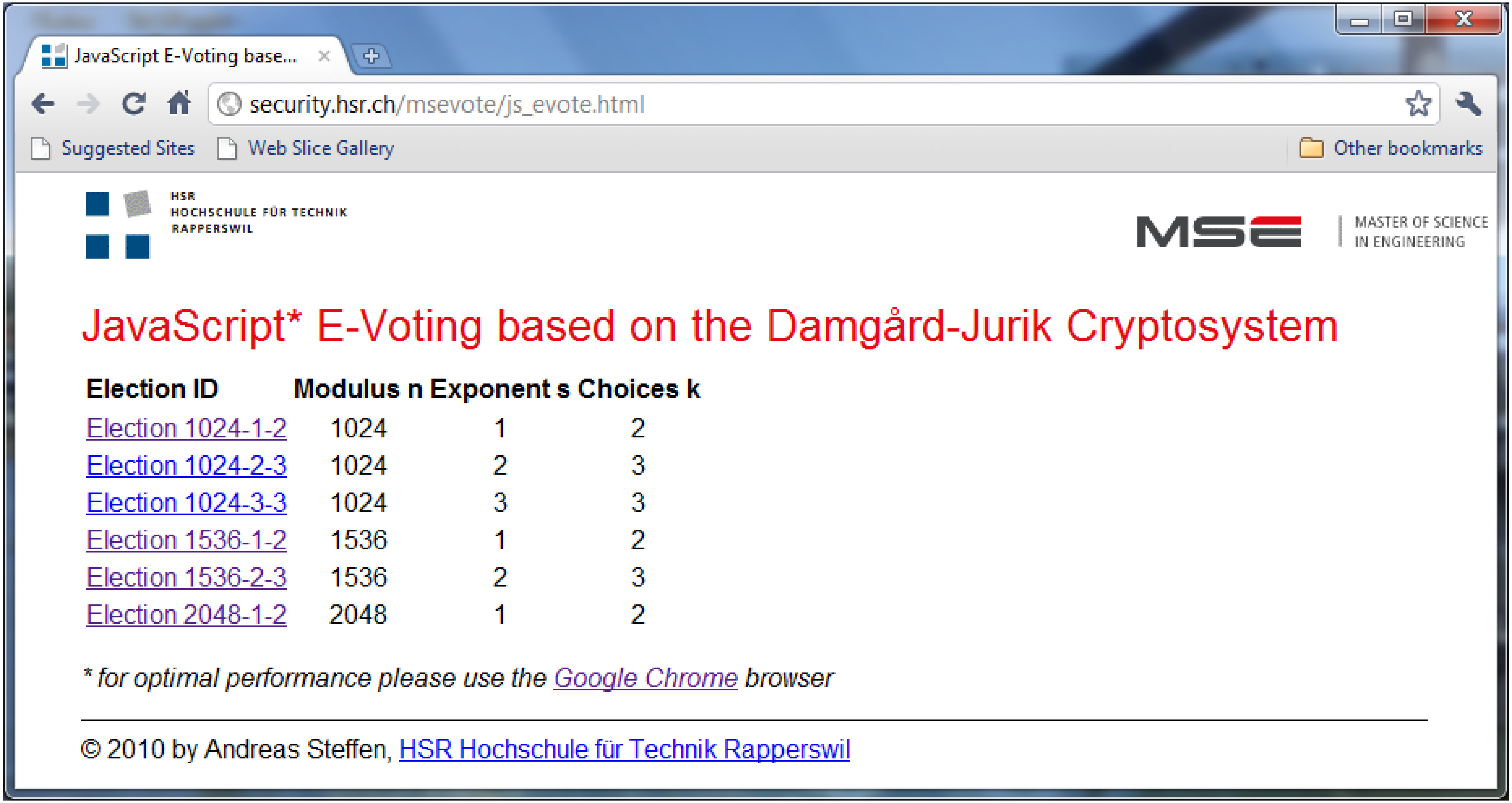
$$\boxed{(m,r) \quad \mapsto \quad g^{m} \cdot r^{n^{s}} \bmod n^{s+1} = c}$$

m: plaintext message, r: random number for semantic security

- Generate the multiplicative subgroup $Z^{*}_{n^{s+1}}$ as $Z_{n^{s}} \times Z^{*}_{n} \mapsto Z^{*}_{n^{s+1}}$
- Generator usually chosen as g = (1+n)

● Size of modulus n:	b bits	(e.g. 1536 bits)	
● Size of message m:	s·b - 1 bits	(s=1: 1535 bits,	s=2: 3071 bits)
● Size of ciphertext c:	(s+1)·b	(s=1: 3072 bits,	s=2: 4608 bits)
● Efficiency:	μ = s/(s+1)	(s=1: 50%, s=2: 67%, s=3: 75%)	

Damgård-Jurik JavaScript E-Voting Client


JavaScript E-Voting base...
security.hsr.ch/msevote/js_evote.html
Suggested Sites
Web Slice Gallery
Other bookmarks
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MSE MASTER OF SCIENCE IN ENGINEERING

JavaScript* E-Voting based on the Damgård-Jurik Cryptosystem

Election ID	Modulus n	Exponent s	Choices k
Election 1024-1-2	1024	1	2
Election 1024-2-3	1024	2	3
Election 1024-3-3	1024	3	3
Election 1536-1-2	1536	1	2
Election 1536-2-3	1536	2	3
Election 2048-1-2	2048	1	2

** for optimal performance please use the Google Chrome browser*

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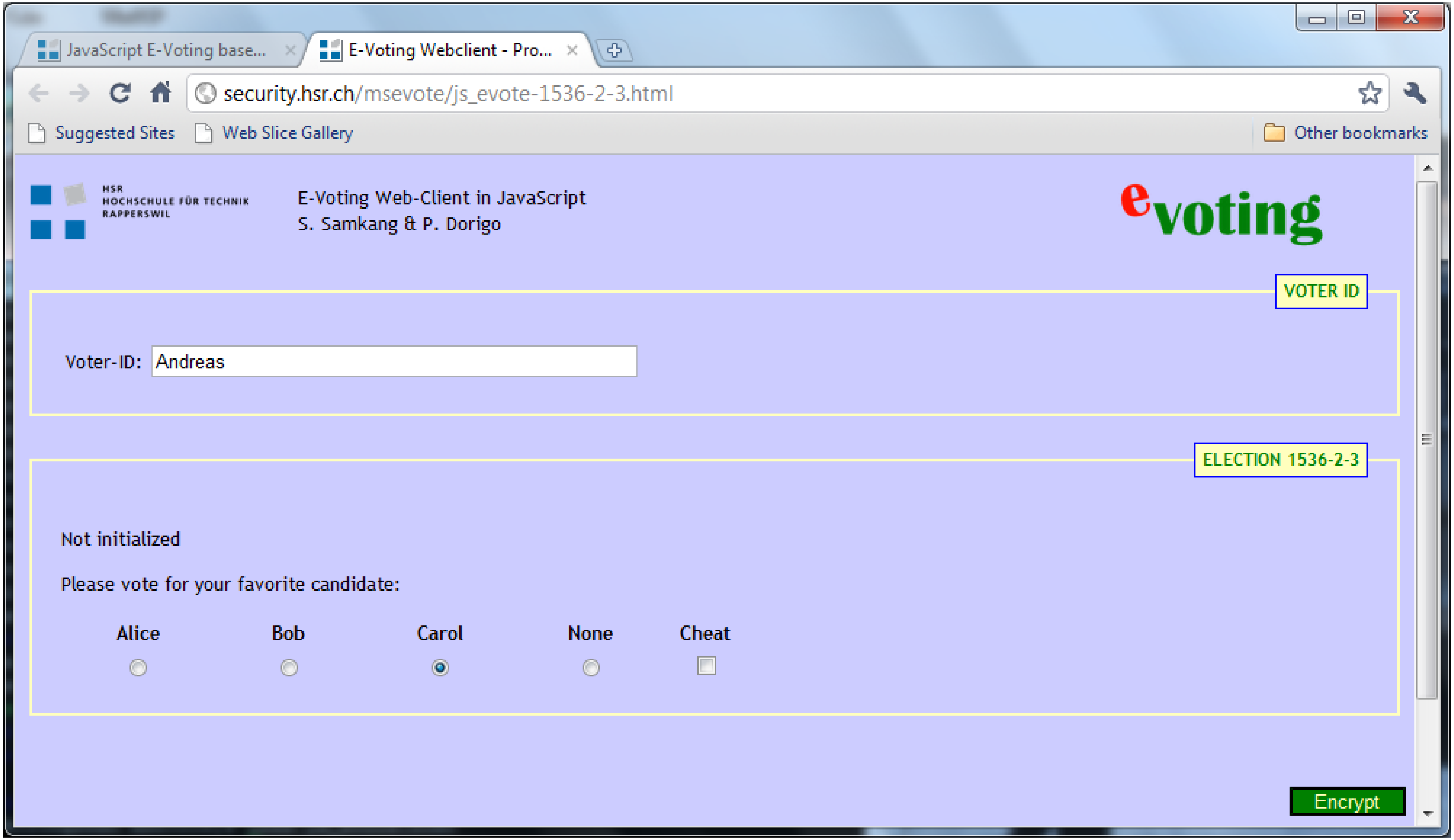
Damgård-Jurik JavaScript E-Voting Client



JavaScript E-Voting base...
E-Voting Webclient - Pro...
security.hsr.ch/msevote/js_evote-1536-2-3.html
Suggested Sites
Web Slice Gallery
Other bookmarks
HSR HOCHSCHULE FÜR TECHNIK RAPPERSWIL
evoting

E-Voting Web-Client in JavaScript
S. Samkang & P. Dorigo

VOTER ID

Voter-ID: Andreas

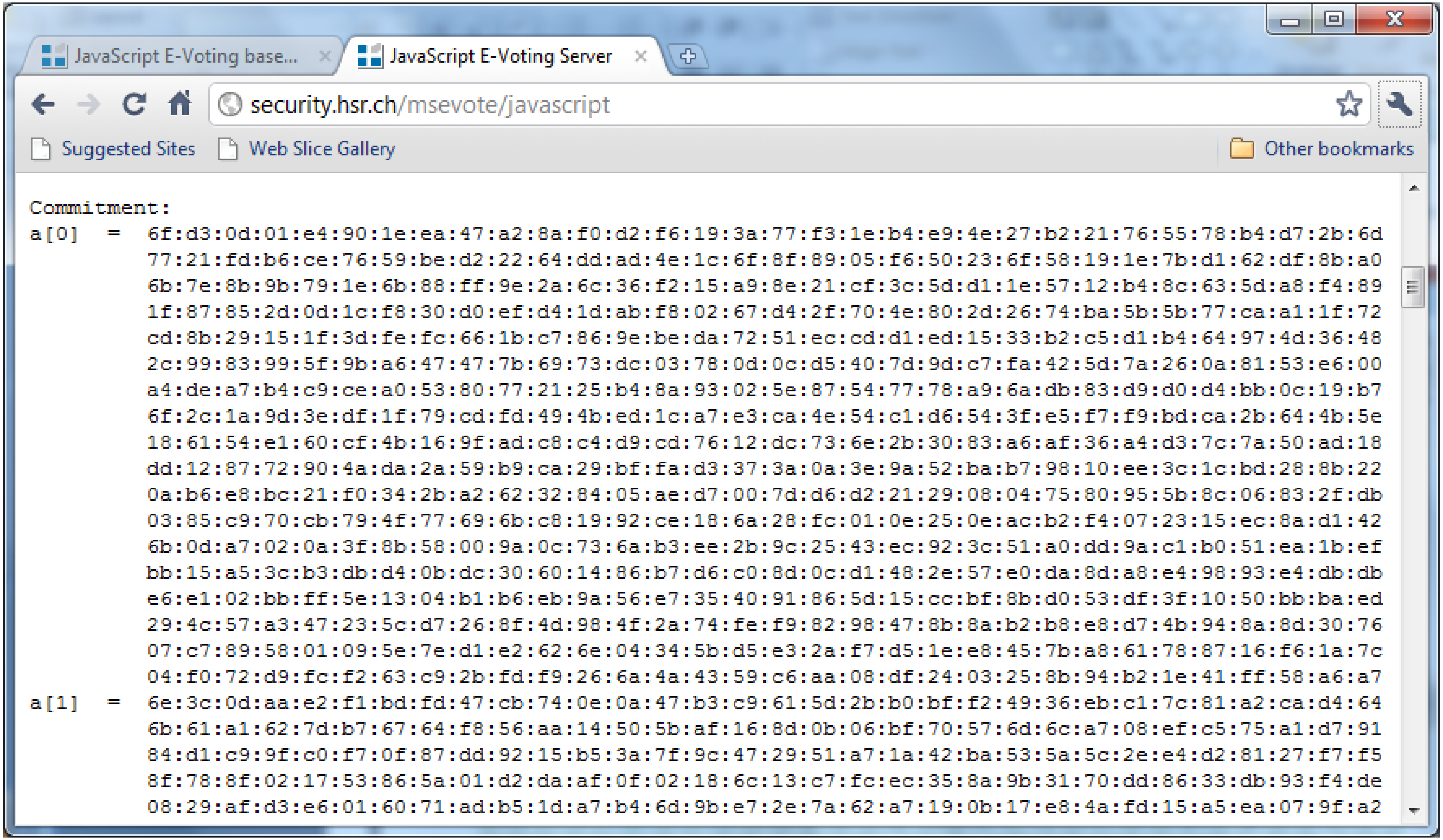
ELECTION 1536-2-3

Not initialized

Please vote for your favorite candidate:

Alice ○ Bob ○ Carol ● None ○ Cheat □

Commitment



JavaScript E-Voting base...
JavaScript E-Voting Server
security.hsr.ch/msevote/javascript
Suggested Sites
Web Slice Gallery
Other bookmarks

```
Commitment:
a[0]   =  6f:d3:0d:01:e4:90:1e:ea:47:a2:8a:f0:d2:f6:19:3a:77:f3:1e:b4:e9:4e:27:b2:21:76:55:78:b4:d7:2b:6d
          77:21:fd:b6:ce:76:59:be:d2:22:64:dd:ad:4e:1c:6f:8f:89:05:f6:50:23:6f:58:19:1e:7b:d1:62:df:8b:a0
          6b:7e:8b:9b:79:1e:6b:88:ff:9e:2a:6c:36:f2:15:a9:8e:21:cf:3c:5d:d1:1e:57:12:b4:8c:63:5d:a8:f4:89
          1f:87:85:2d:0d:1c:f8:30:d0:ef:d4:1d:ab:f8:02:67:d4:2f:70:4e:80:2d:26:74:ba:5b:5b:77:ca:a1:1f:72
          cd:8b:29:15:1f:3d:fe:fc:66:1b:c7:86:9e:be:da:72:51:ec:cd:d1:ed:15:33:b2:c5:d1:b4:64:97:4d:36:48
          2c:99:83:99:5f:9b:a6:47:47:7b:69:73:dc:03:78:0d:0c:d5:40:7d:9d:c7:fa:42:5d:7a:26:0a:81:53:e6:00
          a4:de:a7:b4:c9:ce:a0:53:80:77:21:25:b4:8a:93:02:5e:87:54:77:78:a9:6a:db:83:d9:d0:d4:bb:0c:19:b7
          6f:2c:1a:9d:3e:df:1f:79:cd:fd:49:4b:ed:1c:a7:e3:ca:4e:54:c1:d6:54:3f:e5:f7:f9:bd:ca:2b:64:4b:5e
          18:61:54:e1:60:cf:4b:16:9f:ad:c8:c4:d9:cd:76:12:dc:73:6e:2b:30:83:a6:af:36:a4:d3:7c:7a:50:ad:18
          dd:12:87:72:90:4a:da:2a:59:b9:ca:29:bf:fa:d3:37:3a:0a:3e:9a:52:ba:b7:98:10:ee:3c:1c:bd:28:8b:22
          0a:b6:e8:bc:21:f0:34:2b:a2:62:32:84:05:ae:d7:00:7d:d6:d2:21:29:08:04:75:80:95:5b:8c:06:83:2f:db
          03:85:c9:70:cb:79:4f:77:69:6b:c8:19:92:ce:18:6a:28:fc:01:0e:25:0e:ac:b2:f4:07:23:15:ec:8a:d1:42
          6b:0d:a7:02:0a:3f:8b:58:00:9a:0c:73:6a:b3:ee:2b:9c:25:43:ec:92:3c:51:a0:dd:9a:c1:b0:51:ea:1b:ef
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          07:c7:89:58:01:09:5e:7e:d1:e2:62:6e:04:34:5b:d5:e3:2a:f7:d5:1e:e8:45:7b:a8:61:78:87:16:f6:1a:7c
          04:f0:72:d9:fc:f2:63:c9:2b:fd:f9:26:6a:4a:43:59:c6:aa:08:df:24:03:25:8b:94:b2:1e:41:ff:58:a6:a7
a[1]   =  6e:3c:0d:aa:e2:f1:bd:fd:47:cb:74:0e:0a:47:b3:c9:61:5d:2b:b0:bf:f2:49:36:eb:c1:7c:81:a2:ca:d4:64
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```
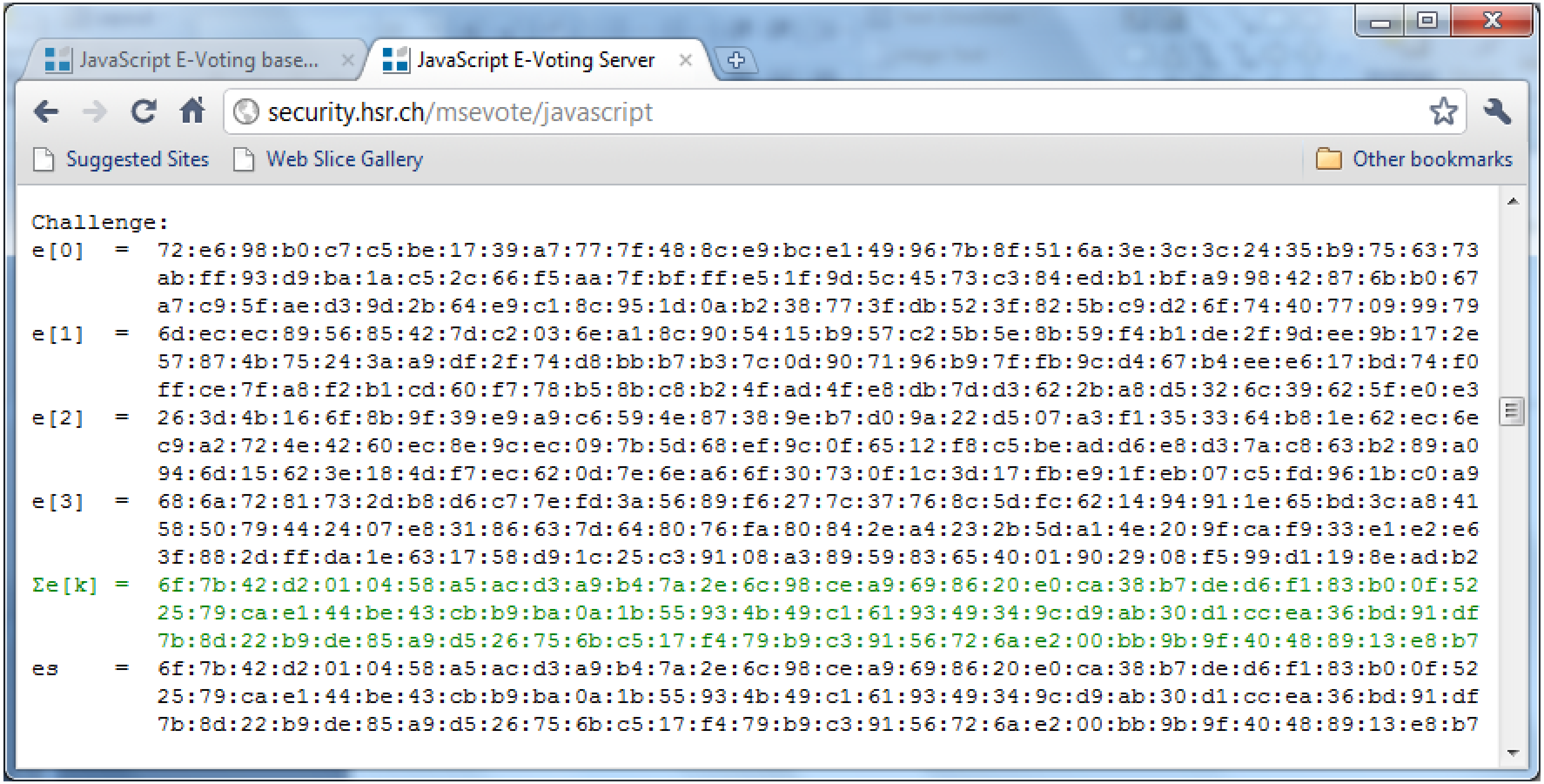

Challenge Verification


JavaScript E-Voting base...
JavaScript E-Voting Server
security.hsr.ch/msevote/javascript
Suggested Sites
Web Slice Gallery
Other bookmarks

```
Challenge:
e[0]  =  72:e6:98:b0:c7:c5:be:17:39:a7:77:7f:48:8c:e9:bc:e1:49:96:7b:8f:51:6a:3e:3c:3c:24:35:b9:75:63:73
         ab:ff:93:d9:ba:1a:c5:2c:66:f5:aa:7f:bf:ff:e5:1f:9d:5c:45:73:c3:84:ed:b1:bf:a9:98:42:87:6b:b0:67
         a7:c9:5f:ae:d3:9d:2b:64:e9:c1:8c:95:1d:0a:b2:38:77:3f:db:52:3f:82:5b:c9:d2:6f:74:40:77:09:99:79
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         58:50:79:44:24:07:e8:31:86:63:7d:64:80:76:fa:80:84:2e:a4:23:2b:5d:a1:4e:20:9f:ca:f9:33:e1:e2:e6
         3f:88:2d:ff:da:1e:63:17:58:d9:1c:25:c3:91:08:a3:89:59:83:65:40:01:90:29:08:f5:99:d1:19:8e:ad:b2
Σe[k] =  6f:7b:42:d2:01:04:58:a5:ac:d3:a9:b4:7a:2e:6c:98:ce:a9:69:86:20:e0:ca:38:b7:de:d6:f1:83:b0:0f:52
         25:79:ca:e1:44:be:43:cb:b9:ba:0a:1b:55:93:4b:49:c1:61:93:49:34:9c:d9:ab:30:d1:cc:ea:36:bd:91:df
         7b:8d:22:b9:de:85:a9:d5:26:75:6b:c5:17:f4:79:b9:c3:91:56:72:6a:e2:00:bb:9b:9f:40:48:89:13:e8:b7
es    =  6f:7b:42:d2:01:04:58:a5:ac:d3:a9:b4:7a:2e:6c:98:ce:a9:69:86:20:e0:ca:38:b7:de:d6:f1:83:b0:0f:52
         25:79:ca:e1:44:be:43:cb:b9:ba:0a:1b:55:93:4b:49:c1:61:93:49:34:9c:d9:ab:30:d1:cc:ea:36:bd:91:df
         7b:8d:22:b9:de:85:a9:d5:26:75:6b:c5:17:f4:79:b9:c3:91:56:72:6a:e2:00:bb:9b:9f:40:48:89:13:e8:b7
```
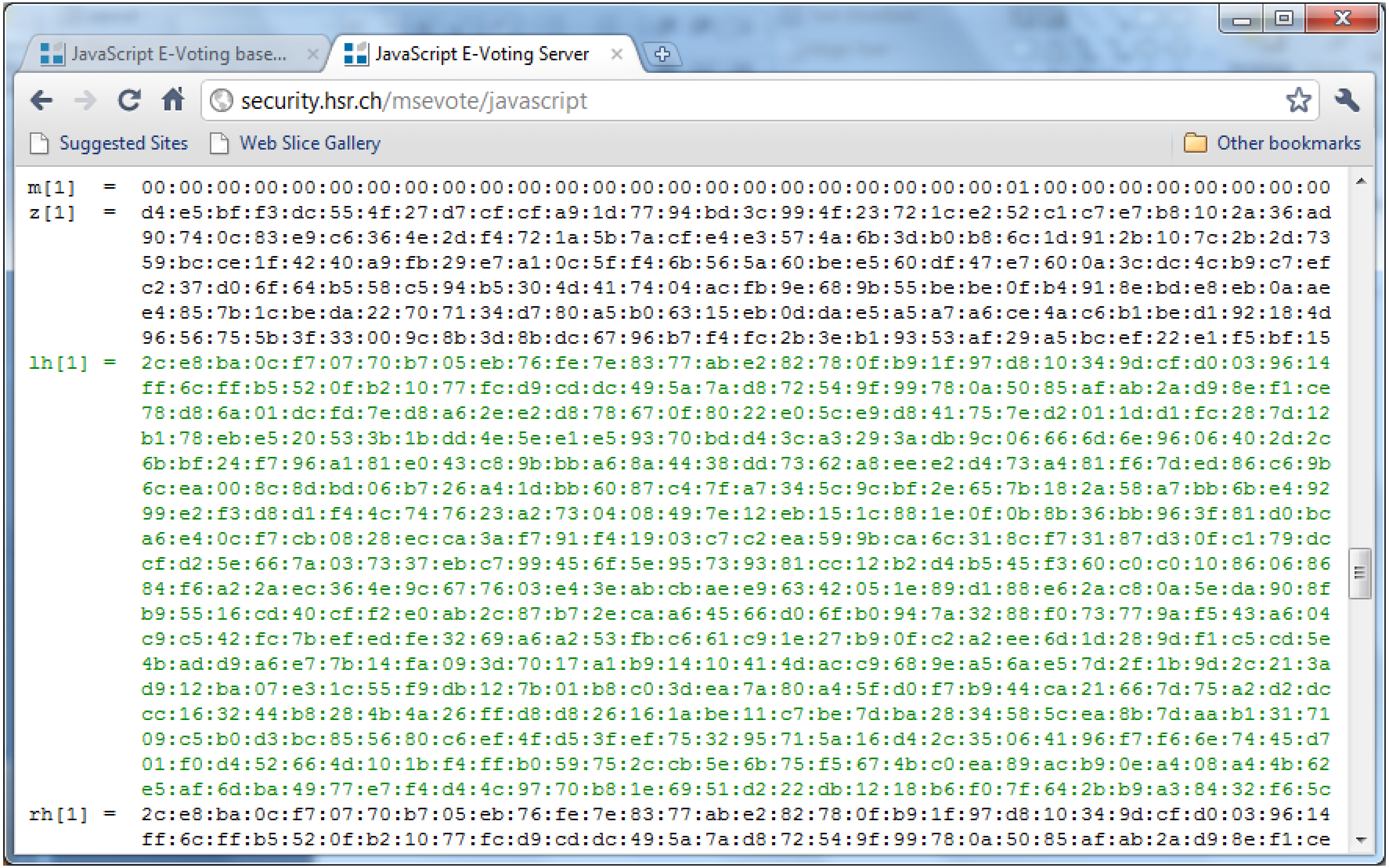
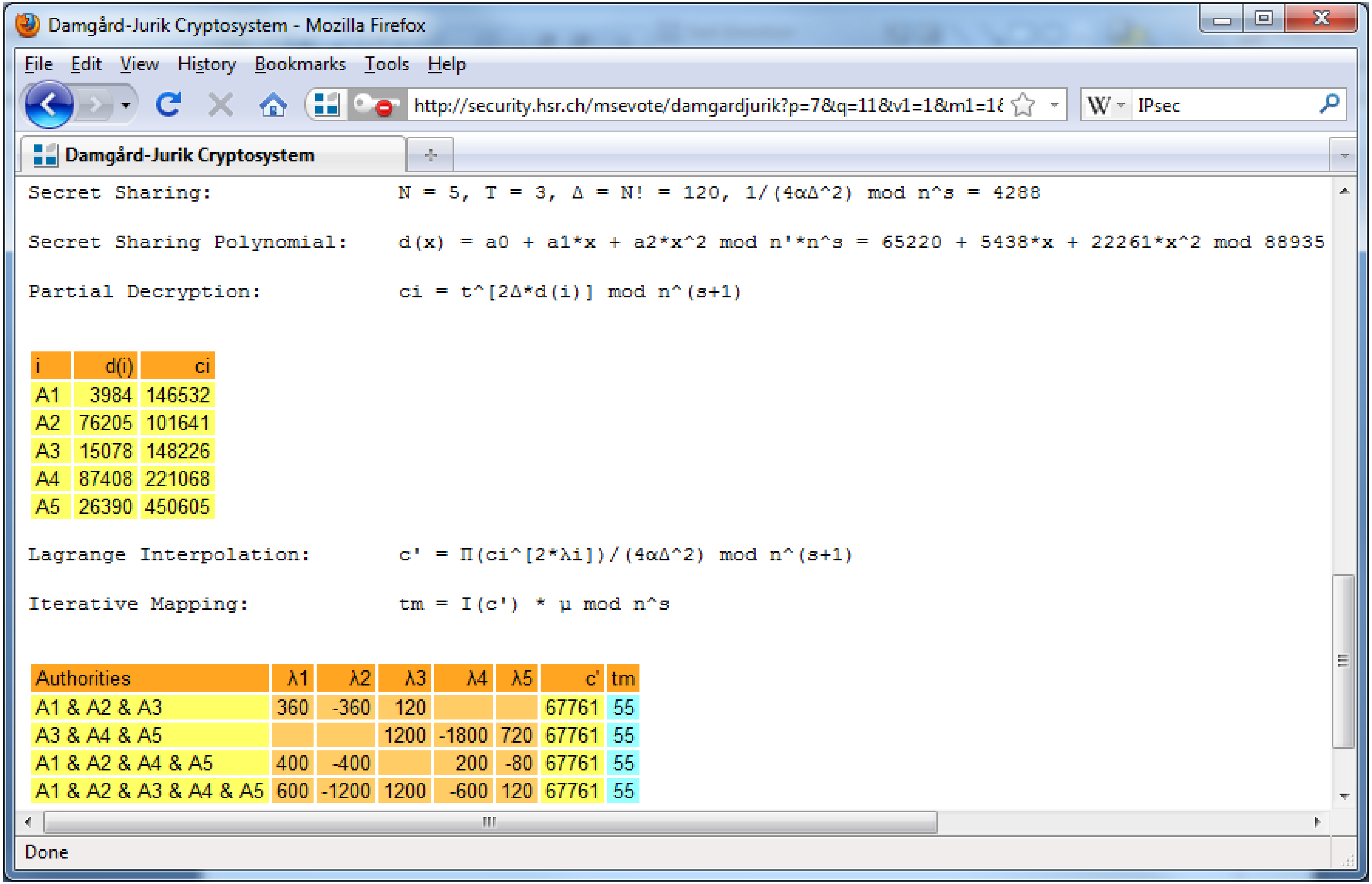

Response Verification


JavaScript E-Voting base...
JavaScript E-Voting Server
security.hsr.ch/msevote/javascript
Suggested Sites
Web Slice Gallery
Other bookmarks

```
m[1]  =  00:00:00:00:00:00:00:00:00:00:00:00:00:00:00:00:00:00:00:00:00:01:00:00:00:00:00:00:00:00
z[1]  =  d4:e5:bf:f3:dc:55:4f:27:d7:cf:cf:a9:1d:77:94:bd:3c:99:4f:23:72:1c:e2:52:c1:c7:e7:b8:10:2a:36:ad
         90:74:0c:83:e9:c6:36:4e:2d:f4:72:1a:5b:7a:cf:e4:e3:57:4a:6b:3d:b0:b8:6c:1d:91:2b:10:7c:2b:2d:73
         59:bc:ce:1f:42:40:a9:fb:29:e7:a1:0c:5f:f4:6b:56:5a:60:be:e5:60:df:47:e7:60:0a:3c:dc:4c:b9:c7:ef
         c2:37:d0:6f:64:b5:58:c5:94:b5:30:4d:41:74:04:ac:fb:9e:68:9b:55:be:be:0f:b4:91:8e:bd:e8:eb:0a:ae
         e4:85:7b:1c:be:da:22:70:71:34:d7:80:a5:b0:63:15:eb:0d:da:e5:a5:a7:a6:ce:4a:c6:b1:be:d1:92:18:4d
         96:56:75:5b:3f:33:00:9c:8b:3d:8b:dc:67:96:b7:f4:fc:2b:3e:b1:93:53:af:29:a5:bc:ef:22:e1:f5:bf:15
lh[1] =  2c:e8:ba:0c:f7:07:70:b7:05:eb:76:fe:7e:83:77:ab:e2:82:78:0f:b9:1f:97:d8:10:34:9d:cf:d0:03:96:14
         ff:6c:ff:b5:52:0f:b2:10:77:fc:d9:cd:dc:49:5a:7a:d8:72:54:9f:99:78:0a:50:85:af:ab:2a:d9:8e:f1:ce
         78:d8:6a:01:dc:fd:7e:d8:a6:2e:e2:d8:78:67:0f:80:22:e0:5c:e9:d8:41:75:7e:d2:01:1d:d1:fc:28:7d:12
         b1:78:eb:e5:20:53:3b:1b:dd:4e:5e:e1:e5:93:70:bd:d4:3c:a3:29:3a:db:9c:06:66:6d:6e:96:06:40:2d:2c
         6b:bf:24:f7:96:a1:81:e0:43:c8:9b:bb:a6:8a:44:38:dd:73:62:a8:ee:e2:d4:73:a4:81:f6:7d:ed:86:c6:9b
         6c:ea:00:8c:8d:bd:06:b7:26:a4:1d:bb:60:87:c4:7f:a7:34:5c:9c:bf:2e:65:7b:18:2a:58:a7:bb:6b:e4:92
         99:e2:f3:d8:d1:f4:4c:74:76:23:a2:73:04:08:49:7e:12:eb:15:1c:88:1e:0f:0b:8b:36:bb:96:3f:81:d0:bc
         a6:e4:0c:f7:cb:08:28:ec:ca:3a:f7:91:f4:19:03:c7:c2:ea:59:9b:ca:6c:31:8c:f7:31:87:d3:0f:c1:79:dc
         cf:d2:5e:66:7a:03:73:37:eb:c7:99:45:6f:5e:95:73:93:81:cc:12:b2:d4:b5:45:f3:60:c0:c0:10:86:06:86
         84:f6:a2:2a:ec:36:4e:9c:67:76:03:e4:3e:ab:cb:ae:e9:63:42:05:1e:89:d1:88:e6:2a:c8:0a:5e:da:90:8f
         b9:55:16:cd:40:cf:f2:e0:ab:2c:87:b7:2e:ca:a6:45:66:d0:6f:b0:94:7a:32:88:f0:73:77:9a:f5:43:a6:04
         c9:c5:42:fc:7b:ef:ed:fe:32:69:a6:a2:53:fb:c6:61:c9:1e:27:b9:0f:c2:a2:ee:6d:1d:28:9d:f1:c5:cd:5e
         4b:ad:d9:a6:e7:7b:14:fa:09:3d:70:17:a1:b9:14:10:41:4d:ac:c9:68:9e:a5:6a:e5:7d:2f:1b:9d:2c:21:3a
         d9:12:ba:07:e3:1c:55:f9:db:12:7b:01:b8:c0:3d:ea:7a:80:a4:5f:d0:f7:b9:44:ca:21:66:7d:75:a2:d2:dc
         cc:16:32:44:b8:28:4b:4a:26:ff:d8:d8:26:16:1a:be:11:c7:be:7d:ba:28:34:58:5c:ea:8b:7d:aa:b1:31:71
         09:c5:b0:d3:bc:85:56:80:c6:ef:4f:d5:3f:ef:75:32:95:71:5a:16:d4:2c:35:06:41:96:f7:f6:6e:74:45:d7
         01:f0:d4:52:66:4d:10:1b:f4:ff:b0:59:75:2c:cb:5e:6b:75:f5:67:4b:c0:ea:89:ac:b9:0e:a4:08:a4:4b:62
         e5:af:6d:ba:49:77:e7:f4:d4:4c:97:70:b8:1e:69:51:d2:22:db:12:18:b6:f0:7f:64:2b:b9:a3:84:32:f6:5c
rh[1] =  2c:e8:ba:0c:f7:07:70:b7:05:eb:76:fe:7e:83:77:ab:e2:82:78:0f:b9:1f:97:d8:10:34:9d:cf:d0:03:96:14
         ff:6c:ff:b5:52:0f:b2:10:77:fc:d9:cd:dc:49:5a:7a:d8:72:54:9f:99:78:0a:50:85:af:ab:2a:d9:8e:f1:ce
```

Threshold Scheme with a Trusted Dealer


Damgård-Jurik Cryptosystem - Mozilla Firefox
File Edit View History Bookmarks Tools Help
http://security.hsr.ch/msevote/damgardjurik?p=7&q=11&v1=1&m1=1&
IPsec
Damgård-Jurik Cryptosystem
+
Done

```
Secret Sharing:              N = 5, T = 3, Δ = N! = 120, 1/(4αΔ^2) mod n^s = 4288

Secret Sharing Polynomial:   d(x) = a0 + a1*x + a2*x^2 mod n'*n^s = 65220 + 5438*x + 22261*x^2 mod 88935

Partial Decryption:          ci = t^[2Δ*d(i)] mod n^(s+1)
```

i	d(i)	ci
A1	3984	146532
A2	76205	101641
A3	15078	148226
A4	87408	221068
A5	26390	450605

```
Lagrange Interpolation:      c' = Π(ci^[2*λi])/(4αΔ^2) mod n^(s+1)

Iterative Mapping:           tm = I(c') * μ mod n^s
```

Authorities	λ1	λ2	λ3	λ4	λ5	c'	tm
A1 & A2 & A3	360	-360	120			67761	55
A3 & A4 & A5			1200	-1800	720	67761	55
A1 & A2 & A4 & A5	400	-400		200	-80	67761	55
A1 & A2 & A3 & A4 & A5	600	-1200	1200	-600	120	67761	55

Threshold Scheme without a Trusted Dealer

- Practical threshold RSA signatures without a trusted dealer
  Ivan Damgard, Maciej Koprowski, 2001
- The distributed generation of an RSA private key required by a Threshold Paillier Cryptosystem is much more complex than the simple independent partial private key generation possible with the El Gamal Cryptosystem.