

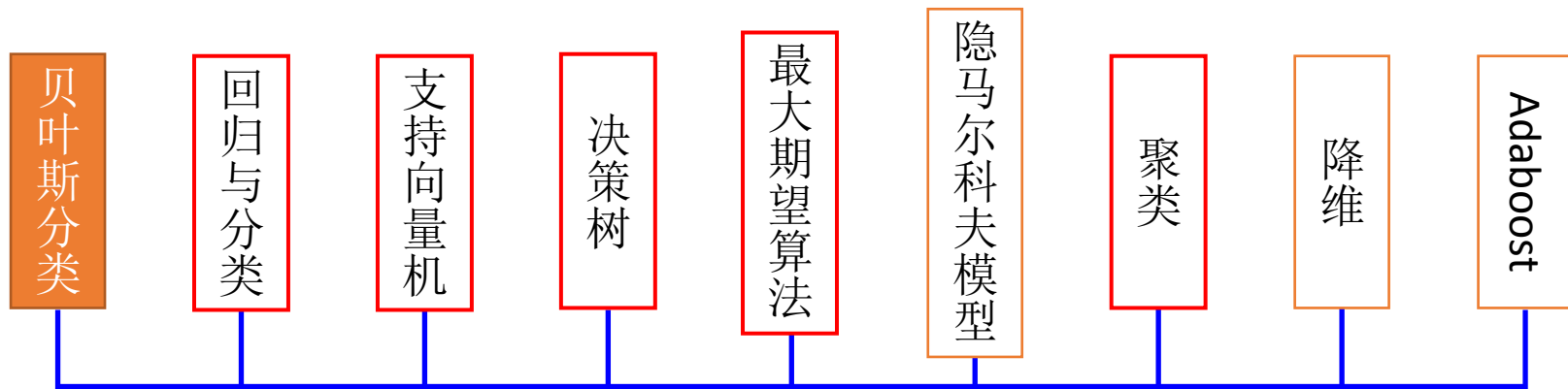
# 贝叶斯分类

——贝叶斯决策与朴素贝叶斯



主讲人 霍博士





9大算法

## 机器学习

矩阵论

概率论

优化

垃圾邮件分类

广告分类

文档分类

食品分类

客户分类

.....

识别=分类?

分类  
问题

解决  
方案



绳子?  
柱子?  
蒲扇?

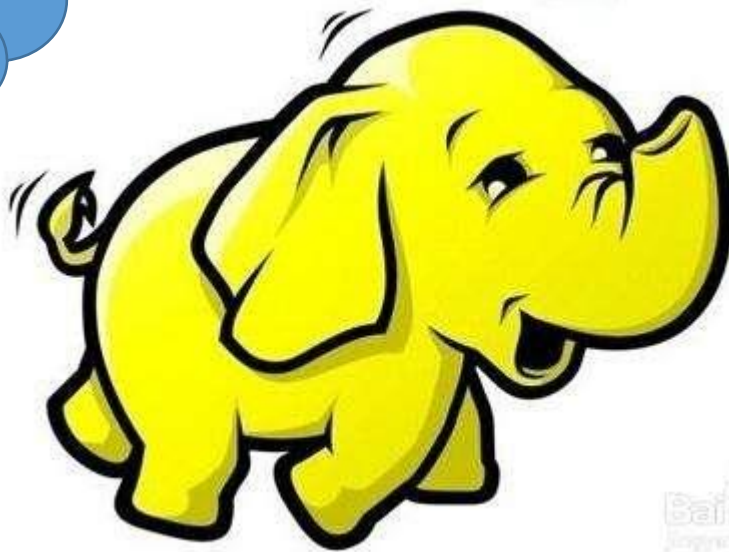
.....

What's this?

$p(\text{绳子}|\text{细长})$

$p(\text{柱子}|\text{粗壮})$

$p(\text{蒲扇}|\text{宽阔})$



Hadoop?

$p(\text{Hadoop}|\text{logo})$

$p(\text{大象}|\text{尾巴} = \text{细长}, \text{腿} = \text{粗壮}, \text{耳朵} = \text{宽阔}, \dots)$

- ✓ 贝叶斯决策理论基础
- ✓ 朴素贝叶斯分类器
- ✓ 鸢尾花分类实践

## 条件概率

- $P(B|A) = \frac{P(AB)}{P(A)}$

## 全概率公式 $\bigcup_{i=1}^n B_i = \Omega$

- $P(A) = P(A|B_1)P(B_1) + \dots + P(A|B_n)P(B_n)$

$$= \sum_{i=1}^n P(A|B_i)P(B_i)$$

## 贝叶斯公式

- $P(B_i|A) = \frac{P(A|B_i)P(B_i)}{P(A)} = \frac{P(A|B_i)P(B_i)}{\sum_{j=1}^n P(A|B_j)P(B_j)}$

# 贝叶斯决策理论基础

样本  $\mathbf{x}$

类别集合  $Y = \{c_1, c_2, \dots, c_K\}$

条件风险(Conditional Risk)

$$Risk(c_i|\mathbf{x}) = \sum_{j=1}^K \lambda_{ij} P(c_j|\mathbf{x})$$

$\lambda_{ij}$ : 将一个真实类别为  $c_j$  的样本误分为  $c_i$  产生的期望损失

贝叶斯风险

$$R(f) = E_{\{\mathbf{x}\}}[Risk(f(\mathbf{x})|\mathbf{x})]$$

**贝叶斯判定准则(Bayesian Decision Rule)**

为最小化总体风险，只需在每个样本上选择能使条件风险最小的类别标记

贝叶斯最优分类器

$$f^*(\mathbf{x}) = \underset{c \in Y}{\operatorname{argmin}} R(c|\mathbf{x})$$

# 贝叶斯决策理论基础

## 条件风险(Conditional Risk)

$$Risk(c_i|\mathbf{x}) = \sum_{j=1}^K \lambda_{ij} P(c_j|\mathbf{x})$$

若目标函数是最小化分类错误率, 则

$$\lambda_{ij} = \begin{cases} 0, & \text{if } i = j \\ 1, & \text{otherwise} \end{cases}$$

$$Risk(c|\mathbf{x}) = 1 - P(c|\mathbf{x})$$

## 贝叶斯最优分类器

$$f^*(\mathbf{x}) = \operatorname{argmin}_{c \in Y} R(c|\mathbf{x})$$

$$f^*(\mathbf{x}) = \operatorname{argmax}_{c \in Y} \{P(c|\mathbf{x})\}$$

后验概率

最小错误率的贝叶斯决策=选择具有最高概率的决策



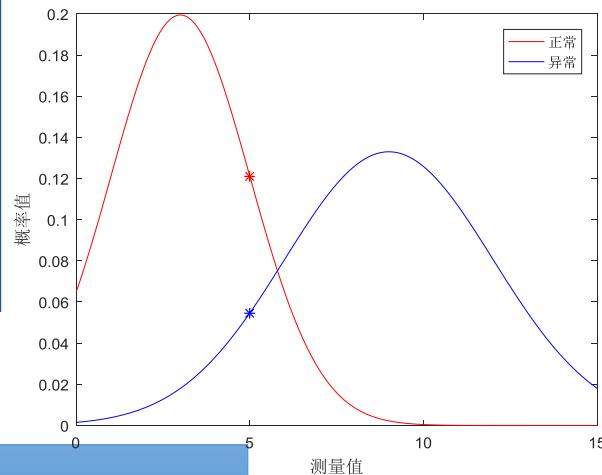
# 贝叶斯决策理论基础

例 某体能指标正常和异常两类的先验概率分别为：

$$P(\text{正常}) = 0.95, P(\text{异常}) = 0.05.$$

小张的该项指标测量值 $x = 5$ ，由每类的条件概率密度分布曲线可得， $P(x = 5|\text{正常}) = 0.12$ ， $P(x = 5|\text{异常}) = 0.05$ 。  
小张的该项体能指标正常吗？

解：判断 $P(\text{正常}|x = 5)$ 与 $P(\text{异常}|x = 5)$ 的大小。  
利用贝叶斯公式分别计算正常和异常的后验概率：



## 先验起主导作用

因此， $P(\text{正常}|x = 5) > P(\text{异常}|x = 5)$ ，根据贝叶斯决策规则，合理的决策是把小张该项体能指标归类于正常。

条件风险(Conditional Risk)

$$Risk(c_i|\mathbf{x}) = \sum_{j=1}^K \lambda_{ij} P(c_j|\mathbf{x})$$

贝叶斯风险

$$R(f) = E_{\{\mathbf{x}\}}[Risk(f(\mathbf{x})|\mathbf{x})]$$

贝叶斯最优分类器

$$f^*(\mathbf{x}) = \underset{c \in Y}{\operatorname{argmin}} Risk(c|\mathbf{x})$$

最小风险的贝叶斯决策

# 贝叶斯决策理论基础

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小张的该项体能指标正常吗？

决策 \ 状态 损失	正常	异常
	0	10
正常		
异常	1	0

解：后验概率： $P(\text{正常}|x = 5) \approx 0.98$ ， $P(\text{异常}|x = 5) \approx 0.02$

条件风险： $R(\text{正常}|x = 5) = \lambda_{11}P(\text{正常}|x = 5) + \lambda_{12}P(\text{异常}|x = 5) \approx 0.2$ ，

$R(\text{异常}|x = 5) = \lambda_{21}P(\text{正常}|x = 5) + \lambda_{22}P(\text{异常}|x = 5) \approx 0.98 > R(\text{正常}|x = 5)$

因此，  
指标归

“损失”起主导作用

项体能

# 贝叶斯决策理论基础

生成式模型

$$P(c|\mathbf{x}) = \frac{P(\mathbf{x}, c)}{P(\mathbf{x})} \xrightarrow{\text{贝叶斯定理}} = \frac{P(c)P(\mathbf{x}|c)}{P(\mathbf{x})} \propto P(c)P(\mathbf{x}|c)$$

先验概率

条件概率/似然

归一化因子

样本空间中各类样本所占的比例:  $P(c) \xrightarrow{\text{大数定律}} \frac{N_c}{N}$

$P(\mathbf{x}|c)$ ?

# 贝叶斯决策理论基础



“未被观测到”

$\neq$

“出现概率为0”

- ✓ 贝叶斯决策理论基础
- ✓ 朴素贝叶斯分类器
- ✓ 鸢尾花分类实践

# 朴素贝叶斯分类器

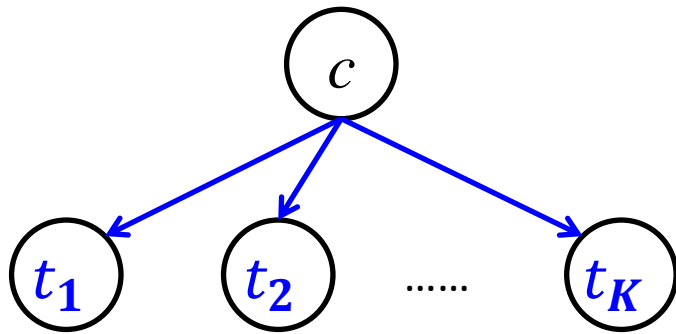
$$P(c|\mathbf{x}) \propto P(c)P(\mathbf{x}|c)$$

$\mathbf{x} = (t_1, t_2, \dots, t_K)$

属性条件独立性假设

MAP?

$$P(c|\mathbf{x}) \propto P(c) \prod_{i=1}^K P(t_i|c)$$



朴素贝叶斯分类器  
(Naïve Bayesian Classifier)

$$f_{nbc}(\mathbf{x}) = \operatorname{argmax}_{c \in Y} \left\{ P(c) \prod_{i=1}^K P(t_i|c) \right\}$$

# 朴素贝叶斯分类器

朴素贝叶斯分类器  
(Naïve Bayesian Classifier)

$$f_{nbc}(\mathbf{x}) = \operatorname{argmax}_{c \in Y} \left\{ P(c) \prod_{i=1}^K P(t_i|c) \right\}$$

样本集合  $X = \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N\}$ , 类别集合  $Y = \{c_1, c_2, \dots, c_K\}$

$$P(c) = \frac{N_c}{N}$$

$$P(t_i|c) = \frac{N_{(c,t_i)}}{N_c}$$

$N_{(c,t_i)}$  表示第  $c$  类中在属性取值为  $t_i$  的样本个数



# 朴素贝叶斯分类器

西瓜数据集 SL1.0

编号	色泽	根蒂	敲声	纹理	脐部	触感	好瓜
1	青绿	蜷缩	沉闷	稍糊	稍凹	硬滑	否
2	青绿	蜷缩	沉闷	清晰	凹陷	硬滑	否
3	青绿	蜷缩	清脆	清晰	凹陷	硬滑	是
4	青绿	蜷缩	沉闷	稍糊	稍凹	硬滑	否
5	青绿	蜷缩	沉闷	稍糊	稍凹	硬滑	否
6	青绿	蜷缩	清脆	清晰	凹陷	硬滑	是
7	青绿	蜷缩	清脆	清晰	凹陷	硬滑	是
8	青绿	蜷缩	沉闷	稍糊	稍凹	硬滑	否
9	青绿	蜷缩	沉闷	稍糊	稍凹	硬滑	否
10	青绿	蜷缩	沉闷	稍糊	稍凹	硬滑	否
11	青绿	蜷缩	沉闷	稍糊	稍凹	硬滑	否
12	青绿	蜷缩	沉闷	稍糊	稍凹	硬滑	否
13	青绿	蜷缩	沉闷	稍糊	稍凹	硬滑	否
14	青绿	蜷缩	沉闷	稍糊	稍凹	硬滑	否
15	青绿	蜷缩	沉闷	稍糊	稍凹	硬滑	否
16	青绿	蜷缩	沉闷	稍糊	稍凹	硬滑	否
17	青绿	蜷缩	沉闷	稍糊	稍凹	硬滑	否

- For each training sample
  - If 样本标签==c\_i
    - c\_i类别样本个数增加1
    - For each feature t\_i
      - 如果特征t\_i出现在样本中, 该特征t\_i在c\_i类别出现的次数加1
- For each class
  - For each feature
    - 将该特征在该类别出现的次数除以该类别样本个数
- 返回每个类别的条件概率 $P(t_i|c_i)$

$$P_{\text{青绿}|\text{是}} = P(\text{色泽} = \text{青绿} | \text{好瓜} = \text{是}) = \frac{3}{8} = 0.375,$$

$$P_{\text{青绿}|\text{否}} = P(\text{色泽} = \text{青绿} | \text{好瓜} = \text{否}) = \frac{3}{9} \approx 0.333,$$

$$P_{\text{硬滑}|\text{否}} = P(\text{触感} = \text{硬滑} | \text{好瓜} = \text{否}) = \frac{6}{9} \approx 0.667,$$

测0 青绿 蜷缩 浊响 清晰 凹陷 硬滑

# 朴素贝叶斯分类器

$$P_{\text{青绿}|\text{是}} = P(\text{色泽} = \text{青绿} | \text{好瓜} = \text{是}) = \frac{3}{8} = 0.375,$$

$$P_{\text{青绿}|\text{否}} = P(\text{色泽} = \text{青绿} | \text{好瓜} = \text{否}) = \frac{3}{9} \approx 0.333,$$

$$P_{\text{蜷缩}|\text{是}} = P(\text{根蒂} = \text{蜷缩} | \text{好瓜} = \text{是}) = \frac{5}{8} = \mathbf{0.625}$$

$$P_{\text{蜷缩}|\text{否}} = P(\text{根蒂} = \text{蜷缩} | \text{好瓜} = \text{否}) = \frac{3}{9} \approx 0.333,$$

$$P_{\text{浊响}|\text{是}} = P(\text{敲声} = \text{浊响} | \text{好瓜} = \text{是}) = \frac{6}{8} = 0.750,$$

$$P_{\text{浊响}|\text{否}} = P(\text{敲声} = \text{浊响} | \text{好瓜} = \text{否}) = \frac{4}{9} \approx 0.444,$$

$$P_{\text{清晰}|\text{是}} = P(\text{纹理} = \text{清晰} | \text{好瓜} = \text{是}) = \frac{7}{8} = 0.875,$$

$$P_{\text{清晰}|\text{否}} = P(\text{纹理} = \text{清晰} | \text{好瓜} = \text{否}) = \frac{2}{9} \approx 0.222,$$

$$P_{\text{凹陷}|\text{是}} = P(\text{脐部} = \text{凹陷} | \text{好瓜} = \text{是}) = \frac{6}{8} = 0.750,$$

$$P_{\text{凹陷}|\text{否}} = P(\text{脐部} = \text{凹陷} | \text{好瓜} = \text{否}) = \frac{2}{9} \approx 0.222,$$

$$P_{\text{硬滑}|\text{是}} = P(\text{触感} = \text{硬滑} | \text{好瓜} = \text{是}) = \frac{6}{8} = 0.750,$$

$$P_{\text{硬滑}|\text{否}} = P(\text{触感} = \text{硬滑} | \text{好瓜} = \text{否}) = \frac{6}{9} \approx 0.667,$$

周志华, 机器学习, P84,152

$$P(\text{好瓜} = \text{是}) = \frac{8}{17} \approx 0.471,$$

$$P(\text{好瓜} = \text{否}) = \frac{9}{17} \approx 0.529,$$

测0	青绿	蜷缩	浊响	清晰	凹陷	硬滑
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$$P(\text{好瓜} = \text{是}) \times P_{\text{青绿}|\text{是}} \times P_{\text{蜷缩}|\text{是}} \times P_{\text{浊响}|\text{是}} \\ \times P_{\text{清晰}|\text{是}} \times P_{\text{凹陷}|\text{是}} \times P_{\text{硬滑}|\text{是}} \approx 0.041$$

$$P(\text{好瓜} = \text{否}) \times P_{\text{青绿}|\text{否}} \times P_{\text{蜷缩}|\text{否}} \times P_{\text{浊响}|\text{否}} \\ \times P_{\text{清晰}|\text{否}} \times P_{\text{凹陷}|\text{否}} \times P_{\text{硬滑}|\text{否}} \\ \approx 8.562e-4$$

0.041 > 8.562e-4  $\xrightarrow{\text{朴素贝叶斯}}$  好瓜

# 朴素贝叶斯分类器

表 4.3 西瓜数据集 3.0

编号	色泽	根蒂	敲声	纹理	脐部	触感	密度	含糖率	好瓜
1	青绿	蜷缩	浊响	清晰	凹陷	硬滑	0.697	0.460	是
2	乌黑	蜷缩	沉闷	清晰	凹陷	硬滑	0.774	0.376	是
3	乌黑	蜷缩	浊响	清晰	凹陷	硬滑	0.634	0.264	是
4	青绿	蜷缩	沉闷	清晰	凹陷	硬滑	0.608	0.318	是
5	浅白	蜷缩	浊响	清晰	凹陷	硬滑	0.556	0.215	是
6	青绿	稍蜷	浊响	清晰	稍凹	软粘	0.403	0.237	是
7	乌黑	稍蜷	浊响	稍糊	稍凹	软粘	0.481	0.149	是
8	乌黑	稍蜷	浊响	清晰	稍凹	硬滑	0.437	0.211	是
9	乌黑	稍蜷	沉闷	稍糊	稍凹	硬滑	0.666	0.091	否
10	青绿	硬挺	清脆	清晰	平坦	软粘	0.243	0.267	否
11	浅白	硬挺	清脆	模糊	平坦	硬滑	0.245	0.057	否
12	浅白	蜷缩	浊响	模糊	平坦	软粘	0.343	0.099	否
13	青绿	稍蜷	浊响	稍糊	凹陷	硬滑	0.639	0.161	否
14	浅白	稍蜷	沉闷	稍糊	凹陷	硬滑	0.657	0.198	否
15	乌黑	稍蜷	浊响	清晰	稍凹	软粘	0.360	0.370	否
16	浅白	蜷缩	浊响	模糊	平坦	硬滑	0.593	0.042	否
17	青绿	蜷缩	沉闷	稍糊	稍凹	硬滑	0.719	0.103	否

$$P_{\text{密度: 0.697}|\text{是}} = p(\text{密度} = 0.697 | \text{好瓜} = \text{是})$$

$$= \frac{1}{\sqrt{2\pi} \cdot 0.129} \exp\left(-\frac{(0.697 - 0.574)^2}{2 \cdot 0.129^2}\right) \approx 1.959,$$

$$P_{\text{密度: 0.697}|\text{否}} = p(\text{密度} = 0.697 | \text{好瓜} = \text{否})$$

$$= \frac{1}{\sqrt{2\pi} \cdot 0.195} \exp\left(-\frac{(0.697 - 0.496)^2}{2 \cdot 0.195^2}\right) \approx 1.203,$$

$$P_{\text{含糖: 0.460}|\text{是}} = p(\text{含糖率} = 0.460 | \text{好瓜} = \text{是})$$

$$= \frac{1}{\sqrt{2\pi} \cdot 0.101} \exp\left(-\frac{(0.460 - 0.279)^2}{2 \cdot 0.101^2}\right) \approx 0.788,$$

$$P_{\text{含糖: 0.460}|\text{否}} = p(\text{含糖率} = 0.460 | \text{好瓜} = \text{否})$$

$$= \frac{1}{\sqrt{2\pi} \cdot 0.108} \exp\left(-\frac{(0.460 - 0.154)^2}{2 \cdot 0.108^2}\right) \approx 0.066.$$

最大似然估计

周志华, 机器学习, P84,151,152

测 1	青绿	蜷缩	浊响	清晰	凹陷	硬滑	0.697	0.460	?
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# 朴素贝叶斯分类器

测 1	青绿	蜷缩	浊响	清晰	凹陷	硬滑	0.697	0.460	?
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$$P(\text{好瓜} = \text{是}) \times P_{\text{青绿}|\text{是}} \times P_{\text{蜷缩}|\text{是}} \times P_{\text{浊响}|\text{是}} \times P_{\text{清晰}|\text{是}} \times P_{\text{凹陷}|\text{是}} \\ \times P_{\text{硬滑}|\text{是}} \times P_{\text{密度: 0.697}|\text{是}} \times P_{\text{含糖: 0.460}|\text{是}} \approx 0.063$$

$$P(\text{好瓜} = \text{否}) \times P_{\text{青绿}|\text{否}} \times P_{\text{蜷缩}|\text{否}} \times P_{\text{浊响}|\text{否}} \times P_{\text{清晰}|\text{否}} \times P_{\text{凹陷}|\text{否}} \\ \times P_{\text{硬滑}|\text{否}} \times P_{\text{密度: 0.697}|\text{否}} \times P_{\text{含糖: 0.460}|\text{否}} \approx 6.80 \times 10^{-5}.$$

由于  $0.063 > 6.80 \times 10^{-5}$ , 因此, 朴素贝叶斯分类器将测试样本“测 1”判别为“好瓜”。

周志华, 机器学习, P151,153

# 朴素贝叶斯分类器

拉普拉斯修正

测3	青绿	蜷缩	清脆	清晰	凹陷	硬滑	0.697	0.460	?
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$$P(\text{好瓜} = \text{是}) \times P_{\text{青绿}|\text{是}} \times P_{\text{蜷缩}|\text{是}} \times P_{\text{清脆}|\text{是}} \times P_{\text{清晰}|\text{是}} \times P_{\text{凹陷}|\text{是}} \\ \times P_{\text{硬滑}|\text{是}} \times P_{\text{密度: 0.697}|\text{是}} \times P_{\text{含糖: 0.460}|\text{是}} \approx 0.063 = 0?$$

作用：避免其他属性所携带的信息被训练集中未出现的属性值“抹去”

$$\hat{P}(c) = \frac{N_c + 1}{N + K}$$

$$P(t_i|c) = \frac{N_{(c,t_i)} + 1}{N_c + |t_i|}$$

$t_i$ 可能的取值数

# 朴素贝叶斯分类器

$P_{\text{青绿}|\text{是}} = P(\text{色泽} = \text{青绿} | \text{好瓜} = \text{是}) =$

$P_{\text{青绿}|\text{否}} = P(\text{色泽} = \text{青绿} | \text{好瓜} = \text{否}) =$

$P_{\text{蜷缩}|\text{是}} = P(\text{根蒂} = \text{蜷缩} | \text{好瓜} = \text{是}) =$

$P_{\text{蜷缩}|\text{否}} = P(\text{根蒂} = \text{蜷缩} | \text{好瓜} = \text{否}) =$

$P_{\text{浊响}|\text{是}} = P(\text{敲声} = \text{浊响} | \text{好瓜} = \text{是}) =$

$P_{\text{浊响}|\text{否}} = P(\text{敲声} = \text{浊响} | \text{好瓜} = \text{否}) =$

$P_{\text{清晰}|\text{是}} = P(\text{纹理} = \text{清晰} | \text{好瓜} = \text{是}) =$

$P_{\text{清晰}|\text{否}} = P(\text{纹理} = \text{清晰} | \text{好瓜} = \text{否}) =$

$P_{\text{凹陷}|\text{是}} = P(\text{脐部} = \text{凹陷} | \text{好瓜} = \text{是}) =$

$P_{\text{凹陷}|\text{否}} = P(\text{脐部} = \text{凹陷} | \text{好瓜} = \text{否}) =$

$P_{\text{硬滑}|\text{是}} = P(\text{触感} = \text{硬滑} | \text{好瓜} = \text{是}) =$

$P_{\text{硬滑}|\text{否}} = P(\text{触感} = \text{硬滑} | \text{好瓜} = \text{否}) =$

?

$P(\text{好瓜} = \text{是}) =$

$P(\text{好瓜} = \text{否}) =$

?

测2

青绿

蜷缩

清脆

清晰

凹陷

硬滑

?

测3

青绿

蜷缩

清脆

清晰

凹陷

硬滑

0.697

0.460

?

拉普拉斯修正的朴素贝叶斯

?

?

$$\begin{aligned}
 P_{\text{青绿}|\text{是}} &= \frac{3+1}{8+3} \approx 0.364 \\
 P_{\text{青绿}|\text{否}} &= \frac{3+1}{9+3} \approx 0.333 \\
 P_{\text{蜷缩}|\text{是}} &= \frac{5+1}{8+3} \approx 0.545 \\
 P_{\text{蜷缩}|\text{否}} &= \frac{3+1}{9+3} \approx 0.333 \\
 P_{\text{清脆}|\text{是}} &= \frac{0+1}{8+3} \approx 0.091 \\
 P_{\text{清脆}|\text{否}} &= \frac{2+1}{9+3} = 0.25 \\
 P_{\text{浊响}|\text{是}} &= \frac{6+1}{8+3} \approx 0.636 \\
 P_{\text{浊响}|\text{否}} &= \frac{4+1}{9+3} \approx 0.417 \\
 P_{\text{清晰}|\text{是}} &= \frac{7+1}{8+3} \approx 0.727 \\
 P_{\text{清晰}|\text{否}} &= \frac{2+1}{9+3} = 0.25 \\
 P_{\text{凹陷}|\text{是}} &= \frac{6+1}{8+3} \approx 0.636 \\
 P_{\text{凹陷}|\text{否}} &= \frac{2+1}{9+3} = 0.25 \\
 P_{\text{硬滑}|\text{是}} &= \frac{6+1}{8+2} = 0.7 \\
 P_{\text{硬滑}|\text{否}} &= \frac{6+1}{9+2} \approx 0.636
 \end{aligned}$$

$$\hat{P}(\text{好瓜} = \text{是}) = \frac{8+1}{17+2} \approx 0.474$$

$$\hat{P}(\text{好瓜} = \text{否}) = \frac{9+1}{17+2} \approx 0.526$$

测2

青绿

蜷缩

清脆

清晰

凹陷

硬滑

$$\begin{aligned}
 &P(\text{好瓜} = \text{是}) \times P_{\text{青绿}|\text{是}} \times P_{\text{蜷缩}|\text{是}} \times P_{\text{清脆}|\text{是}} \\
 &\quad \times P_{\text{清晰}|\text{是}} \times P_{\text{凹陷}|\text{是}} \times P_{\text{硬滑}|\text{是}} \approx 0.003
 \end{aligned}$$

$$\begin{aligned}
 &P(\text{好瓜} = \text{否}) \times P_{\text{青绿}|\text{否}} \times P_{\text{蜷缩}|\text{否}} \times P_{\text{清脆}|\text{否}} \\
 &\quad \times P_{\text{清晰}|\text{否}} \times P_{\text{凹陷}|\text{否}} \times P_{\text{硬滑}|\text{否}} \approx 0.001
 \end{aligned}$$

0.003 > 0.001      拉普拉斯修正的朴素贝叶斯

测2是好瓜

$$\begin{aligned}
 P_{\text{青绿}|\text{是}} &= \frac{3+1}{8+3} \approx 0.364 \\
 P_{\text{青绿}|\text{否}} &= \frac{3+1}{9+3} \approx 0.333 \\
 P_{\text{蜷缩}|\text{是}} &= \frac{5+1}{8+3} \approx 0.545 \\
 P_{\text{蜷缩}|\text{否}} &= \frac{3+1}{9+3} \approx 0.333 \\
 P_{\text{清脆}|\text{是}} &= \frac{0+1}{8+3} \approx 0.091 \\
 P_{\text{清脆}|\text{否}} &= \frac{2+1}{9+3} = 0.25 \\
 P_{\text{浊响}|\text{是}} &= \frac{6+1}{8+3} \approx 0.636 \\
 P_{\text{浊响}|\text{否}} &= \frac{4+1}{9+3} \approx 0.417 \\
 P_{\text{清晰}|\text{是}} &= \frac{7+1}{8+3} \approx 0.727 \\
 P_{\text{清晰}|\text{否}} &= \frac{2+1}{9+3} = 0.25 \\
 P_{\text{凹陷}|\text{是}} &= \frac{6+1}{8+3} \approx 0.636 \\
 P_{\text{凹陷}|\text{否}} &= \frac{2+1}{9+3} = 0.25 \\
 P_{\text{硬滑}|\text{是}} &= \frac{6+1}{8+2} = 0.7 \\
 P_{\text{硬滑}|\text{否}} &= \frac{6+1}{9+2} \approx 0.636
 \end{aligned}$$

$$\hat{P}(\text{好瓜} = \text{是}) = \frac{8+1}{17+2} \approx 0.474$$

$$\hat{P}(\text{好瓜} = \text{否}) = \frac{9+1}{17+2} \approx 0.526$$

测3

青绿

蜷缩

清脆

清晰

凹陷

硬滑

0.697

0.460

?

$$\begin{aligned}
 &P(\text{好瓜} = \text{是}) \times P_{\text{青绿}|\text{是}} \times P_{\text{蜷缩}|\text{是}} \times P_{\text{清脆}|\text{是}} \\
 &\quad \times P_{\text{清晰}|\text{是}} \times P_{\text{凹陷}|\text{是}} \times P_{\text{硬滑}|\text{是}} \\
 &\quad \times P_{0.697|\text{是}} \times P_{0.460|\text{是}} \approx 0.005
 \end{aligned}$$

$$\begin{aligned}
 &P(\text{好瓜} = \text{否}) \times P_{\text{青绿}|\text{否}} \times P_{\text{蜷缩}|\text{否}} \times P_{\text{清脆}|\text{否}} \times P_{\text{清晰}|\text{否}} \\
 &\quad \times P_{\text{凹陷}|\text{否}} \times P_{\text{硬滑}|\text{否}} \\
 &\quad \times P_{0.697|\text{否}} \times P_{0.460|\text{否}} \approx 7.0 \times 10^{-5}
 \end{aligned}$$

0.005 > 7.0 \* 10<sup>-5</sup> 拉普拉斯修正的朴素贝叶斯

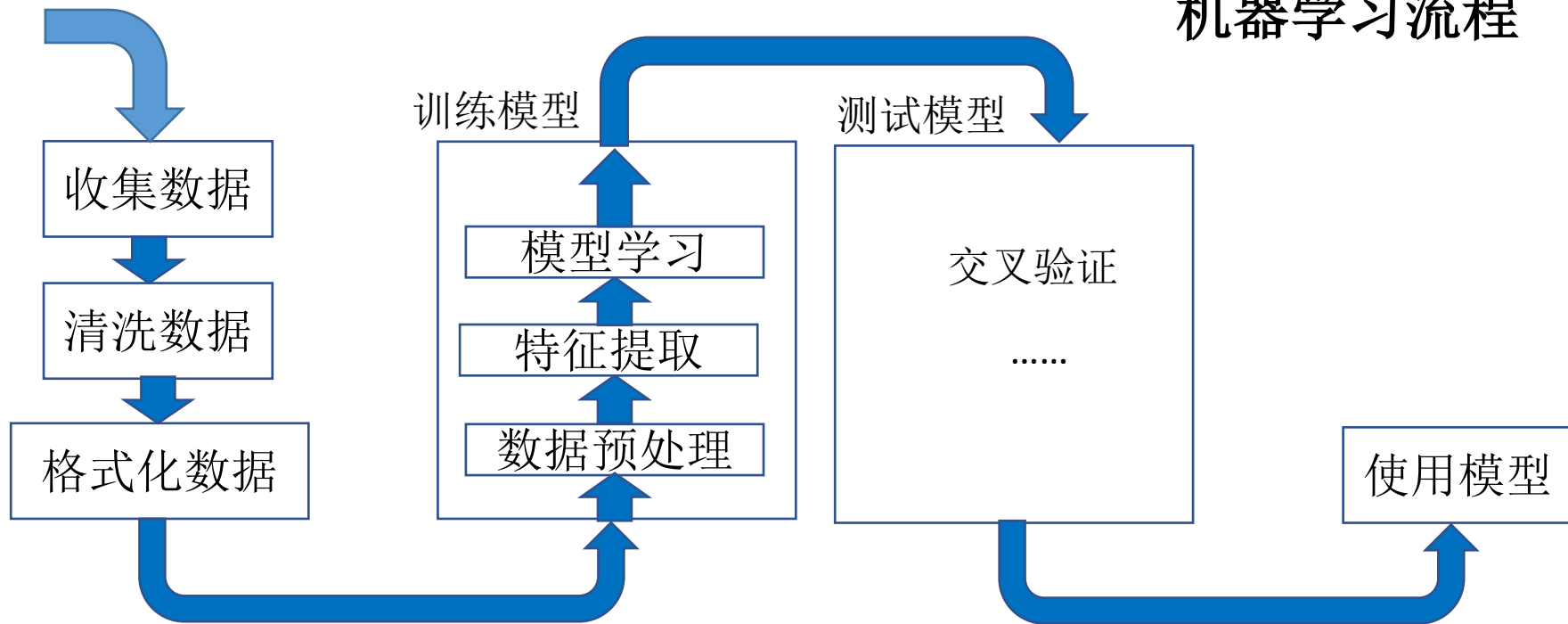
测3是好瓜



- ✓ 贝叶斯决策理论基础
- ✓ 朴素贝叶斯分类器
- ✓ 鸢尾花分类实践

# 鸢尾花分类实践

## 机器学习流程



# 鸢尾花分类实践

## Iris Data Set



数目：150=50\*3

<http://archive.ics.uci.edu/ml/assets/MLimages/Large53.jpg>

数据下载地址：<http://archive.ics.uci.edu/ml/datasets/Iris>

特征/属性 (cm)：

1. sepal length — 花萼长度
2. sepal width — 花萼宽度
3. petal length — 花瓣长度
4. petal width — 花瓣宽度

类别：

- Iris Setosa — 山鸢尾
- Iris Versicolour — 多彩鸢尾
- Iris Virginica — 弗吉尼亚鸢尾

# 鸢尾花分类实践

## Documentation

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### Classification

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Fisher's Iris Data

Linear and Quadratic Discriminant  
Analysis

Naive Bayes Classifiers

Decision Tree

Conclusions

## Classification

This example shows how to perform classification using discriminant analysis, naive Bayes classifiers, and decision trees. Suppose you have a data set containing observations with measurements on different variables (called predictors) and their known class labels. If you obtain predictor values for new observations, could you determine to which classes those observations probably belong? This is the problem of classification.

[Open This Example](#)

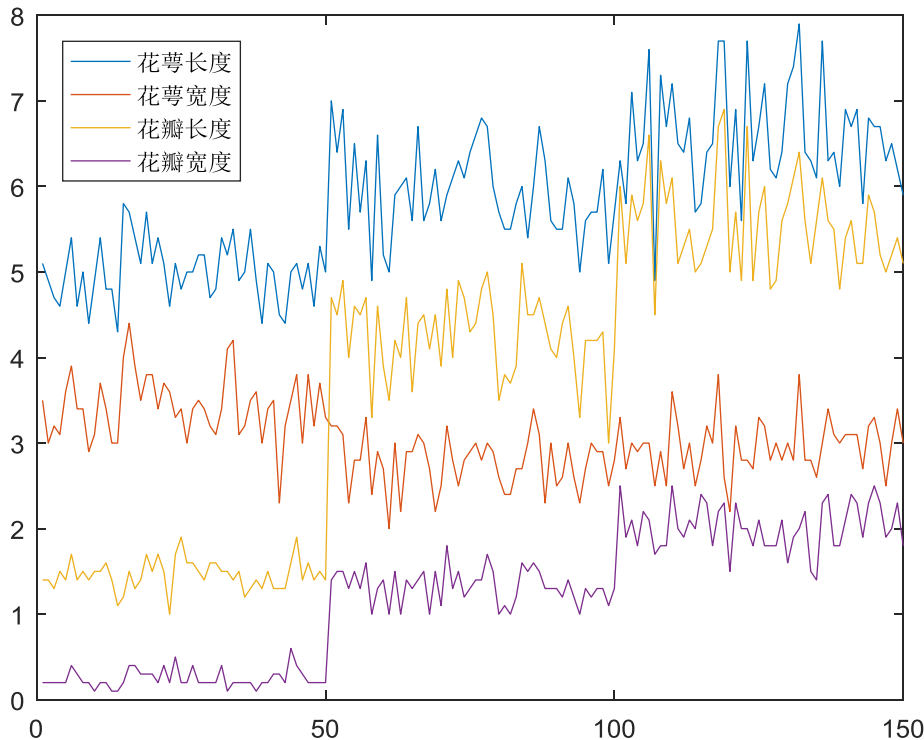
### Fisher's Iris Data

Fisher's iris data consists of measurements on the sepal length, sepal width, petal length, and petal width for 150 iris specimens. There are 50 specimens from each of three species. Load the data and see how the sepal measurements differ between species. You can use the two columns containing sepal measurements.

```
load fisheriris
gscatter(meas(:,1), meas(:,2), species, 'rgb', 'osd');
xlabel('Sepal length');
ylabel('Sepal width');
N = size(meas,1);
```

# 鸢尾花分类实践

```
clear all;close all;clc;  
rng('default');  
%%  
% 导入Fisher's Iris data (鸢尾花数据)  
load fisheriris;  
% 显示特征取值  
figure;  
plot(meas);  
legend('花萼长度','花萼宽度','花瓣长度','花  
瓣宽度','Location','NorthWest');
```



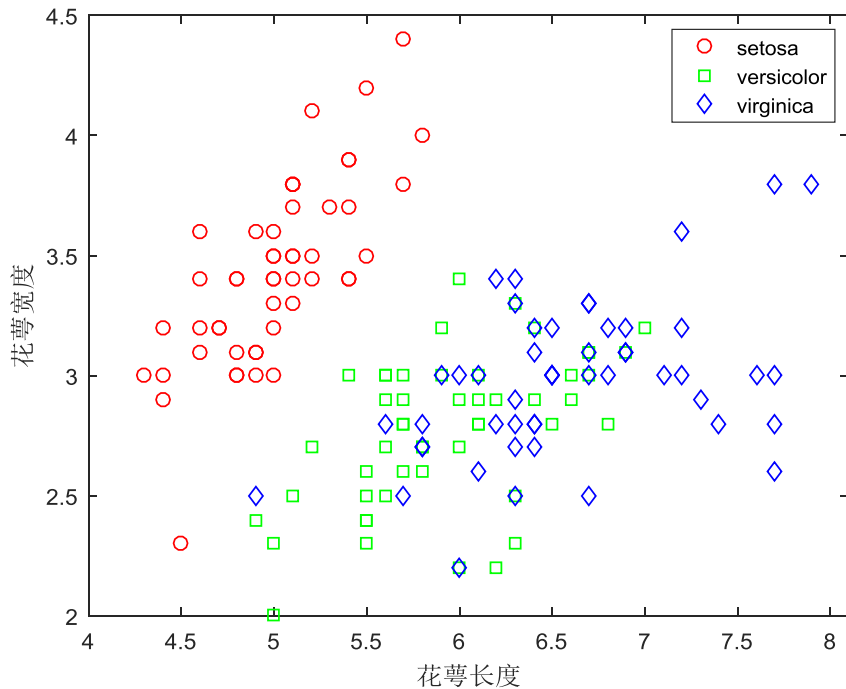
# 鸢尾花分类实践

## 特征相关性分析

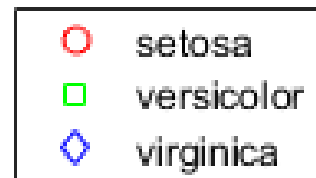
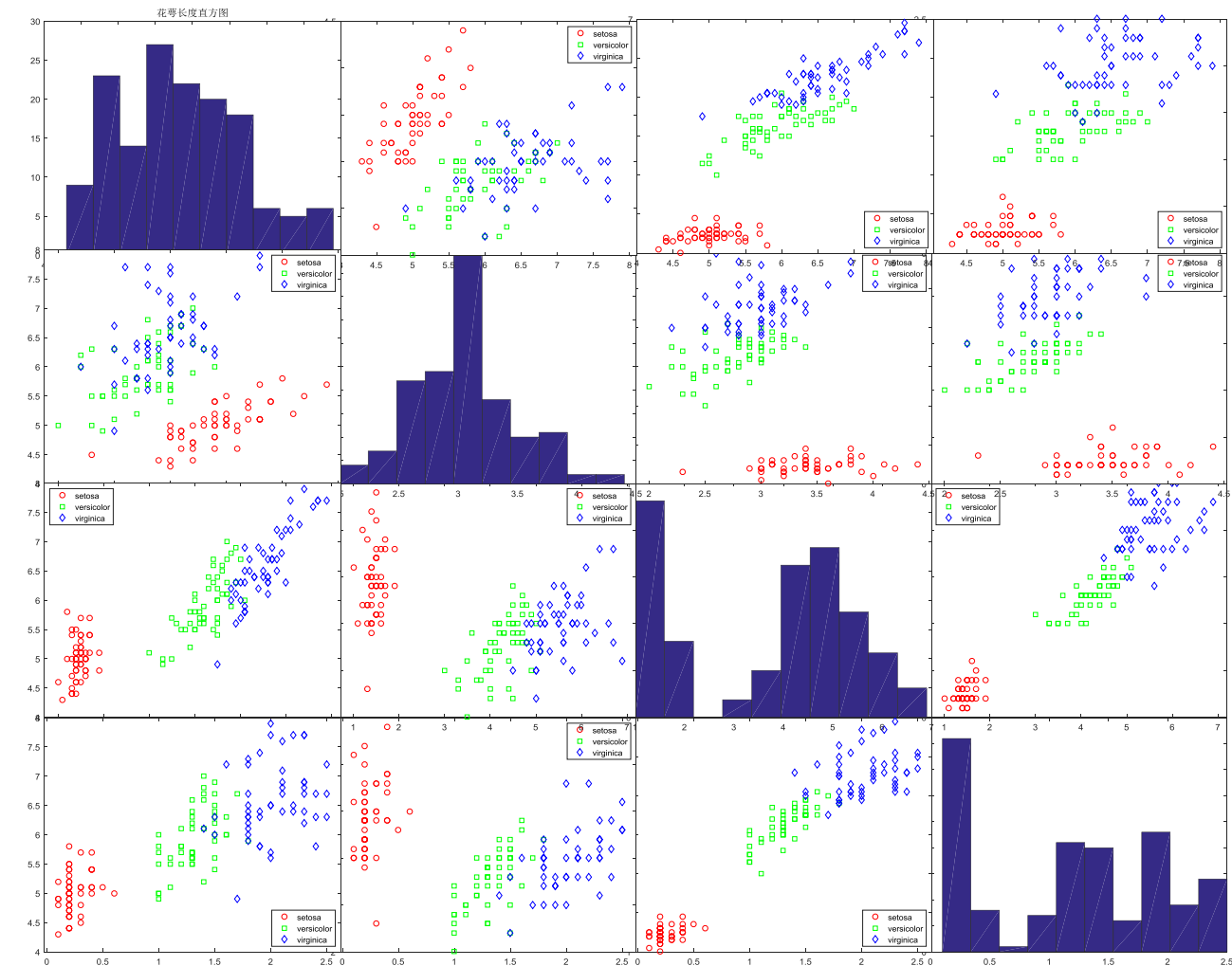
```
gscatter(meas(:,1), meas(:,2), species,  
        'rgb','osd'); %探索数据  
xlabel('Sepal length');  
ylabel('Sepal width');
```

4个特征的协方差矩阵

0.6857	-0.0424	1.2743	0.5163
-0.0424	0.1900	-0.3297	-0.1216
1.2743	-0.3297	3.1163	1.2956
0.5163	-0.1216	1.2956	0.5810



花萼长度直方图



```

for i=1:4
    for j=1:4
        if i==j
            figure;hist(meas(:,i));drawnow;
        else
            figure;
            gscatter(meas(:,i), meas(:,j), ...
                    species,'rgb','osd');
        end
    end
end

```

# 鸢尾花分类实践

## 分离训练和测试数据

训练数据各类别数据比例

Value	Count	Percent
setosa	33	33.00%
virginica	29	29.00%
versicolor	38	38.00%

测试数据各类别数据比例

Value	Count	Percent
versicolor	12	24.00%
setosa	17	34.00%
virginica	21	42.00%

$$P(c) = \frac{N_c}{N}$$

```
% 打乱数据排序, 并保持标签对应
N = size(meas,1); %全部数据个数
randpN = randperm(N);
randp_meas = meas(randpN,:);
randp_species = species(randpN,:);
% 分离训练2/3和测试1/3数据,
train_datas = randp_meas(1:N/3*2,:);
train_labels = randp_species(1:N/3*2);
test_datas = randp_meas(1+N/3*2:end,:);
test_labels = randp_species(1+N/3*2:end);
disp('训练数据各类别数据比例');
tabulate(train_labels)
disp('测试数据各类别数据比例');
tabulate(test_labels)
```



# 鸢尾花分类实践

训练模型

$$P(c) = \frac{N_c}{N}$$

$$\hat{P}(c) = \frac{N_c + 1}{N + K}$$

$$P(t_i|c) = \frac{1}{\sqrt{2\pi}\sigma_{c,i}} e^{-\frac{(t_i - \mu)^2}{2\sigma_{c,i}^2}}$$
$$r(t_i|c) = \frac{N_{(c,t_i)}}{N_c + |t_i|}$$

MLE

# 鸢尾花分类实践

Matlab自带Naïve Bayes分类器函数的用法

MODEL=fitcnb (TBL,Y)

MODEL=fitcnb(X,Y,'PARAM1',val1,'PARAM2',val2,...)

```
'DistributionNames'
    'normal','kernel','mvnmn','mn'
'Kernel'           'normal','box','triangle', or 'epanechnikov'.
'Support'          'unbounded' , 'positive' , [L,U]
'Width'            scalar, row vector, column vector, matrix
'CategoryalPredictors' - List of categorical predictors.
'ClassNames'       - Array of class names.
'Cost'              - Square matrix, where COST(I,J) is the
                     cost of classifying a point into class J if its
                     true class is I.
'CrossVal'          'on', 'off'
                     - If 'on', performs 10-fold cross-validation.
'CVPartition'       - A partition created with CVPARTITION to use
                     the cross-validated tree.
```

# 鸢尾花分类实践

'Holdout' - Holdout validation uses the specified fraction of the data for test, and uses the rest of the data for training. Specify a numeric scalar between 0 and 1.

'KFold' - Number of folds to use in cross-validated tree, a positive integer. Default: 10

'Leaveout' - Use leave-one-out cross-validation by setting 'on'.

'PredictorNames' - A cell array of names for the predictor variables, in the order in which they appear in

'Prior' - Prior probabilities for each class.

'ResponseName' - Name of the response variable Y, a string.

'ScoreTransform' - Function handle for transforming scores, string representing a built-in transformation function.

'symmetric', 'invlogit', 'ismax', 'symmetricismax', 'none', 'logit', 'doublelogit', 'symmetriclogit', and 'sign'.

'Weights' - Vector of observation weights, one weight per observation.

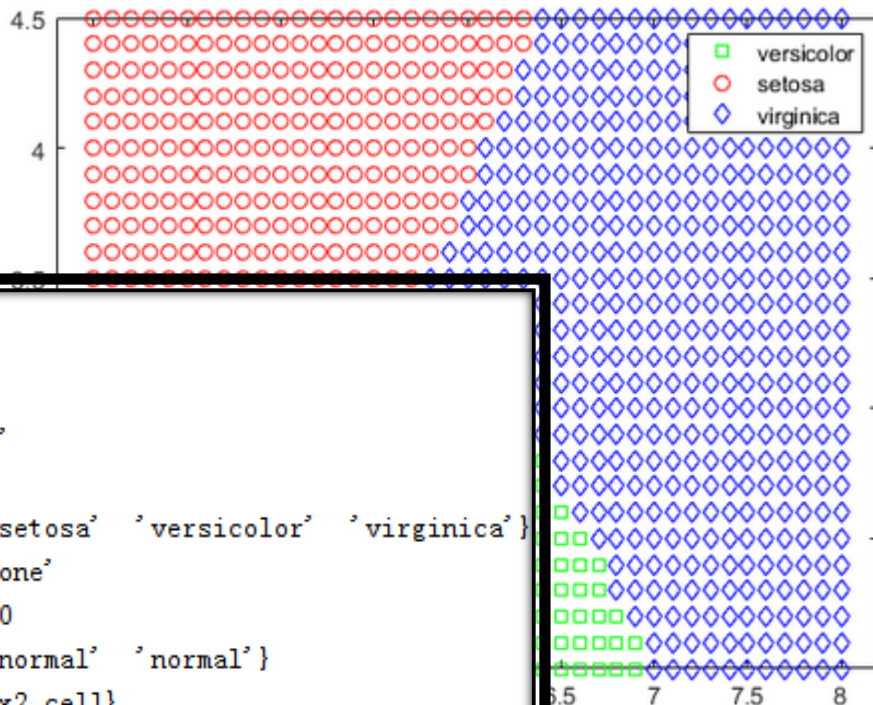
# 鸢尾花分类实践

```
nbGau = fitcnb(meas(:,1:2), species);  
nbGauResubErr = resubLoss(nbGau)  
nbGauCV = crossval(nbGau, 'CVPartition',cp);  
nbGauCVERr = kfoldLoss(nbGauCV)
```

```
labels = predict(nbGau, meas(:,1:2));  
gscatter(x,y,labels,'grb')
```

```
nbGauResubErr =  
    0.2200  
nbGauCVERr =  
    0.2200
```

```
nbGau =  
ClassificationNaiveBayes  
    ResponseName: 'Y'  
    CategoricalPredictors: []  
    ClassNames: {'setosa' 'versicolor' 'virginica'}  
    ScoreTransform: 'none'  
    NumObservations: 150  
    DistributionNames: {'normal' 'normal'}  
    DistributionParameters: {3x2 cell}
```



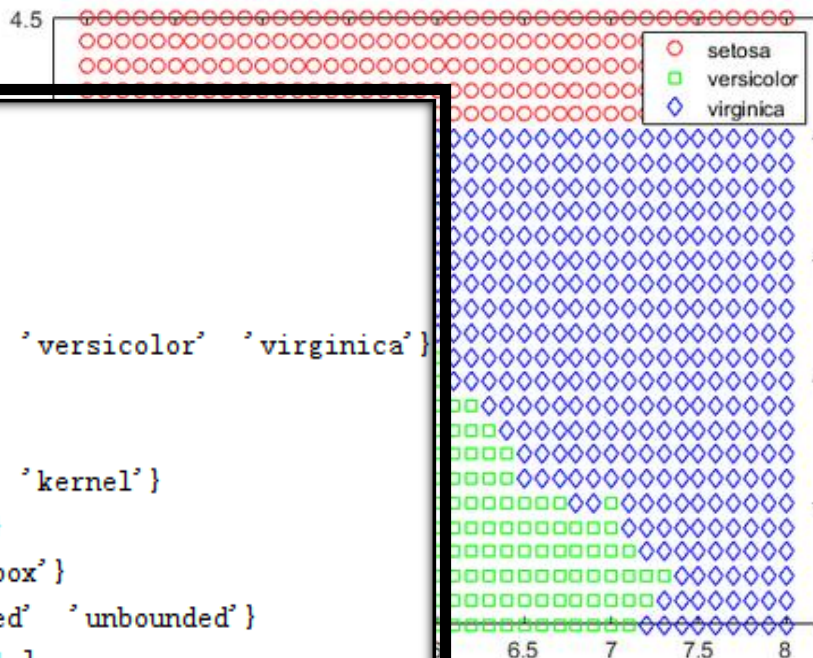
# 鸢尾花分类实践

```
nbKD = fitcnb(meas(:,1:2), species,  
'DistributionNames','kernel','Kernel','box');  
nbKDResubErr = resubLoss(nbKD) nbKDCV =  
crossval(nbKD, 'CV', 'kfoldLoss', nbKDCV)  
gscatter(x,y,labels)
```

```
nbKDResubErr =  
0.2067  
nbKDCVErr =  
0.2133
```

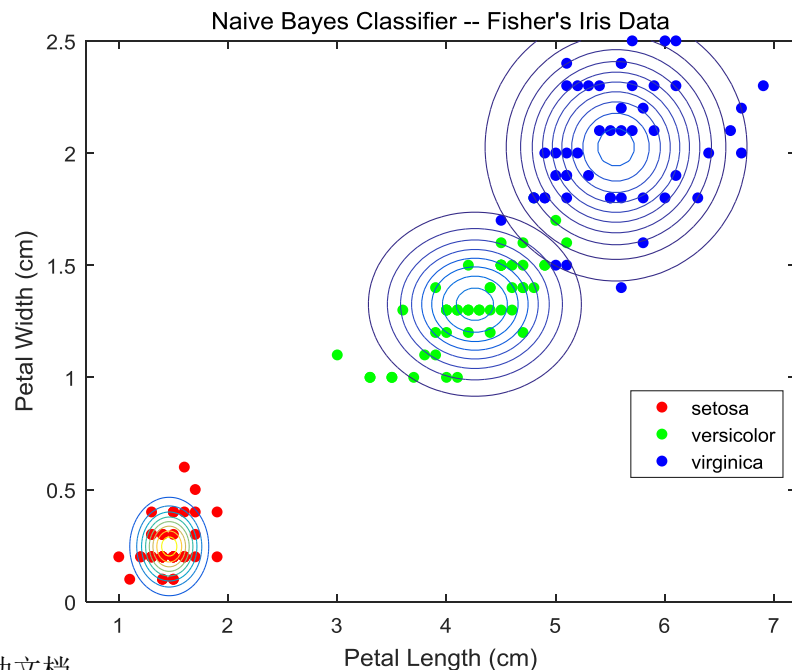
## ClassificationNaiveBayes

```
ResponseName: 'Y'  
CategoricalPredictors: []  
ClassNames: {'setosa' 'versicolor' 'virginica'}  
ScoreTransform: 'none'  
NumObservations: 150  
DistributionNames: {'kernel' 'kernel'}  
DistributionParameters: {3x2 cell}  
Kernel: {'box' 'box'}  
Support: {'unbounded' 'unbounded'}  
Width: [3x2 double]
```



# 鸢尾花分类实践

\Documents\MATLAB\Examples\TrainANaiveBayesClassifierFitcnbExample\  
TrainANaiveBayesClassifierFitcnbExample.m



朴素贝叶斯分类器  
(Naïve Bayesian Classifier)

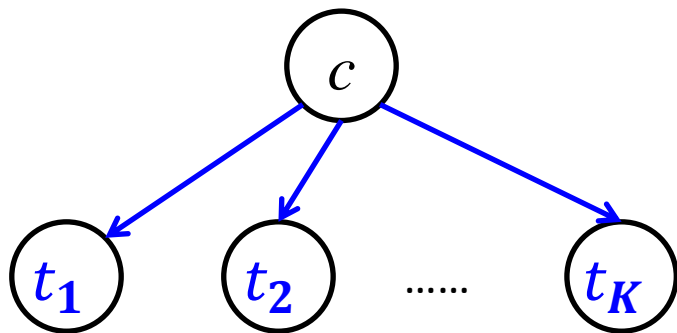
$$f_{nbc}(\mathbf{x}) = \operatorname{argmax}_{c \in Y} \left\{ P(c) \prod_{i=1}^K P(t_i|c) \right\}$$

优点:

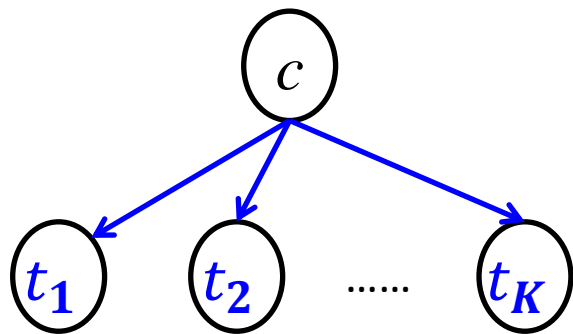
1. 只需要计算组合概率, 所需估计的参数较少
2. 对数据较少/缺失数据的鲁棒性好
3. 能够充分利用领域知识和样本数据
4. 能够学习变量间的因果关系
5. 具有自我纠正能力

缺点:

- ① 对于输入数据的准备方式较为敏感
- ② 独立假设条件在实际中可能不成立
- ③ 不能学习特征间的交互关系

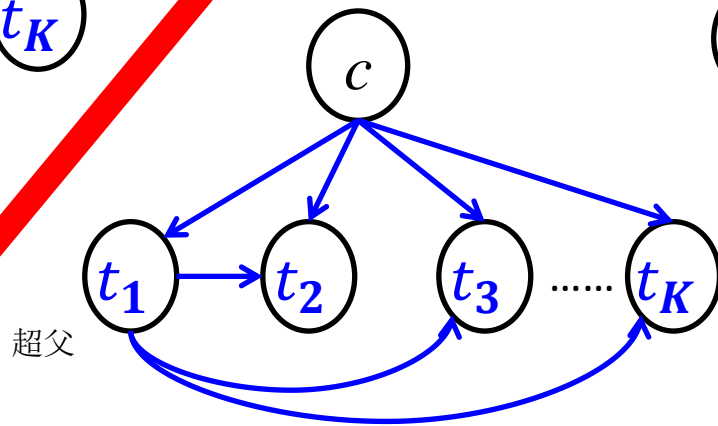


# 扩展

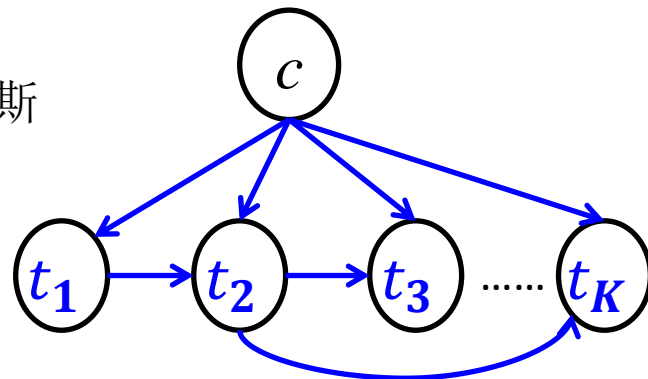


朴素贝叶斯

半朴素贝叶斯



SPODE



强相关属性的依赖

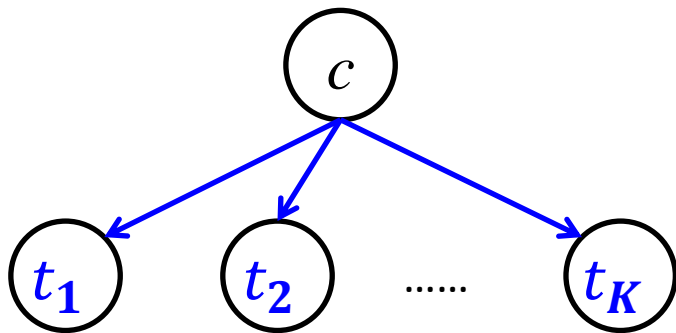
TAN



# Q&A

朴素贝叶斯分类器  
(Naïve Bayesian Classifier)

$$f_{nbc}(\mathbf{x}) = \operatorname{argmax}_{c \in Y} \left\{ P(c) \prod_{i=1}^K P(t_i | c) \right\}$$





**感谢各位聆听 !**  
Thanks for Listening ●



## 附录： 二维高斯分布示意图

