Solutions: Problem set 1

1. A box contains three marbles: one red, one green, and one blue. Consider an experiment that consists of taking one marble from the box then replacing it in the box and drawing a second marble from the box. What is the sample space? If, at all times, each marble in the box is equally likely to be selected, what is the probability of each point in the sample space?

   Solution: S = {(R, R),(R, G),(R, B),(G, R),(G, G),(G, B),(B, R),(B, G),(B, B)}. The probability of each event is 1/9.

2. Repeat Exercise 1 when the second marble is drawn without replacing the first marble.

   Solution: S = {(R, G),(R, B),(G, R),(G, B),(B, R),(B, G)} with each event having a probability of 1/6.

3. A coin is to be tossed until a head appears twice in a row. What is the sample space for this experiment? If the coin is fair, what is the probability that it will be tossed exactly four times?

Solution: S = {HH, T HH, HT HH, T T HH, T HT HH, HT T HH, . . .}
P{4 tosses} = P{(T, T, H, H)} + P{(H, T, H, H)} = 2 (1/2)^4 = 1/8 .

4. Suppose that 5 percent of men and 0.25 percent of women are colour-blind. A colour-blind person is chosen at random. What is the probability of this person being male? Assume that there are an equal number of males and females.

   Solution: Let C be the event of a person being colour-blind.
   p(C|Male) = 0.05

   p(Male) = 0.5

   p(C|Female) = 0.0025

   By applying Bayes' theorem:
   p(M|C) = p(C|Male)p(Male)/ (p(C|Male)p(Male) + p(C|Female)p(Female))

5. Two cards are randomly selected from a deck of 52 playing cards.
(a) What is the probability they constitute a pair (that is, that they are of the same denomination)?
(b) What is the conditional probability they constitute a pair given that they are of different suits?

Solution: a. p(pair) = 52 * 1/52 * 3/51 = 3/51
b. Since, you know that they are different suits, the second card is chosen from a deck consisting of just 13 × 3 = 39 cards. Thus:
p(pair|different suit) = 52 * 1/52 * 3/39 = 1/13

6. The probability that Bob will fail a certain statistics examination is 0.5 and the probability that Alice will fail the same examination is 0.2. The probability that both Bob and Alice will fail the examination is 0.1.
   a. What is the probability that, between Bob and Alice, at least one of them will fail?
   b. What is the probability that neither Bob not Alice will fail?
   c. Are the event "Bob will fail the examination" and "Alice will fail the examination" disjoint?
   d. What is the probability that, between Bob and Alice, exactly one of them will fail the exam?
   e. Bob and Alice are also taking an advanced mathematic class whose midterm exam is scheduled right after the statistics exam. The probability that Bob will fail this second exam is 0.7 and the probability that Alice will fail is 0.4. Determine the minimal and maximal possible values of the probability that both Bob and Alice will fail this second exam.

Solution:
a. (0.5-0.1)+(0.2-0.1)+0.1=0.6
b. By using inclusion-exclusion, 0.5+0.8-0.9=0.4
c. No, since the probability of both failing is not 0.
d. (0.5-0.1)+(0.2-0.1)=0.5
e. The minimal value of the probability of both failing would be 0.1. The maximal value would be 0.4, and this would happen when Alice's set is a strict subset of Bob's set.

7. An ordinary die is converted into a biased die in which the probability that a given number of dots appears is proportional to the number of dots. What is the probability, in one roll of the die, that an even number will turn up?

Solution: 1+2+3+4+5+6 = 21, and if the probabilities that a given number of dots appears is proportional to the number of dots, then getting even dots is simply 2/21+4/21+6/21 = 12/21.