Robust Attitude Consensus Control of Multi-Spacecraft with Stochastic Links Failure
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Abstract: This paper presents an adaptive attitude consensus controller for a group of spacecrafts subject to stochastic communication links failure and external disturbances. By leveraging the sliding-mode control technique and the super-martingales convergence method, the proposed adaptive controller is robust to the bounded but known disturbances and ensures almost sure consensus on the attitude among multi-spacecraft, respectively. Moreover, when compared with existing results dealing with attitude consensus control with indeterministic communication topology, our approach can drive the attitude of multi-spacecraft to a desired attitude of a virtual spacecraft. To verify the effectiveness of the proposed approach, an attitude consensus control of a group of six spacecraft with a virtual leader is carried out.

Keywords: *Attitude consensus, Communication links failures, Adaptive control, Multi-spacecraft Systems*

1 Introduction

In the recent decades, the development and application of a single spacecraft are constrained due to the gradually increased volume, weight and cost. Due to the low cost, small volume, light weight, low energy consumption, the multi-spacecraft system is developed to compensate for the disadvantages of a single spacecraft. Multi-spacecraft system can achieve or even exceed the capabilities of a single spacecraft. Moreover, failure of a spacecraft will not lead to the failure of the whole system. Thus, the multi-spacecraft system has attracted considerable attention. In order to accomplish complex space missions, it is critical for the multi-spacecraft systems to maintain accurate relative attitude. Attitude consensus control of multi-spacecraft systems is one of the key factors for the success of space missions, especially for earth observation and universal exploration. For instance, the traditional single spacecraft is hard to conduct the large area Earth observation owing to its limited field of view. Therefore, it is worthwhile further investigating attitude consensus control for multi-spacecraft coordinated missions.

There have been numerous attitude consensus control schemes proposed to make attitude converge to a common attitude. In [1-4], adaptive attitude controllers using different techniques (e.g., sliding mode control, backstepping control, robust control, et al) were designed to achieve accurate relative orientation control among spacecraft in either the undirected or directed topology, despite the existence of inertia uncertainties and external disturbances. When there exist input saturation constraints, terminal sliding mode-based control methods were proposed to achieve attitude consensus [5-7]. In [8,9], relative position cooperative control problem was studied, where the proposed methods guarantee finite-time error convergence. However, the modeling uncertainties and disturbances are not considered, leading to limited robustness and degraded control performance.

On the other hand, in order to achieve reliable attitude consensus control, it is of vital importance for spacecraft to transmit information through inter-satellite communication links, and maintain a certain configuration and relative attitude. In the leader-follower control strategy, the spacecraft that is selected as pilot one needs to reach the expected attitude, while the rest spacecraft are regarded as the follower ones to track the pilot one so as to complete the attitude consensus mission. In this control strategy, it is essential that the information of the pilot spacecraft can be transferred to any follower spacecrafts through a deterministic communication link. In [10-12], the leader tracks the common attitude and the attitude information of the leader can be obtained by the followers to achieve the cooperative attitude control. In the virtual structure control strategy, the whole spacecraft tracks the virtual spacecraft which transfers the desired attitude. In [13-16]，the multi-spacecraft system maneuvers to the specified attitude according to information transferred by virtual leader through communication links. Based on consensus theory, spacecraft members utilize their own attitude information to cooperatively achieve the same attitude through communication link. In [17-19], the attitude consensus problem was addressed and a desired attitude can be reached through an undirected communicating link. Although the aforementioned works can tackle the attitude consensus problem, the communication link failure is not considered in the controller design.

In practice, communication links are vulnerable to interference from various uncertain sources, which may lead to intermediate failure of the communication link. Under this situation, communication links may fail and rebuilt stochastically over time. In [20], the attitude consensus problem of multi-spacecraft system under stochastic communication link failure in a directed topology composed of four spacecraft was solve, where a stochastic variable associated with the link interconnection probability was used to model the information transmission of the spacecraft topology. Recently, taking into account the uncertain inertia matrix, a robust controller was designed under the directed communication topology composed of six spacecraft to make the attitude almost surely consensus in [21]. However, these works can only achieve attitude consensus but cannot converge to a desired attitude.

In this paper, an adaptive attitude consensus control strategy for multi-spacecraft systems in the presence of stochastic communication links failure and interference of environmental disturbances is proposed and studied. The spacecraft topology consists of a virtual leader and a group of followers, where the motion of the virtual leader is governed by a desired attitude and the followers are to reach consensus and track the desired attitude. Based on the sliding mode control and super-martingales convergence theorem, the designed controller not only achieves almost sure consensus, but also suppresses the external disturbances. In contrast to the existing works in [20,21], the proposed approach can handle a more complex communication topology and converge the attitude of all spacecraft to a specific desired attitude despite the existence of the environmental disturbances. Finally, numerical simulation results are given to verify the efficiency of the proposed attitude consensus controller.

The structure of this paper is organized as follows. In Section 2, preliminaries are introduced. In Section 3, the detailed problem is stated. In Section 4, the controller design and stability analysis are developed. In Section 5, an example case study is given to illustrate the effectiveness and performance of the proposed controller design method. Finally, conclusions are presented in Section 6.

2 Preliminaries

A. Notations

In the rest of this paper, the following notations are used. $\mathbb{R}$ represents the set of real numbers. $\mathbb{R}_+$ represents the set of positive real numbers. $\|\cdot\|$ is the standard Euclidean norm. If a square matrix D is symmetric positive definite, the matrix D is represented as $D > 0$. We define $D = diag(D_1, D_2, \cdots, D_n)$ as a block diagonal matrix with entries consisting of the matrices D_1, $D_2, \cdots$, D_n. I_n denotes an $n \times n$ identity matrix. $\otimes$ represents the Kronecker product. $\mathrm{sgn}(\cdot)$ denotes the sign function. We use $\mathrm{rank}(\cdot)$ to denote the rank of a matrix and use $\mathrm{Null}(\cdot)$ to denote the null space.

B. Spacecraft attitude dynamics

The unit-quaternion form of attitude dynamics is used to describe the multi-spacecraft systems. The attitude kinematics and dynamics of the i-th spacecraft can be expressed as:

$$\begin{aligned} \dot{q}_i &= Z(q_i)\omega_i \\ J_i\dot{\omega}_i &= S(\omega_i)J_i\omega_i + \tau_i + d_i, \end{aligned} \tag{1}$$

where the subscript $i \in \{1, 2, \cdots, n\}$ with n being the number of spacecraft in the multi-spacecraft system, $q_i = [q_{1i} \quad q_{2i} \quad q_{3i} \quad q_{4i}]^\top \in \mathbb{R}^4$ is the unit-quaternion of the body fixed frame $\mathfrak{B}$ relative to the inertial frame $\mathfrak{I}$ and satisfies the property $\|q_i\| = 1$, $\omega_i = [\omega_{xi} \quad \omega_{yi} \quad \omega_{zi}]^\top \in \mathbb{R}^3$ is angular velocity of $\mathfrak{B}$ relative to $\mathfrak{I}$, $Z(q_i) \in \mathbb{R}^{4\times3}$ is in the form of

$$Z(q_i) = \frac{1}{2}\begin{bmatrix} q_{4i} & -q_{3i} & q_{2i} \\ q_{3i} & q_{4i} & -q_{1i} \\ -q_{2i} & q_{1i} & q_{4i} \\ -q_{1i} & -q_{2i} & -q_{3i} \end{bmatrix},$$

$J_i \in \mathbb{R}^{3\times3} > 0$ is inertia matrix, $\tau_i \in [\tau_{xi} \quad \tau_{yi} \quad \tau_{zi}]^\top \in \mathbb{R}^3$ is the control torque, $d_i = [d_{xi} \quad d_{yi} \quad d_{zi}]^\top \in \mathbb{R}^3$ is the unknown but bounded disturbance torque, $S(\omega_i)$ is the skew-symmetric matrix that is defined as follows:

$$S(\omega_i) = \begin{bmatrix} 0 & \omega_{zi} & -\omega_{yi} \\ -\omega_{zi} & 0 & \omega_{xi} \\ \omega_{yi} & -\omega_{xi} & 0 \end{bmatrix}.$$

Assumption 1. The disturbance torque is bounded by $\|d_i\| \le d_{i,max}$, where $d_{i,max} \in \mathbb{R}_+$ is a positive constant.

C. Stochastic processes

A stochastic process $X = \{X(t), t \geq 0\}$ is to express the change in time of a stochastic phenomenon. The stochastic process can be described by the probability triple $(\Omega, \mathcal{F}, \mathbb{P})$, where Ω is a set of the events, $\mathcal{F}$ is a σ-algebra and belongs to a subspace of Ω, and $\mathbb{P}$ is the probability of event with $0 \leq \mathbb{P}\{\cdot\} \leq 1$ and $\mathbb{P}\{\Omega\} = 1$. Moreover, a filtration $\{\mathcal{F}_t, t \geq 0\}$ on $(\Omega, \mathcal{F}, \mathbb{P})$ is a set of sub σ-algebras of $\mathcal{F}$ and satisfies the following conditions:

$$\mathcal{F}_0 \subset \mathcal{F}_1 \subset \cdots \subset \mathcal{F}_{\mathrm{n}} \subset \cdots \subset \mathcal{F}.$$

If the stochastic process X is $\mathcal{F}_t$-measurable for all $t \geq 0$, then it is said to be adapted to the filtration $\{\mathcal{F}_t\}$ In addition, a process X is a super-martingale if the following conditions are satisfied [20]:

1) X is adapted to the filtration $\{\mathcal{F}_t\}$;
2) $\mathbb{E}\{|X(t)|\} < \infty, \forall t$;
3) $\mathbb{E}\{X(t) \mid \mathcal{F}_s\} \leq X(s), t > s$.

Now, it is ready to define the almost surely convergence of the stochastic variable $X(t)$ to a finite X_f if the following condition is satisfied:

$$\mathbb{P}\left\{\lim_{t\to\infty} X(t) = X_f\right\} = 1,$$

which is further equivalently defined as

$$\lim_{t\to\infty} X(t) \overset{\text{a.s}}{\to} X_f.$$

Then, the following super-martingales convergence lemma would be useful for the convergence analysis of stochastic variables.

Lemma 1. [22] If the stochastic process $X = \{X(t), t \geq 0\}$ is a nonnegative super-martingale, then there exists a finite X_f such that $\lim_{t\to\infty} X(t) \overset{\text{a.s}}{\to} X_f$.

3 Problem statement

As shown in Figure. 1, we assume that there are n spacecrafts in a multi-spacecraft system. The multi-spacecraft form a cyclic topology in which the i-th spacecraft follows the $(i+1)$-th one and spacecraft $i+1$ can obtain attitude information from spacecraft i, and the arrows in Figure. 1 indicate the information flow. It is worth noting that the $(n+1)$-th spacecraft is referred to spacecraft 1. In addition, we also assume that there is a virtual leader, i.e., spacecraft 0, whose attitude is denoted by $q_d \in \mathbb{R}^{4\times 1}$. Here, the virtual leader only transmits information to spacecraft 1 and does not receive feedback from spacecraft 1. A controller is designed for the n spacecrafts to achieve consensus and achieve the desired attitude transmitted by the virtual spacecraft.

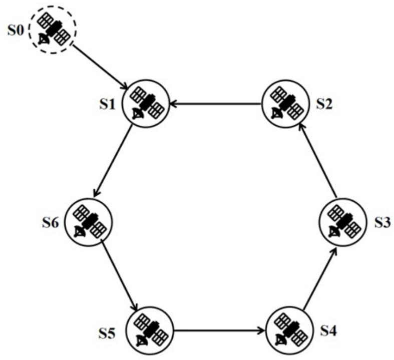


Fig. 1 The network topology of the multi-spacecraft systems

We suppose that the communication link between each two spacecrafts excluding the virtual leader is not deterministic. The parameter $p_i(t) \in [0,1]$ is used to describe the probability of information transferred from Spacecraft i to Spacecraft j. In addition, we define a stochastic binary variable $a_i(p)$ as follows:

$$a_i(p) = \begin{cases} 1 & \text{with probability } p_i(t) \\ 0 & \text{with probability } 1 - p_i(t) \end{cases}$$

It indicates that $a_i(p) = 0$ if there is no information transfer between two spacecrafts with probability $1 - p_i(t)$, while $a_i(p) = 1$ means that the communication link from spacecraft $i + 1$ to i is connected with probability $p_i(t)$.

The purpose of this paper is to design an attitude controller, so that the attitude of multi-spacecraft system subject to communication failure and external disturbances can achieve consensus and stabilize to a desired attitude with

$$\begin{aligned} &\lim_{t\to\infty}(q_i - q_d) \xrightarrow{a.s.} \mathbf{0}_{4\times1}, \\ &\lim_{t\to\infty}(q_i - q_j) \xrightarrow{a.s.} \mathbf{0}_{4\times1}, \\ &\lim_{t\to\infty}\omega_i \xrightarrow{a.s.} \mathbf{0}_{3\times1}, \end{aligned} \tag{2}$$

where $i, j \in \{1,2,\ldots,N\}$. It is worth noting that q_i and $-q_i$ stand for the same attitude.

4 Adaptive attitude consensus controller

In this section, we propose a control law such that the attitude of multi-spacecraft system is able to achieve attitude consensus under the communication topology as follows:

$$\tau_i = -\gamma Z(q_i)^\top a_i(p)(q_i - q_{i+1}) - k_i \,\mathrm{sgn}(\omega_i) - \hat{d}_{i,max}\frac{\omega_i}{\|\omega_i\|} - \beta K Z(q_1)^\top (q_1 - q_d) \tag{3}$$

with

$$\beta = \begin{cases} 1 & \text{if } i = 1 \\ 0 & \text{if } i \in \{2, \cdots, n\} \end{cases} \tag{4}$$

where $\gamma, K \in \mathbb{R}_+$ are positive constants, $q_d \in \mathbb{R}^{4\times 1}$ is the desired attitude of the virtual leader with the property that $\dot{q}_d = 0$, the constant $k_i \in \mathbb{R}_+$ is designed later, $\hat{d}_{i,max} \in \mathbb{R}$ is the estimate of the boundary of disturbance torque, which is obtained by

$$\dot{\hat{d}}_{i,max} = r\|\omega_i\|, \qquad i \in \{1,2,\dots,n\} \tag{5}$$

where the constant $r \in \mathbb{R}_+$ determines the convergence rate of the parameter estimate. In the above proposed attitude controller, the adaptive control term $-\hat{d}_{i,max}\frac{\omega_i}{\|\omega_i\|}$ is designed to counteract effects of the external disturbances, and the term $-\beta K Z(q_1)^\top (q_1 - q_d)$ is used to make the attitude point to the same desired attitude q_d.

The main result of this paper is summarized in the following theorem:

Theorem 1. Under the described stochastic communication topology, the attitude of multi-spacecraft systems can almost surely achieve consensus under the controller designed above, if the following conditions are satisfied:

$$k_i > \gamma, \quad p_i(t) \neq 0, \quad i \in \{1,2,\dots,N\}. \tag{6}$$

Proof. To prove the Theorem 1, a candidate Lyapunov function is constructed as follows:

$$\begin{aligned} V &= \frac{1}{2}\omega^\top J \omega + \frac{1}{2}\gamma q^\top (\mathcal{T}^\top \otimes I_4) q \\ &+ \frac{1}{2r}\sum_{i=1}^{N}(\hat{d}_{i,max} - d_{i,max})^2 + \frac{K}{2}(q_1 - q_d)^\top (q_1 - q_d) \end{aligned} \tag{7}$$

where $\omega = [\omega_1^\top, \omega_2^\top, \cdots, \omega_N^\top]^\top \in \mathbb{R}^{3N\times 1}$, $J = \mathrm{diag}(J_1, J_2, \cdots, J_N) \in \mathbb{R}^{3N\times 3N}$, $q = [q_1^\top, q_2^\top, \cdots, q_N^\top]^\top \in \mathbb{R}^{4N\times 1}$, and the variable $\mathcal{T}$ is defined as

$$\mathcal{T} = \begin{bmatrix} 1 & -1 & 0 & \dots & 0 \\ 0 & 1 & -1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & 0 & 0 & \dots & 1 \end{bmatrix}.$$

In the foregoing Lyapunov condition, the term $\frac{1}{2}\gamma q^\top (\mathcal{T}^\top \otimes I_4) q$ represents the attitude errors between two spacecrafts, the term $\frac{1}{2r}\sum_{i=1}^{N}(\hat{d}_{i,max} - d_{i,max})^2$ is the error between the boundary of the disturbance and its estimate, the last term $\frac{K}{2}(q_1 - q_d)^\top (q_1 - q_d)$ describes the error between the attitude and the desired attitude.

Then, the time derivation of V is calculated to be:

$$\dot{V} = \omega^\top J \dot{\omega} + \gamma q^\top (\mathcal{T}^\top \otimes I_4)\dot{q} + \frac{1}{r}\sum_{i=1}^{N}(\hat{d}_{i,max} - d_{i,max})\dot{\hat{d}}_{i,max} + K(q_1 - q_d)^\top \dot{q}_1. \tag{8}$$

Substituting the equation of motion of multi-spacecraft into above yields:

$$\begin{aligned} \dot{V} &= \omega^\top S(\omega) J \omega + \omega^\top \tau + \omega^\top d + \gamma q^\top (\mathcal{T}^\top \otimes I_4) Z(q)\omega \\ &+ \sum_{i=1}^{N}\big(\hat{d}_{i,max} - d_{i,max}\big)\|\omega_i\| + K(q_1 - q_d)^\top Z(q_1)\omega_1, \end{aligned} \tag{9}$$

where the stacked variables $Z(q) = diag(Z(q_1),\ Z(q_2),\ \cdots,\ Z(q_N)), S(\omega) = diag(S(\omega_1),\ S(\omega_2),\ \cdots,\ S(\omega_N))$, $\tau = [\tau_1^\top,\ \tau_2^\top, \cdots, \tau_N^\top]^\top$, and $d = [d_1^\top,\ d_2^\top,\ \cdots, d_N^\top]$ are used. Since $S(\omega)$ is skew symmetric, one can further have that:

$$\omega^\top S(\omega) J \omega = 0$$
$$\gamma q^\top (\mathcal{T}^\top \otimes I_4) Z(q)\omega = \gamma \omega^\top Z(q)^\top (\mathcal{T} \otimes I_4) q$$
$$K(q_1 - q_d)^\top Z(q_1)\omega_1 = K\omega_1^\top Z(q_1)^\top (q_1 - q_d).$$

Now, substituting the controller into $\dot{V}$ results in:

$$\begin{aligned}\dot{V} = -\gamma\omega^\top Z(q)^\top (\mathcal{L}(p) \otimes I_4) q - \omega^\top (k \otimes I_3)\,\mathrm{sgn}(\omega) - \sum_{i=1}^{N} \omega_i^\top \hat{d}_{i,max} \frac{\omega_i}{\|\omega_i\|} \\ -K\omega_1^\top Z(q_1)^\top (q_1 - q_d) + \omega^\top d + \gamma\omega^\top Z(q)^\top (\mathcal{T} \otimes I_4) q \\ + \textstyle\sum_{i=1}^{N} \big(\hat{d}_{i,max} - d_{i,max}\big)\|\omega_i\| + K\omega_1^\top Z(q_1)^\top (q_1 - q_d),\end{aligned} \tag{10}$$

where $k = \mathrm{diag}(k_1, k_2, \ldots, k_N)$ and a coupling matrix relating to the communication topology is

$$\mathcal{L}(p) = \begin{bmatrix} a_1(p) & -a_1(p) & 0 & \ldots & 0 \\ 0 & a_2(p) & -a_2(p) & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_N(p) & 0 & 0 & \ldots & a_N(p) \end{bmatrix}.$$

Then, the above equation can be rewritten as

$$\begin{aligned}\dot{V} = \gamma\omega^\top Z(q)^\top ((\mathcal{T} - \mathcal{L}(p)) \otimes I_4) q - \omega^\top (k \otimes I_3)\,\mathrm{sgn}(\omega) - \sum_{i=1}^{N} \hat{d}_{i,max}\|\omega_i\| \\ +\omega^\top d + \textstyle\sum_{i=1}^{N} (\hat{d}_{i,max} - d_{i,max})\|\omega_i\|.\end{aligned} \tag{11}$$

Now we define a vector $\mu = (\mu_1^\top,\ \mu_2^\top, \cdots, \mu_N^\top)^\top = \Big(\big(\mathcal{T} - \mathcal{L}(p)\big) \otimes I_4\Big) q \in \mathbb{R}^{4N\times 1}$, where

$$\mathcal{T} - \mathcal{L}(p) = \begin{bmatrix} 1 - a_1(p) & -1 + a_1(p) & 0 & \ldots & 0 \\ 0 & 1 - a_2(p) & -1 + a_2(p) & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 + a_N(p) & 0 & 0 & \ldots & 1 - a_N(p) \end{bmatrix}$$

Consequently, we have

$$\begin{aligned}\dot{V} = \gamma\omega^\top Z(q)^\top \mu - \omega^\top (k \otimes I_3)\,\mathrm{sgn}(\omega) + \omega^\top d - \sum_{i=1}^{N} d_{i,max}\|\omega_i\| \\ = \textstyle\sum_{i=1}^{N} (\gamma\omega_i^\top Z(q_i)^\top \mu_i - k_i\omega_i^\top\,\mathrm{sgn}(\omega_i) + \omega_i^\top d_i - d_{i,max}\|\omega_i\|).\end{aligned} \tag{12}$$

Since $a_i(p) \in \{0,1\}$, $\|q_i\| = 1$, and $\|Z(q_i)^\top Z(q_i)\|$=1/4, we can conclude that $\|Z(q_i)^\top \mu_i\| \le 1$. As a result, it is clear that

$$\dot{V} \le \textstyle\sum_{i=1}^{N} (\gamma\|\omega_i\| - k_i\omega_i^\top\,\mathrm{sgn}(\omega_i) + \omega_i^\top d_i - d_{i,max}\|\omega_i\|). \tag{13}$$

According to the assumption that $\|d_i\| \le d_{i,max}$ and $\|\omega_i\| \le \omega_i^\top\,\mathrm{sgn}(\omega_i)$, we have

$$\dot{V} \le \textstyle\sum_{i=1}^{N} (\gamma - k_i)\omega_i^\top\,\mathrm{sgn}(\omega_i). \tag{14}$$

From this inequality, we can see that as long as the condition $k_i > \gamma$ is satisfied, $\dot{V}$ is negative semidefinite. By employing the generalized invariance principle, it is obtained that:

$$\lim_{t\to\infty} \omega_i \equiv \mathbf{0}_{3\times 1}, i \in \{1,2,\ldots,N\} \tag{15}$$

Substituting the conclusion into the attitude dynamics model and controller leads to

$$\lim_{t\to\infty} Z(q_i)^\top a_i(p)(q_i - q_{i+1}) = \mathbf{0}_{3\times 1}, i \in \{2, \dots, N\} \tag{16}$$

Since $Z(q_i) \in \mathbb{R}^{4\times 3}$ and $Z(q_i)^\top Z(q_i) = \frac{1}{4} I_3$, it can be obtained that $\operatorname{rank}(Z(q_i)^\top) = 3$. Therefore, the following conclusion can be drawn:

$$a_i(p)(q_i - q_{i+1}) \in \operatorname{Null}(Z(q_i)^\top)$$

Moreover, $Z(q_i)$ satisfies $Z(q_i)^\top q_i = 0$. As a consequence, we can transform the above condition as follows:

$$\lim_{t\to\infty} a_i(p)(q_i - q_{i+1}) = \eta_i q_i \tag{17}$$

where η_i is a constant. Due to that $a_i(p)$ is uncertain, we make use of expectation to deal with the formula on both sides:

$$\lim_{t\to\infty} \mathbb{E}\{a_i(p)(q_i - q_{i+1})\} = \mathbb{E}\{\eta_i q_i\}. \tag{18}$$

According to the property of expectation, we further have

$$\lim_{t\to\infty} \mathbb{E}\{a_i(p)\}(q_i - q_{i+1}) = \eta_i q_i,$$

which results in

$$\lim_{t\to\infty} (\mathbb{E}\{a_i(p)\} - \eta_i) q_i - \mathbb{E}\{a_i(p)\} q_{i+1} = \mathbf{0}_{4\times 1} \tag{19}$$

Because of the definition of $a_i(p)$, we have

$$\mathbb{E}\{a_i(p)\} = 1 \times p_i + 0 \times (1 - p_i) = p_i$$

where $p_i(t)$ denotes the probability of the connection between agents. If $p_i(t) \neq 0$, we can obtain that:

$$\mathbb{E}\{a_i(p)\} \neq 0$$

Thus, we have:

$$\lim_{t\to\infty} ((1 - \mathbb{E}\{a_i(p)\}^{-1}\eta_i) q_i - q_{i+1}) = \mathbf{0}_{4\times 1}$$

It also can be rewritten as follows:

$$\lim_{t\to\infty} ((1 - \mathbb{E}\{a_i(p)\}^{-1}\eta_i) q_i^\top - q_{i+1}^\top) = \mathbf{0}_{1\times 4}$$

Multiplying $(1 - \mathbb{E}\{a_i(p)\}^{-1}\eta_i) q_i + q_{i+1}$ on both sides, it can be transformed into:

$$\lim_{t\to\infty} ((1 - \mathbb{E}\{a_i(p)\} - \eta_i)^2 q_i^\top q_i - q_{i+1}^\top q_{i+1}^\top) = \mathbf{0}_{1\times 4} \tag{20}$$

On account of unit quaternion vector satisfying the property $q_i^\top q_i = 1$ and $q_{i+1}^\top q_{i+1} = 1$, we have

$$\lim_{t\to\infty} (1 - \mathbb{E}\{a_i(p)\}^{-1}\eta_i)^2 = 1$$

Hence, we can obtain two situations based on above condition:

$$\lim_{t\to\infty} \eta_i = 0 \tag{21}$$

or

$$\lim_{t\to\infty} \eta_i = 2\mathbb{E}\{a_i(p)\} \tag{22}$$

Then, we analyze the two situations respectively:

1) For the first situation:

If η_i satisfies the equation $\lim_{t\to\infty} \eta_i = 0$, then we have:

$$\lim_{t\to\infty} \mathbb{P}\{\|q_i - q_{i+1}\| > \varepsilon\} = 0 \tag{23}$$

Now, we define filtration $\mathcal{F}_t = \{[q(\rho)^\top \omega(\rho)^\top], 0 < \rho < t\}$. The following three conditions can be obtained in the first situation:

a) For spacecraft $2, \cdots, N$, owing to that the external disturbance has been compensated, we regard Lyapunov candidate $V(t) = \frac{1}{2}\omega^\top J\omega + \frac{1}{2}\gamma q^\top (\mathcal{T}^\top \otimes I_4) q$ as a stochastic process. $V(t)$ is $\mathcal{F}_t$-measurable for each time t. Apart from $q(t)$ and $\omega(t)$, $V(t)$ is also determined by their history, so $V(t)$ only depends on $\{\mathcal{F}_s, 0 \leq s \leq t\}$. As a consequence, $V(t)$ is adapted to the filtration $\mathcal{F}_t$.

b) Because of time derivation of Lyapunov candidate $\dot{V}(t) \leq 0$, $V(t)$ is getting smaller. In addition,$q(t)$ and $\omega(t)$ are bounded, consequently, $V(t)$ is also bounded. Thus, we know that $\mathbb{E}\{V(t)\}$ is also bounded.

c) Due to that $\dot{V}(t) \leq 0$, we know $V(t) \leq V(s)$ if $t \geq s$. In view of the fact that $V(t)$ is measurable for each t and the property of expectation, we have:

$$\mathbb{E}\{V(t)|\mathcal{F}_s\} \leq V(s), t \geq s$$

As a consequence, the above conditions yield that $V(t)$ is super-martingale. Then, making use of the conclusion in Lemma 1, we know that

$$\lim_{t\to\infty} V(t) \to V_f \tag{24}$$

where V_f is a finite nonnegative real number. From the equation $V(t) = \frac{1}{2}\omega^\top J\omega + \frac{1}{2}\gamma q^\top(\mathcal{T}^\top \otimes I_4)q$ and $\lim_{t\to\infty} \omega_i = \mathbf{0}_{3\times1}$, it is clear that:

$$\lim_{t\to\infty} \frac{1}{2}\gamma q^\top(\mathcal{T}^\top \otimes I_4)q \xrightarrow{a.s.} V_f \tag{25}$$

Due to the fact that $\frac{1}{2}\gamma q^\top(\mathcal{T}^\top \otimes I_4)q$ is the attitude errors between spacecrafts and $\lim_{t\to\infty} \mathbb{P}\{\|q_i - q_{i+1}\| > \varepsilon\} = 0$,it follows that $V_f = 0$. Therefore, we have

$$\lim_{t\to\infty}(q_i - q_{i+1}) \overset{a.s.}{\to} 0, \forall i = 1, \cdots, n \tag{26}$$

It also can be said that:

$$\mathbb{P}\left\{\lim_{t\to\infty}(q_i - q_{i+1}) = 0\right\} = 1, \forall i = 1, \cdots, n \tag{27}$$

which implies that the attitude errors between spacecraft almost surely converge to zero.

2) For the second situation:

If η_i satisfies the equation $\lim_{t\to\infty} \eta_i = 2\mathbb{E}\{a_i(p)\}$, then we have:

$$\lim_{t\to\infty} \mathbb{P}\{\|q_i + q_{i+1}\| > \varepsilon\} = 0 \tag{28}$$

We also define a filtration $\mathcal{F}_t = \{[q(\rho)^\top \omega(\rho)^\top], 0 < \rho < t\}$. Similar to the first situation, the following three conditions can be obtained:

a) We regard Lyapunov candidate $\bar{V}(t) = \frac{1}{2}\omega^\top J\omega + \frac{1}{2}\gamma q^\top(M^\top \otimes I_4)q$ as a stochastic process with

$$M = \begin{bmatrix} 1 & 1 & 0 & \dots & 0 \\ 0 & 1 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \dots & 1 \end{bmatrix}.$$

$\bar{V}(t)$ is $\mathcal{F}_t$-measurable for each time t and is also adapted to the filtration $\mathcal{F}_t$.

b) $\mathbb{E}\{\bar{V}(t)\}$ is also bounded.

c) In addition, we can get that:

$$\mathbb{E}\{\bar{V}(t)|\mathcal{F}_s\} \leqslant \bar{V}(s), \ \ t \geq s.$$

As a consequence, the above conditions yield that $\bar{V}(t)$ is super-martingale. Then, make use of the conclusion in Theorem 2, we know that:

$$\lim_{t\to\infty} \bar{V}(t) \to \bar{V}_F, \tag{29}$$

where V_F is a finite nonnegative real number. Combined with the Lyapunov candidate and the conclusion $\lim_{t\to\infty} \omega_i = \mathbf{0}_{3\times1}$, it yields that:

$$\lim_{t\to\infty}\frac{1}{2}\gamma q^{\top}(M^{\top}\otimes I_4)q \xrightarrow{a.s.} \bar{V}_F. \tag{30}$$

Due to the fact that $\frac{1}{2}\gamma q^{\top}(M^{\top}\otimes I_4)q$ is with respect to (q_i+q_{i+1}), $\forall i=1,\cdots,n$ and $\lim_{t\to\infty}\mathbb{P}\{\|q_i+q_{i+1}\|>\varepsilon\}=0$, it follows that $\bar{V}_F=0$. Therefore, we have

$$\lim_{t\to\infty}(q_i+q_{i+1}) \overset{a.s.}{\to} 0, \forall i=1,\cdots,n. \tag{31}$$

It also can be said that:

$$\mathbb{P}\left\{\lim_{t\to\infty}(q_i+q_{i+1})=0\right\}=1, \forall i=1,\cdots,n \tag{32}$$

which implies that the attitude errors between spacecrafts almost surely converge to zero. It means that spacecraft $i+1$ can transfer information to spacecraft i, and spacecraft i is capable of tracking spacecraft $i+1$, resulting in attitude consensus.

Since $Z(q_1)$ satisfies the property $Z(q_1)^{\top}q_1=0$, we have:

$$\lim_{t\to\infty}Z(q_1)^{\top}q_d=\mathbf{0}_{3\times1}. \tag{33}$$

Hence, we know that $\mathrm{Null}(Z(q_1)^{\top})=\bar{\eta}_1 q_d$ with $\bar{\eta}_1$ being a constant. As a result, we obtain that as $t\to\infty$, we have

$$q_d=\eta_1 q_1.$$

Since $\|q_d\|=1$ and $\|q_1\|=1$, we can further obtain $\eta_1=1$. Therefore, it is clear that

$$\lim_{t\to\infty}(q_1-q_d)\to\mathbf{0}_{4\times1}. \tag{34}$$

Moreover, since $\lim_{t\to\infty}(q_i-q_{i+1}) \overset{a.s.}{\to} 0$, $\forall i=1,\cdots,n$, it is obtained that

$$\lim_{t\to\infty}(q_i-q_d) \overset{a.s.}{\to} \mathbf{0}_{4\times1}, \forall i=1,\cdots,n. \tag{35}$$

This implies that the attitude errors between spacecraft and virtual spacecraft almost surely converge to zero, and all spacecraft are capable of achieving the desired attitude almost surely. This completes the proof.

5 Simulation results

In this section, we carry out numerical simulation of a multi-spacecraft system including 6 spacecraft in a ring topology to verify the effectiveness of the proposed adaptive sliding mode control law and analyze its performance. For each spacecraft in the system, the inertia matrix and external disturbances are assumed as follows:

$$J_i=\begin{bmatrix}60 & 0 & -5\\ 0 & 65 & 0\\ -5 & 0 & 70\end{bmatrix}\mathrm{kg}\cdot\mathrm{m}^2,\ \ i\in\{1,2,\cdots,6\}$$

$$d_i=10^{-2}\begin{pmatrix}-1+3\sin\left(0.1t+\dfrac{\pi}{2}\right)+4\sin(0.03t)\\ 1.5-1.5\sin(0.02t)-3\sin(0.05t+\dfrac{\pi}{2})\\ 1+2\sin(0.1t)-1.5\sin(0.04t+\dfrac{\pi}{2})\end{pmatrix},\ \ i\in\{1,2,\cdots,6\}$$

The probabilities of information transmission between six spacecraft and a virtual spacecraft are shown in Figure 2. The initial attitudes and initial angular velocities are illustrated in Table 1. For the proposed controller, the control gains are selected with $\gamma = 65$, $K = 110$, $k_i = 65$, $r = 0.02, i \in \{1,2,\ldots,6\}$. The desired attitude of the virtual is set as $q_d = [0.1105, -0.468, -0.854, 0.1433]^{\top}$. In the following, two simulation cases are considered.

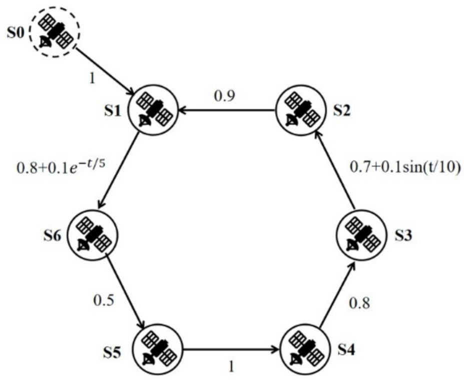


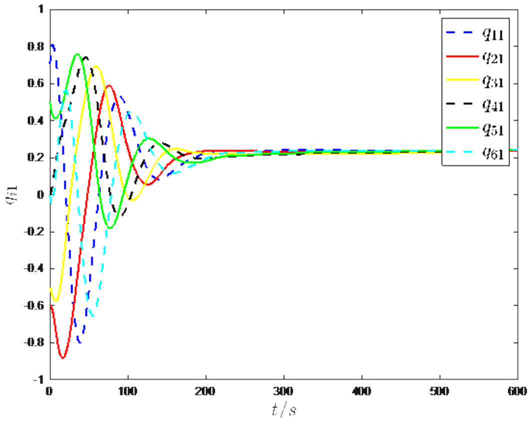
Fig. 2 The probabilities of the connectivity of the multi-spacecraft systems

Table 1. Initial condition of each spacecraft.

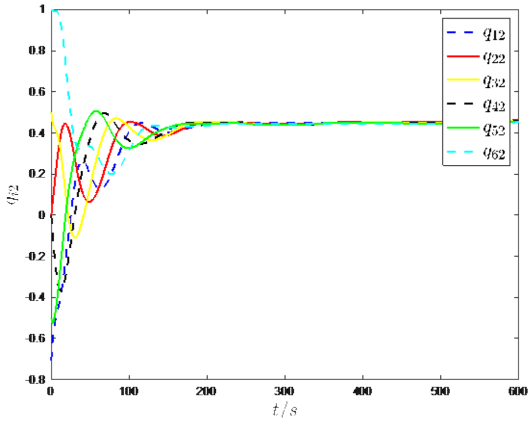
spacecraft	${q_i}^{\top}$	${\omega_i}^{\top}$[rad/s]
S1	$[\sqrt{2}/2 \ \ -\sqrt{2}/2 \ \ 0 \ \ 0]$	$[0.1 \ \ -0.05 \ \ 0.2]$
S2	$[-0.6 \ \ 0 \ \ 0 \ \ -0.8]$	$[0 \ \ 0.06 \ \ 0.2]$
S3	$[-0.5 \ \ 0.5 \ \ 0.5 \ \ 0.5]$	$[-0.02 \ \ -0.05 \ \ -0.1]$
S4	$[0 \ \ 0 \ \ -1 \ \ 0]$	$[0.1 \ \ 0 \ \ 0.04]$
S5	$[0.5 \ \ -0.5 \ \ -0.5 \ \ 0.5]$	$[-0.01 \ \ -0.03 \ \ 0.1]$
S6	$[0 \ \ 1 \ \ 0 \ \ 0]$	$[0 \ \ -0.1 \ \ -0.1]$

- *Case 1: Attitude consensus of multi-spacecraft system without a virtual leader*

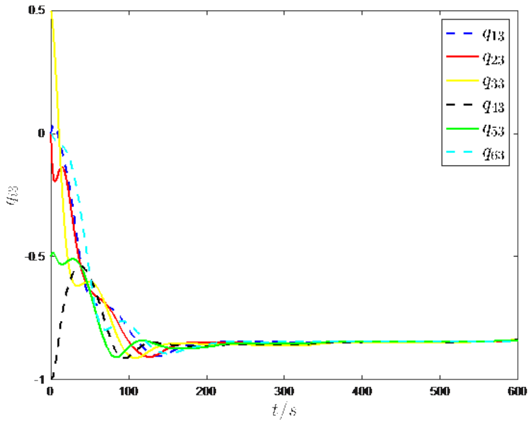
For comparison, the attitude controller in [21] are also simulated. To have a fair comparison, the leaderless attitude consensus of the multi-spacecraft system is considered in Case 1. As shown in Figure 3 and Figure 4, the attitude errors among spacecrafts under the proposed controller can gradually converge to the desired attitude with a high accuracy, while the steady-state consensus errors under the existing controller in [21] are much worse than that under our proposed approach. This is due to the fact that the existing controller in [21] does not consider external disturbances in the design process, and hence has limited robustness. The time history of angular velocity and norm of the commanded control torque of each spacecraft in the topology under the two controllers are depicted in Figure 5 and Figure 6, from which it is clear that the proposed attitude consensus control approach achieves better angular velocity convergence accuracy than that of the existing control approach in [21].



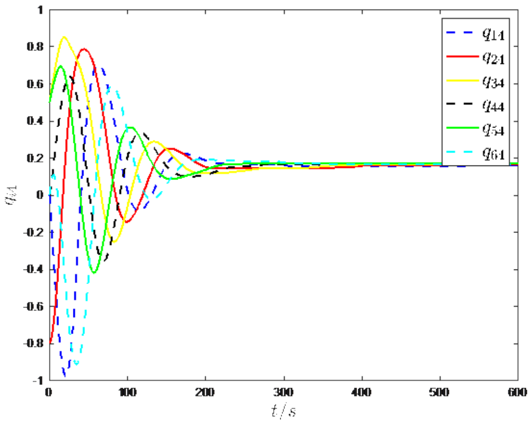
(a) Attitude q_{i1}



(b) Attitude q_{i2}

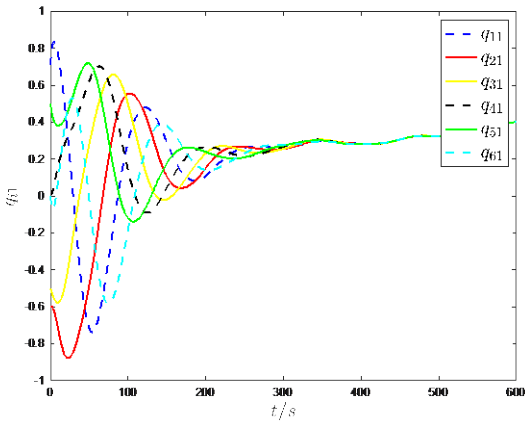


(c) Attitude q_{i3}

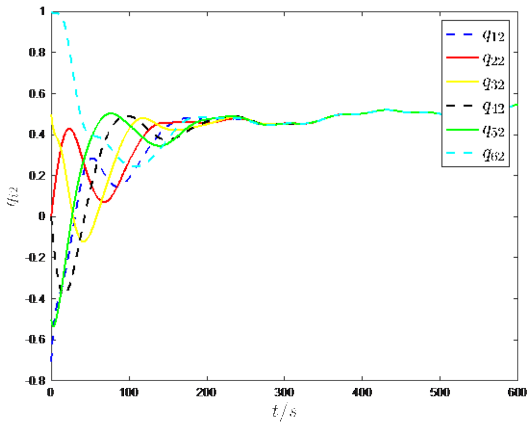


(d) Attitude q_{i4}

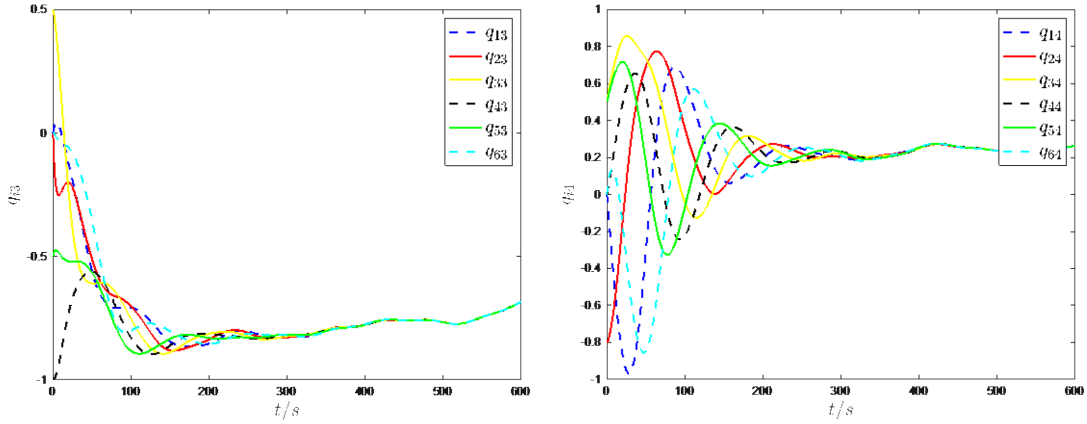
Figure 3. Attitude under the proposed attitude controller



(a) Attitude q_{i1}

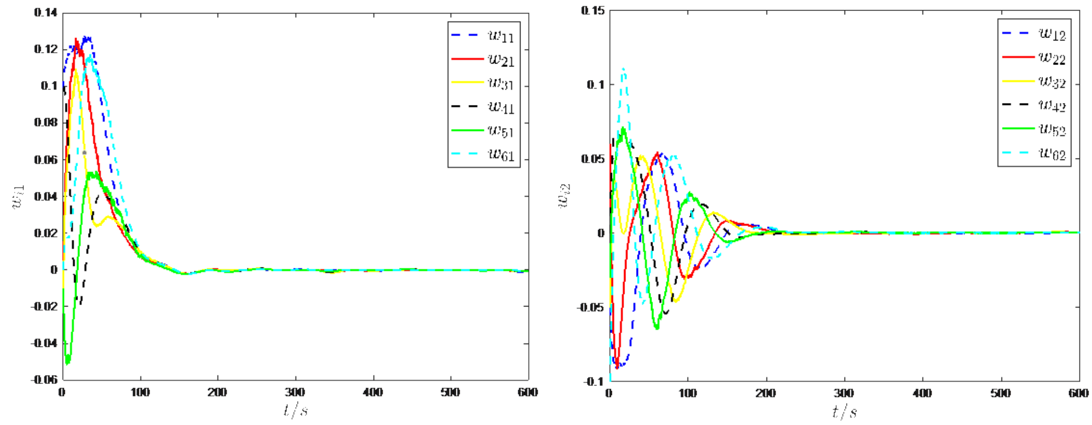


(b) Attitude q_{i2}

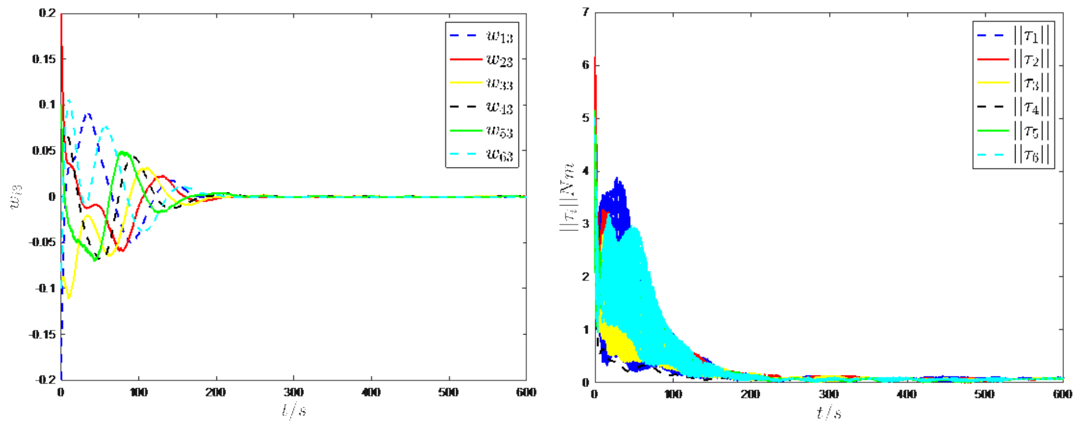


(c) Attitude q_{i3} (d) Attitude q_{i4}

Figure 4. Attitude under the existing controller in [21]

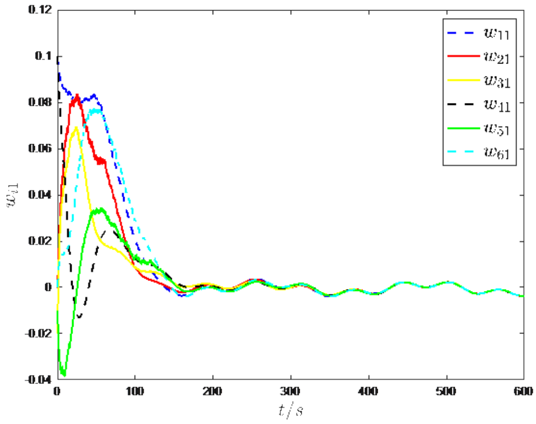


(a) Angular velocity ω_{i1} (b) Angular velocity ω_{i2}

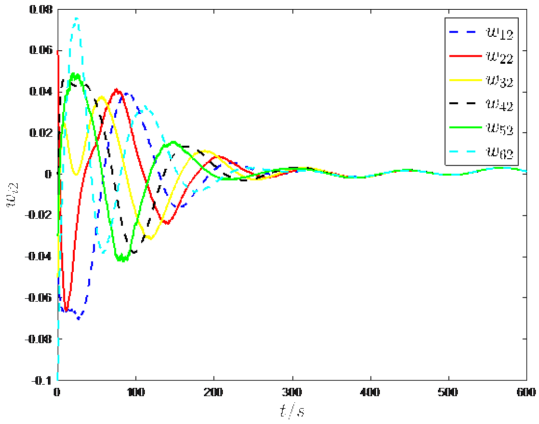


(c) Angular velocity ω_{i3} (d) Norms of commanded control torque $\|\tau_i\|$

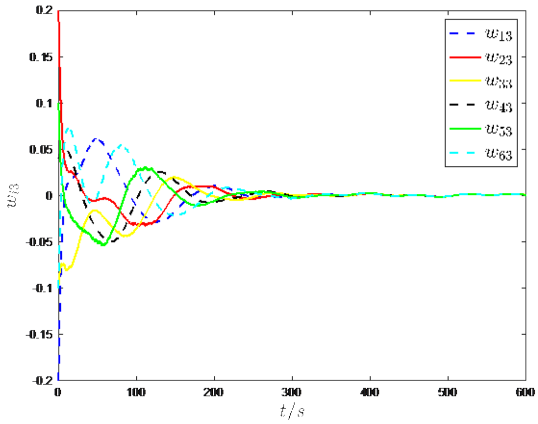
Figure 5. Angular velocities and norms of commanded control inputs under the proposed attitude controller



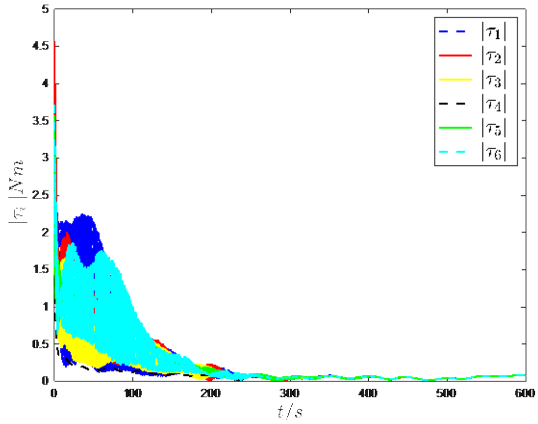
(a) Angular velocity ω_{i1}



(b) Angular velocity ω_{i2}



(c) Angular velocity ω_{i3}

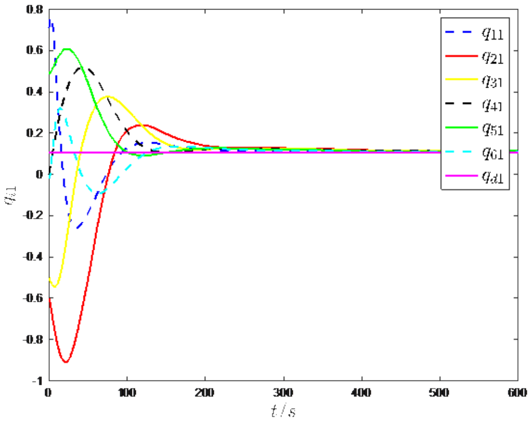


(d) Norms of commanded control torque $\|\tau_i\|$

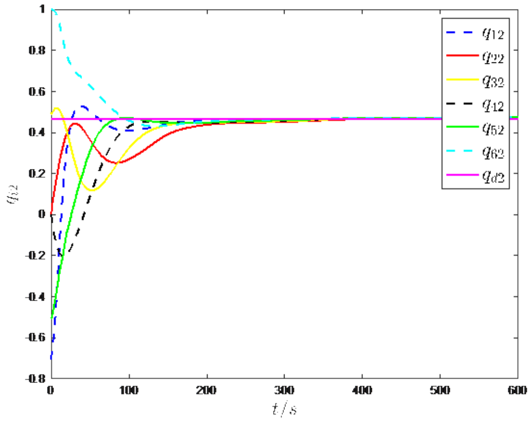
Figure 6. Angular velocities and norms of commanded control inputs under the existing controller in [21]

- *Case 2: Attitude consensus of multi-spacecraft system with a virtual leader*

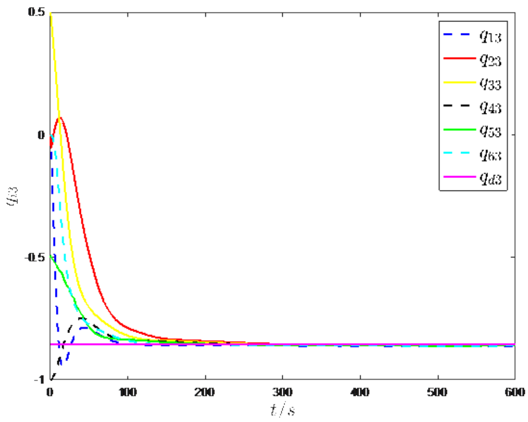
In this case, we consider the spacecraft topology that there is a virtual leader connected with the spacecraft 1, as the topology shown in Figure 2. Under this spacecraft topology, the time history of the attitude with each spacecraft is shown in Figure 7. We can see that the proposed controller can ensure the asymptotic convergence of attitude errors between follower spacecrafts and the virtual spacecraft. Figure 8 show the angular velocity and the norms of the commanded control torques of each spacecraft. With proposed controller, we can see that the angular velocity of the multi-spacecraft system tends to be stable over time. In the simulation, the control torque curve of each follower spacecraft is shown. In addition, it can be observed that the control torque is only large in the initial stage and there is a certain oscillation. When the attitude consensus errors become small, the control torque can be kept in a small range. Due to the physical limitation of actuators, we add a constraint that the control input torque cannot exceed $10\ N \cdot m$ in the simulation process.



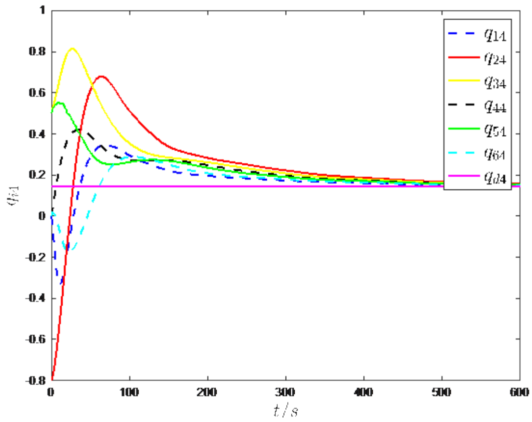
(a) Attitude q_{i1}



(b) Attitude q_{i2}

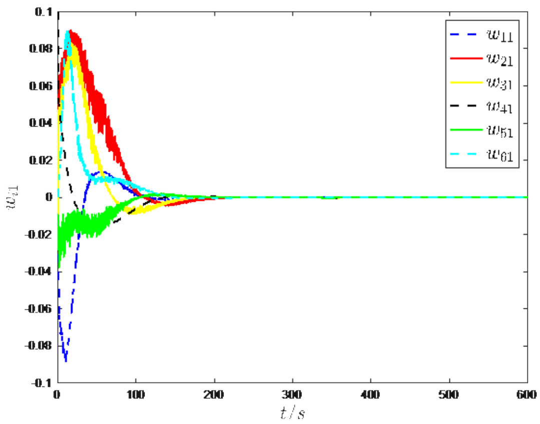


(c) Attitude q_{i3}

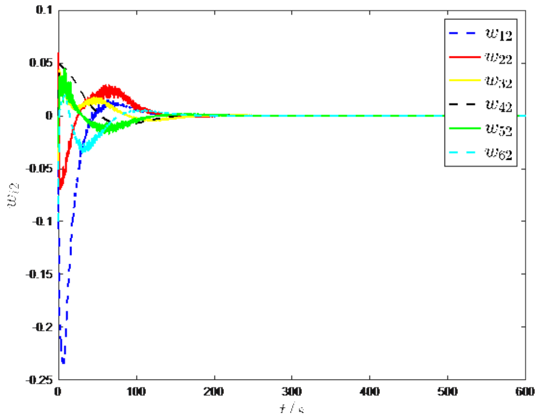


(d) Attitude q_{i4}

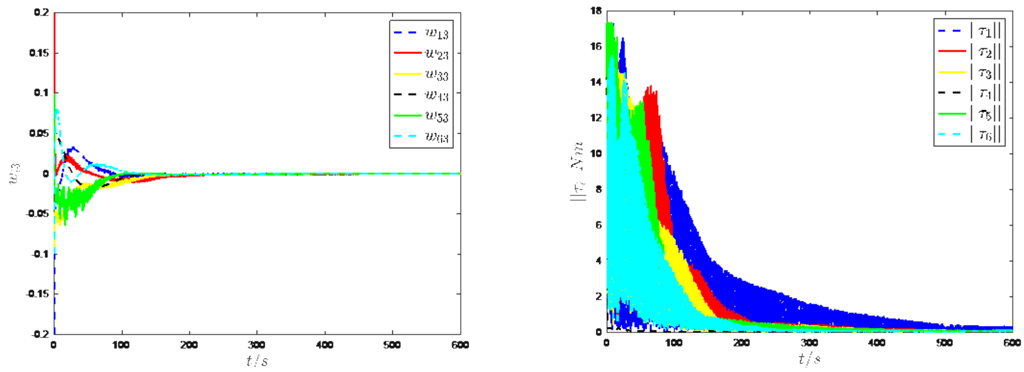
Figure 7. Attitude under the proposed attitude controller when there is a virtual leader in the topology



(a) Angular velocity ω_{i1}



(b) Angular velocity ω_{i2}



(c) Angular velocity ω_{i3} (d) Norms of commanded control torque $\|\tau_i\|$

Figure 8. Angular velocity and norms of the commanded control torque under the proposed attitude controller when there is a virtual leader in the topology

6 Conclusion and future work

An adaptive attitude consensus control strategy is proposed for multi-spacecraft systems subject to stochastic communication links failure and environmental disturbances. The spacecraft topology of interest is made up with multiple follower spacecraft and a virtual leader. Taking advantages of the sliding mode control technique and super-martingales convergence theorem, the proposed adaptive attitude controller guarantees that the attitudes of all follower spacecraft almost surely converge to a desired attitude in spite of the disturbances and indeterministic information exchange channels. Simulation results for a group of six spacecraft with a virtual leader is shown to validate the proposed attitude consensus controller with an improved performance. In the future, inertia uncertainties and a more general communication topology can be incorporated in the adaptive stochastic attitude consensus controller design.

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