Problem set 2:

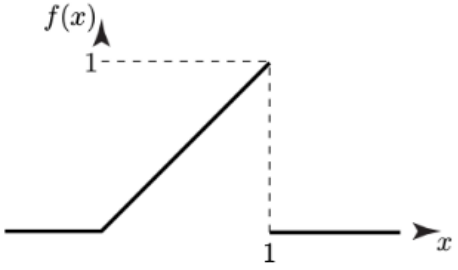
1. Throw a (6-face) fair dice twice independently. Let A be the point we get in the first throw, and B be the point we get in the second throw. Let X = A − B be the difference between two points, and Y = A+B be the sum of the two points.
   i. Find the range of X and Y.
   ii. Find $p_{X|Y}$ (x|Y=8).
   iii. Find E[Y |X ≥ 4].

2. If the probability density function (pdf) of a random variable is given by:

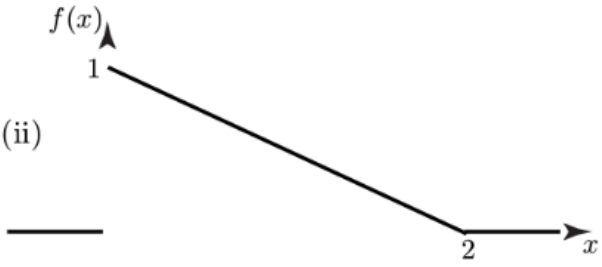
$$p(x) = \begin{cases} \frac{1}{2}x & \text{for} 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

Find **a)** the expectation E[x], **b)** variance Var[x]

(i)



(ii)



(iii) $f(x) = \begin{cases} x^2 - 4x + \frac{10}{3}, & 0 \le x \le 3 \\ 0, & \text{elsewhere} \end{cases}$

3. Which of the following are not probability density functions. State why?

4. If cumulative density function (cdf) of a random variable is given by:

$$F(x) = \begin{cases} 1 - e^{-2x} & \text{for} x \ge 0 \\ 0 & \text{for} x < 0 \end{cases}$$

Find **a)** the probability density function (pdf) p(x), **b)** the cdf F(x > 2) and **c)** F(-3 ≤ x ≤ 4)

5. If x and y have the joint probability density function (pdf) given by:

$$p(x,y) = \begin{cases} \frac{3}{4} + xy & \text{for} 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find **a)** the conditional probability p(y | x),
b) the cumulative density $F(y > 0.5 \mid x = 0.5)$